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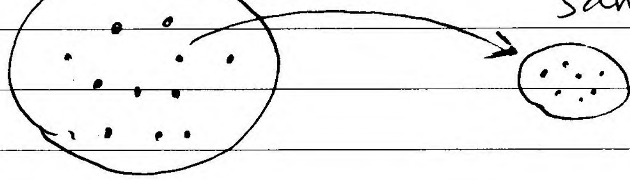
4 Probability Distributions

- population

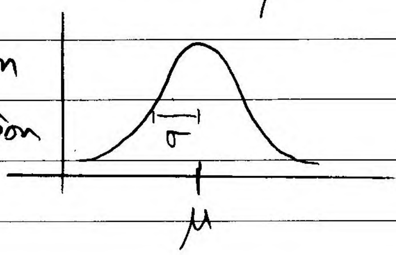
sample



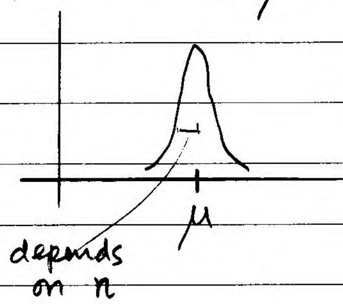
n = sample size

$$X \sim N(\mu, \sigma^2) \qquad \overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

population distribution of X



sampling distribution of $\overline{X}$



$$\Rightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1) \qquad Z = \frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

$$Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi^2_{(1)} \qquad Z^2 \sim \left(\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}\right)^2 \sim \chi^2_{(1)}$$

- Question

$$S^2 = \frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1} \rightarrow \text{what's its distribution?}$$

$$\frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n}\left(\frac{X_i-\overline{X}}{\sigma}\right)^2 \longrightarrow$$

appears to be $\chi^2_{(n)}$

it's actually $\chi^2_{(n-1)}$.

$$= \underbrace{\sum_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2}_{\sim \chi^2_{(n)}} - \underbrace{\left(\frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}}\right)^2}_{\sim \chi^2_{(1)}}$$

$$\sim \chi^2_{(n-1)}$$

$$\Rightarrow \underbrace{(n-1)\frac{S^2}{\sigma^2} \sim \chi^2_{(n-1)}}$$

$$\left(E\left[(n-1)\frac{S^2}{\sigma^2}\right] = (n-1) \Rightarrow E(S^2) = \sigma^2 \right)$$

- <u>Question</u>

$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

$$\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim \text{?} \rightarrow \text{what is its distribution?}$$

$$\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} = \frac{\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}}{\frac{\frac{S}{\sqrt{n}}}{\frac{\sigma}{\sqrt{n}}}} = \frac{Z}{\sqrt{\frac{S^2}{\sigma^2}}} = \frac{Z}{\sqrt{\frac{Y}{n-1}}} \sim t_{(n-1)}$$

$$Y = (n-1)\frac{S^2}{\sigma^2} \sim \chi^2_{(n-1)}$$

$$\frac{S^2}{\sigma^2} = \frac{Y}{n-1} \sim \frac{\chi^2_{(n-1)}}{n-1}$$

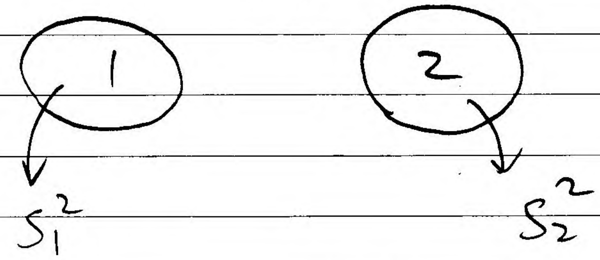
Note: the df of $t_{(n-1)}$ follows the df of $\chi^2_{(n-1)}$

$$\Rightarrow \underbrace{t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{(n-1)}}$$

$se(\bar{X}) \rightarrow$ standard error

- <u>Question</u>

consider 2 populations



$\dfrac{S_1^2}{S_2^2} \sim$? what's its distribution ?

$\because \; \underbrace{(n_1-1)\dfrac{S_1^2}{\sigma_1^2}}_{Y_1} \sim \chi^2_{(n_1-1)}$

$\underbrace{(n_2-1)\dfrac{S_2^2}{\sigma_2^2}}_{Y_2} \sim \chi^2_{(n_2-1)}$

$$\Rightarrow \quad \frac{S_1^2}{\sigma_1^2} = \frac{Y_1}{n_1-1} \sim \frac{\chi^2_{(n_1-1)}}{n_1-1}$$

$$\frac{S_2^2}{\sigma_2^2} = \frac{Y_2}{n_2-1} \sim \frac{\chi^2_{(n_2-1)}}{n_2-1}$$

if $\sigma_1^2 = \sigma_2^2$

$$\frac{S_1^2}{S_2^2} = \frac{\dfrac{S_1^2}{\sigma_1^2}}{\dfrac{S_2^2}{\sigma_2^2}} = \frac{\dfrac{Y_1}{n_1-1}}{\dfrac{Y_2}{n_2-1}} \sim F_{(n_1-1,\, n_2-1)}$$

$Y_1 \to \chi^2_{(n_1-1)}$

$Y_2 \to \chi^2_{(n_2-1)}$

equal

- 4 basic tests

	1 population	2 populations
mean	$H_0 : \mu = 10$	$H_0 : \mu_1 = \mu_2$
variance	$H_0 : \sigma^2 = 20$	$H_0 : \sigma_1^2 = \sigma_2^2$

① $H_0 : \mu = 10 \rightarrow \mu_0$

$$t \text{ statistic} = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} \sim t_{(n-1)} \xrightarrow{n \to \infty} N(0,1)$$

② $H_0 : \sigma^2 = 20 \rightarrow \sigma_0^2$

$$\chi^2 \text{ statistic} = (n-1)\frac{S^2}{\sigma_0^2} \sim \chi^2_{(n-1)}$$

③ $H_0 : \mu_1 = \mu_2$ (assuming $\sigma_1^2 = \sigma_2^2$)

$$t \text{ statistic} = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}$$

$$\left(S = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} \right)$$

④ $H_0 : \sigma_1^2 = \sigma_2^2$

$$F \text{ statistic} = \frac{S_1^2}{S_2^2} \sim F_{(n_1-1, n_2-1)}$$