CSC336

Numerical Methods

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CONTENTS

Part I

Part 1

CHAPTER

INTRODUCTION 1

1.1 Motivation

$$\text{Math Textbook} + \text{Laptop} + \text{Coding} \overset{?}{\Longrightarrow} \text{Compute Accurate Solution}$$

Consider the McLaurin series expansion of the function $f(x) = e^x$:

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \end{aligned}$$

The issue is that we cannot compute to infinity. We need to introduce partial sums

$$S_n = \sum_{i=0}^{n} \frac{x^i}{i!}$$

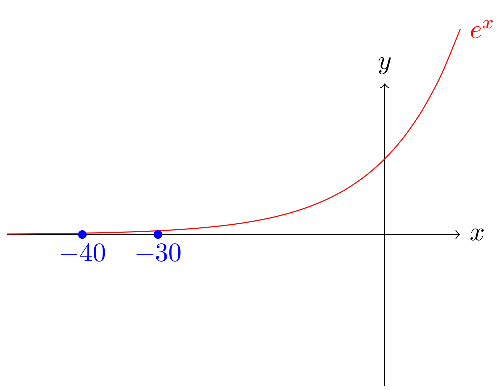
We could iterate over n until $|S_n - S_{n-1}| <$ tolerance.

x	0	1	10	20	40
Num Terms to "Converge"	2	13	42	69	104

We observe that the running time is dependent on the value of x. We need to find a better way to compute the sum – with more consistent running time.

Using the python program,

- When $x = -30$, convergence happened after 97 terms, to -6.0×10^{-5}.
- When $x = -40$, convergence happened after 124 terms, to approximately -5.9×10^{0}.



Clearly, we have inaccuracy when $x = -40$, as $0 < e^x < 1$ for all $x < 0$. The math textbooks' s techniques does not always provide good computational algorithms.

Course goal:

Show computational algorithms and discuss why they are good.

Example (e^x Better Algorithm)**.** A better algorithm is as follows

- Find k such that $r = \frac{x}{k}$ exactly with $\|r\| < 1$.
- Compute $e^r = e^{x/k}$ using the McLaurin series.
- Then, $e^x = (e^r)^k$.

♢

> **Remark** Error due to Catasrophic Cancellation
> When we subtract two numbers that are very close to each other, we lose precision.

1.2 Topics

- Computer Arithmetic and Computational Errors (Chap. 1)
  - Floating Point Arithmetic
  - Two Concepts
    - The conditioning of a math problem
    - the numerical stability of an algorithm
- Solving Systems of Linear Equations (Chap. 2)
  - Solve $Ax = b$ for x
- Solving Non-linear Equations (Chap. 5)
  Fine x s.t. $f(x) = 0$ or $g(x) = 0$ or $f(x) = g(x)$.
- Interpolation (Chap. 7)
  - Given the set of data
    $$\{(t_i, y_i)\}_{i=0}^n \qquad \text{or} \qquad \{(t_i, f(t_i))\}_{i=0}^n$$
    come up with a function $g(t)$ that approximates the data.

Part II

Appendices

BIBLIOGRAPHY

[1] Danelnov, *Plantilla latex*, https://github.com/Danelnov/Plantilla-latex, 2022.

[2] Material Design. "The color system." Section: Tools for Picking Colors. (2024), [Online]. Available: https://m2.material.io/design/color/the-color-system.html#tools-for-picking-colors.