```
Algorithm foo(n)
c1 { i ← 1
     k ← 1
   while i < n do {
c2 {   if i = k then {
           A[i] ← k
           k ← k+1
           i ← 1
       }
       i ← i+1
   }
```

Let us count the number of iterations performed by the while loop:

	# iterations
$k=1,\ i=\underline{1}$	1
$k=2,\ i=1\rightarrow\underline{2}$	1
$k=3,\ i=1\rightarrow\underline{2}\rightarrow\underline{3}$	2
$k=4,\ i=1\rightarrow\underline{2}\rightarrow\underline{3}\rightarrow\underline{4}$	3
$k=5,\ i=1\rightarrow\underline{2}\rightarrow\underline{3}\rightarrow\underline{4}\rightarrow\underline{5}$	4
$\vdots$	
$k=n-1,\ i=1\rightarrow\underline{2}\rightarrow\underline{3}\rightarrow\underline{4}\rightarrow\cdots\rightarrow\underline{n-1}$	$n-2$
$k=n,\ i=1\rightarrow\underline{2}\rightarrow\underline{3}\rightarrow\underline{4}\rightarrow\cdots\rightarrow\underline{n-1}\rightarrow n$	$n-2$

$\text{total} = 1+1+2+3+\cdots+n-2+n-2$

$= 1+2+3+\cdots+n-2+n-1$

Each underlined number means one iteration of the while loop where the value of k is compared with the underlined value

The total number of iterations is $1+2+3+\cdots+n-2+n-1 = \underbrace{\sum_{j=1}^{n-1} j}_{\text{Arithmetic sum}} = \frac{n(n-1)}{2} = \frac{n^2}{2} - \frac{n}{2}$

The time complexity of the algorithm is then $c_1 + c_2 \times \#\text{iterations} = c_1 - \frac{c_2}{2} n + \frac{c_2}{2} n^2$ is $O(n^2)$

$Kf(n)$ is $O(f(n))$ for any constant $K>0$:

Find constants $c>0$, $n_0 \geq 1$ such that

$$Kf(n) \leq \underline{\underline{c}}\, f(n) \quad \forall n \geq \underline{\underline{n_0}}$$

$$0 \leq cf(n) - Kf(n) = (c-K)f(n), \quad \forall n \geq \underline{\underline{n_0}}$$

$\boxed{c=K}$: $\quad 0 \leq 0 f(n) = 0, \quad \forall n \geq \boxed{n_0 = 1}$

$f(n) + g(n)$ is $O(\max\{f(n), g(n)\})$:

Find constants $c > 0$, $n_0 \geq 1$ such that

$$f(n) + g(n) \leq c \cdot \max\{f(n), g(n)\}, \quad \forall n \geq n_0$$

$$f(n) \leq \max\{f(n), g(n)\}, \quad \forall n \geq 1$$

$$+ \quad g(n) \leq \max\{f(n), g(n)\}, \quad \forall n \geq 1$$

$$f(n) + g(n) \leq 2 \max\{f(n), g(n)\}, \quad \forall n \geq 1$$

$c = 2$, $n_0 = 1$