R로 하는 구조방정식 연습
- 측정동일성(Measurement Invariance)[1]

신종화(SHIN Jong-Hwa)[2]

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[1]한국여가문화학회 2013년 콜로키움의 자료로 준비되었습니다. 행사장에서는 일부 내용의 개선이 반영된 자료가 배포될 수 있습니다.

[2]서울과학종합대학원(aSSIST) 레저경영주임교수/ 한국 R 사용자모임(KRUG) 커미티
shin.jonghwa@leisure.or.kr/ jhshin@assist.ac.kr

[**상황**] 인근에 자리잡은 두 학교, Grant-White와 Pasteur를 두고, 학부형들끼리 논쟁을 벌이고 있다. 두 학교 중에 어느 학교가 학생을 잘 가르치는가에 대한 문제를 두고 말이다. 평가영역은 '시각', '문장', '반응성'이다. 이 평가 영역에 관한 9개의 항목이 x1, x,2, x3, x4, x5, x6, x7, x8, x9다.

```
> require(lavaan)
> data(HolzingerSwineford1939)
> dim(HolzingerSwineford1939)

[1] 301  15

> names(HolzingerSwineford1939)

 [1] "id"     "sex"    "ageyr"  "agemo"  "school" "grade"  "x1"     "x2"
 [9] "x3"     "x4"     "x5"     "x6"     "x7"     "x8"     "x9"

> head(HolzingerSwineford1939)

  id sex ageyr agemo  school grade       x1   x2    x3       x4   x5        x6
1  1   1    13     1 Pasteur     7 3.333333 7.75 0.375 2.333333 5.75 1.2857143
2  2   2    13     7 Pasteur     7 5.333333 5.25 2.125 1.666667 3.00 1.2857143
3  3   2    13     1 Pasteur     7 4.500000 5.25 1.875 1.000000 1.75 0.4285714
4  4   1    13     2 Pasteur     7 5.333333 7.75 3.000 2.666667 4.50 2.4285714
5  5   2    12     2 Pasteur     7 4.833333 4.75 0.875 2.666667 4.00 2.5714286
6  6   2    14     1 Pasteur     7 5.333333 5.00 2.250 1.000000 3.00 0.8571429
        x7   x8       x9
1 3.391304 5.75 6.361111
2 3.782609 6.25 7.916667
3 3.260870 3.90 4.416667
4 3.000000 5.30 4.861111
5 3.695652 6.30 5.916667
6 4.347826 6.65 7.500000

> summary(HolzingerSwineford1939)

       id             sex            ageyr        agemo
 Min.   :  1.0   Min.   :1.000   Min.   :11   Min.   : 0.000
 1st Qu.: 82.0   1st Qu.:1.000   1st Qu.:12   1st Qu.: 2.000
 Median :163.0   Median :2.000   Median :13   Median : 5.000
 Mean   :176.6   Mean   :1.515   Mean   :13   Mean   : 5.375
 3rd Qu.:272.0   3rd Qu.:2.000   3rd Qu.:14   3rd Qu.: 8.000
 Max.   :351.0   Max.   :2.000   Max.   :16   Max.   :11.000

         school        grade            x1               x2
 Grant-White:145   Min.   :7.000   Min.   :0.6667   Min.   :2.250
 Pasteur    :156   1st Qu.:7.000   1st Qu.:4.1667   1st Qu.:5.250
                   Median :7.000   Median :5.0000   Median :6.000
                   Mean   :7.477   Mean   :4.9358   Mean   :6.088
                   3rd Qu.:8.000   3rd Qu.:5.6667   3rd Qu.:6.750
                   Max.   :8.000   Max.   :8.5000   Max.   :9.250
                   NA's   :1.000
       x3              x4              x5              x6
 Min.   :0.250   Min.   :0.000   Min.   :1.000   Min.   :0.1429
 1st Qu.:1.375   1st Qu.:2.333   1st Qu.:3.500   1st Qu.:1.4286
 Median :2.125   Median :3.000   Median :4.500   Median :2.0000
```

```
 Mean   :2.250   Mean   :3.061   Mean   :4.341   Mean   :2.1856
 3rd Qu.:3.125   3rd Qu.:3.667   3rd Qu.:5.250   3rd Qu.:2.7143
 Max.   :4.500   Max.   :6.333   Max.   :7.000   Max.   :6.1429

       x7              x8               x9
 Min.   :1.304   Min.   : 3.050   Min.   :2.778
 1st Qu.:3.478   1st Qu.: 4.850   1st Qu.:4.750
 Median :4.087   Median : 5.500   Median :5.417
 Mean   :4.186   Mean   : 5.527   Mean   :5.374
 3rd Qu.:4.913   3rd Qu.: 6.100   3rd Qu.:6.083
 Max.   :7.435   Max.   :10.000   Max.   :9.250
```

```
> ?HolzingerSwineford1939
```

Chapter 1

어떤 분석 방법을 사용할 것인가?

1. 두 학교의 전교생 모두를 대상으로 평가조사를 할까?: 전수조사에 대한 고민
2. 수리영역, 언어영역, 운동능력 등을 어떻게 측정할 것인가?: 차원축소(요인)에 대한 고민
3. 기술통계 vs. 추리통계
   (a) (탐색적) 요인분석
      i. 평균차이검정
         A. 정규분포인 경우: t-test
         B. 비모수 차이검정?
   (b) 확인적 요인분석

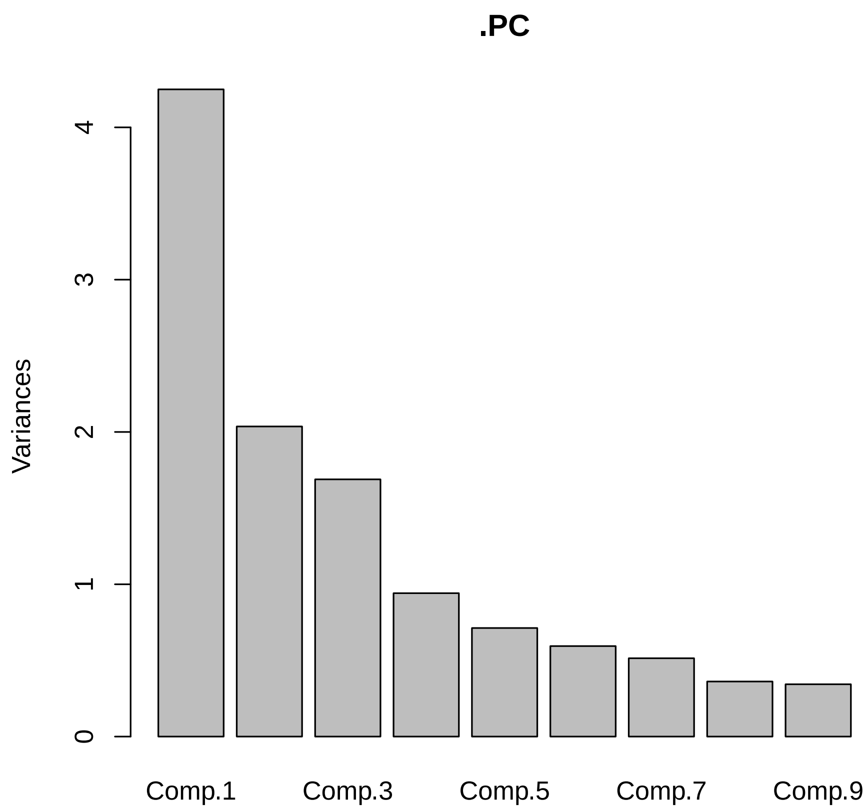
Part I

탐색적 요인분석(Exploratory Factor Analysis)

Chapter 2

주성분분석(Principal Component Analysis, PCA)

```
> .PC <- princomp(HolzingerSwineford1939[, 7:15])
> screeplot(.PC)
```



Chapter 3

탐색적 요인분석(Exploratory Component Analysis, EFA)

```
> .FA <- factanal(HolzingerSwineford1939[, 7:15], factors=3, rotation="varimax", scores="regression")
> .FA

Call:
factanal(x = HolzingerSwineford1939[, 7:15], factors = 3, scores = "regression",     rotation = "vari

Uniquenesses:
   x1    x2    x3    x4    x5    x6    x7    x8    x9
0.513 0.749 0.543 0.279 0.243 0.305 0.502 0.469 0.543

Loadings:
   Factor1 Factor2 Factor3
x1  0.277   0.623   0.151
x2  0.105   0.489
x3          0.663   0.130
x4  0.827   0.165
x5  0.861
x6  0.801   0.212
x7                  0.696
x8          0.162   0.709
x9  0.132   0.406   0.524

               Factor1 Factor2 Factor3
SS loadings      2.185   1.343   1.327
Proportion Var   0.243   0.149   0.147
Cumulative Var   0.243   0.392   0.539

Test of the hypothesis that 3 factors are sufficient.
The chi square statistic is 22.38 on 12 degrees of freedom.
The p-value is 0.0335

> head(.FA$scores)

         Factor1    Factor2      Factor3
[1,]  0.08247152 -0.7258616 -0.001591841
```

```
[2,] -1.21273720  0.5161999  0.822875023
[3,] -1.76841308 -0.2476878 -1.088698544
[4,] -0.04449707  0.7666871 -0.769782192
[5,] -0.06666738 -0.5306562  0.344127073
[6,] -1.53029546  0.3499646  1.148005610
```

```
> HolzingerSwineford1939$F1 <- .FA$scores[,1]
> HolzingerSwineford1939$F2 <- .FA$scores[,2]
> HolzingerSwineford1939$F3 <- .FA$scores[,3]
```

- x4, x5, x6 -> Factor1
- x1, x2, x3 -> Factor 2
- x7, x8, x9 -> Factor 3
- (탐색적) 요인분석은 다변량 정보를 단변량화 시키는 차원 축소의 영역이다.

3.1 집단별 평균차이 검정(t-test)

```
> t.test(F1~school, alternative='two.sided', conf.level=.95, var.equal=FALSE, data=HolzingerSwineford

        Welch Two Sample t-test

data:  F1 by school
t = 5.5448, df = 298.819, p-value = 6.478e-08
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.3663486 0.7694722
sample estimates:
mean in group Grant-White     mean in group Pasteur
                0.2943323                -0.2735781

> t.test(F2~school, alternative='two.sided', conf.level=.95, var.equal=FALSE, data=HolzingerSwineford

        Welch Two Sample t-test

data:  F2 by school
t = -1.3478, df = 298.994, p-value = 0.1788
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.31020508  0.05802032
sample estimates:
mean in group Grant-White     mean in group Pasteur
              -0.06535020                0.06074218

> t.test(F3~school, alternative='two.sided', conf.level=.95, var.equal=FALSE, data=HolzingerSwineford

        Welch Two Sample t-test

data:  F3 by school
t = -2.9541, df = 291.524, p-value = 0.003392
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
```

```
 -0.4712071 -0.0943852
sample estimates:
mean in group Grant-White     mean in group Pasteur
               -0.1465654                 0.1362307
```

- t-test는 조사된 객관적 점수들-기준점이 필요없는-을 바탕으로 비교 집단들의 상대적 우열을 추론한다.

Part II

확인적 요인분석(Confirmatory Factor Analysis, CFA)

Chapter 4

분석의 조건과 언어

- 탐색적 요인분석
  - 의미있는 특징을 발견하고, 수리적으로 정리하는 데 사용한다.
  - 귀납적 연구방법
- 확인적 요인분석
  - 경험적인 직관이나 지식에 기반한 추론의 적절성 여부
  - 적절성 판단이후에 수리적인 논거를 제시하는 데 사용한다.
  - 연역적 연구방법

4.1 다변량 정규성에 대한 확인

```
> require(psych)
> ?mardia
> require(mvnormtest)
> ?mshapiro.test
```

- normality vs. Partial Least Squares(PLS)

4.2 모형적합성이란?

- 연역적 논거의 설계가 설득적인가에 대한 입장 표명
  - 설계된 모형과 수집된 표본의 분석 결과가 차이가 있는가의 여부: χ^2검정

Chapter 5

확인적 요인분석 기초

5.1 모형의 설계

```
> cfa.model1 <- ' F1 =~ x1 + x2 +x3
+ F2 =~ x4 + x5 + x6
+ F3 =~ x7 + x8 + x9 '
```

5.2 모형의 적합성 판단

```
> cfa.model1.fit <- cfa(cfa.model1, data=HolzingerSwineford1939, estimator="MLR")
> cfa.model1.fit
```

```
lavaan (0.5-11) converged normally after  41 iterations

  Number of observations                           301

  Estimator                                         ML      Robust
  Minimum Function Test Statistic               85.306      87.132
  Degrees of freedom                                24          24
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.979
    for the Yuan-Bentler correction
```

```
> summary(cfa.model1.fit, fit.measures=TRUE)
```

```
lavaan (0.5-11) converged normally after  41 iterations

  Number of observations                           301

  Estimator                                         ML      Robust
  Minimum Function Test Statistic               85.306      87.132
  Degrees of freedom                                24          24
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.979
    for the Yuan-Bentler correction

Model test baseline model:
```

```
  Minimum Function Test Statistic              918.852     880.082
  Degrees of freedom                                36          36
  P-value                                        0.000       0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.931       0.925
  Tucker-Lewis Index (TLI)                       0.896       0.888

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3737.745   -3737.745
  Scaling correction factor                                  1.093
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -3695.092   -3695.092
  Scaling correction factor                                  1.043
    for the MLR correction

  Number of free parameters                         30          30
  Akaike (AIC)                                7535.490    7535.490
  Bayesian (BIC)                              7646.703    7646.703
  Sample-size adjusted Bayesian (BIC)         7551.560    7551.560

Root Mean Square Error of Approximation:

  RMSEA                                          0.092       0.093
  90 Percent Confidence Interval          0.071  0.114       0.073  0.115
  P-value RMSEA <= 0.05                          0.001       0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.060       0.060

Parameter estimates:

  Information                                 Observed
  Standard Errors                   Robust.huber.white

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.553    0.132    4.191    0.000
    x3                0.729    0.141    5.170    0.000
  F2 =~
    x4                1.000
    x5                1.113    0.066   16.946    0.000
    x6                0.926    0.061   15.089    0.000
  F3 =~
    x7                1.000
    x8                1.180    0.130    9.046    0.000
    x9                1.082    0.266    4.060    0.000
```

```
Covariances:
  F1 ~~
    F2                0.408    0.099    4.110    0.000
    F3                0.262    0.060    4.366    0.000
  F2 ~~
    F3                0.173    0.056    3.081    0.002

Intercepts:
    x1                4.936    0.067   73.473    0.000
    x2                6.088    0.068   89.855    0.000
    x3                2.250    0.065   34.579    0.000
    x4                3.061    0.067   45.694    0.000
    x5                4.341    0.074   58.452    0.000
    x6                2.186    0.063   34.667    0.000
    x7                4.186    0.063   66.766    0.000
    x8                5.527    0.058   94.854    0.000
    x9                5.374    0.058   92.546    0.000
    F1                0.000
    F2                0.000
    F3                0.000

Variances:
    x1                0.549    0.156
    x2                1.134    0.112
    x3                0.844    0.100
    x4                0.371    0.050
    x5                0.446    0.057
    x6                0.356    0.047
    x7                0.799    0.097
    x8                0.488    0.120
    x9                0.566    0.119
    F1                0.809    0.180
    F2                0.979    0.121
    F3                0.384    0.107
```

5.3 집단별 모형 적합성 판단: 기초

- HolzingerSwineford1939 데이터셋에는 집단변수로 'school'이 포함되어 있다.

```
> cfa.model1.fit.group <- cfa(cfa.model1, data=HolzingerSwineford1939, estimator="MLR", group="school
> cfa.model1.fit.group

lavaan (0.5-11) converged normally after  63 iterations

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              115.851     121.741
  Degrees of freedom                                48          48
```

```
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.952
    for the Yuan-Bentler correction

Chi-square for each group:

  Pasteur                                       64.309      67.578
  Grant-White                                   51.542      54.162
```

> summary(cfa.model1.fit.group, fit.measures=TRUE)

```
lavaan (0.5-11) converged normally after  63 iterations

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              115.851     121.741
  Degrees of freedom                                48          48
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.952
    for the Yuan-Bentler correction

Chi-square for each group:

  Pasteur                                       64.309      67.578
  Grant-White                                   51.542      54.162

Model test baseline model:

  Minimum Function Test Statistic              957.769     934.309
  Degrees of freedom                                72          72
  P-value                                        0.000       0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.923       0.914
  Tucker-Lewis Index (TLI)                       0.885       0.872

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3682.198   -3682.198
  Scaling correction factor                                  1.099
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
  Scaling correction factor                                  1.033
    for the MLR correction

  Number of free parameters                         60          60
  Akaike (AIC)                                7484.395    7484.395
  Bayesian (BIC)                              7706.822    7706.822
  Sample-size adjusted Bayesian (BIC)         7516.536    7516.536
```

```
Root Mean Square Error of Approximation:

  RMSEA                                         0.097       0.101
  90 Percent Confidence Interval         0.075  0.120       0.078  0.124
  P-value RMSEA <= 0.05                         0.001       0.000

Standardized Root Mean Square Residual:

  SRMR                                          0.068       0.068

Parameter estimates:

  Information                                 Observed
  Standard Errors                   Robust.huber.white

Group 1 [Pasteur]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.394    0.197    1.996    0.046
    x3                0.570    0.201    2.838    0.005
  F2 =~
    x4                1.000
    x5                1.183    0.106   11.213    0.000
    x6                0.875    0.093    9.388    0.000
  F3 =~
    x7                1.000
    x8                1.125    0.262    4.298    0.000
    x9                0.922    0.297    3.099    0.002

Covariances:
  F1 ~~
    F2                0.479    0.144    3.334    0.001
    F3                0.185    0.076    2.449    0.014
  F2 ~~
    F3                0.182    0.076    2.402    0.016

Intercepts:
    x1                4.941    0.095   52.249    0.000
    x2                5.984    0.098   60.949    0.000
    x3                2.487    0.093   26.778    0.000
    x4                2.823    0.092   30.689    0.000
    x5                3.995    0.105   38.183    0.000
    x6                1.922    0.079   24.321    0.000
    x7                4.432    0.087   51.181    0.000
    x8                5.563    0.078   71.214    0.000
    x9                5.418    0.079   68.440    0.000
    F1                0.000
    F2                0.000
```

```
    F3                0.000

Variances:
    x1                0.298    0.338
    x2                1.334    0.179
    x3                0.989    0.154
    x4                0.425    0.073
    x5                0.456    0.086
    x6                0.290    0.055
    x7                0.820    0.132
    x8                0.510    0.101
    x9                0.680    0.126
    F1                1.097    0.378
    F2                0.894    0.160
    F3                0.350    0.140



Group 2 [Grant-White]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.736    0.190    3.870    0.000
    x3                0.925    0.216    4.281    0.000
  F2 =~
    x4                1.000
    x5                0.990    0.089   11.179    0.000
    x6                0.963    0.091   10.588    0.000
  F3 =~
    x7                1.000
    x8                1.226    0.162    7.560    0.000
    x9                1.058    0.250    4.228    0.000

Covariances:
  F1 ~~
    F2                0.408    0.115    3.535    0.000
    F3                0.276    0.101    2.731    0.006
  F2 ~~
    F3                0.222    0.105    2.115    0.034

Intercepts:
    x1                4.930    0.095   51.696    0.000
    x2                6.200    0.092   67.416    0.000
    x3                1.996    0.086   23.195    0.000
    x4                3.317    0.093   35.625    0.000
    x5                4.712    0.096   48.986    0.000
    x6                2.469    0.094   26.277    0.000
    x7                3.921    0.086   45.819    0.000
    x8                5.488    0.087   63.174    0.000
    x9                5.327    0.085   62.571    0.000
```

```
    F1                0.000
    F2                0.000
    F3                0.000

Variances:
    x1                0.715    0.182
    x2                0.899    0.142
    x3                0.557    0.122
    x4                0.315    0.066
    x5                0.419    0.071
    x6                0.406    0.076
    x7                0.600    0.100
    x8                0.401    0.159
    x9                0.535    0.140
    F1                0.604    0.194
    F2                0.942    0.162
    F3                0.461    0.120
```

Chapter 6

확인적 요인분석: 측정동일성

6.1 group.equal=c("loadings")

```
> cfa.model1.fit.group.loadings <- cfa(cfa.model1, data=HolzingerSwineford1939, estimator="MLR", grou
> summary(cfa.model1.fit.group.loadings, fit.measures=TRUE)

lavaan (0.5-11) converged normally after  46 iterations

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              124.044     125.997
  Degrees of freedom                                54          54
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.984
    for the Yuan-Bentler correction

Chi-square for each group:

  Pasteur                                       68.825      69.909
  Grant-White                                   55.219      56.089

Model test baseline model:

  Minimum Function Test Statistic              957.769     934.309
  Degrees of freedom                                72          72
  P-value                                        0.000       0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.921       0.917
  Tucker-Lewis Index (TLI)                       0.895       0.889

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3686.294   -3686.294
```

```
  Scaling correction factor                                  1.082
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
  Scaling correction factor                                  1.033
    for the MLR correction

  Number of free parameters                         54          54
  Akaike (AIC)                                7480.587    7480.587
  Bayesian (BIC)                              7680.771    7680.771
  Sample-size adjusted Bayesian (BIC)         7509.514    7509.514

Root Mean Square Error of Approximation:

  RMSEA                                          0.093       0.094
  90 Percent Confidence Interval          0.071  0.114       0.073  0.116
  P-value RMSEA <= 0.05                          0.001       0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.072       0.072

Parameter estimates:

  Information                                 Observed
  Standard Errors                   Robust.huber.white

Group 1 [Pasteur]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.599    0.140    4.285    0.000
    x3                0.784    0.152    5.146    0.000
  F2 =~
    x4                1.000
    x5                1.083    0.069   15.609    0.000
    x6                0.912    0.069   13.283    0.000
  F3 =~
    x7                1.000
    x8                1.201    0.135    8.884    0.000
    x9                1.038    0.207    5.022    0.000

Covariances:
  F1 ~~
    F2                0.416    0.135    3.080    0.002
    F3                0.169    0.066    2.564    0.010
  F2 ~~
    F3                0.176    0.061    2.905    0.004

Intercepts:
    x1                4.941    0.095   52.249    0.000
```

```
    x2                5.984    0.098   60.949    0.000
    x3                2.487    0.093   26.778    0.000
    x4                2.823    0.092   30.689    0.000
    x5                3.995    0.105   38.183    0.000
    x6                1.922    0.079   24.321    0.000
    x7                4.432    0.087   51.181    0.000
    x8                5.563    0.078   71.214    0.000
    x9                5.418    0.079   68.440    0.000
    F1                0.000
    F2                0.000
    F3                0.000

Variances:
    x1                0.551    0.175
    x2                1.258    0.165
    x3                0.882    0.129
    x4                0.434    0.072
    x5                0.508    0.080
    x6                0.266    0.057
    x7                0.849    0.113
    x8                0.515    0.095
    x9                0.658    0.118
    F1                0.805    0.226
    F2                0.913    0.139
    F3                0.305    0.082


Group 2 [Grant-White]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.599    0.140    4.285    0.000
    x3                0.784    0.152    5.146    0.000
  F2 =~
    x4                1.000
    x5                1.083    0.069   15.609    0.000
    x6                0.912    0.069   13.283    0.000
  F3 =~
    x7                1.000
    x8                1.201    0.135    8.884    0.000
    x9                1.038    0.207    5.022    0.000

Covariances:
  F1 ~~
    F2                0.437    0.107    4.067    0.000
    F3                0.314    0.096    3.253    0.001
  F2 ~~
    F3                0.226    0.099    2.288    0.022
```

```
Intercepts:
    x1                4.930    0.095   51.696    0.000
    x2                6.200    0.092   67.416    0.000
    x3                1.996    0.086   23.195    0.000
    x4                3.317    0.093   35.625    0.000
    x5                4.712    0.096   48.986    0.000
    x6                2.469    0.094   26.277    0.000
    x7                3.921    0.086   45.819    0.000
    x8                5.488    0.087   63.174    0.000
    x9                5.327    0.085   62.571    0.000
    F1                0.000
    F2                0.000
    F3                0.000

Variances:
    x1                0.645    0.170
    x2                0.933    0.146
    x3                0.605    0.113
    x4                0.329    0.065
    x5                0.384    0.072
    x6                0.437    0.075
    x7                0.599    0.095
    x8                0.406    0.144
    x9                0.532    0.126
    F1                0.722    0.176
    F2                0.906    0.146
    F3                0.475    0.113
```

```
> anova(cfa.model1.fit.group, cfa.model1.fit.group.loadings)
```

```
Scaled Chi Square Difference Test (test = yuan.bentler)

                              Df    AIC    BIC  Chisq Chisq diff Df diff
cfa.model1.fit.group          48 7484.4 7706.8 115.85
cfa.model1.fit.group.loadings 54 7480.6 7680.8 124.04     7.6798       6
                              Pr(>Chisq)
cfa.model1.fit.group
cfa.model1.fit.group.loadings     0.2625
```

6.2 group.equal=c("loadings", "intercepts")

```
> cfa.model1.fit.group.loadings.intercepts <- cfa(cfa.model1, data=HolzingerSwineford1939, estimator=
> summary(cfa.model1.fit.group.loadings.intercepts, fit.measures=TRUE)
```

```
lavaan (0.5-11) converged normally after  63 iterations

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              164.103     166.748
```

```
  Degrees of freedom                                60          60
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.984
    for the Yuan-Bentler correction

Chi-square for each group:

  Pasteur                                       90.210      91.664
  Grant-White                                   73.893      75.084

Model test baseline model:

  Minimum Function Test Statistic              957.769     934.309
  Degrees of freedom                                72          72
  P-value                                        0.000       0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.882       0.876
  Tucker-Lewis Index (TLI)                       0.859       0.851

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3706.323   -3706.323
  Scaling correction factor                                  1.095
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
  Scaling correction factor                                  1.033
    for the MLR correction

  Number of free parameters                         48          48
  Akaike (AIC)                                7508.647    7508.647
  Bayesian (BIC)                              7686.588    7686.588
  Sample-size adjusted Bayesian (BIC)         7534.359    7534.359

Root Mean Square Error of Approximation:

  RMSEA                                          0.107       0.109
  90 Percent Confidence Interval          0.088  0.127       0.089  0.129
  P-value RMSEA <= 0.05                          0.000       0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.082       0.082

Parameter estimates:

  Information                                 Observed
  Standard Errors                   Robust.huber.white

Group 1 [Pasteur]:
```

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.576    0.131    4.395    0.000
    x3                0.798    0.173    4.627    0.000
  F2 =~
    x4                1.000
    x5                1.120    0.067   16.623    0.000
    x6                0.932    0.064   14.652    0.000
  F3 =~
    x7                1.000
    x8                1.130    0.133    8.488    0.000
    x9                1.009    0.207    4.865    0.000

Covariances:
  F1 ~~
    F2                0.410    0.135    3.043    0.002
    F3                0.178    0.067    2.661    0.008
  F2 ~~
    F3                0.180    0.062    2.889    0.004

Intercepts:
    x1                5.001    0.094   53.050    0.000
    x2                6.151    0.087   70.867    0.000
    x3                2.271    0.095   24.016    0.000
    x4                2.778    0.086   32.173    0.000
    x5                4.035    0.103   39.095    0.000
    x6                1.926    0.075   25.751    0.000
    x7                4.242    0.079   53.702    0.000
    x8                5.630    0.075   75.002    0.000
    x9                5.465    0.072   76.114    0.000
    F1                0.000
    F2                0.000
    F3                0.000

Variances:
    x1                0.555    0.184
    x2                1.296    0.161
    x3                0.944    0.147
    x4                0.445    0.073
    x5                0.502    0.081
    x6                0.263    0.058
    x7                0.888    0.128
    x8                0.541    0.089
    x9                0.654    0.115
    F1                0.796    0.239
    F2                0.879    0.135
    F3                0.322    0.092
```

```
Group 2 [Grant-White]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.576    0.131    4.395    0.000
    x3                0.798    0.173    4.627    0.000
  F2 =~
    x4                1.000
    x5                1.120    0.067   16.623    0.000
    x6                0.932    0.064   14.652    0.000
  F3 =~
    x7                1.000
    x8                1.130    0.133    8.488    0.000
    x9                1.009    0.207    4.865    0.000

Covariances:
  F1 ~~
    F2                0.427    0.107    4.003    0.000
    F3                0.329    0.101    3.268    0.001
  F2 ~~
    F3                0.236    0.097    2.423    0.015

Intercepts:
    x1                5.001    0.094   53.050    0.000
    x2                6.151    0.087   70.867    0.000
    x3                2.271    0.095   24.016    0.000
    x4                2.778    0.086   32.173    0.000
    x5                4.035    0.103   39.095    0.000
    x6                1.926    0.075   25.751    0.000
    x7                4.242    0.079   53.702    0.000
    x8                5.630    0.075   75.002    0.000
    x9                5.465    0.072   76.114    0.000
    F1               -0.148    0.140   -1.053    0.292
    F2                0.576    0.119    4.841    0.000
    F3               -0.177    0.104   -1.712    0.087

Variances:
    x1                0.654    0.177
    x2                0.964    0.152
    x3                0.641    0.129
    x4                0.343    0.065
    x5                0.376    0.073
    x6                0.437    0.074
    x7                0.625    0.106
    x8                0.434    0.147
    x9                0.522    0.125
    F1                0.708    0.186
    F2                0.870    0.143
    F3                0.505    0.125
```

```
> anova(cfa.model1.fit.group.loadings, cfa.model1.fit.group.loadings.intercepts)

Scaled Chi Square Difference Test (test = yuan.bentler)

                                         Df    AIC    BIC  Chisq Chisq diff
cfa.model1.fit.group.loadings            54 7480.6 7680.8 124.04
cfa.model1.fit.group.loadings.intercepts 60 7508.6 7686.6 164.10     26.706
                                         Df diff Pr(>Chisq)
cfa.model1.fit.group.loadings
cfa.model1.fit.group.loadings.intercepts       6  0.0001644 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> anova(cfa.model1.fit.group, cfa.model1.fit.group.loadings.intercepts)

Scaled Chi Square Difference Test (test = yuan.bentler)

                                         Df    AIC    BIC  Chisq Chisq diff
cfa.model1.fit.group                     48 7484.4 7706.8 115.85
cfa.model1.fit.group.loadings.intercepts 60 7508.6 7686.6 164.10     37.616
                                         Df diff Pr(>Chisq)
cfa.model1.fit.group
cfa.model1.fit.group.loadings.intercepts      12  0.0001774 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- cfa.model1.fit.group.loadings와 cfa.model1.fit.group.loadings.intercepts의 차이가 분산분석으로 드러난다.

6.3 modIndices()

- 관찰변수 중에서 mi 값이 큰 두 개를 선택하고자 한다.
- 171번과 175번을 주목한다.

```
> modindices(cfa.model1.fit.group.loadings.intercepts)

    lhs op rhs group     mi mi.scaled    epc sepc.lv sepc.all sepc.nox
1    F1 =~  x1     1  1.716     1.744  0.259   0.231    0.199    0.199
2    F1 =~  x2     1  0.302     0.307 -0.051  -0.045   -0.036   -0.036
3    F1 =~  x3     1  1.611     1.637 -0.131  -0.117   -0.097   -0.097
4    F1 =~  x4     1  0.286     0.291  0.045   0.040    0.035    0.035
5    F1 =~  x5     1  2.799     2.844 -0.152  -0.136   -0.107   -0.107
6    F1 =~  x6     1  1.181     1.200  0.077   0.069    0.068    0.068
7    F1 =~  x7     1  4.795     4.872 -0.249  -0.222   -0.202   -0.202
8    F1 =~  x8     1  0.737     0.749 -0.092  -0.082   -0.084   -0.084
9    F1 =~  x9     1  8.416     8.552  0.300   0.268    0.270    0.270
10   F2 =~  x1     1  7.705     7.829  0.323   0.303    0.260    0.260
11   F2 =~  x2     1  0.997     1.013 -0.109  -0.102   -0.082   -0.082
12   F2 =~  x3     1  5.639     5.730 -0.242  -0.227   -0.189   -0.189
13   F2 =~  x4     1  0.002     0.002 -0.003  -0.003   -0.003   -0.003
14   F2 =~  x5     1  2.507     2.548  0.087   0.082    0.064    0.064
15   F2 =~  x6     1  2.450     2.489 -0.073  -0.069   -0.068   -0.068
```

16	F2	=~	x7	1	0.396	0.402	0.061	0.057	0.052	0.052
17	F2	=~	x8	1	0.507	0.515	-0.065	-0.061	-0.063	-0.063
18	F2	=~	x9	1	0.041	0.042	0.018	0.017	0.017	0.017
19	F3	=~	x1	1	0.488	0.496	0.158	0.090	0.077	0.077
20	F3	=~	x2	1	1.217	1.237	-0.233	-0.132	-0.106	-0.106
21	F3	=~	x3	1	0.000	0.000	-0.004	-0.002	-0.002	-0.002
22	F3	=~	x4	1	0.003	0.003	0.008	0.004	0.004	0.004
23	F3	=~	x5	1	1.122	1.140	-0.160	-0.091	-0.071	-0.071
24	F3	=~	x6	1	0.895	0.909	0.112	0.063	0.063	0.063
25	F3	=~	x7	1	0.008	0.008	-0.020	-0.011	-0.010	-0.010
26	F3	=~	x8	1	0.026	0.027	0.023	0.013	0.013	0.013
27	F3	=~	x9	1	0.004	0.004	-0.008	-0.004	-0.004	-0.004
28	x1	~~	x1	1	0.000	0.000	0.000	0.000	0.000	0.000
29	x1	~~	x2	1	0.634	0.645	-0.083	-0.083	-0.057	-0.057
30	x1	~~	x3	1	0.196	0.199	-0.065	-0.065	-0.046	-0.046
31	x1	~~	x4	1	3.983	4.047	0.121	0.121	0.090	0.090
32	x1	~~	x5	1	0.564	0.573	-0.049	-0.049	-0.033	-0.033
33	x1	~~	x6	1	0.358	0.364	0.030	0.030	0.026	0.026
34	x1	~~	x7	1	1.298	1.318	-0.092	-0.092	-0.072	-0.072
35	x1	~~	x8	1	0.379	0.385	-0.044	-0.044	-0.039	-0.039
36	x1	~~	x9	1	2.005	2.038	0.102	0.102	0.089	0.089
37	x2	~~	x2	1	0.000	0.000	0.000	0.000	0.000	0.000
38	x2	~~	x3	1	2.273	2.309	0.154	0.154	0.102	0.102
39	x2	~~	x4	1	3.002	3.051	-0.125	-0.125	-0.087	-0.087
40	x2	~~	x5	1	0.007	0.007	0.006	0.006	0.004	0.004
41	x2	~~	x6	1	0.721	0.733	0.051	0.051	0.040	0.040
42	x2	~~	x7	1	8.913	9.057	-0.284	-0.284	-0.207	-0.207
43	x2	~~	x8	1	0.000	0.000	-0.001	-0.001	-0.001	-0.001
44	x2	~~	x9	1	2.376	2.414	0.130	0.130	0.105	0.105
45	x3	~~	x3	1	0.000	0.000	0.000	0.000	0.000	0.000
46	x3	~~	x4	1	0.409	0.415	-0.042	-0.042	-0.030	-0.030
47	x3	~~	x5	1	5.727	5.820	-0.168	-0.168	-0.110	-0.110
48	x3	~~	x6	1	0.862	0.876	0.050	0.050	0.041	0.041
49	x3	~~	x7	1	0.363	0.369	-0.052	-0.052	-0.039	-0.039
50	x3	~~	x8	1	0.000	0.000	-0.001	-0.001	-0.001	-0.001
51	x3	~~	x9	1	2.701	2.745	0.125	0.125	0.105	0.105
52	x4	~~	x4	1	0.000	0.000	0.000	0.000	0.000	0.000
53	x4	~~	x5	1	5.563	5.653	0.165	0.165	0.113	0.113
54	x4	~~	x6	1	7.522	7.643	-0.162	-0.162	-0.139	-0.139
55	x4	~~	x7	1	7.514	7.636	0.170	0.170	0.134	0.134
56	x4	~~	x8	1	1.363	1.385	-0.062	-0.062	-0.055	-0.055
57	x4	~~	x9	1	1.346	1.368	-0.064	-0.064	-0.056	-0.056
58	x5	~~	x5	1	0.000	0.000	0.000	0.000	0.000	0.000
59	x5	~~	x6	1	0.124	0.126	0.023	0.023	0.018	0.018
60	x5	~~	x7	1	1.780	1.809	-0.090	-0.090	-0.064	-0.064
61	x5	~~	x8	1	0.353	0.359	-0.034	-0.034	-0.028	-0.028
62	x5	~~	x9	1	0.285	0.290	0.032	0.032	0.025	0.025
63	x6	~~	x6	1	0.000	0.000	0.000	0.000	0.000	0.000
64	x6	~~	x7	1	0.009	0.009	0.005	0.005	0.004	0.004
65	x6	~~	x8	1	1.623	1.649	0.057	0.057	0.057	0.057
66	x6	~~	x9	1	0.054	0.055	-0.011	-0.011	-0.011	-0.011
67	x7	~~	x7	1	0.000	0.000	0.000	0.000	0.000	0.000

68	x7	~~	x8	1	1.821	1.851	0.113	0.113	0.105	0.105
69	x7	~~	x9	1	0.865	0.878	-0.073	-0.073	-0.067	-0.067
70	x8	~~	x8	1	0.000	0.000	0.000	0.000	0.000	0.000
71	x8	~~	x9	1	0.217	0.220	-0.039	-0.039	-0.040	-0.040
72	x9	~~	x9	1	0.000	0.000	0.000	0.000	0.000	0.000
73	F1	~~	F1	1	0.000	0.000	0.000	0.000	0.000	0.000
74	F1	~~	F2	1	0.000	0.000	0.000	0.000	0.000	0.000
75	F1	~~	F3	1	0.000	0.000	0.000	0.000	0.000	0.000
76	F2	~~	F2	1	0.000	0.000	0.000	0.000	0.000	0.000
77	F2	~~	F3	1	0.000	0.000	0.000	0.000	0.000	0.000
78	F3	~~	F3	1	0.000	0.000	0.000	0.000	0.000	0.000
79	x1	~1		1	4.485	4.557	-0.133	-0.133	-0.114	-0.114
80	x2	~1		1	6.634	6.741	-0.165	-0.165	-0.132	-0.132
81	x3	~1		1	17.717	18.002	0.248	0.248	0.206	0.206
82	x4	~1		1	1.816	1.846	0.058	0.058	0.050	0.050
83	x5	~1		1	1.316	1.337	-0.054	-0.054	-0.042	-0.042
84	x6	~1		1	0.028	0.028	-0.007	-0.007	-0.007	-0.007
85	x7	~1		1	13.681	13.902	0.205	0.205	0.186	0.186
86	x8	~1		1	3.864	3.926	-0.099	-0.099	-0.102	-0.102
87	x9	~1		1	1.322	1.343	-0.058	-0.058	-0.059	-0.059
88	F1	~1		1	0.000	0.000	0.000	0.000	0.000	0.000
89	F2	~1		1	0.000	0.000	0.000	0.000	0.000	0.000
90	F3	~1		1	0.000	0.000	0.000	0.000	0.000	0.000
91	F1	=~	x1	2	1.198	1.217	-0.181	-0.152	-0.130	-0.130
92	F1	=~	x2	2	0.302	0.307	0.051	0.043	0.039	0.039
93	F1	=~	x3	2	1.611	1.637	0.131	0.110	0.105	0.105
94	F1	=~	x4	2	0.699	0.710	0.069	0.058	0.053	0.053
95	F1	=~	x5	2	2.588	2.630	-0.146	-0.123	-0.101	-0.101
96	F1	=~	x6	2	0.825	0.839	0.078	0.066	0.060	0.060
97	F1	=~	x7	2	5.478	5.566	-0.243	-0.204	-0.192	-0.192
98	F1	=~	x8	2	1.208	1.227	-0.112	-0.094	-0.091	-0.091
99	F1	=~	x9	2	11.518	11.704	0.334	0.281	0.276	0.276
100	F2	=~	x1	2	0.365	0.371	0.057	0.053	0.045	0.045
101	F2	=~	x2	2	0.844	0.857	0.076	0.071	0.065	0.065
102	F2	=~	x3	2	1.588	1.614	-0.099	-0.092	-0.088	-0.088
103	F2	=~	x4	2	0.001	0.001	0.002	0.002	0.002	0.002
104	F2	=~	x5	2	2.507	2.548	-0.087	-0.081	-0.067	-0.067
105	F2	=~	x6	2	2.450	2.489	0.073	0.068	0.063	0.063
106	F2	=~	x7	2	1.962	1.993	-0.101	-0.094	-0.089	-0.089
107	F2	=~	x8	2	1.488	1.512	-0.086	-0.080	-0.077	-0.077
108	F2	=~	x9	2	6.975	7.088	0.181	0.169	0.166	0.166
109	F3	=~	x1	2	0.004	0.004	-0.010	-0.007	-0.006	-0.006
110	F3	=~	x2	2	0.244	0.248	-0.066	-0.047	-0.043	-0.043
111	F3	=~	x3	2	0.163	0.166	0.051	0.036	0.035	0.035
112	F3	=~	x4	2	0.033	0.033	-0.017	-0.012	-0.011	-0.011
113	F3	=~	x5	2	0.159	0.162	0.040	0.029	0.024	0.024
114	F3	=~	x6	2	0.065	0.067	-0.025	-0.018	-0.016	-0.016
115	F3	=~	x7	2	0.003	0.003	0.007	0.005	0.005	0.005
116	F3	=~	x8	2	0.026	0.027	-0.023	-0.016	-0.016	-0.016
117	F3	=~	x9	2	0.004	0.004	0.008	0.005	0.005	0.005
118	x1	~~	x1	2	0.000	0.000	0.000	0.000	0.000	0.000
119	x1	~~	x2	2	0.060	0.061	-0.022	-0.022	-0.017	-0.017

120	x1	~~	x3	2	1.843	1.873	-0.163	-0.163	-0.134	-0.134
121	x1	~~	x4	2	0.060	0.061	0.014	0.014	0.011	0.011
122	x1	~~	x5	2	0.059	0.060	-0.015	-0.015	-0.011	-0.011
123	x1	~~	x6	2	0.016	0.017	-0.008	-0.008	-0.006	-0.006
124	x1	~~	x7	2	7.662	7.785	-0.197	-0.197	-0.159	-0.159
125	x1	~~	x8	2	0.211	0.214	0.031	0.031	0.025	0.025
126	x1	~~	x9	2	8.604	8.743	0.197	0.197	0.166	0.166
127	x2	~~	x2	2	0.000	0.000	0.000	0.000	0.000	0.000
128	x2	~~	x3	2	3.301	3.354	0.143	0.143	0.125	0.125
129	x2	~~	x4	2	0.432	0.439	0.039	0.039	0.032	0.032
130	x2	~~	x5	2	0.989	1.005	-0.064	-0.064	-0.048	-0.048
131	x2	~~	x6	2	0.000	0.000	0.000	0.000	0.000	0.000
132	x2	~~	x7	2	1.276	1.296	-0.083	-0.083	-0.072	-0.072
133	x2	~~	x8	2	0.006	0.007	0.005	0.005	0.005	0.005
134	x2	~~	x9	2	0.193	0.196	0.030	0.030	0.027	0.027
135	x3	~~	x3	2	0.000	0.000	0.000	0.000	0.000	0.000
136	x3	~~	x4	2	0.155	0.157	0.021	0.021	0.018	0.018
137	x3	~~	x5	2	1.189	1.209	-0.062	-0.062	-0.049	-0.049
138	x3	~~	x6	2	1.571	1.597	0.069	0.069	0.061	0.061
139	x3	~~	x7	2	1.489	1.513	-0.080	-0.080	-0.072	-0.072
140	x3	~~	x8	2	0.155	0.158	-0.024	-0.024	-0.022	-0.022
141	x3	~~	x9	2	0.850	0.864	0.057	0.057	0.053	0.053
142	x4	~~	x4	2	0.000	0.000	0.000	0.000	0.000	0.000
143	x4	~~	x5	2	0.987	1.003	-0.074	-0.074	-0.055	-0.055
144	x4	~~	x6	2	2.422	2.461	0.092	0.092	0.076	0.076
145	x4	~~	x7	2	0.594	0.604	0.039	0.039	0.033	0.033
146	x4	~~	x8	2	5.071	5.153	-0.106	-0.106	-0.093	-0.093
147	x4	~~	x9	2	0.458	0.466	0.032	0.032	0.029	0.029
148	x5	~~	x5	2	0.000	0.000	0.000	0.000	0.000	0.000
149	x5	~~	x6	2	0.369	0.375	-0.040	-0.040	-0.031	-0.031
150	x5	~~	x7	2	0.411	0.418	0.035	0.035	0.027	0.027
151	x5	~~	x8	2	0.853	0.867	0.047	0.047	0.037	0.037
152	x5	~~	x9	2	0.191	0.194	0.023	0.023	0.018	0.018
153	x6	~~	x6	2	0.000	0.000	0.000	0.000	0.000	0.000
154	x6	~~	x7	2	0.021	0.021	-0.008	-0.008	-0.007	-0.007
155	x6	~~	x8	2	0.444	0.451	-0.033	-0.033	-0.029	-0.029
156	x6	~~	x9	2	0.053	0.053	-0.012	-0.012	-0.010	-0.010
157	x7	~~	x7	2	0.000	0.000	0.000	0.000	0.000	0.000
158	x7	~~	x8	2	8.278	8.411	0.214	0.214	0.194	0.194
159	x7	~~	x9	2	3.049	3.098	-0.119	-0.119	-0.110	-0.110
160	x8	~~	x8	2	0.000	0.000	0.000	0.000	0.000	0.000
161	x8	~~	x9	2	1.383	1.405	-0.089	-0.089	-0.084	-0.084
162	x9	~~	x9	2	0.000	0.000	0.000	0.000	0.000	0.000
163	F1	~~	F1	2	0.000	0.000	0.000	0.000	0.000	0.000
164	F1	~~	F2	2	0.000	0.000	0.000	0.000	0.000	0.000
165	F1	~~	F3	2	0.000	0.000	0.000	0.000	0.000	0.000
166	F2	~~	F2	2	0.000	0.000	0.000	0.000	0.000	0.000
167	F2	~~	F3	2	0.000	0.000	0.000	0.000	0.000	0.000
168	F3	~~	F3	2	0.000	0.000	0.000	0.000	0.000	0.000
169	x1	~1		2	4.484	4.557	0.133	0.133	0.114	0.114
170	x2	~1		2	6.634	6.741	0.165	0.165	0.151	0.151
171	x3	~1		2	17.717	18.002	-0.248	-0.248	-0.238	-0.238

```
172  x4 ~1          2  1.816     1.846 -0.058  -0.058   -0.053   -0.053
173  x5 ~1          2  1.316     1.337  0.054   0.054    0.044    0.044
174  x6 ~1          2  0.028     0.028  0.007   0.007    0.006    0.006
175  x7 ~1          2 13.681    13.902 -0.205  -0.205   -0.193   -0.193
176  x8 ~1          2  3.864     3.926  0.099   0.099    0.096    0.096
177  x9 ~1          2  1.322     1.343  0.058   0.058    0.057    0.057
178  F1 ~1          2  0.000     0.000  0.000   0.000    0.000    0.000
179  F2 ~1          2  0.000     0.000  0.000   0.000    0.000    0.000
180  F3 ~1          2  0.000     0.000  0.000   0.000    0.000    0.000
```

6.4 group.partial=c("x3~1", "x7~1")

```
> cfa.model1.fit.group.loadings.intercepts.group.partial <- cfa(cfa.model1, data=HolzingerSwineford19
> summary(cfa.model1.fit.group.loadings.intercepts.group.partial, fit.measures=TRUE)
```

```
lavaan (0.5-11) converged normally after  65 iterations

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              129.422     130.994
  Degrees of freedom                                58          58
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.988
    for the Yuan-Bentler correction

Chi-square for each group:

  Pasteur                                       71.170      72.034
  Grant-White                                   58.253      58.960

Model test baseline model:

  Minimum Function Test Statistic              957.769     934.309
  Degrees of freedom                                72          72
  P-value                                        0.000       0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.919       0.915
  Tucker-Lewis Index (TLI)                       0.900       0.895

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3688.983   -3688.983
  Scaling correction factor                                  1.086
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
  Scaling correction factor                                  1.033
    for the MLR correction
```

```
  Number of free parameters                         50          50
  Akaike (AIC)                                7477.966    7477.966
  Bayesian (BIC)                              7663.322    7663.322
  Sample-size adjusted Bayesian (BIC)         7504.750    7504.750

Root Mean Square Error of Approximation:

  RMSEA                                          0.090       0.091
  90 Percent Confidence Interval          0.070  0.111       0.071  0.112
  P-value RMSEA <= 0.05                          0.001       0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.073       0.073

Parameter estimates:

  Information                                 Observed
  Standard Errors                   Robust.huber.white

Group 1 [Pasteur]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.606    0.146    4.166    0.000
    x3                0.791    0.159    4.978    0.000
  F2 =~
    x4                1.000
    x5                1.120    0.067   16.633    0.000
    x6                0.932    0.064   14.655    0.000
  F3 =~
    x7                1.000
    x8                1.200    0.134    8.952    0.000
    x9                1.041    0.208    5.006    0.000

Covariances:
  F1 ~~
    F2                0.404    0.135    2.981    0.003
    F3                0.168    0.066    2.558    0.011
  F2 ~~
    F3                0.172    0.060    2.896    0.004

Intercepts:
    x1                4.914    0.095   51.601    0.000
    x2                6.087    0.080   76.352    0.000
    x3                2.487    0.093   26.778    0.000
    x4                2.778    0.086   32.172    0.000
    x5                4.035    0.103   39.087    0.000
    x6                1.926    0.075   25.749    0.000
```

```
    x7                4.432    0.087   51.181    0.000
    x8                5.569    0.074   75.596    0.000
    x9                5.409    0.070   77.093    0.000
    F1                0.000
    F2                0.000
    F3                0.000

Variances:
    x1                0.560    0.181
    x2                1.267    0.166
    x3                0.879    0.131
    x4                0.446    0.073
    x5                0.502    0.081
    x6                0.263    0.058
    x7                0.850    0.113
    x8                0.516    0.095
    x9                0.656    0.118
    F1                0.796    0.230
    F2                0.879    0.135
    F3                0.304    0.083



Group 2 [Grant-White]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    x1                1.000
    x2                0.606    0.146    4.166    0.000
    x3                0.791    0.159    4.978    0.000
  F2 =~
    x4                1.000
    x5                1.120    0.067   16.633    0.000
    x6                0.932    0.064   14.655    0.000
  F3 =~
    x7                1.000
    x8                1.200    0.134    8.952    0.000
    x9                1.041    0.208    5.006    0.000

Covariances:
  F1 ~~
    F2                0.426    0.106    4.027    0.000
    F3                0.312    0.097    3.224    0.001
  F2 ~~
    F3                0.223    0.097    2.303    0.021

Intercepts:
    x1                4.914    0.095   51.601    0.000
    x2                6.087    0.080   76.352    0.000
    x3                1.955    0.109   17.907    0.000
    x4                2.778    0.086   32.172    0.000
```

```
    x5                4.035    0.103   39.087    0.000
    x6                1.926    0.075   25.749    0.000
    x7                3.992    0.092   43.463    0.000
    x8                5.569    0.074   75.596    0.000
    x9                5.409    0.070   77.093    0.000
    F1                0.051    0.133    0.384    0.701
    F2                0.576    0.119    4.841    0.000
    F3               -0.071    0.089   -0.806    0.420

Variances:
    x1                0.651    0.172
    x2                0.939    0.149
    x3                0.603    0.114
    x4                0.343    0.065
    x5                0.377    0.073
    x6                0.437    0.074
    x7                0.599    0.095
    x8                0.407    0.145
    x9                0.531    0.127
    F1                0.715    0.179
    F2                0.870    0.143
    F3                0.475    0.114
```

- Grant-White(Group 2)의 intercepts에서 요인들(F1, F2, F3)의 점수를 살펴보고, 유의확률로 근거를 확보한다.
- "Grant-White의 F2의 상수값 0.576이 유의확률을 바탕으로 Pasteur 학교보다 높다고 말할 수 있다."

참고문헌

```
> citation()

To cite R in publications use:

  R Development Core Team (2011). R: A language and environment for
  statistical computing. R Foundation for Statistical Computing,
  Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org/.

A BibTeX entry for LaTeX users is

  @Manual{,
    title = {R: A Language and Environment for Statistical Computing},
    author = {{R Development Core Team}},
    organization = {R Foundation for Statistical Computing},
    address = {Vienna, Austria},
    year = {2011},
    note = {{ISBN} 3-900051-07-0},
    url = {http://www.R-project.org/},
  }

We have invested a lot of time and effort in creating R, please cite it
when using it for data analysis. See also 'citation("pkgname")' for
citing R packages.

> citation("lavaan")

To cite lavaan in publications use:

  Yves Rosseel (2012). lavaan: An R Package for Structural Equation
  Modeling. Journal of Statistical Software, 48(2), 1-36. URL
  http://www.jstatsoft.org/v48/i02/.

A BibTeX entry for LaTeX users is

  @Article{,
    title = {{lavaan}: An {R} Package for Structural Equation Modeling},
    author = {Yves Rosseel},
    journal = {Journal of Statistical Software},
    year = {2012},
    volume = {48},
    number = {2},
    pages = {1--36},
```

```
  url = {http://www.jstatsoft.org/v48/i02/},
}
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