Complex Flows - Hand in 3

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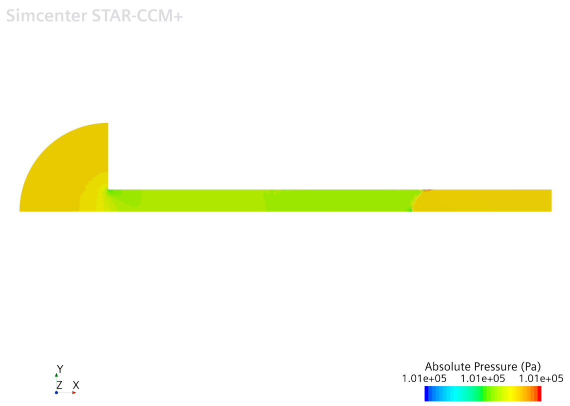
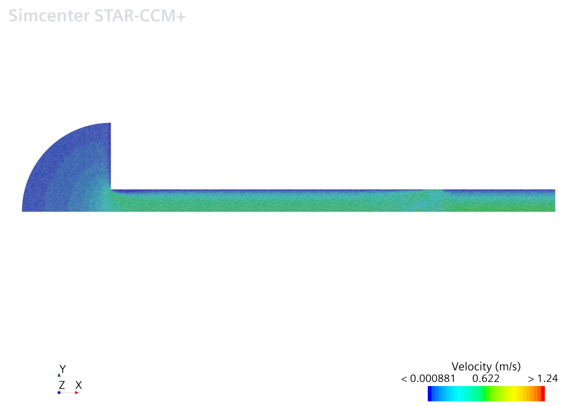
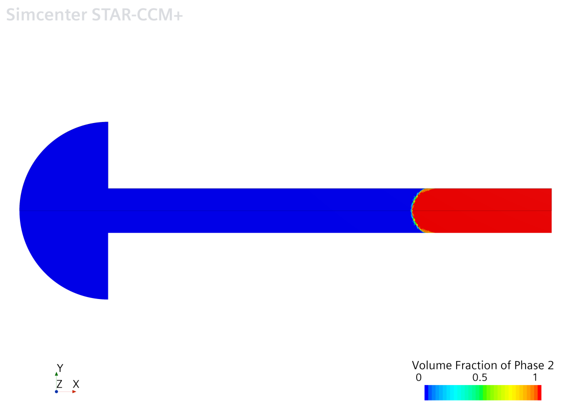
Modelling the scenario in *Star-CCM+* provides the results on figure 3.



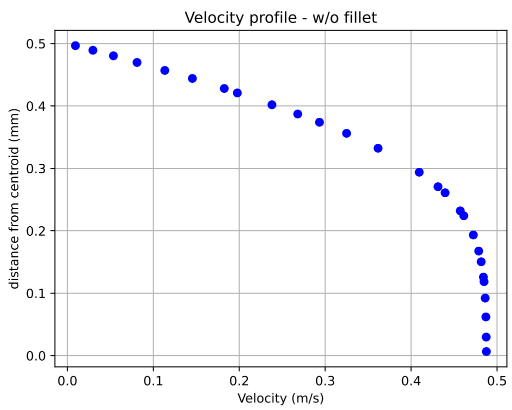
(a) Pressure distribution at t = 0.02s



(b) Velocity field at t = 0.02s



(c) Volume fraction at t = 0.02s



(d) Velocity profile 2mm inside channel at t = 0.02s

Figure 1: Caption

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The filling rate of a capillary is estimated by Lucas-Washburn to be equation (1) which gives the position of the fluid as a function of time, equation (2).

$$Q_{\mathrm{HP}} = \frac{H^3 w}{12\mu} \frac{\partial p}{\partial x} \tag{1}$$

$$\frac{dl(t)}{dt} = V(t) = \frac{Q}{wH} \approx \frac{H^2}{12\mu} \frac{\Delta p_{YL}}{l(t)} \tag{2}$$

Solving for $l(t)$ provides an insight into capillary flows.

The Lucas-Washburn solution is given with equation (3) [1].

$$l(t) = \sqrt{\frac{H\gamma\cos(\theta)}{3\mu}} = H\sqrt{\frac{t}{\tau_{\text{adv}}}} \tag{3}$$

Where the characteristic time is $\tau_{\text{adv}} = \frac{3\mu H}{\gamma\cos(\theta)}$. Since equation (3) does not include the effects of momentum change C.H. Bosanquet proposed a altered solution to equation (2). The Bosanquet solution is given by equation (4) [2].

$$l(t)^2 = \frac{2A^2}{B}\left(t - \frac{1}{B}\left(1 - \exp(-Bt)\right)\right) \tag{4}$$

Where $A = \sqrt{\frac{2\gamma\cos(\theta)}{\rho H}}$ and $B = \frac{12\mu}{\rho H^2}$. The two solutions are plotted on figure 2 along with the simulated data. The position of the liquids is normalized to the length of the capillary so the vertical scale of figure 2 represents position as a ratio of the total capillary length. It can be seen that the Lewis-Washburn equation results in a final liquid position that exceeds the length of the capillary.

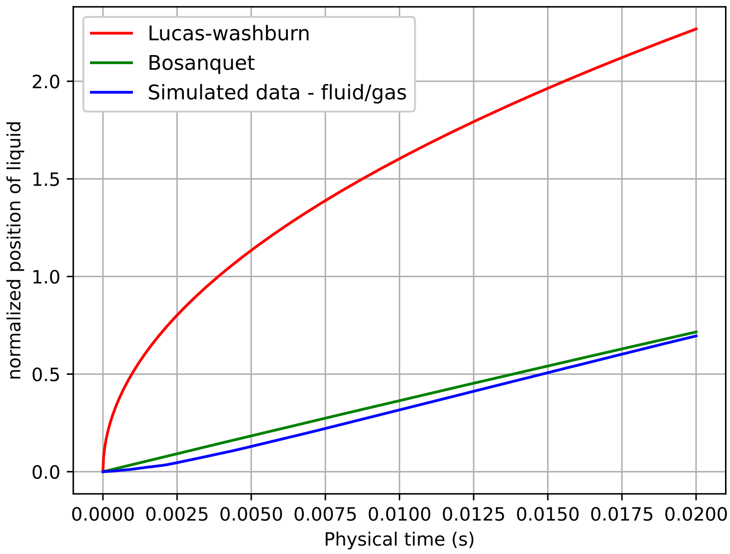

Figure 2: Position of liquid

As seen on figure 2, the Bosanquet solution is closer to the simulated data. The effect of the missing velocity retardant can clearly be seen in figure 2 as the initial velocity at $t = 0$ s is infinite for the Bosanquet solution. The small discrepancy between the Bosanquet model and the simulated data is likely due to the exclusion of the gas' effect on the liquid flow.

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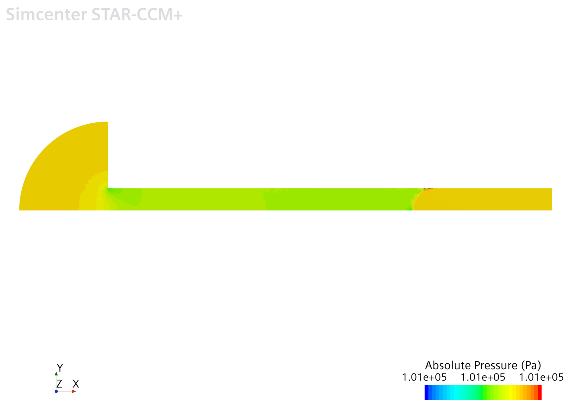
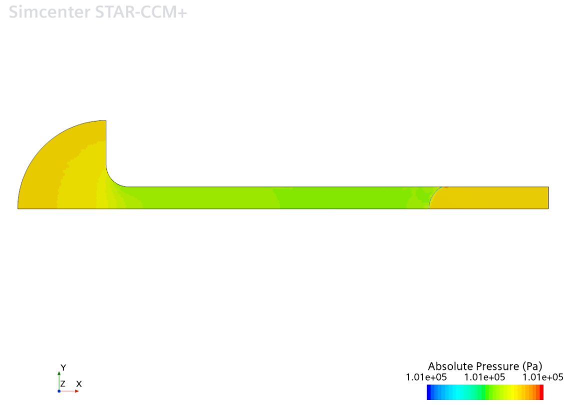
A side by side comparison of the simulated pressure scene shows that in terms of the pressure, only a slight difference in the reservoir part of the capillary can be observed. However, the pressure distribution in figure 3b causes a change in the final value of $l(t)$.



(a) Pressure distribution at t = 0.02s - w/o fillet



(b) Pressure distribution at t = 0.02s - fillet

Figure 3: A side by side comparison of the pressure distributions

At $t = 0.02$, the two simulated values for the positions of the liquid in the capillary and filleted capillary are $l_{\text{w/o fillet final}} = 0.0069544$ and $l_{\text{fillet final}} = 0.0074026$ respectively, showing an increase of around 6.445%. The slight increase can be seen on figure 4 where the two position functions are compared.

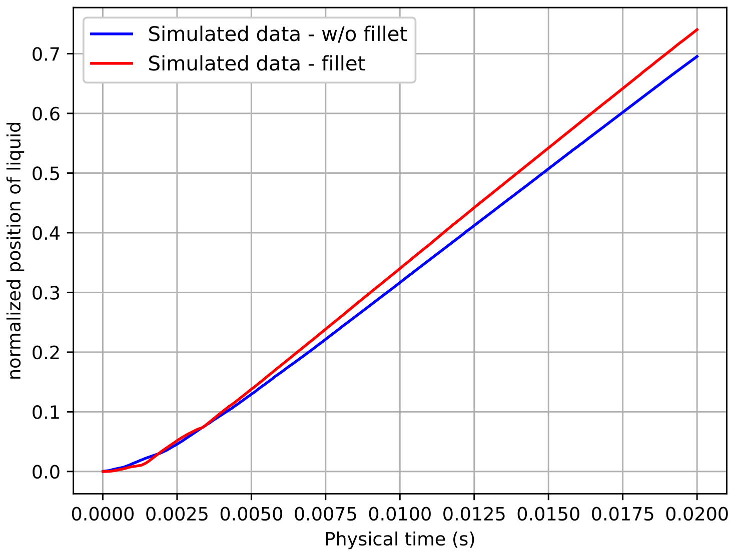


Figure 4: Comparison of position of liquid for fillet and w/o fillet

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If the capillary is closed at the end two scenarios have to be considered, either the gas is assumed incompressible or not. In the incompressible case, the pressure within the gas will not change significantly as the gas will not compress and the volume will not decrease. The expected position of the liquid is very close to the start of the capillary. The effect of the surface tension might be seen as the forces due to it will drive a volume of the liquid further along the wall of the capillary. Since the gas is assumed incompressible, the displaced gas might drive the liquid down where the forces due to the pressure difference are dominant.

If the gas is assumed to be compressible, the position of the liquid is likewise determined by the pressure difference. However an increase in pressure on the liquid's side will cause a compression of the gas, countering its displacement. Since the end of the capillary is closed, the imbition of the two phases is expected to oscillate more than in the incompressible case as these two effects balance out. The effect of the compressibility is likely to raise the final position of the liquid from it's initial position.

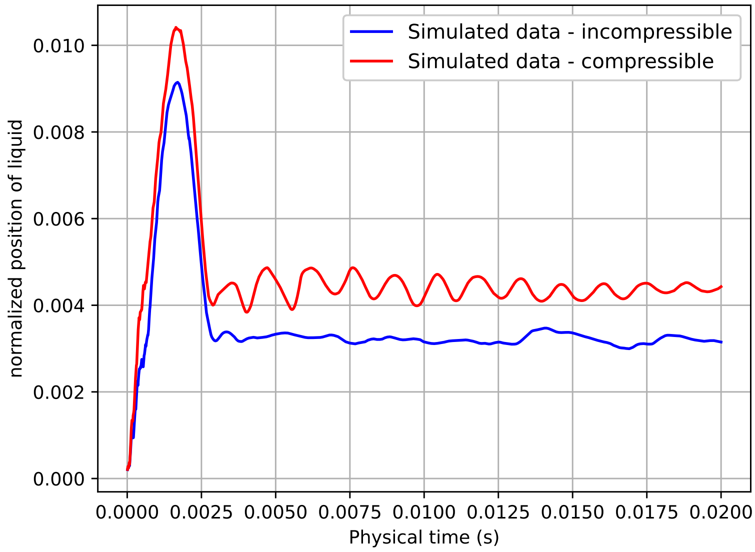

Figure 5: The liquids position with a closed capillary - incompressible and compressible gas

The two position lines on figure 5 show that the simulated positions are as expected, along with an initial spike. The initial spike is consistent with both cases as it is with the open ended capillary. It is likely a result of the initial rise of the liquid due to surface tension. The scale of these position changes shows that even with compressible gas, the liquids position does not change significantly. On figure 6 the liquids position can be seen very close to the beginning of the capillary and a part of it even dips below it's initial position. This shows that the effect of displaced gas is not negated completely by making in compressible.

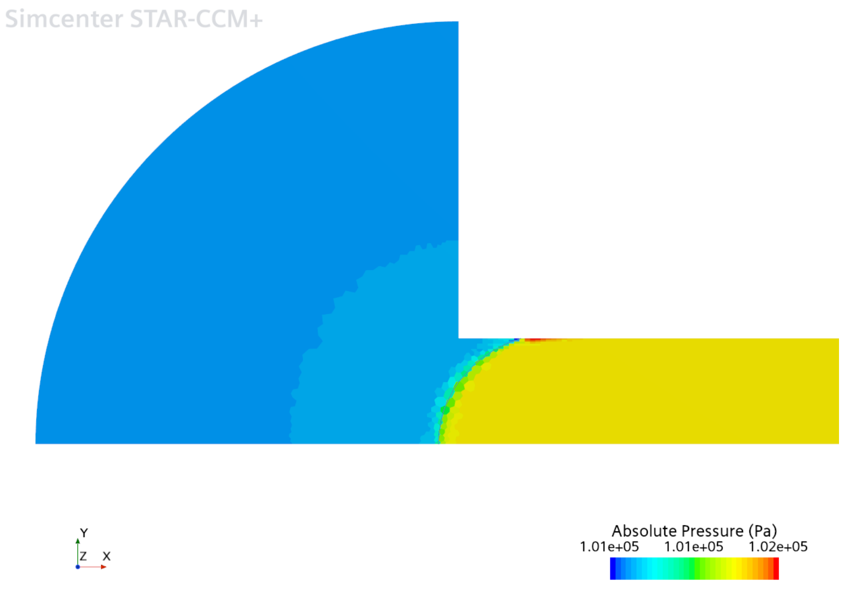

Figure 6: A closer look at the liquid position in a capillary with compressible gas at $t = 0.02$

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The missing inclusion of the gas' effect on the capillary flow can be supplemented in the Lucas-Washburn equations by introducing the factor Λ, defined in equation (5) where $h = \frac{H}{2}$ [3].

$$\Lambda = \frac{\mu_g L}{\mu_l h} \tag{5}$$

$$\tilde{t} = \frac{t\gamma \cos(\theta)}{2\mu_l h} \tag{6}$$

With equation (5) and a time dependent variable equation (6), a new solution to equation (2) is formulated in equation (7) [3].

$$l(t) = \left(\sqrt{\tilde{t} + \Lambda^2} - \Lambda\right) \cdot h \tag{7}$$

The model is plotted along with previosly mentioned models on figure 7

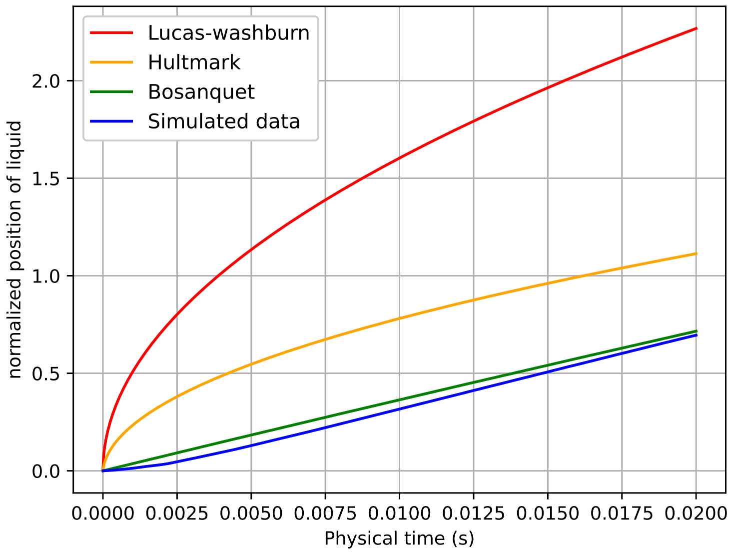

Figure 7: Position of liquid including the Hultmark model

The slope development of the Hultmark model is similar to the Lucas-Washburn model but the effects of the gas dampens the velocity of the flow. Hultmark seems to model the advancement of the liquids position better than the Lucas-Washburn model while the Bosanquet model stays more relevant initially. The Lucas-Washburn model has an infinite initial velocity while the initial velocity in the Hultmark model can be resolved as a constant as shown in equation (8) [3].

$$\left.\frac{dl}{d\tilde{t}}\right|_{t=0} = \frac{1}{2\Lambda} \tag{8}$$

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If the both fluids are liquid, in this case water, none of the models can predict the accurate fluids' interface position. Neither the Lucas-Washburn nor the Bosanquet equations include the effects of the secondary fluid, which will in this case be highly relevant. Even though the Hultmark model includes the effect of the gas, simply exchanging the properties of the gas in for the same liquid as the model assumes that $\rho_l \gg \rho_g$. Assuming that the resistance that the protruding liquid meets will be more the expected final position is closer to the initial position than the fluid/gas setup. This is shown in figure 8.

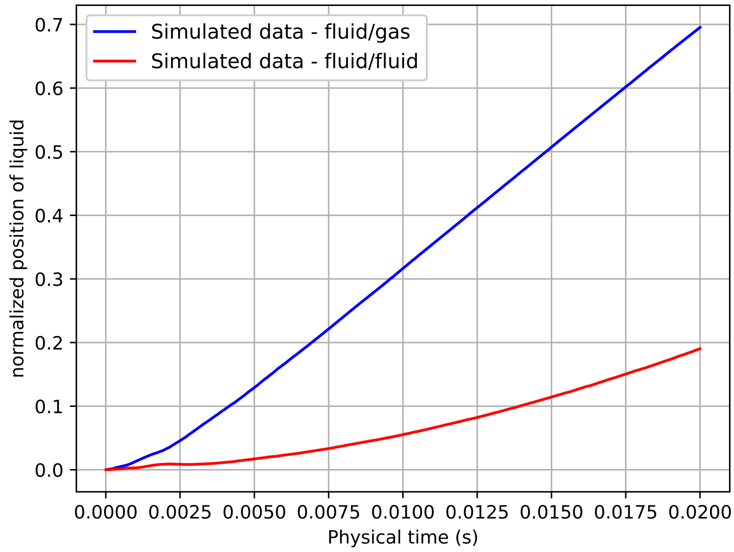

Figure 8: Position of the liquids interface

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The governing equation is equation (9).

$$\frac{d}{dt}\left(L\frac{dl(t)}{dt}\right) + BL\frac{dl(t)}{dt} = A_I^2 \tag{9}$$

Where $A_I = \sqrt{\frac{2\gamma\cos(\theta)}{\rho H}}$, $B = \frac{12\mu}{\rho H^2}$ and assuming that $l(0) = 0$ and $\frac{dl}{dt} = 0$ the solution for $l(t)$ can be determined. By assuming these conditions the solution will model a heavy displaced liquid.

$$l(t) = A_I^2 \frac{e^{-Bt} + Bt - 1}{B^2 L} \tag{10}$$

This new solution is expected to model the liquid/liquid scenario as shown on figure 9 where equation (10) is plotted as *Bosanquet 2*.

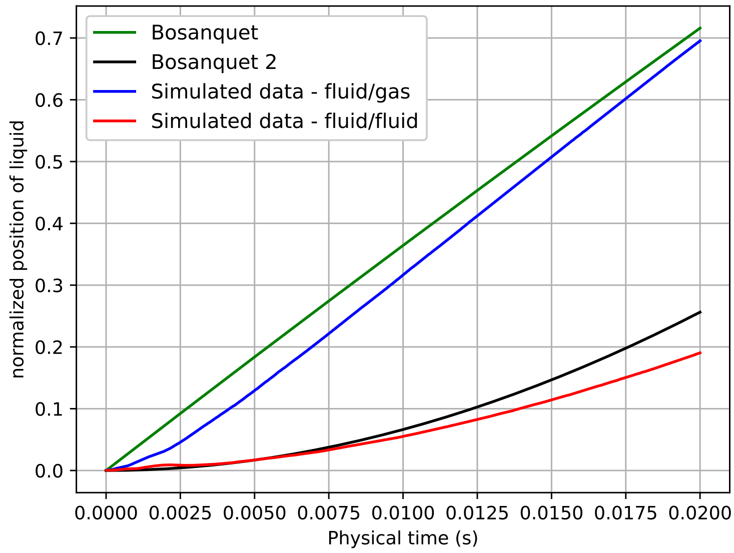

Figure 9: Fluid interface position including New Bosanquet model

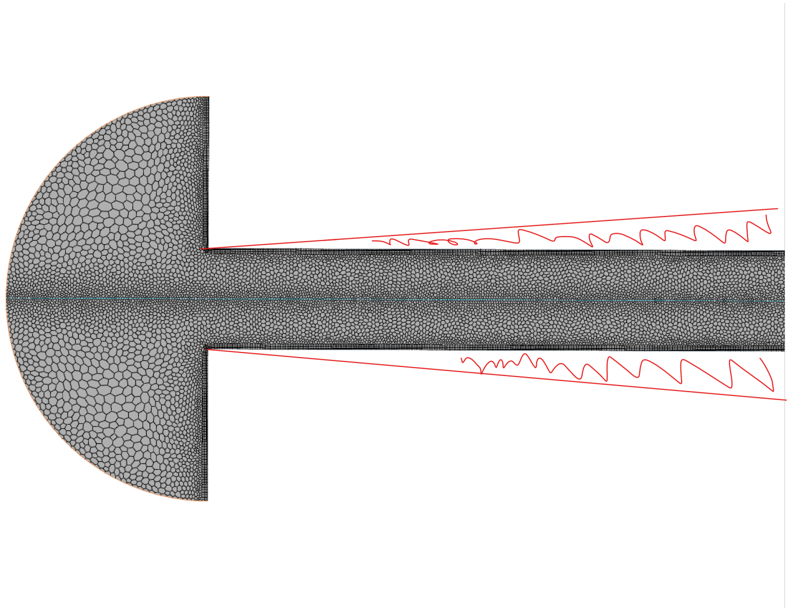

Figure 10: Suggested action

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By slanting the capillary tube so that the radii increases along the length of it the pressure will not allow for the fluid to flow, as per the Young-Laplace equation, equation (11) [4].

$$\Delta p_{surf} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\gamma \tag{11}$$

The suggested action is shown in figure 10.

Bibliography

[1] Richard Lucas. “Ueber das Zeitgesetz des kapillaren Aufstiegs von Flüssigkeiten”. In: *Koll. - Zeitsch.* (1918), pp. 15–22.

[2] C.H. Bosanquet. “On the flow of liquids into capillary tubes”. In: *Phil. Mag* 45 (1923), pp. 525–531.

[3] Marcus Hultmark, Jeffrey M. Aristoff, and Howard A. Stone. “The influence of the gas phase on liquid imbibition in capillary tubes”. In: *J. Fluid Mech* (2011).

[4] Clayton T. Crow et al. *Multiphase Flows with Droplets and Particles*. 2012.