An Investigation of Faraday Rotation

PHYS 441

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1 Abstract

The purpose of this experiment is to investigate the magneto-optical phenomenon known as Faraday rotation. Specifically, we will aim to determine how angle of polarization changes due to magnetic field strength, sample material, and wavelength. This will ultimately allow us to extract and compare the Verdet constants, which is a measure of the strength of the Faraday effect in a particular material [1]. For our experiment, we will be using SF-59 glass and distilled water for sample variation as well as red, green, and violet lasers for wavelength variation. Our results show that a sample of SF-59 glass has Verdet constants of $(23.75 \pm 0.05)\text{radT}^{-1}\text{m}^{-1}$, $(7.7 \pm 0.6) \times 10^{-1}\text{radT}^{-1}\text{m}^{-1}$, and $(1.14 \pm 0.06) \times 10^{-1}\text{radT}^{-1}\text{m}^{-1}$ for a red, green, and violet laser respectively. Furthermore, distilled water has Verdet constants $(3.34 \pm 0.03)\text{radT}^{-1}\text{m}^{-1}$ and $(7 \pm 1) \times 10^{-3}\text{radT}^{-1}\text{m}^{-1}$ for red and green lasers respectively.

2 Introduction

The Faraday effect was first discovered by Michael Faraday in 1845 [2]. Based on a series of experiments conducted by William Thomson eleven years prior, Faraday passed a ray of polarized light through a piece of glass placed in a strong magnetic field. He found that the plane of polarization of the ray had rotated. This was the first experimental confirmation of the relationship between light and electromagnetism.

This phenomenon is relevant for study in the modern day due to its practicality. Faraday rotation has many applications for remote current sensors, optical filters, lasers, and measuring behavior of interstellar particles.

In order to understand Faraday Rotation, we must begin with a brief discussion of light. Linearly polarized light can be thought of as a superposition of two counter rotating, circularly polarized components with equal amplitude. The direction of electric field from by this light will thus also be circular, and force charged particles in a material to move as such. These moving ions will create their own magnetic field. This field will either be parallel or anti-parallel to the externally applied magnetic field. This causes one component of the beam to slow, while the other speeds up, resulting in a phase shift. When these two components recombine, the light is linearly polarized again, but the plane of polarization has shifted. This phenomenon is known as the Faraday effect or Faraday rotation.

The Verdet constant is an indicator of how strongly a material exhibits Faraday rotation. For our experiment, we will be measuring changes to polarization angle in combination with magnetic field strength and wavelength of light in order to determine the Verdet constants of different materials. Our methodology will be similar to that outlined in the TeachSpin Faraday Rotation laboratory manual [3], as well as experiments conducted by Tripathy et al [4] and Loeffler [5].

3 Theory

3.1 Manual/DC Methods

The direction of rotation due to the Faraday effect is independent of the direction of propagation of the light. The angle of rotation θ_0 is a function of magnetic field strength, B; length of the material, L; and the Verdet constant, V. Both wavelength and temperature are implicitly carried by the Verdet constant. Then, θ_0 becomes

$$\theta_0 = VBL. \tag{1}$$

Rearranging Eq. (1) to express Vertdet constant, we find that

$$V = \frac{\theta_0}{BL}. \tag{2}$$

If the assumption that ideal solenoids do not have fringe effects is employed, calculating magnetic field from supplied current is fairly simple:

$$B = \mu n I, \tag{3}$$

where μ is permeability, n is turn density, and I is supplied current. However, we are not using an ideal solenoid and thus fringe effects must be accounted for. To do this, we will be using a Hall probe (DRV 5055 A2) to measure magnetic field for various positions along the solenoid. A Hall probe does not output magnetic field directly, however. Instead, a Hall probe will output a voltage corresponding to magnetic field strength. This output voltage is given as

$$V_{out} = V_Q + B\Big[S\big(1 + S_{TC}(T_A - 25°C)\big)\Big]. \tag{4}$$

In this expression, V_{out} is output voltage, V_Q is quiescent voltage, S is sensitivity at a given temperature, S_{TC} is a sensitivity temperature compensation factor, and T_A is ambient temperature. All temperatures are in degrees Celsius. Eq. (4) can be rearranged to solve for B, yielding

$$B = \frac{V_{out} - V_Q}{S(1 + S_{TC}[T_A - 25°C])}. \tag{5}$$

Eq. (5) allows the Hall probe to give the magnetic field.

3.2 Lock-in/AC Methods

For a magnetic field resulting from an AC current, angle of rotation, θ_0 can be found as

$$\theta_0 = \frac{1}{2}\frac{\Delta\Phi}{\Phi_0}, \tag{6}$$

where Φ_0 and $\Delta\Phi$ are intensities [4]. In practice, we will measure two output voltage values. The Φ_0 voltage corresponds to the initial intensity output from the laser. The $\Delta\Phi$ voltage corresponds to the root mean square value of intensity. We can easily convert RMS values to actual change in intensity using the following:

$$\Delta\Phi = \Phi_{rms}\sqrt{2}. \tag{7}$$

We can relate voltages V_0 and supplied current I to intensity through power P knowing that

$$P = IV_0, \tag{8}$$

and

$$\Phi = \frac{P}{A}. \tag{9}$$

Here, A is area. Note that because the geometry of our system never changes and that θ_0 depends on a ratio of intensities, we can ultimately ignore any constant terms such as the area.

4 Apparatus & Procedures

Our basic apparatus is shown in Fig. (1). We began trials with a red HeNe laser with a nominal wavelength of 650nm and a 10cm long rod of SF-59 glass for our sample material. This HeNe laser was connected to a 4V power supply; in this case, a TeachSpin Power Audio Amplifier (PAA1-A). The 1370-tun, 15cm, copper wire solenoid was attached to a Volteq variable DC power supply. The laser light will travel through the solenoid, glass rod, and polarizing analyzer filter. Then, intensity of the laser light will be captured via a photodiode sensor with adjustable resistor, and read as voltage from an Agilent digital multimeter (34401A). Subsequent trials replaced the glass rod with a 25cm long tube filled with distilled water as well as exchanging the red laser with either a green laser or a violet laser, all in various combinations.

When moving the glass rod from the solenoid, a long, soft swab was used to prevent the surfaces from becoming scratched. Any marring to the glass' surface could cause unwanted effects, such as reflection and thus introduce error.

The adjustable resistor on the photodiode was used to ensure that the photodiode was not over saturated. The max output voltage of the photodiode was around 0.3V, so the resistor was set such that laser would not cause saturation.

The solenoid can be damaged by heat and so its driving current must be carefully managed. The maximum continual operating current of the solenoid is 3A, however we ran it at that current for no

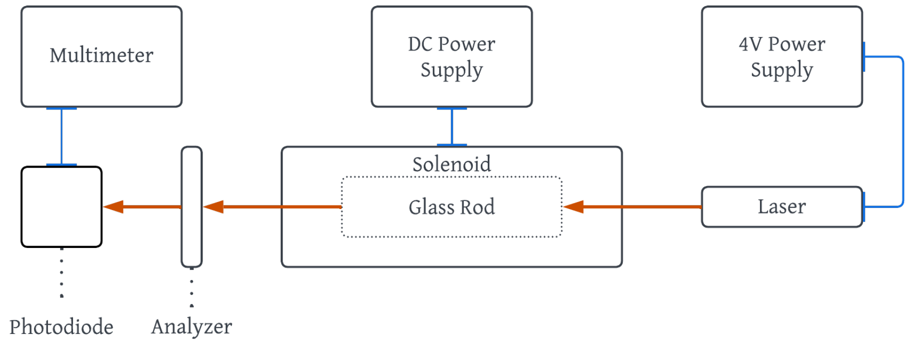

Figure 1: Experimental apparatus for extinction and to-45 methods (DC current). A laser passes light though the sample, a SF-59 glass rod, located within a solenoid. The light then traverses a polarizing analyzer filter before being detected by a photodiode.

more than thirty seconds at a time. The magnetic field produced by a solenoid at a given current is dependent on the temperature of the solenoid. For measurements at lower currents, we would often run the solenoid for several minutes at that current to equilibrate its temperature, preventing temperature drift during measurements from affecting the results. At higher currents, care was taken to measure values quickly and give time for the solenoid to cool down.

4.1 Extinction Method

The extinction method relies on finding an angle perpendicular to (90° from) the initial polarizer angle. First, we applied no current to our solenoid so magnetic field would be zero. Next, we found the angles corresponding to the maximum and minimum output. We then adjusted the polarizer angle such that a minimum output voltage was achieved. In our case, this was 95°. Next, we increased the current supplied to our solenoid in 0.50A steps, and again adjusted the analyzer angle to minimize intensity. This process was repeated for every 0.50A increment between 0.00A and 6.00A.

4.2 To-45 Method

Instead of finding angles of extinction, the "to-45" method seeks an angle such that the output voltage from the system with a magnetic field applied matches that with no magnetic field. To do this, we first found the angles corresponding to the maximum and minimum output. Then, we set the analyzer to 45° with respect to the initial polarizer angle. In our case, this was 50°. Without applying any magnetic field, we then measured the output voltage. Like before, we then increased the current in our solenoid by 0.50A. However, instead of adjusting our polarizer to find minimum intensity, we adjusted the angle such that the output matched our output voltage corresponding to no magnetic field. This process was also repeated for 0.50A steps up to 6.00A.

We expect the to-45 method to yield higher accuracy results. This is because we are matching to a value that corresponds to the highest rate of change of attenuation. Thus, any change in angle away from the ideal will cause a much more noticeable deviance from the target voltage value.

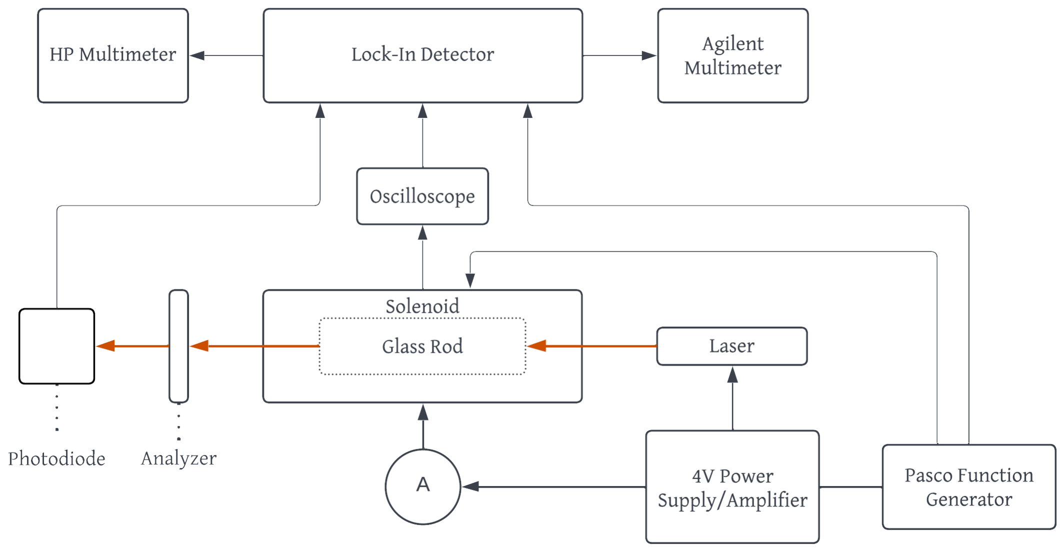

Figure 2: Setup for Lock-In detection methods (AC Current). The laser, solenoid, polarizing analyzer filter, and photodiode are unchanged from the DC current apparatus, however a Lock-in detector has replaced the multimeter connected to the photodiode. Additionally, an AC current provides powers the solenoid rather than a DC power supply.

4.3 Lock-In Detection Method

In this portion of the lab, we supplied an AC voltage to our solenoid. To do this, a Pasco digital function generator was connected to our solenoid and power supply. An ammeter was added between our power supply and solenoid so that current could be monitored. Our solenoid was also connected to an oscilloscope. This allowed us to see the waveform of our magnetic field and adjust amplification to prevent distortion. The oscilloscope, photodiode, and function generator were all connected to a Stanford Research Systems lock-in detector (SR530). Outputs of our lock-in detector were fed through a Hewlett Packard multimeter (3478A) and an Agilent Digital multimeter (34401A), as shown in Fig 2.

Instead of measuring the angle of rotation "manually", we calculate it from a ratio of intensities as shown in Eq. (5). Utilizing a lock-in detector, we measured output voltages corresponding to intensities Φ_0 and $\Delta\Phi$ shown on the Agilent multimeter and HP multimeter, respectively. Trials were conducted for various combinations of laser color and sample material: red with glass, red with water, green with glass, green with water, and purple with glass.

4.4 Measuring the Magnetic Field

To accurately model the fringe effects of the magnetic field, we performed a special trial using the solenoid and a Hall probe. We ran a current of 2.00A through the solenoid and waited until its temperature reached an equilibrium, $(40.5 \pm 0.5)°$C in our case. Then the Hall probe, which was placed at the end of a stick long enough to reach through to the middle of the solenoid, was placed throughout the solenoid to characterize its magnetic field. The Hall probe also measured the magnetic field several centimeters beyond the edge of the solenoid. Since the solenoid is effectively symmetrical, data was only taken for one half of the solenoid.

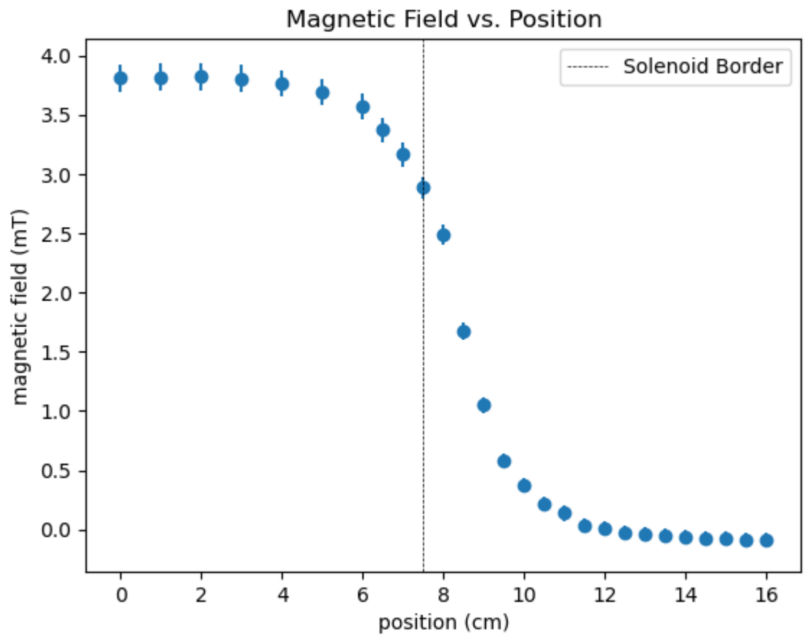

Figure 3: The magnetic field strength as a function of the position from the solenoid. The origin is defined to be at the centre of the solenoid and outward is in the positive direction. A vertical line is placed at $(7.5.0 \pm 0.1)$cm, the edge of the solenoid.

5 Results

5.1 Magnetic Field and Solenoid Fringe Effects

As seen in Fig. (3), the magnetic field was roughly constant at (3.81 ± 0.11)mT near the centre of the solenoid before it quickly decayed to zero as it reached beyond the edge of the solenoid at (7.5 ± 0.1)cm. Since (0.0 ± 0.1)cm was defined to be the centre of the solenoid, the magnetic field was symmetric about the y-axis.

The glass rod was placed at the centre of the solenoid where fringe effects were minimal. As such, the value for the magnetic field B was taken to be the constant value of (3.8 ± 0.1)mT. The water-filled glass rod extended 5cm beyond the edge of the solenoid so taking the magnetic field to be constant is a poor approximation. The magnetic field is linearly related the Verdet constant, so averaging the magnetic field over the glass rod's position will yield an accurate value. The average magnetic field from (0.0 ± 0.1)cm to (12.5 ± 0.1)cm was found to be (1.55 ± 0.08)mT.

The Hall probe used to find the magnetic field caused the majority of the error, yet the error remains small. Each variable in Eq. (5) has an associated error, however all are at most a few relative percent and thus they contribute only a small amount of error. Standard propagation of errors was used to find the total error in magnetic field. The values for the errors of the DRV 5055 A2 are supplied by the manufacturer.

5.2 DC Manual Detection Findings

Using the extinction method, the Verdet constant was found to be $(29 \pm 2)\text{radT}^{-1}\text{m}^{-1}$ and the to-45 method's Verdet constant was $(26 \pm 3)\text{radT}^{-1}\text{m}^{-1}$. Both methods used the 650nm red laser. Fig. (4) shows the polarization angle versus the magnetic field for the extinction method and Fig. (5) shows the same for the to-45 method. The Verdet constants were obtained through Eq. (2), by

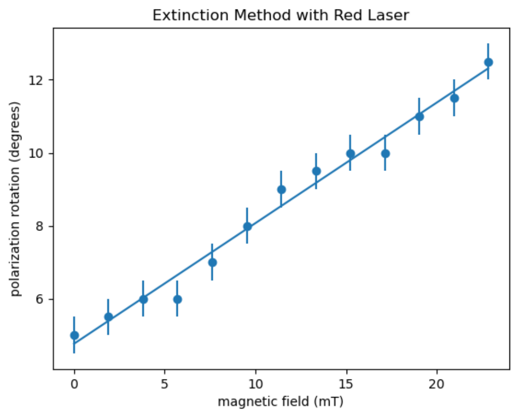

Figure 4: Polarization angle versus magnetic field strength for the extinction method. The data has been overlaid with a best-fit line.

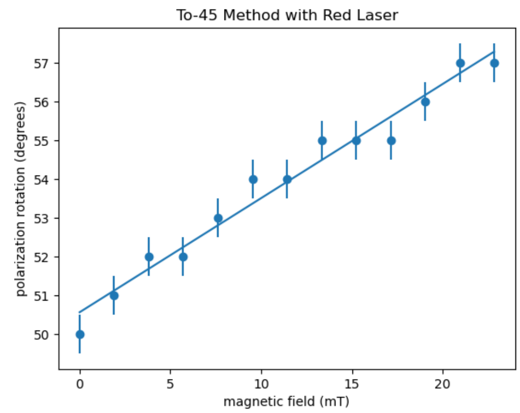

Figure 5: Polarization angle versus magnetic field strength for the to-45 method. The data has been overlaid with a best-fit line.

using the slope of the best fit line of polarization angle versus magnetic field and L, the length of the glass rod.

While the slope of the best-fit line was used to find the Verdet constant, the line's y-intercept is irrelevant. The laser's pre-polarized light and the polarizer used in the experiment were not perfectly aligned. In other words, there was a relative angle between the laser's pre-polarized light and the polarizer's measured angle. This is the cause of the non-zero intercept. The data could be easily transformed to have a zero intercept by subtracting the relative angle from the measured angle. Fig. (4) and Fig. (5) both have a relative angle of $(5 \pm 1)^\circ$. This transform would lead to a polarization angle at (0.0 ± 0.1)cm of $(0 \pm 1)^\circ$ and $(45 \pm 1)^\circ$ for the extinction and to-45 methods respectively.

The error associated with the Verdet constant has three main sources: the polarizer, ambient light, and the magnetic field.

The first of the three main sources of error, the polarizer, was loose and tended to slide which caused inaccurate readings. The polarizer only had tick marks every 5° and thus lacked precision. This lack of precision is evident in Fig. (4) and Fig. (5) where the polarization angle's resolution is no better than a single degree. Both of these sources are random errors.

While a piece of felt was used to protect the photodiode from ambient light, this method was inconsistent and lacking. Even with the felt, intensity readings would fluctuate. Furthermore, the placement of the felt affected the results and it needed to be moved after every measurement in order to reach and adjust the polarizer. The ambient light itself causes systematic error that would increase the measured intensity but the placement of the felt caused random error.

The final major source of error would be the magnetic field caused by the solenoid. While the Hall probe measurements themselves are accurate and precise, the magnetic field produced by the solenoid is dependent on its temperature. The temperature of the solenoid would drift as it was run at high current. The magnetic field produced is inversely proportional to its temperature, and so this introduced a systematic error that would increase the Verdet constant. Since the bore diameter of the solenoid was small, a thermometer could not be present within the solenoid concurrently to performing the experiment without blocking the laser.

While the determined values for the Verdet constant are not within uncertainty of the values found through the AC method in the following section, the aforementioned sources of error are not captured within the uncertainty since they cannot readily be quantified. Fortunately, these three sources of errors are eliminated in the AC Lock-in trials.

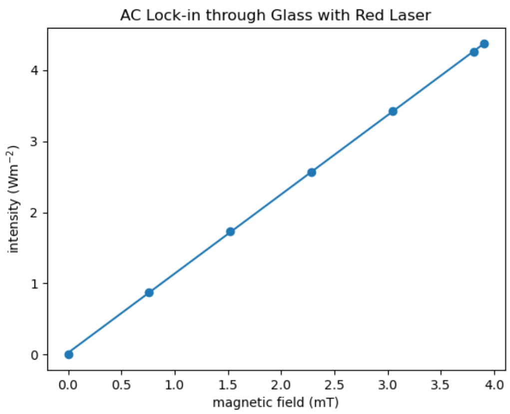

Figure 6: Intensity versus magnetic field for the red laser through glass. The data is overlaid with a best-fit line.

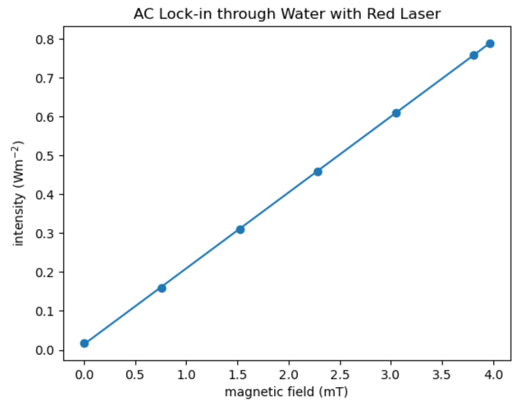

Figure 7: Intensity versus magnetic field for the red laser through water. The data is overlaid with a best-fit line.

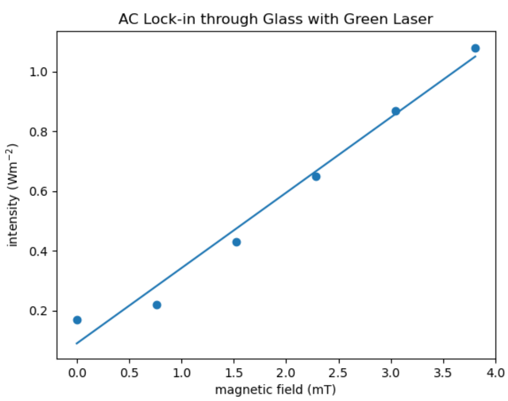

Figure 8: Intensity versus magnetic field for the green laser through glass. The data is overlaid with a best-fit line.

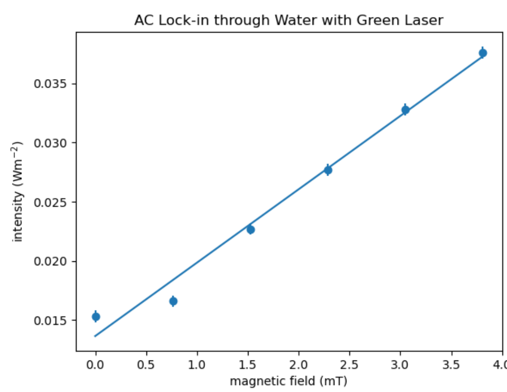

Figure 9: Intensity versus magnetic field for the green laser through water. The data is overlaid with a best-fit line.

5.3 AC Lock-In Findings

As seen in Fig. (6) and Fig. (7), the AC Lock-in detection found that with a red laser the Verdet constant through the glass rod was $(23.75 \pm 0.05)\mathrm{radT^{-1}m^{-1}}$ and for the water tube was $(3.34 \pm 0.03)\mathrm{radT^{-1}m^{-1}}$. Likewise, in Fig. (8) and Fig. (9), the green laser had a Verdet constant in the glass rod and in water of $(7.7 \pm 0.6) \times 10^{-1}\mathrm{radT^{-1}m^{-1}}$ and $(7 \pm 1) \times 10^{-3}\mathrm{radT^{-1}m^{-1}}$ respectively. Finally, as per Fig. (10), the Verdet constant for the violet laser through the glass rod was found to be $(1.14 \pm 0.06) \times 10^{-1}\mathrm{radT^{-1}m^{-1}}$.

The elicited Verdet constants appears to follow the trend where it is proportional to the wavelength. The red laser, which had a wavelength was 650nm, had the greatest Verdet constant in both glass and water trials whereas the 405nm purple laser displayed the smallest Verdet constant. With only three glass rod trials and two water trials, the exact relationship between the Verdet constant and wavelength per material could not be determined; more data points are required to determine statistical significance.

As mentioned previously, the errors in the AC Lock-in are small compared to the manual DC

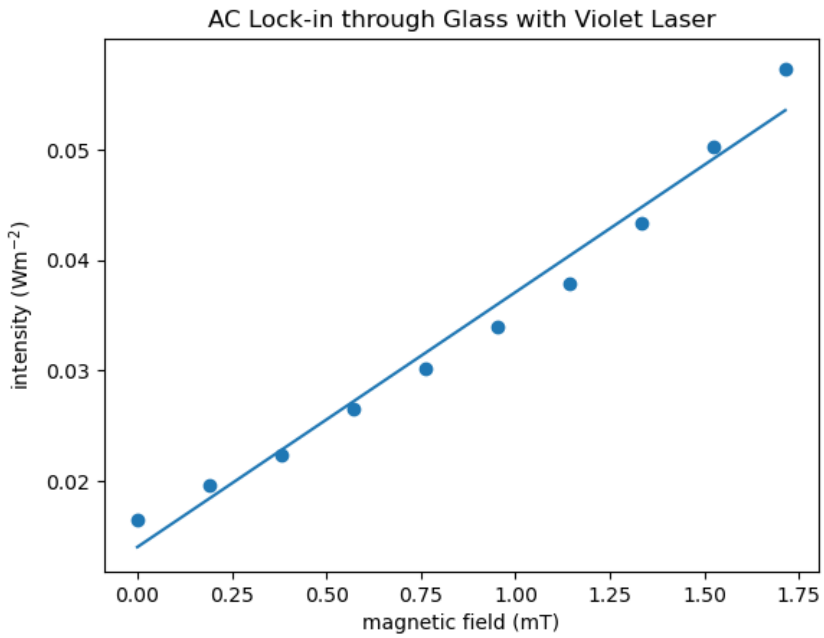

Figure 10: Intensity versus magnetic field for the violet laser through glass. The data is overlaid with a best-fit line.

methods. This part of the experiment used the same polarizer as the previous portion, however the lack of granularity of the polarizer was not an issue due to using a precise multimeter to measure intensity of the laser. While the experiment was not performed in a location that lacked ambient light, the Lock-in detector's lock-in frequency did not match nor was harmonic with the ambient light's frequency. As such, ambient light had minimal effect. Finally, the solenoid was run at comparatively lower currents than in the manual trials and so drifting strength of the magnetic field due to the solenoid's temperature had minimal effect.

A notable source of error arose during the green laser trials. As seen in Fig. (8) and Fig. (9), the data points nearest to the y-intercept do not follow the linear trend present throughout the rest of their trials. Rather, the leftmost data points are roughly equal in intensity as their nearest data points despite and equal step size in magnetic field. The cause of this discrepancy is believed to be a faulty ammeter. The ammeter was capable of measure most values but encounter issues at low currents. The ammeter was replaced, fixing the issue in further trials. Regardless, the discrepant data point increased the value of the slope and thus increased the measured Verdet constant. The data point was nevertheless included due to the already small number of data points.

As a result of the odd behaviour observed during the green laser trials, additional data points were recorded for the violet laser trial. This decision revealed that the trend was not linear, as seen in Fig. (10), instead it appears to have a positive slope. This is likely caused by approximations made in the derivation for Eq. (6) where a sinusoidal function is taken to be roughly linear [4]. It appears that this approximation is not suitable for the wavelength of the purple laser, resulting in a non-linear dependence.

6 Conclusion

Faraday rotation was successfully observed, as was a trend between the Verdet constant and wavelength for a given material. In a sample of SF-59, the Verdet constants were found to be $(23.75 \pm 0.05)\mathrm{radT^{-1}m^{-1}}$ for the red laser, $(7.7 \pm 0.6) \times 10^{-1}\mathrm{radT^{-1}m^{-1}}$ for the green laser, and $(1.14 \pm 0.06) \times 10^{-1}\mathrm{radT^{-1}m^{-1}}$ for the violet laser. In the sample of distilled water, the Verdet constants were found to be $(3.34 \pm 0.03)\mathrm{radT^{-1}m^{-1}}$ for the red laser and $(7 \pm 1) \times 10^{-3}\mathrm{radT^{-1}m^{-1}}$ for the green laser.

If improvements were to be made to this experiment, two points are readily identified. First, a greater number of data points should be taken per trial in order to clearly resolve any interesting behaviour that may arise. Second, a greater variety of wavelengths should be tested in order to quantifiably determine the relationship between the Verdet constant and wavelength.

References

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