Discrete Structures for Computing

CS2214A, winter 2018

Final exam

April 23, 2:00-5:00pm

100 points max

STUDENT NUMBER: ____

STUDENT NAME: ____

NOTE: students can use one cheat-sheet (letter-size page, both sides) without any worked out examples. The cheat-sheets will be collected with the exam. Students can use basic calculators, but no signal emiting electronic devices (phones, laptops, tablets, etc) are allowed.

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10

Problem 1. (10 points) Let $S(x, y)$ be the statement "x sends DNS requests to y" where the domain are all computers at the Computer Science department. Use quantifiers to express each of the statements:

(a) Orion sends DNS requests to all other computers.

(b) All computers send DNS requests to Hal.

(c) No computer sends DNS requests to Bonnie.

(d) Some computer sends DNS request outside the department's network.

(e) Both C-3PO and R2-D2 can send DNS requests to any other computer.

Problem 2. (10 points) Prove that if n is a positive integer, then n is odd if and only if $3n + 7$ is even.

Problem 3. (10 points) Prove that

(a) If $n = a + b$, where a and b are positive integers, then $a \leq \frac{n}{2}$ or $b \leq \frac{n}{2}$.

(b) $|x| + |y| \geq |x + y|$ for all real numbers x and y. Here, $|x|$ denotes the absolute value of x, that is, $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$.

Problem 4. (10 points) Use mathematical induction to prove that for every positive integer n the following equality holds

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) \quad = \quad n(n+1)(n+2)/3$$

Problem 5. (10 points) A message was encrypted using the affine cipher

$$c = (17p + 22) \textbf{ mod } 26$$

Find the decryption function and use it to decrypt the cryptotext message "VAJM". Use the method presented in class, including the Euclidean algorithm to find an inverse with respect to modular multiplication.

Problem 6. (10 points) The English alphabet contains 21 consonants and 5 vowels. How many strings of 7 lowercase letters of the English alphabet (repeats allowed) contain:

(a) exactly one consonant

(b) exactly two vowels

(c) at least one consonant

(d) at least one vowel and at least one consonant

Problem 7. (10 points) Assume that a string of length 9 is a random permutation of letters $\{a, b, c, d, e, f, g, h, i\}$. Answer the following questions (justify).

(a) What is the probability of event A: letter g comes before a ?

(b) What is the probability of event B: letter g comes after b and d ?

(c) What is the probability of event C: letters e and f are before b and d ?

(d) Choose an arbitrary pair of events from $\{A, B, C\}$ above and determine if these two are independent or not.

Problem 8. (10 points) Identify the type of relation (e.g. "equivalence" or any other discussed in class). If you can not recognize the type, state "NR". Justify your answer.

(a) $R_A = \{(x, y) \mid x \equiv y \ (mod\ 4)\}$ on the set of positive integers Z^+

(b) $R_B = \{(x, y) \mid x = 2^y\}$ on the set of positive integers Z^+

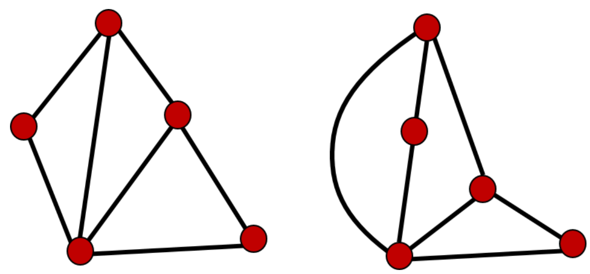
(c) $R_C = \{(x, y) \mid \exists k \in Z^+ : y = kx\}$ on the set of positive integers Z^+

(d) $R_D = \{((x, y), (s, t)) \mid x^2 + y^2 \leq s^2 + t^2\}$ on Z^2 (ordered pairs of integers)

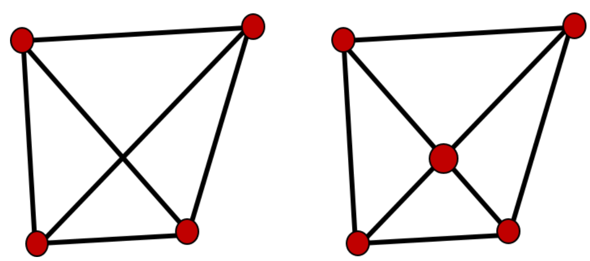
(e) $R_E = \{(\{x, y\}, \{s, t\}) \mid x + y = s + t\}$ on unordered pairs of integers

Problem 9. (10 points) Specify if the following pairs of graphs are isomorphic. Justify your answer.

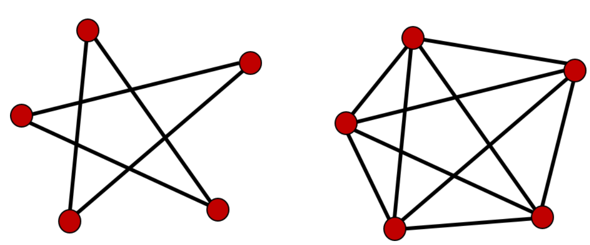
(a) Pair 1



(b) Pair 2

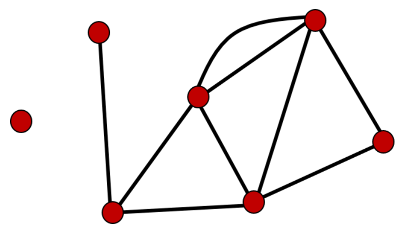


(c) Pair 3

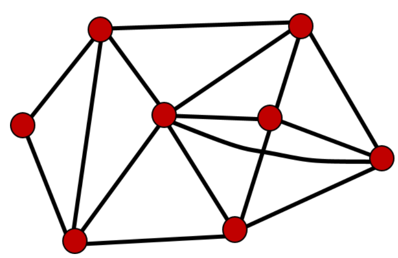


Problem 10. (10 points) Specify if the following graphs have Euler circuit, Euler path, or neither. Justify your answer.

(a) Graph 1



(b) Graph 2



(c) Graph 3