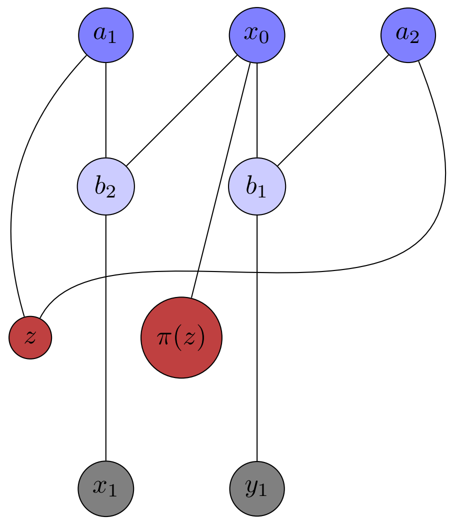
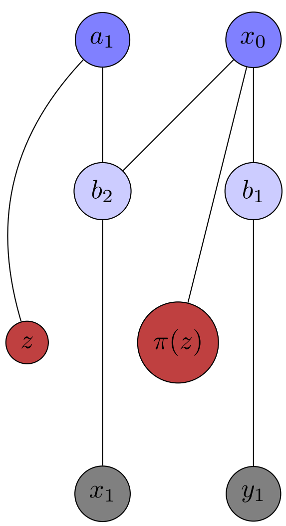
Here is the template for the $P(2,1)$ case, without adding extra vertices.

$k = 2$

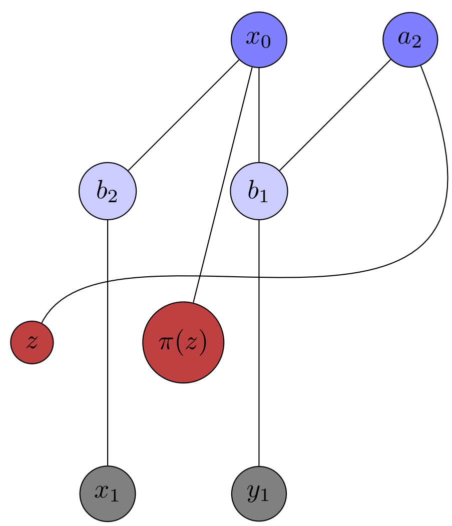


The basic templates for the graphs Γ_1 and Γ_2 are given below.

Γ_1



Γ_2

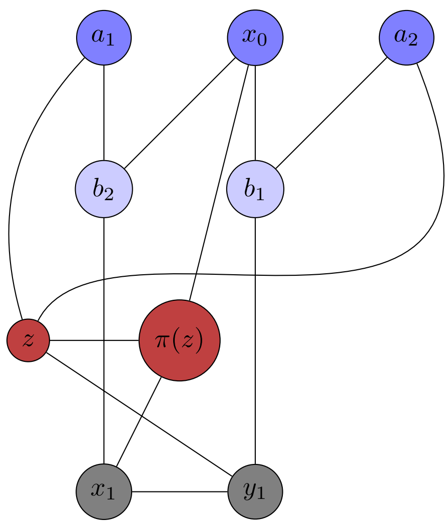


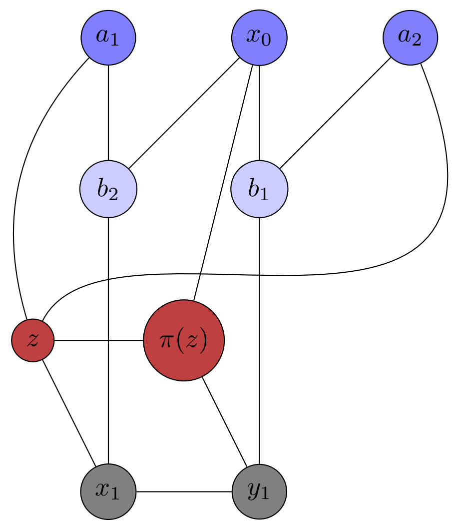
Note that if we join z to x_1 then Γ_2 has a six cycle $x_1, z, a_2, b_1, x_0, b_2$ with the two bumps. Such a cycle occurs in Γ_1 only if we join y_1 to z.

Therefore, if we join z to x_1 but do *not* join z to y_1, the graphs Γ_1 and Γ_2 will be nonisomorphic.

Exploring the smallest $P(2,1)$ graphs

There are two $P(2,1)$ graphs with orbit matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

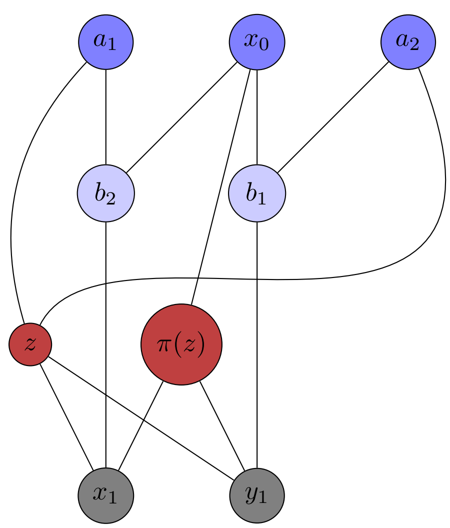




These are isomorphic by a isomorphism that swaps a_1 and a_2.

This graph gives two Kocay graphs, Γ_1 and Γ_2 that are not isomorphic. On has a six cycle $(z, x_1, b_2, x_0, b_1, a_2)$ containing the bumps; one does not.

There is one $P(2,1)$ with orbit matrix $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$



The function $\theta := (a_1\ a_2)(x_1\ y_1)(b_1\ b_2)(z)(x_0)(z_2)$ is an automorphism of this graph. This means that $\Gamma_1 \cong \Gamma_2$ so we are not interested in this case.