Tutorial #6

Problem 1 Use structural induction to prove that $e(T)$, the number of edges of a binary tree T, can be computed via formula

$$e(T) \;=\; 2(n(T) - \ell(T))$$

where $n(T)$ is the number of nodes in T and $\ell(T)$ is the number of *leaves*. Note that a *leaf* is a tree node that does not have descendents (children nodes). You can use the following recursive definition for the set of leaves:

Basis step: If a tree has a single node, then it is a *leaf* (as well as a *root*).
Recursive step: The set of leaves of the tree $T = T_1 \cdot T_2$ is the union of the set of leaves of T_1 and T_2.

You can also use the fact that the tree $T = T_1 \cdot T_2$ adds two new edges when connecting T_1 and T_2 to the new common root. Provide detailed justification.

Problem 2 Consider all *genes* (strings with $\Sigma = \{A, T, C, G\}$) of length 10.
 1. How many genes begin with AGT?
 2. How many genes begin with AG and end with TT?
 3. How many genes begin with AG or end with TT?
 4. How many genes have exactly four A's?
 5. How many genes have exactly four A's non-adjacent to each other?
Provide detailed justification for your answers.

Problem 3 A company has 12 software developers (SD).

(a) How many distinct project teams of five SDs can be selected?

(b) Three of the SDs are valuable experts in HPC and the company can not allow them to work on the same project (to maximize the number of projects where such expertice is available). How many ways can a project team of five SDs be selected using this restriction?

(c) Two SDs are close buddies and want to work on the same project. How many ways a project team of five SDs can be selected so that the two buddies are together (both either in the team or not)?

(d) Suppose that 12 SDs are six men and six women. How many distinct teams of five SDs can be selected with three women and two men? How many distinct teams of five SDs have at least one men?

(e) Suppose that 12 SDs include three teenagers, three people in their 20s, three people in their 30s, and three people that are at least 40 years old. How many distinct teams of five SDs contain at least one representative from each of the age group mentioned?

Provide detailed justification for your answers.

Problem 4