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naive driving model with/out obstacles& horizon

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May 11, 2025

1 naive driving model (with/out obstacles& horizon)

1.1 trajectory definition

Firstly consider several types of functions to simulate possible trajectories in real life: straight line(trivial case), ring(circle and ellipse), cubic Bezier Curve and multiple polynomial routes.

Ring route can be depicted by under the polar coordinates.

Bezier Curve is defined by

$$q(\tau_i) = \sum_{k=0}^{m} \binom{m}{k} (1-\tau_i)^{m-k} \tau_i^k, \quad \tau_i \in [0,1]$$

Here m is the order of Bezier Curve, $q(\tau_i)$ is the interpolation value at parameter τ_i, and P_k is the kth control point. By taking value of τ_i from$[0,1]$ to generate interpolation points from the first control point P_0 to the last control point P_m.

Take m=3, get Cubic Bezier Curve

$$B(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3$$

To get every two sections smooth at the connected point,i.e. the last points from last curve should be the same with the first point of next curve, and the slope should be collinear.

1.2 basic trajectory following model

With the defined several types of trajectory above, initiate a basic trajectory following model to see, under the situation without considering of both obstacles and horizon, how fast and safe it is for the driving car.

For the trajectory following part, it would involve control variables: $x(t)$, $y(t)$, define the position of the car. $v(t)$, $\dot{v(t)} = \alpha(t)$,define the velocity and acceleration or deceleration of the vehicle. $\delta(t)$ defines the steering angle, ψ defines the yaw angle. Since they are variables depend on time, the whole system would only be ordinary differential equations. Moreover, it would include objective model: consisting of measurement of both how fast it is and how safe it is. Under the situation without any obstacles, the latter term will only depends on the quantity of how much the offset from center line is. Then one can construct objective function as:

$$J = -\sum_{k=1}^{N} v_k + w_{tracking} \sum_{k=1}^{N} ((x_k - x_{target})^2 + (y_k - y_{target})^2) + w_{control} \sum_{k=1}^{N} (a_k^2 + \delta_k^2)$$

Here $w_{tracking}$, $w_{control}$ are all parameters to bargain between "high speed" and "safety".

As well as no speed limits, and steering angles limits are given by the performance of constructed car model.(HIGHLIGHT: it depends on the model constructed, to be REWRITTEN later to see the limits of angle). Conclude from early double track model,

$$-\delta_{max} \leq \delta \leq \delta_{max}$$

1.3 numerical methods for solving

1.4 simulation with CasADi on Matlab

With a given enough long trajectory separately as well as the combination of several types of the routes above, simulate the performance. Assume the car must start from the start point to the target point to finish the whole path. Given arbitrary initial value , one can calculate the highest speed it can get with CasADi in Matlab.