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Energy Project D

Group 10

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1 Introduction

Electricity is a very unusual commodity in the commodity framework because it cannot be easily stored. This characteristic significantly affects not only the infrastructure and organization of transmission but also the markets, as it leads to volatile behavior with price spikes and seasonality.

In this intricate domain of Energy Finance, the calibration and pricing of exotic options associated with commodity and electricity swaps are of growing importance in recent times. As the energy markets undergo continual transformation, the complex nature of commodities and electricity swaps, introduces both challenges and prospects for discerning market participants.

The aim of this project is to exploit the HJM NIG model on French electricity swaps in order to calibrate and price some structured payoff options. The data we had to deal with were provided through an Excel file with 4 sheets. The first sheet contains all liquid maturities for French power swaps; the second and third sheets contain the option prices (implied volatility quotes) on the 2026 and the 2028 futures (monitoring period over the year, delivery at the end) on November 4th 2024; the fourth sheet contains the market discount factor curve.

In order to achieve the stated goals, we largely exploited the energy finance book "Stochastic Modelling of Electricity and Related Markets".

2 Admissible range for the model parameters and drift condition

We start by introducing the general HJM model for the swap. This is its risk-neutral dynamics under the measure $\mathbb{Q}$:

$$F(t,\tau_1,\tau_2) = F(0,\tau_1,\tau_2)exp\Big(\int_0^t A(u,\tau_1,\tau_2)du + \sum_{k=1}^{p}\int_0^t \Sigma_k(u,\tau_1,\tau_2)dW_k(u) + \sum_{j=1}^{n}\int_0^t Y_j(u,\tau_1,\tau_2)dJ_j(u)\Big)$$

Here, $A(u,\tau_1,\tau_2)$, $\Sigma_k(t,\tau_1,\tau_2)$ and $Y_j(t,\tau_1,\tau_2)$ with i=k,...,p and j=1,...,n , are real-valued continuous ($\Sigma_k(t,\tau_1,\tau_2)$ also positive) functions where $0 \leq \tau_1 \leq \tau_2$, with τ_1, τ_2 being respectively the starting and the ending dates of the delivery. In particular in our case we consider the case with no brownian motion, i.e p=0 and the driver being a NIG process ($J(t)$), therefore n=1. Moreover we consider $Y_j(u,\tau_1,\tau_2)$ to be constant, then the model can be stated as:

$$F(t,\tau_1,\tau_2) = F(0,\tau_1,\tau_2)exp\Big(\int_0^t A(u,\tau_1,\tau_2)du + YJ(t)\Big) \tag{1}$$

Regarding the ranges of the model parameters, these can be obtained imposing appropriate conditions in order to reasonably model the swap prices.
First of all it's important to state that given a positive initial condition $F(0,\tau_1,\tau_2)$ the positivity of the swap prices is guaranteed by the model, moreover also the exponential integrability condition, which in our case is:

$$\int_0^{\tau_1}\int_{|z|\geq 1} e^{Yz}\nu(dz)du < \infty$$

is a condition already satisfied since the NIG is a Lévy process and $\nu(z)$ its Lévy measure.
We should ensure that the forward prices are martingales and here we can also write explicitly the drift condition which is:

$$\int_0^t A(u,\tau_1,\tau_2)du + Y\int_0^t d\gamma(u) + \int_0^t\int_{\mathbb{R}}(e^{Yz} - 1 - Yz\mathbb{1}_{|z|<1})\nu(dz)du = 0$$

Leveraging on the fact that the last two integrals in the expression above constitute the Lévy-Kintchine representation of the NIG Lévy process, we obtain from the explicit NIG characteristic exponent the following drift condition:

$$-\int_0^t A(u,\tau_1,\tau_2)du = \Psi_{NIG}(-iY)t = t\Big(\frac{1}{k} - \frac{1}{k}\sqrt{1 - Y^2\sigma^2 k - 2\theta Y k}\Big) \tag{2}$$

Thanks to this computation we have directly written the integral of A, this will be crucial for the simulation of the swap price being an explicit writing of the drift, it enables us to avoid numerical integration.

At this stage we can also state the explicit formula for the characteristic function.
If we define the process

$$X_t = \int_0^t A(u, \tau_1, \tau_2)du + YJ_t$$

we can compute its characteristic function as

$$\phi_{X_t}(u) = E[exp(iuX_t)] = exp(iu(\int_0^t A(u, \tau_1, \tau_2)du + t\Psi_{NIG}(uY)))$$

we can now substitute using the relationship found at 2 and we get

$$\phi_{X_t}(u) = exp(-iut\Psi_{NIG}(-iY) + t\Psi_{NIG}(uY))$$

This formula will be crucial for calibration, as we can use the FFT (Carr-Madan) algorithm to price the call option by directly utilizing this expression for the characteristic function.
Finally we impose the argument of the square root in 2 has to be greater than 0 since otherwise we would have the integral of A with an imaginary part, but this would lead to a complex drift which is unreasonable, therefore:

$$Y^2\sigma^2 k + 2\theta Y k \leq 1$$

This is the range of Y, the coefficient in front of the stochastic integral:

$$\frac{-\theta k - \sqrt{\theta^2 k^2 + \sigma^2 k}}{\sigma^2 k} \leq Y \leq \frac{-\theta k + \sqrt{\theta^2 k^2 + \sigma^2 k}}{\sigma^2 k}$$

It's important to notice that the argument of the 2 square roots is positive since σ is the volatility of the brownian motion and k is the variance of the subordinator, that's why both have to be non-negative: $\sigma \geq 0$ and $k \geq 0$, while θ being the drift of the brownian motion can in principle assume every value in $\mathbb{R}$. These are the ranges of the 4 parameters characterizing our HJM NIG model.

3 Calibration 2026 French option prices

Our aim is to calibrate the model on the 2026 French option prices, in order to price derivatives. To do that, we have to find a suitable set of parameters that works properly and rightly describes the dynamic of the underlying. Under HJM NIG framework, using the previously descripted formulas, we had to calibrate four parameters: [k,σ,θ,Y].

We were given in Excel file *DATA˙FREEX.xlsx* the implied volatility of the market, so, first of all, we import the data and plot the surface. Then we obtained the market option prices using Matlab function *blkprice*.

The option price surface can be found below and regarding it there's an important phenomenon to analyze: the option prices rise as the strike raises, as we can see for strikes above 300. Given that these are call options and it happens also for equal tenors, there's a clear arbitrage opportunity. This phenomenon, though not uncommon for commodities, is likely attributed to the illiquidity of the market.

Moreover, as we said, we derived this surface using the Black formula which, together with all the theory we exploit in this project, leverages on the Absence of Arbitrage assumption on the market. The lack of occurence of this assumption, as we will see, will heavily affect the quality of our calibration.

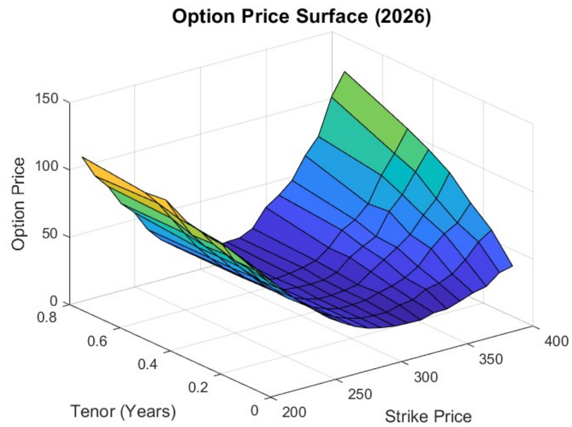


Figure 1: Option Price Surface (2026)

To perform the calibration we have created a function that minimizes the difference between model prices (passed as function handle with respect [k,σ,θ,Y]) and market prices (previously computed). As loss function we use a weighted MSE:

$$\text{wMSE} = \frac{1}{n} \cdot \sum_{i=1}^{n} w_i \cdot \left(p_{\text{model},i} - p_{\text{market},i}\right)^2$$

where

$$w_i = \frac{1}{(F_0 - strike)^2}$$

We decided to apply weights to penalize strikes that are far from the actual value of F_0. This adjustment led to a reduction in the error, but the model was unable to capture the unusual behavior of the market for very OTM strikes. As a result, we decided to limit the range to strikes between 230 and 330, since the underlying swap starts from $F_0 = 284.35$.

In order to obtain suitable parameters, we performed several simulation and several attempt, using different Matlab function, in particular *fmincon, lsqnonlin* and *fminsearch*.

This method relies on the wMSE distance definition, so all the functions try to minimize this error. Moreover, since we are in Carr-Madan (FFT) framework, we have two discretization parameters: A and *Npow*.
Performing the calibration, our goal was to obtain robust result, that despite a small or big changing in discretization parameters, remains more or less stable.

We selected the initial point [0.05, 0.9, -1.3, 1] for the optimization process. It's important to emphasize that choosing a different starting point can significantly affect both the resulting parameter values and the computational time.

This variability arises because the starting point determines the proximity to a local minimum, which influences the efficiency of the optimization search and its convergence behavior. In the minimization function we restrict the parameter calibration imposing positivity to σ and k and we also add the nonlinear constraints, relying on the range of Y explained in the previous point.

Now we report the result using *fmincon* and *lsqnonlin*. We avoid to report the *fminsearch* results, since they were not satisfying at all. We started reporting what we obtained using *fmincon*.

Discretization Parameters	**Time to run**	**k**	$\boldsymbol{\sigma}$	θ	**Y**
Npow=16 A=1000	106.58s	1.7309e-05	2.4937e+05	61348	-1.3214e-06

We are not satisfied about this calibration since the parameters found are very different and not comparable in terms of order of magnitude and it's also very computationally expensive. We considered this value unacceptable and therefore we did not proceed and focused on *lsqnonlin*

As we can see in this case the parameters seems to be more realistic and meaningfull, they are stable with respect to the discretization of FFT and the elapsed time is much better.
In the following, we show the result and the comparison between the market and the model surface.

Discretization Parameters	**Time to run**	**k**	$\boldsymbol{\sigma}$	θ	**Y**	**wMSE**
Npow=16 A=1000	34.27s	1.3564e-05	1.1925	-0.9776	0.27673	0.4055
Npow=14 A=600	6.84s	1.3572e-05	1.1914	-1.0004	0.27685	0.4052
Npow=12 A=400	2.07s	6.7899e-05	1.1919	-0.98787	0.27683	0.4054

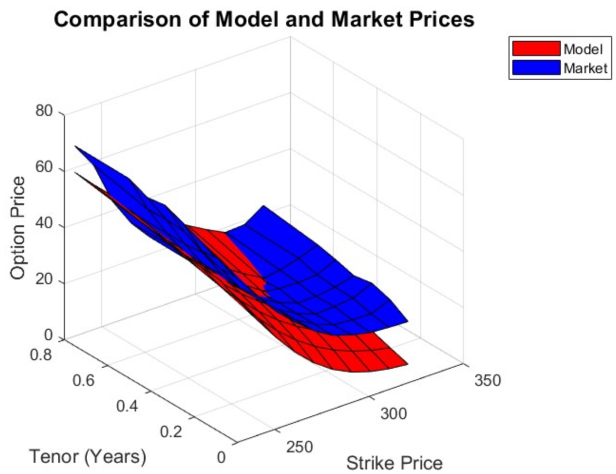


Figure 2: Surfaces Comparison (2026)

4 Pricing path dependent Option

In this section, we are tasked with pricing a Lookback Call option with the following payoff:

$$\left(\max_{t\in[0,1]} \left(F(t,\tau_1,\tau_2)\right) - K\right)^+$$

where $F(t,\tau_1,\tau_2)$ represents the swap dynamic, τ_1 is the delivery start date set to 1/1/2026, τ_2 is the delivery end date set to 31/12/2026, and $K = 300$ is the strike price of the option.

4.1 Monte Carlo Simulation

To price the option, we employ a Monte Carlo approach, which is always feasible for this type of problem. The main building blocks of the simulation are the swap dynamics, described by equation 1, and the drift condition given by equation 2. The initial value of the swap dynamic is $F(0,\tau_1,\tau_2) = 284.35$, which corresponds to the price of the future with a delivery period throughout the entire year of 2026.

For the simulation, we set the following parameters: the number of simulations is $N = 1$ million and the number of monitoring dates is $M = 252$, corresponding to daily monitoring.

The dynamics of the swap price can be rewritten as:

$$F(t,\tau_1,\tau_2) = F(0,\tau_1,\tau_2)\exp(X_t)$$

For each monitoring date, the process is updated as follows:

$$X_{i+1} = X_i + \text{drift} \cdot \Delta t + Y \cdot \left(\theta \cdot \Delta S + \sigma \cdot \sqrt{\Delta t} \cdot Z\right)$$

where:

- X_i is the previous value of the process.
- drift $= \Psi_{\text{NIG}}(-iY)$.
- Δt is the time interval between monitoring dates.
- σ, θ, k, Y are the NIG parameters obtained from the calibration performed earlier.
- ΔS is a value sampled from an Inverse Gaussian distribution with mean Δt and shape parameter $\frac{\Delta t^2}{k}$.
- Z is a value sampled from a standard normal distribution $N(0, 1)$.

At the end of the simulation, we check the martingale property of the process X by computing $E[\exp(X)]$, which should be equal to $\phi_X(-i) = 1$.

During the simulation, we track the maximum value reached by the process in order to compute the payoff. Finally, the price of the Lookback option is the discounted mean of the simulated payoff.

To compute a confidence interval for the price, we use the MATLAB function *normfit*, which provides the 95% confidence interval.

We also implement the Antithetic Variable technique, a variance reduction method that improves the efficiency of Monte Carlo simulations. For the standard normal variable, we simulate two processes using Z and $-Z$ respectively.

For the Inverse Gaussian variable, within the MATLAB function *fastInverseGaussian*, we sample a Uniform random variable U, and use U and $1 - U$ as antithetic variables for the first and second process.

It is important to note that we perform these techniques one at a time, as we must preserve the correlation and independence between the variables.

The table below summarizes the results of our simulation:

	Price	IC	Δ
MC	61.895	$[61.756, 62.035]$	0.27827
MC AV (ΔS)	61.956	$[61.817, 62.095]$	0.27806
MC AV (Z)	61.775	$[61.707, 61.843]$	0.13597

Table 1: Lookback option Prices

At the end, we performed a double-check to verify the accuracy of our simulation. Specifically, we priced a call option using both the Monte Carlo method and the Carr-Madan algorithm. The results we obtained are as follows:

	Price
MC	29.53
Carr-Madan	29.563

Table 2: Call option Prices

Since the results are very close, it seems that our simulation is functioning correctly. Additionally, the fact that the price of the call option is lower than the lookback option is consistent, as the lookback option considers the maximum value over the entire year.

4.2 Closed Formula

We also attempted to explore a closed-form solution, as the Monte Carlo method is computationally expensive. However, we encountered two main challenges. The first is related to the construction of the payoff, as it is a path-dependent option. The second challenge arises from the fact that our underlying asset follows a NIG model, which is more complex than, for example, the Black-Scholes model, where a closed-form solution is available.

5 Calibration 2026 and 2028 French option prices

In this section, we were asked to calibrate our model taking into account both 2026 and 2028 surfaces. Under the HJM framework, we consider all swap contracts to be driven by their own random sources. Indeed, respecting the framework, we would calibrate 2 sets of 4 parameters for each curve, which is equivalent to conducting 2 separate calibrations. This approach is further discussed in Section 6.1.

However, we decided to bend the framework's rules to access a close formula for the option introduced in Section 6, more details will be introduced in the proper setting. In this adjusted framework, we consider the two swap contracts to be modelled as follows:

$$F(t,\tau_1,\tau_2) = F(0,\tau_1,\tau_2)\exp(X_t)$$
$$F(t,\tau_3,\tau_4) = F(0,\tau_3,\tau_4)\exp(X_t)$$

Indeed, swaps are driven by indistinguishable Lévy processes. This is a stretch, mainly because we are assuming a perfect correlation between their behaviours. Utilizing this approach, the join calibration results in the estimation of the usual 4 parameters using the 2 price surfaces.

We performed this calibration using, as before, *lsqnonlin*. However, in this case, we also add upper and lower bounds for the four parameters in order to reduce the computational time needed to complete the calibration. We set LB = [0, 0, -10, -10] and UB = [10, 10, 10, 10] respectively for [k,σ,θ,Y].

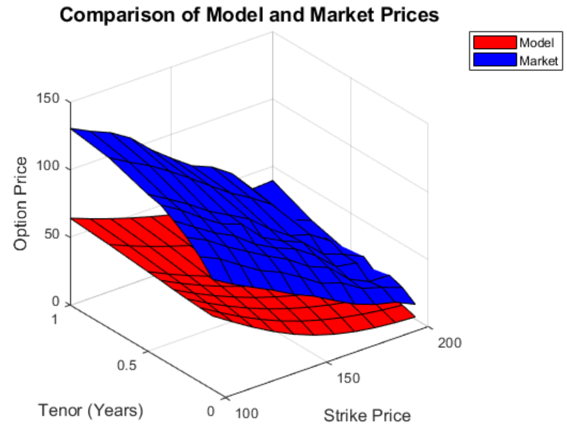
Discretization Parameters	Time to run	k	σ	θ	Y	wMSE
Npow=16 A=1000	124.73s	1.0312e-05	5.8588	-3.1424	0.11376	15.2051
Npow=14 A=600	6.88s	2.6528e-06	4.3523	-2.8333	0.15311	15.2083
Npow=12 A=400	6.24	1.0264e-05	5.6273	-2.9215	0.11844	15.2068

Given the possibility of multiple local minimums, after multiple trials we selected a single initial point; it is worth noting that even if the model parameters were different between starting points, the resulting surfaces were similar to the ones reported. It is worth noting that the model struggles to capture the price surface. We were approximately considering double the amount of fitting points, using the same number of parameters; in addition to the previously mentioned calibrating issues.

Comparing the surfaces we can observe that the 2026 surface Model prices are generally higher than the market ones, whereas in the 2028 case model prices are dominated by the market ones. It is clear that the model struggles to capture those surfaces together. Given the risk of misspricing options, a user utilizing the fitted model must be aware of these issues.

The two surfaces are shown in the following plots:

(a) Surfaces Comparison (2026)



(b) Surfaces Comparison (2028)

Figure 3: Joint calibration comparison

6 Pricing Basket Option

In this section, we are tasked with pricing a Basket Option, whose payoff depends on two electricity swap contracts:

$$(\max(F(1, \tau_1, \tau_2), F(1, \tau_3, \tau_4)) - K)^+$$

where $F(t, \tau_1, \tau_2)$ represents the swap contract with τ_1 set to 1/1/2026 and τ_2 set to 31/12/2026 and where $F(t, \tau_3, \tau_4)$ represents the swap contract with τ_3 set to 1/1/2028 and τ_4 set to 31/12/2028. The $K = 300$ is the strike price of the option and $T = 1y$ is the time to maturity.

6.1 Calibration and Monte Carlo Simulation

It's important to recall that the HJM framework considers all swap contracts to be driven by their own random sources. Indeed, respecting the framework, we would calibrate 2 sets of 4 parameters for each curve as described previously, which is equivalent to conducting 2 separate calibrations.

For the parameters concerning the 2026 curve we refer to section 3, whereas in the next table, we report only the values for 2028 since the calibration procedure is analogous with respect the one performed at section 3. In this case we keep the strike between 100 and 200 since $F(0, \tau_3, \tau_4) = 154.98$ and we drop the options with tenor greater than 1y since we are interested in pricing an option with maturity 1y.

Discretization Parameters	**Time to run**	**k**	$\boldsymbol{\sigma}$	θ	**Y**	**wMSE**
Npow=16 A=1000	13.16s	2.8728e-06	1.5519	1.5712	1.0444	0.70221
Npow=14 A=600	4.16s	2.8758e-06	1.5514	1.5678	1.0447	0.70221
Npow=12 A=400	0.79s	2.874e-06	1.5511	1.5649	1.045	0.70229

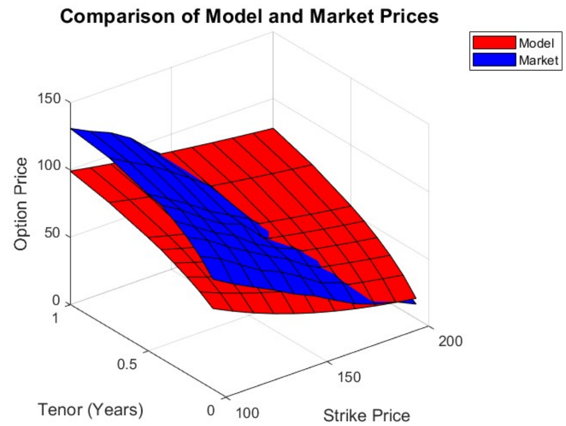


Figure 4: Surfaces Comparison (2028)

To price this option, we proceed through Monte Carlo simulation for the two swap contracts. Specifically, we assume that the two underlying processes $F(t, \tau_1, \tau_2)$ and $F(t, \tau_3, \tau_4)$ are independent. The independence assumption simplifies the correlation structure, enabling the Monte Carlo method to simulate each process separately while ensuring computational efficiency. We recall that the dynamics, under NIG HJM model, are described as follows:

$$F(t, \tau_1, \tau_2) = F(0, \tau_1, \tau_2) \exp\Big(-t\Big(\frac{1}{k_1} - \frac{1}{k_1}\sqrt{1 - Y_1^2\sigma_1^2 k_1 - 2\theta_1 Y_1 k_1}\Big) + Y_1 J_1(t)\Big)$$

$$F(t, \tau_3, \tau_4) = F(0, \tau_3, \tau_4) \exp\Big(-t\Big(\frac{1}{k_2} - \frac{1}{k_2}\sqrt{1 - Y_2^2\sigma_2^2 k_2 - 2\theta_2 Y_2 k_2}\Big) + Y_2 J_2(t)\Big)$$

where the set of parameters ($k_1 = 1.3564e-05, \sigma_1 = 1.1925, \theta_1 = -0.9776, Y_1 = 0.27673$) for the the contract with delivery period in 2026 is obtained thanks to the calibration computed in section **3)**. Similarly, for the dynamics of the contract with delivery in 2028, an additional calibration is required.

This calibration is carried out analogously to the approach in Section **3)**, yielding the set of parameters ($k_2 = 2.8728e-06, \sigma_2 = 1.5519, \theta_2 = 1.5712, Y_2 = 1.0444$). Once all the necessary parameters are obtained, it is then possible to proceed with the MC simulation of the processes. Since in this case, the option to be priced is not path-dependent, and the payoff depends exclusively on the values of the underlying at maturity, the simulation proves to be computationally simpler compared to the case in Section **4)**, as no monitoring procedures are required:

$$X_T^{(1,2)} = \text{drift}^{(1,2)} \cdot T + Y^{(1,2)} \cdot \Big(\theta^{(1,2)} \cdot \Delta S^{(1,2)} + \sigma^{(1,2)} \cdot \sqrt{T} \cdot Z^{(1,2)}\Big)$$

$$F_T^{(1,2)} = F_0^{(1,2)} \exp(X_T^{(1,2)})$$

For the independency of the two processes, we simulate $Z^{(1)}$ and $Z^{(2)}$ in MATLAB as two uncorrelated standard Gaussian and $\Delta S^{(1)}$ and $\Delta S^{(2)}$ as two independent Inverse Gaussian distribution with mean T and shape parameter $\frac{T^2}{k^{(1,2)}}$. As done in step 4, the simulations are carried out using the antithetic variables technique, and the results are shown in the table below:

	Price	IC	Δ
MC	92.408	[91.427, 93.389]	1.9618
MC AV (ΔS)	91.949	[91.011, 92.887]	1.8758
MC AV (Z)	92.541	[91.857, 93.225]	1.3681

Table 5: Best of Option Price

Even in this case, the contribution from the antithetic variables technique (S) is almost negligible, while the method using antithetic variables Z significantly improves the confidence interval.

As before we double check the result computing the single call option on $F(1, \tau_1, \tau_2)$ and $F(1, \tau_3, \tau_4)$ because we know that the Best Of Option has to be greater than the maximum between the two calls and lower than the sum.

	Price
MC	29.563
Carr-Madan	29.563

(a) Call Option on $F(1, \tau_1, \tau_2)$

	Price
MC	65.612
Carr-Madan	65.168

(b) Call Option on $F(1, \tau_3, \tau_4)$

Lower Bound	Price	Upper Bound
65.294	92.408	94.845

(c) Range

Table 6: Check

6.2 Closed Formula

Finding a closed formula in the HJM framework is complex, even using the independence of the swaps contracts assumed by the model, it is difficult to reconduct ourselves to know distributions or characteristic functions for the maximum. On the other hand, addressing the problem in the alternative framework proposed in Section 5 is rather simple. Indeed, the two contracts have the same dynamics and evolution due to the hypothesis of indistinguishable processes. Therefore the contract that has a higher valuation in 0 dominates the other one (in our case the 2026 swap), therefore the basket option reduces itself on a normal call on that swap. Ultimately, we are pricing a European call with a tenor of 1 year, strike 300 and $F_0 = 284$.

Since in the calibrated surface, we do not consider call options with a tenor of 1, we compare this result with the market price of the call with the same underlying and strike and maturity of 9 months. Indeed, the price of this second call is 20.15, significantly lower than the price obtained with the model shown in the table below. This shift in price seems too large with respect to the increase in time to maturity, further underlying the fitting issues of this model and approach. In the table, we report its value, under this approach:

	Price
Carr-Madan	65.576

Table 7: Closed formula

6.3 Evaluation of the 2 approaches

The key difference observed between the two approaches is that the one that assumes independence leads to a higher price than the one that considers the processes to be indistinguishable. This is perfectly reasonable since in the latter approach we are forcing the swap that starts from a lower price at t=0, i.e $F(t, \tau_3, \tau_4)$, to be under the higher one, $F(t, \tau_1, \tau_2)$, in every trajectory, and $\forall t \in [0, 1]$, but this is not very realistic. On the other hand, the first approach introduces another source of randomness and allows trajectories where $F(1, \tau_3, \tau_4) > F(1, \tau_1, \tau_2)$, which switches on the $\max\{\cdot\}$ in the payoff. In other words, in this situation, the derivative has more probability of being in the money at maturity, which justifies the higher price.

6.4 Brief Conclusion

In conclusion, it is important to stress that the presented market, and more generally the energy market, breaks the assumption of non-arbitrage opportunities. Considering the described adjustments, the presented model fits well the surfaces of 2026 and 2028 and seems to capture the price distribution of the options, for a fixed tenor. The assumption of independence between contracts of the HJM seems strong, considering we are dealing with contracts with the same underlying. Still, it produces more coherent results than the opposite approach presented in Section 5.