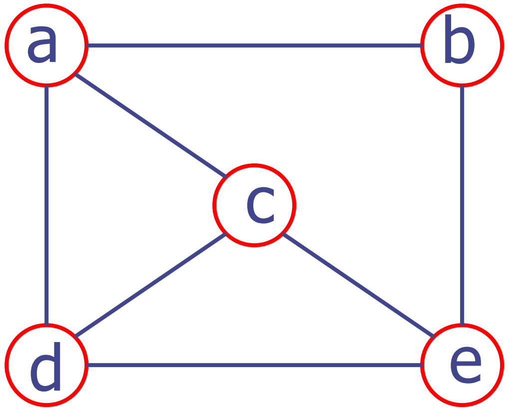
Graphs

A graph is a pair (V, E), where

- V is a set of nodes or vertices
- E is a collection of pairs of vertices (u,v), called edges, links, or arcs



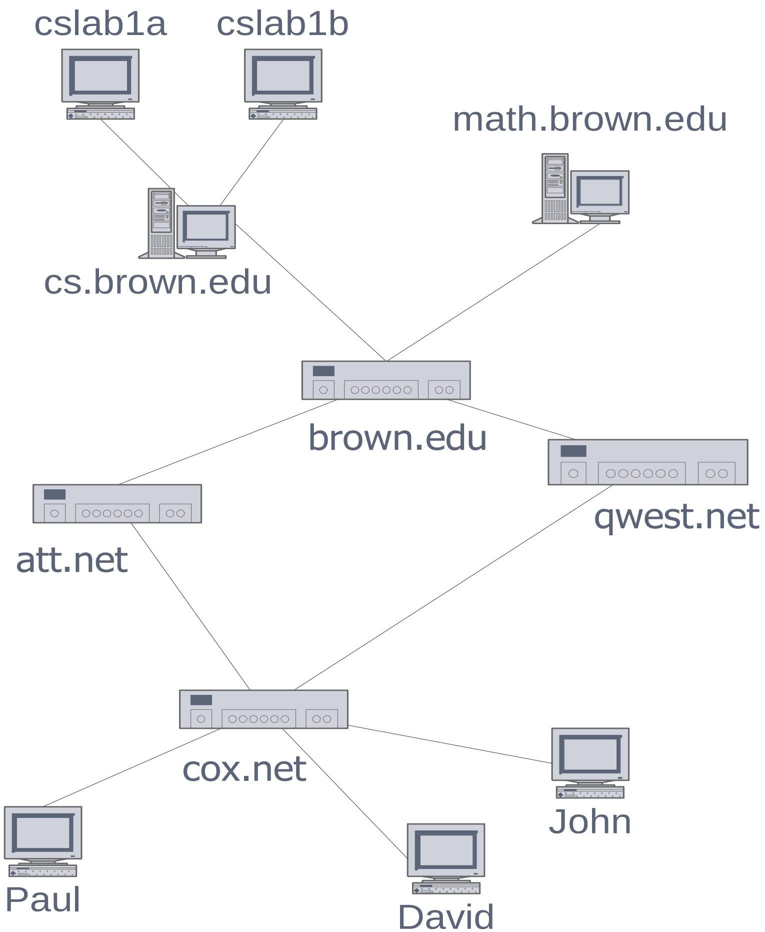
V = {a,b,c,d,e}
E = {(a,b),(a,c),(a,d),
(b,e),(c,d),(c,e),(d,e)}

Edge Types

- Directed edge
  - ordered pair of vertices (u,v)
  - first vertex u is the origin
  - second vertex v is the destination
- Undirected edge
  - unordered pair of vertices (u,v)
- Directed graph or digraph
  - all the edges are directed
- Undirected graph
  - all the edges are undirected
- Mixed graph
  - directed and undirected edges

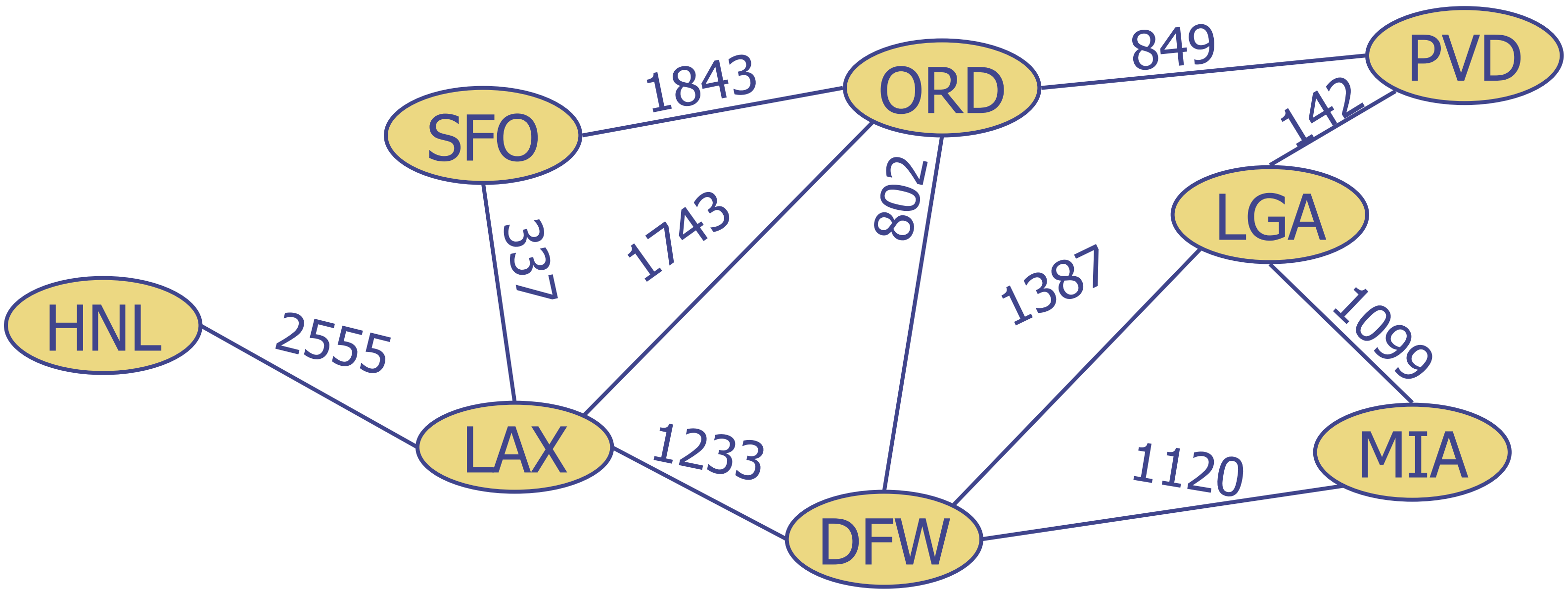
Applications

- Computer networks



Applications

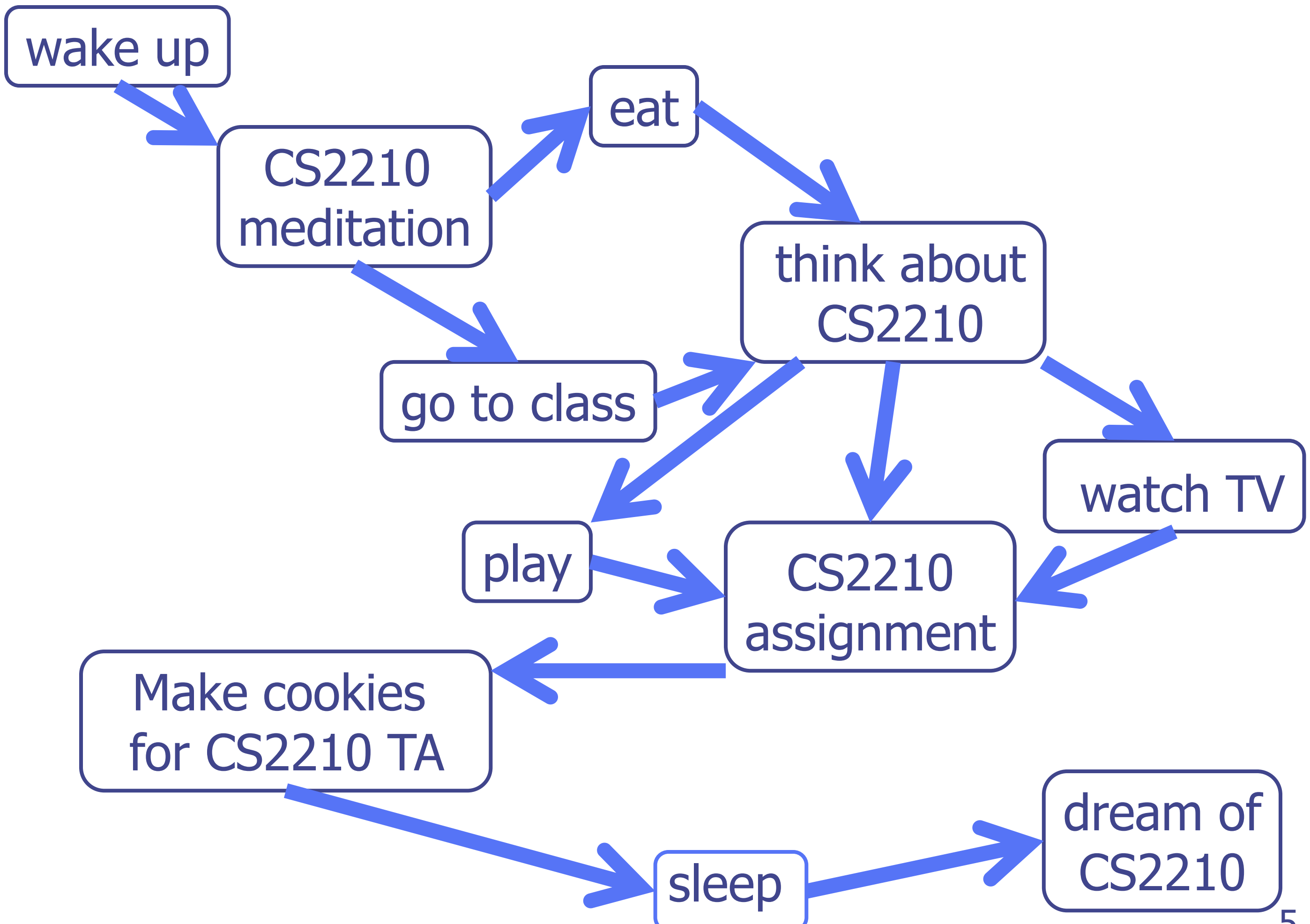
- Transportation networks



Applications

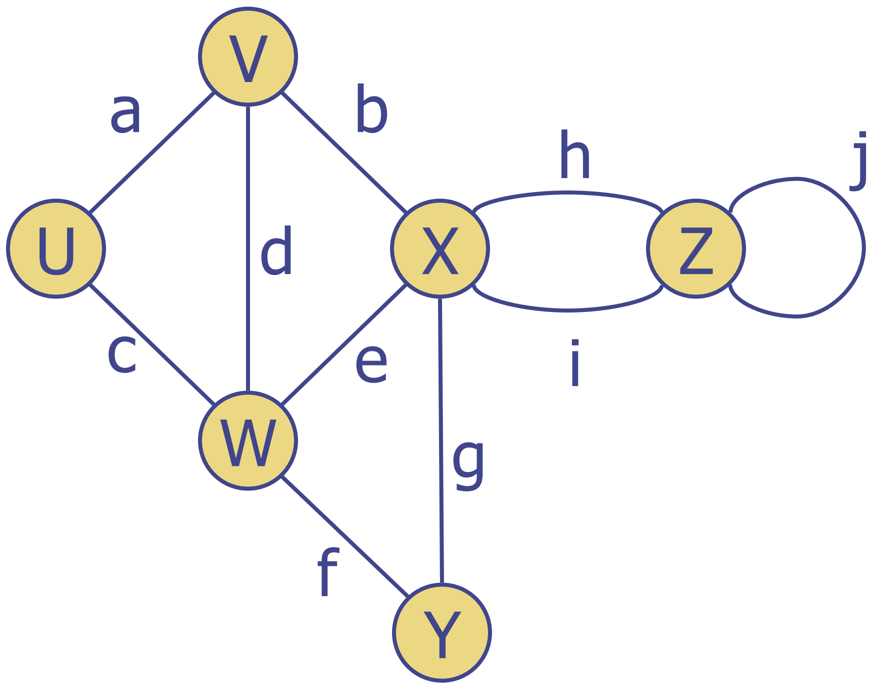
- Scheduling tasks

A typical student day



Terminology

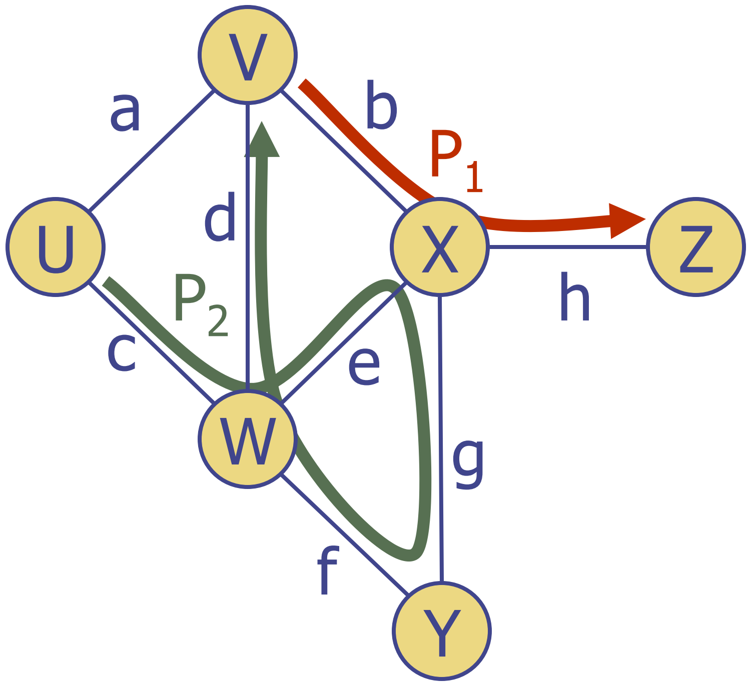
- End vertices (or endpoints) of an edge
  - U and V are the endpoints of a
- Edges incident on a vertex
  - a, d, and b are incident on V
- Adjacent vertices
  - U and V are adjacent
- Degree of a vertex
  - X has degree 5
- Parallel edges
  - h and i are parallel edges
- Self-loop
  - j is a self-loop



A graph without parallel edges or self-loops is called a simple graph

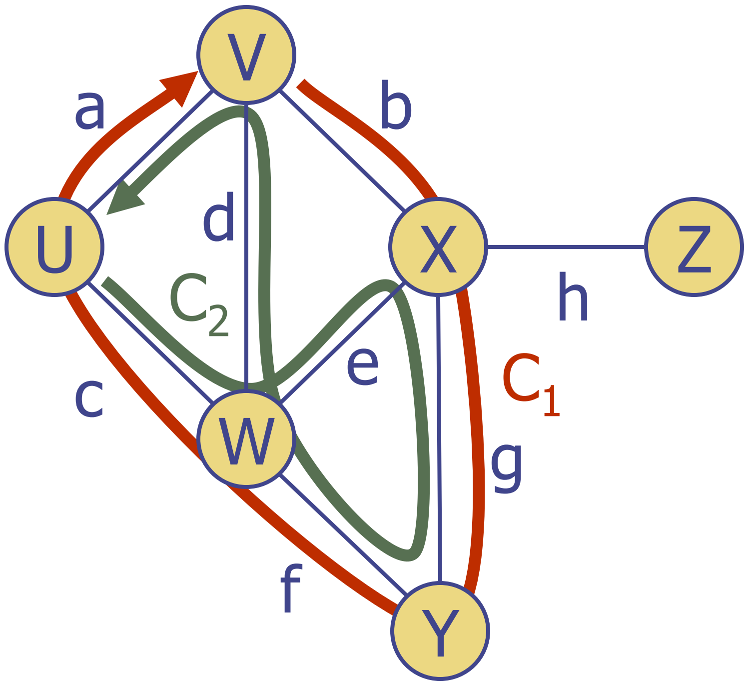
Terminology

- Path
  - sequence of adjacent vertices
- Simple path
  - path such that all its vertices and edges are distinct
- Examples
  - P_1=(V,X,Z) is a simple path
  - P_2=(U,W,X,Y,W,V) is a path that is not simple



Terminology

- Cycle
  - circular sequence of adjacent vertices
- Simple cycle
  - cycle such that all its vertices are distinct (except first and last)
- Examples
  - C_1=(V,X,Y,W,U,V) is a simple cycle
  - C_2=(U,W,X,Y,W,V,U) is a cycle that is not simple



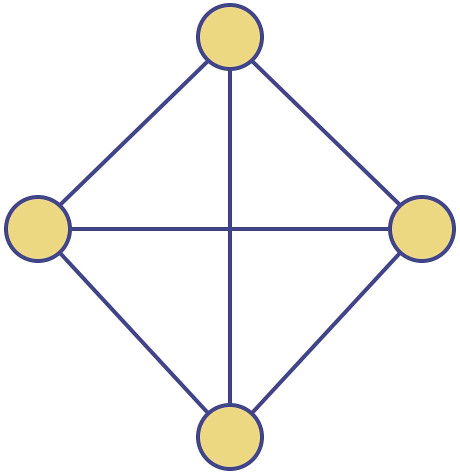
Properties

Notation

n	number of vertices
m	number of edges
$\deg(v)$	degree of vertex v

Property 1

$\Sigma_v \deg(v) = 2m$

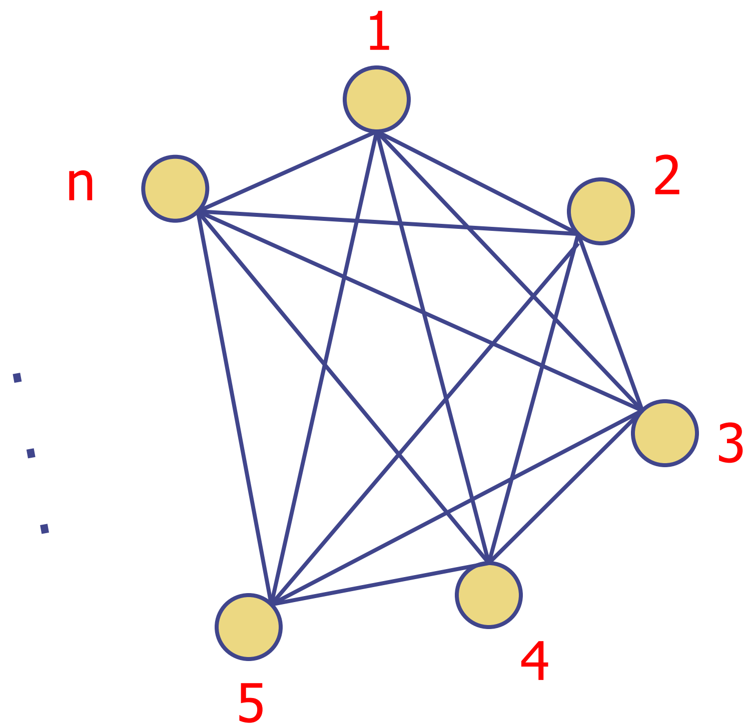


Example

- $n = 4$
- $m = 6$
- $\deg(v) = 3$
- $\Sigma_v \deg(v) = 12$

Complete Graph or Clique

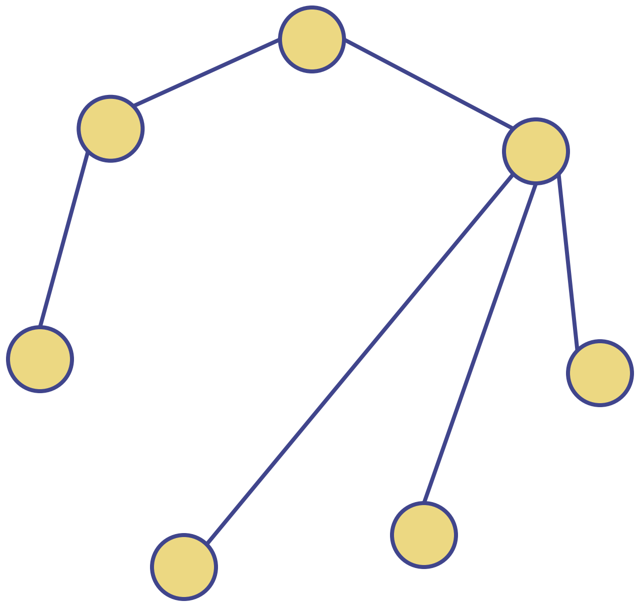
Each vertex is connected to every other vertex.



$m = n(n-1)/2$

Trees

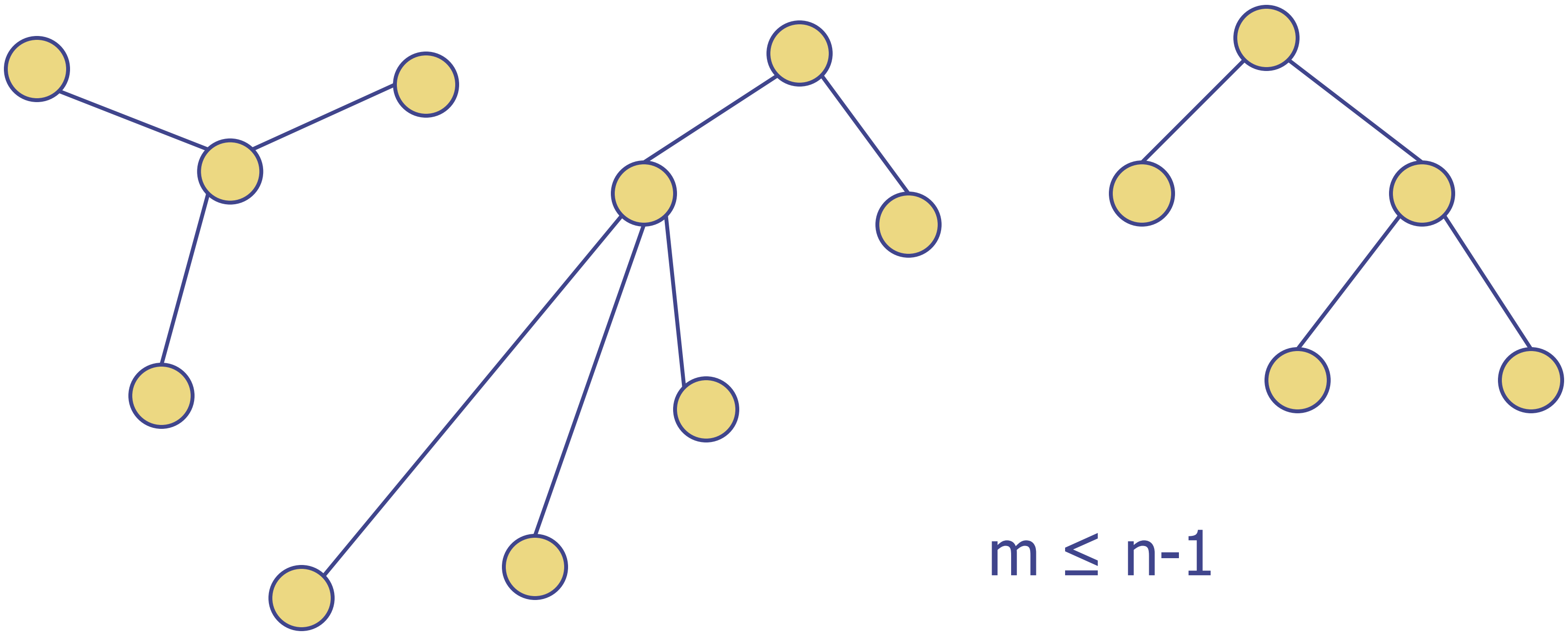
A tree is a graph without cycles.



$m = n-1$

Forest

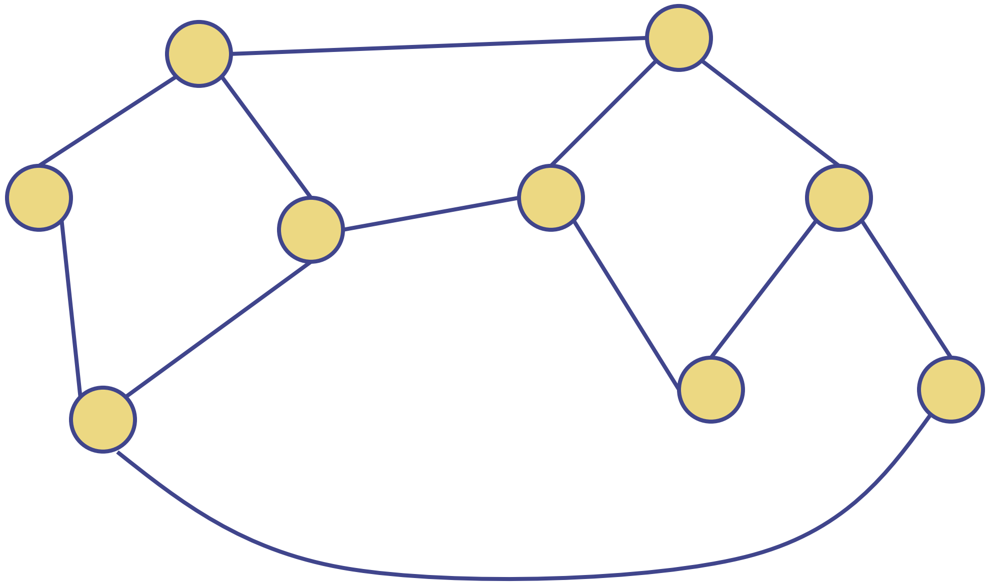
A forest is a set of trees.



$m \leq n-1$

Subgraph

A subgraph is a subset of vertices and edges that forms a graph.



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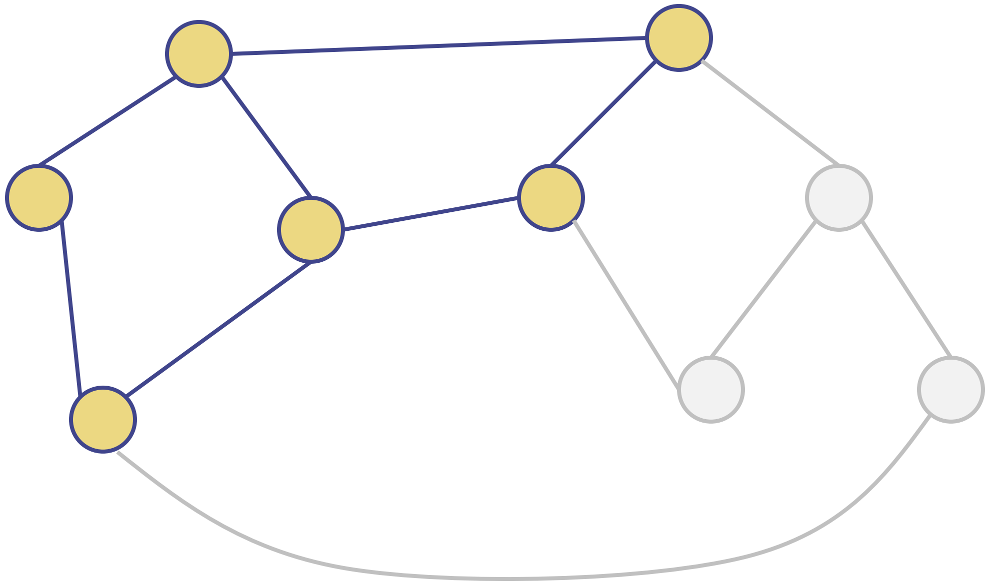
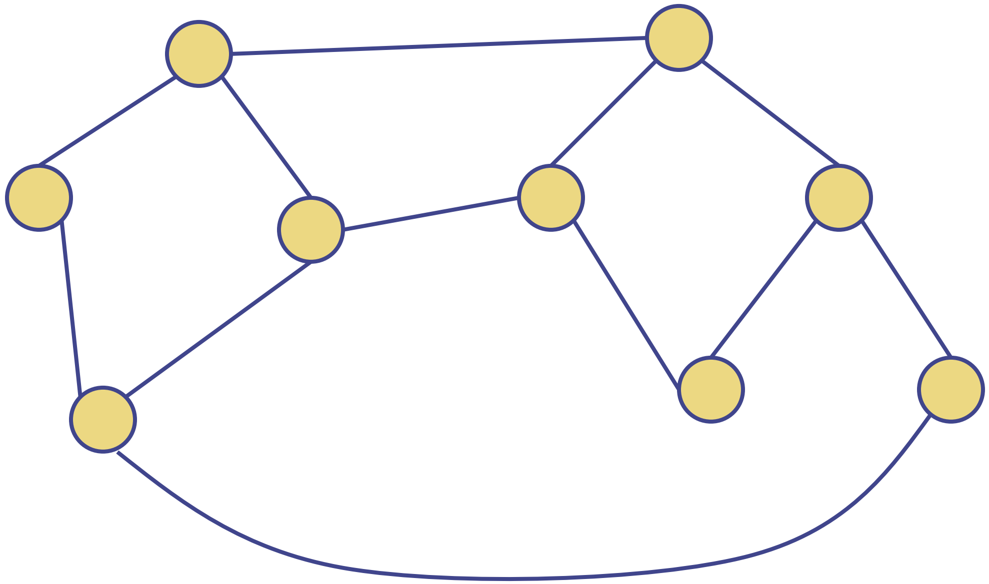


Connected Graph

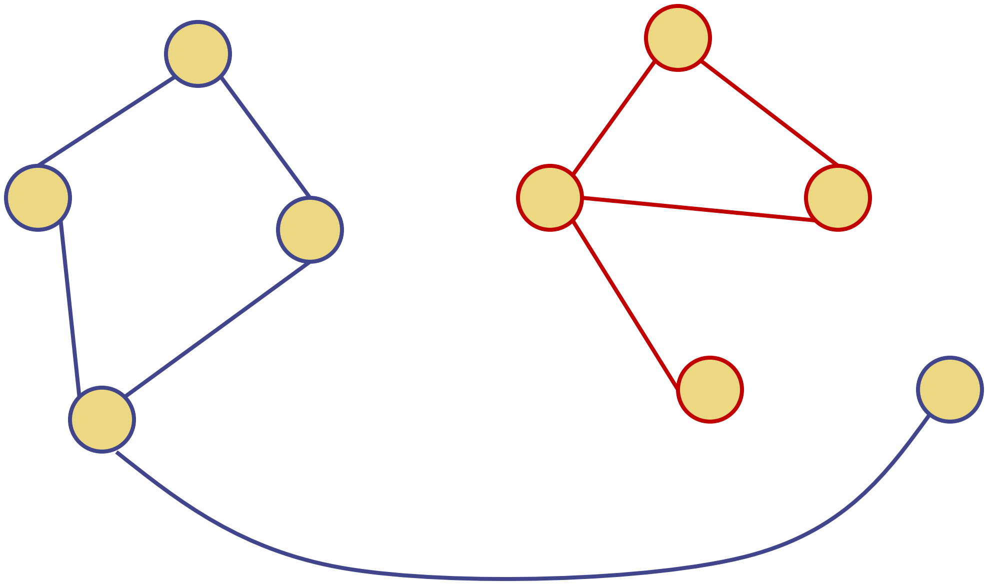
A graph is connected if there is a path from each vertex to every other vertex



Connected Component

A connected component is a maximal connected subgraph.

2 connected components



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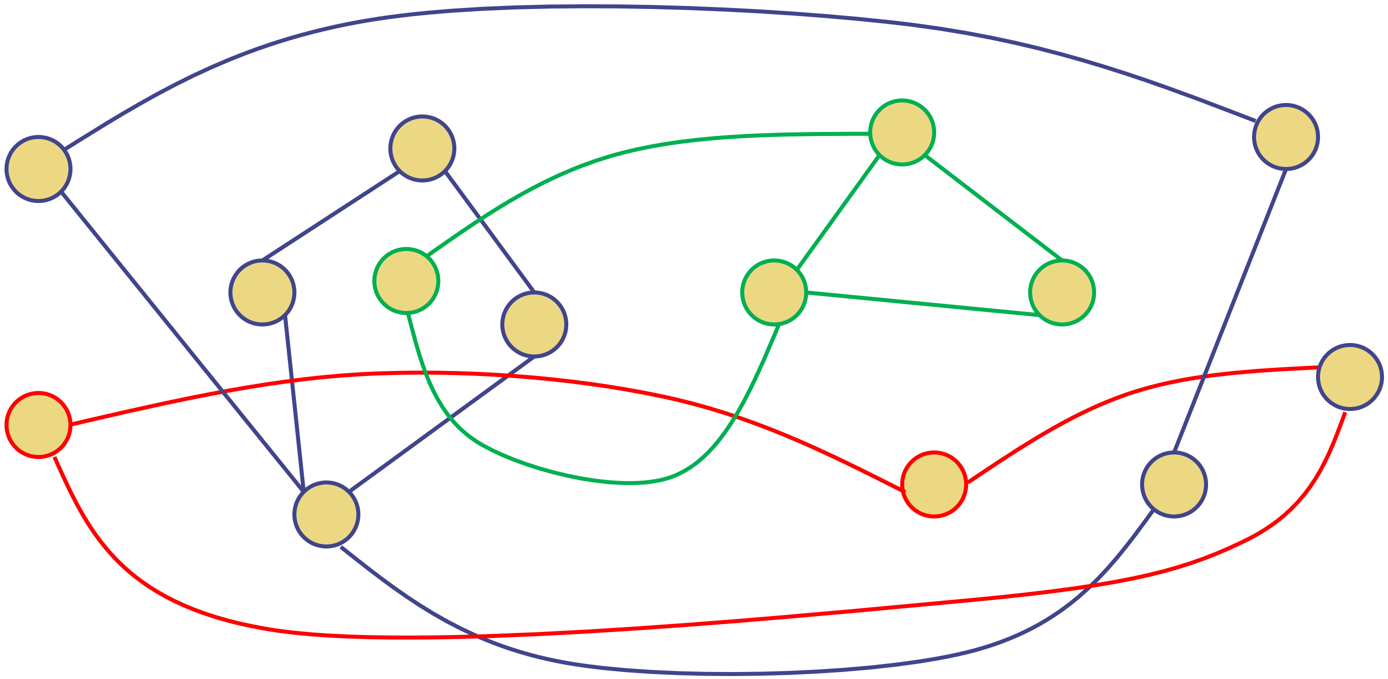


3 connected components

Graph ADT

numVertices(): number of vertices of the graph

getEdge(u,v): returns the edge between vertices u and v

opposite(v,e): returns the vertex other than v that is incident on e

insertVertex(x): creates and returns a new vertex storing value x

insertEdge(u,v,x): creates an edge between u and v soring value x

removeVertex(v): removes vertex v and all edges incident on it

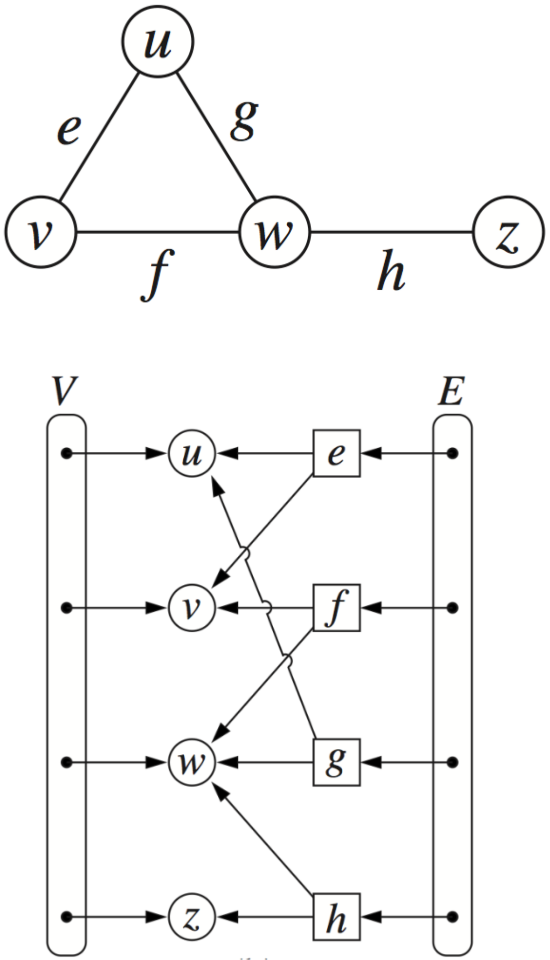
removeEdge(e):   removes edge e

areAdjacent(u,v): returns true is u and v are adjacent; false otherwise

incidentEdges(u): returns an iterator of all edges incident on vertex u.

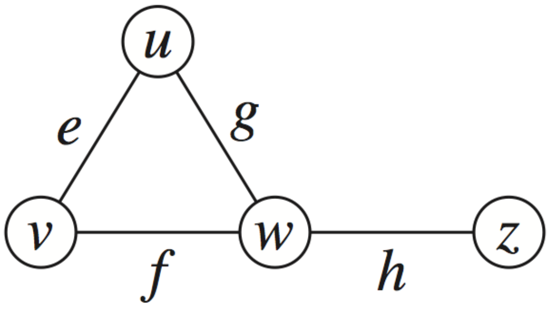
Edge List Structure

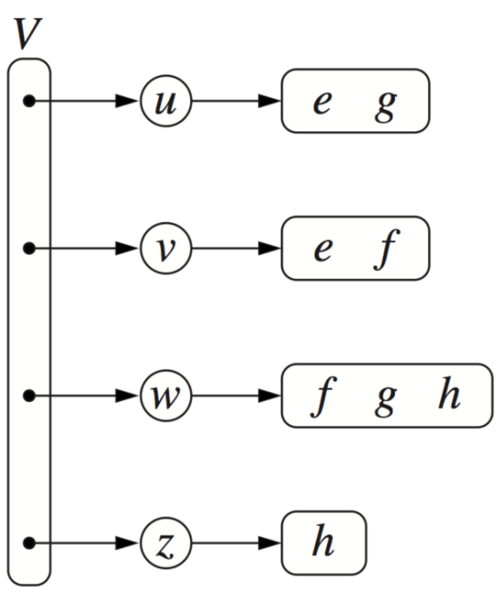
- Vertex object
  - element
- Edge object
  - element
  - origin vertex object
  - destination vertex object
- Vertex sequence
  - sequence of vertex objects
- Edge sequence
  - sequence of edge objects



Adjacency List Structure

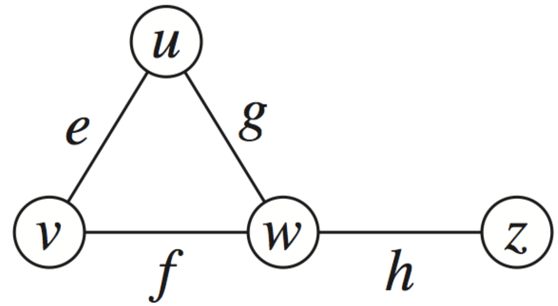
- Incidence sequence for each vertex
  - list of incident edges





Adjacency Matrix Structure

- 2D-array adjacency array
  - Reference to edges for adjacent vertices
  - Null for non nonadjacent vertices
- The “old fashioned” version just has 0 for no edge and 1 for edge



		0	1	2	3
$u \rightarrow$	0		e	g	
$v \rightarrow$	1	e		f	
$w \rightarrow$	2	g	f		h
$z \rightarrow$	3			h	

Performance

▪ n vertices, m edges	Edge List	Adjacency List	Adjacency Matrix
Space	$O(n + m)$	$O(n + m)$	$O(n^2)$
incidentEdges(v)	$O(m)$	$O(\deg(v))$	$O(n)$
areAdjacent (v, w)	$O(m)$	$O(\min\{\deg(v), \deg(w)\})$	$O(1)$
insertVertex(o)	$O(1)$	$O(1)$	$O(n^2)$
insertEdge(v, w, o)	$O(1)$	$O(1)$	$O(1)$
removeVertex(v)	$O(m)$	$O(\deg(v))$	$O(n^2)$
removeEdge(v,w)	$O(m)$	$O(\deg(u)+\deg(v))$	$O(1)$