"Birthday Twins"

Given N people, what is the probability that two people have the same birthday.

Number of possible birthdays = 366 (leap year)

Consider probability of all N having different birthdays, call this p, then probability of (at least) 2 people having the same birthday = (1-p)

Probability of N people having different birthdays

1st person has a birthday on any day

Prob. 2nd having b'day different to previous = $1 - \frac{1}{366}$

= $\frac{365}{366}$

Prob. of 3rd having same b'day as 1st = $\frac{1}{366}$

also prob. of 3rd " " as 2nd = $\frac{1}{366}$

so prob. of 3rd having either same as 1st or 2nd = $\frac{1}{366} + \frac{1}{366} = \frac{2}{366}$

tf. prob. 3rd having b'day different to previous b'days = $1 - \frac{2}{366} = \frac{364}{366}$

Prob 4th " " = $1 - \frac{3}{366}$

= $\frac{363}{366}$

…

prob. Nth " " = $1 - \frac{N-1}{366}$

= $\frac{366-(N-1)}{366}$

Prob N people having all different birthdays

(Multiply all the probabilities above e.g. Prob of two heads = $\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$)

$$= \frac{365}{366} * \frac{364}{366} * \frac{363}{366} * \dots * \frac{366-(N-1)}{366}$$

$$= \frac{365 * 364 * \dots 366-(N-1)}{366^{N-1}}$$

$$= \frac{(366-1) * (366-2) * \dots * (366-(N-1))}{366^{N-1}}$$

Example: N = 23

$$\frac{(366-1)*(366-2)*\ldots*(366-(23-1))}{366^{23-1}}$$

$$= \frac{365*364*\ldots*344}{366^{22}}$$

$$= \frac{123{,}034{,}458{,}606{,}683{,}264{,}934{,}098{,}143{,}075{,}536{,}318{,}524{,}051{,}780{,}468{,}736{,}000{,}000}{249{,}220{,}566{,}387{,}204{,}098{,}009{,}877{,}496{,}558{,}393{,}544{,}293{,}430{,}769{,}946{,}781{,}024{,}256}$$

This simplifies to (lowest reducible fraction)

$$= \frac{496{,}768{,}798{,}820{,}224{,}409{,}065{,}512{,}997{,}908{,}133{,}946{,}070{,}296{,}875}{1{,}006{,}262{,}821{,}062{,}572{,}246{,}849{,}093{,}750{,}912{,}932{,}693{,}143{,}409{,}161}$$

$$\approx \frac{496*10^{42}}{1{,}006*10^{42}}$$

≈ 0.493 (Prob. of 23 <u>not</u> having the same birthday)

therefore,

in a group of 23 people, the probability of at least 2 people having the same birthday

= 1 - 0.493

= 0.507

i.e. In a group of 23 people, there is a 51% chance that (at least) 2 people have the same birthday.

Birthday Probability Calculation

Construct a Java function, `double calculate_probability(int n)`

to calculate

$$\frac{(366-1)*(366-2)*\ldots*(366-(n-1))}{366^{n-1}}$$

We can rewrite this expression as

$$\frac{(366-1)}{366}*\frac{(366-2)}{366}*\ldots*\frac{(366-(n-1))}{366}$$

$$\frac{365}{366}*\frac{364}{366}*\ldots\frac{k}{366}\ldots*\frac{(366-(n-1))}{366}$$

Java function to calculate probability of n people, all having different birthdays:

```
double calculate_probability(int n)
{
    double p = 1.0;
    int last = 366 - n;
    for (int k = 365; k > last; k = k-1)
        p = p * k / 366;
    return p;
} // calculate_probability
```

Class Birthday_Twin

```
class Birthday_Twin
{
    Birthday_Twin()
    {
        int n;
        double p;

        TextIO.put("Enter size of group :  " );
        n = TextIO.getInt();
        TextIO.putln("In a group of " + n + " people");
        p = 1 - calculate_probability(n);
        TextIO.putln("Probability of a birthday twin = " +  p);
    } // Birthday_Twin

    double calculate_probability(int n)
    {
        double p = 1.0;
        int last = 366 - n;
        for (int k = 365; k > last; k = k-1)
            p = p*k/366;
        return p;
    } // calculate_probability

} // class Birthday_Twin
```