History of logic

- An example of an **Aristotelian syllogism**:
  All humans are mortal.
  Socrates is human.
  Therefore, Socrates is mortal.

- A **syllogism** is a kind of logical argument in which one proposition (the conclusion) is inferred from two or more others (the premises) of a specific form.

- This syllogism is an example of good reasoning - constitutes a good argument - because it is truth-preserving.

- If the first two sentences (premises) are true, then the third sentence (conclusion) must also be true.

Natural vs. Logic language

- Natural languages are ambiguous.
- For example, the word "any" can have different meanings depending on the context:
- **Any = some**
  - She will be happy if she can solve any question.
  - She will be happy if she can solve ~~every~~ question.
    some
- **Any = all**
  - Any student knows this.
  - Every student knows this.

Natural language (cont'd)

- Natural language treats some of the "connectives" differently

  - or I can have some cake <u>or</u> a cookie.

    [Normal interpretation: choose one of cake / cookies.]

- if ... then   If it rains then I won't go golfing.

[Normal interpretation:

raining / won't golf

but usually we add:

not raining / will golf

and sometimes add:

not golfing — because of the rain

and The cowboy jumped on the horse and rode off into the sunset.

[Normal interpretation: both conjuncts happen but the first happens before the second; so the sentence does not have the same meaning

as

The cowboy rode off into the sunset and jumped on the horse.]

We will want to give precise meaning to objects in our propositional language that are formed from propositions connected with these connectives.

So what we will do is define "or", "and", and "if ... then" precisely and sensibly (i.e., governed by our normal natural language interpretation)

Propositions

A proposition is a declarative sentence that can be true or false in a world.

Examples of propositions:

It is hot outside today.

It is cold outside today.

[Note: one of these is true in our world and one is false in our world.]

[Note: there are other possible worlds in which the first proposition is true and other possible worlds in which it is false. Likewise, for the second proposition.]

I will eat lunch with my friends in the restaurant around the corner.

[Note: the declarative sentence can be complex.]

It is hot and I will sweat.

[Note: propositions can contain connectives.]

So, we will divide propositions into:

<u>atomic propositions</u>

(those propositions that don't contain any connectives)

and <u>compound propositons</u>

(propositions that do contain a connective)

Atomic proposition examples:

It is hot.

I am sweating.

Compound proposition examples:

It is hot and I am sweating.

If it is hot then I am sweating.

From propositions to propositional variables

In logic we are interested in the structure of propositions and the structure of arguments we will abstract away from the natural language propositions to abstract symbols

We map atomic propositions to "propositional variables"

These variables will come from the beginning of the alphabet $(A, B, C, \ldots)$ and will be uppercase or from the middle of the alphabet (p, q, r)

How propositions are mapped to propositional variables is somewhat arbitrary:

A: It is hot.

B: I am sweating.

Going back to a previous example:

A: It is hot outside today.

B: It is cold outside today.

Note: A and B can both be true or both be false. We didn't say that possible worlds need be sensical.

If we want to rule out some possible worlds, we will need to make our mapping to propositional variables somewhat less arbitrary (more on this later).

Now that we have propositional variables to represent atomic propositions, we can connect these atomic propositions with connectives to give us compound propositions!

We will have the following connectives:

$\wedge$ – "similar" to and

$\vee$ – "similar" to or

$\rightarrow$ – "similar" to if...then

$\neg$ – not

So, if we have the atomic propositions

A: It is hot.

B: I am sweating.

we can have the compound proposition

It is hot and I am sweating

which is represented as

$$A \wedge B$$

It is hot or I am sweating.

$$A \vee B$$

If it is hot then I am sweating.

$$A \rightarrow B$$

It is not hot.

$$\neg A$$

Remember our goal — deduction

From It is hot and I am sweating

we will probably want to deduce

It is hot.

So from $A \wedge B$ we will want to deduce A.

But from A: It is cold.

B: I am freezing

It is cold and I am freezing we want to deduce It is cold ; i.e. from $A \wedge B$ deduce A

So, what is important is the structure of the proposition not what the propositional variables represent

So, now we can simply look at the abstract propositional language.

Symbols

Propositional variables A, B, C (or p, q, r)

Unary connective $\neg$

Binary connectives $\wedge, \vee, \rightarrow$

Punctuation $($ $)$

Formulas (also called expressions)

Any sequence of symbols Eg $(AB($

$(A \wedge B)$

We will want to refer to formulas using a meta-language

Our meta-language will consist of symbols that will refer to formulas; we will use Greek letters for these symbols (e.g., α, β); $=$ to be read as "represents"; and object level formulas

Eg. $\alpha = ($A$($ is to be read as α represents the formula (A($

Well-formed formulas (WFFs)

This subset of formulas are the sequences of symbols that are meaningful for our purposes.

We will give an inductive definition of the set of well-formed formulas (note: this set is infinite in size)

WFFs

Let $\mathcal{P}$ be a set of propositional variables
The set of WFFs over $\mathcal{P}$ is defined by:

1. Any formula consisting of a single propositional variable in $\mathcal{P}$ is a WFF.
2. If α is a WFF then $(\neg\alpha)$ is a WFF.
3. If α and β are WFFs then all of $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, and $(\alpha \to \beta)$ are WFFs
4. Nothing else is a WFF.

How to read rules 2 & 3:

These rules use a mixture of symbols from the meta-language and from the object language

E.g. $(\alpha \wedge \beta)$ is to read as "the concatenation of the symbols $($, "the WFF that α represents", $\wedge$, "the WFF that β represents", $)$

So, if α represents the WFF A $(\alpha = A)$ —rule 1— and β represents the WFF B $(\beta = B)$ —rule 1— then rule 3 says that $(A \wedge B)$ is a WFF.

Examples of wffs

1. $p \quad q \quad r \quad s$ — 4 wffs - from Rule 1
2. $(\neg p)$ — from Rule 2 and #1
3. $(r \wedge q)$ — from Rule 3 and #1
4. $((\neg p) \rightarrow s)$ — from Rule 3 and #1, #2
5. $((r \wedge q) \vee ((\neg p) \rightarrow s))$ — from Rule 3 and #3, #4
6. $(\neg (r \wedge q))$ — from Rule 2 and #3