Semantic entailment

(or simply, entailment)

A relation between a set of wffs and a wff

The symbol for the entailment relation (double turnstyle)

$$\models$$

$$\{wff_1, wff_2, \ldots, wff_n\} \models wff$$

Meaning of entailment

(using the previous relation)

whenever every $wff_1, \ldots, wff_n$ is true,

wff is true

(or in all worlds in which $wff_1, \ldots, wff_n$ are all true, wff is also true)

(we say "$wff_1, \ldots, wff_n$ entails wff")

equivalent to: if $wff_1, \ldots, wff_n$ are true then wff is true

Some interesting entailments

$\{\} \models \alpha$ means that α is a tautology

every world makes this set true so α has to be true in every world

$\{\beta, \neg\beta\} \models \alpha$

and $\{\beta, \neg\beta\} \models \neg\alpha$

means that both α and $\neg\alpha$ are true

no world makes this set true so α is true in all of those worlds and so is $\neg\alpha$

Example of an entailment

$$\{(p \to q), p\} \models q$$

every world in which these are true

means that q is also true

Let's use a truth table

	p	q	$(p \to q)$
** *	T	T	T
	T	F	F
*	F	T	T
*	F	F	T

* these are worlds in which $(p \to q)$ is true

◯ these are worlds in which q is true

** these are worlds in which both are true — note q is true in all of those worlds

Another example of entailment

$$\{(p \to q), (q \to r)\} \models (p \to r)$$

Let's use a truth table

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

You've seen this last example before

$$\{\} \models \underbrace{((p \to q) \wedge (q \to r))}_{} \to (p \to r)$$

Note that whenever the conjuncts are true the conjunction is true

[Set notation and conjunction are used interchangebly when it makes sense to do so]

Entailment and tautology seem to be connected through implication ($\to$)

(They are!)

From inconsistent set of wffs, every wff is entailed.

Let us use the implication form

$(\alpha \wedge \neg\alpha) \to \beta$ $\qquad$ $(\alpha \wedge \neg\alpha) \to \neg\beta$

α	β	$\neg\alpha$	$(\alpha \wedge \neg\alpha)$		$\neg\beta$	
T	T	F	F	T	F	T
T	F	F	F	T	T	T
F	T	T	F	T	F	T
F	F	T	F	T	T	T

[paraconsistent logics]