I. Physikalisches Anfängerpraktikum

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Report for the Experiment

Resonanz (eng. »Reconance«)

(Experiment 13)

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FEZ

Inhaltsverzeichnis

I. Introduction

1.1. Objective and Motivation

The aim of this experiment is to investigate free, damped, and forced oscillations of a torsional pendulum. The focus is on determining the damping constant δ and the natural frequency ω_0 using different methods. First, the period of undamped oscillations is measured, then the decay of the amplitude in damped oscillations is analyzed, and finally the resonance curves of forced oscillations are studied. Comparing the results from these different approaches allows an evaluation of the measurement methods and provides deeper insight into the dynamics of damped and driven oscillations.

1.2. Theoretical Background

A torsional pendulum can be modeled as a harmonic oscillator, which in practice is always subject to damping due to friction or other resistive forces. In this experiment, damping is introduced by an eddy current brake: electromagnets generate a magnetic field in which the moving pendulum induces eddy currents. According to Lenz's law, these currents oppose the motion and decelerate the rotation of the pendulum.

1.2.1. Free Harmonic Oscillator

A free harmonic oscillator has no damping or external driving force. It is described by

$$\ddot{x} + \omega_0^2 x = 0, \tag{1}$$

with the general solution

$$x(t) = a\cos(\omega_0 t + \varphi), \tag{2}$$

where a and φ are constants determined by the initial conditions. The motion is purely sinusoidal with angular frequency ω_0 and period

$$T = \frac{2\pi}{\omega_0}. \tag{3}$$

In this case, the energy of the system is conserved and oscillates between kinetic and potential energy without losses.

1.2.2. Damped Oscillator

Real oscillators are always damped due to resistive forces. Adding damping to the equation of motion yields

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = 0. \tag{4}$$

Here δ is the damping constant, which quantifies energy loss per cycle.

Three regimes can be distinguished:

- **Underdamped** ($\delta < \omega_0$): Oscillations occur with exponentially decaying amplitude. This is the relevant case for our experiment.
- **Critically damped** ($\delta = \omega_0$): The system returns to equilibrium without oscillating, and in the shortest possible time.
- **Overdamped** ($\delta > \omega_0$): The system also returns without oscillating, but more slowly than in the critical case.

For the underdamped case, the solution is

$$x(t) = e^{-\delta t} a\cos(\omega_f t + \varphi), \tag{5}$$

where the damped angular frequency is

$$\omega_f = \sqrt{\omega_0^2 - \delta^2}. \tag{6}$$

The amplitude decays exponentially according to

$$a(t) = a_0 e^{-\delta t}. \tag{7}$$

The damping constant can be extracted from the half-life $t_{1/2}$ of the amplitude:

$$\delta = \frac{\ln 2}{t_{1/2}}. \tag{8}$$

1.2.3. Forced Oscillator

When an external periodic driving torque acts on the system, the equation of motion becomes

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = A\omega_0^2 \cos(\omega t), \tag{9}$$

with A the driving amplitude and ω the driving angular frequency.

The general solution consists of:

- A **transient part**, proportional to $e^{-\delta t}$, which decays after a short time.
- A **steady-state part**, which oscillates with the driving frequency ω and dominates after the transient vanishes.

The steady-state solution is

$$x(t) = b(\omega)\cos(\omega t - \epsilon(\omega)), \tag{10}$$

with amplitude

$$b(\omega) = \frac{A\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\omega)^2}}. \tag{11}$$

The maximum amplitude occurs at the resonance frequency

$$\omega' = \sqrt{\omega_0^2 - 2\delta^2}. \tag{12}$$

This value shifts below ω_0 as damping increases.

1.2.4. Resonance Curve

The resonance curve $b(\omega)$ describes how the oscillation amplitude depends on the driving frequency. Key parameters:

- **Resonance frequency:** ω', which decreases with stronger damping.
- **Resonance amplitude:** $b(\omega')$, which is smaller for stronger damping.
- **Half-width:**

$$H = \omega_2 - \omega_1 \approx 2\delta, \tag{13}$$

a measure of frequency selectivity.

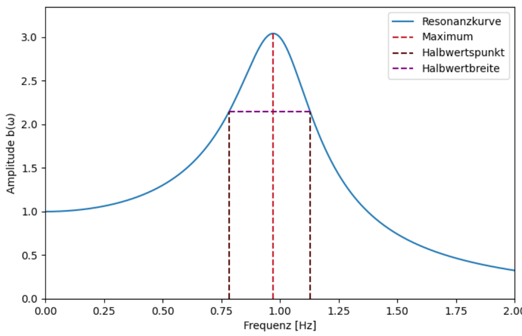

Abbildung I.1.: Schematic resonance curve $b(\omega)$ of a damped, forced oscillator.

The resonance amplification is

$$\frac{b(\omega')}{b(\omega \to 0)} \approx \frac{\omega_0}{2\delta}. \tag{14}$$

1.2.5. Quality Factor

The sharpness of resonance is described by the quality factor

$$Q = \frac{\omega_0}{H} = \frac{\omega_0}{2\delta}. \tag{15}$$

Large Q means weak damping, narrow resonance, and strong amplification. Small Q means strong damping, broad resonance, and weak amplification.

1.2.6. Forces Acting on the System

The pendulum dynamics are governed by four torques:

- **Restoring torque:** from torsional stiffness, proportional to angular displacement.
- **Damping torque:** from the eddy current brake, proportional to angular velocity.
- **Driving torque:** periodic torque applied by the motor in forced oscillation experiments.
- **Inertial torque:** due to the moment of inertia, opposing angular acceleration.

Their interplay determines whether the pendulum undergoes free, damped, or forced oscillation.

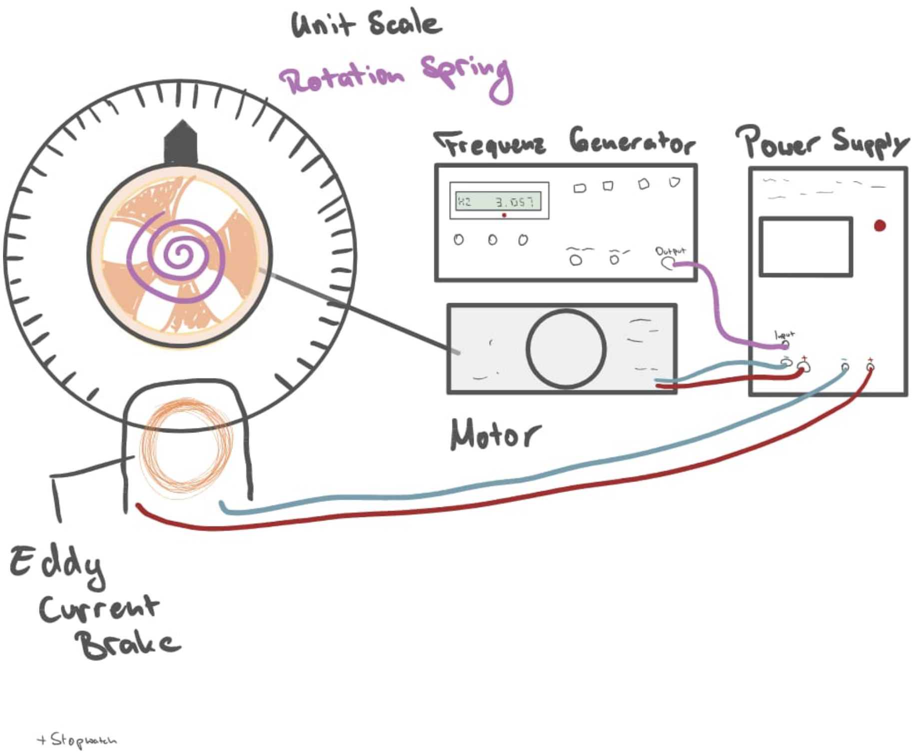

Abbildung I.2.: Schematic setup of the experiment

September 8th 2025

13 — Resonance

By Annika Künstle and Finn Zeumer

Table 1)

Messurment index	time t [s]	period [s]
1	35,37 ± 0,20	1,7685 ± 0,01
2	35,53 ± 0,20	1,7765 ± 0,01
3	35,29 ± 0,20	1,7645 ± 0,01

Amplitude: 10 Units

Table 2)

current value [mA]	Periods	Setteling time [s]
340 ± 10	15	26,81 ± 0,20
440 ± 10	10	17,42 ± 0,20

Amplitude: 18 Units

Table 3)

Period	340 [Units] 1th	2th	440mA [Units] first	second
1	15	14,6	14,2	13,2
2	13	12,2	10,4	10,6
3	11	10,2	8,0	7,4
4	8,8	8,6	5,8	5,2
5	7,6	6,8	4,2	3,8
6	6,4	5,8	3,6	2,6
7	5,2	4,6	2,6	1,8
8	4,6	3,8	2,0	1,4
9	3,8	3,0	1,0	0,8
10	3,2	2,4	1,0	0,4
11	2,6	1,8	—	
12	2,2	1,4	—	
13	1,8	1,0	—	
14	1,6	0,8	—	
15	1,2	0,4	—	

$T_{340} = (1,5873 \pm 0,013)\,s$

$T_{440} = (1,7420 \pm 0,02)\,s$

Tabel 4) 340 mA

Frequency [Hz]	Stady state - Amplitude [Units]
301 ± 2	0,6 ± 0,1
500	0,6
701	0,6
899	0,7
1100	0,7
1300	0,8
1500	0,9
1700	1,0
1898	1,3
2112 ± 4	2,3
2304	4,8
2524	1,6
2700	1,1
2908	0,8
3100	0,7
3307	0,6
3517	0,5
3697	0,5
3912	0,4
2158	3,0
2208	4,0
2249	5,4
2304	4,8
2350	3,4
2399	2,6
2458	2,0

Function Generator
1k → 10k

Tabel 5) 440 mA

Frequency [Hz]	Stady state - Amplitude [Units]
301 ± 2	0,6 ± 0,1
500	0,6
698	0,7
902	0,7
1100	0,8
1303	0,8
1504	0,9
1703	1,0
1906	1,3
2108 ± 4	2,1
2310	3,2
2502	1,6
2708	1,0
2905	0,8
3095	0,7
3297	0,6
3495	0,6
3705	0,5
3894	0,4
2153	2,5
2200	2,9
2255	3,4
2304	3,2
2358	2,6
2400	2,2
2449	1,8

Equipment:

- Stopwatch: Digets: 0,01 s error: unkown
- Damping-coil :

II. Execution

2.1. Experimental Setup

The experiment consists of a rotary pendulum whose oscillations can be damped by an eddy current brake. The brake is controlled by adjustable currents through the inductors. A stepper motor driven by a function generator is used to excite forced oscillations. The stepper motor rotates by 0.72° per control pulse, with 500 steps required for a full rotation. Due to microstepping, one full step requires eight impulses, so a control signal of 4000 Hz corresponds to one rotation per second of the motor shaft. The function generator must be set to TTL-level square wave output in the frequency range "1 k."

2.2. Measuring Procedure

Task 1: Sketch of the Experimental Setup

Draw a schematic diagram of the experimental arrangement, including the pendulum, eddy current brake, stepper motor, and function generator.

Task 2: Determination of the Natural Period T_0

Deflect the undamped pendulum and measure its oscillation period. Perform three measurements, each over 20 oscillations, and calculate T_0.

Task 3: Determination of the Settling Time for Different Damping

Switch on the eddy current brake. Observe qualitatively how the oscillation amplitude depends on the current through the inductors. Set the two current values indicated on the aperture. For these values, the oscillation amplitude decreases to 5% of its initial value after 10 and 15 oscillations, respectively. Record the time required for the amplitude to reach 5% in each case. These times are the settling times and will be needed in Task 5. *Note: It is recommended to complete Tasks 4 and 5 for one current setting before switching to the other value.*

Task 4: Free Oscillations with Damping

For the two current values from Task 3, measure the oscillation period T_f and the decrease of the amplitude. At $t = 0$, release the pendulum at its turning point. After each oscillation, record the amplitude at the reversal point. Repeat this measurement twice for each damping value. If necessary, work with a partner to ensure accurate recording.

Task 5: Forced Oscillations

Excite the pendulum using the stepper motor and function generator. Measure the stationary amplitude as a function of the excitation frequency between 300 Hz and 4 kHz for both damping values. Use frequency steps of 200 Hz, but reduce to 50 Hz near the resonance frequency. At each frequency, wait until transient oscillations decay (settling times from Task 3) before recording the stationary amplitude. Measure the amplitude exactly at the resonance frequency and at the half-width frequencies where $b = 0.7b_{\max}$. Additionally, observe the phase difference between the excitation and the pendulum at low, high, and resonance frequencies.

It is woth mentoning that we switched between »1K« and »10K« mode, therefore the values were less accurate. The switch is marked in the protocol.

III. Evaluation

Error Analysis

For the statistical evaluation of n measured values x_i, the following quantities are defined [Wag25]:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{Arithmetic mean} \tag{1}$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \text{Variance} \tag{2}$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \text{Standard deviation} \tag{3}$$

$$\Delta\bar{x} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{x} - x_i)^2} \qquad \text{Error of the mean} \tag{4}$$

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2} \qquad \text{Gaussian error propagation for } f(x,y) \tag{5}$$

$$\Delta f = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad \text{Error for } f = x + y \tag{6}$$

$$\Delta f = |a|\Delta x \qquad \text{Error for } f = ax \tag{7}$$

$$\frac{\Delta f}{|f|} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \qquad \text{Relative error for } f = xy \text{ or } f = x/y \tag{8}$$

$$\sigma = \frac{|a_{\text{lit}} - a_{\text{meas}}|}{\sqrt{\Delta a_{\text{lit}}^2 + \Delta a_{\text{meas}}^2}} \qquad \text{Calculation of significant deviation} \tag{9}$$

3.1. Pendulum Oscillation Period

First, the period of an undamped harmonic oscillator is determined. The measured data are shown in Table 1:

Index	t [s]	Δt [s]	T [s]	ΔT [s]
1	35.37	0.20	1.7685	0.01
2	35.53	0.20	1.7765	0.01
3	35.29	0.20	1.7645	0.01

Tabelle III.1.: Measured oscillation times for 20 oscillations and calculated periods. Uncertainties in time and period are indicated by Δt and ΔT, respectively.

The time uncertainty Δt is based on an estimated human reaction time of 200 ms. The stopwatch error is negligible.

From Table 1, the average period is calculated as

$$\boxed{\bar{T_0} = (1.77 \pm 0.01)\,\mathrm{s}} \tag{10}$$

Having the period we can also calculate the natural frequency using equation 1.3:

$$\omega_0 = \frac{2\pi}{\overline{T_0}} = 3.5498222. \tag{11}$$

Furthermore we need to use Gaussian error propagation:

$$\Delta\omega_0 = \frac{\Delta\overline{T_0}}{\overline{T_0}}\omega_0 = 0.0200554, \tag{12}$$

to get to the resulting natural frequenzy of the pole:

$$\boxed{\omega_0 = (3.54952 \pm 0.02006)s^{-1}} \tag{13}$$

3.2. Calculation of the Damping Constant δ

With the period T_0 known, the next step is to determine the damping constant δ using graphical analysis. Data from the protocol (Tables 2 and 3) are used to construct a decay curve for each. You will finde the results in figure 3.1. We fit a line and an error line to each dataset and determine the number of oscillations after which the amplitude is only one fourth of the starting amplitude. This is double the amount of oscillations it takes for the amplitude to halve ($n_{\frac{1}{2}}$).

Next we need to calculate the half-life time $t_{\frac{1}{2}}$ and its uncertainty. Therefore we need the oscillation periods T_f and its uncertainty ΔT_f, or rather T_{340}, T_{440} and their uncertainties $\Delta T_{340} = 0.013$ and $\Delta T_{440} = 0.02$. The halftime is calculated using

$$t_{\frac{1}{2}} = n_{\frac{1}{2}} \cdot T_f, \tag{14}$$

and its uncertainty using

$$\Delta t_{\frac{1}{2}} = \sqrt{(T_f \cdot \Delta n_{\frac{1}{2}})^2 + (n_{\frac{1}{2}} \cdot \Delta T_f)^2}. \tag{15}$$

First we get the results for the T_f

I **[mA]**	**Periods**	t [s]
340 ± 10	15	26.81 ± 0.20
440 ± 10	10	17.42 ± 0.20

Tabelle III.2.: Measured settling times and number of periods for two different current values. Uncertainties are indicated for current and time.

The corresponding periods for the two damping currents are calculated as:

$$\boxed{\begin{aligned} T_{340\,\mathrm{mA}} &= (1.787 \pm 0.013)\,\mathrm{s} \\ T_{440\,\mathrm{mA}} &= (1.742 \pm 0.020)\,\mathrm{s} \end{aligned}} \tag{16}$$

We get the results of Tabel 3 from the protocol:

Nb	340 mA [Un] $A_{w,Avg}$	$\Delta A_{w,Avg}$	440 mA [Un] $A_{s,Avg}$	$\Delta A_{s,Avg}$
1	14.8	2.48	13.70	2.33
2	12.6	1.89	10.20	1.40
3	10.6	1.36	7.70	0.70
4	8.7	0.85	5.50	0.14
5	7.2	0.45	4.00	0.26
6	6.1	0.16	3.10	0.50
7	4.9	0.16	2.20	0.70
8	4.2	0.35	1.70	0.90
9	3.4	0.56	0.90	1.09
10	2.8	0.73	0.70	1.14
11	2.2	0.89	-	-
12	1.8	0.99	-	-
13	1.4	1.10	-	-
14	1.2	1.15	-	-
15	0.8	1.26	-	-

Tabelle III.3.: Combined results for weak (w) damping (340 mA) and strong (s) damping (440 mA). The first 15 rows show measured amplitudes and uncertainties, while the last four rows provide derived quantities for 340 mA. Dashes indicate values not available for the strong damping case.

The data from this table is displayed in the figure 3.1, include the error lines.

Now we can use equation 1.8 to calculate the properties of the weak and the strong damping. We once again use gaussian propagation to calculate the uncertainty

$$\Delta\delta = \frac{\Delta t_{\frac{1}{2}}}{t_{\frac{1}{2}}} \cdot \delta. \tag{17}$$

The results of all calculations can be found in the table 3.4 below

The important result are the dumping constants that were calculate by using the data in

Property	**340 mA (w)** Res	Δ Res	**440 mA (s)** Res	Δ Res
$n_{\frac{1}{2}}$	3.15	0.25	1.93	0.09
T_f [s]	1.787	0.013	1.742	0.020
$t_{\frac{1}{2}}$ [s]	5.6	0.4	3.36	0.16
δ [s^{-1}]	0.123	0.009	0.206	0.010

Tabelle III.4.: Comparison of derived properties for weak (340 mA) and strong (440 mA) damping.

the table:

$$\boxed{\begin{aligned} \delta_{340} &= (0.123 \pm 0.009)s^{-1} \\ \delta_{440} &= (0.206 \pm 0.010)s^{-1} \end{aligned}} \tag{18}$$

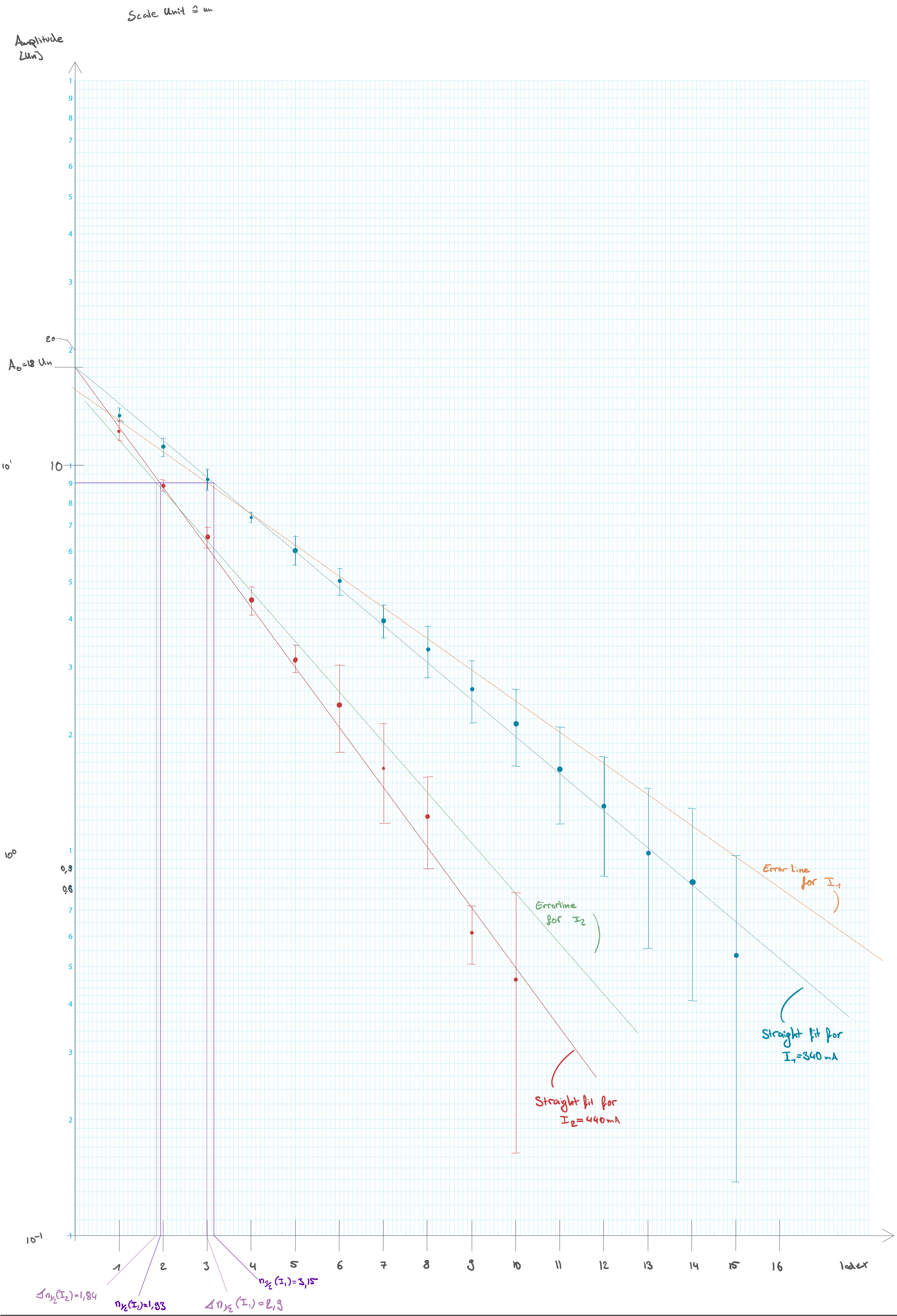

Abbildung III.1.: Amplitude vs. number of oscillations on semi-logarithmic axes. Blue (weak damping, 340mA) and red (strong damping, 440mA) markers show the data with orange and green error bars, respectively. Best-fit lines are included, and purple/magenta markers indicate half-amplitude points.

3.3. Finding the damping constant δ using the resonance curve

In this section we once again want to determine the damping constant δ, but this time, we plot the resunance curve. We do this twice, once for the strong and once for the weak dumping. We will need two different values: the resonance frequency and the half-width for each dumping. We will use the data from Table 4 and 5 from the protocol. Some values where analysed graphical, hence you find the resonance curve strong dumping and for weak dumping.

Resonance Frequency

Let us start by determining the frequency of both our curves. To determine the resonance frequency from our plots, we identify the generator frequency f' at which the maximum of the resonance curve occurs. Since the measurements around the resonance were taken in steps of 50 Hz, we can estimate the uncertainty as $\Delta f' = 40\,\text{Hz}$.

In order to obtain the angular resonance frequency of the driving motor, the generator frequency must first be converted. According to the experiment manual, the motor frequency is related to the generator frequency by a factor of 3900. Additionally, we need to apply the standard conversion factor 2π between frequency and angular frequency. Thus, the resonance angular frequency is calculated as

$$\omega' = 2\pi \frac{f'}{3900}. \tag{19}$$

Applying Gaussian error propagation, the corresponding uncertainty is given by

$$\Delta\omega' = 2\pi \frac{\Delta f'}{3900}. \tag{20}$$

Doing this will lead to the results in the tabel 3.5:

Property	340 mA (w)		440 mA (s)	
	Res.	Δ Res	Res	Δ Res
f' [Hz]	2240	40	2225	40
ω' [s^{-1}]	3.61	0.06	3.58	0.06
σ_{ω_0}	0.96σ		0.48σ	

Tabelle III.5.: Comparison of resonance frequencies for weak (340 mA) and strong (440 mA) damping.

I also added the significant deviation (3.9) from ω_0.

Half-Width

To determine the damping constant δ, we first use the half-width of the resonance curve. Specifically, we identify in Figures III.2 and Figures III.3 the points where the resonance curve drops to $1/\sqrt{2}$ of its maximum value. These points, marked in the figures, define the frequency difference corresponding to the half-width H' of the curve. The associated uncertainty is estimated as $\Delta H' = 40\,\text{Hz}$.

$$H = 2\pi \frac{H'}{3900}, \tag{21}$$

with the corresponding uncertainty obtained through Gaussian error propagation:

$$\Delta H = 2\pi \frac{\Delta H'}{3900}. \tag{22}$$

Finally, from equation 1.13, the damping constant can be expressed as

$$\delta = \frac{H}{2}, \qquad \Delta\delta = \frac{\Delta H}{2}. \tag{23}$$

Doing this will lead to the results in the tabel 3.6:

Resonance Amplification

Lastly, we will use the reosnace amplification (1.14) to determine the damping constant. Therefore, we need $b(\omega')$ and $b(\omega \to 0)$, as

Property	**340 mA (w)**		**440 mA (s)**	
	Res.	Δ Res	Res	Δ Res
H' [Hz]	120	40	260	40
H [s^{-1}]	0.19	0.06	0.42	0.06
δ [s^{-1}]	0.095	0.03	0.21	0.03

Tabelle III.6.: Comparison of half-width for weak (340 mA) and strong (440 mA) damping.

Property	**340 mA (w)**		**440 mA (s)**	
	Res.	Δ Res	Res	Δ Res
$b(\omega')$ [Units]	5.4	0.1	3.4	0.1
δ [s^{-1}]	0.13	0.03	0.21	0.05

Tabelle III.7.: Values for dumping using resonance amplification.

well as ω_0. The uncertainty for $b(\omega)$ is assumed to be 0.1 [Units]. Since the curve is quite flat at the beginning, we can assume that $b(\omega \to 0) = (0.4 \pm 0.1) Units$. We use equation 1.12 and rearange it:

$$\delta = \frac{\omega_0 \cdot b(\omega \to 0)}{2 \cdot b(\omega')}, \tag{24}$$

and calculate its uncertainty using gaussian error propagation

$$\Delta\delta = \sqrt{\left(\frac{\Delta\omega_0}{\omega_0}\right)^2 + \left(\frac{\Delta b(\omega \to 0)}{b(\omega \to 0)}\right)^2 + \left(\frac{\Delta b(\omega')}{b(\omega')}\right)^2} \cdot \delta. \tag{25}$$

We will set the results into a table to compate one again weak and strong daming:

Comparison of the Calculated Damping Values.

Now we have three different damping values and want to compare all of them. Here are the results from (Tabel 3.4), named δ_g, (Tabel 3.6), named δ_h, and (Tabel 3.7), named δ_a for weak (w) and strong (s) damping:

$$\delta_{g,w} = (0.123 \pm 0.009)\,\mathrm{s}^{-1} \tag{26}$$
$$\delta_{g,s} = (0.206 \pm 0.010)\,\mathrm{s}^{-1} \tag{27}$$

$$\delta_{h,w} = (0.095 \pm 0.03)\,\mathrm{s}^{-1} \tag{28}$$
$$\delta_{h,s} = (0.210 \pm 0.03)\,\mathrm{s}^{-1} \tag{29}$$

$$\delta_{a,w} = (0.130 \pm 0.03)\,\mathrm{s}^{-1} \tag{30}$$
$$\delta_{a,s} = (0.208 \pm 0.05)\,\mathrm{s}^{-1} \tag{31}$$

Now I will calculate all possible significant deviation: First for all weak dumping values diviations:

$$\delta_{g,h,w} = 0.89\sigma \tag{32}$$
$$\delta_{g,a,w} = 0.22\sigma \tag{33}$$
$$\delta_{h,a,w} = 0.82\sigma \tag{34}$$
$$\tag{35}$$

and for the strong dumpings:

$$\delta_{g,h,s} = 0.13\sigma \tag{36}$$
$$\delta_{g,a,s} = 0.04\sigma \tag{37}$$
$$\delta_{h,a,s} = 0.03\sigma \tag{38}$$
$$\tag{39}$$

These values are all increadbly close, especially for the strong dumping.

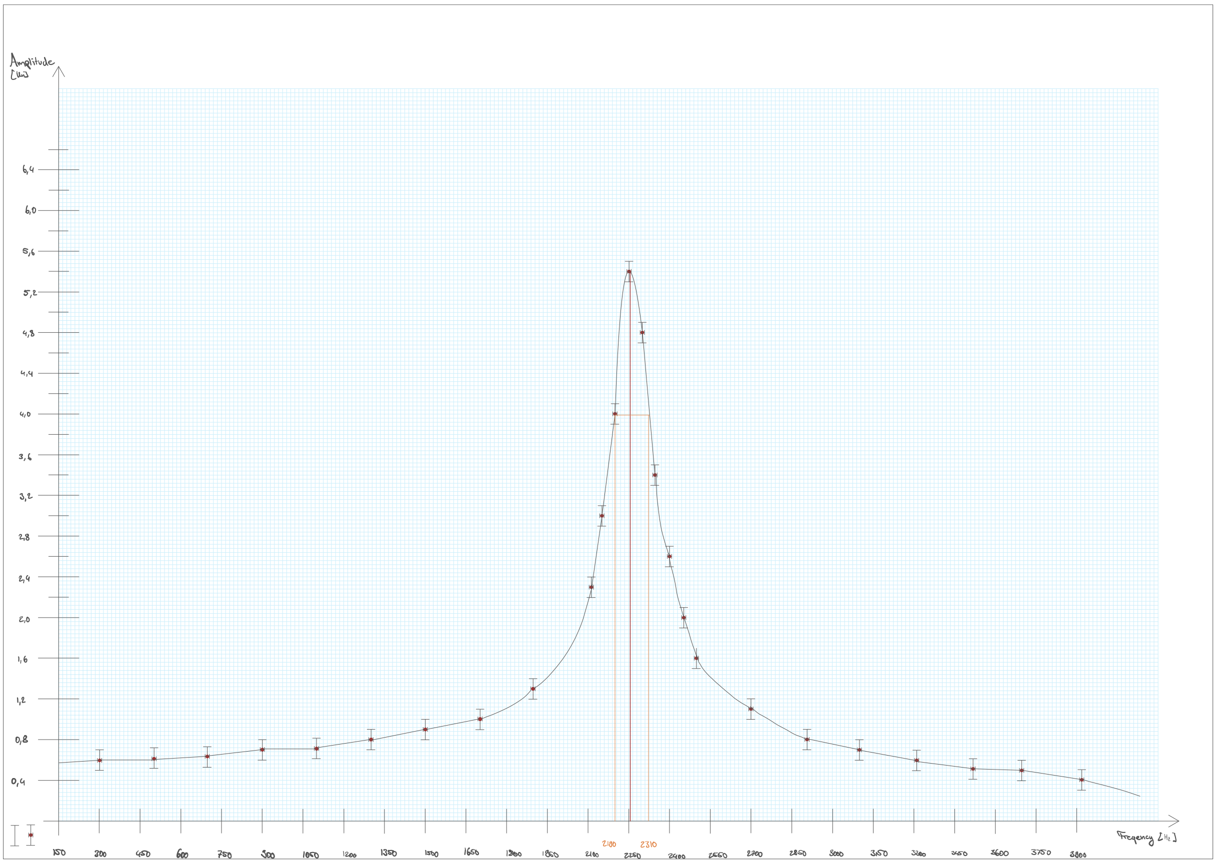

Abbildung III.2.: Resonance curve for stong dumping (440mA). The red line is the maximum ω' and orange is the half-width H.

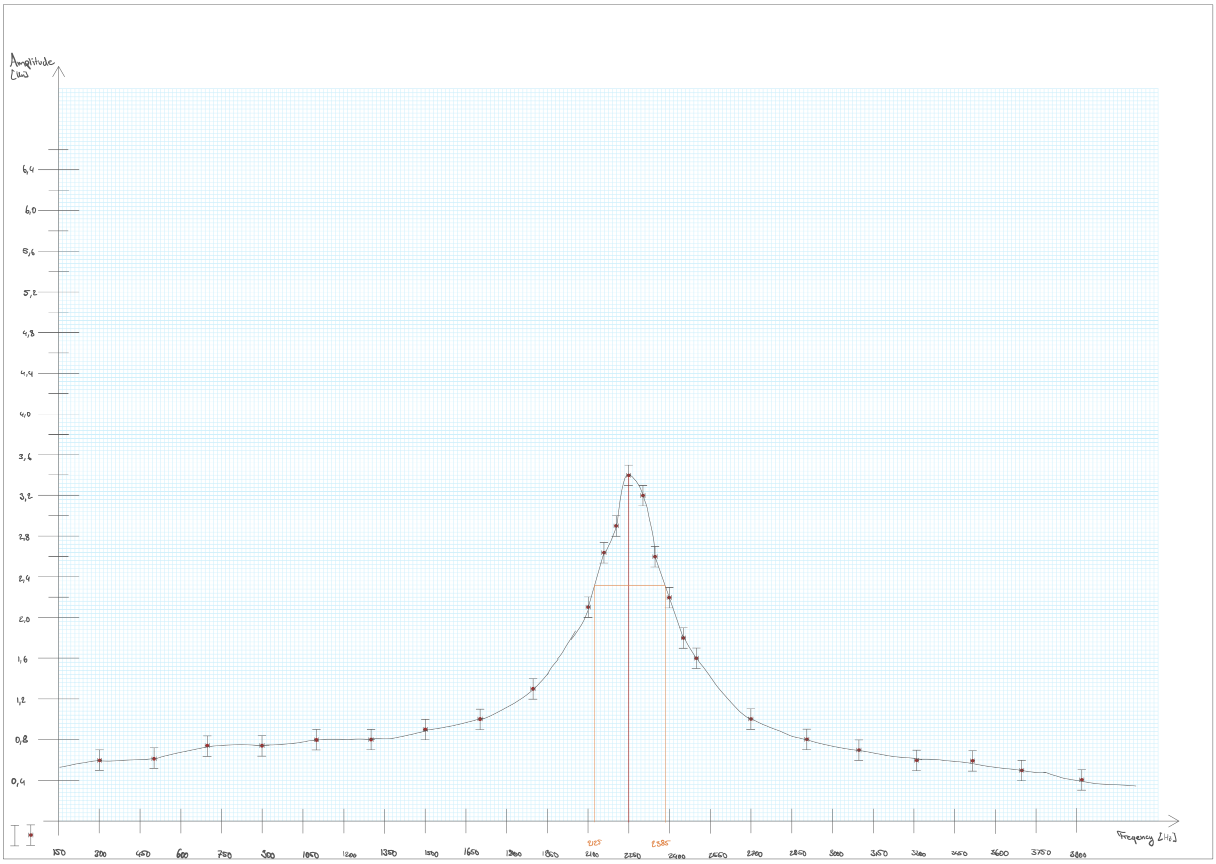

Abbildung III.3.: Resonance curve for weak dumping (340mA). The red line is the maximum ω' and orange is the half-width H.

IV. Discussion

Summary of Results

In this experiment, the oscillation period and damping properties of a pendulum with an eddy current brake were measured and analyzed. The period of the undamped pendulum was found to be

$$\bar{T_0} = (1.77 \pm 0.01)\,\mathrm{s},$$

which corresponds to a natural angular frequency of

$$\omega_0 = (3.55 \pm 0.02)\,\mathrm{s}^{-1} \quad \text{(Table III.1)}.$$

Damping constants were determined using three different methods: logarithmic decrement, the half-width of the resonance curve, and resonance amplification. The results for weak damping (340 mA) are

$$\delta_{g,w} = 0.123 \pm 0.009\,\mathrm{s}^{-1},$$
$$\delta_{h,w} = 0.095 \pm 0.03\,\mathrm{s}^{-1},$$
$$\delta_{a,w} = 0.130 \pm 0.03\,\mathrm{s}^{-1},$$

and for strong damping (440 mA) they are

$$\delta_{g,s} = 0.206 \pm 0.010\,\mathrm{s}^{-1},$$
$$\delta_{h,s} = 0.210 \pm 0.03\,\mathrm{s}^{-1},$$
$$\delta_{a,s} = 0.208 \pm 0.05\,\mathrm{s}^{-1}.$$

These values show good agreement across all methods, with deviations generally below 1σ, particularly for the strong damping case.

Discussion of Results

The data confirm that increasing the eddy current leads to higher damping constants, consistent with theoretical expectations. The logarithmic decrement method and the resonance-based methods provide consistent results, though uncertainties are slightly larger for weak damping. This can be attributed to the smaller amplitudes and fewer oscillations available for analysis, which naturally increase measurement variability. In the case of strong damping, all three methods are in excellent agreement, suggesting that the experimental procedure is reliable when damping is significant.

The measured resonance frequencies ω' were found to be very close to the natural frequency ω_0, and the deviations observed are well within the expected uncertainties (Table III.5). The analysis of the resonance curves, shown in Figures III.2 and III.3, demonstrates the expected behavior: as damping increases, the resonance peak broadens and its amplitude decreases. This trend aligns with theoretical predictions and further validates the experiment.

Possible Criticisms and Sources of Error

Despite the overall consistency of the results, some limitations should be noted. The measurement of the pendulum period relied on manual timing, with an estimated reaction time of 200 ms. Although this uncertainty was included in the analysis, it could still introduce a small systematic bias, especially for shorter measurement intervals. Another limitation arises from the finite frequency resolution of the resonance curves. Frequency steps of 50 Hz may not fully capture the peak, particularly in the weak damping case, which reduces the precision of δ_h and δ_a. The experiment also assumed ideal harmonic motion, neglecting minor effects such as friction, air resistance, and non-uniform magnetic fields, which may slightly affect the damping constants. Additionally, some amplitude measurements for strong damping were either missing or unreliable at small amplitudes, which could influence the logarithmic decrement calculation to a minor degree.

Conclusion

In conclusion, the experiment successfully measured both the natural frequency and the damping properties of the pendulum under varying eddy currents. The results are in agreement with theoretical expectations and show the anticipated trends for weak and strong damping. While minor uncertainties due to timing limitations and data resolution exist, these do not significantly impact the overall conclusions. The experiment thus provides a clear and reliable characterization of the damping behavior of the pendulum system.

Abbildungsverzeichnis

Tabellenverzeichnis

Literaturverzeichnis

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