Generative Radial Geometry (GRG)

A π-Free Model for Circular Area and Circumference

Stefano Ferragina

Abstract

This paper introduces Generative Radial Geometry (GRG), a novel method to compute the area and circumference of a circle without using the constant π. GRG is based on radial quantization and projection principles, not on classical integration or limit theory. Although GRG's area formula resembles a Riemann sum, the approach is original and emerged independently, without prior knowledge of Riemann methods. We present the foundations, formulas, convergence behavior, and conceptual implications of this model.

1. Introduction

Generative Radial Geometry (GRG) proposes a novel method for computing the area and circumference of a circle using only the radius and a chosen number of subdivisions. Unlike traditional approaches, GRG entirely avoids the use of π by relying on discrete summation techniques. By subdividing either the radius (for area) or the central angle (for circumference), GRG approximates geometric quantities with increasing precision. As the number of subdivisions increases, the results converge exactly to classical formulas, offering a rigorous yet π-free computational alternative.

2. Motivation

The classical concept of π has proven effective for centuries, yet it remains an external, transcendental constant. GRG was born from the intuition that the circle's area and perimeter could be reconstructed from within—using only radial measurements and discrete logic. Instead of approximating π, GRG builds the circle as a sum of generative radial components.

3. Method and Definitions

Let r be the circle's radius, and N the number of subdivisions.

3.1 Area Calculation (Radial Projection)

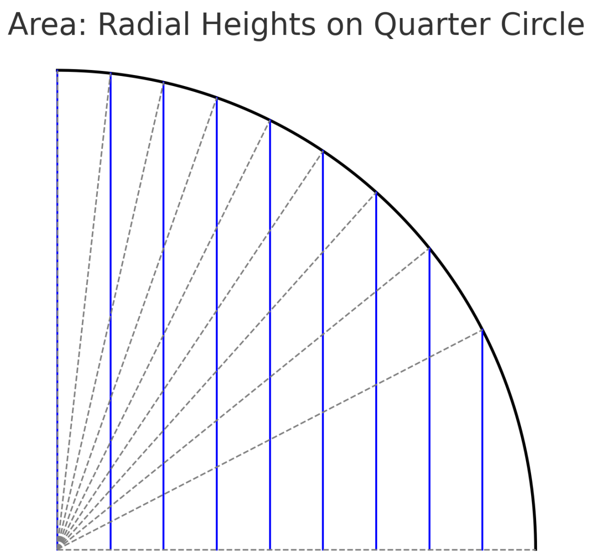


Figure 1: Enter Caption

Each step i corresponds to a rectangle of width $\Delta r = \frac{r}{N}$ and height:

$$y_i = \sqrt{r^2 - \left(i \cdot \frac{r}{N}\right)^2}$$

Thus, the quarter-circle area is:

$$A_{\text{quarter}} = \sum_{i=1}^{N} y_i \cdot \Delta r$$

$$A = 4 \cdot A_{\text{quarter}}$$

3.2 Circumference Calculation (Angular Subdivision)

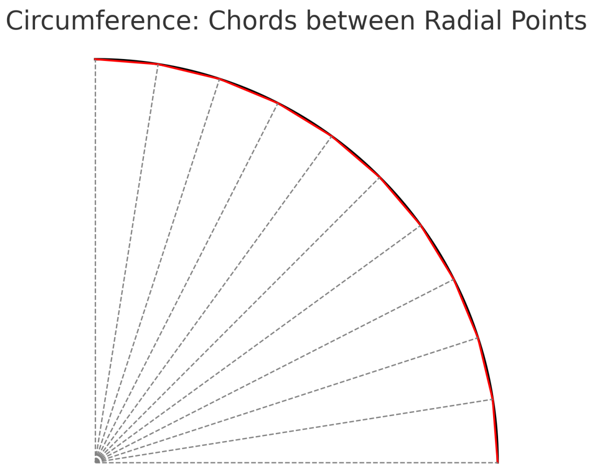


Figure 2: Enter Caption

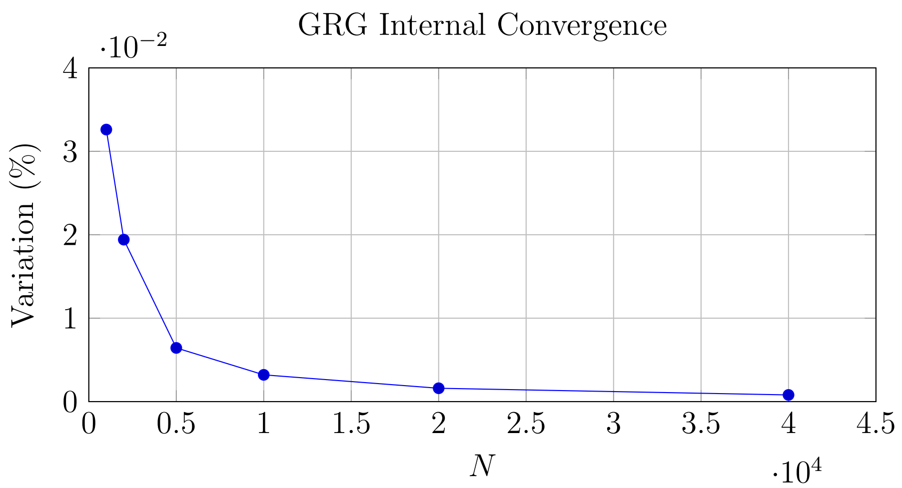
$$c = 2r \cdot \sin\left(\frac{90^\circ}{2N}\right)$$

$$C_{\text{quarter}} = N \cdot c = 2Nr \cdot \sin\left(\frac{90^\circ}{2N}\right)$$

$$C = 4 \cdot C_{\text{quarter}} = 8Nr \cdot \sin\left(\frac{90^\circ}{2N}\right)$$

4. Internal Convergence of GRG

Subdivisions (N)	**Comparison**	**Variation %**
1000 → 2000	GRG vs GRG	0.032607
2000 → 5000	GRG vs GRG	0.019414
5000 → 10000	GRG vs GRG	0.006435
10000 → 20000	GRG vs GRG	0.003207
20000 → 40000	GRG vs GRG	0.001600



5. Comparison with Classical Geometry

Feature	**Classical Geometry**	**GRG**
Fundamental Constant	π	None
Computation Method	Continuous formulas	Discrete sums
Precision	Limited by π approximation	Improves with N
Geometric Philosophy	Analytic	Generative
Use Case	Universal	Didactic, Computational, Theoretical

6. Precision and Principle

GRG follows a principle similar to a digital compass: each point on the circle is a projected result of radial subdivision. This makes the method geometrically exact and computationally scalable.

7. Conclusion

GRG offers a coherent alternative to traditional circle geometry. It avoids π, employs discrete summation, and demonstrates strong convergence. Its autonomous, generative structure may serve as a foundation for educational, theoretical, and computational geometry.

8. Author's Note on AI Assistance

This manuscript was written by the author as an independent work based on original research and mathematical exploration. Generative AI tools (such as ChatGPT) were used solely to improve the formatting, structure, and language clarity of the manuscript, not for the generation of mathematical ideas, results, or conclusions. All content and theoretical development are original contributions by the author.

9. Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

10. Declaration of Interests

The author declares no competing interests.

Keywords: radial geometry, discrete computation, pi-free circle, mathematical modeling, generative methods

The author previously introduced the GRG (Generative Radial Geometry) method in a non-academic format on Medium as a preliminary public disclosure. This article represents the first formal scientific presentation of the GRG framework. The original post is available at: https://medium.com/@stefano.ferragina/grg-generative-radial-geometry-35e2f2bcaa03