School of Computer Science and Engineering, UNSW
COMP 3121/3821/9101/9801
Midterm 2017

Each problem is worth 10 points. Write legibly and describe in detail everything that you are doing.

1. You have a processor that can operate 24 hours a day, every day. People have submitted n requests to run daily jobs on the processor. Each such job comes with a start time s_i and an finishing time f_i; if the job is accepted to run on the processor, it must run continuously, every day, for the period between its start and finishing times (Note that certain jobs can begin before midnight and end after midnight!). Given a list of n such jobs, your goal is to accept as many jobs as possible (regardless of their length), subject to the constraint that the processor can run at most one job at any given point in time. Provide an algorithm to do this with a running time that is polynomial in n. You may assume for simplicity that no two jobs have the same start or end times. Prove that your algorithms is correct and estimate its asymptotic running time.

2. Assume that you got a fabulous job and you wish to repay your student loan as quickly as possible. Unfortunately, the bank *Western Road Robbery* which gave you the loan has the condition that you must start your monthly repayments by paying off \$1 and then each subsequent month you must pay either double the amount you paid the previous month, the same amount as the previous month or a half of the amount you paid the previous month. On top of these conditions, your schedule must be such that the last payment is \$1. Design an algorithm which, given the size of your loan, produces a payment schedule which minimises the number of months it will take you to repay your loan while satisfying all of the banks requirements. If optimality of your solution is obvious from the algorithm description, you do not have to provide any further correctness proof.

3. (a) Compute the DFT of the sequence $(1, -1, -1, 1)$ by **any method** you wish.
   (b) Compute the linear convolution of sequences $(1, -1, -1, 1)$ and $(-1, 1, 1, -1)$ by **any method** you wish.

4. Find the square of a complex number $a + i\,b$ where a and b are real numbers, using only two multiplications and at most 3 additions or subtractions. In other words, you have to find two real numbers c and d such that $c + i\,d = (a + i\,b)^2$ using only two multiplications and at most 3 additions or subtractions of real numbers.

Extended classes (comp3821/9801) only:

5. You are given a fair coin, i.e., a coin with equally likely head and tail outcomes. Design an algorithm which uses such a coin to simulate a 6 sided fair dice, i.e., a dice with 6 equally likely outcomes. Compute the expected number of coin tosses needed to obtain an outcome of throwing such a dice once?

Good luck!!