Assignment #1
Due: Feb. 12, 2017, by 23:55
Submission: on the OWL web site of the course

Format of the submission. You must submit a **single** file which must be in **PDF** format. All other formats (text or Miscrosoft word format) will be **ignored** and considered as **null**. You are strongly encouraged to type your solutions using a text editor. To this end, we suggest the following options:

1. Miscrosoft word and convert your document to PDF
2. the typesetting system LaTeX; see `https://www.latex-project.org/` and `https://en.wikipedia.org/wiki/LaTeX#Example` to learn about LaTeX; see `https://www.tug.org/begin.html` to get started
3. using a software tool for typing mathematical symbols, for instance `http://math.typeit.org/`
4. using a Handwriting recognition system such as those equipping tablet PCs

Hand-writing and scanning your answers is allowed but not encouraged:

1. if you go this route please use a scanning printer and **do not take a picture of your answers with your phone**,
2. if the quality of the obtained PDF is too poor, your submission will be **ignored** and considered as **null**.

Problem 1 (Proving properties about the integers) *[15 marks]* Prove or disprove the following properties:

1. For every integer n we have $n \leq n^2$.
2. For every integer n, the integer $n^2 + n + 1$ is odd.

Problem 2 (Proving properties about real numbers) *[15 marks]* Prove or disprove the following properties:

1. For every real number x, if $x \leq 0$ or $1 \leq x$ holds, then $x \leq x^2$ holds as well.
2. For all real number x we have $\lfloor 2x \rfloor = 2\lfloor x \rfloor$

Problem 3 (Properties of preimage sets) *[20 marks]* Let f be a function from a set A to a set B. Let S and T be two subsets of B. Prove the following properties:

1. $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
2. $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

Problem 4 (Properties of functions) *[30 marks]* Which of the functions below is injective? surjective? When the function is bijective, determine its inverse

1. $f_1 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ n & \longmapsto & 2019n + 1 \end{array}$

2. $f_2 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ n & \longmapsto & \lfloor n/2 \rfloor + \lceil n/2 \rceil \end{array}$

3. $f_3 : \begin{array}{ccc} [1,2) & \to & [0,1) \\ x & \longmapsto & x - \lfloor x \rfloor \end{array}$

4. $f_4 : \begin{array}{ccc} [1,2) & \to & [0,1) \\ x & \longmapsto & (f_3(x))^2 \end{array}$

Problem 5 (Properties of functions) *[20 marks]* Let f be a surjective function from a set A to a set B and a g be a function from B to a set C. Prove or disprove the following properties:

1. if g is surjective then so is gof.
2. if f and g are both injective, then so is gof.