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CS 2209 Assignment #1
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QUESTION 1

$$[(P \rightarrow Q) \rightarrow P] \rightarrow P$$

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \rightarrow P$	$(P \rightarrow Q) \rightarrow P \rightarrow P$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

This is a tautology because all the values are true.

Question 2

$$\overbrace{(((P \wedge Q) \vee (P \wedge S)) \vee ((R \wedge Q) \vee (R \wedge S)))}^{\text{Distributivity Left Side}} \equiv ((P \vee R) \wedge (Q \vee S))$$
$$(((P \wedge (Q \vee S)) \vee ((R \wedge Q) \vee (R \wedge S))) \quad \text{Distributivity Left Side}$$
$$((P \wedge (Q \vee S)) \vee (R \wedge (Q \vee S))) \quad \text{Distributivity Right Side}$$
$$((P \wedge (Q \vee S)) \vee (R \wedge (Q \vee S))) \quad \text{Rewriting}$$
$$(Q \vee S) \wedge (P \vee R) \quad \text{Distributivity}$$
$$(P \vee R) \wedge (Q \vee S) \quad \text{Commutativity}$$

Question 3

$$((P \leftrightarrow Q) \rightarrow (P \wedge r))$$

Question 3

a

$$(((P \rightarrow Q) \wedge (Q \rightarrow P)) \rightarrow (P \wedge r))$$

Removing
 \leftrightarrow

$$(((\neg P \vee Q) \wedge (\neg Q \vee P)) \rightarrow (P \wedge r))$$

$A \rightarrow B$ equivalence

$$(\neg((\neg P \vee Q) \wedge (\neg Q \vee P)) \vee (P \wedge r))$$

Removing
 \rightarrow
equivalence.

$$((\neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)) \vee (P \wedge r))$$

DeMorgan's
to \vee

$$((P \wedge \neg Q) \vee (Q \wedge \neg P) \vee (P \wedge r))$$

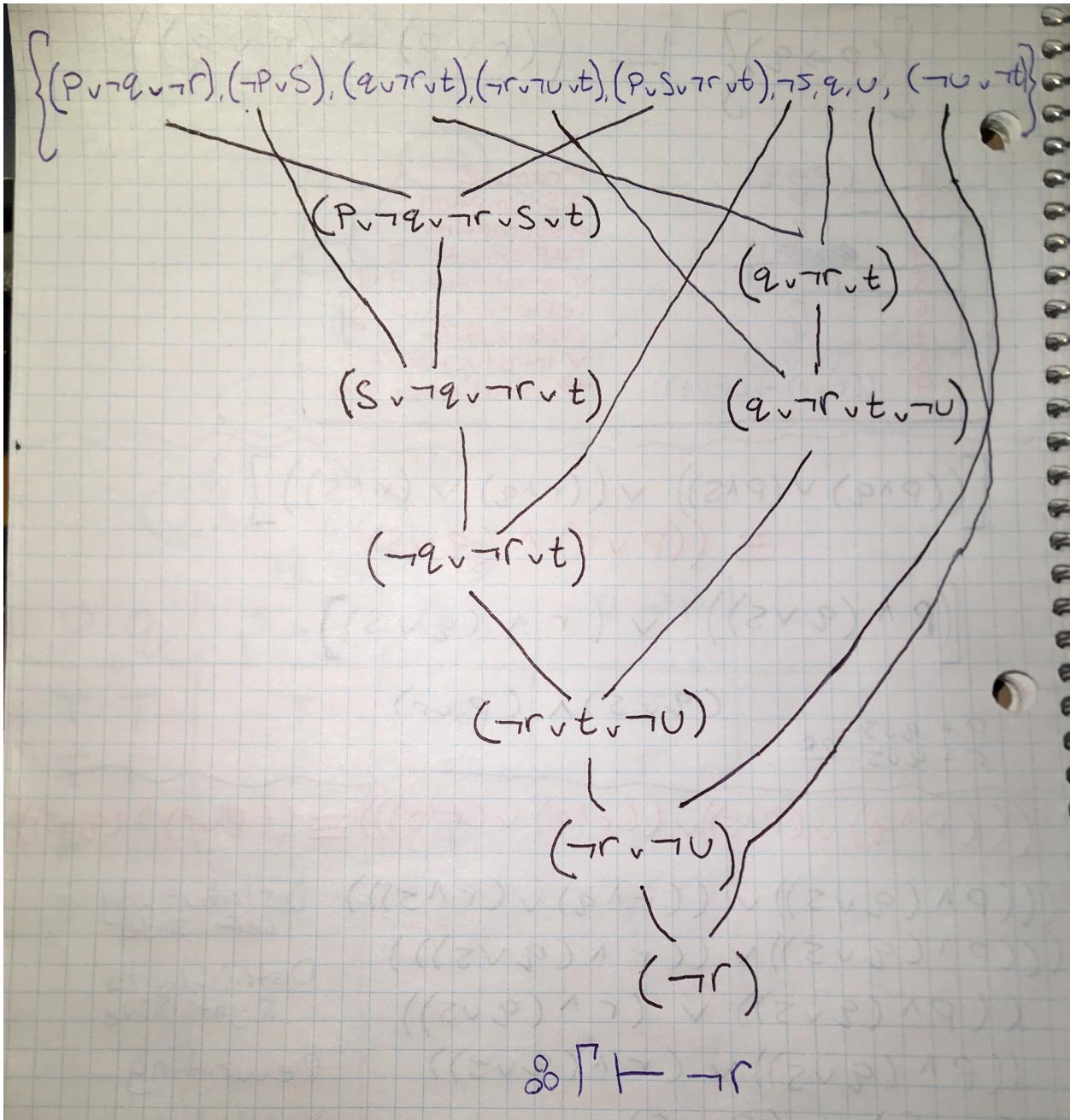
Demorgan to
 \wedge * + Double
Negation

Question 4

$$\{(P \rightarrow Q)\} \vdash ((r \vee P) \rightarrow (r \vee Q))$$

1	($P \rightarrow Q$)	Premise
2	P	ASSumption
3	r	ASSumption
4	P	reflexivity: 2
5	($r \vee P$)	\checkmark introduction: 3-4
6	r	reflexivity: 3 \checkmark
7	q	ASSumption
8	($r \vee q$)	\checkmark introduction: 6-7
9	(($r \vee P$) \rightarrow ($r \vee q$))	\rightarrow introduction: 5, 8

Question 5



Question 6

Question 6 }

$$C = \{ \}$$

$$C = \{ d \}$$

$$C = \{ d, a \}$$

$$C = \{ d, a, c \}$$

$$C = \{ d, a, c, e \}$$

$$C = \{ d, a, c, e, b \}$$

$$C = \{ d, a, c, e, b, g \}$$

$$C = \{ d, a, c, e, b, g, f \}$$

$$C = \{ d, a, c, e, b, g, f, h \}$$

Question 7

Question 7)

yes $\leftarrow h$

yes $\leftarrow f \wedge g$

yes $\leftarrow f \wedge a \wedge c$

yes $\leftarrow f \wedge a \wedge a$

yes $\leftarrow f \wedge d \wedge d$

yes $\leftarrow f$

yes $\leftarrow d \wedge e \wedge b$

yes $\leftarrow d \wedge e \wedge e$

yes $\leftarrow d \wedge c \wedge c$

yes $\leftarrow d \wedge a \wedge a$

yes $\leftarrow d \wedge d \wedge d$

yes \leftarrow

$\bullet \bullet \vdash h$