Department of Computer Science

CS 2209A — Applied Logic for Computer Science

Assignment 1

Assigned: 1 October 2018

Due: 15 October 2018 (on OWL before midnight)

1. For the following proposition, give the unique readability tree and the truth table derived from that tree. What type of statement (tautology, contradiction, satisfiable) is it? Why?

$$\left(\left((p \to q) \to p \right) \to p \right)$$

2. Show the following logical equivalence using the logical identities and tautologies such as the law of the excluded middle.

$$\left(\left((p \wedge q) \vee (p \wedge s) \right) \vee \left((r \wedge q) \vee (r \wedge s) \right) \right) \equiv \left((p \vee r) \wedge (q \vee s) \right)$$

3. Convert the following proposition to Conjunctive Normal Form. Show the steps required to get your answer.

$$\left((p \leftrightarrow q) \to (p \wedge r) \right)$$

4. Show the following using natural deduction.

$$\left\{ (p \to q) \right\} \vdash \left((r \vee p) \to (r \vee q) \right)$$

5. With the clauses given by Γ show $\Gamma \vdash \neg r$ using resolution.

$$\Gamma = \left\{ (p \vee \neg q \vee \neg r), (\neg p \vee s), (q \vee \neg r \vee t), (\neg r \vee \neg u \vee t), (p \vee s \vee \neg r \vee t), \neg s, q, u, (\neg u \vee \neg t) \right\}$$

6. Show a bottom up derivation of $\Gamma \vdash h$ given the definite clauses (written as head $\leftarrow$ body) in set Γ.

$$\Gamma = \left\{ \begin{array}{l} h \leftarrow f \wedge g \\ g \leftarrow a \wedge c \\ d \\ e \leftarrow c \\ a \leftarrow d \\ f \leftarrow d \wedge e \wedge b \\ b \leftarrow e \\ c \leftarrow a \end{array} \right\}$$

7. Show a top down derivation of $\Gamma \vdash h$ given the definite clauses (written as head $\leftarrow$ body) in set Γ.

$$\Gamma = \left\{ \begin{array}{l} h \leftarrow f \wedge g \\ g \leftarrow a \wedge c \\ d \\ e \leftarrow c \\ a \leftarrow d \\ f \leftarrow d \wedge e \wedge b \\ b \leftarrow e \\ c \leftarrow a \end{array} \right\}$$