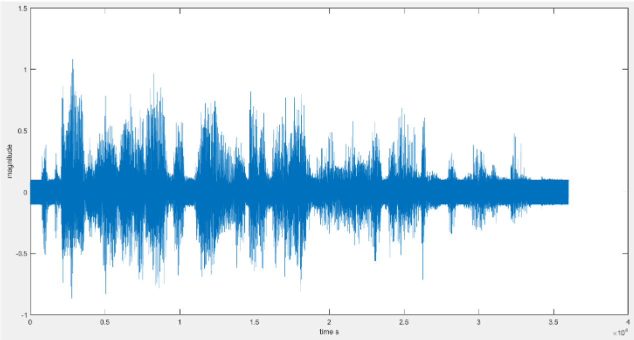
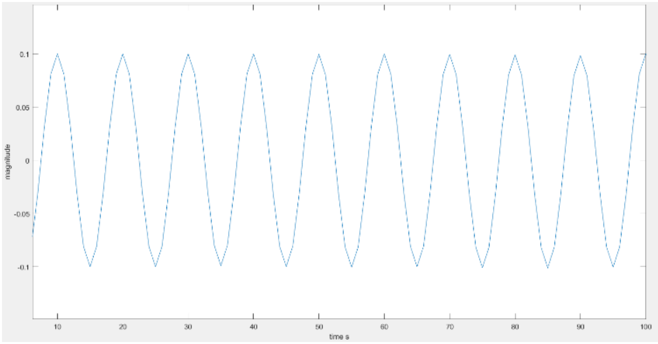
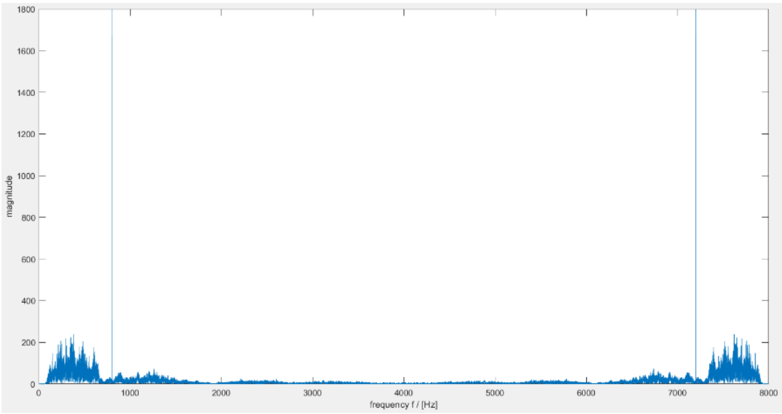
EE320 Semester 1 Assignment

1) **(Time domain analysis of interference frequency):**
   When plotting the amplitude of the signal x1, the part at the start of the signal, dominated by the interference, had a period of 10 samples.
   With the sampling frequency of 8000 Hz, the sampling period is 1/8000 Hz = 1.25_{x10}^{-4} s.
   A span of 10 of this sampling period results in a wave period of 1.25_{x10}^{-3} s.
   This value creates a frequency of 800 Hz.

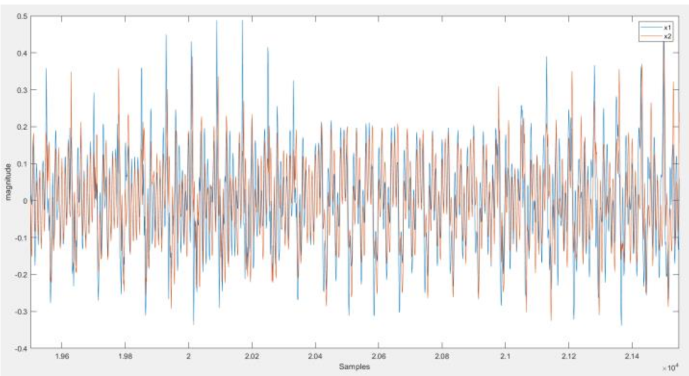


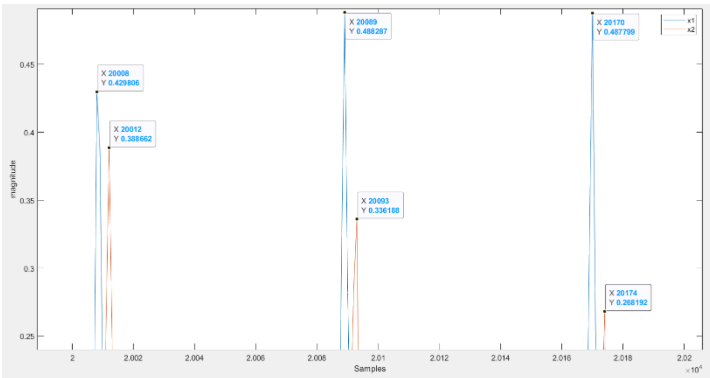


2) **(Frequency domain analysis of interference frequency):**
   The Fourier transform of the signal plotted shows peak values at 800 Hz and 7200 Hz. This implies that there is a consistent value of 800 Hz throughout the frequency range (matching Q1). The 7200 Hz spike comes from the periodic nature of an aperiodic signal in the Fourier domain.



3) **(Delay estimation in the time domain):**
   Plotting the two signals against each other within the range of samples 19,500 – 21,548 gives the following graph on the left.
   Measuring matching parts of both signals $x_1[n]$ and $x_2[n]$ within this range proves that there is a delay of 4 samples (or 5_{x10}^{-4} s) between the signals This delay was estimated using 3 different points of the signal and the result was the same each time.





4) (Delay and gain analysis in the Fourier domain):

Due to the slightly varying signal amplitudes in the range of the previous question (which is in the time domain), and because of the Fourier transform's linear property, the value:

$$G(\Omega) = \left|\frac{X_2(e^{j\Omega})}{X_1(e^{j\Omega})}\right|,$$

(which concerns the signals' discrete time Fourier transforms) should be equal to some constant value because the time domain equivalent is the same, with the amplitude of x_2/x_1.

The Fourier transform's linearity and time shifting properties together specify:

$$x_2[\mathrm{n}] = \alpha x_1[\mathrm{n} - t_0],$$

$$X_2\left[e^{j\Omega}\right] = \alpha X_1\left[e^{j\Omega}\right] \cdot e^{-j\Omega t_0} \qquad (1)$$

Therefore, when $t_0 = 4,$ and subbing in (1):

$$G(\Omega) = \left|\frac{X_2\left[e^{j\Omega}\right]}{X_1\left[e^{j\Omega}\right]}\right| \quad ,$$

$$G(\Omega) = \left|\frac{\alpha X_1\left[e^{j\Omega}\right] \cdot e^{-j\Omega \cdot 4}}{X_1\left[e^{j\Omega}\right]}\right| = \alpha \cdot e^{-j\Omega \cdot 4} \quad , \alpha < 1$$

For the following equation:

$$A(\Omega) = \angle\left\{\frac{\mathrm{X}_2(\mathrm{e}^{j\Omega})}{\mathrm{X}_1(\mathrm{e}^{j\Omega})}\right\},$$

$$x_2[\mathrm{n}] = \alpha x_1[\mathrm{n} - t_0],$$

$$X_2\left[e^{j\Omega}\right] = \alpha X_1\left[e^{j\Omega}\right] \cdot e^{-j\Omega t_0} \qquad (1)$$

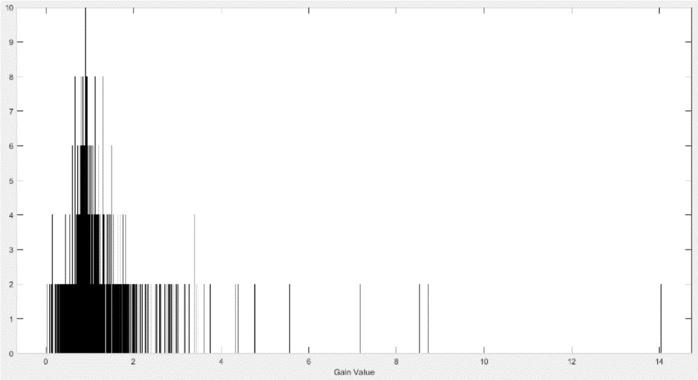
$$A(\Omega) = \angle\left\{\frac{\alpha X_1\left[e^{j\Omega}\right] \cdot e^{-j\Omega t_0}}{\mathrm{X}_1(\mathrm{e}^{j\Omega})}\right\}$$

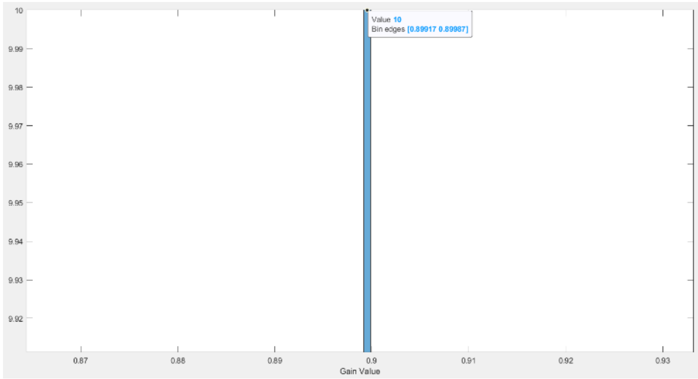
When $t_0 = 5_{x10}{}^{-4}$ s:

$$A(\Omega) = \angle\left\{\frac{X_1\left[e^{j\Omega}\right] \cdot e^{-j\Omega \cdot 4}}{\mathrm{X}_1(\mathrm{e}^{j\Omega})}\right\} = \angle\left\{e^{-j\Omega \cdot 4}\right\} = \Omega \cdot 4$$

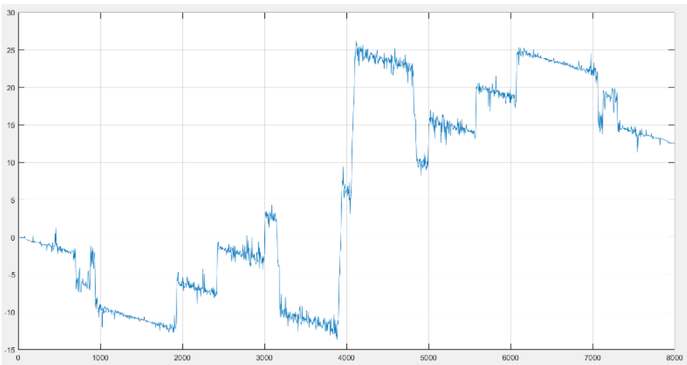
5) **(Delay and gain estimation in the Fourier domain):**

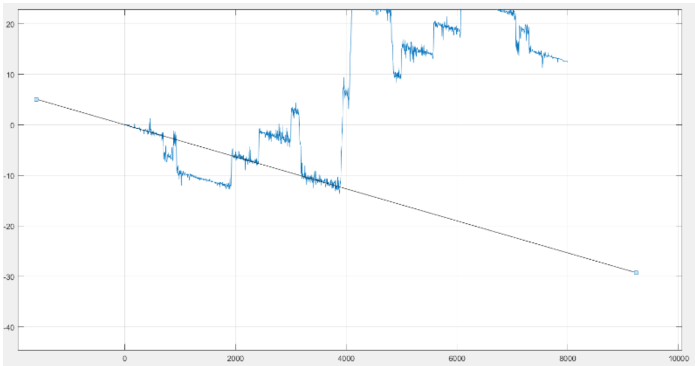
By plotting a histogram of the absolute value of the ratio $\frac{X_2}{X_1}$ against the frequency values in the specified sample range (19501:21548), the most common gain value is shown to be just below 1, specifically 0.899.





In order to find the delay between $x_1[n]$ and $x_2[n]$, the gradient of the unwrapped plot of the phase angle must be calculated with respect to the normalised angular frequency beginning from the origin.

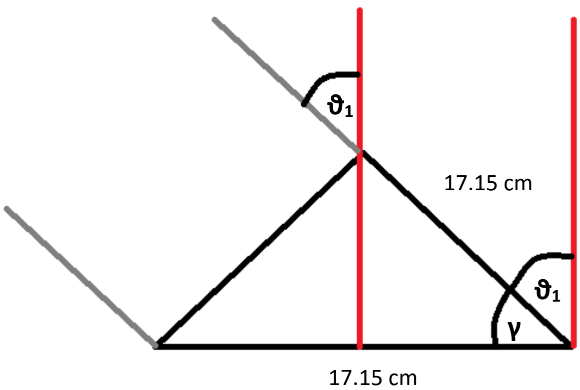




$$Gradient = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-25-0}{2\pi - 0} = -3.979 \;\cong 4 \; samples \; delay$$

This sample delay matches the one found from the time domain plot.

To find the angle ϑ_1 from the figure (in degrees), the sample delay must first be translated into seconds. This is done by multiplying it by the sample period (1/8000), giving $\mathbf{5_{x10}^{-4}}$ **s**. Then a simple distance = speed x time calculation is carried out with the speed of sound in air = 343 ms^{-1}, giving **17.15 cm** more distance travelled than the source to the first microphone. The diagram below was created from this knowledge.



The angle **γ** can be found through basic trigonometry:

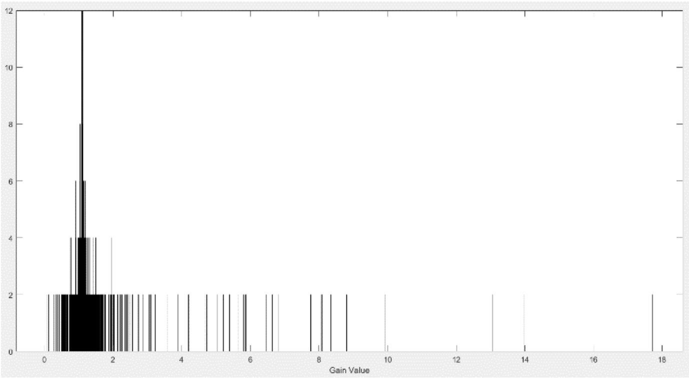
$$\gamma \ = \ \cos^{-1}\left(\frac{17.15}{17.15}\right) = 0°$$

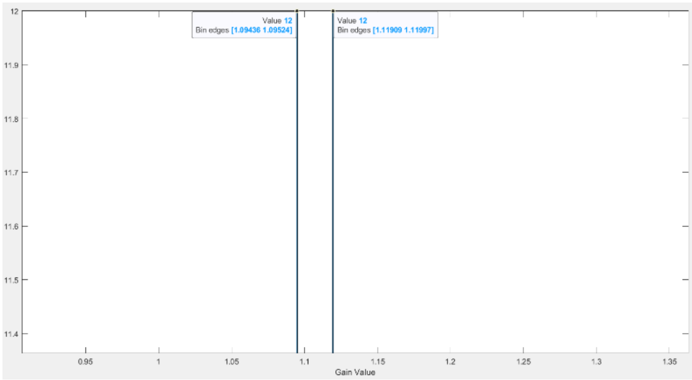
From this angle, a simple subtraction can find the required angle of **ϑ$_1$**:

$$\boldsymbol{\vartheta 1} \ = \ 90° - 0° = 90°$$

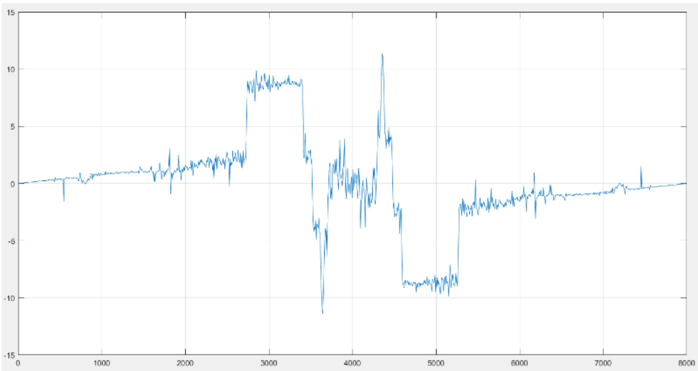
6) (Estimation for 2nd speaker):

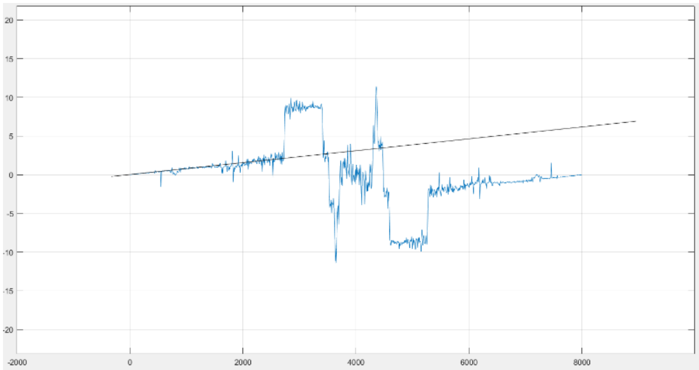
By plotting a histogram of the absolute value of the ratio $\frac{X_2}{X_1}$ against the frequency values in the specified sample range (24800:25824), the most common gain values are shown to be just above 1, specifically 1.094 and 1.119.





In order to find the new delay between x$_1$[n] and x$_2$[n], the gradient of the unwrapped plot of the phase angle must be calculated again at the new range, with respect to the normalised angular frequency beginning from the origin.

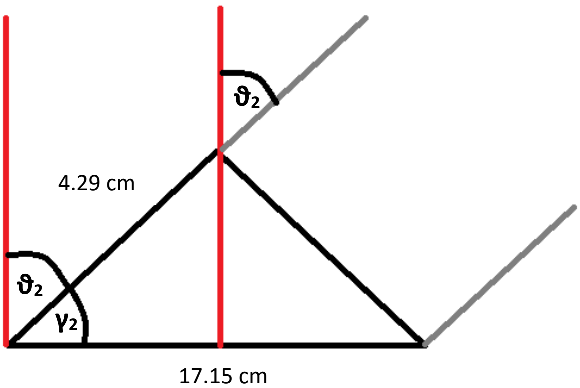




$$Gradient = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 0}{2\pi - 0} = 1.114 \ \cong 1 \ sample \ delay \ or \ 1.25_{x10}{}^{-4} \ \mathrm{s}$$

Distance = speed x time for this delay gives **4.29 cm** more distance travelled than the source to the first microphone.

The original trigonometric calculation was done for this value too and gave:



With the angle $\boldsymbol{\gamma_2}$ coming from:

$$\boldsymbol{\gamma_2} = \cos^{-1}\left(\frac{4.29}{17.15}\right) = 75.5139°$$

From this angle, a simple subtraction can find the required angle of $\boldsymbol{\vartheta_2}$:

$$\boldsymbol{\vartheta_2} = 90° - 75.5139° = 14.49°$$

7) (Estimation of relative transfer functions):

For the first range of samples: $\frac{X_2}{X_1}$ had a gain of 0.899 and sample delay of -4

For the second range of samples: $\frac{X_2}{X_1}$ had a gain of 1.094 and sample delay of 1

$$X_2 = X_1 \cdot 0.899 \cdot z^{-4} \quad => \quad X_2 = X_1 \cdot 0.899 \cdot z^{-4}$$

$$X_1 = X_2 \cdot \frac{1}{1.094 \cdot z^{+1}} \quad => \quad X_1 = X_2 \cdot \frac{1}{1.094} \cdot z^{-1}$$

$$H(z) = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{1.094} \cdot z^{-1} \\ 0.899 \cdot z^{-4} & 1 \end{bmatrix}$$

8) (Construction of separation filters):

$$G(z) = H^{-1}(z)$$

$$\boldsymbol{G}(z) = \frac{1}{D(z)} \begin{bmatrix} H_{2,2}(z) & -H_{1,2}(z) \\ -H_{2,1}(z) & H_{1,1}(z) \end{bmatrix}$$

$$D(z) = H_{1,1} \cdot H_{2,2} - H_{2,1} \cdot H_{1,2} = (1) \cdot (1) - (0.899 \cdot z^{-4}) \cdot (\frac{1}{1.094} \cdot z^{-1})$$

$$D(z) = 1 - \frac{0.899}{1.094} \cdot z^{-5}$$

Therefore:

$$G(z) = \frac{1}{1 - \frac{0.899}{1.094} \cdot z^{-5}} \cdot \begin{bmatrix} 1 & -\frac{1}{1.094} \cdot z^{-1} \\ -0.899 \cdot z^{-4} & 1 \end{bmatrix}$$

$$G(z) = \frac{1}{1 - 0.8217 \cdot z^{-5}} \cdot \begin{bmatrix} 1 & -0.9141 \cdot z^{-1} \\ -0.899 \cdot z^{-4} & 1 \end{bmatrix}$$

$$G(z) = \begin{bmatrix} \frac{1}{1 - 0.8217 \cdot z^{-5}} & -\frac{0.91418 \cdot z^{-1}}{1 - 0.8217 \cdot z^{-5}} \\ -\frac{0.899 \cdot z^{-4}}{1 - 0.8217 \cdot z^{-5}} & \frac{1}{1 - 0.8217 \cdot z^{-5}} \end{bmatrix}$$

The output signals y_1' and y_2' were both the separated speakers with the interference signal still playing over the top of each of them.

9) N/A

10) (Notch filter analysis):

$$Q(z) = \frac{(1-e^{j\Omega_0}z^{-1})(1-e^{-j\Omega_0}z^{-1})}{(1-\gamma e^{j\Omega_0}z^{-1})(1-\gamma e^{-j\Omega_0}z^{-1})}$$

<u>numerator:</u>

$$(1)\times(1) + (1)(-e^{-j\Omega_0}z^{-1}) + (-e^{j\Omega_0}z^{-1})(1) + (-e^{j\Omega_0}z^{-1})(-e^{-j\Omega_0}z^{-1})$$

$$= 1 - e^{-j\Omega_0}z^{-1} - e^{j\Omega_0}z^{-1} + z^{-2} \quad = 1 - (e^{-j\Omega_0} + e^{j\Omega_0})z^{-1} + z^{-2}$$

$$= 1 - (2\cos\Omega)z^{-1} + z^{-2}$$

<u>denominator:</u>

$$(1)\times(1) + (1)(-\gamma e^{-j\Omega_0}z^{-1}) + (-\gamma e^{j\Omega_0}z^{-1})(1) + (-\gamma e^{j\Omega_0}z^{-1})(-\gamma e^{-j\Omega_0}z^{-1})$$

$$= 1 - \gamma e^{-j\Omega_0}z^{-1} \cancel{+} - \gamma e^{j\Omega_0}z^{-1} + \gamma^2 z^{-2}$$

$$= 1 - (\gamma e^{-j\Omega_0} + \gamma e^{j\Omega_0})z^{-1} + \gamma^2 z^{-2} \quad = 1 - (2\gamma\cos(\Omega_0))z^{-1} + \gamma^2 z^{-2}$$

$$Q(z) = \frac{1 + (-e^{-j\Omega_0} - e^{j\Omega_0})z^{-1} + z^{-2}}{1 + (-\gamma e^{-j\Omega_0} - \gamma e^{j\Omega_0})z^{-1} + \gamma^2 z^{-2}}$$

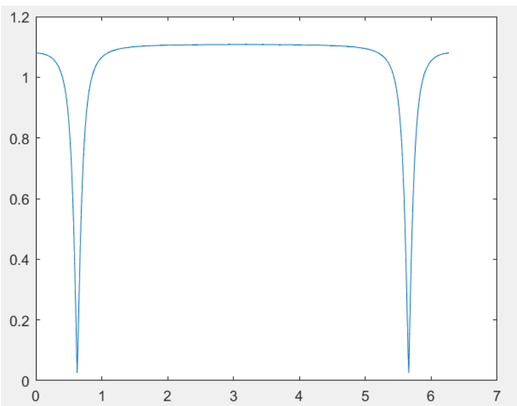
$$Q(z) = \frac{1 + (-2\cos(\Omega_0))z^{-1} + z^{-2}}{1 + (-2\gamma\cos(\Omega_0))z^{-1} + \gamma^2 z^{-2}}$$

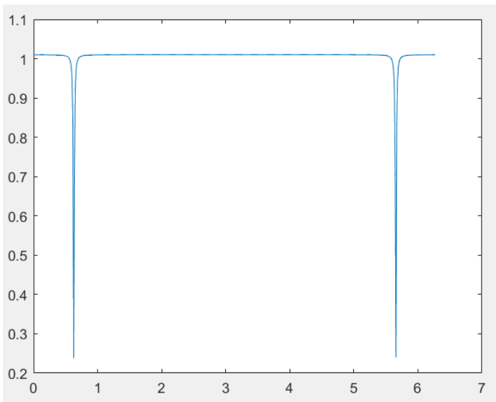
From these calculations, the following coefficients can be found and are as listed below:

$$Q(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

b_0 = 1; **b_1** = $-2\cos(\Omega_0)$; **b_2** = 1
a_0 = 1; **a_1** = $-2\Upsilon\cos(\Omega_0)$; **a_2** = Υ^2

The resulting magnitude response plots, using these coefficients when $\Upsilon = 0.9$(left), and when $\Upsilon = 0.99$(right), are shown below:





11) (Notch filter implementation and application):

The signals filtered with the Notch filter when gamma = 0.9 sound slightly tinny, however, the interference tone has been removed nonetheless.

The signals filtered when gamma = 0.99 sound cleaner and fuller with the interference tone also removed.

This difference is likely due to the wider band of the Notch filter when gamma = 0.9, acting as a high pass filter and creating an equalizer effect on the vocals.

12) (Filter sequence):

The Notch filters can be switched with the unmixing system because they are both linear, time-invarient systems. This means that the same effect will be had from input to output regardless of order.