UWO CS2214

Tutorial #1

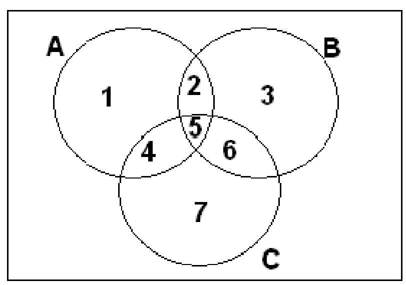
Problem 1 Give a direct proof of the following: "If p is a prime number larger than 2 then p^2 is odd".

Clearly separate the hypotheses from the conclusion and provide detailed justification for your answer.

Solution 1 Assume that p is a prime number larger than 2. Let us prove that p^2 is odd. We saw in class that if an integer number is odd, then so is is square. The prime p (being greater than 2) is odd. Hence p^2 is odd.

Problem 2 Show Venn's diagram for the general case of three sets A, B, C in universal set U. Also, use shading to highlight the set $\bar{A} \cap (B \cup C)$.

Solution 2



The set $\bar{A} \cap (B \cup C)$ corresponds to the regions labelled $3, 6, 7$.

Problem 3 Show that

$$((p \to q) \land (q \to r)) \to (p \to r) \quad \equiv \quad T$$

i.e. that the compound proposition on the left of $\equiv$ is a tautology. You should do it in two different ways using

(a) truth tables

(b) logical equivalences

HINT: first replace all "$\rightarrow$" using $a \rightarrow b \equiv \neg a \vee b$. Then, use Morgan's laws and other standard logical equivalence laws (slides 48-50 in propositional logic).

Solution 3 Our proposed solution combines both types of techniques. We have the following equivalences:

$$\begin{array}{rc} ((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) & \iff \\ ((\neg p \vee q) \wedge (\neg q \vee r)) \rightarrow (\neg p \vee r) & \iff \\ \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r) & \iff \\ \neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r) & \iff \\ (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r). & \end{array}$$

Consider now the first parenthesized expression, namely $(p \wedge \neg q)$. Observe that if $p = \mathsf{true}$ and $q = \mathsf{false}$, then the whole proposition is true, whatever is r. Assume now that either $p = \mathsf{false}$ or $q = \mathsf{true}$:

- if $p = \mathsf{false}$, then $(\neg p \vee r)$ is true, whatever is r,
- if $q = \mathsf{true}$, then $(q \wedge \neg r)$ is true, whatever is r.

Finally, whatever are p, q, r, the formula is true. This concludes the proof.