Project 1
Quantum Walks and Monte Carlo

Ceară Filip-Ioan, Ilie Cristian, Popescu Daniel

August, 2025

1. Introduction

The article *Universal Statistical Simulator* by Mark Carney and Ben Varcoe introduces a new quantum algorithm designed to efficiently simulate different types of statistical processes. The core idea is based on a quantum version of the Galton board—a device originally used to demonstrate how the normal distribution forms when balls fall through rows of pegs. In this quantum version, superposition and entanglement allow all possible outcomes to be represented at once, instead of one path at a time. This makes the simulation much faster and useful for statistical sampling using quantum computers.

2. Quantum Galton Board Method

The main focus of the paper is the development of a "quantum peg," a special circuit element that imitates the random behavior of a peg in a classical Galton board. The setup uses basic quantum gates like Hadamard, controlled-SWAP, and Pauli-X, along with extra qubits called ancillas that help keep track of position. In the unbiased case, each peg puts the particle in an equal superposition of going left or right. Thanks to its layered structure, the circuit simulates the full Galton board efficiently, creating a quantum state with many possible paths. When measured, these states collapse into classical values following the intended statistical pattern.

One of the big improvements in this design is the use of $RX(\theta)$ rotation gates instead of Hadamard gates. These rotations let the algorithm change how likely the particle is to go left or right at each peg. This turns the Galton board into a flexible tool that can mimic various probability distributions—not just the normal one. The authors also show that you can fine-tune the bias for each individual peg, which adds even more versatility to the model.

3. Simulations and Dealing with Noise

To test the algorithm, the authors ran simulations using IBM's Qiskit platform, both with ideal conditions and on actual noisy quantum hardware. The results show that the unbiased version of the quantum Galton board closely matches the normal distribution, and by changing the bias, it can model other types of distributions too. Compared to previous models, this one uses fewer gates and has shorter circuits, which helps reduce errors. Still, noise from complex multi-qubit gates remains a challenge when running the algorithm on current quantum systems.

To process the measurement results, the authors used Monte Carlo sampling, a method that helps translate quantum outputs into familiar distribution graphs. They combined different techniques like block summation and rescaling to match the data with well-known distributions such as binomial and exponential ones. These tests highlight both the strengths and current limitations of quantum hardware in performing detailed statistical simulations.

4. Applications and Final Thoughts

This quantum Galton board method could be applied in fields like finance (to model stock movements), quantum cryptography (which depends on randomness), and machine learning (especially with stochastic models). The circuit design is modular, meaning it can be adjusted for different distribution outputs—whether you want a simple random sample or something more specific.

Overall, the paper presents a clear and well-tested algorithm that connects statistical modeling with quantum computing, showing speedups and a wide range of possibilities. While there are still some limitations because of hardware constraints like gate noise and low qubit counts, the approach has room to grow. Future work could focus on improving noise reduction, expanding the model to more complex setups, and creating more specialized biased distributions that could bring real quantum advantages.