CS 2210 Data Structures and Algorithms

Roberto Solis-Oba

Introduction

In this and next lectures we introduce the two core components of every computer program:

- Data structures
- Algorithms

A Fundamental Problem

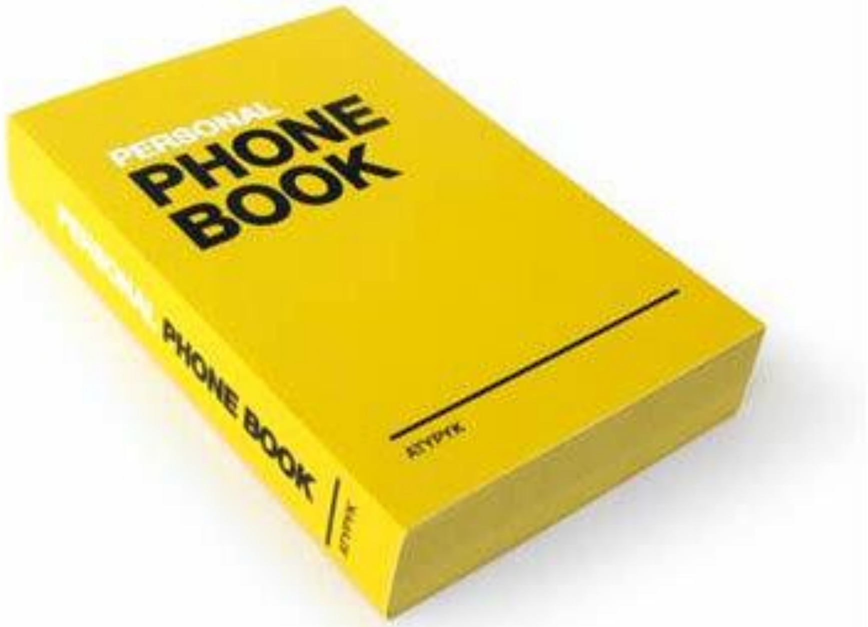
Given a set S of elements and a particular element x the search problem is to decide whether x is in S.

The Search Problem

This problem has a large number of applications:

- S = Names in a phone book

  x = name of a person

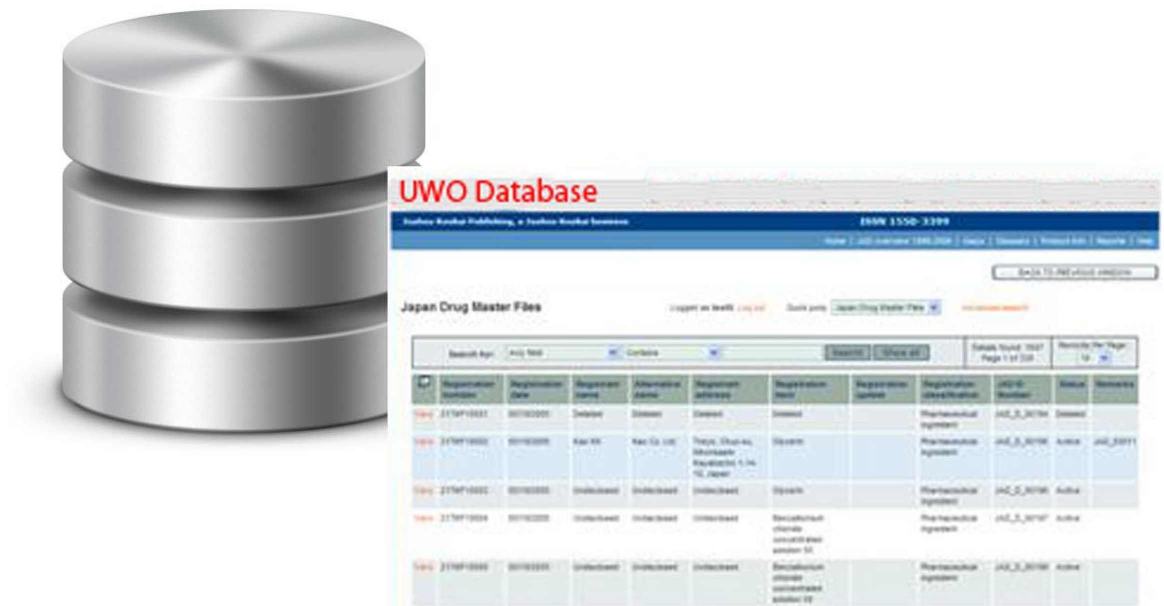


The Search Problem

This problem has a large number of applications:

- S = Student records

  x = student ID



The Search Problem

This problem has a large number of applications:

- S = Variables in a program

  x = name of a variable

```
/* When the user has selected a play, this method is invoked to
   process the selected play */
public void actionPerformed(ActionEvent event) {
    if(event.getSource() instanceof JButton) { /* Some position of the
                                                  board was selected */
        int row = -1, col = -1;
        PosPlay pos;

        if (game_ended) System.exit(0);
        /* Find out which position was selected by th eplayer */
        for (int i = 0; i < board_size; i++) {
            for (int j = 0; j < board_size; j++)
                if(event.getSource() == board[i][j]) {
                    row = i;
                    col = j;
                    break;
                }
            if (row != -1) break;
        }
```

The Search Problem

This problem has a large number of applications:

- S = Web host names

  x = URL

Solving a Problem

The solution of a problem has 2 parts:

- How to organize data

- How to solve the problem

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  Algorithm:

  - a step-by-step procedure for performing some task in finite time

A First Solution

For simplicity, let us assume that S is a set of n different integers stored in non-decreasing order in an array L.

L	3	9	11	17	18	26	29	43	48	55
	0	1	2	3	4	5	6	7	8	9

x

Algorithm LinearSearch (L,n,x)

Input: Array L of size n and value x

Output: Position i, $0 \le i < n$, such that L[i] = x, if x in L, or -1, if x not in L

```
i ← 0
while (i < n) and (L[i] ≠ x) do
    i ← i+1
if i=n then return -1
else return i
```

Proving the Correctness of an Algorithm

To prove that an algorithm is correct we need to show 2 things:

- The algorithm terminates
- The algorithm produces the correct output

Correctness of Linear Search

Termination

- i takes values 0, 1, 2, 3, …
- The while loop cannot perform more than n iterations because of the condition ($i < n$)

Correctness of Linear Search

Correct Output

- The algorithm compares x with L[0], L[1], L[2], …
- Hence, if x is in L then x = L[i] in some iteration of the **while** loop; this ends the loop and then the algorithm correctly returns the value i
- If x is not in L then in some iteration $i = n$; this ends the loop and the algorithm returns -1.

Algorithm BinarySearch (L,x, first, last)

Input: Array L of size n and value x

Output: Position i, $0 \le i < n$, such that $L[i] = x$, if x in L,or -1, if x not in L

if first > last **then return** -1

else mid $\leftarrow \lfloor (first + last)/2 \rfloor$

if x = L[mid] **then return** mid

else if x < L[mid] **then**

return BinarySearch (L,x,first,mid -1)

else return BinarySearch (L,x,mid +1,last)

Correctness of Binary Search

Termination

- If x = L[*mid*] the algorithm terminates
- If x < L[*mid*] or x > L[*mid*], the value L[*mid*] is discarded from the next recursive call. Hence, in each recursive call the size of L decreases by at least 1.
- After a finite # recursive calls the size of L is zero and the algorithm ends

Correctness of Binary Search

Correct Output

- If $x = L[mid]$ the algorithm correctly returns *mid*
- The algorithm only discards values different from x so if all values of L are discarded (so L is empty) it is because x is not in L and the algorithm correctly returns -1.

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- Space (memory) usage

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We are particularly interested in two computer resources: memory and time.

Consequently, we define two types of complexity functions:

- Space complexity: amount of memory that the algorithm needs.
- Time complexity: amount of time needed by the algorithm to complete.

Complexity Function

The complexity of an algorithm is a non-decreasing function on the size of the input .

Types of Complexity Functions

For both kinds of complexity functions we can define 3 cases:

- **Best case**: Least amount of resources needed by the algorithm to solve an instance of the problem of size n.

Types of Complexity Functions

For both kinds of complexity functions we can define 3 cases:

- **Worst case**: Largest amount of resources needed by the algorithm to solve an instance of the problem of size n.

Types of Complexity Functions

For both kinds of complexity functions we can define 3 cases:

- **Average case**:

$$\frac{\text{amount of resources to solve instance 1 of size n} + \text{amount of resources to solve instance 2 of size n} + \ldots + \text{amount of resources to solve last instance of size n}}{\text{number of instances of size n}}$$

Types of Complexity Functions

In this course we will study worst case complexity.

How do we compute the time complexity of an algorithm?

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We need a clock to measure time.

Experimental way of measuring the time complexity

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We need:

- a computer

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- a compiler for the programming language in which the algorithm will be implemented
- an operating system

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Drawbacks

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- Results depend on the input selected
- Results depend on the particular implementation

Computing the time complexity

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- We wish to compute the time complexity of an algorithm without having to implement it.
- We want the time complexity to characterize the performance of an algorithm on ALL inputs and all implementations (i.e. all computers and all programming languages).

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Primitive Operations

A basic or primitive operation is an operation that requires a constant amount of time in any implementation.

Examples:

$\leftarrow$, $+$, $-$, x, $/$, $<$, $>$, $=$, $\leq$, $\geq$, $\neq$

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Constant, means independent from the size of the input.

When do we need to compute the time complexity function?

Assume a computer with speed 10^8 operations per second.

Time Complexity	Time			
$f(n) = n$				
$f(n) = n^2$				
$f(n) = n^3$				
$f(n) = 2^n$				

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$f(n) = n$	10^{-7} s			
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$f(n) = 2^n$	10^{-5} s	10^{-1} s		

When do we need to compute the time complexity function?

Assume a computer with speed 10^8 operations per second.

Time Complexity	Time			
	n = 10	n = 20	n = 1000	
$f(n) = n$	10^{-7} s	$2x10^{-7}$ s	10^{-5} s	
$f(n) = n^2$	10^{-6} s	$4x10^{-6}$ s	10^{-2}s	
$f(n) = n^3$	10^{-5} s	$8x10^{-5}$ s	10 s	
$f(n) = 2^n$	10^{-5} s	10^{-1} s	10^{293} yrs	

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$f(n) = n^2$	10^{-6} s	4×10^{-6} s	10^{-2}s	2.4 hrs
$f(n) = n^3$	10^{-5} s	8×10^{-5} s	10 s	360 yrs
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Asymptotic Notation

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The mathematical notation used to express time complexities is the asymptotic notation:

Asymptotic or Order Notation

Let f(n) and g(n) be functions from $\mathbb{I}$ to $\mathbb{R}$. We say that f(n) is O(g(n)) (read ``f(n) is big-Oh of g(n)'' or ''f(n) is of order g(n)'') if there is a real constant $c > 0$ and an integer constant $n_0 \geq 1$ such that

$$f(n) \leq c \times g(n) \quad \text{for all } n \geq n_0$$

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We sometimes write f(n) = O(g(n)) or f(n) ϵ O(g(n)).