

Firas Aboushamalah
250 920 750
CS 2209 Assignment #2
Robert Mercer

QUESTION 1

1.a) $\exists x \forall y L(x, y)$

$$\begin{array}{c} \exists x \\ \forall y \\ L(x, y) \\ [L(x, y)] \end{array}$$

b) $\forall x \exists y L(y, x)$

$$\begin{array}{c} \forall x \exists y \\ L(y, x) \\ \forall x \\ L(f(x), x) \\ [L(f(x), x)] \end{array}$$

c) $\forall z \left\{ Q(z) \rightarrow \left\{ \neg \forall x \exists y [P(y) \rightarrow P(g(z, x))] \right\} \right\}$

$$\begin{array}{c} \forall z \left\{ Q(z) \rightarrow \left\{ \neg \forall x \exists y [P(y) \rightarrow P(g(z, x))] \right\} \right\} \\ \forall z \left\{ Q(z) \rightarrow \left\{ \neg \forall x \exists y [\neg P(y) \vee P(g(z, x))] \right\} \right\} \\ \forall z \left\{ \neg Q(z) \vee \left\{ \neg \forall x \exists y [\neg P(y) \vee P(g(z, x))] \right\} \right\} \\ \forall z \left\{ \neg Q(z) \vee \left\{ \forall y [P(y) \vee \neg P(g(z, c_2))] \right\} \right\} \end{array}$$

$$(\neg Q(z) \vee P(y)) \wedge (\neg Q(z) \vee \neg P(g(z, c_2)))$$

$$[\neg Q(z) \vee P(y)] [\neg Q(z) \vee \neg P(g(z, c_2))]$$

Question 2

$$2. \Gamma = \{\neg B(x) \vee C(x), \neg C(a) \vee D(b), \neg C(c) \vee E(d), \neg D(w) \vee E(y)\}$$

1. $\neg B(x) \vee C(x)$ Premise
2. $\neg C(a) \vee D(b)$ Premise
3. $\neg C(c) \vee E(d)$ Premise
4. $\neg D(w) \vee E(y)$ Premise
5. $B(z)$ Prove $\exists x \neg B(x)$
6. $C(x)$ 1,5 $(z/x, x/x)$ + Standardize var. apart
7. $D(b)$ 2,6 (x/a)
8. $\neg E(y)$ 2,4 (w/b) + Standardize var. apart
9. $\neg C(c)$ 3,8 (y/d)
10. \square 6,9 (x/c)
 $(x, x/\exists)$

$\{ \{ (x, x/\exists) \} \leftarrow (w, b) \{ \{ E(x/b) \} \leftarrow (y, d) \} \} \vdash A$

$\{ \{ (x, x/\exists) \} \leftarrow (w, b) \{ \{ E(x/b) \} \leftarrow (y, d) \} \} \vdash A$

$((w, b) \{ \{ E(x/b) \} \leftarrow (y, d) \}) \vdash A$

$((w, b) \{ \{ E(x/b) \} \leftarrow (y, d) \}) \vdash A$

Question 3

3. $\forall x P(x) \rightarrow \exists y P(y)$

$$\neg (\forall x P(x) \rightarrow \exists y P(y))$$

$$\neg (\neg (\forall x P(x) \vee \exists y P(y)))$$

$$\forall x P(x) \wedge \neg [\exists y P(y)]$$

$$\forall x P(x) \wedge \forall y \neg P(y)$$

$$P(x) \wedge \neg P(y)$$

$$[P(x)] [P(y)]$$

1. $P(x)$ (conclusion \neg)

2. $P(y)$ (Conclusion)

3. \square 1,2 (x/y)

Question 4 / 5

4.

$$\begin{aligned} C &= \{\} \\ &= \{P(a_4, a_3)\} \\ &= \{P(a_1, a_4), P(a_4, a_3)\} \\ &= \{Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \\ &= \{Q(a_1, a_3), Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \\ &= \{P(a_3, a_2), Q(a_1, a_3), Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \\ &= \{Q(a_1, a_2), P(a_3, a_2), Q(a_1, a_3), Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \\ &= \{P(a_2, a_1), Q(a_1, a_2), P(a_3, a_2), Q(a_1, a_3), Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \\ &= \{Q(a_1, a_1), P(a_2, a_1), Q(a_1, a_2), P(a_3, a_2), Q(a_1, a_3), Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \\ &= \{P(a_1, a_1), Q(a_1, a_1), P(a_2, a_1), Q(a_1, a_2), P(a_3, a_2), Q(a_1, a_3), Q(a_1, a_4), P(a_1, a_4), P(a_4, a_3)\} \end{aligned}$$

5.

$$y_0 \leftarrow R(a_1, a_1)$$

$$\begin{aligned} \text{Yes}(A, B) &\leftarrow R(A, B) \\ \text{Yes}(A, B) &\leftarrow Q(B, A) \\ \text{Yes}(A, B) &\leftarrow Q(B, z) \wedge P(z, A) \\ \text{Yes}(a_1, B) &\leftarrow Q(B, a_2) \wedge P(a_2, a_1) \\ \text{Yes}(a_1, B) &\leftarrow Q(B, a_2) \\ \text{Yes}(a_1, B) &\leftarrow Q(B, a_3) \wedge P(a_3, a_2) \\ \text{Yes}(a_1, B) &\leftarrow Q(B, a_3) \\ \text{Yes}(a_1, B) &\leftarrow Q(B, a_4) \wedge P(a_4, a_3) \\ \text{Yes}(a_1, B) &\leftarrow Q(B, a_4) \\ \text{Yes}(a_1, B) &\leftarrow P(B, a_4) \\ \text{Yes}(a_1, a_1) &\leftarrow P(a_1, a_1) \end{aligned}$$