UNIVERSITY OF WESTERN ONTARIO

Computer Science 2214, 2014-15

Discrete Structures for Computing

MAKE-UP FINAL EXAM

The use of a reference sheet is allowed that is letter-size, two-sided, handwritten by student, with neither flaps nor fully worked-out proofs/examples
No electronic devices of any kind, including cell phones, are allowed
This exam has 10 equally-weighted questions

Name _____
ID _____

(1) Let p, q, and r be the propositions

p: You get an A on the final exam.

q: You do every exercise in the textbook.

r: You get an A in this class.

Write the following sentences in the language of propositional logic using p, q, r as propositional variables, as well as logical connectives.

(a) You get an A in this class, but you do not do every exercise in the textbook.

(b) You get an A on the final, you do every exercise in the textbook, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) Getting an A on the final and doing every exercise in the textbook is sufficient for getting an A in this class.

(e) You will get an A in this class if and only if you either do every exercise in the textbook or you get an A on the final.

(2) Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck", "x is one of my poultry", "x is an officer", and "x is willing to waltz", respectively. Express each of the following statements using quantifiers, logical connectives and $P(x), Q(x), R(x)$ and $S(x)$. The universe of discourse is the set of all living beings.

(a) No ducks are willing to waltz.

(b) No officers ever decline to waltz.

(c) All my poultry are ducks.

(d) My poultry are not officers.

(e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

(3) *(i)* Prove that if $n = a \cdot b$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

(ii) Prove that $|x| \cdot |y| = |x \cdot y|$, for all real numbers x and y, where $|x|$ denotes the absolute value of x ($|x|$ equals x if $x \geq 0$ and equals $-x$ if $x < 0$.)

(4) A single line divides the plane into two regions. Two lines (by crossing) can divide the plane into four regions; three lines can divide it into seven regions, etc.. Let $P(n)$ be the maximum number of regions into which n lines divide a plane, where n is a positive integer.

(a) Derive a recurrence relation for $P(k)$ in terms of $P(k-1)$ for all integers $k \geq 2$.

(b) Use iteration to find an explicit (closed) formula for $P(n)$.

(c) Into how many regions can 100 lines divide the plane?

(5) A plaintext message was encrypted using the affine cipher

$$f(x) = (17x + 22) \bmod 26.$$

Find the decryption function and use it to decrypt the cryptotext message "QHAR". Use the method presented in class, including the Euclidean algorithm to find an inverse with respect to modular multiplication.

(6) Use mathematical induction to prove that, for every positive integer n, the following equality holds:

$$1 + 4 + 7 + 10 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$$

(7) The English alphabet contains 21 consonants and 5 vowels. How many strings of six lowercase letters of the English alphabet (repeats allowed) contain:

(a) exactly one vowel?
(b) exactly two vowels?
(c) at least one vowel?
(d) at least two vowels?

Provide detailed justifications of your answers.

(8) In a roulette, a wheel with 38 numbers is spun. Of these, 18 are red, and 18 are black. The other two numbers, which are neither black nor red, are 0 and 00. The probability that when the wheel is spun it lands on any particular number is 1/38.

(a) What is the probability that the wheel lands on a red number?

(b) What is the probability that the wheel lands on a black number twice in a row?

(c) What is the probability that the wheel lands on 0 or 00?

(d) What is the probability that in five spins the wheel never lands on either 0 or 00?

Provide detailed justifications of your answers.

(9) Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time (this is a "false positive" test result).

(a) What is the probability that a randomly selected bicylist who tests positive for steroids actually uses steroids?

(b) What is the probability that a randomly selected bicyclist who tests negative for steroids did not use steroids?

(10) For *each* of the following two graphs:

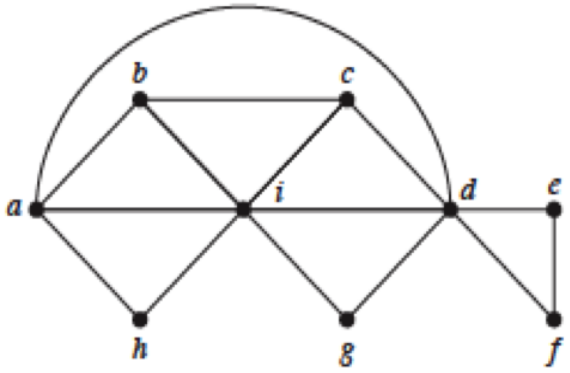
(a) Determine whether or not it has an Euler circuit. Justify your answer.

(b) If the graph has an Euler circuit, use the algorithm described in class to find it, including drawings of any intermediate subgraphs.

(c) If no Euler circuit exists, determine whether the graph has an Euler path. Justify your answer.

(d) Construct an Euler path if one exists.

(I)



(II)