# Reconfirming the Mean Lifetime of Muons

*PHYS 441*

Skyler E Alderson [redacted]

## 1 Abstract

This experiments seeks to reconfirm the mean lifetime of the muon, an elementary particle. By measuring the time differences between muons entering our apparatus and their decays, a histogram of decay times can be created. From this data, the mean lifetime can be extracted. The mean lifetime of a muon was found to be $(2.23 \pm 0.01)\mu$s.

## 2 Introduction

Muons are elementary particles with the same charge and spin as an electron but are significantly more massive. Muons were first discovered 1936 when Anderson, C. and Neddermeyer, S. observed a particle move through a magnetic field at a trajectory similar to but less curved than that of an electron [1]. The discovery was then corroborated in 1937 by Street, J. and Stevenson, E. who observed the particle in a cloud chamber [2].

After the discovery of a material, be it a metallic compound or a particle, research teams will attempt to measure its fundamental properties, allowing the material to be characterised. Without knowing this characterisation of the material, we would never know its full breadth of uses. Even well after the material has been documented, teams will continue to quantify its properties in hopes of validating previous findings as well as make more precise measurements. This experiment seeks to continue that tradition and confirm the measurement of the mean lifetime of a muon.

A prominent source of muons are from when a cosmic proton, mostly those created from the sun, strikes a nucleus in the upper atmosphere [3]. This creates a pion which decays after a mean travel time on the order of a meter and become a muon and a muon neutrino. While the distance between the upper atmosphere and the ground is two orders of magnitude larger than what the expected lifetime of a muon would allow it to travel, the muons' relativistic speeds cause length contraction, allowing them to reach the surface. Our experiment will be measuring the decay times of the muons created by cosmic protons.

## 3 Theory

### 3.1 Muons

The lifetime and decay of muons is governed by the radioactive decay formula. If $N$ is the number of decay events and $t$ is time, then

$$N(t) = -\frac{dN}{dt}. \tag{1}$$

The solution to the differential that is Eq. (1) is an exponential and takes on a similar form to the Poisson distribution. The equation is solved to find

$$N(t) = N_0 e^{-\frac{t}{\tau_\mu}}, \tag{2}$$

where $N_0$ if the initial number of particles and $\tau_\mu$ is the mean lifetime of a muon. We can transform Eq. (2) into a linear equation by taking the natural logarithm of both sides,

$$\ln(N(t)) = -\frac{1}{\tau_\mu} t + \ln(N_0). \tag{3}$$

The slope of Eq. (3) is $m = -\frac{1}{\tau_\mu}$ and the intercept is $b = \ln(N_0)$. By plotting the natural logarithm of counts as a function of time, one may extract the slope in order to find the mean lifetime of a muon.

### 3.2 Scintillator

A scintillator is a piece of equipment that fluoresces when exposed to high energy particles, such as a muon. If the scintillator is covered by light-blocking cloth or other material, it becomes isolated from light and so can only be fluoresced by particles, such as muons, which have a large penetration depth.

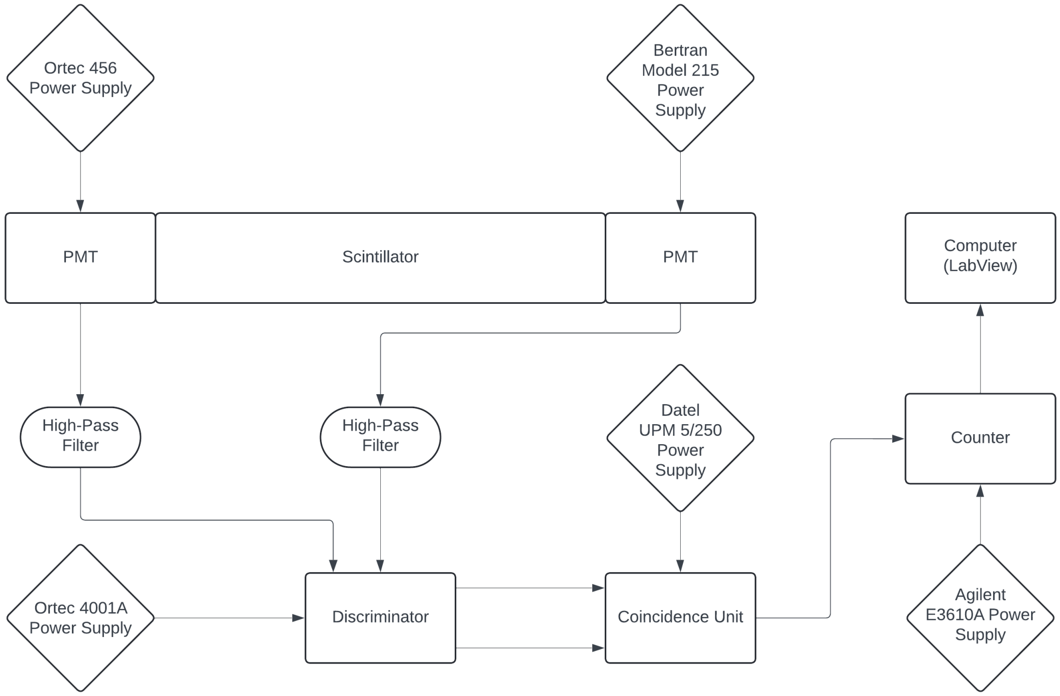


Figure 1: The apparatus used to detect muons. The scintillator detects the capture or decay of a muon by releasing photons which are amplified in the PMTs. The PMTs each output a signal through a high-pass filter before it reaches the discriminator. The discriminator then passes the signals to the coincidence unit then onto the counter. Finally, the signal is captured and recorded by LabView.

There are two types of events that can causes fluorescence from a muon: a muon entering the scintillator, which we call a capture event; and a muon decaying within the scintillator, which we call a decay event. The photons created by both of these events are able to travel to the ends of the scintillator and be measured by other equipment.

### 3.3 Photo-Multiplier Tubes

Photo-Multiplier Tubes (PMTs) are used to amplify the signal from a photon. When a photon strikes the photocathode, it ejects electrons. These electrons are accelerated by a large voltage difference and strike a dynode, releasing more electrons. These new electrons are once again accelerated by a large voltage difference, striking a new dynode. This process repeats until the electrons reach the anode; since each dynode has multiplied the number of electrons, this creates an amplified signal.

PMTs allow weak signals, such as the low energy photons emitted by the capture or decay of muons in a scintillator, to be greatly amplified. Without such equipment, electronics which rely on a LO signal of 0 volts and a HI signal of 5 volts would be entirely unable to register such otherwise weak signals.

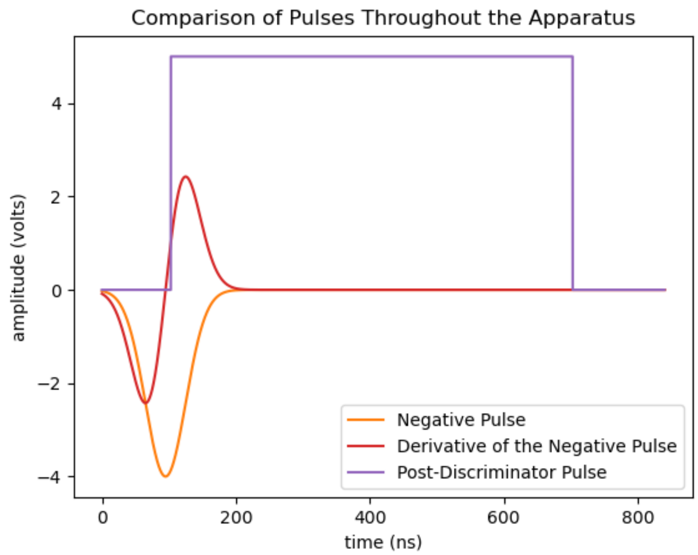


Figure 2: An amplitude versus time plot of the three predominant types of pulses present within the apparatus. The orange curve represents the signal from the PMT. The red curve represents the signal after the high-pass filter. The purple curve represents the signal sent by the discriminator; the purple curve reaches its HI value after the red curve reaches the positive threshold voltage. The values shown are not accurate, rather they are for demonstrative purposes.

# 4 Apparatus

To detect muons, a scintillator is used. When a muon enters the scintillator or when a muon decays within the scintillator, it will release photons. These photons have insufficient energy to be easily measured so a pair of PMTs are used to amplify the signal. Each PMT has its own power supply but the voltage has been chosen such that the amplitude of both signals produces are comparable; both voltages are near 2.1kV. The PMT on the left of Fig. (1) is supplied by a Ortec 456 power supply and the PMT of the right is supplied by a Bertran Model 215 power supply.

The PMTs outputs a small negative pulse whose amplitude is on the order of a few hundred millivolts. The later circuitry requires a positive signal that ranges from zero to five volts. To map the pulses to that range, we first pass the outputs of the PMTs through high-pass filters. In effect, high-pass filters take the derivative of the input signal; thus, a negative input signal corresponds to a negative and positive pulse, each at half the width. After the high-pass filter, the signals go into a discriminator. Discriminators detect any positive pulse which is greater than some threshold voltage and output a square pulse with a five volt amplitude. This new pulse from the discriminator is significantly wider than the input pulse. The difference in the these three pulses can be seen in Fig. (2). The high-pass filters do not need a power supply and the discriminator is supplied by a Ortec 4001A power supply.

The scintillator and PMTs are very sensitive devices, so unintentional firings of the PMTs are inevitable. Among the ways of reducing the occurrences of false positives is cloth and casing draped over the apparatus. The apparatus has two PMTs thus two signals which are also used to ensure that only true events are detected. Both events, those being muons capture and muons decay, will emit photons spatially uniformly. As such, if both PMTs emit a signal simultaneously, then it very likely corresponds to a true event. To detect this simultaneity, a coincidence unit is used. When this unit receives an signal at one of its inputs, it will listen on the other input for a second signal within some small time window. If within that window the second channel receives an input, it will output a signal; if it does not receive a second signal, the coincidence unit does nothing. This unit is used to correlate the events detected from both PMTs; the timing window is required due to propagation delay. If a muon does not enter at the exact centre of the scintillator, the photons will not reach the PMTs at the same time; if the cabling from each PMT has different lengths, the signals will not reach the coincidence unit at the same time. This experiment did not have access to a proper coincidence unit and so a NAND gate was used. NAND gates–or equivalently for this purpose, an AND gate–will only output if it receives a signal at both of its inputs, just like a coincidence unit. While a NAND gate lacks a timing window, the pulse extension caused by the discriminator compensates by increasing the likelihood that the signals overlap. The combination of a NAND gate and the lengthened pulse from the discriminator acted like an ad-hoc coincidence unit. The NAND gate was supplied by an Agilent E3610A power supply.

A principle challenge of detecting muon decays with this apparatus is determining which observed events correspond to decay events. The two events detected by the scintillator and PMTs–muon capture and muon decay–produce identical signals. We know that if a muon were to decay within the scintillator, that it would happen within a few microseconds (within its expected lifetime) of the muon entering, thus we would be looking for two sequential events occurring in such a time frame. The counter, the final logical step of the apparatus, will only increase the count if it receives two signals in a very short time span, thereby only measuring decay events. Upon receiving such a signal, it will record the difference in time between the capture event and the decay event. The counter was supplied by a Datel UPM 5/250 power supply.

Finally, a digital acquisition device was used to record the data to LabView where it was later exported and analysed using Python. The data exported was a list of decay times in microseconds for valid decay events; that is, the time between the muon entering the scintillator and the muon decaying.

## 5 Procedure

The procedure to collect data is simple. After ensuring all the connections are correct and that power is being supplied, wait. Once the apparatus has been setup, it requires neither additional action nor supervision. We collected data for five weeks, checking in every week to inspect the data. The exception of this was the fourth week, where we let the experiment run unchecked for two weeks. The precision of this experiment is directly related to the number of counts it detects, thus the length for which the experiment is left to run.

While the experiment was running, a Tektronix TBS1104 oscilloscope was used to inspect the signals. By connecting it to the outputs of any combination of after the PMTs, high-pass filters, or the discriminator, the detected events could be seen. Each PMT detected on the order of five events per second. For an example of the shape of these pulses, see Fig. (2).

After the data was collected, it was combined into a single histogram. The histogram compared decay times and had a bin size of one microsecond. One microsecond was chosen since it resolved the greatest amount of detail without introducing artefacts from low counts. The natural logarithm of the counts was taken so, by Eq. (3), the system could be described by a linear equation. Then the mean lifetime of the muon $\tau_\mu$ was extracted by performing a linear fit and obtaining the value of the slope.

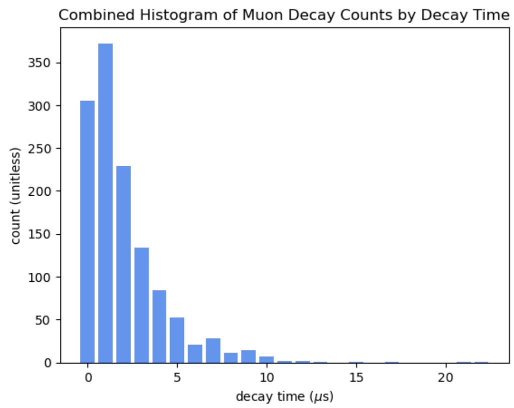


Figure 3: A histogram of decay events binned by decay times in microseconds. The bin size is one microsecond and the largest bin occurs at one microsecond. The histogram shows exponential decay, but it loses coherence near the tail.

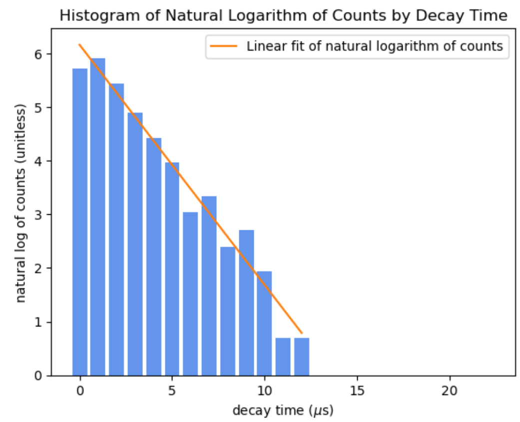


Figure 4: A histogram of the natural logarithm of decay times in microseconds. A linear trend line with slope $m = (-0.45 \pm 0.03)\mu s^{-1}$ has been fit to the data. Notice that counts of one have vanished due to the natural logarithm.

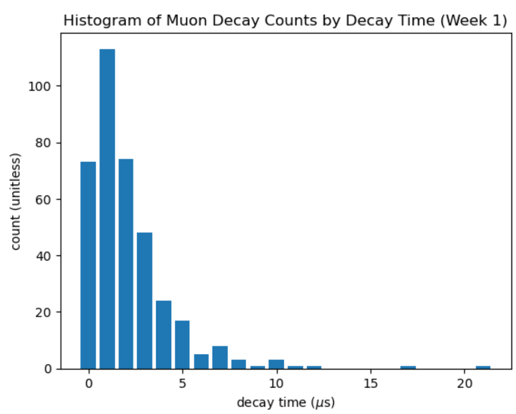


Figure 5: A histogram of decay events binned by decay times in microseconds for the first week of data.

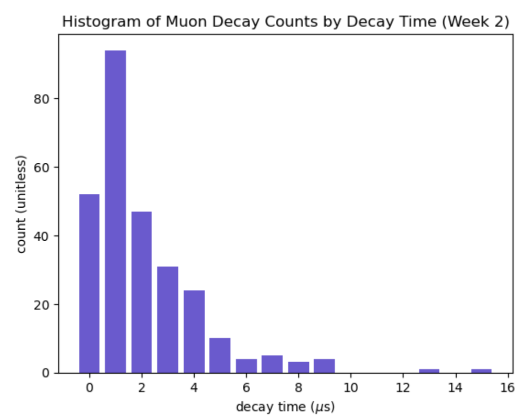


Figure 6: A histogram of decay events binned by decay times in microseconds for the second week of data.

# 6 Results

## 6.1 Mean Muon Lifetime

The mean lifetime of the muon was found to be $\tau_\mu = (2.23 \pm 0.01)\mu$s, which is not within uncertainty of the accepted value $\tau_\mu = (2.196980 \pm 0.000002)\mu$s [4]. Fig. (3) shows all measured counts whereas Fig. (4) shows the logarithm of counts with the best fit line. The error associated with $\tau_\mu$ was obtained by talking the square-root of the variance of the best-fit line's slope, as found during the linear fit; then, standard propagation of errors was used to convert it into the uncertainty on $\tau_\mu$.

An advantage of collective data in batches is that each batch may be individually inspected. Since each batch contains relatively few counts, artefacts are more visible than in the combined

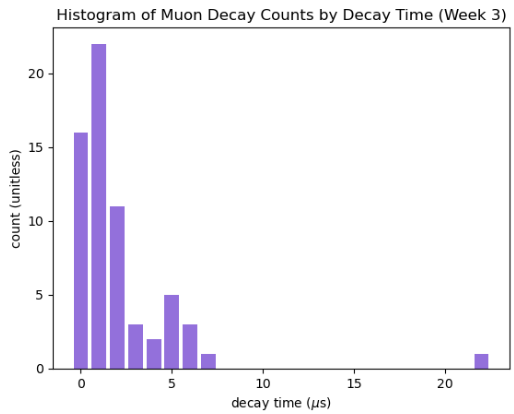


Figure 7: A histogram of decay events binned by decay times in microseconds for the third week of data.

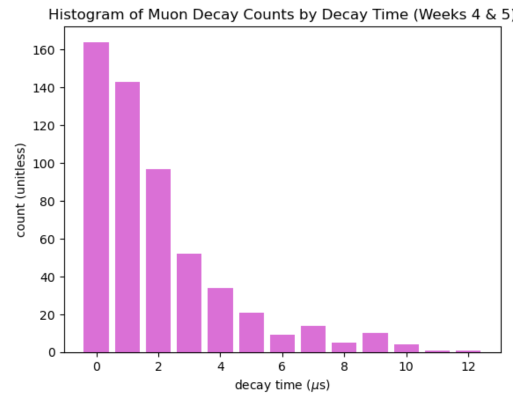


Figure 8: A histogram of decay events binned by decay times in microseconds for the fourth and fifth weeks of data.

histogram of Fig. (3). In Fig. (5), Fig. (6), and Fig. (7) there are several bins with one or two counts well beyond the cutoff of the tail; oddly enough, the trailing counts do not occur in Fig. (8) despite it overall having the greatest number of counts. These trailing counts could be caused by electronic issues, two consecutive capture events interpreted as a decay, or the unlikely event that they truly are decays. Regardless of the cause, these trailing counts account for a negligible proportion of overall counts. Additionally, counts of one in the combined histogram were ignored since they likely corresponded to outliers, so these trailing counts do not affect the results.

There is another oddity when comparing Fig. (8), the fourth and fifth weeks of data, to the rest. The bin at decay time of $0\mu$s typically ranges from half to two-thirds the height of the bin at decay time of $1\mu$s, except in the aforementioned weeks four and five where it is the bin with the greatest count. Why the fourth and fifth weeks appear to be an outlier is unclear; that said, the fourth and fifth weeks are closest to the expected curve. Given that Eq. (2) dictates this relationship should follow a decaying exponential curve, then the bin at $0\mu$s should be exponentially larger than the bin at $1\mu$s. The cause of the $0\mu$s bin being small is likely because of the discriminator's pulse width. The pulse width of the discriminator is approximately 600ns, which is about 60% of the bin's width. If the PMT were two send an event pair–a capture and a decay–separated by less than the discriminator's pulse width, then the discriminator would only output a single pulse. In other words, if a decay event occurs less than 600ns after the corresponding capture event, it is ignored. This causes the measured size of the bin at $0\mu$s to be about 40% too small. Accounting for this issue would cause the data to be significantly closer to following a decaying exponential curve.

However, that is not the sole reason why the data in the $0\mu$s bin is unexpected. As seen in Fig. (9), by choosing a bin size of $0.1\mu$s, there is a notable peak at $0\mu$s followed by bins of zero until the predicted bin of $0.5\mu$s. By the previous explanation, such data should be impossible. This data is likely due to the electronics and the edge cases of its logic, since it is located near the minimum time spans that is can process data; the counter can only measure values with a resolution of $0.1\mu$s. Additionally, such analysis performed by taking increasingly small bins should be heeded with caution, as artefacts from the random distribution of decay times become predominant.

A final note to make about the individual batches of data is the particularly small set of counts during the third week. Its largest bin is nearly a quarter as large as the data from week two, which is the week with the second smallest count in its respective largest bin. The apparatus was unchanged from previous weeks of data collection, but given that the mean counts had been decreases from the first week onward, it is likely that an electrical connection became increasingly loose in this time.

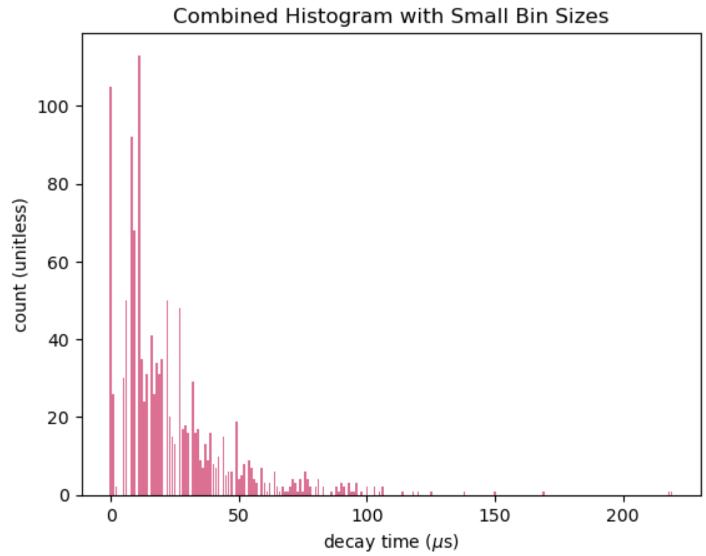


Figure 9: The same histogram as Fig. (3) except with a bin size of $0.1\mu$s. The exponential decay shape is significantly less obvious.

After seeing the particularly low count, the apparatus was inspected which likely remedied the loose connection, thus allowing the fourth and fifth weeks to show reasonable counts.

## 6.2 Counting Rates

A reoccurring issue during this experiment was a low rate of counts. The accuracy and precision of this method is directly proportional to the counts of decay events since this experiment relies on the probabilistic nature of particle decay. Due to the law of large numbers and since decays are both independent and identically distributed events, the more decays observed, the closer the measured distribution will approach the true distribution. As such, this low counting rate has impeded this experiment by introducing unmitigated random error.

After the duration of this experiment, a total of 1268 decays were observed. The experiment ran for approximately five weeks, which leads to roughly 36 counts per day. Then, the rate at which decays were observed was $r = 4 \times 10^{-4}\text{s}^{-1}$. This number should be taken to be an estimate, but it serves to juxtapose against the total events observed by the PMT; the PMT would output a signal on the order of five times a second. Judging by this brief order-of-magnitude calculation, only 0.01% of event did the apparatus find to be decay events.

There are two plausible causes for the low counting rate within the apparatus. The first is that an insignificant fraction of observed events were decay events, and the second is that the coincidence unit's time window was too strict to be able to correctly correlate signals from both PMTs. There are other potential causes that have been ruled out, namely: the discriminator's threshold voltage being too low to observe decay events, and the counter's timing window being too strict as to be unable to correlate sequential events to be a decay event.

It is possible that scintillator's orientation caused an insignificant fraction of observed events to be decay events. Since the scintillator used was a long and narrow cylinder, then a highly-directional distribution of incoming muons would greatly affect the ratio of observed captures versus decays. If the mean direction is perpendicular to the long axis of the cylinder, then the greater cross-sectional area would cause the scintillator to capture more muons; however, each muon would travel a shorter depth through the scintillator, which would decrease the likelihood of the scintillator observing that muon's decay. Conversely, should the mean direction be parallel to the long axis, then less captures would be observed but the greater depth would cause a larger proportion of decays to be observed. An additional experiment is required to determine whether the orientation of the scintillator was the cause of the low count rates; such an experiment would involve comparing the count of decay events measured versus the orientation of the scintillator. Such an experiment was outside our time scope.

A more likely culprit for the low counting rate was the coincidence unit. The coincidence unit the lab previously had access to had been damaged and so a NAND gate was used for this experiment. NAND gates lack the timing window required of a coincidence unit. While the discriminator increased the pulse width to about 600ns, it is possible that this is two narrow to be able to correlate the signals from both PMTs as an event. That said, it is unlikely that propagation delay within the scintillator or the wires had a significant impact given that if propagation speed is taken to be $s = \frac{2}{3}c$, a two meter difference in wire lengths would only account for a 10ns discrepancy. The coincidence unit has been attributed to be the most likely culprit since previous experiments using the same apparatus, save for a proper coincidence unit, saw rates which were at least an order of magnitude greater.

# 7 Conclusion

This experiment was successfully able to measure the mean lifetime of a muon to be $\tau_\mu = (2.23 \pm 0.01)\mu$s but our findings were unable to reconfirm the accepted value.

There are three primary ways to improve this experiment, all involve improving the counting rate of muon decays. The first is to perform the experiment over a significantly longer duration, thereby ensuring a more accurate and precise result. The second way is to improve the electronics and equipment to ensure greater counting rates. The third way is to perform an additional experiment to find the optimal orientation of the scintillator for the best ratio of muon captures to muon decays.

# References

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