Politecnico di Milano

COMPUTATIONAL FINANCE

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Portfolio Management Exam C

Group 10

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1 Assets review

The aim of this report is to construct portfolios based on various allocation strategies and evaluate their performances. The investment universe consists of 11 sector indices and 5 factor indices of the S&P 500. For our analysis, we consider returns of the main components of the **S&P500** during the the year of 2023. It's important to recall that the index performed well in this time frame, reporting growth over the year of approximately **19%**. Following we plotted the presented drivers of the index, divided as cyclical(in red), defensive (in green) and sensitive (in blue).

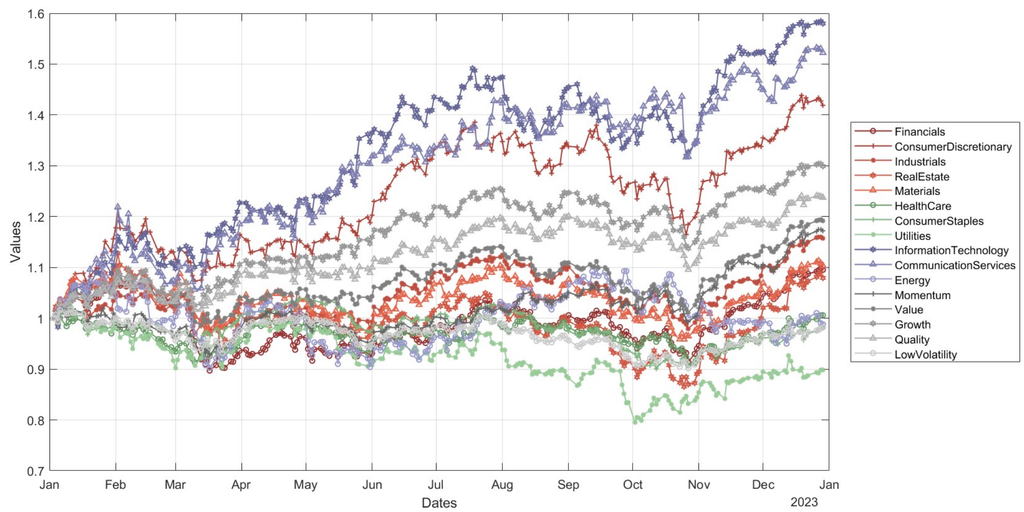


Figure 1: Frontier Comparison

As we can see in the previous figure most of the assets grew during the selected period. Notably the Information-Technology sector grew by almost 60% due to the recent developments in AI technology, Communication services sector value increased by more than 50% mainly attributed to social networks and streaming services and lastly Consumer Discretionary grew by approximately 40%. Overall 2023 is regarded as a positive year for the market, indeed just 4 out of the 16 asset classes brought negative average returns during the time period, those assets accounting only for a small percentage of the total index.

Name	**Expected Return**	**Volatility**
Financials	3.6538×10^{-4}	1.0273×10^{-2}
Consumer Discretionary	1.4047×10^{-3}	1.2695×10^{-2}
Industrials	5.8904×10^{-4}	9.3022×10^{-3}
Real Estate	3.0657×10^{-4}	1.2647×10^{-2}
Materials	3.9526×10^{-4}	1.0580×10^{-2}
Health Care	2.3782×10^{-5}	7.2870×10^{-3}
Consumer Staples	-7.8101×10^{-5}	7.0843×10^{-3}
Utilities	-4.3223×10^{-4}	1.1247×10^{-2}
Information Technology	1.8367×10^{-3}	1.2046×10^{-2}
Communication Services	1.6878×10^{-3}	1.3625×10^{-2}
Energy	-4.9390×10^{-5}	1.4417×10^{-2}
Momentum	6.3784×10^{-4}	8.2189×10^{-3}
Value	7.0543×10^{-4}	8.4410×10^{-3}
Growth	1.0511×10^{-3}	8.4560×10^{-3}
Quality	8.5551×10^{-4}	7.8718×10^{-3}
Low Volatility	-7.3012×10^{-5}	6.8757×10^{-3}

Table 1: Expected returns and volatility by sector

Moreover, it is worth noting that most of the sectors seem to present positive correlations between each other. Below we report a heatmap for the correlation matrix.

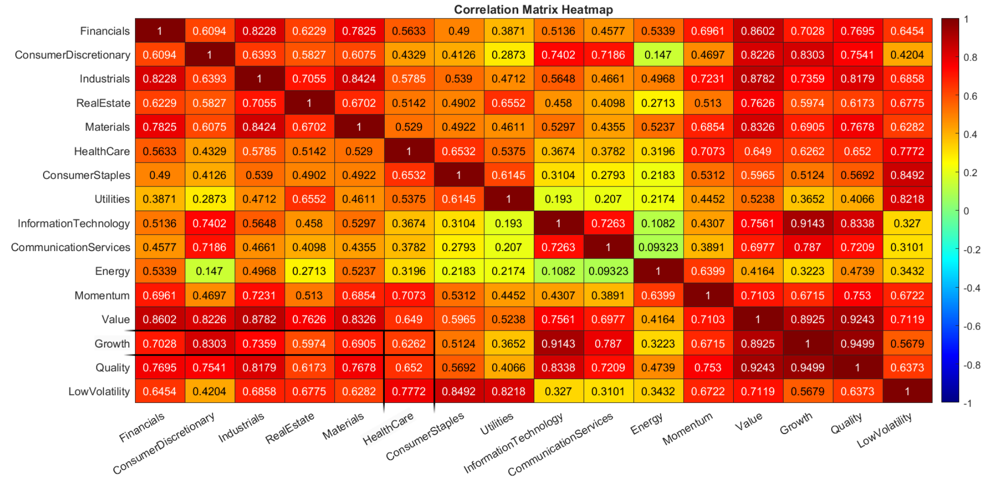


Figure 2: Heatmap visualization

From the correlation matrix, we can clearly see a lack of assets with negative correlation. Sectors in the cyclical subsection present a substantial positive correlation (first 5 rows) and the same can be said about the defensive sector (6th row to 8th row).

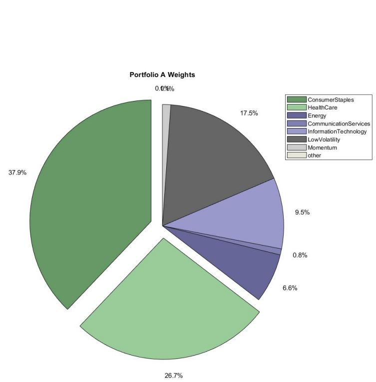
2 Part A

2.1 Efficient frontier under the standard constraints

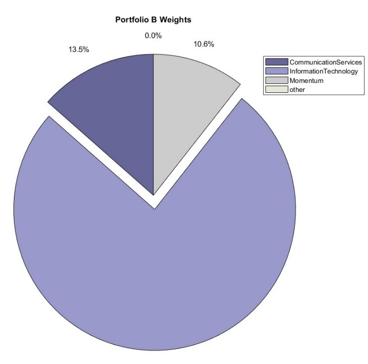
To compute the efficient frontier under the standard constraints using asset prices from 01/01/2023 to 31/12/2023, we begin by selecting the relevant time period from the *prices.xlsx* file. Next, we impose the standard constraints, which require the sum of the asset weights to be equal to 1 and each individual weight to be non-negative. These constraints are applied using the MATLAB function *setDefaultConstraints*. Then we build the efficient frontier, estimate the weights, and derive portfolio risks and returns. We have to compute 2 particular portfolios: the Minimum Variance Portfolio, named Portfolio A, and the Maximum Sharpe Ratio Portfolio, named Portfolio B.

The Portfolio A is the first one obtained in the frontier. On the other hand, for the second one, we have computed the Sharpe Ratio for all the frontier portfolios and selected the one that maximized the ratio.

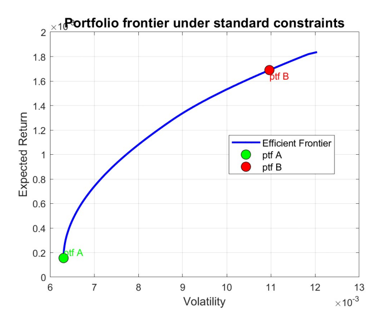
We have plotted this figure:



(a) Portfolio A



(b) Portfolio B



(c) Efficient Frontier under standard constraints

2.2 The efficient frontier under non-standard constraints

In the second point, we have to compute the efficient frontier under non-standard constraints, to be considered all at the same time:

• Standard constraints
• The total exposure to sensible sectors has to be lower than 50% and the total exposure on defensive sectors has to be greater than 30%
• Limit the exposure of the most volatile sectors (Communication Services and Energy ($0.05 \leq w_i \leq 0.1$)
• The total exposure of cyclical has to be equal to the defensive sectors.

Also in this case, our aim is to obtain the Minimum Variance Portfolio, named Portfolio C, and the Maximum Sharpe Ratio Portfolio, named Portfolio D.

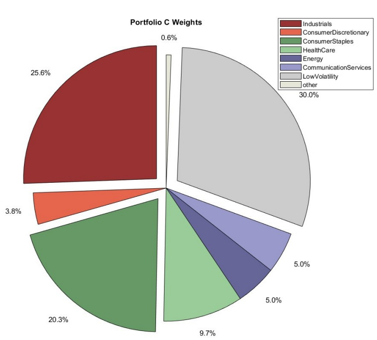
Starting from the second constraint, we implemented the matrix A, where we impose 1 in the index that correspond to sensible sectors (1,5,8). Then in vector named b we impose the corresponding limitation, in this case, the exposure has to be lower than *50%*.
We repeat this, using a similar procedure, for the defensive sectors, with the main difference that we impose -1 in matrix A, since in this case the total exposure has to be greater than *30%*.

To compute the third constraint, we have proceeded in the same way as before, selecting the index of the most volatile sectors, Communication Services and Energy, and imposing in the corresponding index of A, -1 or 1 if we want the corresponding weight smaller or larger than *0.05* and *0.1*. Finally, we put together these two conditions in one matrix.

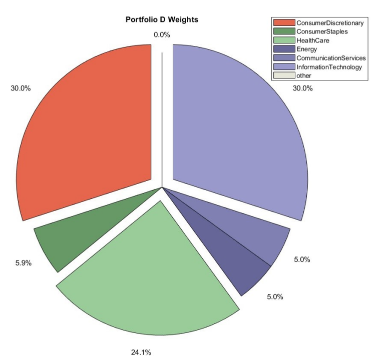
The last constraints requires ensuring that the total exposure of cyclicals is equal to the exposure to defensive sectors. In the case, we compute the equality imposing the index that corresponds to cyclical sectors equal to 1, and the second one equal to -1.

Then we proceeded as in the previous point, building an efficient frontier, estimating weights and portfolio risks and returns. In this case, using Matlab function *addEquality* and *addInequality*, we add to the computation the new non-standard constraints previously obtained. Also in this case we derive the Minimum Variance Portfolio and the Maximum Sharpe Ratio Portfolio.

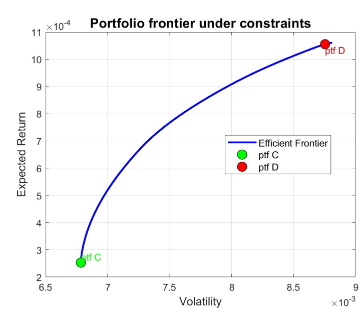
The following plots show the efficient frontier and the comparison between the new one and the one obtained via standard constraints. We also show the new weights of both portfolios C and D.



(a) Portfolio C



(b) Portfolio D



(c) Efficient Frontier under additional constraints

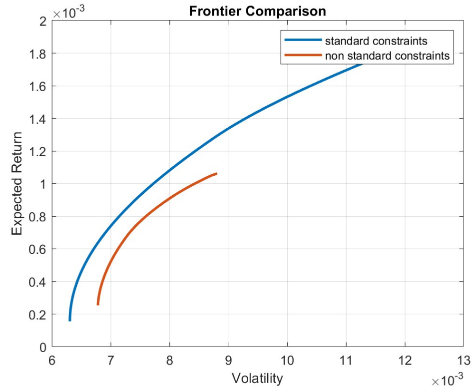


Figure 5: Frontier Comparison

2.3 Resampling method

2.3.1 Robust Frontier Construction

The third point asks us to compute the efficient frontiers under standard and additional constraints using the resampling method to obtain two robust frontiers.

As we can see, the portfolios (A, B) are concentrated in a few assets, which fails to meet the diversification principle. Our goal is to derive a more resilient, less sensitive-to-variations frontier.

This can be achieved by either adding extra constraints (as we did in the previous point) or using a technique known as frontier resampling.

The main idea behind frontier resampling is to repeat the optimization procedure using sampled expected returns and covariance matrices. Specifically, we set the number of simulations to $N = 100$. For each simulation, we sample daily returns for the 16 assets from a Multivariate Normal distribution. We then compute the mean and the covariance matrix of these simulated returns, which are used to calculate the efficient frontier for each resampling iteration.

At the end of the process, the robust frontier (thicker line) is obtained by averaging all 100 simulated frontiers as shown in the following figure.

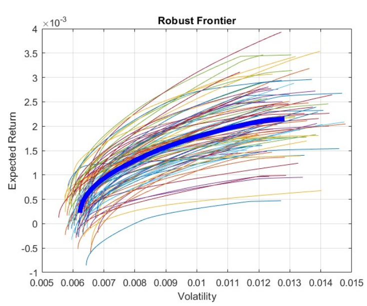


Figure 6: sampled frontier under standard constraints

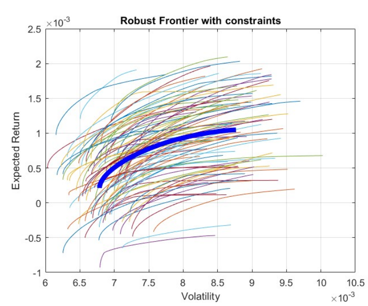


Figure 7: sampled frontier under additional constraints

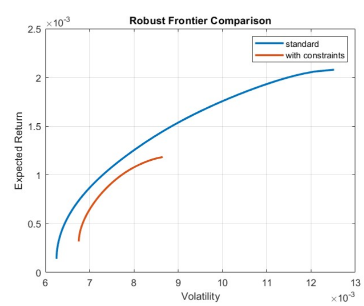
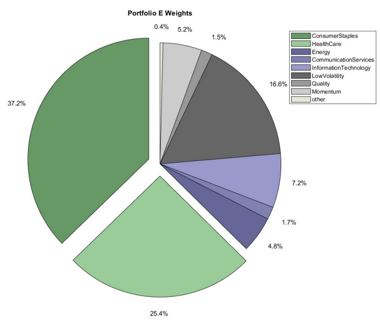


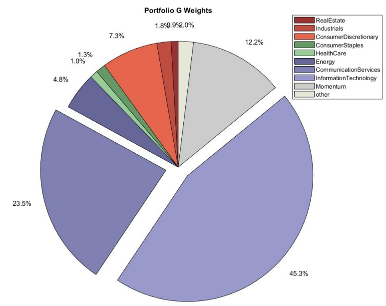
Figure 8: robust frontier comparison

Finally, we determine the Minimum Variance Portfolio (referred to as Portfolios E and F) by selecting the portfolio with the lowest variance from the mean frontier. The Maximum Sharpe Ratio Portfolio (Portfolios G and H) is identified by calculating the Sharpe Ratio for each portfolio along the frontier and selecting the one with the highest Sharpe Ratio.

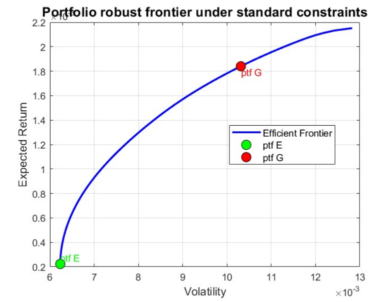
For each portfolio, we also compute the weights as the average of the weights for the same portfolio across all simulated frontiers at that specific index.



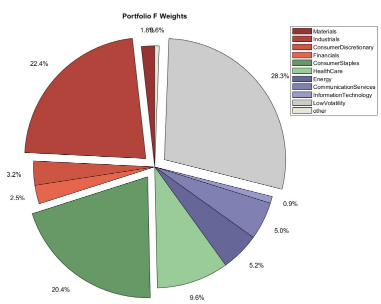
(a) Portfolio E



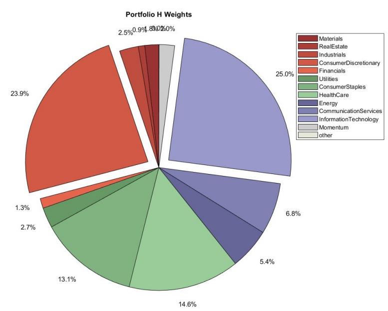
(b) Portfolio G



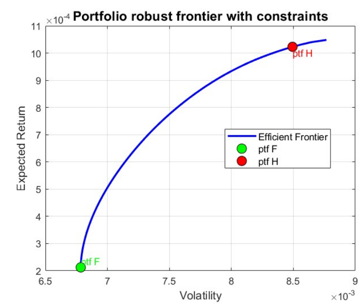
(c) Resampling frontier under standard constraints



(a) Portfolio F



(b) Portfolio H



(c) Resampling frontier under standard constraints

2.3.2 Frontier Comparison

We also perform a graphical comparison between the frontier obtained respectively at the first and second point with the two robust frontier.

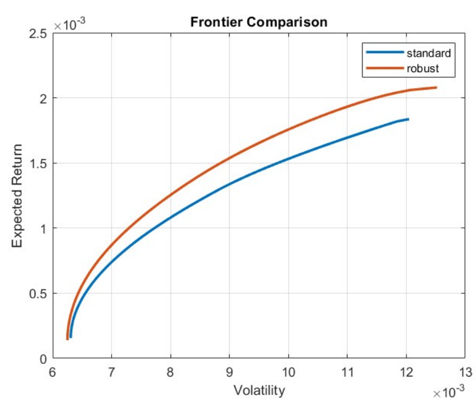


Figure 11: Standard constraints comparison

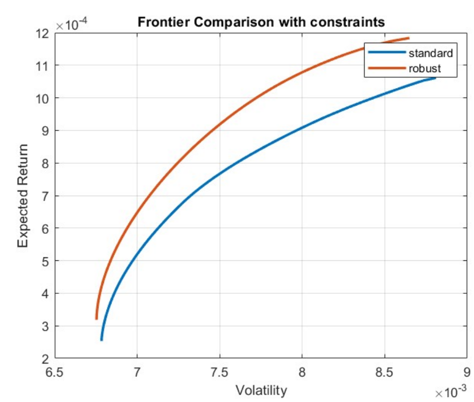


Figure 12: Additional constraints comparison

2.4 Black Litterman

2.4.1 BL Frontier Construction

We calculate the efficient portfolio frontier using the Black-Litterman model, a sophisticated approach that combines subjective investor views with the market equilibrium condition derived from the market capitalization indices provided. This methodology allows portfolio optimization by integrating personalized insights while maintaining consistency with the broader market dynamics, offering a balanced framework for decision-making.

In practical terms, we address the four building blocks: Prior Distribution, View Distribution, Likelihood Distribution and Posterior Distribution:

- **Prior Distribution:** The market variable X is assumed to be driven by the capitalization-weighted index. Specifically, X is considered to follow a Gaussian distribution with mean μ and variance $\tau\Sigma$: $X \sim \mathcal{N}(\mu, \tau\Sigma)$. Where μ is the Implied Equilibrium Return Vector obtained from the maximization of the quadratic utility function, $\mu = \lambda\Sigma w_{mkt}$ (λ is the risk aversion coefficient), and τ is a parameter chosen as the inverse of the number of observations.
- **Views Distribution:** In our study the investor views are:
1) Energy Annual Performance of 3 %
2) Quality factor outperforms the Momentum factor of 1 %
which are expressed through a linear relation: $q = P\mu$.
- **Likelihood Distribution:** We compute the conditional distribution of the investor's view given the market assuming a Gaussian distribution: $V|P\mu \sim \mathcal{N}(P\mu, \Omega)$ where $\Omega = \tau P \Sigma P^T$
- **Posterior Distribution:** By the Bayesian theorem combining the Prior Distribution with the Likelihood Distribution we conclude that $X|v \sim \mathcal{N}(\mu_{BL}, \Sigma_{BL})$

Asset Names	Prior Belief on Exp Ret	BL Exp Ret
InformationTechnology	0.023715	0.036062
Financials	0.019587	0.02892
HealthCare	0.011845	0.013978
ConsumerDiscretionary	0.024499	0.034836
CommunicationServices	0.023694	0.035225
Industrials	0.018416	0.02676
ConsumerStaples	0.010029	0.012614
Energy	0.01424	0.029913
Utilities	0.012762	0.015017
RealEstate	0.020663	0.028201
Materials	0.019704	0.029373
Momentum	0.014878	0.017526
Value	0.019216	0.028018
Growth	0.019293	0.028154
Quality	0.018043	0.02749
LowVolatility	0.011278	0.013877

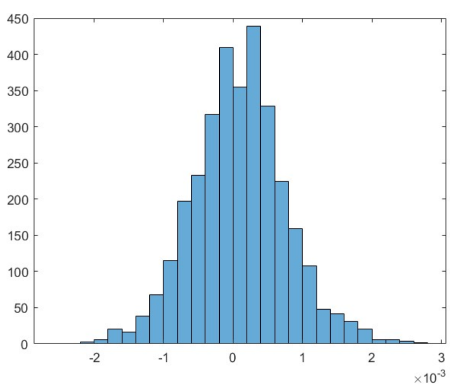


Figure 13: Prior Distribution

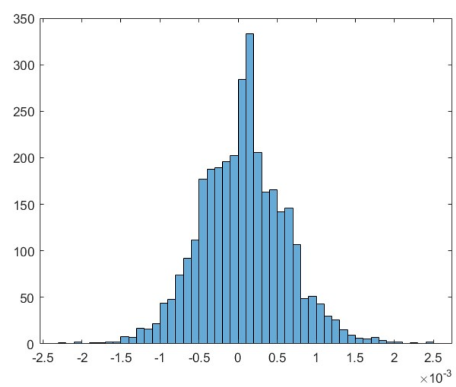
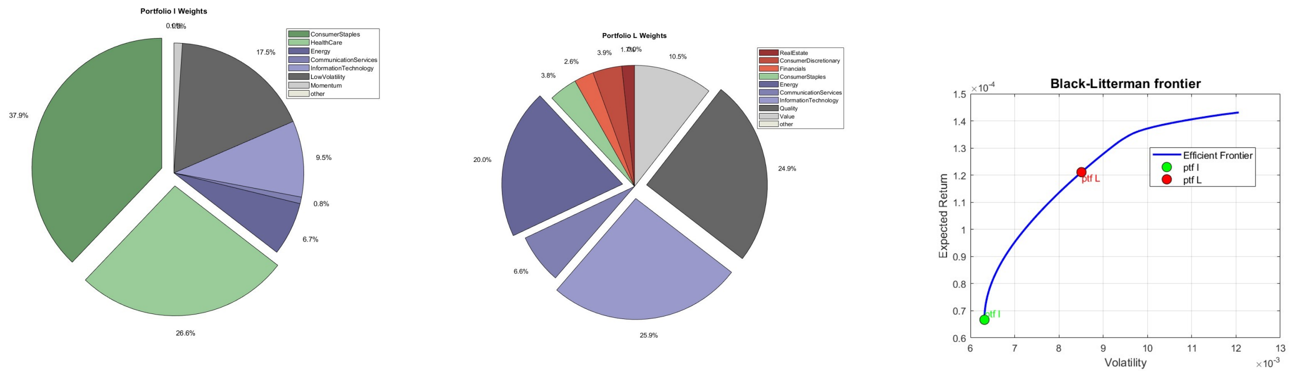


Figure 14: Posterior Distribution

Finally, accordingly with the Posterior distribution we have the distribution of the asset returns $r \sim \mathcal{N}(\mu_{BL}, \Sigma_{BL} + \Sigma)$. We then proceed to calculate the new portfolio frontier and the portfolios I and L, replicating the approach used for portfolios A and B, with the difference that we set the moments in accordance with the distribution of the asset returns described above.



(a) Portfolio I (b) Portfolio L (c) Black Litterman Frontier

2.4.2 Frontier Comparison

In the following graph, we have plotted the frontier calculated in point A.1 using the classical method, the one obtained through the robust method, and finally, the frontier derived using the BL approach. As expected, it can be observed that the first two frontiers are very similar to each other. On the other hand, the third frontier is noticeably different from the others, which might suggest that the investor's views have a significant impact on the shape of the frontier. In particular, the Black-Litterman frontier diverges because it incorporates a subjective component: the investor's views, which alter asset weights and risk-return estimates. If these views suggest that certain assets will perform very differently from what historical data indicates, the resulting frontier can look markedly different from the classical one.

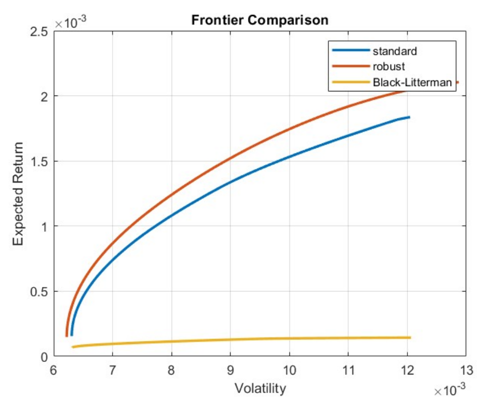


Figure 16: Frontier Comparison

2.5 Maximum Diversified Portfolio and the Maximum Entropy

In this section, we operate according to the standard constraints:

$$\sum_{i=1}^{N=16} \omega_i = 1 \quad \text{with} \quad 0 \leq \omega_i \leq 1, \quad \forall i \in [1, \dots, N]$$

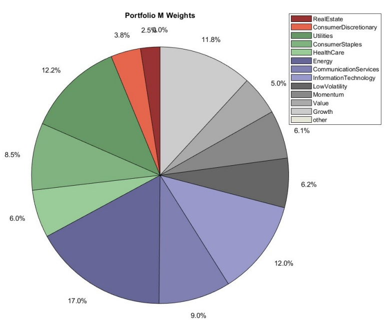
Moreover, we also add a constraint concerning the defensive sector:

$$\omega_3 + \omega_7 + \omega_9 < 0.2$$

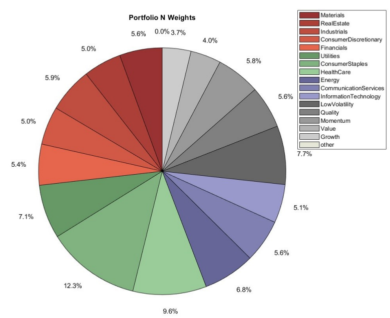
And the last one concerns the distance between the weights of the optimal portfolio ω_{opt} and the benchmark portfolio, i.e the capitalization-weighted one, ω_{mkt} which we write as:

$$\sum_{i=1}^{N=16} |\omega_{opt} - \omega_{mkt}|^2 = 0.09$$

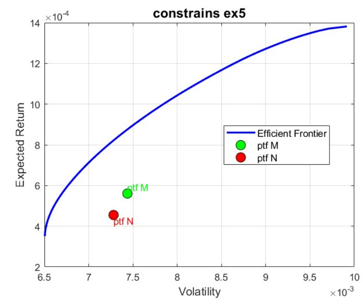
Then we computed through the fmincon function in Matlab, the Maximum Diversified Portfolio (PORTFOLIO M) and the Maximum Entropy portfolio (PORTFOLIO N). These portfolios focus on the diversification and mitigation of risk, using two different measures for diversification. Portfolio M maximizes the logarithm of the diversification ratio, i.e $\omega_{opt} = max(ln(\frac{w^T\sigma}{\sigma_{ptf}}))$, where in the numerator there's just the weighted sum of the volatilities of the single assets, while in the denominator we take into account their correlation through the portfolio volatility . Therefore the higher it is, the more diversified is the portfolio. For what concerns the Portfolio N, we maximize the entropy as a function of the risk contributions which we define as function of the portfolio weights as: $RC_i = \frac{|\omega_i \dot{(V\omega)}_i|}{|\sigma_{ptf}|}$, where V is the covariance matrix, therefore the optimization problem we solve is: $\omega_{opt} = max(-\sum_{i=1}^{N=16} \frac{RC_i}{\sigma_{ptf}} ln(\frac{RC_i}{\sigma_{ptf}}))$. The compositions of the portfolios M and N can be found in the pie charts below. As we can clearly see they are very diversified, especially the portfolio N which is really close to the equally weighted, this is how the 2 Portfolios position with respect to the efficient frontier:



(a) Portfolio M



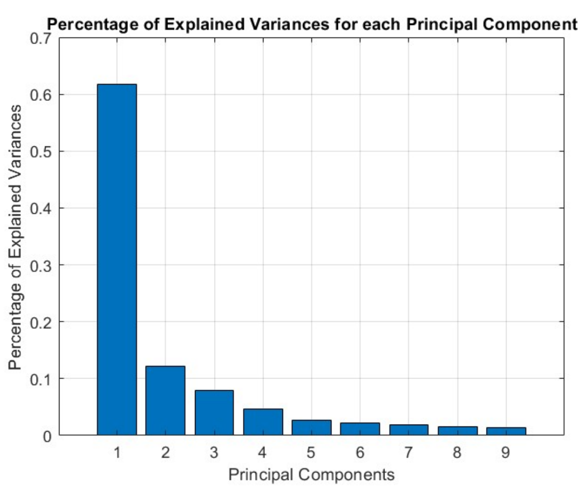
(b) Portfolio N

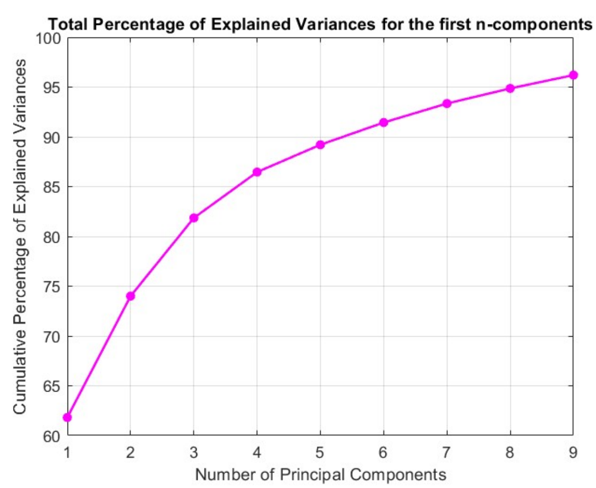


(c) Maximum Diversified Frontier

2.6 Principal Component Analysis

In this section, we exploited Principal Component Analysis to reduce the dimensionality of our problem and limit our choices to those factors that together explain more than the 95% of the cumulative variance. The PCA methodology yielded the following graphs about the percentage of explained variance by each component and the cumulative explained variance through which we selected the appropriate number of factors, which in our case is 9:





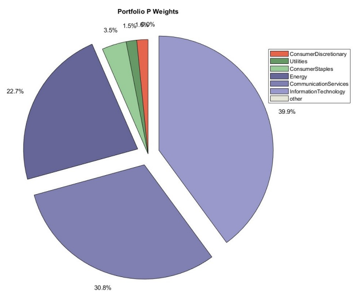
Now that we have our 9 factors, we consider the following constraints, the usual standard ones:

$$\sum_{i=1}^{N=9} \omega_i = 1 \quad \text{with} \quad 0 \leq \omega_i \leq 1, \quad \forall i \in [1, \ldots, N]$$

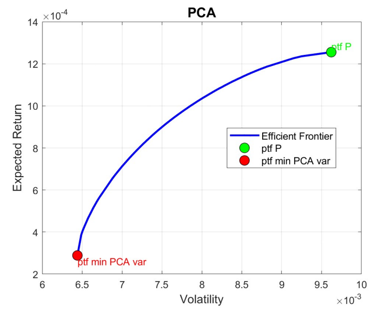
Moreover, we also add a constraint related to the volatility of the portfolio and the target volatility :

$$\sigma_{ptf} \leq \sigma_{tgt} = 0.75$$

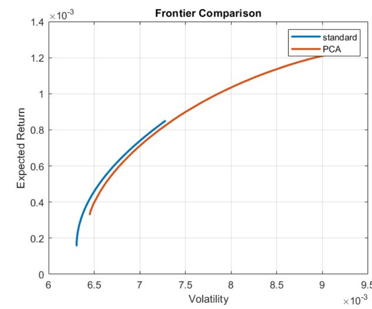
As requested, the constraint was enforced on the reconstruction of the variance based on standardized returns. In contrast, the construction of the frontier was done by rescaling the PCA factors using the volatility of the assets, to ensure a fair comparison. The portfolio P we obtained, as we can see from the plot, is highly concentrated on the sensible sectors which reflect global trends, but at the same time exhibit stability due to innovation, therefore the PCA selects them as the most representative assets, the other ones account only for a small portion of the portfolio P. In addition, we added the plots of the portfolio P which despite its concentration belongs to the efficient frontier and a comparison between the frontier obtained through the PCA and the original one which, as we expected are really close since the explained variance through the PCA is very high.



(a) Portfolio P



(b) PCA Efficient Frontier



(c) Frontier Comparison

2.7 Expected Shortfall-modified Sharpe Ratio

In this section, we employed a modified version of the Sharpe Ratio. Here we use the Expected Shortfall, computed through the historical method, as a way to measure risk instead of the classical formula that adopts the volatility. Here we considered 2 confidence levels that are 95% and 99%, the first being the one that we use in our optimization problem, here we are subjected to standard constraints, but we added a slight modification, i.e as lower bound for the portfolio weights we consider 0.001 instead of the classical 0:

$$\sum_{i=1}^{N=16} \omega_i = 1 \quad \text{with} \quad 0.001 \leq \omega_i \leq 1, \quad \forall i \in [1, \ldots, N]$$

This is the pie chart for the obtained Portfolio Q:

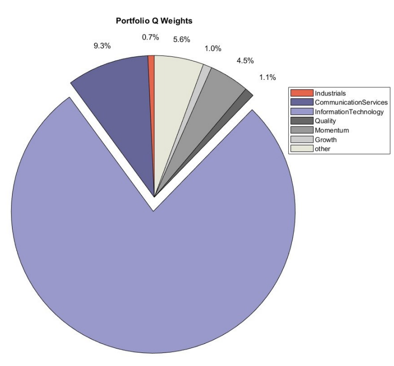


Figure 20: Portfolio Q

Moreover, as a check of our solution, we computed the expected shortfall at 95% confidence level which in our case is $-7e-3$ which is what we expected: at the numerator of our objective function we have the expected return, which is strictly positive, while at the denominator we have the expected shortfall without its normalization factor, since it doesn't change the optimization problem. Given that we are computing this risk measure on the returns and we are minimizing the objective function we expect this measure to be negative and close to 0 as we have retrieved.

2.8 Metrics Analysis

We divided our analysis by evaluating performance metrics, assessing diversification, and finally visualizing the performance trends of the various portfolios.

2.8.1 Performance Metrics

Portfolio	Annual Return	Annual Volatility	Sharpe Ratio	MaxDD	Calmar
MKT PORTFOLIO	0.23897	0.12512	1.91	-0.19826	1.2053
PORTFOLIO EW	0.16950	0.12233	1.3856	-0.15801	1.0728
PORTFOLIO A	0.05135	0.09953	0.5159	-0.09718	0.52835
PORTFOLIO B	0.52890	0.17636	2.999	-0.35612	1.4852
PORTFOLIO C	0.07379	0.10778	0.6847	-0.10696	0.68992
PORTFOLIO D	0.32537	0.14418	2.2567	-0.25226	1.2898
PORTFOLIO E	0.05467	0.09966	0.5486	-0.09525	0.57396
PORTFOLIO F	0.07670	0.10793	0.7106	-0.10765	0.71249
PORTFOLIO G	0.44092	0.16022	2.7519	-0.31247	1.4111
PORTFOLIO H	0.28655	0.13645	2.1001	-0.22902	1.2512
PORTFOLIO I	0.05135	0.09953	0.5160	-0.09720	0.52833
PORTFOLIO L	0.28132	0.13645	2.0617	-0.22354	1.2585
PORTFOLIO M	0.17296	0.11932	1.4496	-0.15973	1.0828
PORTFOLIO N	0.13467	0.11612	1.1597	-0.13938	0.96617
PORTFOLIO P	0.39413	0.15647	2.519	-0.28781	1.3694
PORTFOLIO Q	0.51846	0.17627	2.9413	-0.35194	1.4732

Table 3: Performance Metrics for Different Portfolios, in red the worst performance and in green the best performance of every index

In our analysis, we use the Market Portfolio as a reference point, as it represents a widely accepted benchmark reflecting the overall market performance. Its return is calculated as the weighted average of individual equity returns, where the weights correspond to market capitalizations. This provides a baseline for comparison against more complex portfolio construction methods. Using the Market Portfolio as a reference allows us to evaluate the added value of alternative strategies in terms of both performance and diversification.
As anticipated, the portfolios B, D, G, H, and L, which are constructed by maximizing the Sharpe ratio, exhibit high annual returns. Notably, these returns are significantly higher compared to those of the MKT Portfolio, highlighting the effectiveness of Sharpe ratio optimization in enhancing performance. On the other hand, these portfolios are not optimal in terms of volatility, as they exhibit annual volatility significantly higher than our benchmark. This highlights certain limitations of the Sharpe ratio maximization algorithm, which achieves strong performance in terms of returns at the cost of increased variance compared to the benchmark. Moreover, this behavior is clearly demonstrated by portfolio Q, which maximizes the modified Sharpe ratio. For portfolios A, C, E, F and I, which are constructed by minimizing variance, the opposite holds true: they achieve a reduction in annual volatility at the expense of lower performance in terms of annual returns.
Comparing portfolios I and L, constructed using the Black-Litterman model, with portfolios A and B, some notable observations emerge. There is an almost complete alignment between the performance metrics of portfolios A and I, suggesting that the incorporation of investor views has minimal impact on the minimum-variance portfolio. Conversely, portfolios L and B exhibit significant differences; in particular, portfolio L shows a lower annual return compared to portfolio B. This outcome may indicate that the investor's views incorporated into the Black-Litterman model did not align closely with actual market developments.
Moreover the portfolios B and Q share similar metrics, this is reasonable since both maximize the Sharpe Ratio, in fact they show two of the highest ones, despite they have at the denominator different risk measures: rspectively

the volatility and the expected shortfall.

Portfolio	Diversification Ratio	Entropy
MKT PORTFOLIO	1.1666	2.3273
PORTFOLIO EW	1.315	2.7726
PORTFOLIO A	1.2873	1.5157
PORTFOLIO B	1.0808	0.7174
PORTFOLIO C	1.2583	1.7152
PORTFOLIO D	1.2565	1.5322
PORTFOLIO E	1.2677	1.6614
PORTFOLIO F	1.2695	1.8966
PORTFOLIO G	1.2118	1.6126
PORTFOLIO H	1.3137	2.0175
PORTFOLIO I	1.2884	1.5155
PORTFOLIO L	1.2941	1.8501
PORTFOLIO M	1.4344	2.3666
PORTFOLIO N	1.3416	2.7233
PORTFOLIO P	1.342	1.3111
PORTFOLIO Q	1.0773	0.97903

Table 4: Diversification Ratio and Entropy for Different Portfolios, in red the worst performance and in green the best performance of every index

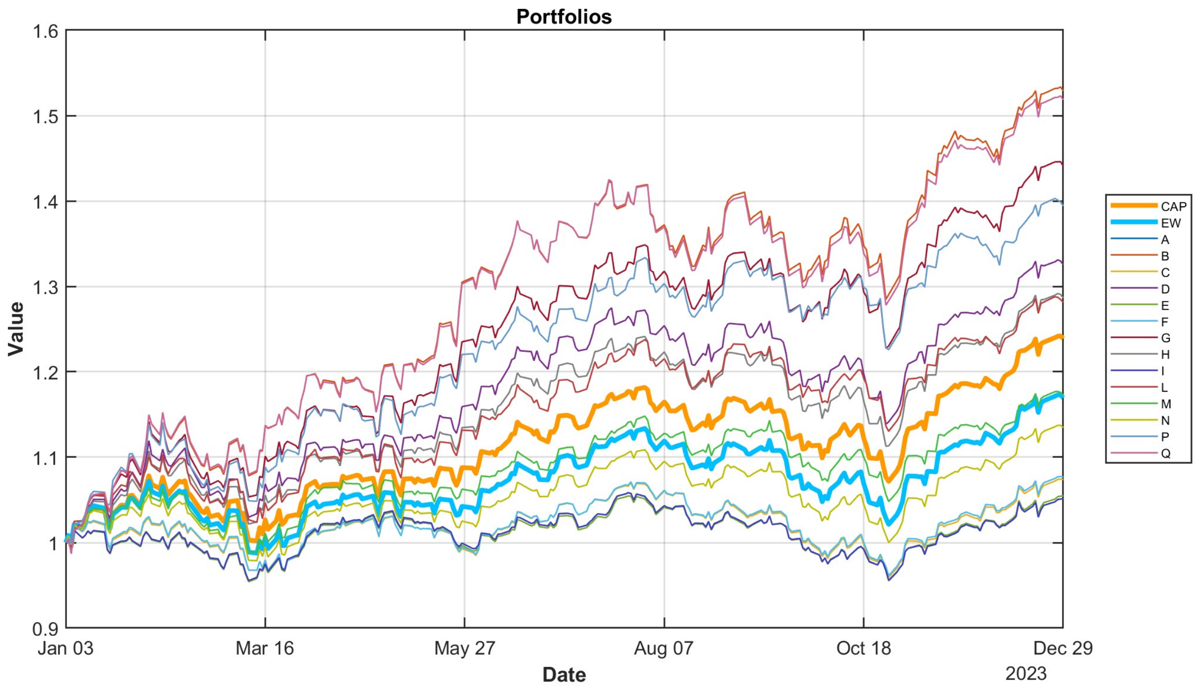
For the diversification analysis, it is natural to use the Equally Weighted Portfolio as a reference point, as it embodies a straightforward and unbiased allocation strategy. By assigning equal weights to all assets, it avoids any assumptions about market dynamics or asset-specific characteristics. Its return is simply the average of the individual equity returns, making it an intuitive and transparent benchmark. This simplicity provides a baseline for assessing the effectiveness of more complex portfolio construction methods in improving diversification and enhancing overall performance.

As expected, Portfolios N and M, given their construction methodologies, exhibit particularly high Diversification Ratios and Entropy levels, closely aligning with those of the Equally Weighted Portfolio (EW). On the other hand, these portfolios demonstrate a lower volatility compared to the Market Portfolio, but at the cost of a relatively low annual return. This trade-off reflects their emphasis on risk reduction and diversification, which comes at the expense of potentially higher returns.

We have specifically observed that portfolios obtained through Sharpe ratio maximization tend to exhibit low diversification, concentrating on the best-performing asset (Information Technology). This results in a higher risk compared to the benchmark, as reflected by increased volatility. However, it should be noted that portfolio D exhibits a relatively high level of diversification, a condition that arises due to the imposition of constraints, particularly those related to the Information Technology sector.

It is interesting to compare the diversification of portfolios B and G, both constructed with standard constraints and through Sharpe ratio maximization. Portfolio G, however, is developed using the resampling method, which leads to greater diversification. This enhanced diversification is attributed to the portfolio's reduced sensitivity to variations.

In the chart below, we have shown the performance of each portfolio during 2023. An overall assessment of the quality of the portfolios largely depends on the investor's level of risk aversion and personal preferences, particularly in evaluating the trade-off between volatility and expected returns. Investors with a low tolerance for risk may prioritize portfolios with reduced volatility, even if it means lower returns, whereas those aiming for higher returns may accept greater levels of risk. This underscores the subjective nature of portfolio evaluation, where the "best" choice is not one-size-fits-all but is instead aligned with individual investment objectives and risk appetite.



3 Part B

Portfolio	Annual Return	Annual Volatility	Sharpe Ratio	MaxDD	Calmar
MKT PORTFOLIO	0.24702	0.12625	1.9566	-0.21568	1.1453
PORTFOLIO EW	0.19922	0.10393	1.9167	-0.18739	1.0631
PORTFOLIO A	0.13789	0.080966	1.703	-0.14214	0.97008
PORTFOLIO B	0.36591	0.20961	1.7457	-0.28877	1.2672
PORTFOLIO C	0.14826	0.086286	1.7183	-0.15559	0.95293
PORTFOLIO D	0.19759	0.13745	1.4375	-0.18199	1.0857
PORTFOLIO E	0.15007	0.08232	1.823	-0.15079	0.9952
PORTFOLIO F	0.15118	0.086258	1.7526	-0.15747	0.96
PORTFOLIO G	0.31304	0.1746	1.7929	-0.25286	1.238
PORTFOLIO H	0.20444	0.12614	1.6208	-0.18584	1.1001
PORTFOLIO I	0.13772	0.080935	1.7016	-0.14202	0.96971
PORTFOLIO L	0.21974	0.12474	1.7616	-0.19965	1.1006
PORTFOLIO M	0.21374	0.10635	2.0097	-0.19191	1.1137
PORTFOLIO N	0.18669	0.096102	1.9426	-0.17966	1.0391
PORTFOLIO P	0.26395	0.15195	1.7371	-0.22315	1.1828
PORTFOLIO Q	0.34915	0.20438	1.7084	-0.27774	1.2571

Table 5: Performance Metrics for Different Portfolios, in red the worst performance and in green the best performance of every index

Regarding the annual returns of the portfolios constructed in Part A of the Assignment for 2024, we observe that only four portfolios outperform our benchmark: portfolios B, G, P, and Q. This performance can be attributed to their composition, which notably involves a concentration of investments in highly performing sectors, such as Information Technology and Momentum.
In our analysis, we also calculated the performance metrics for each individual sector, as shown in the table below.

Sectors	Annual Return	Annual Volatility	Sharpe Ratio	Max DD	Calmar
InformationTechnology	0.37027	0.22888	1.6178	-0.29497	1.2553
Financials	0.23358	0.12402	1.8834	-0.21809	1.071
HealthCare	0.06990	0.10349	0.6754	-0.11854	0.5896
ConsumerDiscretionary	0.14627	0.18534	0.7892	-0.15487	0.9444
CommunicationServices	0.29316	0.19139	1.5318	-0.24818	1.1812
Industrials	0.19791	0.12999	1.5225	-0.20680	0.9570
ConsumerStaples	0.13168	0.09602	1.3714	-0.15710	0.8382
Energy	0.06676	0.17465	0.3823	-0.18588	0.3592
Utilities	0.25079	0.15115	1.6592	-0.27226	0.9212
RealEstate	0.08865	0.15768	0.5622	-0.20908	0.4240
Materials	0.09969	0.13455	0.7409	-0.17210	0.5793
Momentum	0.42740	0.18426	2.3196	-0.31521	1.3559
Value	0.12069	0.09761	1.2364	-0.14729	0.8194
Growth	0.31949	0.17650	1.8102	-0.25585	1.2487
Quality	0.24075	0.12339	1.9512	-0.21892	1.0997
LowVolatility	0.13050	0.08699	1.5001	-0.14462	0.9023

Table 6: Performance Metrics for Different Sectors, in red the worst performance and in green the best performance of every index

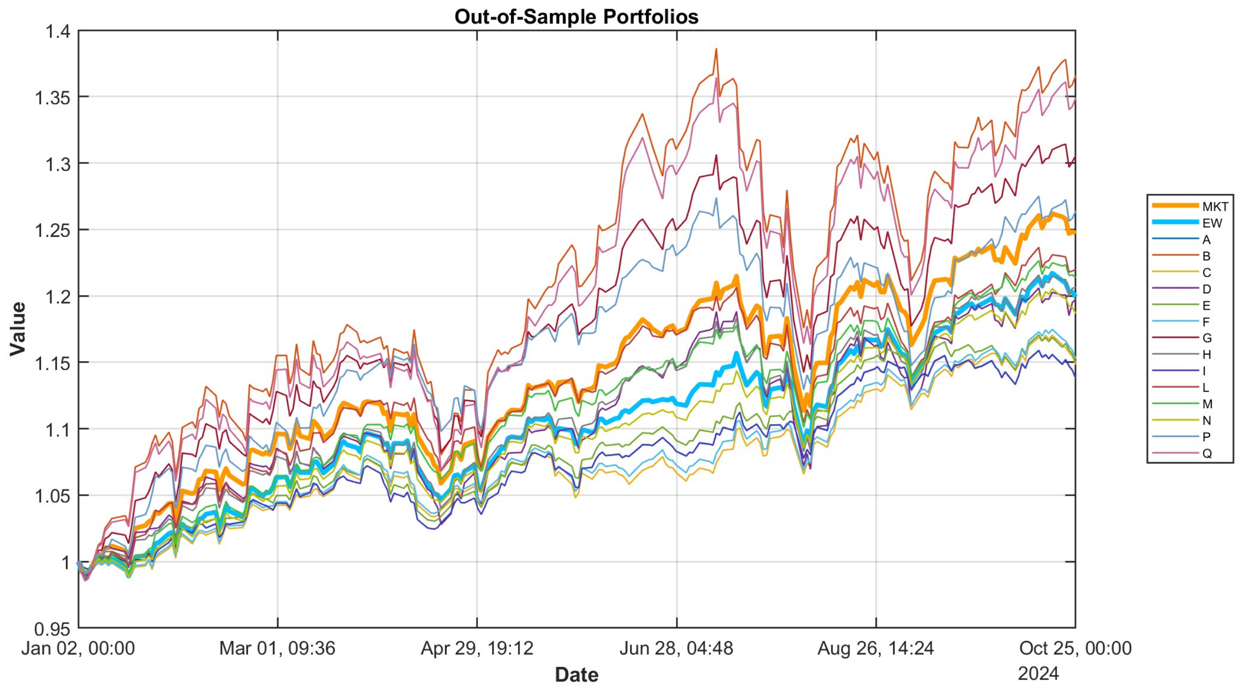
Regarding the plot that describes the trend of the value of the portfolios we build, we can clearly see that the 2024 has been so far a year of big expansion especially, as we said, for the AI related sectors which yielded a high return over 2024, as long as a high volatility as we can see from the related table. However, we note that there's a sudden drop between mid July to the beginning of August, it is mainly influenced by two causes: the first one is related to political instability, the assassination attempt on Donald Trump on July 13th and President Joe Biden's withdrawal from the election shortly afterward added volatility to the markets. The second reason is related to the AI sector overvaluation concerns, the investors were worried that the profits of leading AI-focused companies might not justify their sky-high valuations. The so-called "Magnificent 7" stocks (Tesla, Alphabet, Meta, Amazon, Microsoft, Apple, and Nvidia) fell nearly 12% from their mid-July peak, contributing significantly to the market downturn.

These factors compounded existing fears of a potential recession and led to a broad sell-off across sectors. Moreover the S&P 500's decline from mid-July was also influenced by uncertainty surrounding a potential Federal Reserve interest rate cut. Speculations about this cut arose as the Fed faced mounting pressure to address signs of economic slowdown. While the rate cut did materialize, it came later than some investors had hoped, dampening optimism and contributing to the market's volatility.

The worst-performing portfolio is Portfolio I, which minimizes variance within the Black-Litterman frontier. This can be attributed to the fact that our view predicted the quality factor would outperform momentum by 3%, but in reality, momentum turned out to be the best-performing factor. Additionally, the energy sector in 2024 performed even better than anticipated.
Regarding the diversification ratio, we observe that Portfolios B and Q have the lowest values, primarily due to their concentration, with over 70% of the allocation in the information technology sector, as shown in the last table under the diversification ratio. This exposure resulted in excellent performance due to the strong performance of the sector, but as a trade-off, it also generated high volatility. As in 2023, the most diversified portfolio is the Maximum Diversification Portfolio (Portfolio M), specifically constructed for this purpose.

In the chart below, we have shown the performance of each portfolio from 1/1/2024 to the end of the available data.

In conclusion, as highlighted throughout this report, every portfolio entails a trade-off. The optimal choice ultimately depends on the unique objectives, investment behaviors, and risk tolerance of each individual. Tailoring the portfolio to align with these factors is essential for achieving long-term success and satisfaction.



Portfolio	Diversification Ratio	Entropy
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PORTFOLIO A	1.4344	1.5157
PORTFOLIO B	1.0455	0.7174
PORTFOLIO C	1.3349	1.7152
PORTFOLIO D	1.2853	1.5322
PORTFOLIO E	1.4251	1.6614
PORTFOLIO F	1.3543	1.8966
PORTFOLIO G	1.1705	1.6126
PORTFOLIO H	1.3596	2.0175
PORTFOLIO I	1.4354	1.5155
PORTFOLIO L	1.3484	1.8501
PORTFOLIO M	1.5438	2.3666
PORTFOLIO N	1.4745	2.7233
PORTFOLIO P	1.3345	1.3111
PORTFOLIO Q	1.0607	0.97903

Table 7: Diversification Ratio and Entropy for Different Portfolios, in red the worst performance and in green the best performance of every index