Financial Time Series

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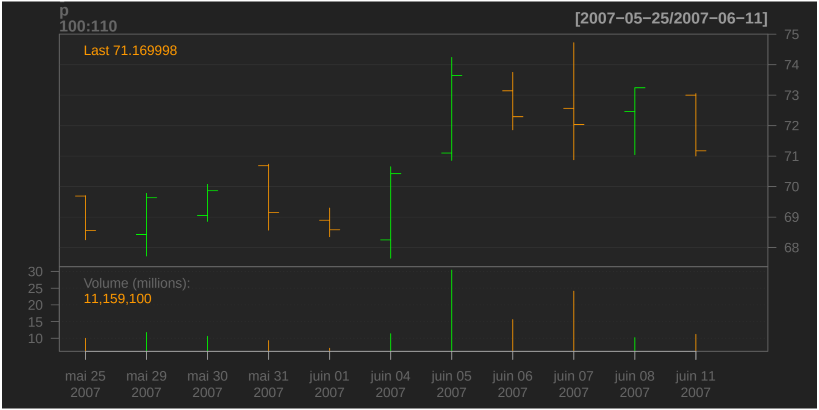
Version: 07 févr. 2024

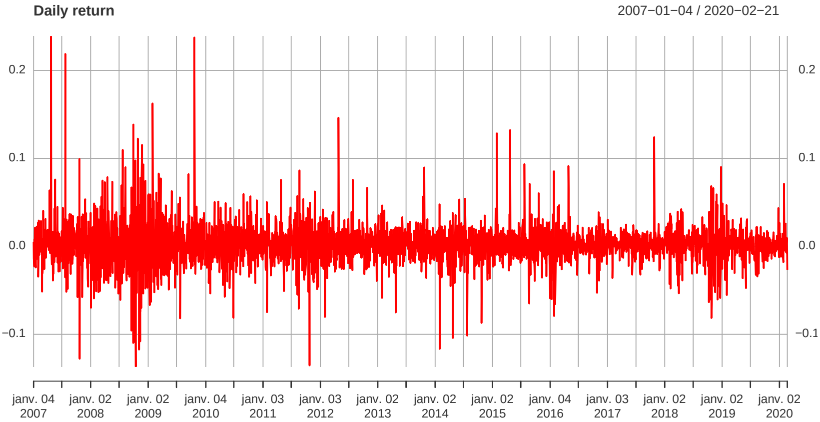
```
## Loading required package: xts

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: TTR

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

##
## Attaching package: 'lubridate'

## The following objects are masked from 'package:base':
```

Financial Time Series (daily OHLC)

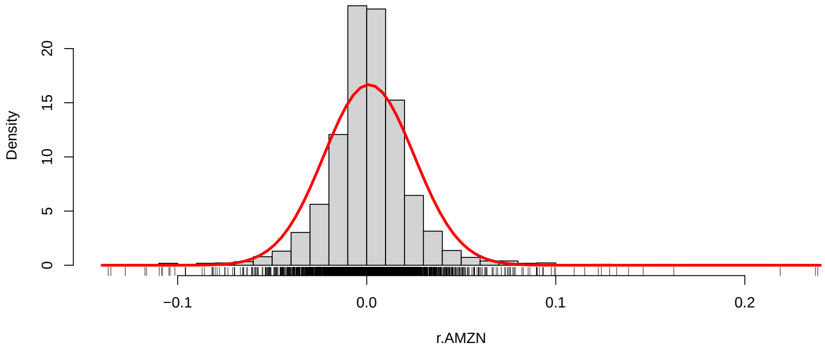


Daily Return - AMZN

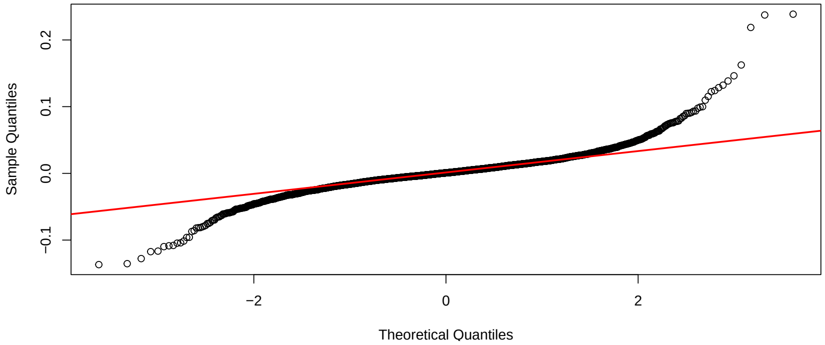
$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$



Histogram of daily return - AMZN



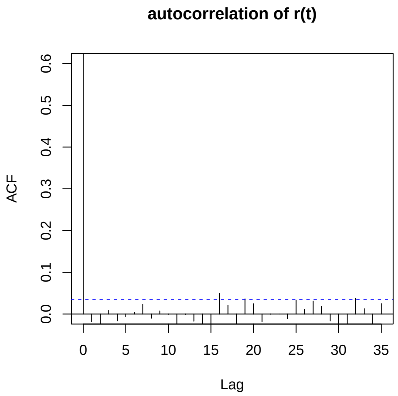
Analysis of return distribution - AMZN

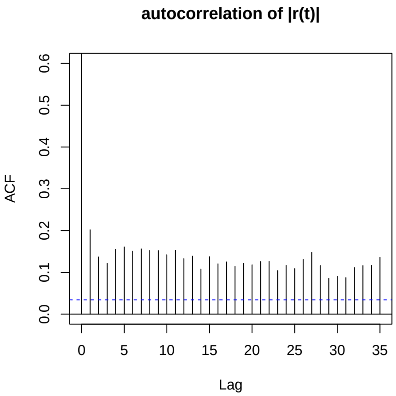


Moments of daily returns

	mean	std dev	skewness	kurtosis
AMZN	0.0012075	0.0239215	0.9530444	12.859279
GOOG	0.0005604	0.0178051	0.5011199	11.504737
AAPL	0.0010301	0.0196726	-0.4681109	7.182435
QQQ	0.0005412	0.0130159	-0.1804299	6.992014
DIA	0.0003516	0.0114022	0.2151165	15.408315
SPY	0.0003397	0.0121101	-0.1558448	14.120228
PG	0.0003238	0.0109203	-0.1052706	7.853350
KO	0.0004433	0.0112445	0.2437227	12.933217

Autocorrelation of Returns (AMZN)





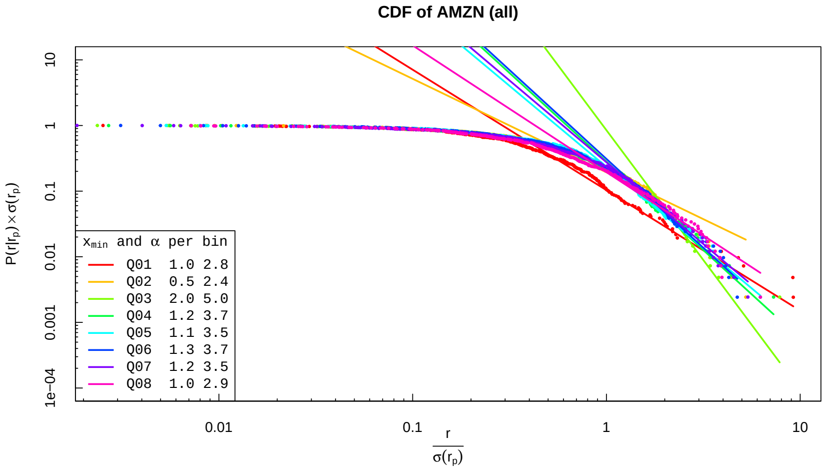
Rescaling daily return by $\sigma(r_{t-1})$ (Chen, Jayaprakash, and Yuan 2008)

$$z_t = \frac{r_t}{\sigma(|r_{t-1}|)}$$

The density of z_t can be approximated by a power law. See paper for details of calculation.

$$\left.\begin{aligned} p(z_t) &= \frac{\alpha - 1}{z_{min}} \left(\frac{z_t}{z_{min}}\right)^{-\alpha} \\ Pr(z_t > x) &= \left(\frac{x}{z_{min}}\right)^{-\alpha+1} \end{aligned}\right\} z_t > z_{min}$$

Rescaling of daily return by $\sigma(|r_{t-1}|)$



Unconditional distribution of return

The Johnson family of distributions is formed by various transformations of the normal density. Let X be the observed data, and define Z by:

$$Z = \gamma + \delta \ln \left(g \left(\frac{X - \xi}{\lambda} \right) \right)$$

where:

$$g(u) = \begin{cases} u & SL \\ u + \sqrt{1 + u^2} & SU \\ \frac{u}{1-u} & SB \\ e^u & SN \end{cases}$$

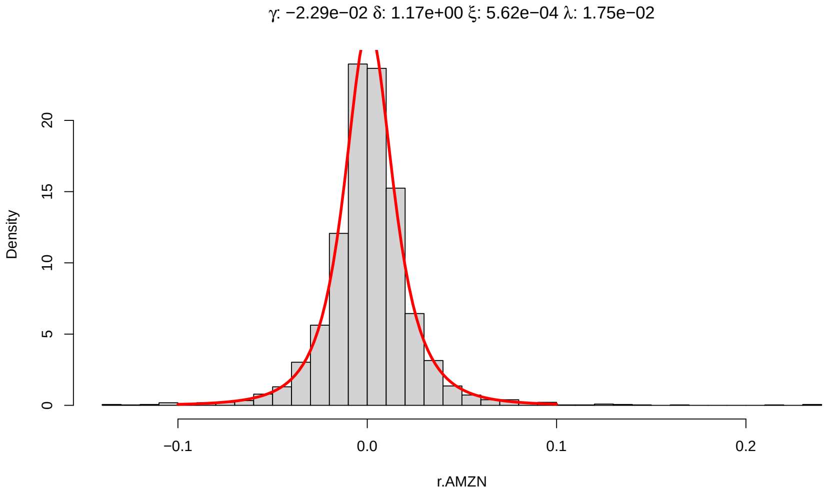
X follows a Johnson distribution if Z is normal.

Fitted Johnson SU distribution - AMZN (1)

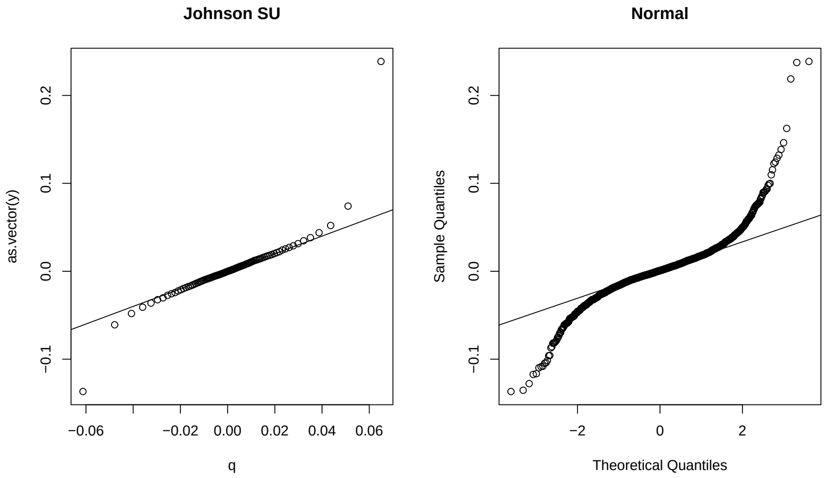
gamma	delta	xi	lambda	type
-0.0228945	1.16685	0.0005621	0.0174527	SU

	sample	johnson
mean	0.0012075	0.0010565
sigma	0.0239179	0.0225752
skew	0.9530444	-0.1098671
kurt	12.8592791	12.3022551

Fitted Johnson SU distribution - AMZN (2)



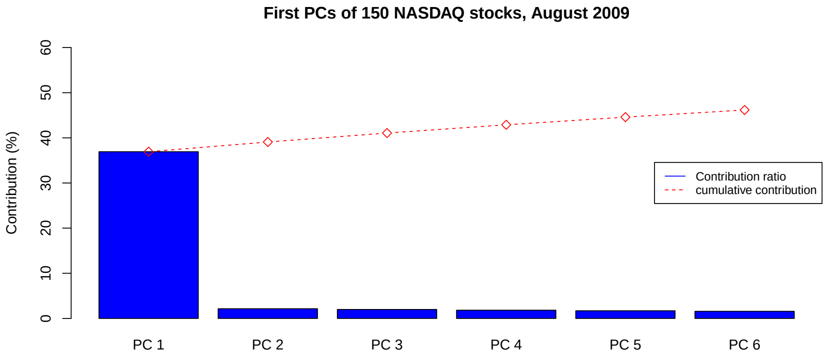
Fitted Johnson SU distribution - AMZN (3)



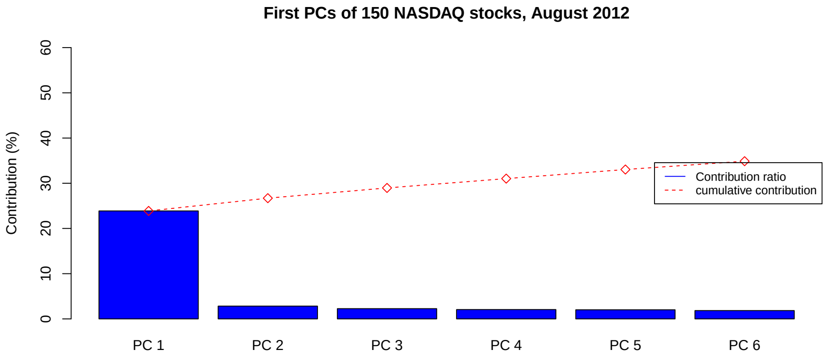
Correlation between assets (NASDAQ)

```
## Loading required package: timeSeries

## Loading required package: timeDate

##
## Attaching package: 'timeDate'

## The following objects are masked from 'package:Performan
##
##     kurtosis, skewness

## The following object is masked from 'package:xtable':
##
##     align

##
## Attaching package: 'timeSeries'

## The following object is masked from 'package:zoo':
```

Correlation between assets

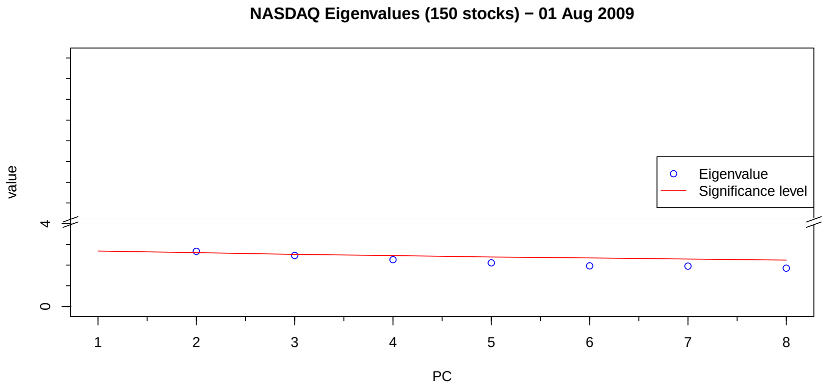


Correlation between assets

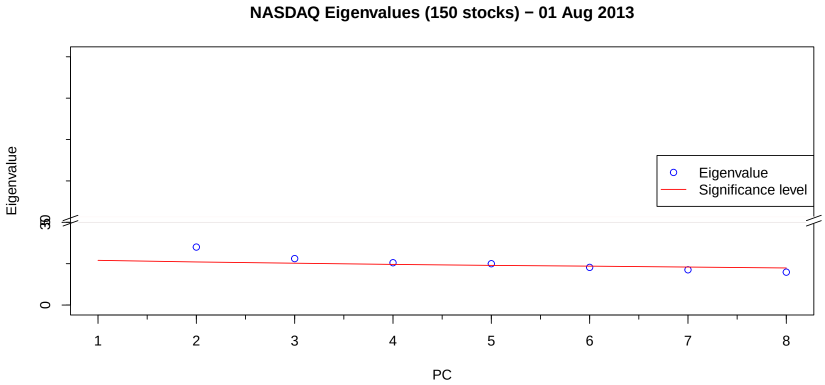


How many dimensions in a market?

Significance level (95%) for eigenvalues (252 observations, 127 variables):



How many dimensions in a market?



Summary

To summarize, empirical observations show that the distribution of returns exhibit features that strongly repart from the classical hypothesis of independence and normality. We find:

1. no evidence of linear autocorrelation of return, however,
2. there is an observable autocorrelation of $|r_t|$ and r_t^2, suggesting autocorrelation in the volatility of return,
3. we also observe large excess kurtosis, which is incompatible with normal density,
4. The rank of a broad stock market such as the NASDAQ is probably much lower than the number of stocks.

Bibliography

Chen, Kan, C Jayaprakash, and Baosheng Yuan. 2008. "Conditional Probability as a Measure of Volatility Clustering in Financial Time Series." *Physica A*, 1–5. https://arxiv.org/abs/0503157v2.