Modelling problems using propositional logic

N-queens

$N = 2$

a	b
d	c

place two queens on a 2x2 chessboard such that no queen attacks another queen

[put another way, one and only one queen can be in a row, a column, and a diagonal]

Satisfiability (3/4)

Eight queens puzzle: general statement

- Go to `https://en.wikipedia.org/wiki/Eight_queens_puzzle` and read.
- For $n = 4$, there are two solutions.
- **Exercise**: how to phrase the search for those solutions into a SAT problem?
- **Hints**:
  - What should the Boolean variables represent?
  - What should the propositional formula represent?
- Remember the rules:
  - at most one (and at least one) queen in every row,
  - at most one (and at least one) queen in every column,
  - at most one queen in every diagonal.

Satisfiability (4/4)

Eight queens puzzle: case $n = 2$

- Associate a Boolean variable with each of the four corners, say a, b, c and d in clock-wise order.
- Exactly one queen on the top row writes: $(a \vee b) \wedge \neg(a \wedge b)$.
- Exactly one queen on the bottom row writes: $(c \vee d) \wedge \neg(c \wedge d)$.
- Exactly one queen on the left column writes: $(a \vee d) \wedge \neg(a \wedge d)$.
- Exactly one queen on the left column writes: $(b \vee c) \wedge \neg(b \wedge c)$.
- No two queens on the same diagonal writes: $\neg(a \wedge c) \wedge \neg(b \wedge d)$.
- We need some help to determine of the conjunction of these 5 formulas is satisfiable!

Try the N-queens with $N = 3$

a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}

(no solution)