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CS 2214 Assignment #1
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Problem 1: Understanding Implication

1) $(p \wedge q) \Longleftrightarrow (\neg q \wedge \neg p)$

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

p	q	¬q ∧ ¬ p
T	T	F
T	F	F
F	T	F
F	F	T

These tautologies are not true because their truth values in the last column are not the same. This is due to the fact that $(\neg q \wedge \neg p)$ is always **false**. There cannot be any value for p or q (true or false) that will make the contrapositive of it true. In fact, they are opposites of each other for the truth values given. Therefore, these tautologies are not true.

1) $(p \vee q) \Longleftrightarrow (\neg q \vee \neg p)$

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

p	q	¬q v ¬ p
T	T	F
T	F	T
F	T	T
F	F	T

In the expression $(p \vee q)$, this is always true if either p or q is true. However, in the last column it is false because both values are false. The contrapositive of this expression is $(\neg q \vee \neg p)$. This is the opposite of (p v q) because it is true for all values except where p or q are false.

2) **(p ↔ q) ⟸⟹ (p ^ q) ^ (q ^ p)**

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

p	q	(p ^ q) ^ (q ^ p)
T	T	T
T	F	F
F	T	F
F	F	F

2) The tautology above is not true because their truth tables in the last column do not match. Therefore they cannot be equal to each other. This is because the second expression is altering the statement to AND so that, in order to reach a true value, both sides of the AND operator MUST be true. The only way to make this happen is if both p and q are true, which only occurs once.

2) **(p ↔ q) ⟸⟹ (p V q) ^ (q V p)**

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

p	q	(p V q) ^ (q V p)
T	T	T
T	F	T
F	T	T
F	F	F

These expressions are also not true anymore after changing the conditional to OR because they produce different values in the final column. This means that they are not true in terms of equality. The OR condition forces the expression (p v q) ^ (q v p) to be true if either of p or q are true, then the entire expression is automatically true. Therefore it is only false for which p and q both equal false. However, the biconditional expression requires ((p → q) ∧ (q → p)). Therefore, this condition cannot be true for p = true while q = false, and p = false while q = true. Therefore, they produce different truth values in the last column and ultimately cannot be a true tautology.

Problem 2: Proving Theorems

$\forall x \forall y \exists z\ (((x \% 2 = 0) \wedge (y \% 2 = 0)) \rightarrow ((x + y) = 2 * z))$

Let's take x = 4 and y = 2.

4 % 2 = 0 and 2 % 2 = 0. Therefore, both numbers are proven to be even. Next, if x and y are both even, then there exists a third integer which, when doubled, is equal to the sum of the first two integers (it can be odd or even). So, let us check to see mathematically if this works out.

$x + y = 2z$
$4 + 2 = 2z$
$6 = 2z$
$6/2 = 2z/2$
$z = 3$

Therefore, there exists a third integer z (even or odd) which is equal to the sum of the first two when doubled. This statement is true and has been proven.

$\forall x \forall y \exists z\ ((\neg(x \% 2 = 0) \wedge \neg(y \% 2 = 0)) \rightarrow ((x + y) = 3 * z))$

Let's take x = 3 and y = 5. x % 2 != 0. This is false. The ¬ operator forces this expression to be true. Similarly to y: 5 % 2 != 0. This is also false. The ¬ operator forces this expression to be true as well. Therefore we have T ^ T so the conditional operator executes the second part which is:

$x + y = 3z$
$3 + 5 = 3z$
$8 = 3z$
$Z = 8/3$

The first part of the conditional is true. However, the second part of the expression is false which creates a T → F implication, which is always false. The reason is because, as shown in the mathematical proof above, there can be no integer which, when tripled, will equal the sum of two odd integers.

Problem 3: Finding a Treasure

SB = There is an old shipwreck near the beach
TC = The treasure buried under a coconut palm tree
CI = There is a coconut palm tree growing at the far end of the island
CC = There is a coconut palm tree growing near the cave
TH = The treasure is hidden in a cave

Premises:

1. $SB \rightarrow TC$	
2. $CI \vee CC$	
3. $SB \vee TH$	
4. $CI \rightarrow \neg SB$	
5. $\neg CC$	
6. $CI \vee \perp$	lines 2, 5, contradiction.
7. CI	V-elimination (from rule (P v $\perp$) = p).
8. $\neg SB$	Modus Ponens, lines 4, 7
9. $\perp \vee TH$	Lines 3, 8, contradiction
10. TH	V-elimination (from rule (P v $\perp$) = p).

Therefore, the treasure is hidden in a cave.

Problem 4: Deciding Consistency

This set of propositions is consistent. For each, they can be translated to propositional variables. Let p denote “The system is in multi user state”, q = “The system is operating normally”, r = “The kernel is functioning”, s = “The system is in interrupt mode”. Now we have the following sentences:

1) $p \Leftrightarrow q$
2) $q \rightarrow r$
3) $\neg r \vee s$
4) $\neg p \rightarrow s$
5) s

In order to make the above propositions true, there must be a value for each variable which will make it consistent. Instead of creating a long truth table, we can prove this by reasoning with cases and understand that if we assign each variable a TRUE value, each sentence will be true. The first one is a biconditional which can be read as $(p \rightarrow q) \wedge (q \rightarrow p)$. If both p and q are true, then this expression is always true based on the truth table for the implication operator. For 2, the same applies: if q and r are both true, then the statement is always true. For 3, because of the OR operator, we only have to prove one of the two to be true in order for the expression to be true. In this case, it is the variable s which is true (r is always true but is negated to be false, which doesn't matter). For 4, $\neg p$ results in a false because we initially want p to be true, however we know that anytime the implication starts with false, the expression is always true. Lastly, for 5, s is taken to be true so this expression is true as well.

Therefore, the set of expressions is consistent because they are all true if we take each variable p, q, r and s to equal true.

Problem 5: Proving Theorems

Question 1)

p	**q**	**r**	$(q \vee \neg p)$ **1*	$p \wedge (q \vee \neg p)$ **2*	$(\neg q \vee \neg r)$ **3*	$p \wedge (q \vee \neg p) \wedge (\neg q \vee \neg r)$ **2 ^ 3*
T	**T**	**T**	**T**	**T**	**F**	**F**
T	**T**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**F**	**F**	**T**	**F**
T	**F**	**F**	**F**	**F**	**T**	**F**
F	**T**	**T**	**T**	**F**	**F**	**F**
F	**T**	**F**	**T**	**F**	**T**	**F**
F	**F**	**T**	**T**	**F**	**T**	**F**
F	**F**	**F**	**T**	**F**	**T**	**F**

Therefore, this is satisfiable because there is at least one combination of values which makes the expression true in the last column.

Question 2)

p	q	(q ∨ ¬p) *1	p ∧ (q ∨ ¬p) *2	(¬q ∨ ¬p) *3	p ∧ (q ∨ ¬p) ∧ (¬q ∨ ¬p) *2 ^ 3
T	**T**	**T**	**T**	**F**	**F**
T	**F**	**F**	**F**	**T**	**F**
F	**T**	**T**	**F**	**T**	**F**
F	**F**	**T**	**F**	**T**	**F**

This is not satisfiable because all of the truth values in the last column lead to false and there is not a single combination which is true. Therefore, it is a contradiction.