Fine Grained Analysis of Algorithms and Data Structures

(Análisis fino para algoritmos y estructuras de datos)

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Abstract

The adaptive analysis of algorithms remedy to the pessimism of the standard worst-case analysis of algorithms by considering other parameters than the input size, in order to better capture the difficulty of the input instance. Developped since the 70's, such an analysis technique has become even more relevant nowadays as the size of data explodes, and with it the gap between the worst case performance and the typical one. We give an introduction of this technique by revisiting classical problems through this analysis, discuss the relations between this analysis technique with output sensitivity and parameterized complexity, and present some preliminary work on problems for which such an analysis seems promising, such as Searching in sorted arrays, Sorting, computing the Convex Hull, computing Optimal Prefix Free Codes (Huffman), etc...

Pedagogical Objectives

After taking this course, the student should be able to

- mention some **examples of fine grained analysis** of some computational complexity;
- **identify the potential** of a fine grained analysis in a problem; and
- **perform a fine grained analysis** of the complexity of some simple problems.

Outline

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1. ☐ Peor caso por n fijado
2. ☐ Peor caso por n, δ fijado
3. ☐ Peor caso por $\delta, n_1, \ldots, n_\delta$ fijado

Outline

1. ☐ Repaso de Covertura Convexa
   - Graham's scan $O(n \lg n)$ [Graham, 1972]
   - Gift Wrapping $O(nh)$
   - Chan $O(n \lg h)$ [Chan, 1996]
2. ☐ Kirkpatrick and Seidel [Kirkpatrick and Seidel, 1986]
   - Median $O(n)$
   - Dominating Edge $O(n)$
   - Convex Hull O(nlg h)
3. ☐ Afshani et al [Afshani et al., 2009]
   - Certificate
   - Lower Bound
   - Upper Bound

1. ☐ Dominating Set -> Instance Optimal DS
2. ☐ MCS Trees -> MCS Splay Trees
3. ☐ Optimal Boxes -> Adaptive Optimal Boxes

1. ☐ Analize disk pile problem in the worst case for n and $\min_i n_i$ fixed.
2. ☐ Analize disk pile problem in the worst case for n and $\max_i n_i$ fixed.
3. ☐ Analize variant of hanoi tower where the disk is removed and inserted in the middle [Barbay, 2016b]

Outline

1. Binary Search -> Doubling Search
2. Binary Encoding -> Gamma Coding
3. (2,4) Search Trees -> Finger Search Tree
4. Splay Arboles
5. Sorted Merge -> Adaptive Sorted Merge
6. Sorted Intersection -> Adaptive Insertion

Outline

1. Insertion Sort -> Local Insertion Sort
2. Heap -> Adaptive Priority queues
3. Heap sort -> Smooth Sort
4. Sorted Merge -> Adaptive Sorted Merge
5. Merge Sort -> Adaptive MergeSort

Outline

1. Synergy Sorting [Barbay et al., 2016]
    - Input Order

Local	Runs
Global	Pivot Positions

    ut Structure Repetitions
    - taking optimally advantage of both at the same time
2. Optimal Prefix Free Codes (Huffman's algorithm)
    - $O(nlg α)$\subset$ O(nlg n) [Barbay, 2016a]
    - OPEN PROBLEM: bounded length Prefix Free Codes
    - OPEN PROBLEM: order restricted optimal prefix free codes (Hu Tucker)
3. Edit Distance
    - Swap Insert [Barbay and Pérez-Lantero, 2015]
    - OPEN PROBLEM: other Edit distances

Similar work by the same Author

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