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naive driving model NLP design with obstacles

Yan Qiu

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1 driving model NLP formulation(with obstacles)

1.1 trajectory definition

Firstly consider several types of functions to simulate possible trajectories in real life: straight line(trivial case), ring(circle and ellipse), cubic Bezier Curve and multiple polynomial routes.

Ring route can be depicted by under the polar coordinates.

Bezier Curve is defined by

$$q(\tau_i) = \sum_{k=0}^{m} \binom{m}{k} (1-\tau_i)^{m-k} \tau_i^k, \quad \tau_i \in [0,1]$$

Here m is the order of Bezier Curve, $q(\tau_i)$ is the interpolation value at parameter τ_i, and P_k is the kth control point. By taking value of τ_i from$[0,1]$ to generate interpolation points from the first control point P_0 to the last control point P_m.

Take m=3, get Cubic Bezier Curve

$$B(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t) t^2 P_2 + t^3 P_3$$

To get every two sections smooth at the connected point,i.e. the last points from last curve should be the same with the first point of next curve, and the slope should be collinear.

1.2 basic trajectory following model

With the defined several types of trajectory above, initiate a basic trajectory following model to see, under the situation without considering of both obstacles and abortion, how fast and safe it is for the driving car.

For the trajectory following part, it would involve control variables: $x(t)$, $y(t)$, define the position of the car. $v(t)$, $\dot{v(t)} = \alpha(t)$,define the velocity and acceleration or deceleration of the vehicle. $\delta(t)$ defines the steering angle, ψ defines the yaw angle. Since they are variables depend on time, the whole system would only be ordinary differential equations. Moreover, it would include objective model: consisting of measurement of both how fast it is and how safe it is. Under the situation with or without countable obstacles, the objective function is given by the lap time within fixed distance

$$J_1 = t_f$$

as well as of the cost of all control efforts:

$$J_2 = \int_0^{t_f} u(t)^2 \, dt$$

. Then one can construct objective function as: Minimize:

$$J = a_1 t_f + a_2 \int_0^{t_f} u(t)^2 \, dt$$

Here a_1, a_2 are all parameters of non-negative weights.

Subject to the dynamics constraints,all of the equations of motions including both non-linear tire and double track model.

Initial and terminal conditions:

$$\begin{aligned} x(0) &= 0, \quad x(t_f) = 100 \\ y(0) &= 0, \quad y(t_f) = 50 \\ \psi(0) &= 0, \quad \psi(t_f) = -\pi \end{aligned}$$

Moreover, there is also Obstacle constraints (for all $k = 1, ..., M$),Given M circular obstacles with centers (X_k, Y_k) and radii R_k, we impose:

$$(x(t) - X_k)^2 + (y(t) - Y_k)^2 \geq \delta R_k^2$$

Here, one may apply a homotopy strategy to solve such problem and it would be a parametric optimal control problem OCP(δ) with a homotopy parameter $\delta \in [0,1]$

Control constraints for $u(t)$.

To ensure the driver is always within track, use the limitation as hard constraints: Assume the reference path is given as a parametric curve:

$$(x_{\text{ref}}(s), y_{\text{ref}}(s)), \quad s \in [0,1]$$

Let the signed lateral deviation from the path be denoted:

$$d(t) = \text{signed\_distance}\big((x(t), y(t)), \text{Track Centerline}\big)$$

Then the vehicle must satisfy the hard constraint:

$$|d(t)| \leq \text{margin}, \quad \forall t \in [0, t_f]$$

Equivalently, for a simpler conservative bound:

$$(x(t) - x_{\text{ref}}(t))^2 + (y(t) - y_{\text{ref}}(t))^2 \leq \text{margin}^2, \quad \forall t$$

This enforces that the vehicle position lies within a tube around the centerline trajectory.

As well as no speed limits, and steering angles limits are given by the performance of constructed car model.(HIGHLIGHT: it depends on the model constructed, to be REWRITTEN later to see the limits of angle). Conclude from early double track model,

$$-\delta_{max} \leq \delta \leq \delta_{max}$$

1.3 numerical methods for solving

With the homotopy strategy to relax the state constraints, define M=5 obstacles first and a solution of the obstacle avoidance problem can be obtained by the this procedure.

1.4 simulation with CasADi on Matlab

With a given enough long trajectory separately as well as the combination of several types of the routes above, simulate the performance. Assume the car must start from the start point to the target point to finish the whole path. Given arbitrary initial value , one can calculate the highest speed it can get with CasADi in Matlab.

2 driving model with emergency abortion