Quiz (Week 10)

Logic

Question 1

Which one (or more) of expressions listed below is a possible formalisation of the phrase: *Not all that glitters is gold*.

1. ✗ $\exists x.\ \ (\mathsf{Glitter}(x) \Rightarrow \mathsf{Gold}(x))$
2. ✗ $\forall x.\ \ (\neg(\mathsf{Glitter}(x)) \Rightarrow \mathsf{Gold}(x))$
3. ✔ $\neg\forall x.\ (\mathsf{Glitter}(x) \Rightarrow \mathsf{Gold}(x))$
4. ✗ $\forall x.\ \ (\neg(\mathsf{Glitter}(x) \Rightarrow \mathsf{Gold}(x)))$

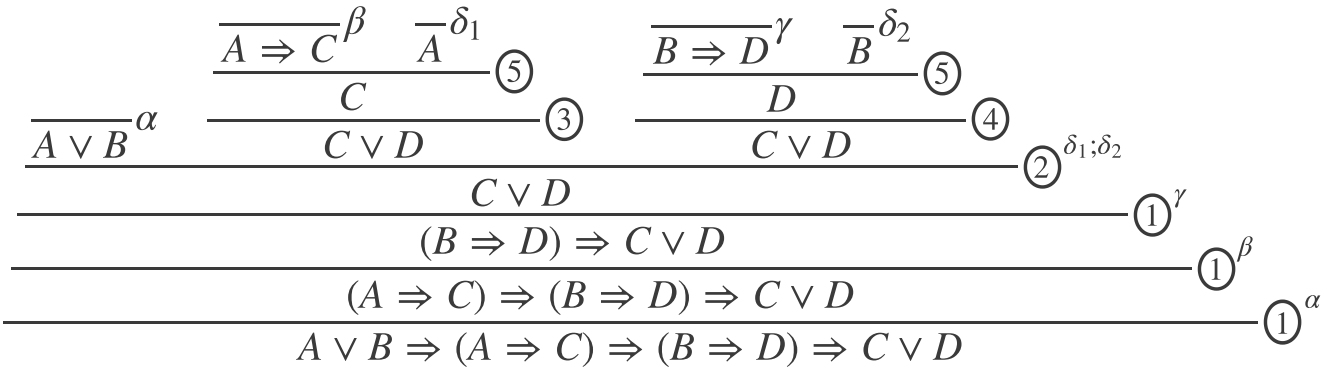
Question 2

Which of the expressions listed below is a possible formalisation of the Abraham Lincoln quote: *You can fool all the people some of the time, and some of the people all the time, but you cannot fool all the people all the time*.

1. ✗ $\forall p.\ \forall t.\ (\mathsf{Fool}(p, t) \wedge \neg\mathsf{Fool}(p, t))$
2. ✗ $(\forall p.\ \exists t.\ \mathsf{Fool}(p, t)) \wedge (\exists p.\ \forall t.\ \mathsf{Fool}(p, t)) \Rightarrow \exists p.\ \exists t.\ \neg\mathsf{Fool}(p, t)$
3. ✗ $(\forall p.\ \exists t.\ \mathsf{Fool}(p, t)) \wedge (\exists p.\ \forall t.\ \mathsf{Fool}(p, t)) \wedge \neg\forall p.\ \forall t.\ \neg\mathsf{Fool}(p, t)$
4. ✔ $(\forall p.\ \exists t.\ \mathsf{Fool}(p, t)) \wedge (\exists p.\ \forall t.\ \mathsf{Fool}(p, t)) \wedge \neg\forall p.\ \forall t.\ \mathsf{Fool}(p, t)$
5. ✗ $(\forall p.\ \exists t.\ \mathsf{Fool}(p, t)) \wedge (\exists p.\ \forall t.\ \mathsf{Fool}(p, t)) \wedge (\mathit{False} \Rightarrow \forall p.\ \forall t.\ \mathsf{Fool}(p, t))$
6. ✗ $(\exists t.\ \forall p.\ \mathsf{Fool}(p, t)) \wedge (\forall t.\ \exists p.\ \mathsf{Fool}(p, t)) \wedge \neg\forall t.\ \forall p.\ \mathsf{Fool}(p, t)$

Question 3

Here is a proof of a logical statement in *natural deduction style*:



The names of the rules used have been replaced with circled numbers. What is the rule used in each position?

1. ✗ ① is ⇒-E; ② is ∨-I; ③ is ∨-E_1; ④ is ∨-E_2; ⑤ is ⇒-I
2. ✗ ① is ⇒-I; ② is ∨-E; ③ is ∨-I_2; ④ is ∨-I_1; ⑤ is ⇒-E
3. ✗ ① is ∨-I; ② is ⇒-E; ③ is ⇒-I_2; ④ is ⇒-I_1; ⑤ is ∨-E
4. ✗ ① is ∨-I; ② is ⇒-E; ③ is ⇒-I_1; ④ is ⇒-I_2; ⑤ is ∨-E
5. ✔ ① is ⇒-I; ② is ∨-E; ③ is ∨-I_1; ④ is ∨-I_2; ⑤ is ⇒-E

Curry-Howard Correspondence

Question 4

Selct all of the following types for which you can write a total, terminating Haskell function.

1. ✔ `(a -> b) -> (b -> c) -> (a -> c)`
2. ✗ `((a, b) -> c) -> (a -> c)`
3. ✔ `(a -> c) -> ((a, b) -> c)`
4. ✗ `((a -> c) -> c) -> a`

Question 5

What is the computational interpretation of the theorem
$(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$?

1. ✔ The function that transforms a *curried* function to an *uncurried* one.
2. ✗ The function that transforms an *uncurried* function to a *curried* one.
3. ✗ The function that creates a tuple of the two given A values and B values.
4. ✗ There is no computational interpretation of this logical formula.

Question 6

Which of the following Haskell programs constitutes a valid proof of the theorem given in Question 3?

1. ✔

```
proof (Left a) f g = Left (f a)
proof (Right b) f g = Right (g b)
```

2. ✗

```
proof (a, b) f g = (f a, g b)
```

3. ✗

```
proof x f g = if x then f x else g x
```

4. ✗

```
proof x f g = x (f x) (g x)
```

5. ✗

```
proof (Left a) f g = Left (g a)
proof (Right b) f g = Right (f b)
```

Question 7

Below is a complicated proof that assuming A and B, we can derive $A \wedge B$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \wedge A}\delta}{A}\wedge\text{-E}_2 \quad \dfrac{\dfrac{}{B \wedge A}\delta}{B}\wedge\text{-E}_1}{A \wedge B}\wedge\text{-I}}{(B \wedge A) \Rightarrow (A \wedge B)}\Rightarrow\text{-I}^{\delta} \quad \dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{A \wedge B}\Rightarrow\text{-E}$$

What is the equivalent program to this proof, in typed lambda calculus (using Haskell-style syntax for pairs)? Assume $a : A$ and $b : B$.

1. ✗ (a, b)
2. ✔ $(\lambda x.\ (\mathsf{snd}\ x, \mathsf{fst}\ x))\ (b, a)$
3. ✗ $(\mathsf{snd}\ (b, a), \mathsf{fst}\ (b, a))$
4. ✗ $(\mathsf{fst}\ (a, b), \mathsf{snd}\ (a, b))$
5. ✗ $(\lambda x.\ (\mathsf{fst}\ x, \mathsf{snd}\ x))\ (a, b)$

Question 8

What proof results from applying the *proof simplification* rule corresponding to β-reduction (and **only** that rule) to the proof from Question 7?

1. ✗

$$\dfrac{\overline{A} \qquad \dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{B}\wedge\text{-E}_1}{A \wedge B}\wedge\text{-I}$$

2. ✗

$$\dfrac{\dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{A}\wedge\text{-E}_2 \qquad \overline{B}}{A \wedge B}\wedge\text{-I}$$

3. ✗

$$\dfrac{\overline{A} \quad \overline{B}}{A \wedge B}\wedge\text{-I}$$

4. ✔

$$\dfrac{\dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{A}\wedge\text{-E}_2 \qquad \dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{B}\wedge\text{-E}_1}{A \wedge B}\wedge\text{-I}$$

5. ✗

$$\dfrac{\dfrac{\dfrac{A \quad B}{A \wedge B}\wedge\text{-I}}{A}\wedge\text{-E}_1 \qquad \dfrac{\dfrac{A \quad B}{A \wedge B}\wedge\text{-I}}{B}\wedge\text{-E}_2}{A \wedge B}\wedge\text{-I}$$

Submission is already closed for this quiz. You can click here to check your submission (if any).