Assignment 3: Asset Swap and CDS

Financial Engineering course

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1 Exercise: Asset Swap

We are given the discounting curve vs Euribor 3m on the 15th of February 2008 at 10:45 C.E.T. and we know the price of a 3y bond for an issuer YY is 1.01 with an annual coupon paid on the swap payment dates equal to 3.9% .
We can compute the Asset Swap spread directly from the formula:

$$s^{asw} = \frac{C(0) - \overline{C}(0)}{BPV^{fl}(0)}$$

In our case we have:

- $\overline{C}(0) = 1.01$, being the price of the fixed CB of the issuer YY;
- $C(0) = 1.0005$, being the price of the "I.B" not-defaultable bond. This term is derived discounting at present time the relative cash-flows;
- $BPV^{fl}(0) = \sum_{i=1}^{N_f} c\delta(t_{i-1}^f, t_i^f)B(t_0^f, t_i^f)$, being the BPV of the floating leg payments.

Through these quantities we get $s^{asw} = -33bps$. We are not expecting to find a negative spread. However it could suggest the presence of an arbitrage; we may think that the bond issued by YY is less risky than the ones traded on the Interbank market and we can suppose that this issuer may be rated AA+ or AAA. This situation can also be due to the period we are considering i.e. 2008 year, in the midst of the financial crisis, characterized by a lot of instability and changes in the regulations.

2 Case Study: : CDS Bootstrap

Given the aforementioned discount factors curve, we now consider ISP as an obligor with:

$$\text{recovery rate } \pi = 0.4$$

and given CDS spreads:

1y	2y	3y	4y	5y	7y
29 bps	32 bps	35 bps	39 bps	40 bps	41 bps

In order to build a complete set of CDS we adopt a **spline interpolation**. We observe interpolating using the last 3 points ensures for a trade-off between accuracy and interpretation, since it is the maximum number of points which ensures an increasing spread structure. The missing 6y CDS spread obtain is:

$$\text{6y spread} = 40.6 \text{ bps}.$$

This value is reasonable since the CDS spread curve is increasing and flattening over the years.

Then we derive the intensities considering λ(t) piecewise constant for the issuer and using the three methods required: first neglecting the "accrual" term, then considering it and finally the Jarrow-Turnbull approximation.

As regards the first two methods, firstly we find the survival probabilities inverting the following formula for every year i:

$$\overline{s_i}\sum_{j=1}^{i}\delta(t_{j-1},t_j)B(t_0,t_j)P(t_0,t_j)+\overline{s_i}\sum_{j=1}^{i}\frac{\delta(t_{j-1},t_j)}{2}B(t_0,t_j)(P(t_0,t_{j-1})-P(t_0,t_j)) =$$

$$= (1-\pi)\sum_{j=1}^{i}B(t_0,t_j)(P(t_0,t_{j-1})-P(t_0,t_j))$$

In particular, the second term (the accrual) is neglected in the first method, while the full formula is used for the second.

Then we recover the intensities from the survival probabilities by:

$$\lambda(t_i) = -\frac{1}{\delta(t_{i-1},t_i)}\ln\left(\frac{P(t_0,t_i)}{P(t_0,t_{i-1})}\right)$$

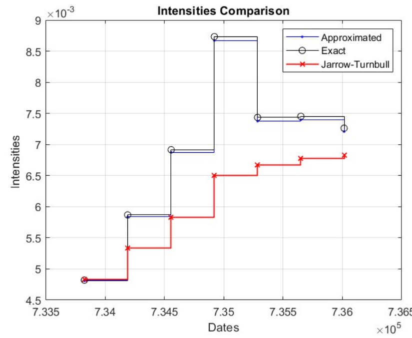
Instead, as for the Jarrow-Turnbull approximation we use:

$$\lambda(t_i) = \frac{\overline{s_i}}{1-\pi}$$

to find the intensities. Then we recover the survival probabilities this way:

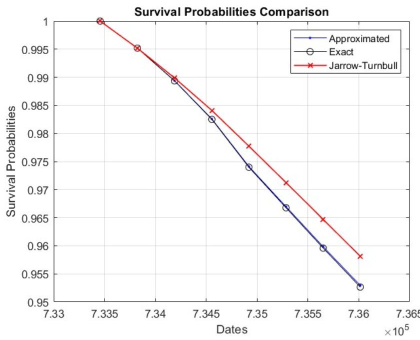
$$P(t_0,t_i) = e^{-\sum_{j=1}^{i}\lambda(t_j)*\delta(t_{j-1},t_j)}$$

Drawing all these values for the three different methods, we have obtained the following plot for the intensities:



We can clearly observe that the "accrual" term is really negligible for the first 2 methods and that the JT approximation is quite precise for the first years and is quite imprecise from year 4 and following.

These are the survival probabilities computed with the three methods:



3 Exercise: Price First to Default

We are now given a set of CDS spread referred to Unicredit Group (UCG) with respective recovery:

$$\text{recovery rate } \pi = 0.45$$

and incomplete spread structure:

1y	2y	3y	4y	5y	7y
34 bps	39 bps	45 bps	46 bps	47 bps	47 bps

In order to build a complete set of CDS, we need to retrieve the CDS spread in 6y by spline interpolation as previously presented for ISP, getting $s_{6y} = 47bps$. The first to default, with maturity 20th of February 2012, that we want to price has been settled in 15th of February 2008.

With our data and exploiting the methods described in section 2, we retrieve the survival probabilities.

In order to price the First to Default, we consider the Li Model with Gaussian Copula to retrieve time to default and correlation term $\rho = 0.2$.

The selected methodology is based on Monte Carlo simulation of N different scenarios. Via simulation of y and consequently of $\underline{\text{u}}$, we are able to simulate the time to default τ_i for both obligors. For each scenario, we record the NPV relative to both the fee and contingent leg. Different situations can happen in each simulation and they are differently treated:

1. **Neither ISP or UCG defaults:**

$NPV^{fee} = \frac{\delta(\tau-1,\tau)}{2} B(t_0, \tau)$
$NPV^{contingent} = 0$

2. **ISP and/or UCG defaults:**

default happens in $\tau \leq 1$:
$NPV^{fee} = \frac{\delta(\tau-1,\tau)}{2} B(t_0, \tau)$
$NPV^{contingent} = (1-\pi) B(t_0, \tau)$

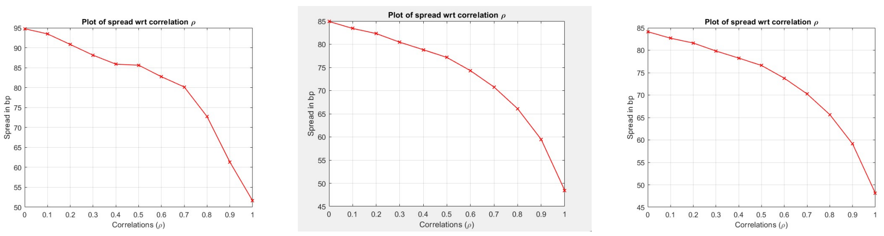
default happens in $\tau > 1$:
$NPV^{fee} = \sum_{j=1}^{j:t_j=\tau} \delta(t_{j-1}, t_j) B(t_0, t_j) + \frac{\delta(\tau-1,\tau)}{2} B(t_0, \tau)$
$NPV^{contingent} = (1-\pi) B(t_0, \tau)$

Once obtained the complete vectors for both NPVs according to all the scenarios, we compute the mean of the NPVs, then considering the ratio:

$$\frac{\overline{NPV^{contingent}}}{\overline{NPV^{fee}}}$$

we obtain the final price.

Finally, we plot the First to default price w.r.t. different values of the correlation ρ, in particular we considered $0 \leq \rho \leq 1$ and notice that as the correlation increases the FtD prices decreases, we expect this since if the 2 obligors are highly correlated it's similar to a CDS on the riskier one while if they are not correlated it's like buying both CDS.



((a)) $2000 simulations$ ((b)) $20000 simulations$ ((c)) $50000 simulations$

Figura 1: Plot of spreads with respect to correlation, varying the number of simulations

The first plot is obtained with N=2000 simulations, we can see it is not very accurate as the one with N=20000 simulations where we see what we expected from the theory i.e $s_{max} \leq s^{FtD} \leq \sum_{i=1}^{n} s_i$, where n here is 2 and is the number of obligors, s_{max} is the maximum of CDS spreads of the 2 obligors at 4 years, in fact here we see that $46 \leq s^{FtD} \leq 85$ the only drawback with so many simulations is the computational time, which drastically increase increasing the number of simulations.

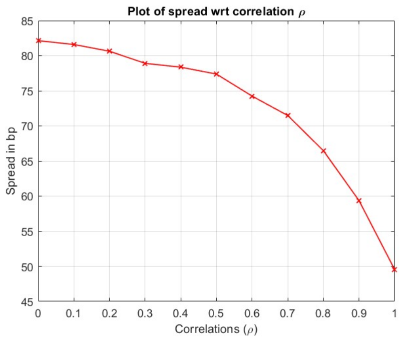


Figura 2: Plot of spreads with respect to correlation, with 10000 simulations

We delivered a code with 10000 simulations since it takes around five minutes to get results, even if they are not much accurate. For better accuracy, it's suggested to increase N, while for a faster code is suggested to decrease N, even if the results will get more inaccurate as more as N get decreased.

Moreover we can see that the FtD price with $\rho = 0.2$ is 81bps which is coherent with what we expected from the considerations above.