COMP3121/3821/9101/9801 Midterm 16s1

1. Assume that you are given a polynomial $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and a polynomial $Q(x) = b_0 + b_1x + b_2x^2 + b_3x^3$ whose coefficients can be arbitrarily large numbers. Let $R(x) = P(x)^2 - Q(x)^2$. Compute the coefficients of $R(x)$ using only 7 large number multiplications.

2. Recall that the DFT of a sequence

$$\langle a_0, a_1, \ldots, a_{n-1} \rangle$$

is the sequence of values

$$\langle P(\omega_n^0), P(\omega_n^1), P(\omega_n^2), \ldots, P(\omega_n^{n-1}) \rangle$$

of the associated polynomial $P(x) = a_0+a_1x+a_2x^2+\ldots+a_{n-1}x^{n-1}$. Compute directly (i.e., without using the FFT, to spare you the trouble) and maximally simplify the DFT of the following sequences:

   (a) $\langle 1, \underbrace{0, \ldots, 0}_{n-1} \rangle$

   (b) $\langle \underbrace{0, \ldots, 0,}_{n-1} 1 \rangle$

   (c) $\langle \underbrace{1, \ldots, 1}_{n} \rangle$

3. Assume you are given a set X of n disjoint intervals $I_1, I_2, \ldots, I_n$ on the real line and a number $k \leq n$. You have to design an algorithm which produces a set Y of at most k intervals $J_1, J_2, \ldots, J_k$ which cover all intervals from X (i.e. $\bigcup_{i=1}^{n} I_i \subseteq \bigcup_{j=1}^{k} J_j$) and such that the total length of all intervals in Y is minimal. (Note that we do not require that $Y \subseteq X$, i.e., intervals J_j can be new. For example, if your set X consists of intervals $[1, 2]$, $[3, 4]$, $[8, 9]$, and $[10, 12]$ and if $k = 2$, then you should choose intervals $[1, 4]$ and $[8, 12]$ of total length $3 + 4 = 7$.)

4. You have a processor that can operate 24 hours a day, every day. People submit requests to run daily jobs on the processor. Each such job comes with a start time and an end time; if the job is accepted to run on the processor, it must run continuously, every day, for the period between its start and end times. (Note that certain jobs can begin before midnight and end after midnight; this makes for a type of situation different from what we saw in the activity selection problem.) Given a list of n such jobs, your goal is to accept as many jobs as possible (regardless of their length), subject to the constraint that the processor can run at most one job at any given point in time. Provide an algorithm to do this with a running time that is polynomial in n. You may assume for simplicity that no two jobs have the same start or end times.

5. Assume that you are given a complete undirected graph $G = (V, E)$ with edge weights $\{w(e) : e \in E\}$ and a proper subset of vertices $U \subset V$, $U \neq V$. Design an algorithm which produces the lightest spanning tree of G in which all nodes of U are leaves (there might be other leaves in this tree as well).