Quantitative Methods for Finance

Solutions - Gujarati Appendix D

D.1

(a) A single numerical value of a (population) parameter is known as a point estimate. An interval estimate provides a range of values that will include the true parameter with a certain degree of confidence (i.e., probability). A point estimator is a formula or rule that tells how to obtain the point estimate.

(b) A null hypothesis (H_0) is the maintained hypothesis which is tested against another hypothesis, called the alternative hypothesis (H_1 or H_a).

(c) Type I error: The error of rejecting a hypothesis when it is true. A type II error is the error of accepting (i.e., not rejecting) a false hypothesis.

(d) The probability of committing a type I error is known as the level of significance (or the size of the test). One minus the probability of committing a type I error is called the confidence coefficient.

(e) The probability of accepting a false hypothesis is called a type II error and (1-prob. of type II error), that is, the probability of not committing a type II error is called the power of the test.

D.2

(a) The two branches of classical statistics, estimation of parameters and testing hypothesis about parameters, constitute statistical inference.

(b) The probability distribution of an estimator.

(c) It is synonymous with a confidence interval.

(d) A statistic used to decide whether a null hypothesis is rejected or not.

(e) That value of the test statistic which demarcates the acceptance region from the rejection region. (Note: demarcate = mark by bounds, set the limits to separate)

(f) It is the probability of committing a type I error.

(g) The exact level of significance of a test statistic.

D.3

(a) If the average, or expected, value of an estimator coincides with the true value of the parameter, that estimator is known as an unbiased estimator.

(b) In a group of competing estimators of a parameter the one with the least variance is called a minimum variance estimator.

(c) In the class of unbiased estimators, the one with the least variance is called an efficient estimator.

(d) An estimator which is a linear function of the observations.

(e) An unbiased linear estimator with the least possible variance.

D.4

(a) True. In classical statistics the parameter is assumed to be some fixed number, although unknown.

(b) False. It is $\mathrm{E}(\hat{\mu}_X) = \mu_X$, where $\hat{\mu_X}$ is an estimator. (It is not $\hat{\mu}_X = \mu_X$.)

(c) True.

(d) False. To be efficient, an estimator must be unbiased and it must have minimum variance.

(e) False. No probabilistic assumption is required for an estimator to be BLUE.

(f) True.

(g) False. A type I error is when we reject a true hypothesis.

(h) False. A type II error occurs when we do not reject a false hypothesis.

Table 3.3 Classifying hypothesis testing errors and correct conclusions

		Reality	
		H_0 is true	H_0 is false
Result of test	Significant (reject H_0)	Type I error $= \alpha$	$\surd$
	Insignificant (do not reject H_0)	$\surd$	Type II error $= \beta$

Type I and type II errors (Brooks, 3ed, p.110)

(i) True. This can be proved formally.

(j) False, generally. Only when the sample size increases indefinitely, the sample mean will be normally distributed. If, however, the sample is drawn from a normal population to begin with, the sample mean is distributed normally regardless of the sample size.

(k) Uncertain. The p-value is the exact level of significance. If the chosen level of significance, say, $\alpha = 5\%$, coincides with the p-value, the two will mean the same thing.

D.5

See Section D.5.

D.6

Disagree. The answer depends on the level of significance (α), the probability of committing a type I error, that one is willing to accept. There is nothing sacrosanct about the 5% or 10% level of significance.

(Note: sacrosanct = too important or too special to be changed.)

D.7

(a) 1.96 (b) 1.65 (c) 2.58 (d) 2.05.

D.8

(a) 3.182 (3 d.f.) (b) 2.353 (3 d.f.) (c) 3.012 (13 d.f.)

(d) 2.650 (13 d.f.) (e) 2.0003 (59 d.f.) (f) 1.972 (199 d.f.)

Note: These critical values have been obtained from electronic statistical tables.

D.9

(a) $\text{P}(-2 \leq Z \leq 2) = 0.9544$.

(b) $\text{P}(Z \geq 2) = 0.0228$.

(c) $\text{P}(Z \leq -2) = 0.0228$.

(d) Yes ($Z = 40$, an extremely high value).

D.10

Note that $X \sim N(1,000, 10)$. (Here: $\sigma^2/n = 10$.)

(a) Practically zero, because $\text{P}(Z \leq -31.6228)$ is negligible.

(b) The 95% confidence interval is: $893.8019 \leq \mu_X \leq 906.1981$. With 95% confidence we can say that the true mean is not equal to 1,000.

(c) Reject the null hypothesis. Use the normal distribution because the sample size is reasonably large.

D.11

(a) Note that $(6.5-\mu)/\sigma = -1.65$ and $(6.8-\mu)/\sigma = 1.28$. Solving these two equations simultaneously, we obtain the two results $\mu = 6.6689$ and $\sigma = 0.1024$.

(b) $Z = (7 - 6.6689)/0.1024 = 3.2333$. Therefore, $\text{P}(Z \geq 3.2333)$ is very small, about 0.0006.

D.12

Note that $\bar{X} = 9$.

(a) If $X \sim N(5, 2)$, then $\bar{X} \sim N(5, 2/10)$. Therefore,

$$Z = \frac{9-5}{0.4472} = 8.9445.$$

Therefore, $\mathrm{P}(Z \geq 8.9445)$ is practically zero. Hence, we reject the null hypothesis that $\mu = 5$. Also, note that the 95% confidence interval for $\bar{X}$ is:

$$\left(5 \pm 1.96 \frac{1.4142}{\sqrt{10}}\right) = (4.1235, 5.8765).$$

This interval does not include $\bar{X} = 9$.

(b) Also reject the null hypothesis.

(c) The p-value is extremely small.

D.13

Use the t distribution, since the true σ^2 is unknown. For 9 d.f., the 5% critical t value is 2.262. Therefore, the 95% CI is:

$$8 \pm 2.262(1.2649) = (5.1388, 10.8612).$$

D.14

(a) $\bar{X} \sim N(8, 36/25)$.

(b) $Z = (7.5 - 8)/1.2 = -0.4167$. Therefore, $\mathrm{P}(Z \leq -0.4167) = 0.3372$.

(c) The 95% CI for $\bar{X} = 8 \pm 1.96(1.2) = (5.6480, 10.3520)$. Since this interval includes the value of 7.5, such a sample could have come from this population.

D.15

Using electronic statistical tables, it can be found that

(a) $p = 0.0492$; (b) $p = 0.0019$; (c) $p = 0.0814$; (d) $p = 0.9400$.

D.16

The answer depends on the level of α, the probability of a type I error. The p-value in this case is about 0.25 (actually, 0.2509). If α is fixed at 0.25, one could reject the relevant null hypothesis at this level of significance.

D.17

(a) $\mathrm{E}(\hat{\mu}_1) = [\mathrm{E}(X_1) + \mathrm{E}(X_2) + \mathrm{E}(X_3)]/3 = 3\mu/3 = \mu$. Hence it is an unbiased estimator. Similarly, it can be shown that $\hat{\mu}_2$ is also unbiased.

(b) $\mathrm{Var}(\hat{\mu}_1) = 1/9[\mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \mathrm{Var}(X_3)] = 3\sigma^2/9 = 0.333\sigma^2$.
$\mathrm{Var}(\hat{\mu}_2) = 7/18\sigma^2 = 0.389\sigma^2$.
So, choose $\hat{\mu}_1$ over $\hat{\mu}_2$.

D.18

(a) 95% confidence interval for the true mean profit:

$900,000 \pm 2.262(100,000/10)$, that is, (828,469, 971,531).

(b) The t distribution, since the true σ^2 is not known.

D.19

(a) 95% confidence interval for σ^2:

$(19)(16)/32.8523 \leq \sigma^2 \leq (19)(16)/8.9065$, that is, (9.2535, 34.1324).

Note: For 19 d.f., $\chi^2_{0.025} = 32.8523$ and $\chi^2_{0.975} = 8.9065$.

(b) Since the preceding interval does not include $\sigma^2 = 8.2$, reject the null hypothesis.

D.20

(a) Note that

$$F = \frac{S_1^2}{S_2^2} = \frac{0.3762}{0.3359} = 1.12.$$

The p-value of obtaining an F value of 1.12 or greater is 0.4146. Therefore, one may not reject the null hypothesis that the two population variances are the same.

(b) The F test is used. The basic assumption is that the two populations are normal.

D.21

$$\mathrm{E}(X^*) = \frac{1}{n+1}\left[\mathrm{E}(X_1) + \cdots + \mathrm{E}(X_n)\right] = \frac{1}{n+1}(n\mu_X) = \frac{n}{n+1}\mu_X \neq \mu_X.$$