CS3331 – Assignment 3
due Nov. 26, 2019, (latest to submit: Nov. 29, 11:55pm)

1. (10pt) Consider the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and define the function $succ : \Sigma^* \to \Sigma^*$, $succ(w)$ is the word immediately following w in lexicographic order. Construct a deterministic Turing machine M that computes the function $succ$, that is, M starts with the initial configuration $(s, \underline{\square}w)$ and halts with the configuration $(h, \underline{\square}succ(w))$. Describe M in details using a directed graph whose edges are labelled by transitions (such as the one in Example 17.2, p. 268 of textbook).

2. (10pt) Construct a deterministic Turing machine M that adds one to its binary input if it is even and subtracts one if it is odd. M starts with the initial configuration $(s, \underline{\square}w)$, where $w \in \{\mathtt{0}, \mathtt{1}\}^*$; the binary input w is interpreted as an integer number. Possible leading 0's have to be removed as well. The machine halts in the appropriate configuration $(h, \underline{\square}(w \pm 1)_{(2)})$, where $w_{(2)}$ is the binary representation of w.

   Here are some examples of M's behaviour:

   $(s, \underline{\square}) \vdash^* (h, \underline{\square}\mathtt{1})$

   $(s, \underline{\square}\mathtt{000}) \vdash^* (h, \underline{\square}\mathtt{1})$

   $(s, \underline{\square}\mathtt{01}) \vdash^* (h, \underline{\square}\mathtt{0})$

   $(s, \underline{\square}\mathtt{111}) \vdash^* (h, \underline{\square}\mathtt{110})$

   $(s, \underline{\square}\mathtt{001100}) \vdash^* (h, \underline{\square}\mathtt{1101})$

   Describe M using the macro language (such as the one in Example 17.8, p. 275 of textbook).

3. (20pt) Construct a Turing Machine M that semidecides, but does *not* decide, each of the following languages over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$:

   (a) $L_1 = \{\mathtt{a}\}$,

   (b) $L_2 = \Sigma^*$,

   (c) $L_3 = \emptyset$.

   In each case, describe M using the macro language.

4. (20pt) Describe in clear English a Turing machine that semidecides the language

   $$L = \{< M >|\ M \text{ accepts the binary encodings of at least 3 prime numbers}\}\ .$$

5. (20pt) Is the set SD closed under:

   (a) Intersection?

   (b) Concatenation?

   Prove your answers. Clear English description of any Turing machines is sufficient. (That is, you don't have to effectively build the machine, instead explain how the machine behaves.)

6. (20pt) Let L_1 and L_2 be two languages that are not decidable.

   (a) Is it possible that $L_1 - L_2$ is regular and $L_1 - L_2 \neq \emptyset$? Prove your answer.

   (b) Is it possible that $L_1 \cup L_2$ is decidable but $L_1 \neq \neg L_2$? Prove your answer.

Note Submit your solution as a (typed) pdf file on `owl.uwo.ca`.