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```
using Pkg;  Pkg.activate("Project.toml")
```

```
begin
    using Optim, Plots
    plotly()
    nothing
end
```

Solving Optimisation Problems For Economics I

Introduction

This notebook lays out the basics of how to solve an economic optimsation problem in Julia. This notebook is aimed at laying out a framework for solving these problems in in the context of a reactive notebook. This notebook covers a single optimisation problem relating to the firm's choice, and is similar to something which you may find in undegraduate or postgraduate problem sets.

Disclaimer: This question was part of a case-study interview question, and was not written by me.

Firm's Choices - Optimal CO_2 Abatement & Cournot Competition

Problemset parameters

```
begin
    Q = 100
    intensity_CO₂ = 0.05
    tax_CO₂ = 120
    discount_rate = 0.05
    nothing
end
```

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1.a – Firm's CO_2 Abatement Choice

- A firm produces 100 units of output per year.
- Producing one unit of output currently leads to 0.05 tonnes of CO_2 emissions
- The government now levies a carbon tax of USD 120 per tonne of CO_2
- Firms can abate a fraction a of their emissions where $a \in [0, 1]$
- The cost of abating a is given by $c(a) = 1000 * a^3$ per tonne of initial CO_2 emissions

Once the carbon tax of USD 120 per ton of CO_2 comes into effect, how much CO_2 will the firm emit and what is the total annual cost of the carbon tax policy to the firm?

Answer

Solving this with Julia is straight forward. All we need to do is specify what our objective function is, and determine what we want to minimise. Once the objective functions has been formulated, we just need to pass in the starting point, and **Optim.jl** finds the closest minima.

```
f_abatement(a) = 1000*a^3;
```

```
f_objective(a) = Q*(Q*intensity_CO₂)*(1-a) + f_abatement(a);
```

```
result = optimize(x->f_objective(x[1]), [0.0]);
```

To access the value which minimzes the our function we simply access the minimizer from our results.

```
a_optim = 0.4082489013671876
```

```
a_optim = result.minimizer[1]
```

We can easily visualise the results for inspection. Note that the starting point can have an impact on the results of the objective have multiple minima, however, for most simple economics problem sets, this is not a major concern.

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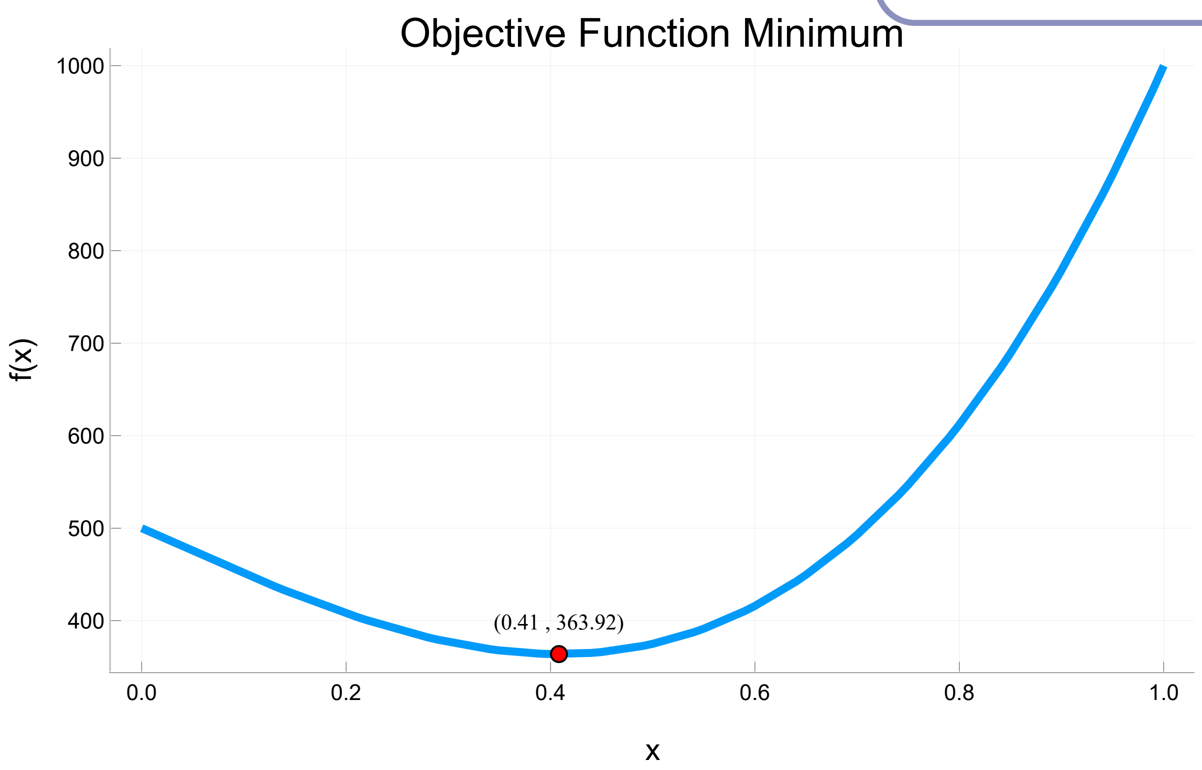


```
begin
    firm_co₂ = Q*intensity_CO₂*(1-a_optim)
    abate_cost = f_abatement(a_optim)
    tax_cost = (Q*intensity_CO₂*(1-a_optim)) * tax_CO₂
    total_cost = (Q*intensity_CO₂*(1-a_optim)) * tax_CO₂ + f_abatement(a_optim)
    nothing
end
```

From the above analysis, we can see that the optimal abatement rate for the firm would be 0.408, costing the firm USD 68.04 in abatement costs. This would mean that after abating their emissions, the firm would emit 2.959 tCO_2 and pay USD 355.05 in taxes; resulting in a total cost of USD 423.09.

1.b – Net Present Value (NPV)

Now suppose the government announces that the above policy will come into effect in 2025 and will stay in place forever. What is today's net present value of the cost to the firm? Assume an annual discount rate of 0.05%.

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Answer

The NPV is a method for determining the current value of all future cash flows. Naturally, we would care more about the present, and discount future events to account for opportunity costs. The NPV of a future cost at time t can be calculated according to:

$$NPV = \frac{R_t}{(1+i)^t}$$

Where R_t is the net cash flows during period t, i is the discount rate (specified at the top of this workbook), and t is the number of time periods. Implementing this to the result from 1.a up to 2025, we get a NPV of the 2025 costs of USD 365.48:

```
NPV_2025 = total_cost/((1+discount_rate)^(2025-2022));
```

The question states that the policy will remain in place forever, consequently, we also need to calculate the NPV over an infinite horizon. Since we know that the costs to the firm will stay constant for perpituity from 2025 onward, we can simply divide our 2025 NPV cost by the discount rate of 0.05. In other words:

$$NPV = \frac{NPV_{2025}}{i}$$

```
NPV = NPV_2025/discount_rate;
```

This means that the firm will face a net present value cost of USD 7309.66.

1.c – Cournot Duopoly

Two firms produce an identical product and compete in a Cournot duopoly. The market demand curve is given by $P = 9 - Q = 9 - q_1 - q_2$. Both firms emit 0.05 tCO_2 per unit of output. They initially both have marginal cost of 0. Firms can abate emissions at cost $c(a) = 1000a^3$. What is the effect of a USD 120 carbon tax on the equilibrium price?

Answer

First, we set up the profit function of each of the firms.

$$\pi_1 = q_1 * (9 - q_1 - q_2) - (t(q_1 * e) + c(a))$$

$$\pi_2 = q_2 * (9 - q_1 - q_2) - (t(q_2 * e) + c(a))$$

Where t us the tax per ton of CO_2, e is the CO_2 intensity per unit of output, $c(a)$ is the firm's abatement cost curve, and q_1 and q_2 represents firm 1 and firm 2's output choices respectively.

Taking the derivative and maximising the profit function, we are l Edit or run this notebook representation of the firm's optimal choice.

$$\frac{\partial \pi_1}{\partial q_1} = 9 - 2q_1 - q_2 - te = 0 \quad \rightarrow \quad q_1 = \frac{9-q_2-te}{2}$$

$$\frac{\partial \pi_2}{\partial q_2} = 9 - q_1 - 2q_2 - te = 0 \quad \rightarrow \quad q_2 = \frac{9-q_1-te}{2}$$

Plugging in q_2 to the first equation, we get:

$$q_1 = \frac{9-(\frac{9-q_1-te}{2})}{2}$$

$$q_1 = \frac{9-q_1-te}{4}$$

$$q_1 = \frac{9-te}{5}$$

and by symmetry:

$$q_2 = \frac{9-te}{5}$$

We can now solve for q_1 and q_2 by plugging in the carbon tax. Initially prior to the tax being implemented, the firm is facing zero marginal cost because $t = 0$. However, after the tax is introduced, the firm needs to pay 120 per ton of CO_2, resulting in a marginal cost of USD 6.0 per unit of output $(t * e)$.

```
f_profit_max_q(;t=tax_CO₂, e=intensity_CO₂) = (9-t*e)/5;
```

The resultant outcome is that prior to the implementation of the carbon tax, both firms produce 1.8 units of output each at an equilibrium price of USD 5.4. After the tax of 120 per ton of CO_2 is introduced, the firms change their production to 0.6 units of output at a new equilibrium price of 7.8. Overall, the carbon tax increased the price by USD 2.4 and reduced the total output produced by both firms by 2.4 units of output.

```
begin
    # Before tax (BT)
    q₁_bt = q₂_bt = f_profit_max_q(t=0)
    Q_bt = q₁_bt+q₂_bt
    p_bt = 9-Q_bt

    # After tax (AT)
    q₁_at = q₂_at = f_profit_max_q(t=tax_CO₂)
    Q_at = q₁_at+q₂_at
    p_at = 9-Q_at

    nothing
end
```

Appendix

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Variable rounding

```
begin
    rounded_a_optim = round(a_optim,digits=3)
    rounded_firm_co₂ = round(firm_co₂,digits=3)
    rounded_abate_cost = round(abate_cost, digits=2)
    rounded_tax_cost = round(tax_cost, digits=2)
    rounded_total_cost = round(total_cost, digits=2)
    rounded_NPV_2025 = round(NPV_2025,digits=2)
    rounded_NPV = round(NPV, digits=2)
    rounded_marginal_cost = round(marginal_cost, digits=2)
    rounded_q₁_bt = round(q₁_bt, digits=2)
    rounded_q₁_at = round(q₁_at, digits=2)
    rounded_p_bt = round(p_bt, digits=2)
    rounded_p_at = round(p_at, digits=2)
    rounded_quantity_decrease = round(quantity_decrease, digits=3)
    rounded_price_increase = round(price_increase, digits=2)
    nothing
end
```

Miscellaneous parameters

```
begin
    #Part 1.c
    marginal_cost = tax_CO₂*intensity_CO₂
    price_increase = p_at - p_bt
    quantity_decrease = Q_bt - Q_at
    nothing
end
```