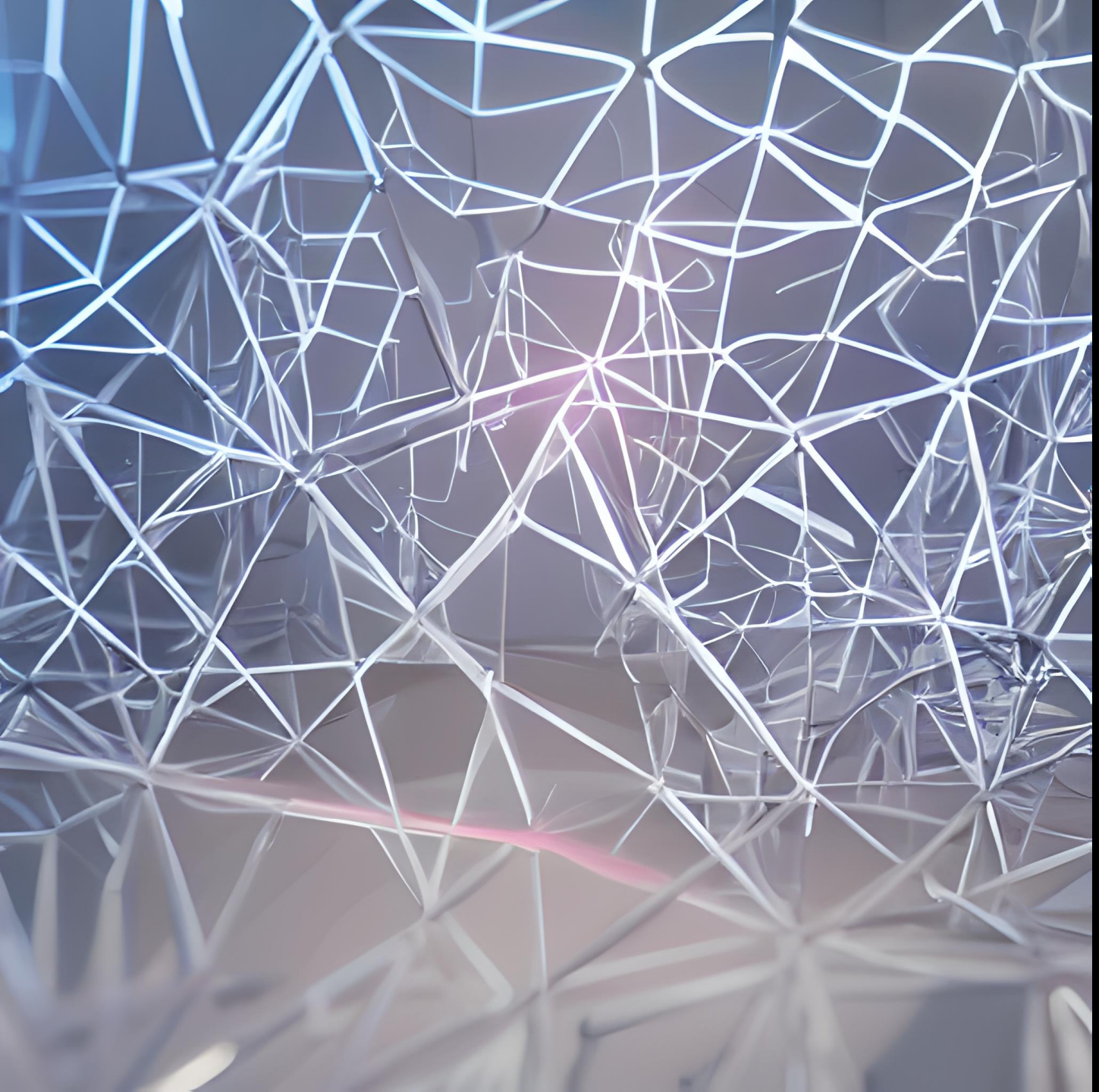

Modern portfolio theory

Fintech society

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Diversification

Diversification is a method which aims to reduce your risk.

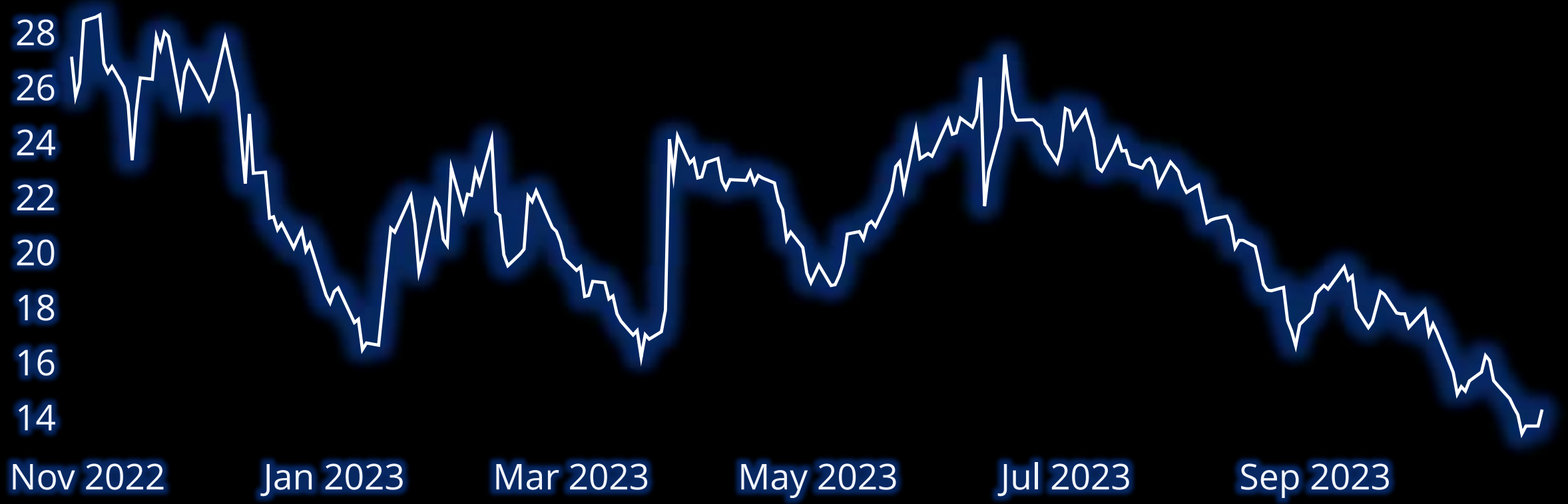
Risk is the uncertainty you face with any action.

We can assume risk is the average movement from the mean
Also known also as standard deviation

A loss is more difficult to recover from. To get the previous price after the asset has taken fallen by 10%, you need an 11% increase to return to the previous level.

The more risky an asset is, the more likely it is to increase **and** decrease in price.

GME stock price over time



Expected return

Expected return is calculated as the sum of the different probabilities of a return occurring.

$$E(x) = \sum P(x) R_x$$

We can also use the historical data as the future performance of the asset, if we are to assume current trends will continue for our holding period.

$$E(x) = \mu_t$$

Risk for assets

If we assume the risk is the standard deviation or variance of the asset, we can calculate it using the sample formula.

When taking a sample, subtract one from the population size to account for the fact it is a sample.

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N - 1}$$

Risk, return and sharpe

Using the average return and standard deviation we can describe stocks very effectively using just those two statistical measures

But a good way to compare assets to each other is to determine the excess risk free rate per unit of risk, also known as sharpe ratio

$$\frac{E(x) - RFR}{\sigma^2}$$

Portfolios

A combination of investments which aim to produce a return.

In Modern portfolio theory it is the selected assets to include in the calculations. The portfolio itself has a return and risk.

A portfolios expected return is the weighted expected return

$$\sum w_i E(x)$$

Portfolio risk

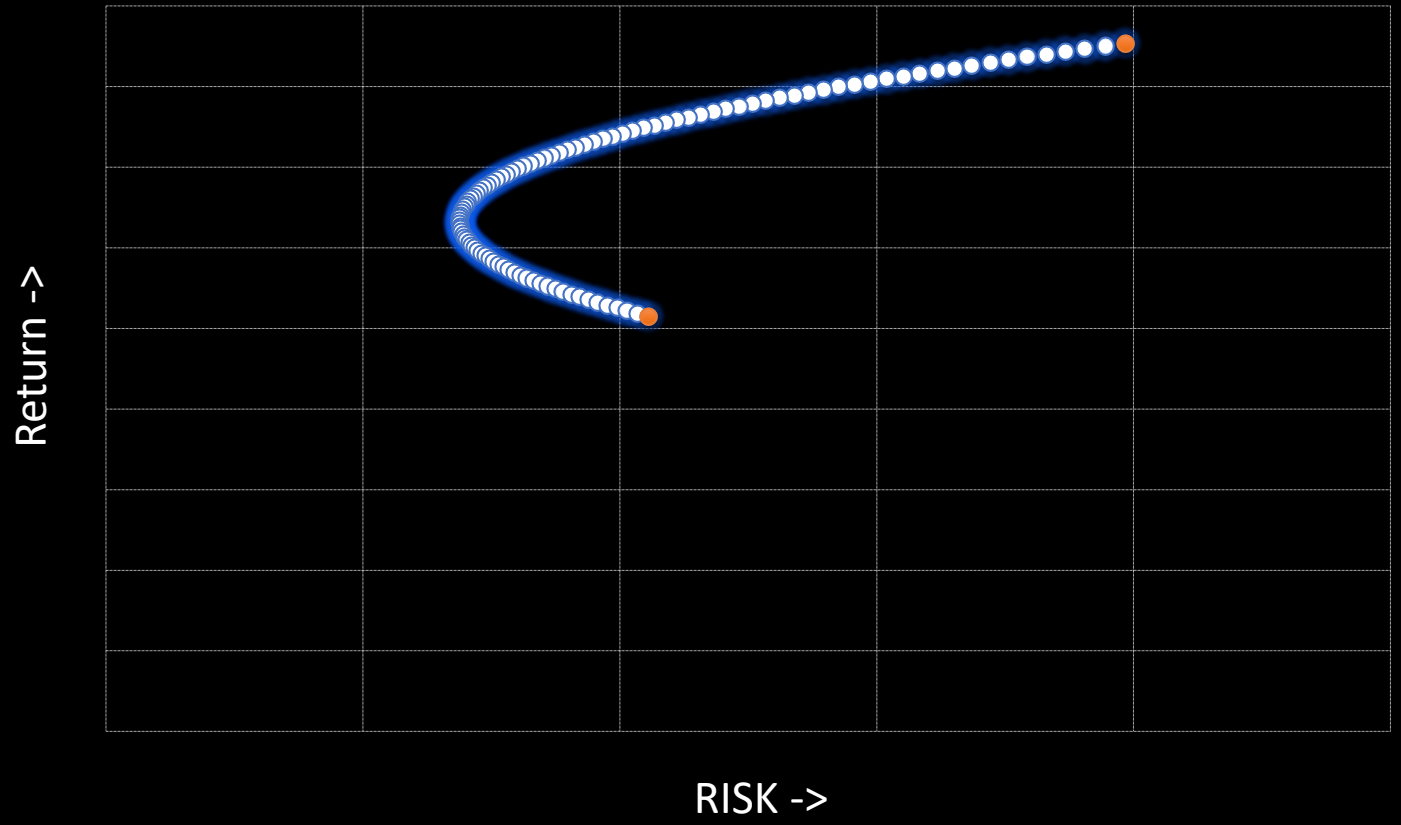
Risk is not just the weighted standard deviation it is also the weighted covariance between the assets

$$\sigma^2 = \sum {w_i}^2 \sigma_i^2 + \sum\sum w_i \, w_j Cov_{ij}$$

The covariance of an asset between existing asset has a greater impact on the risk of the portfolio than the weighted standard deviation of the asset.
To minimise risk, we must minimise the Covariance first.

Efficient frontier

The curve where the portfolios are optimally allocated to minimise the risk for a level of return



Matrices and dot products

$$\sum A_i X_i = A \cdot X$$

Each element in the product matrix is obtained by multiplying corresponding elements from the original matrices and summing them up, following specific rules for matrix dimensions.

$$\begin{bmatrix} a \\ b \end{bmatrix} \cdot [x\ y] = ax + ay + bx + by$$

Matrices for returns in mpt

$$E(x) = \mu_t$$

$$\sum w_i E(x)$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \cdot [E(x)\ E(y)] = w_1 E(x) + w_2 E(y)$$

Matrices for variance in mpt

$$\sigma_p^2 = \sum w_i^2 \sigma_i^2 + \sum\sum w_i w_j Cov_{ij}$$

$$\sigma_p^2 = W \cdot Cov\ Matrix \cdot W^T$$

2 asset variance example

$$[w_1\ w_2] \cdot \left(\begin{bmatrix} \sigma^2(x) & Cov_{xy} \\ Cov_{xy} & \sigma^2(y) \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \right) = [w_1\ w_2] \cdot \begin{bmatrix} w_1\sigma^2(x) + w_1 Cov_{xy} \\ w_2 Cov_{xy} + w_2\sigma^2(y) \end{bmatrix}$$

$$[w_1\ w_2] \cdot \begin{bmatrix} w_1\sigma^2(x) + w_2 Cov_{xy} \\ w_1 Cov_{xy} + w_2\sigma^2(y) \end{bmatrix} = w_1\left(w_1\sigma^2(x) + w_2 Cov_{xy}\right) + w_2\left(w_1 Cov_{xy} + w_2\sigma^2(y)\right)$$

$$= w_1^2\sigma^2(x) + w_1 w_2 Cov_{xy} + w_1 w_2 Cov_{xy} + w_2^2\sigma^2(y)$$

$$= w_1^2\sigma^2(x) + w_2^2\sigma^2(y) + 2w_1 w_2 Cov_{xy}$$

Breaking down the tasks

0. Get data
1. Calculate the returns and variance for each asset
2. Calculate the portfolio from weights and data
3. Minimise the risk by adjusting weights
4. Maximise sharpe ratio by adjusting weights

- Fabozzi, F. J. & Markowitz, H. M. (2011) The Theory and Practice of Investment Management : Asset Allocation, Valuation, Portfolio Construction, and Strategies.