CS 2209A — Applied Logic for Computer Science

Quiz 3

Name:
Stud. No.

1. With the interpretation on the left, give the truth value of each sentence on the right.

$$\mathcal{I} = \left\{ \begin{array}{l} \mathscr{D} = \{\delta_1, \delta_2, \delta_3, \delta_4\} \\ a \to \delta_1 \\ b \to \delta_2 \\ c \to \delta_3 \\ f(\delta_1) \to \delta_4 \\ f(\delta_2) \to \delta_4 \\ f(\delta_3) \to \delta_4 \\ f(\delta_4) \to \delta_4 \\ P \to \{\delta_1, \delta_2, \delta_3, \delta_4\} \\ Q \to \{<\delta_3, \delta_4>, <\delta_4, \delta_3>\} \end{array} \right\}$$

__T__ $\forall x P(x)$

__F__ $P(a) \wedge Q(a, b)$

__T__ $Q(c, f(b))$

__F__ $P(b) \wedge Q(f(a), f(b))$

__T__ $P(f(f(a)))$

__T__ $\forall x \forall y (Q(x, y) \to P(y))$

__F__ $\forall x \forall y (P(y) \to Q(x, y))$

__T__ $\forall x \exists y (Q(x, y) \to P(y))$

__F__ $\exists x \forall y (P(y) \to Q(x, y))$

__T__ $\exists x (P(x) \to Q(x, f(x)))$

2. Using the unification algorithm, find the most general unifier for each of the following pairs of strings (a, b, c, d, e are constants, f, g are functions, and u, v, w, x, y, z are variables):

$P(a, b)$	$P(x, y)$	$x/a, \ y/b$
$P(a, x, f(g(y)))$	$P(z, f(z), f(w))$	$z/a, \ x/f(a), \ w/g(y)$
$P(a, b, u, v))$	$P(x, y, z, b)$	$x/a, \ y/b, \ u/z, \ v/b$ or z/u
$P(x, z, z)$	$P(a, y, b)$	$x/a, \ y/b, \ z/b$
$P(g(y), a)$	$P(g(u), u)$	$y/a, \ u/a$

3. Let Γ be the set of statements that are given below. Propositionalize Γ.

$$\Gamma = \left\{ \begin{array}{l} \forall x \exists y (P(b) \to Q(x,y)) \\ R(a,b) \\ \exists x \forall y S(x,y) \end{array} \right\}$$

Γ has two constants: a, b

$R(a,b)$

$P(b) \to Q(a, c_1)$

$P(b) \to Q(b, c_2)$

$S(d, a)$

$S(d, b)$

What would happen if the second sentence were: $R(a, f(a))$?

In addition to the two constants a, b, Γ has a function f of one argument. Wherever we propositionalize Γ with a, b above we would now need to include $f(-)$ where the $-$ can be any non-variable term (i.e., a constant or function). So, add the following to the proposctionalizd sentences above.

$S(d, f(a))$ $S(d, f(b))$ $P(b) \to Q(f(a), e_{11})$ $P(b) \to Q(f(b), e_{21})$

$S(d, f(f(a)))$ $S(d, f(f(b)))$ $P(b) \to Q(f(f(a)), e_{12})$ $P(b) \to Q(f(f(b)), e_{22})$

$\vdots$ $\vdots$ $P(b) \to Q(f(f(f(a))), e_{13})$ $P(b) \to Q(f(f(f(b))), e_{23})$

                                   $\vdots$ $\vdots$