The Stirling Cycle: Investigated the Real Stirling Cycle against the Idealised Stirling Cycle

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Abstract

The experiment consisted of two stages. The first involved the change of volume, ΔV, and pressure, Δp, which were both measured with respect to time, t, of a Stirling cycle heat engine system. The first stage was further divided into two sets of trials. One set of trials ran the Stirling engine without modification, and the second set of trials added a friction force to the system. A method of Reimann sums was used to determine the area of the p versus V diagram to extract the work, W, performed by the engine, and by using the time taken for one cycle to complete, the power, P, could be computed. The observed p versus V graph agrees with the expected shape of a real diagram, but differs from the simple idealised model; adding a friction force had the expected effect of increasing the work done per cycle but reducing the engine's power output. The second stage of the experiment was likewise divided into two trials, and involved applying an external force to the engine in order to drive a temperature difference. The two parts to this stage involved running the engine in the forward and backward directions.

Introduction

A heat engine is a system that uses a difference in temperature between a reservoir of heat and a heat sink to generate work. Heat is extracted from the reservoir and moved to the sink, and work is extracted by the exchange, as seen in the simple Carnot cycle in Figure 1. An idealised Stirling cycle can be divided into four components, as seen in Figure 2: segment 1 to 2 is an isothermal addition of heat, segment 2 to 3 is an isochoric removal of heat, segment 3 to 4 is the isothermal removal of heat, and segment 4 to 1 is an isochoric addition of heat. The region enclosed by a p versus V diagram is the work done by the system per cycle, and so by recording how p and V change over time as heat moves between the reservoirs, work can be determined. The Stirling cycle is a heat engine that is reversible, meaning that when supplied with mechanical power it becomes a heat pump as it uses the supplied work to move heat between the two reservoirs. Thus, supplying mechanical energy to the system will drive a temperature difference between the two reservoirs. This experiment seeks to observe the difference between the ideal Stirling cycle and examples of real Stirling cycles as well as to investigate the Stirling cycle, by adding a friction force and by supplying mechanical energy rather than heat from a reservoir.

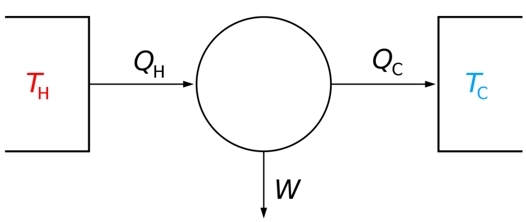

Figure 1: a simple heat engine

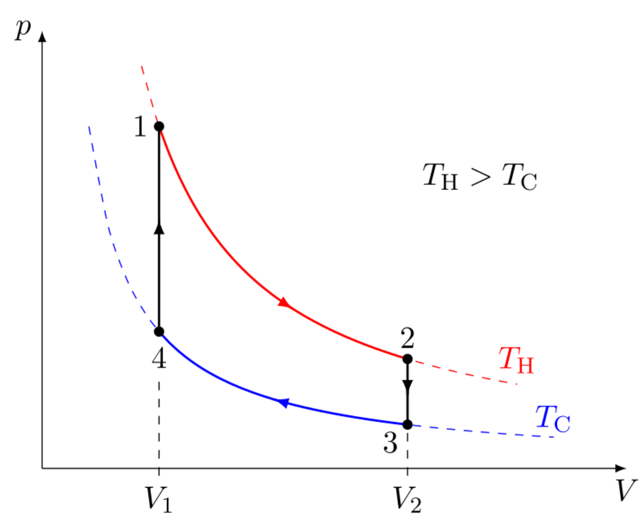

Figure 2: an idealised Stirling engine

Theory

The infinitesimal work done, W, by a infinitesimal change in volume, V, can described as

$$dW = pdV \tag{1}$$

To remove the infinitesimal prefix, this expression can be integrated to get

$$W = \int_{V_i}^{V_f} p\,dV \tag{2}$$

One can approximate the area enclosed by the graph by using a large number of discrete bins. In doing so, the continuous integral is exchanged for a summation over a finite number, where the height of each bin is multiplied by its width. On a graph of pressure, p, versus volume, V, the width of the bin becomes the region over V that is considered and the height becomes the pressure at that point. As seen in Figure 2, the area enclosed by the graph can be described using the pressure at the top, p_t, minus the pressure at the bottom, p_b, for each bin. Thus, the work can be approximated as

$$W \approx \sum_{i=1}^{N} (p_{t,i} - p_{b,i}) V_i \tag{3}$$

where V_i is the width of the i-th bin. This approximation approaches the true value as N, the number of bin, reaches infinity. This will return the work done per cycle of the engine. As such, the power output, P, can be found by dividing the work done per cycle by the period, T, of the cycle.

$$P = \frac{W}{T} \tag{4}$$

The ideal gas law states that

$$\frac{pV}{T} = nR \tag{5}$$

where n the number of moles, R is the gas constant, and T is temperature. The ratio of p and V over T is constant. During segments II and IV, where the Stirling cycle is isochoric and thus volume doesn't change, the work converts a change in pressure to a temperature difference.

Experimental Procedure

In the first stage of the experiment, a beta style Stirling engine was placed atop a cup of boiling water; the water acted as the hot reservoir, and the environment acted as the cool reservoir. Once the Stirling engine's piston had sped up and reached an equilibrium speed, data of time, change in pressure, and change in pressure were recorded. The flywheel had 100 black stripes, and a camera was used to count their passing; one revolution of the flywheel, thus one period of the piston, corresponds to a count of 100 stripes. The count of stripes was converted to an angle of the percent rotation of the flywheel, and that was converted into the height of the piston. The height of the piston was then used to obtain the change in volume as the cross-sectional area of the piston was previously known. The change in pressure was obtained using a barometer that compared the pressure within the mug to the ambient pressure outside the mug in order to obtain a change in pressure. These values were recorded alongside time using an oscilloscope. The friction trials used the same experimental procedure, except a small piece of foam was lightly held in place against the Stirling engine's flywheel. Both trials recorded data over a period of half a second, which allowed the Stirling engine's flywheel to perform multiple complete revolutions. The friction trial was performed first and the no-friction trial was performed immediately after so that the loss of heat from the poorly-isolated mug was minimised.

The second stage of the experiment enclosed the Stirling engine within foam insulation to reduce the effects of environmental changes in temperature. A hand drill equipped with a foam bit was placed against the Stirling engine's flywheel to introduce mechanical energy by forcing the flywheel to spin. A pair of k-type thermocouples were attached to the upper and lower plate of the Stirling engine, which acted as the hot and cold reservoir, and was used to record the temperature difference between the plates. The drill induced spin in the flywheel until the system reached an equilibrium, then the direction of the drill's rotation was reversed and the system was allowed to reach equilibrium once more. The hand drill was turned off and the system was allowed to reach equilibrium with the environment. The difference in temperature and time were recorded by a multimeter.

The oscilloscope and multimeter used in stage one and two of this experiment were connected to LabView, which outputted text files containing all data values. These text files were processed in Python, with which all calculations performed and all plots created.

Results & Discussion

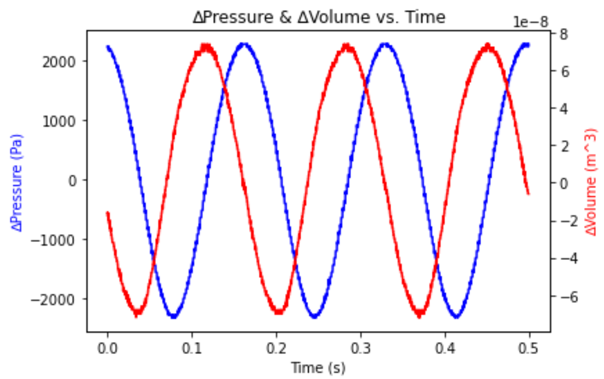

Figure 4: Δp (blue) and ΔV (red) versus t of the no-friction trial

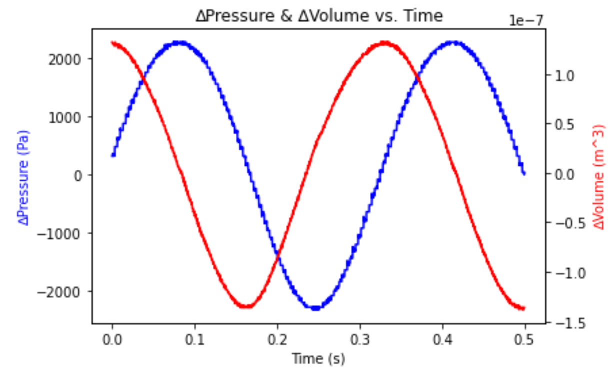

Figure 5: Δp (blue) and ΔV (red) versus t of the friction trial

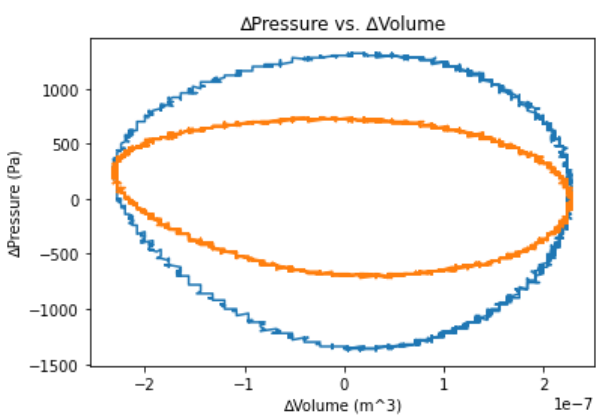

Figure 3: a Δp vs. ΔV diagram with (blue line) and without (orange line) friction

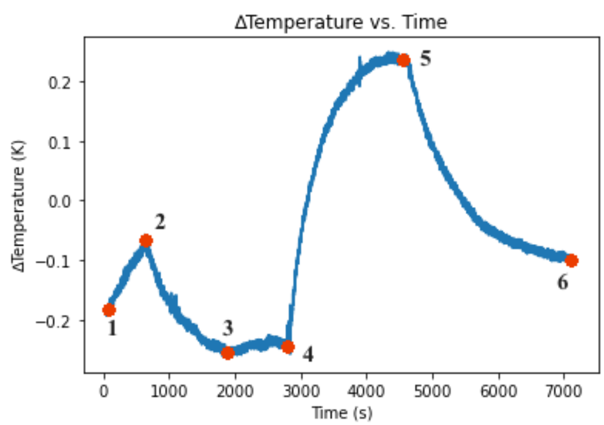

Figure 6: a ΔT vs. t diagram when mechanical energy is supplied to the flywheel

For Stage one of the experiment, both trials (no-friction and friction) had data recorded over 0.50 s. The no-friction trial completed 2.9 cycles and the friction trial completed 1.5 cycles, which respectively correspond to periods of 0.17 s and 0.33 s. The no-friction trial performed 0.50 mJ of work and had an output of 3.0 mW, whereas the friction trial performed 0.93 mJ of work and had an output of 2.8 mW. This corroborates Figure 3, which shows that the friction trial encloses a greater area as the engine spends energy to overcome the added friction force, and Figures 4 (the no-friction trial) and 5 (the friction trial) that display the longer period of the friction trial. Notice that both the no-friction and friction trials depicted in Figure 3 differ in several ways from the ideal Stirling cycle as seen in Figure 2, namely segment 1 to 2 and both of the isochoric segments (2 to 3 and 4 to 1). The two isochoric segments should be vertical (i.e. volume does not change) and the top isothermal segment should be concave up rather than concave down. Since the characteristic p versus V diagram is the result of two sinusoidal, from which it is impossible to produce the sharp edges required by the ideal cycle. As a result, the real p versus V diagram has only very limited regions of the curves where the system is either isothermal or isochoric. While both trials were subject to friction from the flywheel as well as the piston, this would affect both trials similarly and would be insufficient to be the cause of the discrepancy between the ideal and the real Stirling engine. A potential source of error between the trials would be the loss of heat from the hot reservoir, however since the trials were performed in quick succession and water has a high specific heat capacity, this source is negligible.

Figure 6 can be split into five segments corresponding to the six points: segment 1 to 2 is the system reaching an equilibrium temperature with the ambient air, segment 2 to 3 corresponds to when the drill was powered and a temperature difference was driven between the plates, segments 3 to 4 is when the Stirling engine reaches an equilibrium when the drill is run, segments 4 to 5 is when the direction of the drill is reversed such that the flywheel spins in the opposite direction, and segment 5 to 6 is the system returning to ambient equilibrium after the drill had been turned off. Figure 6 shows a clear difference in temperature between the reservoirs, demonstrating that supplying the system with work in the form of mechanical energy can run the Stirling engine in reverse, in a refrigeration cycle. As demonstrated in segment 2 to 5 of Figure 6 by the clear reflection about a ΔT of zero, the direction of the flywheel's spin dictates the direction of flow of the heat between the engine's heat reservoirs. The major source of uncertainty within this stage of the experiment would be the fluctuations in the ambient air temperature. Despite the foam insulation used to try to isolate the Stirling engine, the effects of air temperature fluctuations can be seen, particularly in the segment 3 to 4. These fluctuations alone are too small to alone account for the drastic shift in reservoir temperature difference when the drill is run and when it is reversed, thus it can still be concluded that the mechanical energy is the cause of the temperature difference.

Conclusion

The work done by a Stirling engine is greater when a friction force is applied, no-friction yielded 0.50 mJ versus the friction yield of 0.93 mJ; yet the power output is lower, where the no-friction had an output of 3.0 mW compared to the friction output of 2.8 mW. Similarly, the period of the no-friction trial was 0.17 seconds whereas the friction trial had a period of 0.33 seconds. Neither of these trials lead to a p versus V diagram of the same form as the idealised cycle as seen in Figure 2. The two sinusoidal waves of change in pressure and change in volume are smooth functions and, when combined, are unable to form the sharp corners that form the boundaries between the isothermal and isochoric segments of the idealised Stirling cycle.

Figure 6 shows that a Stirling cycle can be put into a refrigeration cycle, the heat engine can be reversed so that when mechanical energy is supplied to the system, a temperature difference is created between the reservoirs. The refrigeration cycle can be reversed causing the direction of flow of heat between the reservoirs can be reversed.

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