I. Physikalisches Anfängerpraktikum

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Report for the Experiment

Resonanz (eng. »Reconance«)

(Experiment 13)

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Inhaltsverzeichnis

I. Introduction

1.1. Objective and Motivation

The aim of this experiment is to investigate free and forced oscillations of a torsional pendulum and to determine the damping constant δ using different methods. First, the period of undamped oscillations is measured, then the decrease of the amplitude in damped oscillations is analyzed, and finally the resonance curves of forced oscillations are studied. Comparing the values obtained from different approaches allows an evaluation of the measurement methods and provides deeper insight into the behavior of damped and driven oscillations.

1.2. Theoretical Background

[Wag25a] A torsional pendulum can be regarded as a system of harmonic oscillations, which in practice are always damped due to friction or other resistive forces. In this experiment, damping is introduced by an eddy current brake: electromagnets generate a magnetic field in which the moving pendulum induces eddy currents. According to Lenz's law, these currents oppose the motion and decelerate the rotation of the pendulum.

For a free, damped oscillation of the pendulum, the temporal evolution of the amplitude is given by

$$a(t) = a_0\, e^{-\delta t} \sin(\omega_f t), \tag{1}$$

where δ is the damping constant and ω_f the angular frequency of the damped oscillation. At the turning points this simplifies to

$$a(t) = a_0\, e^{-\delta t}. \tag{2}$$

The damping constant δ can be determined from the exponential decay of the amplitude, for example using the half-life $t_{1/2}$:

$$\delta = \frac{\ln 2}{t_{1/2}}. \tag{3}$$

The angular frequencies of the undamped oscillation ω_0 and the damped oscillation ω_f are related by

$$\omega_f = \sqrt{\omega_0^2 - \delta^2}. \tag{4}$$

If the pendulum is driven by a periodic torque, a stationary oscillation with the frequency of the driver is established after transient effects decay. The amplitude of this forced oscillation depends on the driving frequency ω and can be described by the resonance curve:

$$b(\omega) = \frac{A\,\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\delta\,\omega)^2}}. \tag{5}$$

The maximum amplitude occurs at the resonance frequency ω':

$$\omega' = \sqrt{\omega_0^2 - 2\delta^2}. \tag{6}$$

ADD LABELS FOR THE POINTS IN THE FIGURE HERE!

Important quantities to characterize the resonance curve are the half-width H and the resonance enhancement:

$$H = \omega_2 - \omega_1 = 2\delta, \quad \frac{b(\omega')}{b(\omega \to 0)} = \frac{\omega_0}{2\delta}. \tag{7}$$

By measuring free and forced oscillations, the damping constant δ and the natural frequency of the torsional pendulum can be determined in several ways and compared.

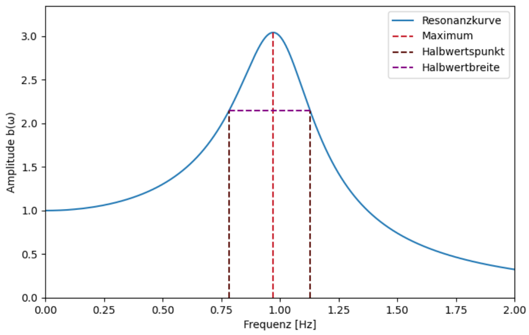

Abbildung I.1.: Schematic resonance curve $b(\omega)$ of a damped, forced oscillator.

Forces Acting on the System

The dynamics of the rotary pendulum are determined by the following torques and forces:

- **Restoring torque:** Generated by the torsional stiffness of the pendulum axis, it acts to return the pendulum to its equilibrium position. It is proportional to the angular displacement.
- **Damping torque:** Produced by the eddy current brake. Eddy currents induced in the conductor oppose the motion (Lenz's law), leading to an angular velocity–dependent resistive torque.
- **Driving torque:** In forced oscillation experiments, the stepper motor provides a periodic torque that excites the pendulum at the frequency set by the function generator.
- **Inertial torque:** Due to the moment of inertia of the pendulum, resisting angular acceleration according to Newton's second law for rotation.

The interplay of these torques determines whether the system undergoes free oscillation, damped oscillation, or driven oscillation.

Influence of Damping on the Resonance Curve

The resonance curve is characterized by the resonance frequency ω', the amplitude at resonance $b(\omega')$, and the half-width H.

- **Resonance frequency:** With increasing damping, the resonance frequency ω' shifts to lower values compared to the natural frequency ω_0.
- **Resonance amplitude:** The maximum amplitude $b(\omega')$ decreases as damping increases, since more energy is dissipated per cycle.
- **Half-width:** The half-width $H = 2\delta$ becomes larger with increasing damping. Thus, the resonance curve broadens as the system becomes less selective in frequency response.

In summary, higher damping reduces the resonance peak, shifts it to lower frequency, and widens the curve.

Quality Factor of a Resonator

The quality factor Q is a dimensionless parameter that characterizes the sharpness of resonance. It is defined as the ratio of the resonance frequency to the half-width of the resonance curve:

$$Q = \frac{\omega_0}{H} = \frac{\omega_0}{2\delta}. \tag{8}$$

A high Q indicates weak damping, a narrow resonance curve, and high resonance amplification. A low Q corresponds to strong damping, a broad resonance curve, and low amplification.

September 8th 2025

13 — Resonance

by Annika Kanstle and Finn Zeumer

Table 1)

Messurment index	time t [s]	period [s]
1	35,37 ± 0,20	1,7685 ± 0,01
2	35,53 ± 0,20	1,7765 ± 0,01
3	35,29 ± 0,20	1,7645 ± 0,01

Amplitude: 10 Units

Table 2)

current value [mA]	Periods	Setteling time [s]
340 ± 10	15	26,81 ± 0,20
440 ± 10	10	17,42 ± 0,20

Amplitude: 18 Units

Table 3)

Period	340 [Units] 1th	340 [Units] 2th	440mA [Units] first	440mA [Units] second
1	15	14,6	14,2	13,2
2	13	12,2	10,4	10,6
3	11	10,2	8,0	7,4
4	8,8	8,6	5,8	5,2
5	7,6	6,8	4,2	3,8
6	6,4	5,8	3,6	2,6
7	5,2	4,6	2,6	1,8
8	4,6	3,8	2,0	1,4
9	3,8	3,0	1,0	0,8
10	3,2	2,4	1,0	0,4
11	2,6	1,8	—	—
12	2,2	1,4	—	—
13	1,8	1,0	—	—
14	1,6	0,8	—	—
15	1,2	0,4	—	—

$T_{340} = (1,5873 \pm 0,013)\,s$

$T_{440} = (1,7420 \pm 0,02)\,s$

Tabel 4) 340 mA

Frequency [Hz]	Steady state - Amplitude [Units]
301 ± 2	0,6 ± 0,1
500	0,6
701	0,6
899	0,7
1100	0,7
1300	0,8
1500	0,9
1700	1,0
1898	1,3
2112 ± 4	2,3
2304	4,8
2524	1,6
2700	1,1
2908	0,8
3100	0,7
3307	0,6
3517	0,5
3697	0,5
3912	0,4
2158	3,0
2208	4,0
2249	5,4
2304	4,8
2350	3,4
2399	2,6
2458	2,0

Function Generator
1k → 10k

Tabel 5) 440 mA

Frequency [Hz]	Steady state - Amplitude [Units]
301 ± 2	0,6 ± 0,1
500	0,6
698	0,7
902	0,7
1100	0,8
1303	0,8
1504	0,9
1703	1,0
1906	1,3
2108 ± 4	2,1
2310	3,2
2502	1,6
2708	1,0
2905	0,8
3095	0,7
3297	0,6
3495	0,6
3705	0,5
3894	0,4
2153	2,5
2200	2,9
2255	3,4
2304	3,2
2358	2,6
2400	2,2
2449	1,8

Equipment:

- Stopwatch: Digits: 0,01 s   error: unknown
- Damping-Coil :

II. Execution

2.1. Experimental Setup

The experiment consists of a rotary pendulum whose oscillations can be damped by an eddy current brake. The brake is controlled by adjustable currents through the inductors. A stepper motor driven by a function generator is used to excite forced oscillations. The stepper motor rotates by 0.72° per control pulse, with 500 steps required for a full rotation. Due to microstepping, one full step requires eight impulses, so a control signal of 4000 Hz corresponds to one rotation per second of the motor shaft. The function generator must be set to TTL-level square wave output in the frequency range "1 k."

2.2. Measuring Procedure

Task 1: Sketch of the Experimental Setup

Draw a schematic diagram of the experimental arrangement, including the pendulum, eddy current brake, stepper motor, and function generator.

Task 2: Determination of the Natural Period T_0

Deflect the undamped pendulum and measure its oscillation period. Perform three measurements, each over 20 oscillations, and calculate T_0.

Task 3: Determination of the Settling Time for Different Damping

Switch on the eddy current brake. Observe qualitatively how the oscillation amplitude depends on the current through the inductors. Set the two current values indicated on the aperture. For these values, the oscillation amplitude decreases to 5% of its initial value after 10 and 15 oscillations, respectively. Record the time required for the amplitude to reach 5% in each case. These times are the settling times and will be needed in Task 5. *Note: It is recommended to complete Tasks 4 and 5 for one current setting before switching to the other value.*

Task 4: Free Oscillations with Damping

For the two current values from Task 3, measure the oscillation period T_f and the decrease of the amplitude. At $t = 0$, release the pendulum at its turning point. After each oscillation, record the amplitude at the reversal point. Repeat this measurement twice for each damping value. If necessary, work with a partner to ensure accurate recording.

Task 5: Forced Oscillations

Excite the pendulum using the stepper motor and function generator. Measure the stationary amplitude as a function of the excitation frequency between 300 Hz and 4 kHz for both damping values. Use frequency steps of 200 Hz, but reduce to 50 Hz near the resonance frequency. At each frequency, wait until transient oscillations decay (settling times from Task 3) before recording the stationary amplitude. Measure the amplitude exactly at the resonance frequency and at the half-width frequencies where $b = 0.7b_{\text{max}}$. Additionally, observe the phase difference between the excitation and the pendulum at low, high, and resonance frequencies.

It is woth mentoning that we switched between »1K« and »10K« mode, therefore the values were less accurate. The switch is marked in the protocol.

III. Evaluation

Error Analysis

For the statistical evaluation of n measured values x_i, the following quantities are defined [Wag25b]:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{Arithmetic mean} \tag{1}$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \text{Variance} \tag{2}$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \text{Standard deviation} \tag{3}$$

$$\Delta\bar{x} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{x} - x_i)^2} \qquad \text{Error of the mean} \tag{4}$$

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2} \qquad \text{Gaussian error propagation for } f(x,y) \tag{5}$$

$$\Delta f = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad \text{Error for } f = x + y \tag{6}$$

$$\Delta f = |a|\Delta x \qquad \text{Error for } f = ax \tag{7}$$

$$\frac{\Delta f}{|f|} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \qquad \text{Relative error for } f = xy \text{ or } f = x/y \tag{8}$$

$$\sigma = \frac{|a_{\text{lit}} - a_{\text{meas}}|}{\sqrt{\Delta a_{\text{lit}}^2 + \Delta a_{\text{meas}}^2}} \qquad \text{Calculation of significant deviation} \tag{9}$$

3.1. Pendulum Oscillation Period

First, the period of an undamped harmonic oscillator is determined. The measured data are shown in Table 1:

Index	t [s]	Δt [s]	T [s]	ΔT [s]
1	35.37	0.20	1.7685	0.01
2	35.53	0.20	1.7765	0.01
3	35.29	0.20	1.7645	0.01

Tabelle III.1.: Measured oscillation times for 20 oscillations and calculated periods. Uncertainties in time and period are indicated by Δt and ΔT, respectively.

The time uncertainty Δt is based on an estimated human reaction time of 200 ms. The stopwatch error is negligible.

From Table 1, the average period is calculated as

$$\boxed{\bar{T_0} = (1.77 \pm 0.01)\,\mathrm{s}} \tag{10}$$

3.2. Calculation of the Damping Constant

With the period T_0 known, the next step is to determine the damping constant δ using graphical analysis. Data from the protocol (Tables 2 and 3) are used to construct a decay curve.

I **[mA]**	**Periods**	t [s]
340 ± 10	15	26.81 ± 0.20
440 ± 10	10	17.42 ± 0.20

Tabelle III.2.: Measured settling times and number of periods for two different current values. Uncertainties are indicated for current and time.

The corresponding periods for the two damping currents are calculated as:

$$\boxed{\begin{aligned} T_{340\,\mathrm{mA}} &= (1.587 \pm 0.013)\,\mathrm{s} \\ T_{440\,\mathrm{mA}} &= (1.742 \pm 0.020)\,\mathrm{s} \end{aligned}} \tag{11}$$

Period	**340 mA** [Units]			**440 mA** [Units]		
	1st	**2nd**	**Avg**	**1st**	**2nd**	**Avg**
1	15.0	14.6	14.8	14.2	13.2	13.7
2	13.0	12.2	12.6	10.4	10.0	10.2
3	11.0	10.2	10.6	8.0	7.4	7.7
4	8.8	8.6	8.7	5.8	5.2	5.5
5	7.6	6.8	7.2	4.2	3.8	4.0
6	6.4	5.8	6.1	3.6	2.6	3.1
7	5.2	4.6	4.9	2.6	1.8	2.2
8	4.6	3.8	4.2	2.0	1.4	1.7
9	3.8	3.0	3.4	1.0	0.8	0.9
10	3.2	2.4	2.8	1.0	0.4	0.7
11	2.6	1.8	2.2			
12	2.2	1.4	1.8			
13	1.8	1.0	1.4			
14	1.6	0.8	1.2			
15	1.2	0.4	0.8			

Tabelle III.3.: Measured amplitudes and their averages over 15 periods for two damping currents (340 mA and 440 mA). Each condition includes two measurements per period. The deflection scale is arbitrary and labeled as »Units«.

Frequency [Hz]	**Amplitude [Units]**
301 ± 0.1	0.6
501	0.6
701	0.6
901	0.7
1100	0.7
1300	0.8
1500	0.9
1700	1.0
1898	1.3
Switch »1K«	to »10K«
2112 ± 4	2.3
2304	4.8
2524	1.6
2700	1.1
2908	0.8
3100	0.7
3307	0.6
3517	0.5
3797	0.5
3912	0.4
Find actual	max amplitude
ADD MISSING VALUES	

Tabelle III.4.: Amplitude measurements at 340 mA across increasing frequencies. The uncertainty at 301 Hz is ±0.1 Vpp, and at 2112 Hz is ±4 Hz. A mode switch in the device occurred at 2112 Hz, indicated by the blue line in the original data.

IV. Discussion

4.1. Summary

4.2. Discussion

4.3. Critic

Abbildungsverzeichnis

Tabellenverzeichnis

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Literaturverzeichnis

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