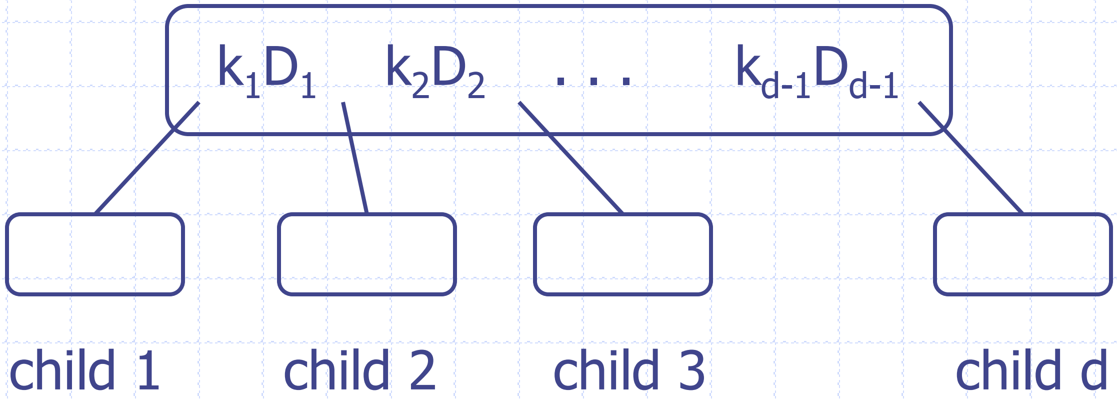
Multi-Way Search Tree

A multi-way search tree is an ordered tree such that

- Each internal node has at least two and at most d children and stores $d-1$ data items (k_i, D_i)

Rule: Number of children = 1 + number of data items in a node



d is the degree or order of the tree

Multi-Way Search Tree

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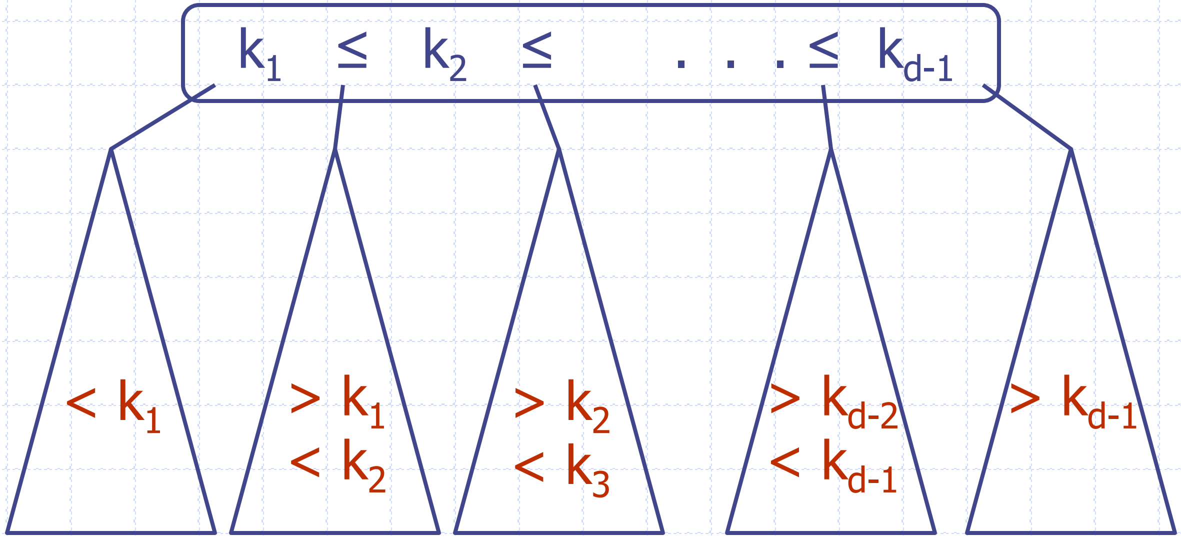
- An internal node storing keys $k_1 \le k_2 \le \ldots \le k_{d-1}$ has d children $v_1\, v_2\, \ldots\, v_d$ such that



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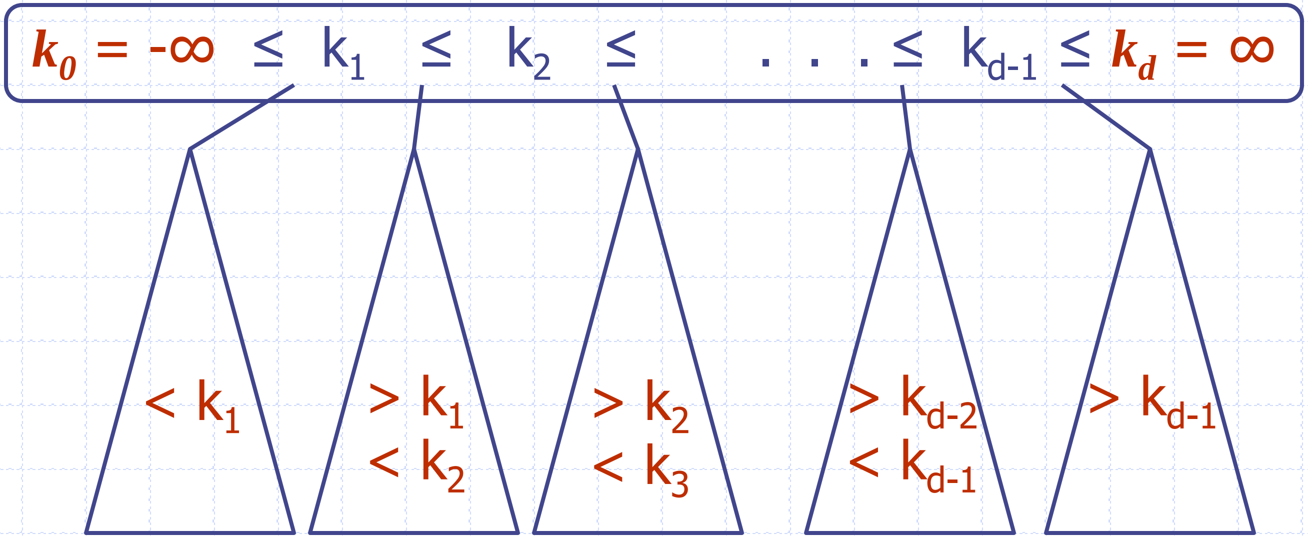
- By convenience we add sentinel keys $k_0 = -\infty$ and $k_d = \infty$



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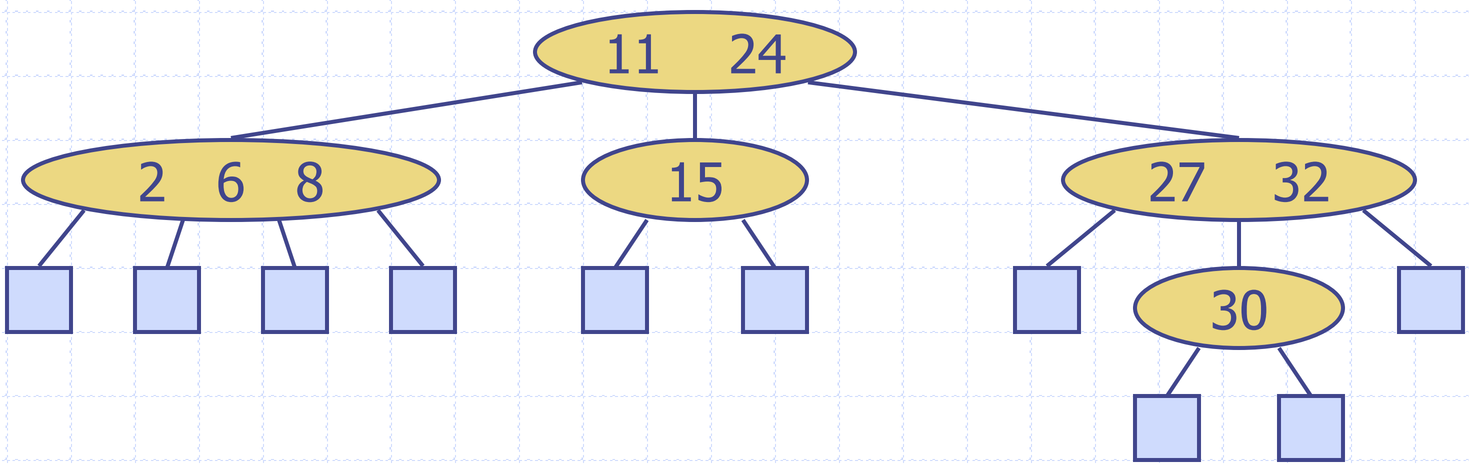
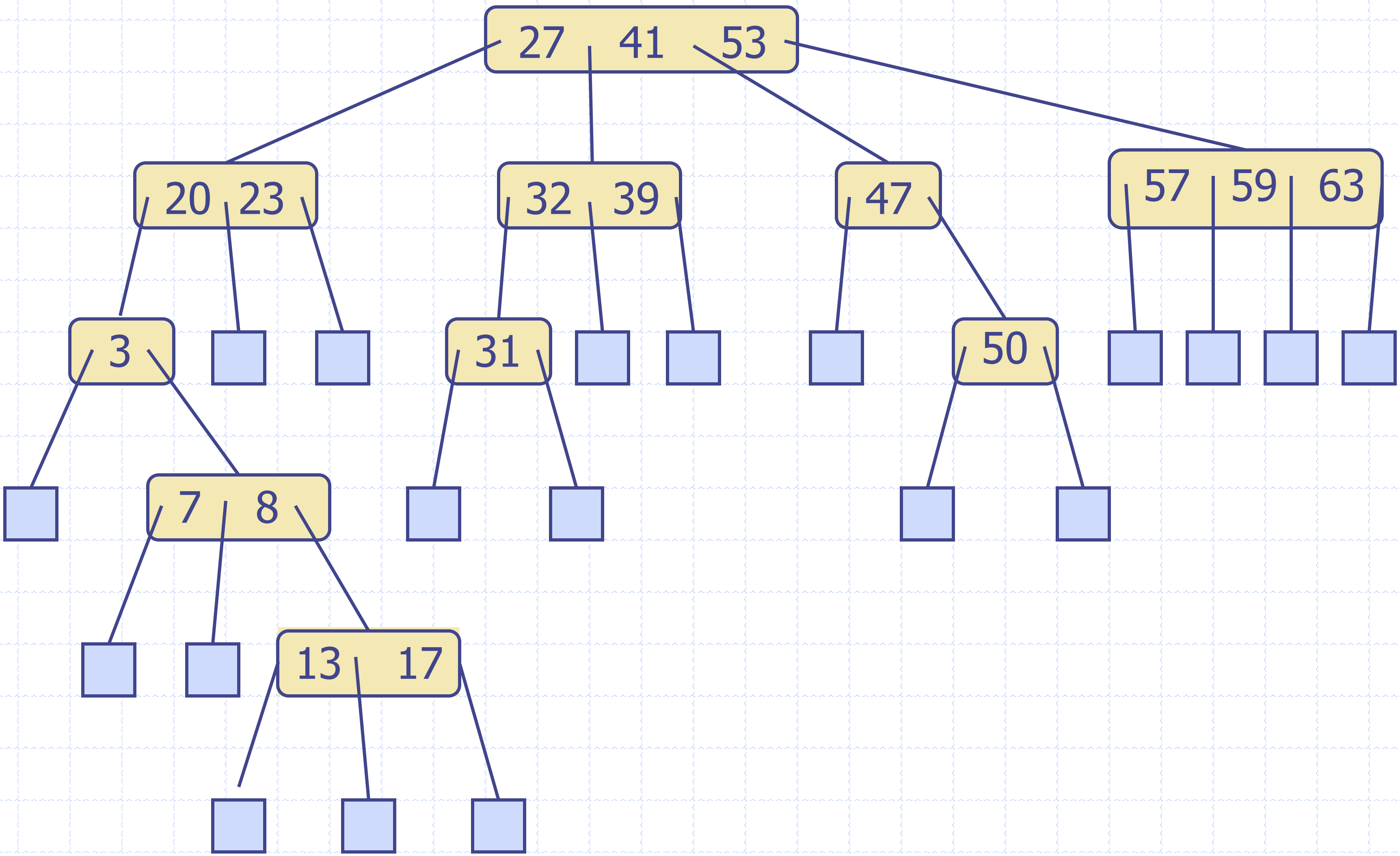
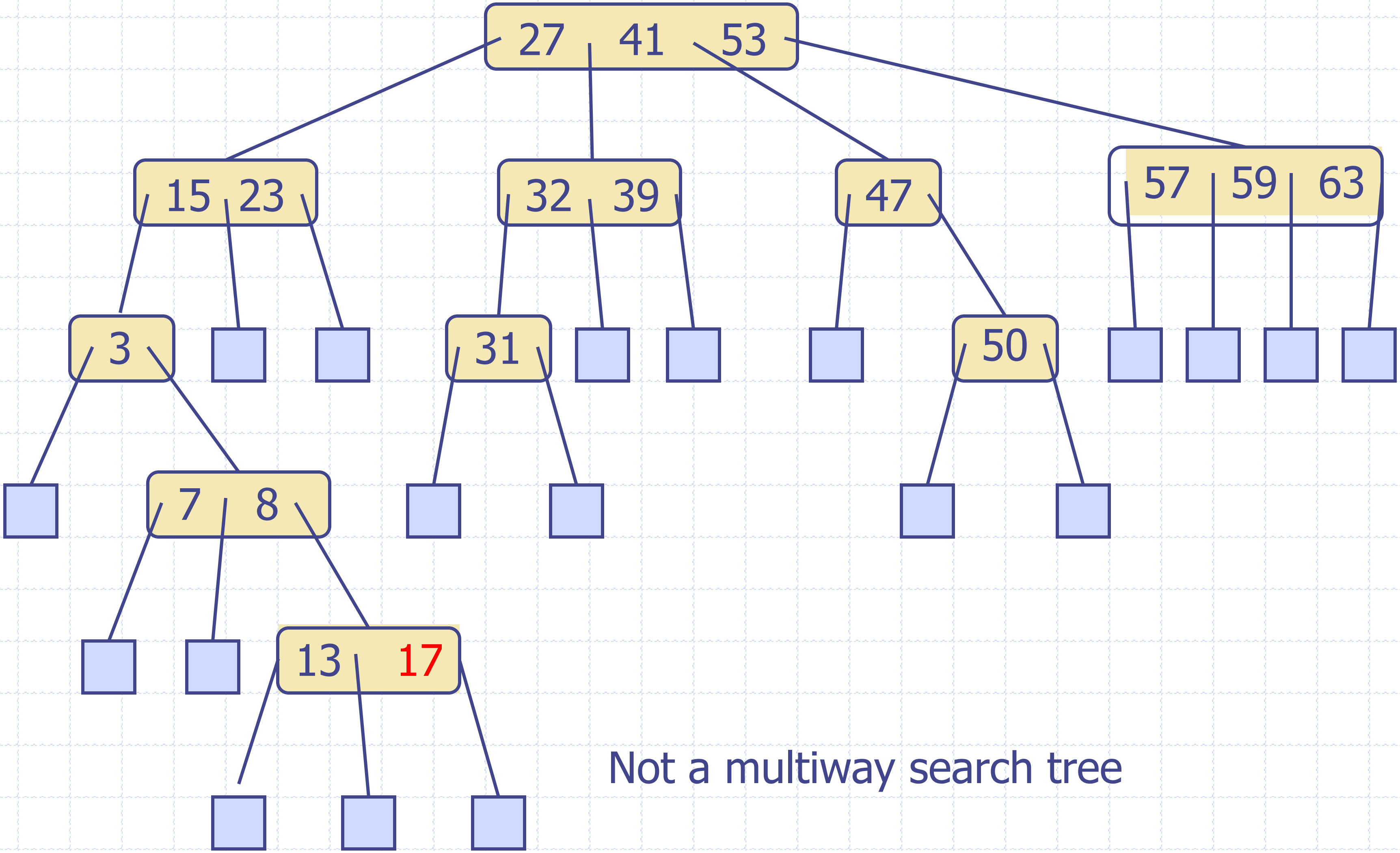
- The leaves store no items and serve as placeholders



Multi-Way Search Tree



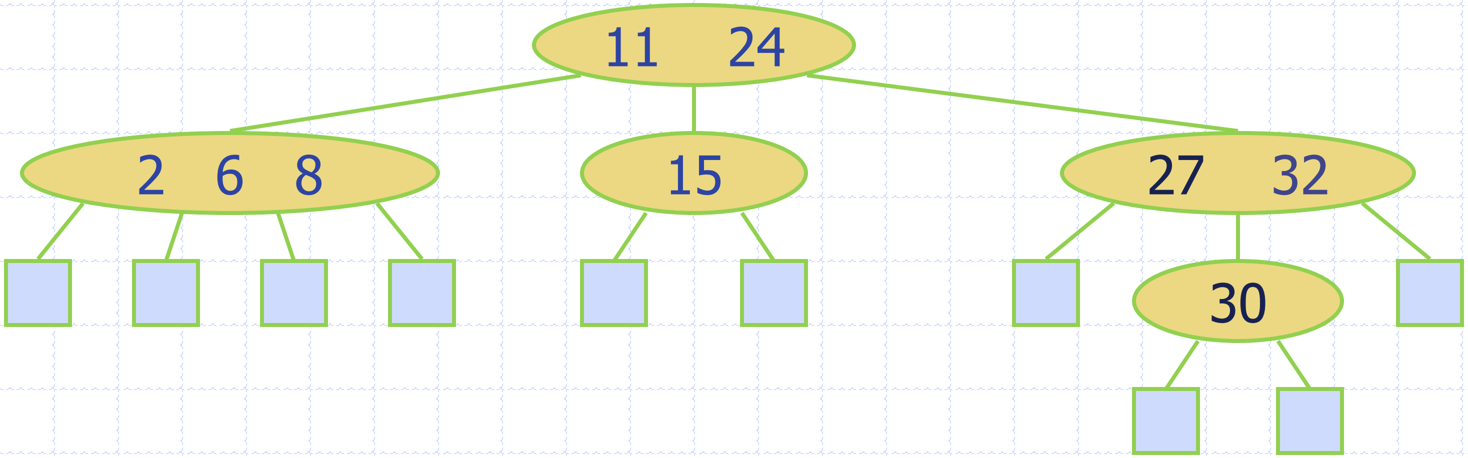
Multi-Way Search Tree



Not a multiway search tree

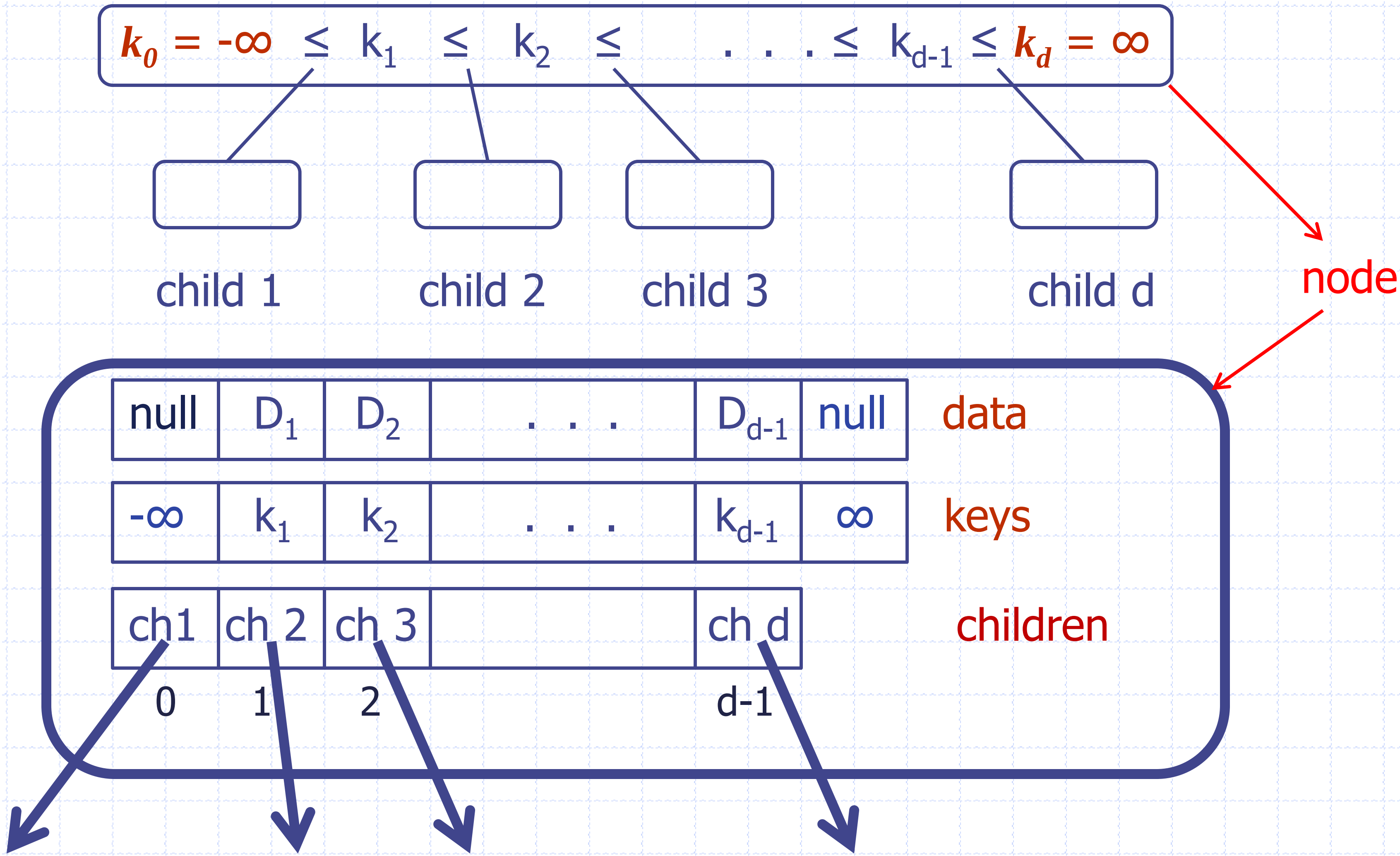
Multi-Way Inorder Traversal

- We can extend the notion of inorder traversal from binary trees to multi-way search trees
- An inorder traversal of a multi-way search tree visits the keys in increasing order

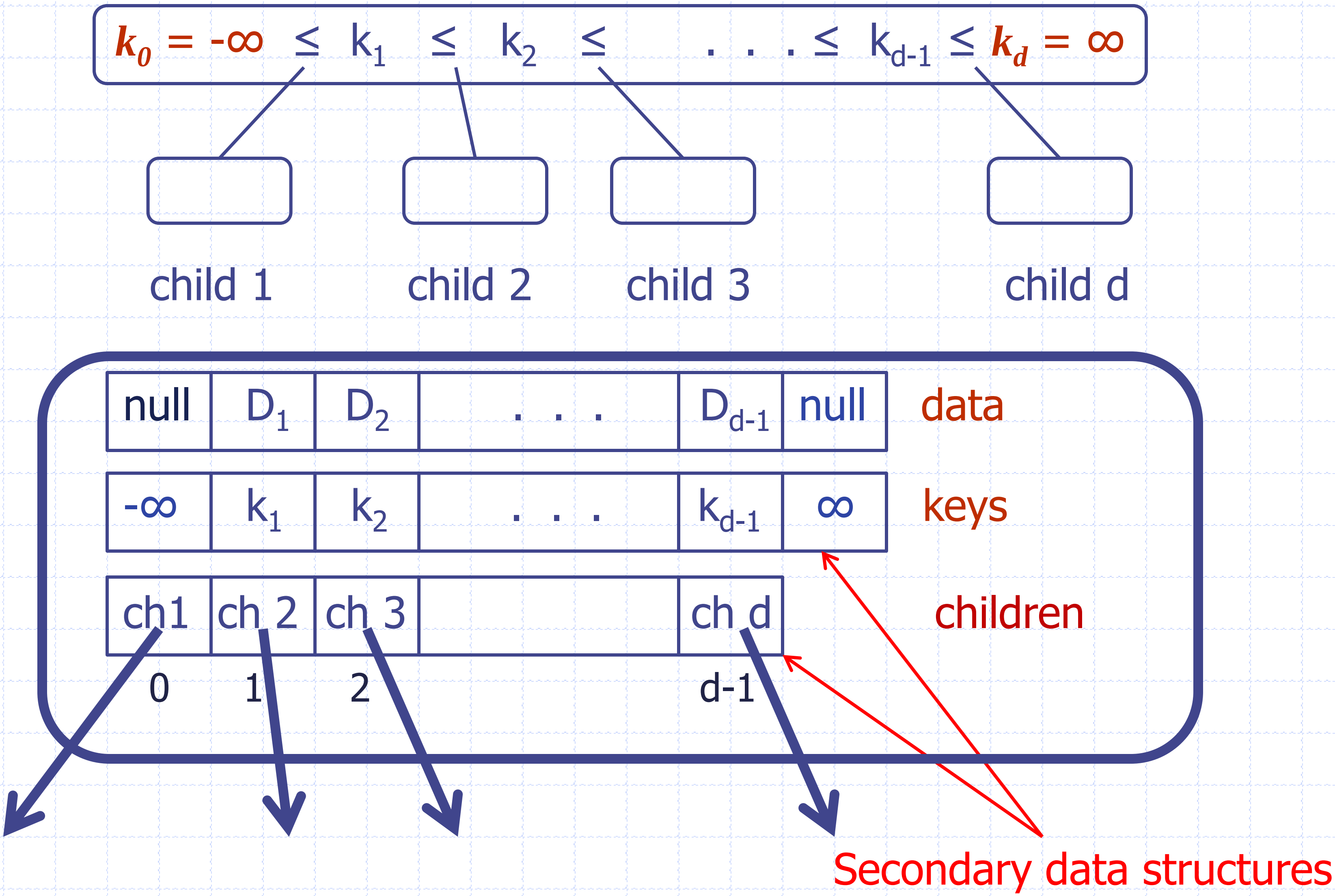


Inorder traversal: 2, 6, 8, 11, 15, 24, 27, 30, 32

Data Structures for Multi-Way Search Trees

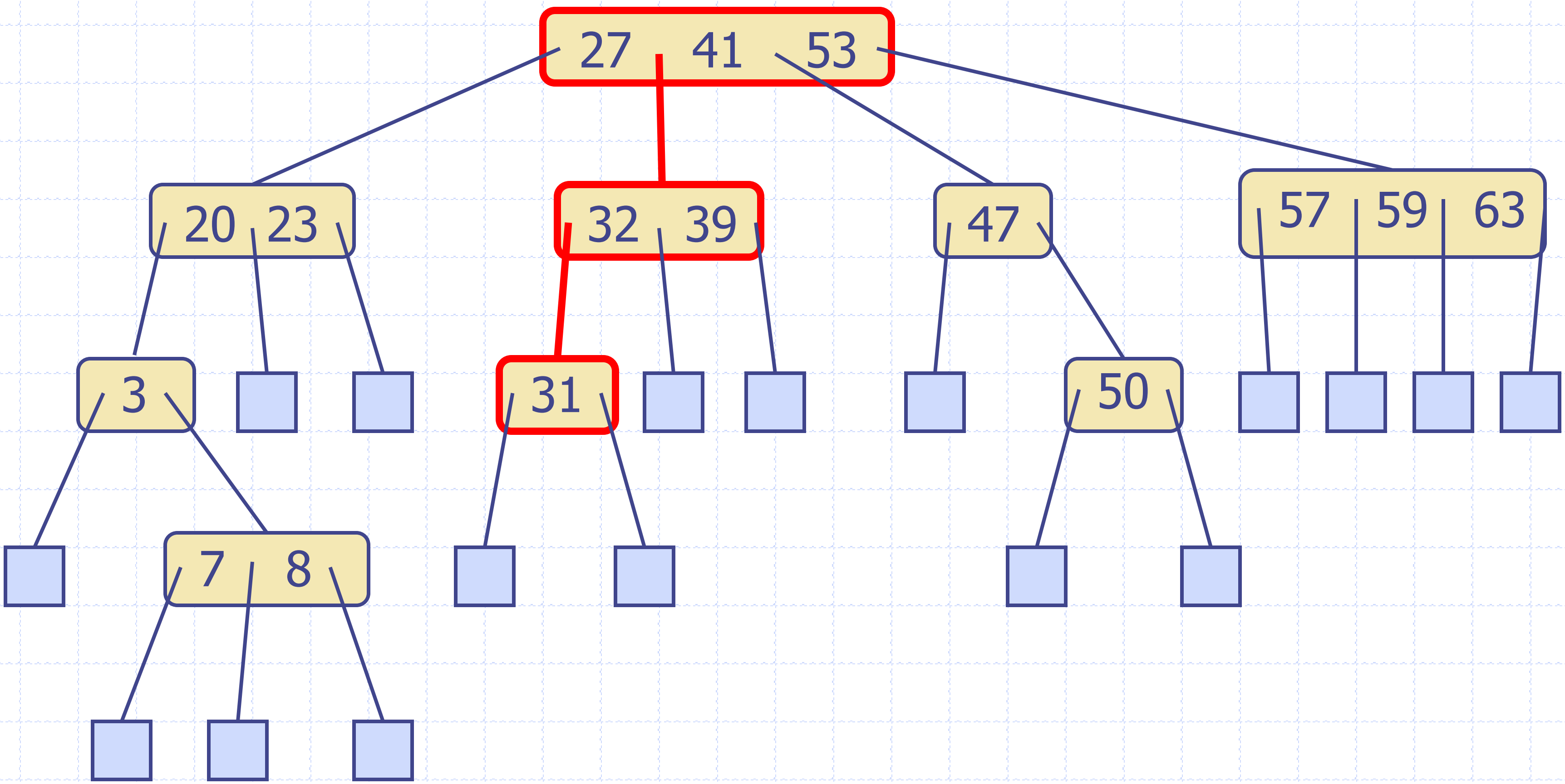


Data Structures for Multi-Way Search Trees



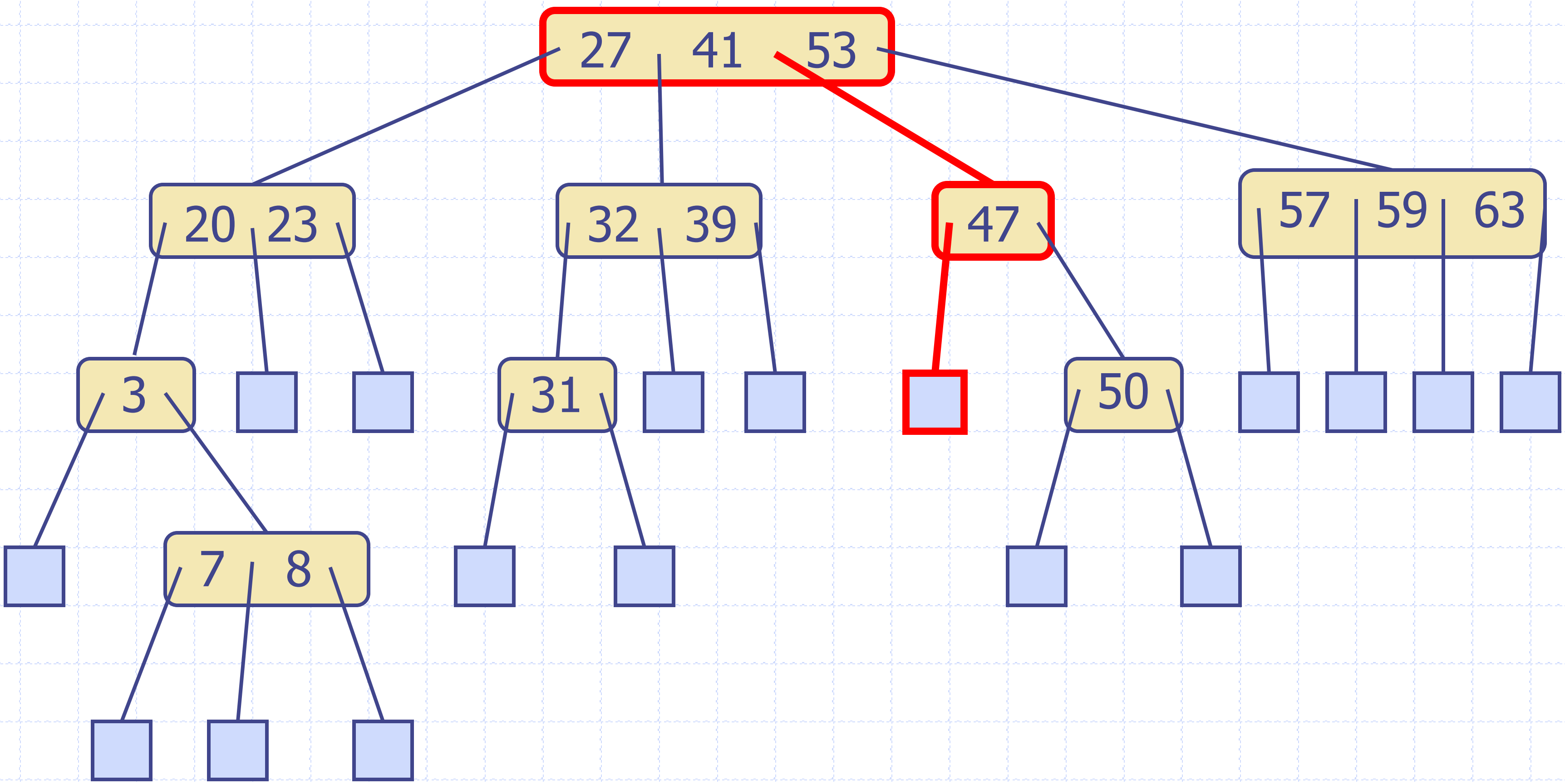
Multi-Way Searching

- Similar to search in a binary search tree
- Example: search for 31



Multi-Way Searching

- Similar to search in a binary search tree
- Example: search for 46



Multi-Way Searching

Algorithm get(r,k)

In: Root r of a multiway search tree, key k

Out: data for key k or null if k not in tree

```
if r is a leaf then return null
else {
    Use binary search to find the index i such that
        r.keys[i] ≤ k < r.keys[i+1]
    if k = r.keys[i] then return r.data[i]
    else return get(r.child[i],k)
}
```

Multi-Way Searching

Algorithm get(r,k)

In: Root r of a multiway search tree, key k

Out: data for key k or null if k not in tree

if r is a leaf **then return** null } c operations

else {

    Use binary search to find the index i such that

        r.keys[i] ≤ k < r.keys[i+1]

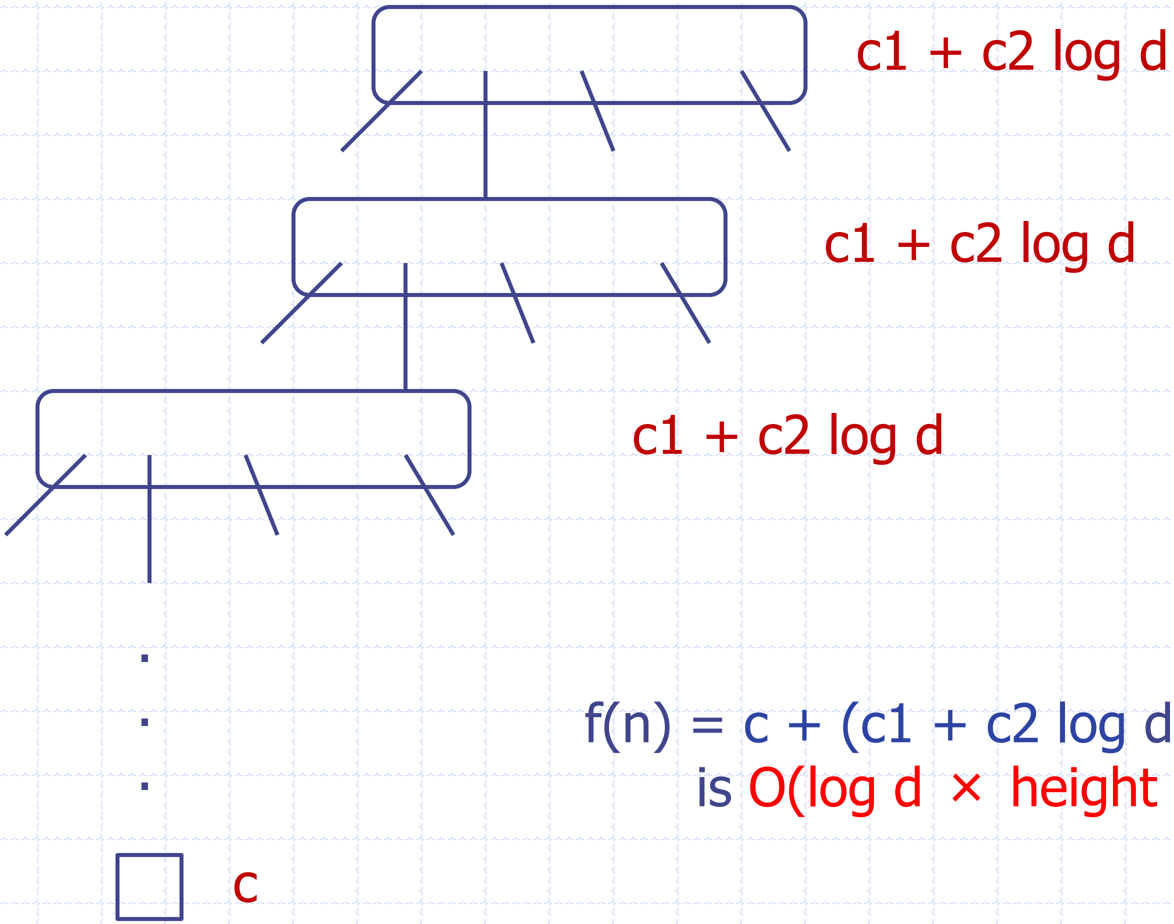
    if k = r.keys[i] **then return** r.data[i]

    else return **get(r,r.child[i])**

}

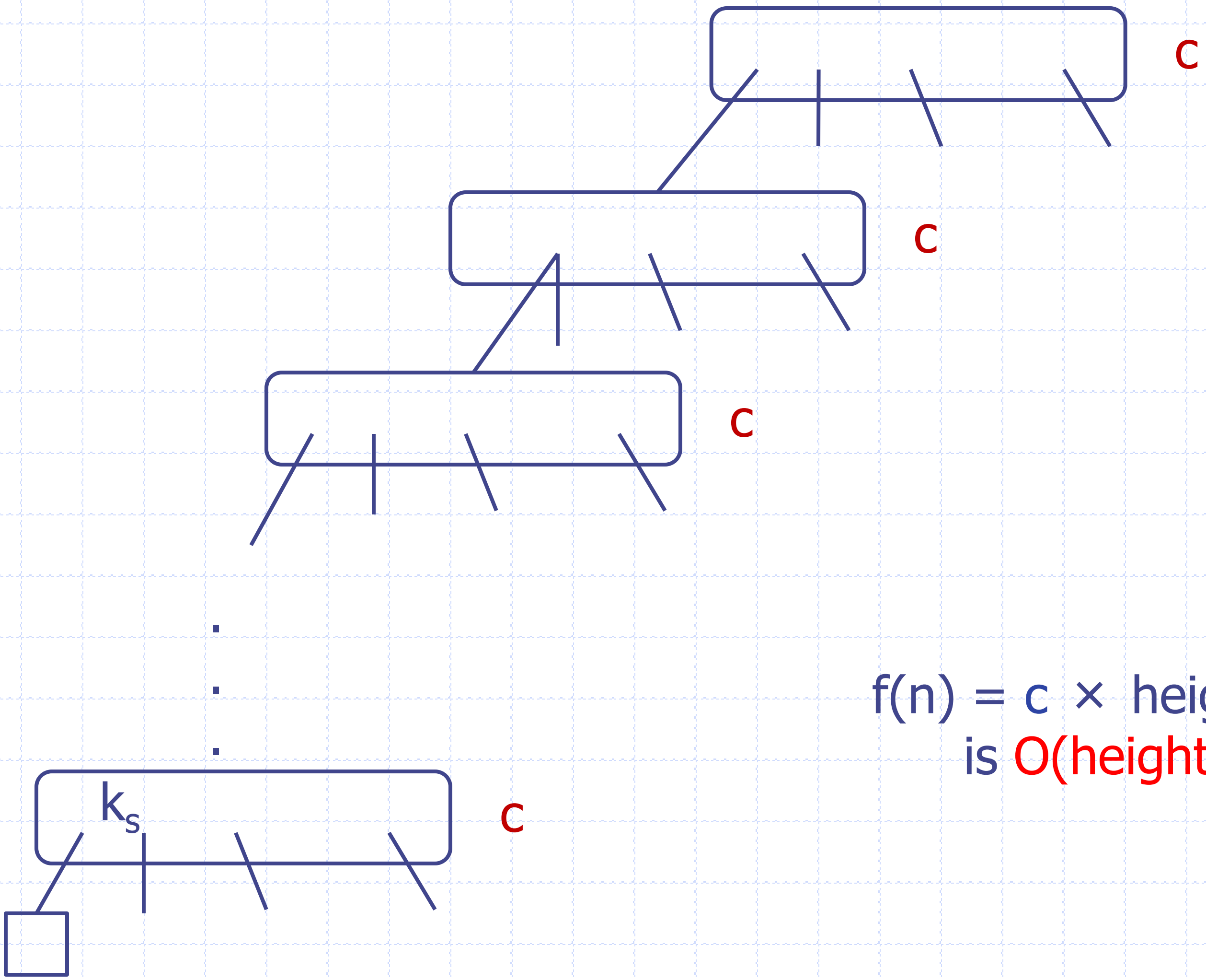
Ignoring recursive calls: $c_1 \log d + c_2$ operations

Time Complexity of get Operation



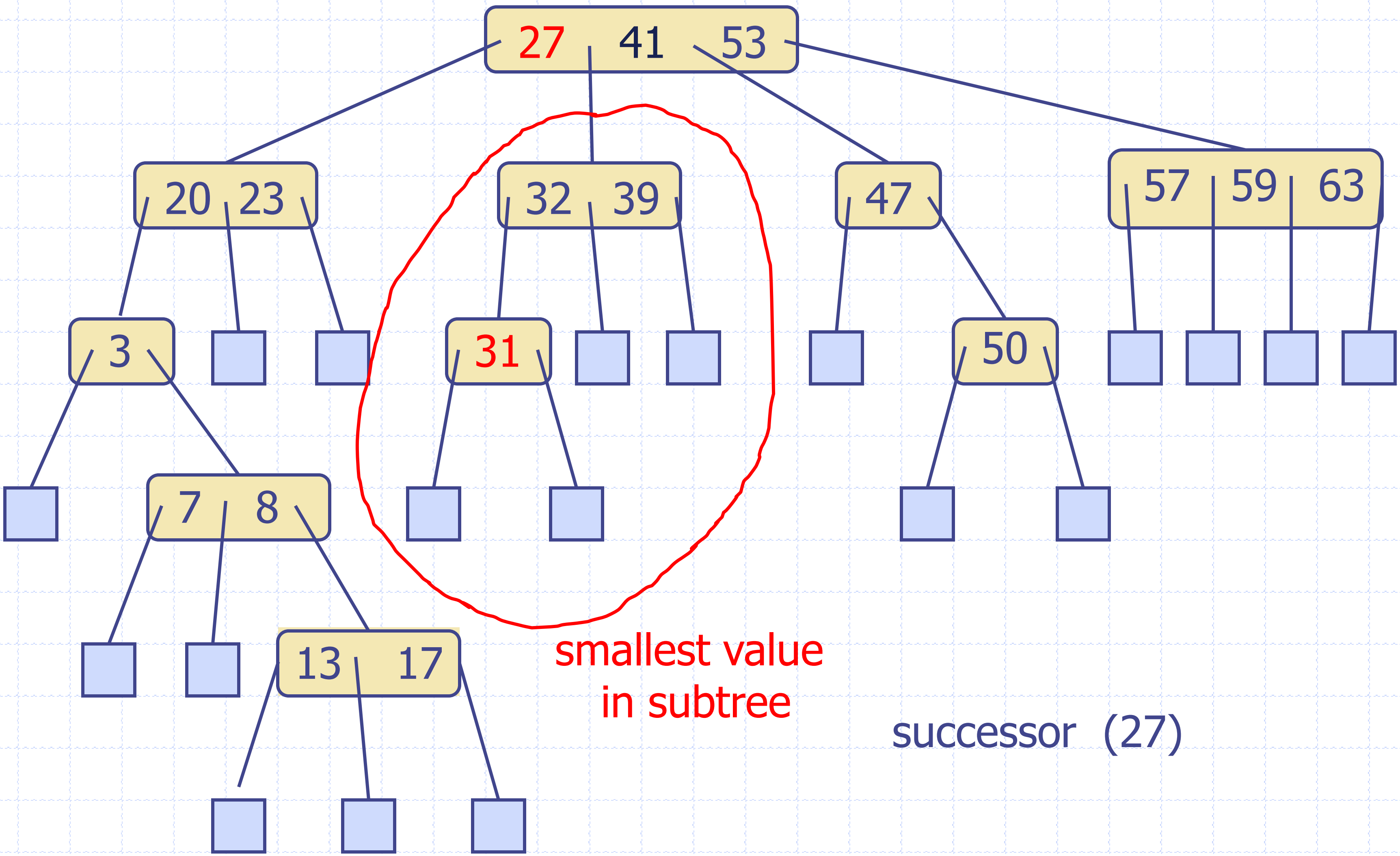
f(n) = c + (c1 + c2 log d) × height of tree
is O(log d × height of tree)

Smallest Operation



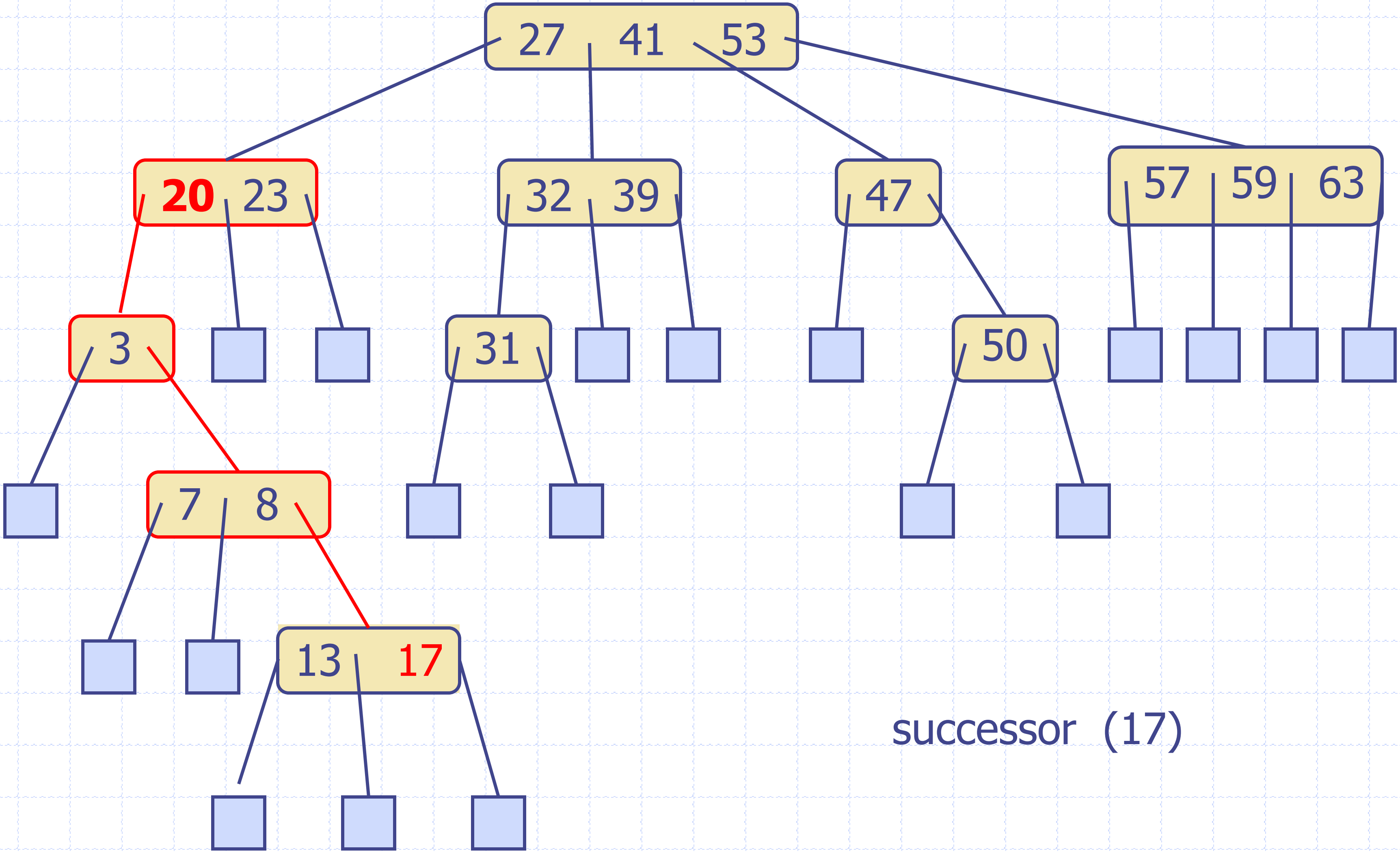
$f(n) = c \times$ height of tree
is O(height of tree)

Successor Operation



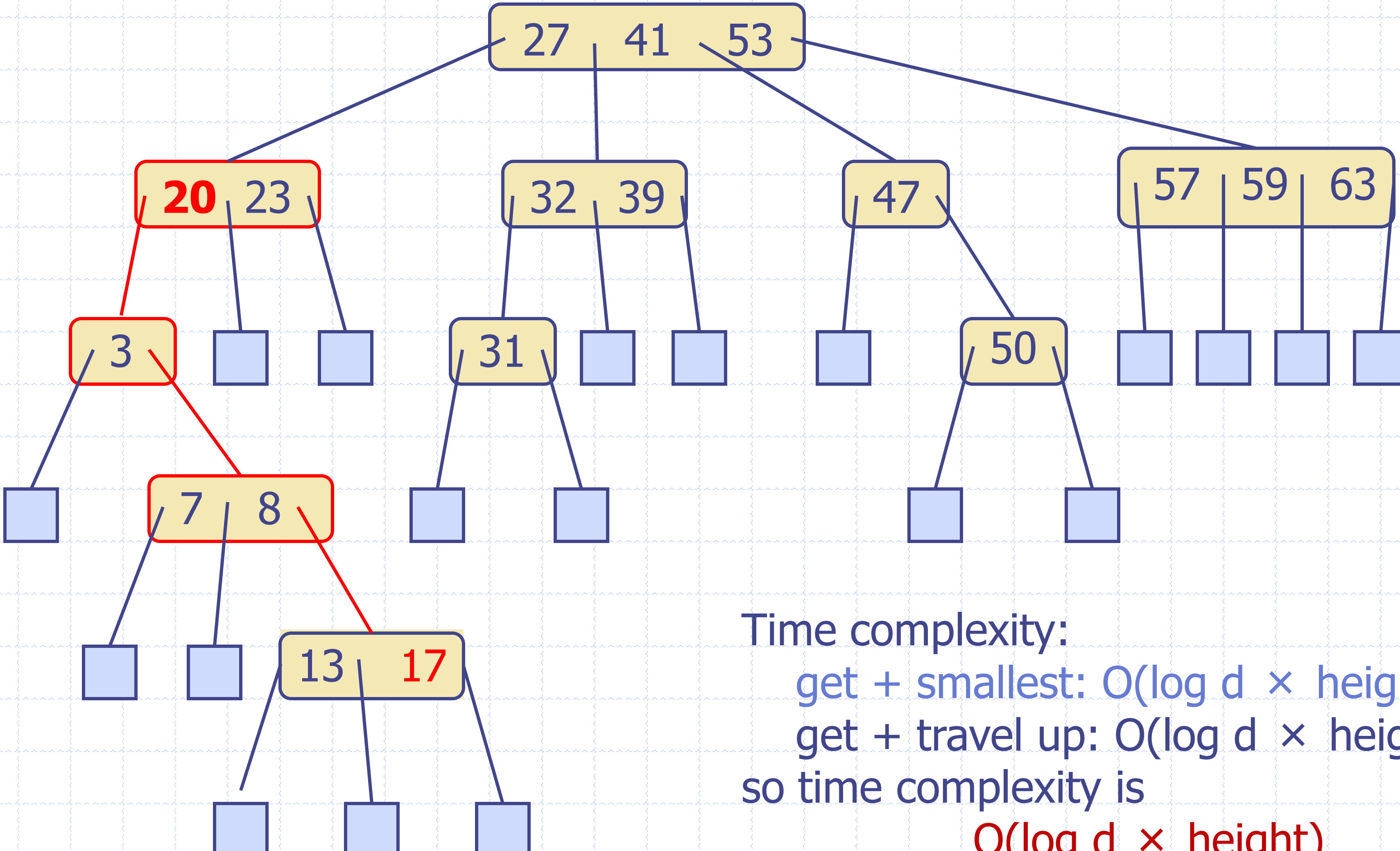
successor (27)

Successor Operation



successor (17)

Successor Operation



Time complexity:

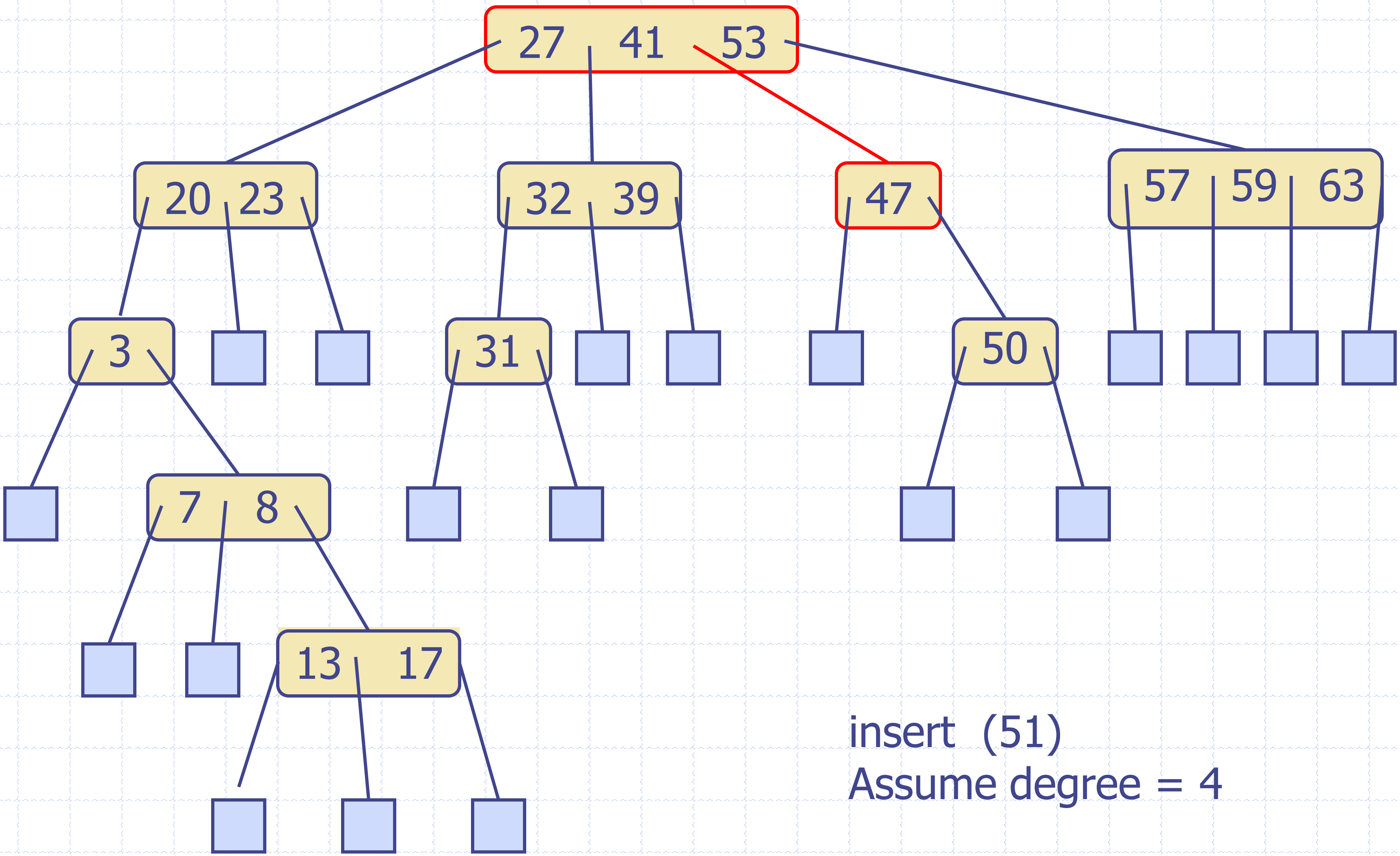
get + smallest: O(log d × height), or

get + travel up: O(log d × height)

so time complexity is

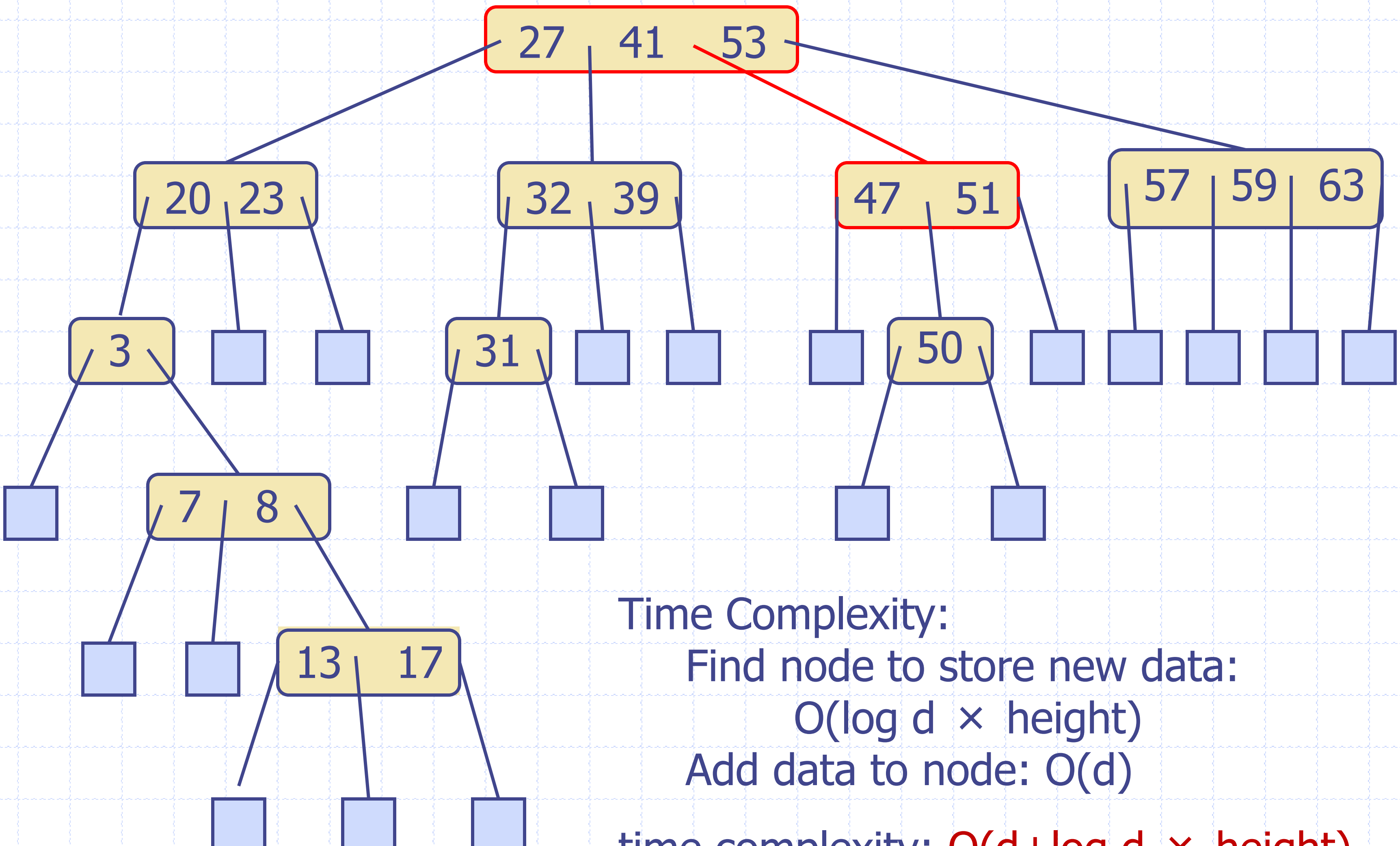
O(log d × height)

Put Operation



insert (51)
Assume degree = 4

Put Operation



Time Complexity:

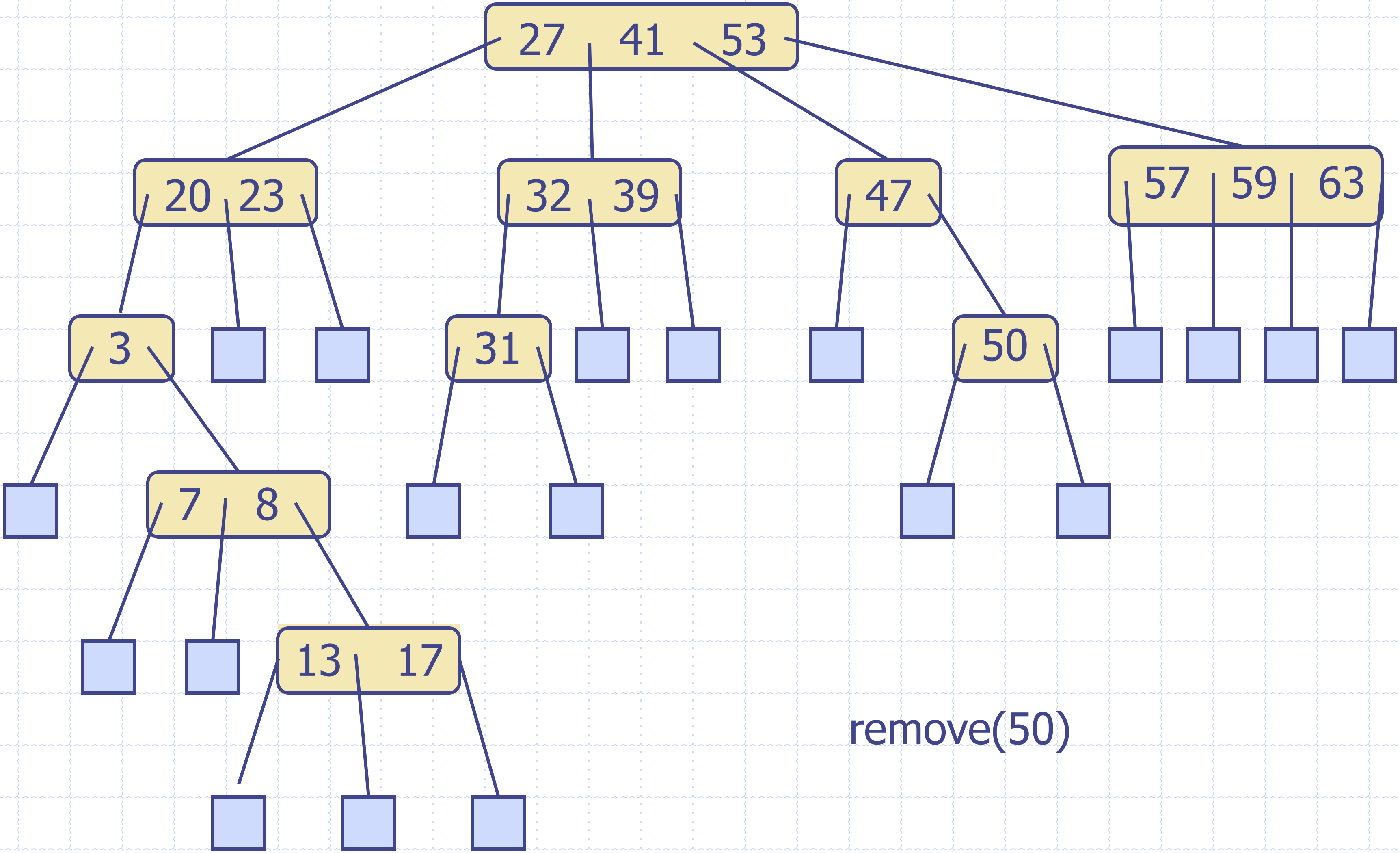
Find node to store new data:

O(log d × height)

Add data to node: O(d)

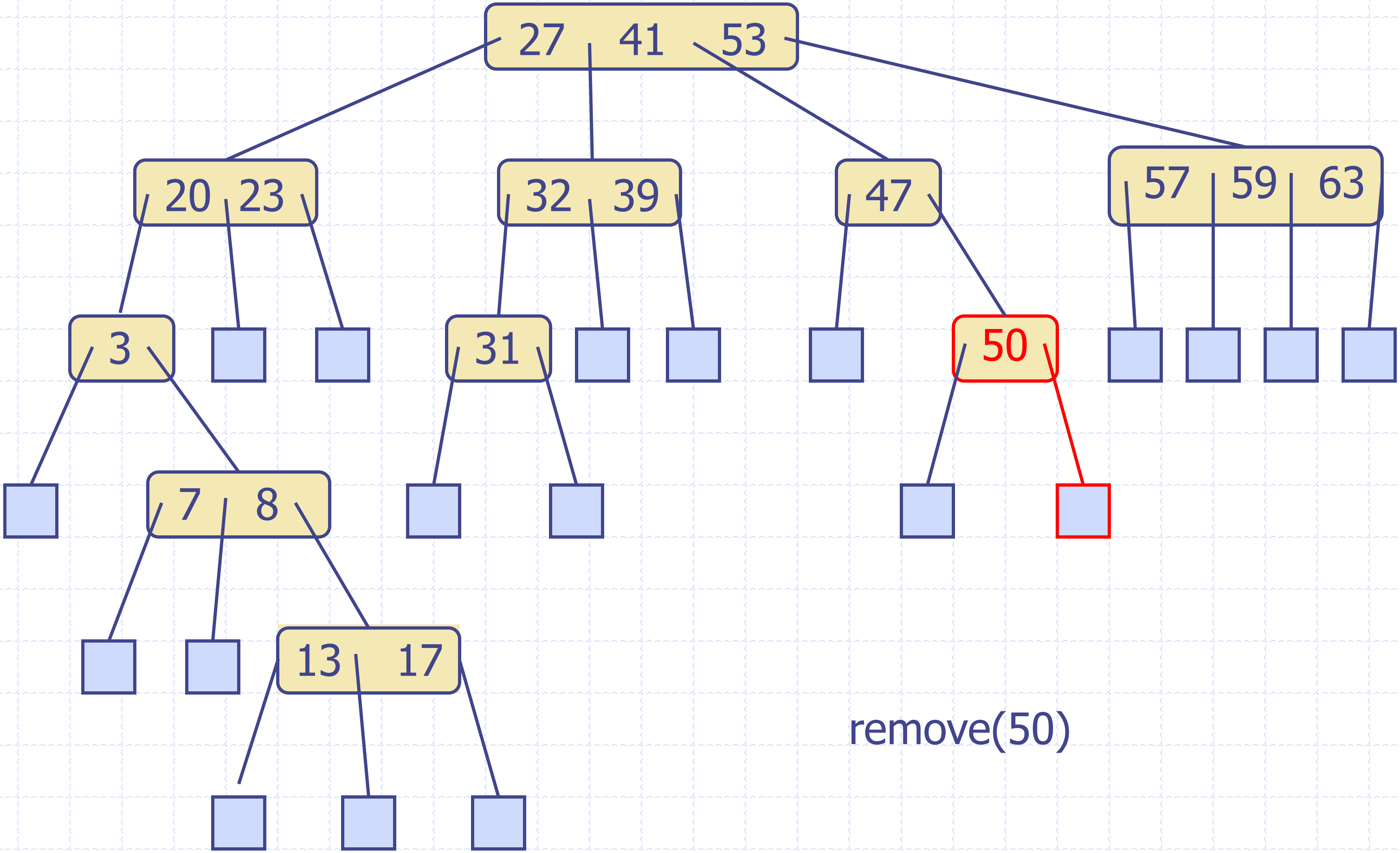
time complexity: O(d+log d × height)

Remove Operation



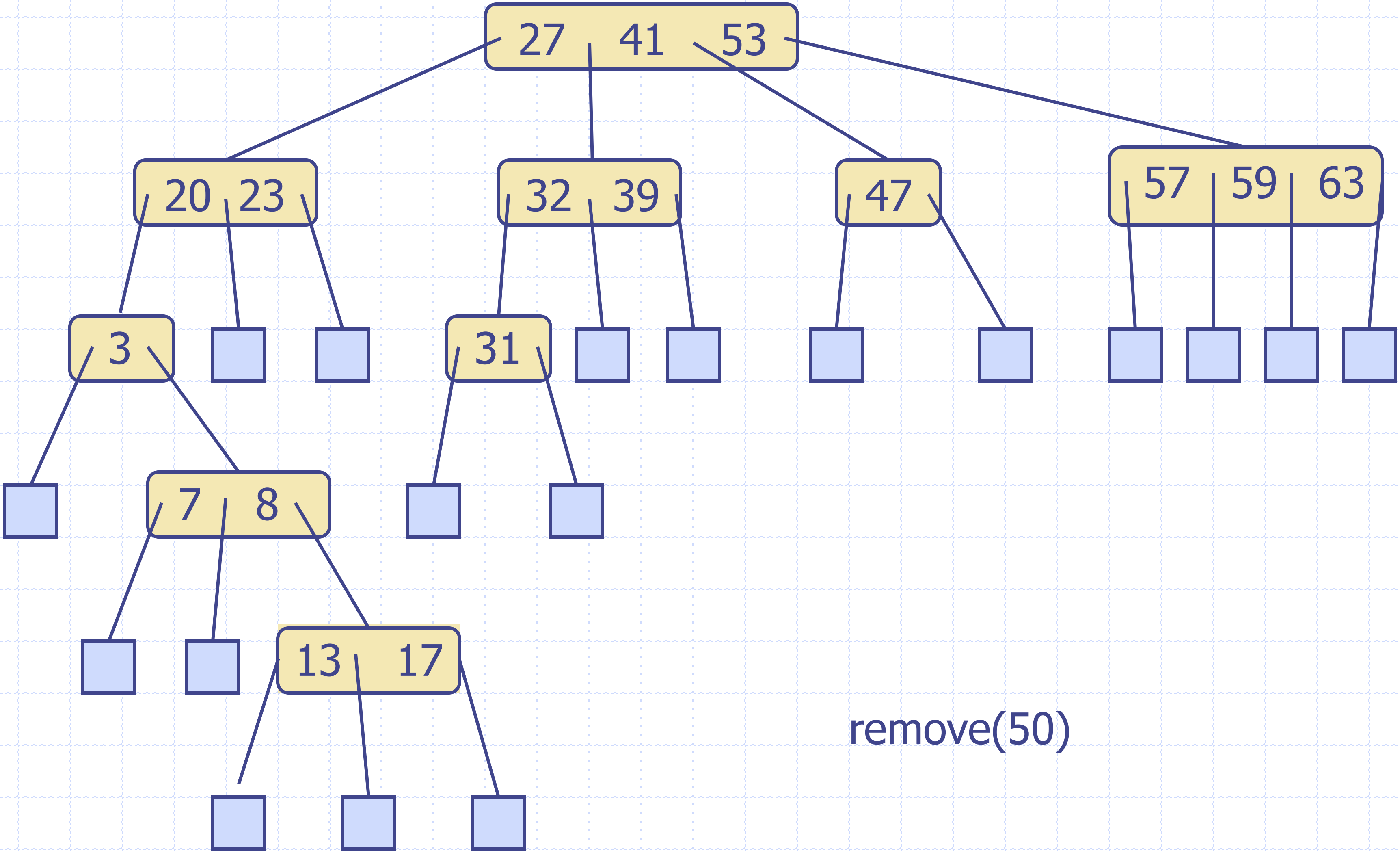
remove(50)

Remove Operation



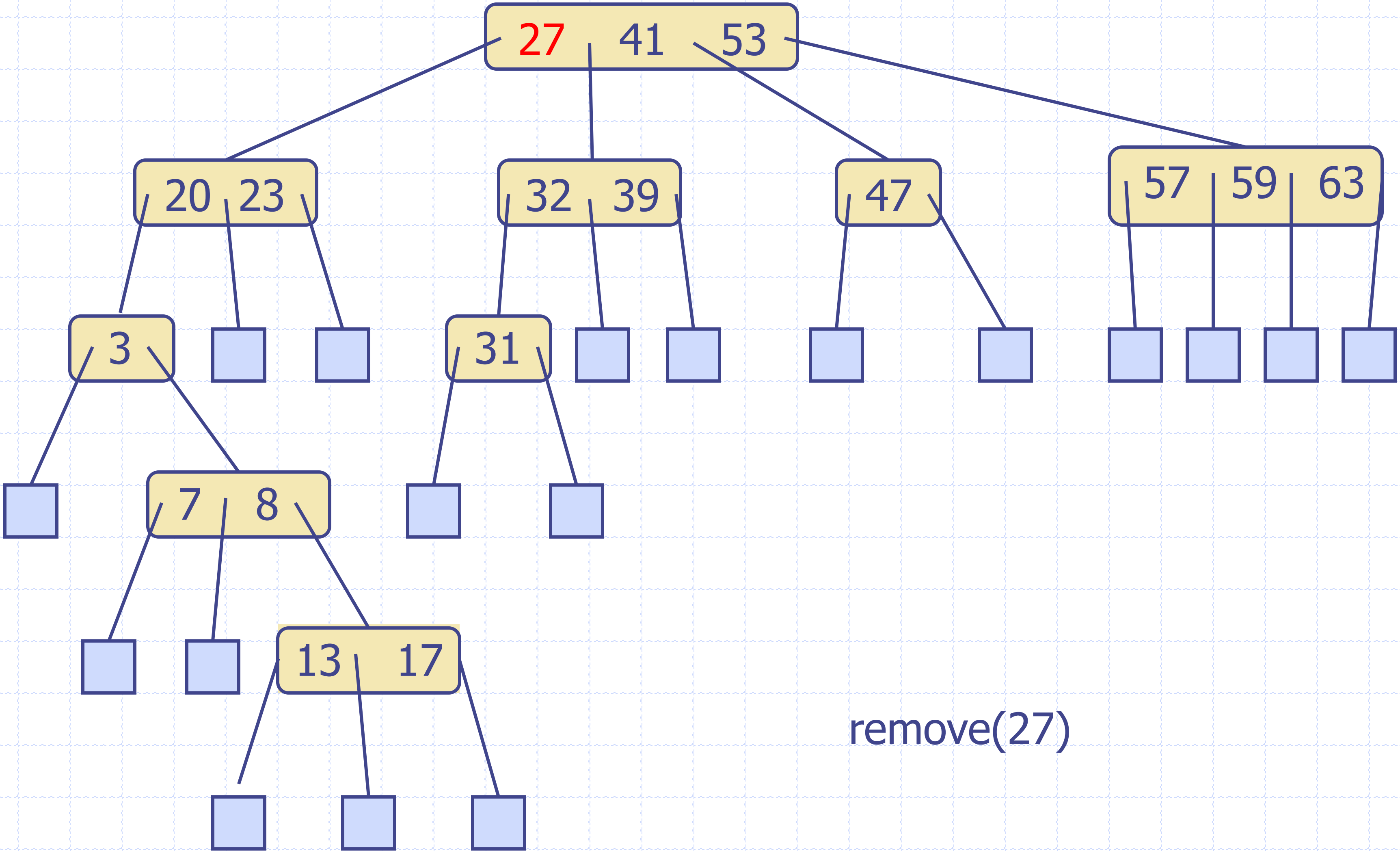
remove(50)

Remove Operation



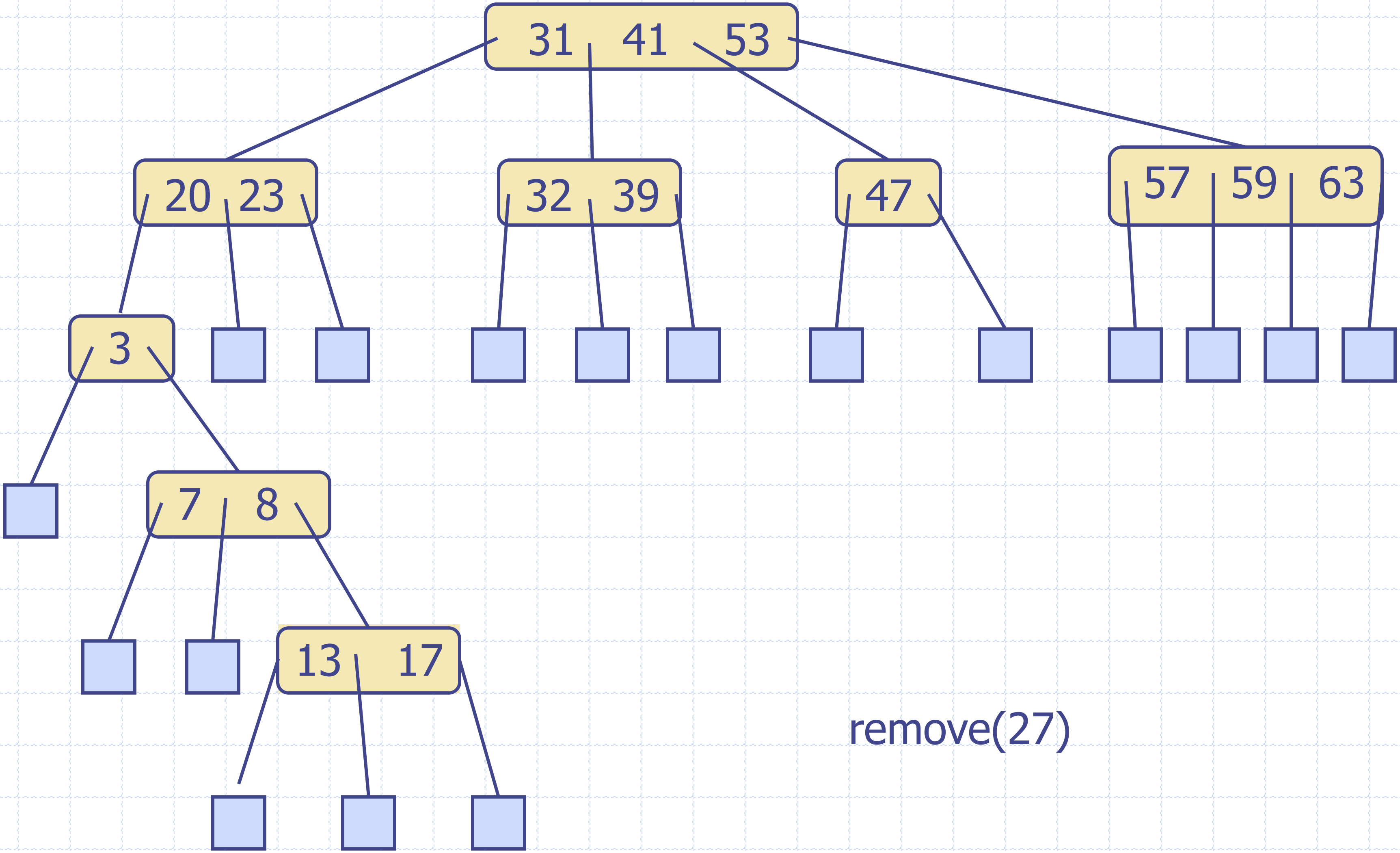
remove(50)

Remove Operation



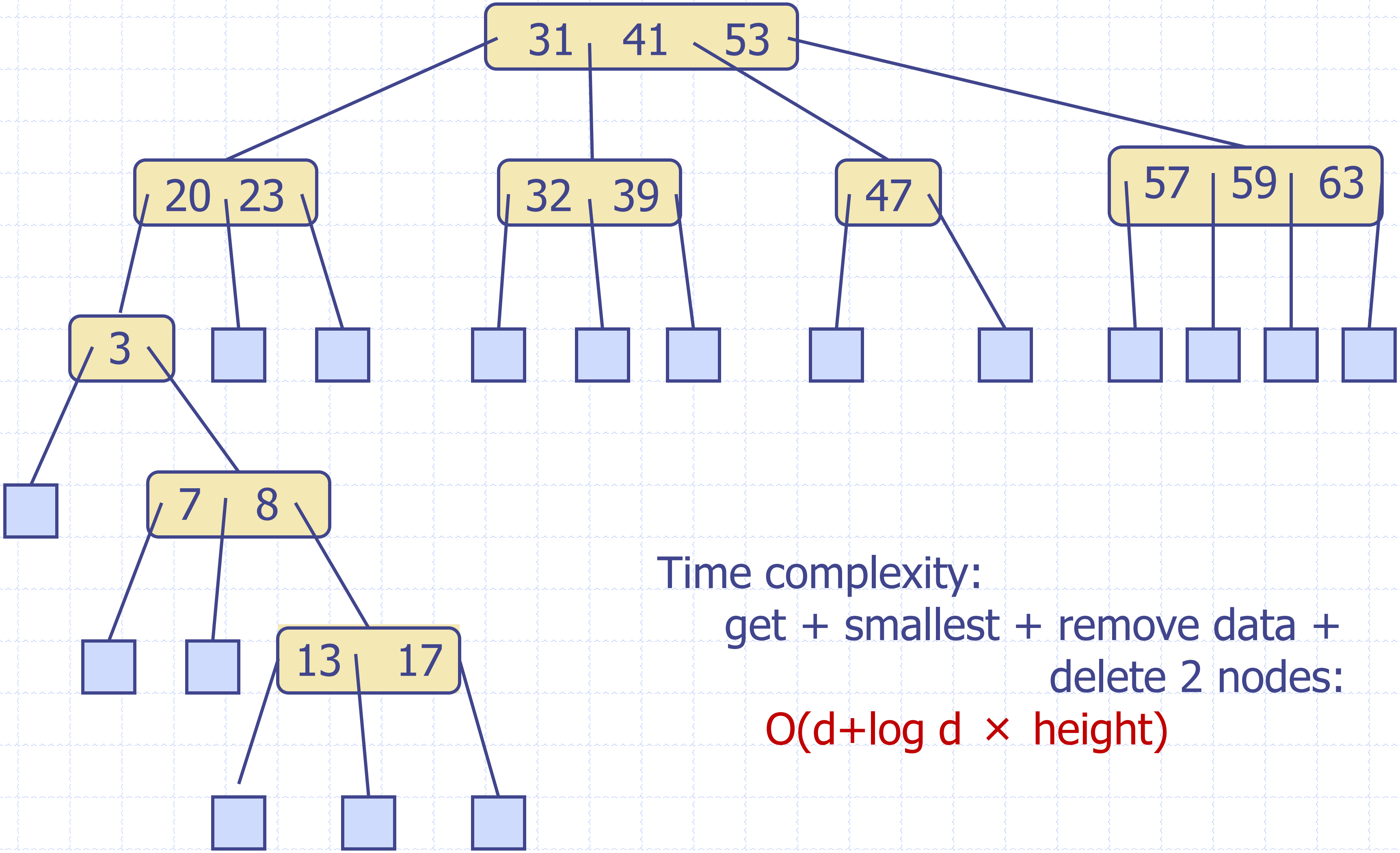
remove(27)

Remove Operation



remove(27)

Remove Operation



Time complexity:
get + smallest + remove data + delete 2 nodes:
O(d+log d × height)