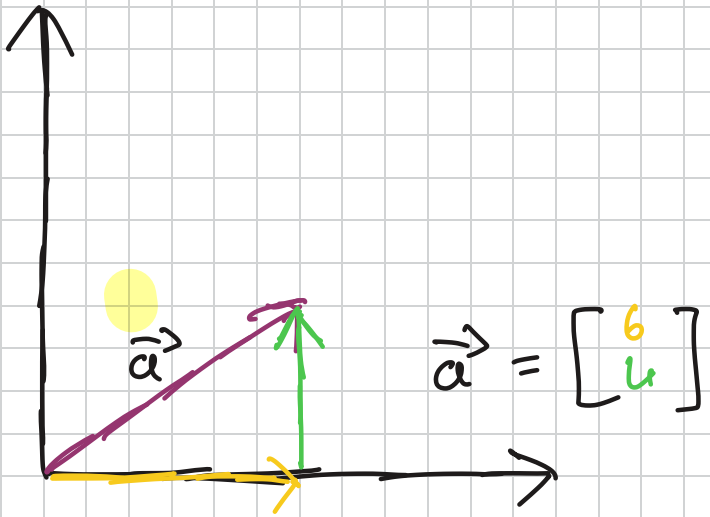
Vector = magnitude + direction

$\vec{v} = (x, y) = \begin{bmatrix} x \\ y \end{bmatrix}$



Real coordinate spaces $\mathbb{R}^2/\mathbb{R}^3/\mathbb{R}^n$

- only real numbers

$\mathbb{R}^2$:

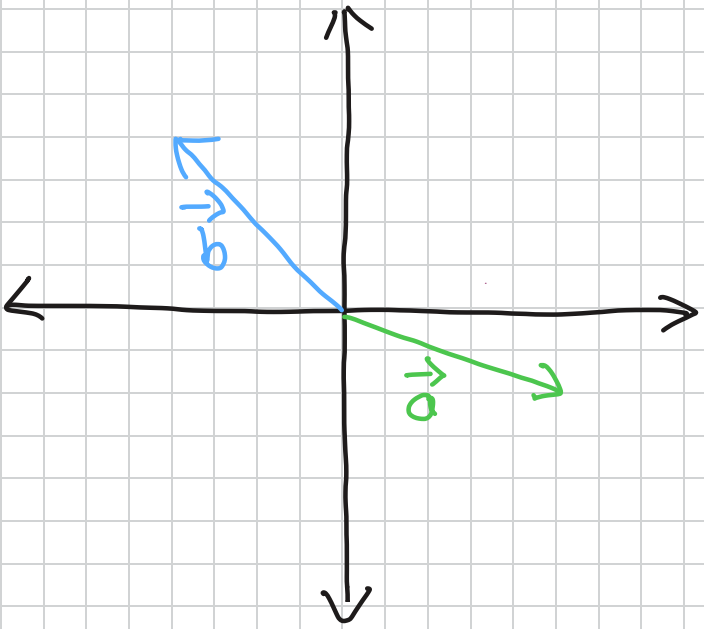
- 2 dimensional $\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad \vec{a} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \Big/ \vec{a} \in \mathbb{R}^2$
- all possible real-valued 2-tuples

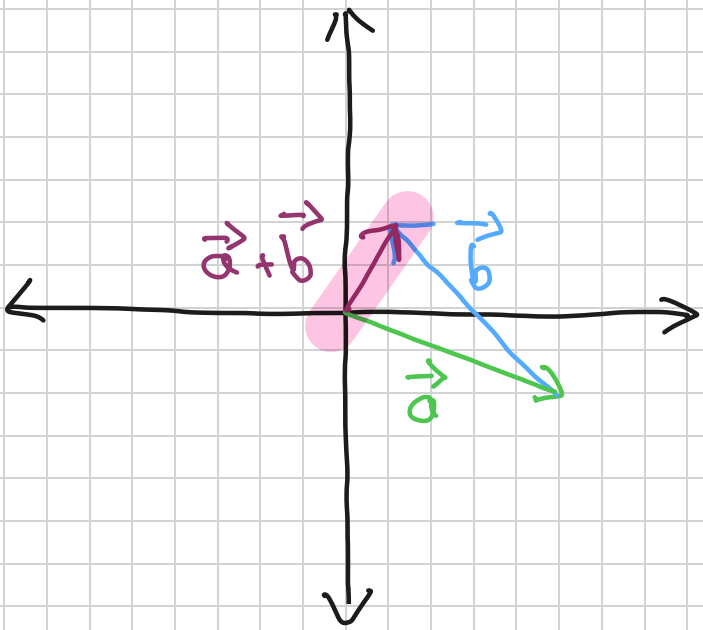
$\mathbb{R}^3$:

- 3 dimensional $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- all possible real-valued 3-tuples

Adding vectors

$\vec{a} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$ $\vec{a} + \vec{b} = \begin{bmatrix} 6 - 4 \\ -2 + 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

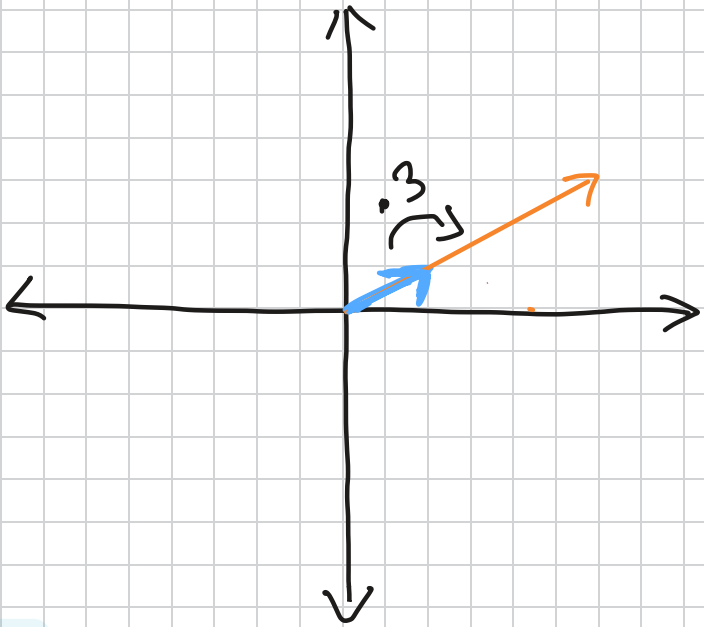




Multiplying a vector by a scalar

$3 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 \\ 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$ same direction, but upscaled

↑ scalar



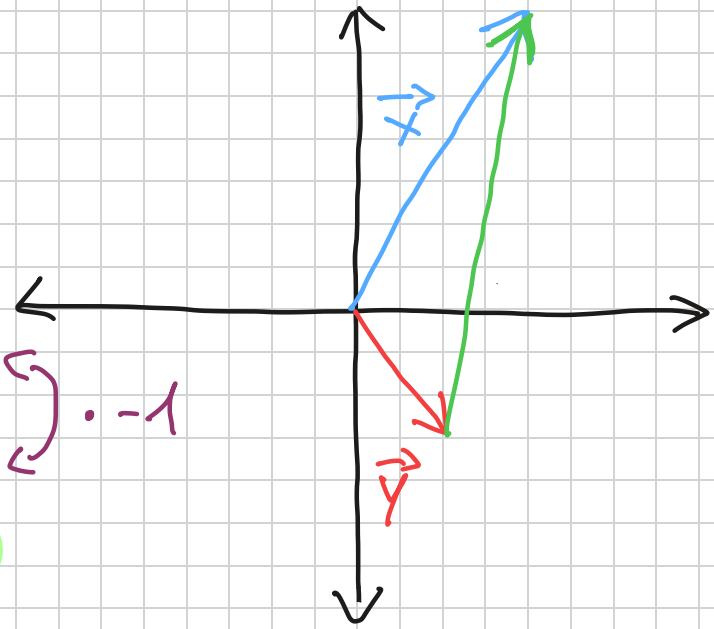
Subtracting vectors

$\vec{x} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ $\vec{y} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$\vec{x} - \vec{y} = \begin{bmatrix} 4-2 \\ 7-(-3) \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$

$\vec{y} - \vec{x} = \begin{bmatrix} 2-4 \\ -3-7 \end{bmatrix} = \begin{bmatrix} -2 \\ -10 \end{bmatrix}$

$\cdot -1$



Unit vectors

$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

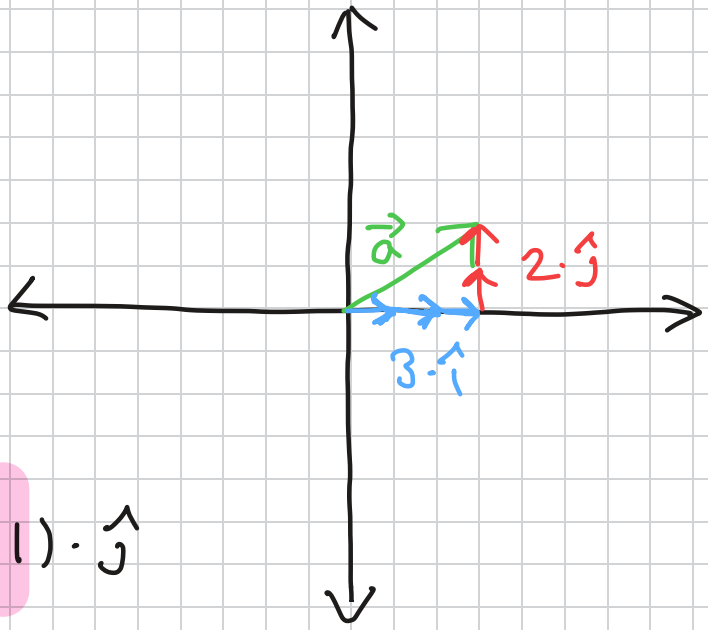
$\vec{a} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 3 \cdot \hat{i} + 2 \cdot \hat{j}$

$\vec{b} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$\vec{a} + \vec{b} = (3+4) \cdot \hat{i} + (2+-1) \cdot \hat{j}$

$= 7 \cdot \hat{i} + 1 \cdot \hat{j}$

$= \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

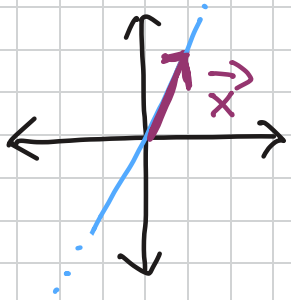


Parametric representations of lines

$L = \{c \cdot \vec{x} \mid c \in \mathbb{R}\}$

Line in the same or opposite direction as $\vec{x}$

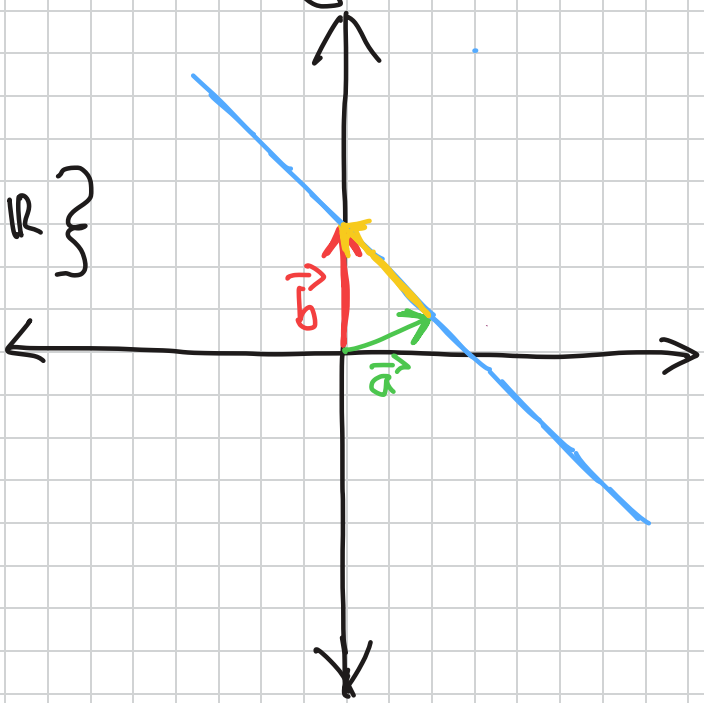
$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$



Line different from the origin

$\vec{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$L = \{\vec{a} + t \cdot (\vec{b} - \vec{a}) \mid t \in \mathbb{R}\}$



$x = 2 + (0-2) \cdot t = -2t + 2$

$y = 1 + (3-1) \cdot t = 2t + 1$

Line $\Rightarrow$ $x = -2t + 2$, $y = 2t + 1$