Robot Arm as a serial manipulator

Built by joints (Gelenke) and links (Glieder)

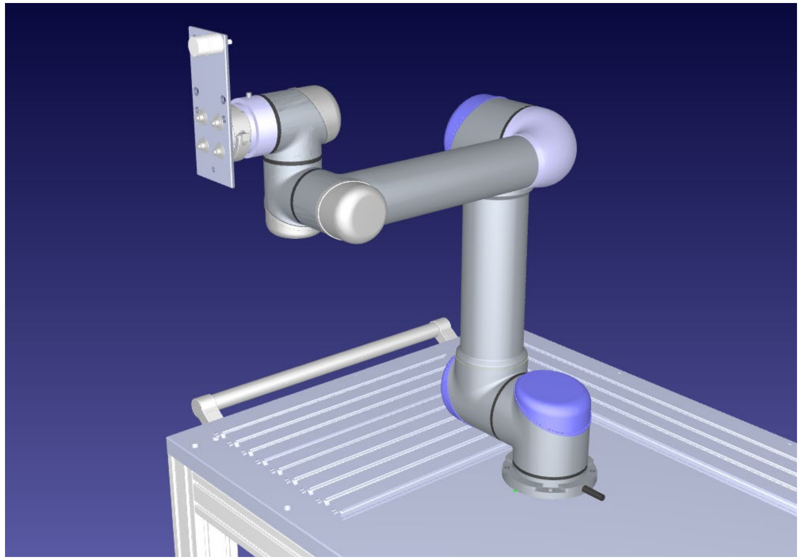
Link: rigid body – spatial relationship between two neighbouring joint axis

Joint: moveable (e.g. rotary) connection between links

Base: assembly of the robot arm

Flange (Flansch): tool assembly

End effector (Endeffektor): Tool



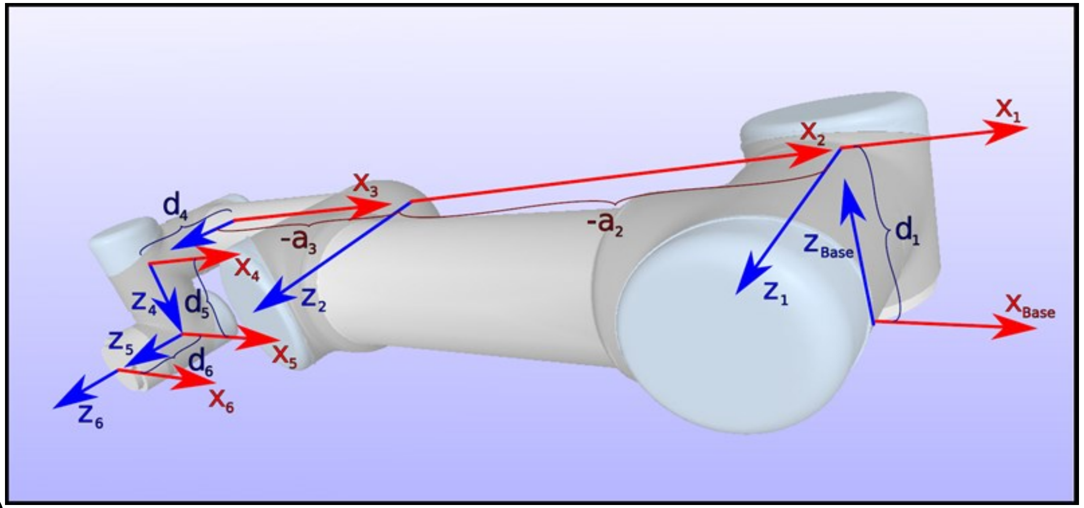
Robot Arm with *n* joints, numbered from 1 to *n*,
has *n* + 1 links numbered from 0 to *n*.
Link 0: Base of the robot

Link *n*: carries the end-effector

Coordinate systems

Coordinate system of the *j*-th link is attached to the far end of the link *j*

- Far end of link *j*
- z_j – rotation axis of joint $j + 1$
- x_j – common perpedicular of the
  z_j and $z_{j\text{-}1}$ axis
  (arbitrary, when the two axes are collinear!)
- y_j – complements a right-handed coordinate system

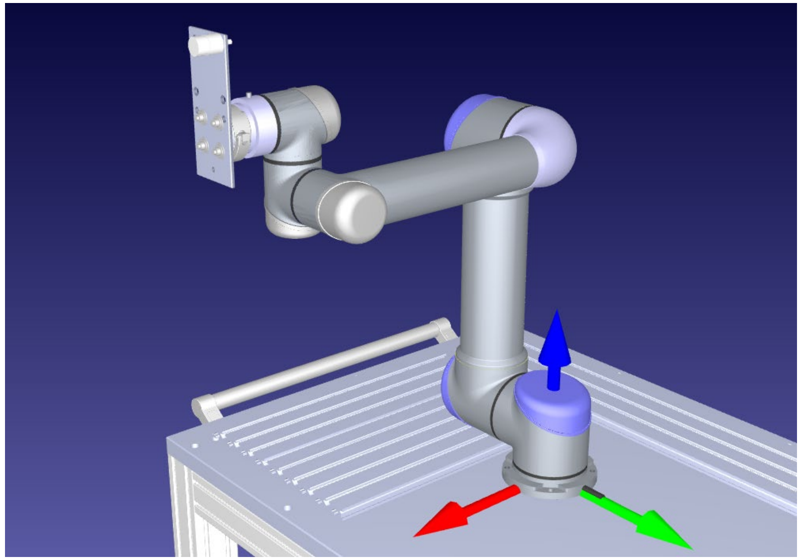


Bildquelle: https://www.universal-robots.com/how-tos-and-faqs/faq/ur-faq/parameters-forcalculations-

Coordinate frames

Basis-frame

- Corresponds to base link
- z - axis ⇨ rotation axis of joint 1
- Orthogonal x, y -plane is the assembly plane
- Direction of x - axis arbitrary
  (cable direction could be an indication)



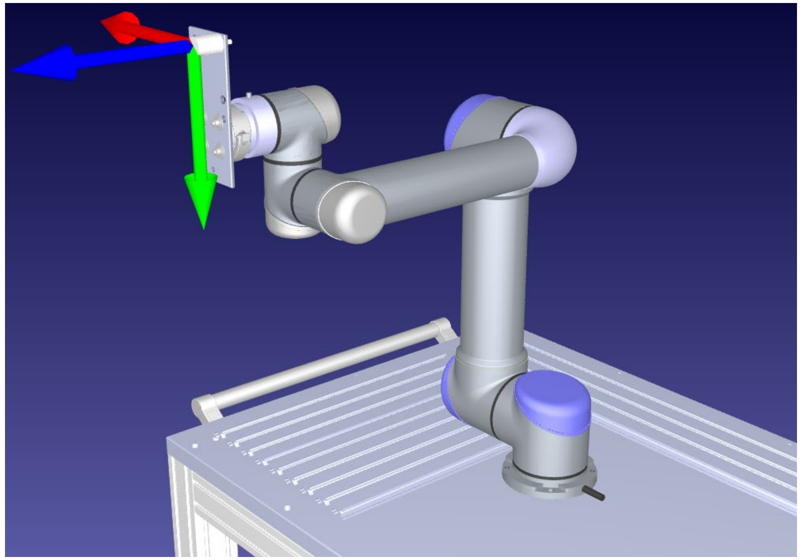
RGB=XYZ

Coordinate frames

End effector‘s frame

- In principle oriented to the tool‘s axes
- Mostly same orientation as the *n*-th link frame

⇨ only translation components between flange and end effector frame

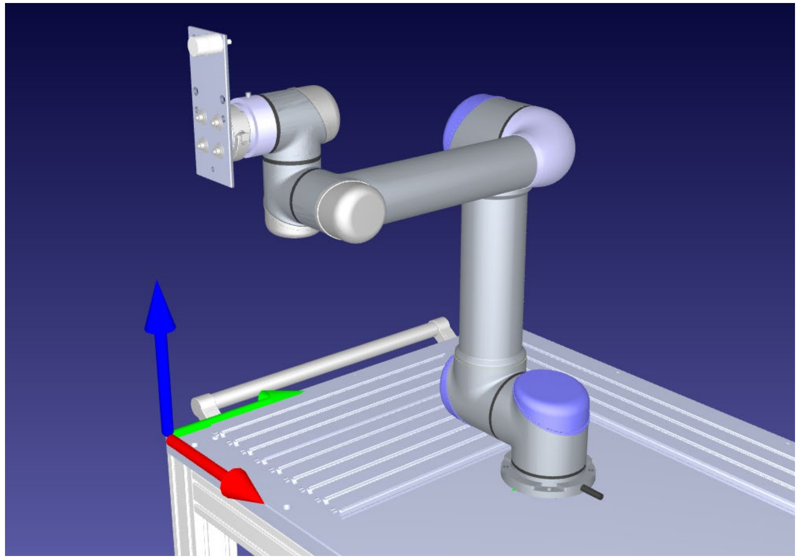


RGB=XYZ

Coordinate frames

World frame

- Superordinate frame
- Determined by:
  - Working area (e.g. support construction)
  - Measurement instrument, like LT



RGB=XYZ

Transformation between consecutive frames

- Frame transformation typically similarity transformation with fixed scale

$$\begin{pmatrix} x^{j-1} \\ y^{j-1} \\ z^{j-1} \end{pmatrix} = \boldsymbol{t}_j^{j-1} + \boldsymbol{R}_j^{j-1} \begin{pmatrix} x^{j} \\ y^{j} \\ z^{j} \end{pmatrix}$$

- Alternative representation:

$$\begin{pmatrix} x^{j-1} \\ y^{j-1} \\ z^{j-1} \\ 1 \end{pmatrix} = \begin{pmatrix} \underset{(3,3)}{\boldsymbol{R}_j^{j-1}} & \underset{(3,1)}{\boldsymbol{t}_j^{j-1}} \\ \underset{(1,3)}{\boldsymbol{0}} & \underset{(1,1)}{1} \end{pmatrix} \begin{pmatrix} x^{j} \\ y^{j} \\ z^{j} \\ 1 \end{pmatrix}$$

Transformation between consecutive frames

- Points and vectors expressed in homogeneous coordinates in frame *j*:

$$\boldsymbol{P}_i^j = \begin{pmatrix} x_i^j \\ y_i^j \\ z_i^j \\ 1 \end{pmatrix} \qquad \boldsymbol{P}_k^j = \begin{pmatrix} x_k^j \\ y_k^j \\ z_k^j \\ 1 \end{pmatrix} \qquad \boldsymbol{v}_{ik}^j = \begin{pmatrix} x_k^j - x_i^j \\ y_k^j - y_i^j \\ z_k^j - z_i^j \\ 0 \end{pmatrix}$$

- Points and vectors transform equally from frame *j* to *j* - 1:

$$\boldsymbol{P}_i^{j-1} = \boldsymbol{H}_j^{j-1}\boldsymbol{P}_i^j \qquad \boldsymbol{v}_i^{j-1} = \boldsymbol{H}_j^{j-1}\boldsymbol{v}_i^j$$

$$\boldsymbol{P}_k^{j-1} = \boldsymbol{H}_j^{j-1}\boldsymbol{P}_k^j$$

Transformation between consecutive frames

- Homogeneous transformation

$$\begin{pmatrix} x^{j-1} \\ y^{j-1} \\ z^{j-1} \\ 1 \end{pmatrix} = \boldsymbol{H}_j^{j-1} \begin{pmatrix} x^{j} \\ y^{j} \\ z^{j} \\ 1 \end{pmatrix}$$

- Consecutive transformations can be represented as follows:

$$\begin{pmatrix} x^{0} \\ y^{0} \\ z^{0} \\ 1 \end{pmatrix} = \boldsymbol{H}_1^{0} \ldots \boldsymbol{H}_{j-1}^{j-2} \boldsymbol{H}_j^{j-1} \begin{pmatrix} x^{j} \\ y^{j} \\ z^{j} \\ 1 \end{pmatrix}$$

Transformation between consecutive frames

- In case of translations only the homogeneous transformation matrix takes the particular form:

$$
\boldsymbol{H}_j^{j-1} = \begin{pmatrix} 1 & 0 & 0 & t_{x\,j}^{\;j-1} \\ 0 & 1 & 0 & t_{y\,j}^{\;j-1} \\ 0 & 0 & 1 & t_{z\,j}^{\;j-1} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} x^{j-1} \\ y^{j-1} \\ z^{j-1} \\ 1 \end{pmatrix} = \begin{pmatrix} \underset{(3,3)}{\boldsymbol{I}_j^{j-1}} & \underset{(3,1)}{\boldsymbol{t}_j^{j-1}} \\ \underset{(1,3)}{\boldsymbol{0}} & \underset{(1,1)}{1} \end{pmatrix} \begin{pmatrix} x^j \\ y^j \\ z^j \\ 1 \end{pmatrix}
$$

- In case of rotation (e.g. z-axis) only the homogeneous transformation matrix takes the particular form :

$$
\boldsymbol{H}_j^{j-1} = \begin{pmatrix} \cos\theta_j & -\sin\theta_j & 0 & 0 \\ \sin\theta_j & \cos\theta_j & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Transformation between consecutive frames

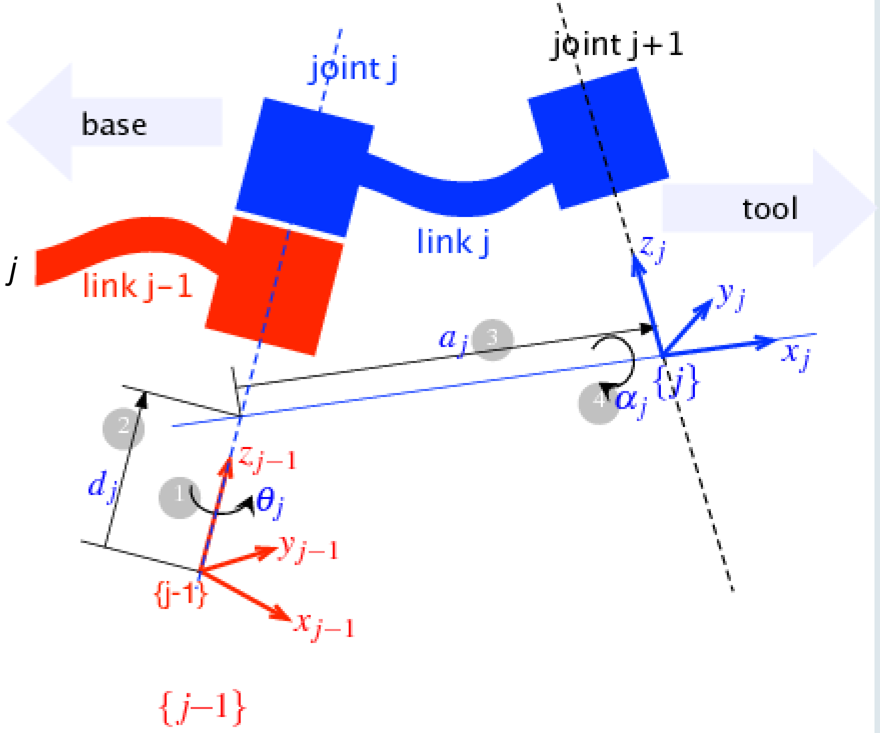
- Therefore, a sequence of transformations involving translation and rotation reads:

$$\boldsymbol{H}_j^{j-1} = \begin{pmatrix} 1 & 0 & 0 & t_{xj}^{\,j-1} \\ 0 & 1 & 0 & t_{yj}^{\,j-1} \\ 0 & 0 & 1 & t_{zj}^{\,j-1} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \boldsymbol{H}_{j-1}^{j-2} = \begin{pmatrix} \cos\theta_{j-1} & -\sin\theta_{j-1} & 0 & 0 \\ \sin\theta_{j-1} & \cos\theta_{j-1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{H}_{j-1}^{j-2}\boldsymbol{H}_j^{j-1} = \begin{pmatrix} \cos\theta_{j-1} & -\sin\theta_{j-1} & 0 & 0 \\ \sin\theta_{j-1} & \cos\theta_{j-1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & t_{xj}^{\,j-1} \\ 0 & 1 & 0 & t_{yj}^{\,j-1} \\ 0 & 0 & 1 & t_{zj}^{\,j-1} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_{j-1} & -\sin\theta_{j-1} & 0 & t_{xj}^{\,j-1}\cos\theta_{j-1} - t_{yj}^{\,j-1}\sin\theta_{j-1} \\ \sin\theta_{j-1} & \cos\theta_{j-1} & 0 & t_{xj}^{\,j-1}\sin\theta_{j-1} + t_{yj}^{\,j-1}\cos\theta_{j-1} \\ 0 & 0 & 1 & t_{zj}^{\,j-1} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Denavit-Hartenberg-Parameters

- Angle between x_{j-1} and x_j about $z_{j\text{-}1}$: θ_j
- Distance from origin of frame j - 1 to x_j axis along z_{j-1} : d_j
- Distance between z_{j-1} and z_j along x_j : a_j
- Angle between z_{j-1} and z_j about $x_{j\text{-}1}$: α_j



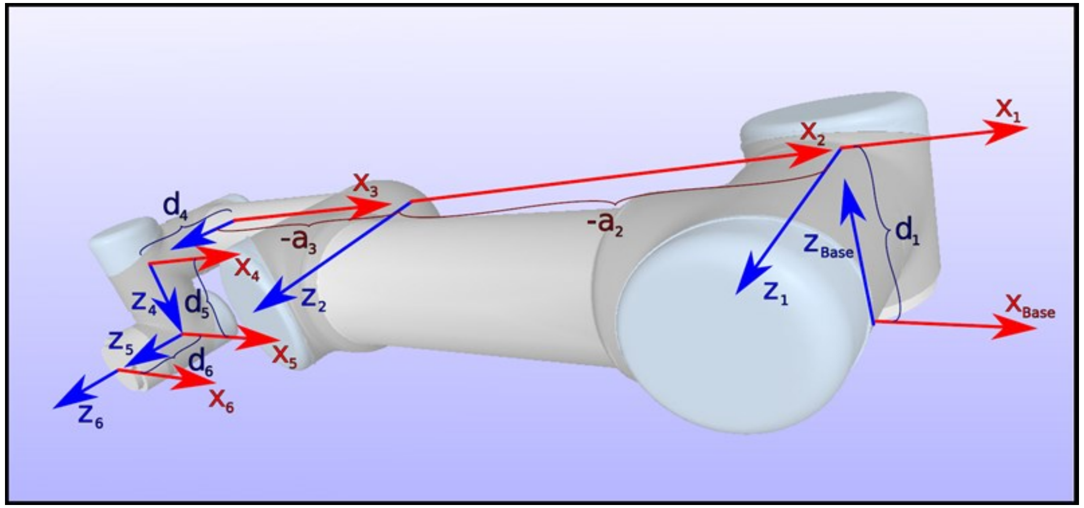
Bildquelle: Corke, Peter: Robotics, Vision And Control : Fundamental Algorithms In MATLAB. 2. Edition. Berlin [u.a.] : Springer, 2013

Denavit-Hartenberg-Parameters UR5

UR5

Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]
Joint 1	0	0	0.089159	π/2
Joint 2	0	-0.425	0	0
Joint 3	0	-0.39225	0	0
Joint 4	0	0	0.10915	π/2
Joint 5	0	0	0.09465	-π/2
Joint 6	0	0	0.0823	0

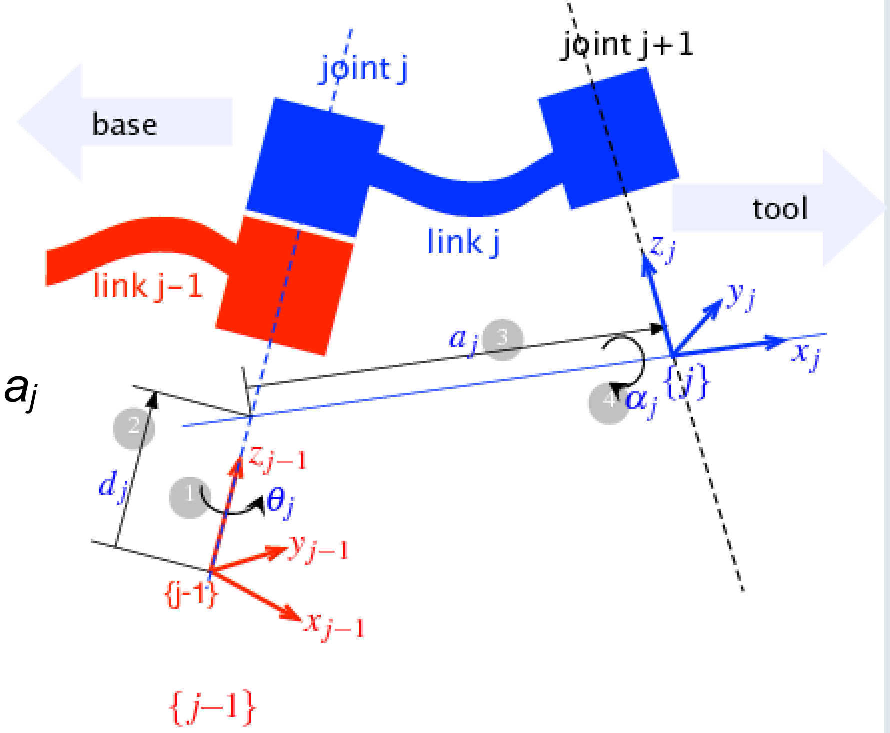
Parameters in joint *j* define the spatial relationship between link frame (*j* - 1) and *j*



Bildquelle: https://www.universal-robots.com/how-tos-and-faqs/faq/ur-faq/parameters-forcalculations-

Transformation between consecutive frames based on Denavit-Hartenberg-Parameters

- Angle between x_{j-1} and x_j about z_{j-1}
  ⇨ Rotation w.r.t. z_{j-1} (corresponds to joint angle): θ_j
- Distance from origin of frame $j-1$ to x_j axis along z_{j-1}
  ⇨ Translation along z_{j-1}: d_j
- Distance between z_{j-1} and z_j along x_j
  ⇨ Translation along x_j (corresponds to the link‘s length): a_j
- Angle between z_{j-1} and z_j about x_{j-1}
  ⇨ Rotation w.r.t. x_j : α_j
- Transformation of frame j in to $(j-1)$:



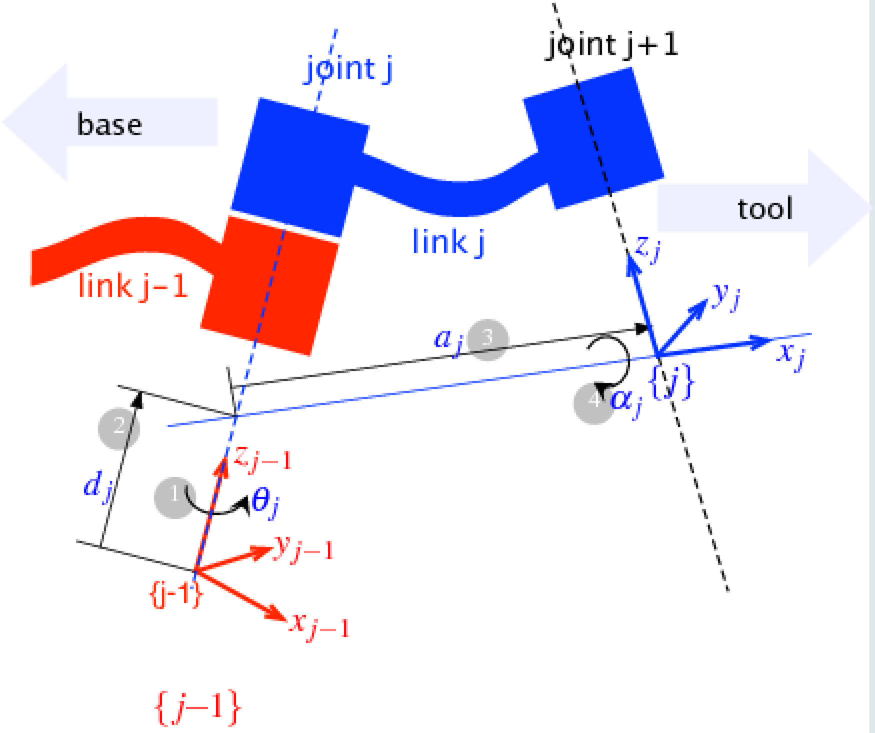
$$T_j^{j-1} = R_z(\theta_j)\tau_z(d_j)\tau_x(a_j)R_x(\alpha_j)$$

Bildquelle: Corke, Peter: Robotics, Vision And Control : Fundamental Algorithms In MATLAB. 2. Edition. Berlin [u.a.] : Springer, 2013

Transformation between consecutive frames based on Denavit-Hartenberg-Parameters

- Resolving the single transformation matrices gives the homogeneous transformation matrix:

$$T_j^{j-1} = \begin{pmatrix} \cos\theta_j & -\sin\theta_j \cos\alpha_j & \sin\theta_j \sin\alpha_j & a_j \cos\theta_j \\ \sin\theta_j & \cos\theta_j \cos\alpha_j & -\cos\theta_j \sin\alpha_j & a_j \sin\theta_j \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Bildquelle: Corke, Peter: Robotics, Vision And Control : Fundamental Algorithms In MATLAB. 2. Edition. Berlin [u.a.] : Springer, 2013

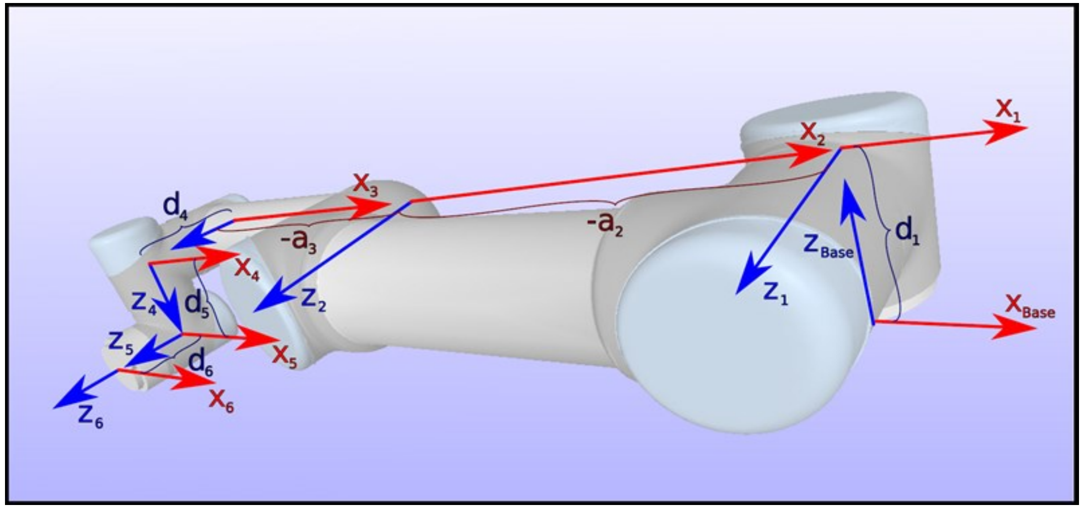
Forward kinematics

Description of the end effector's pose (E) in the world-cs (W) from the (fixed) geometry of the robot links and the (measured) joint parameters

-> In fact a set of cascaded transformations:

$$T_E^W = T_0^W \, T_6^0 \, T_E^6 \; mit$$

$$T_0^6 = \prod_{i=1}^{6} T_i^{i-1} = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5$$



Bildquelle: https://www.universal-robots.com/how-tos-and-faqs/faq/ur-faq/parameters-forcalculations-