UNIVERSITY OF WESTERN ONTARIO

Computer Science 2214

Discrete Structures for Computing

SAMPLE FINAL EXAM

3 hours

The use of a reference sheet is allowed that is letter-size, two-sided, handwritten by the student, with neither flaps nor fully worked-out proofs or examples

No electronic devices of any kind, including cell phones, are allowed

This exam has 10 equally-weighted questions

Name ____________________________

ID ____________________________

(1) Let p, q, and r be the propositions

p: You get an A on the final exam.

q: You do every exercise in the textbook.

r: You get an A in this class.

Write the following sentences in the language of propositional logic using p, q, r as propositional variables, as well as logical connectives.

(a) You get an A in this class, but you do not do every exercise in the textbook.

(b) You get an A on the final, you do every exercise in the textbook, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) Getting an A on the final and doing every exercise in the textbook is sufficient for getting an A in this class.

(e) You will get an A in this class if and only if you either do every exercise in the textbook or you get an A on the final.

(2) Let $C(x)$ be the statement "x has a cat" and let $D(x)$ be the statement "x has a dog", and let $H(x)$ be the statement "x has a hedgehog". Express each of the following sentences in the language of predicate logic, in terms of $C(x)$, $D(x)$, $H(x)$, quantifiers and logical connectives. The domain consists of all students in your class.

(a) A student in your class has a cat, a dog, and a hedgehog.

(b) All students in your class have a cat, a dog or a hedgehog.

(c) Some student in your class has a cat and a hedgehog, but not a dog.

(d) No student in your class has a cat, a dog, and a hedgehog.

(e) For each of these three animals, cats, dogs and hedgehogs, there is a student in your class who has this animal as a pet.

(3) *(i)* Prove that if $n = a \cdot b$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

(ii) Prove that $|x| \cdot |y| = |x \cdot y|$, for all real numbers x and y, where $|x|$ denotes the absolute value of x ($|x|$ equals x if $x \geq 0$ and equals $-x$ if $x < 0$.)

(4) An employee joined a company on January 1st, 2012 with a starting salary of \$70,000. Every year this employee receives a raise: \$1,000 bonus plus 5% of the salary of the previous year.

(a) Set up a recurrence relation for the salary of this employee n years after 2012.

(b) What will the salary of this employee be at the end of 2014?

(c) Find an explicit (closed) formula for the salary of this employee n years after 2012.

Provide detailed justifications of your answers.

(5) A plaintext message was encrypted using the affine cipher

$$f(x) = (17x + 3) \bmod 26.$$

Find the decryption function and use it to decrypt the received cryptotext message "H R D V". Use the method presented in class, including the Euclidean algorithm to find an inverse with respect to modular multiplication. Provide detailed justifications of your answers.

(6). Use mathematical induction to prove that, for every positive integer n, the following equality holds:

$$1 \cdot 2 + 2 \cdot 3 + \ldots + n(n+1) = n(n+1)(n+2)/3.$$

(7). How many strings of **six** lowercase letters from the English alphabet contain

(a) The letter a?

(b) The letters a and b?

(c) The letters a and b in consecutive positions with a preceding b, with all the letters distinct?

(d) The letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?

Provide detailed justifications of your answers.

(**8**) What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

(a) The permutation consists of the letters in reverse alphabetic order.

(b) z is the first letter of the permutation.

(c) z precedes a in the permutation.

(d) a immediately precedes m, which immediately precedes z in the permutation.

(e) m, n and o are in their original places in the permutation.

Provide detailed justifications of your answers.

(9) When a test for steroids is given to soccer players, 98% of the players taking steroids test positive, and 12% of the players not taking steroids test positive. Suppose that 5% of the soccer players take steroids.

(a) What is the probability that a randomly selected soccer player who tests positive for steroids actually takes steroids?

(b) What is the probability that a randomly selected soccer player who tests negative for steroids did not use steroids?

Provide detailed justifications of your answers.

(10) For *each* of the following two graphs:

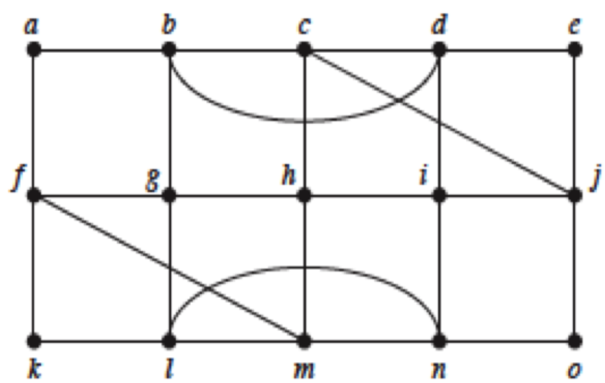
(a) Determine whether or not it has an Euler circuit. Justify your answer.

(b) If the graph has an Euler circuit, use the algorithm described in class to find it, including drawings of any intermediate subgraphs.

(c) If no Euler circuit exists, determine whether the graph has an Euler path. Justify your answer.

(d) Construct an Euler path if one exists.

(I)



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