Chapter One

Introduction to Data Structures and Algorithms

Introduction

A program

- A set of instruction which is written in to solve a problem.
- A solution to a problem actually consists of two things:
    - ✓ A way to organize the data
    - ✓ Sequence of steps to solve the problem
- The way data are organized in a computers memory is said to be **Data Structure.**
- The sequence of computational steps to solve a problem is said to be an **Algorithm.**
- Therefore, a *program* is **Data structures** plus **Algorithm.**

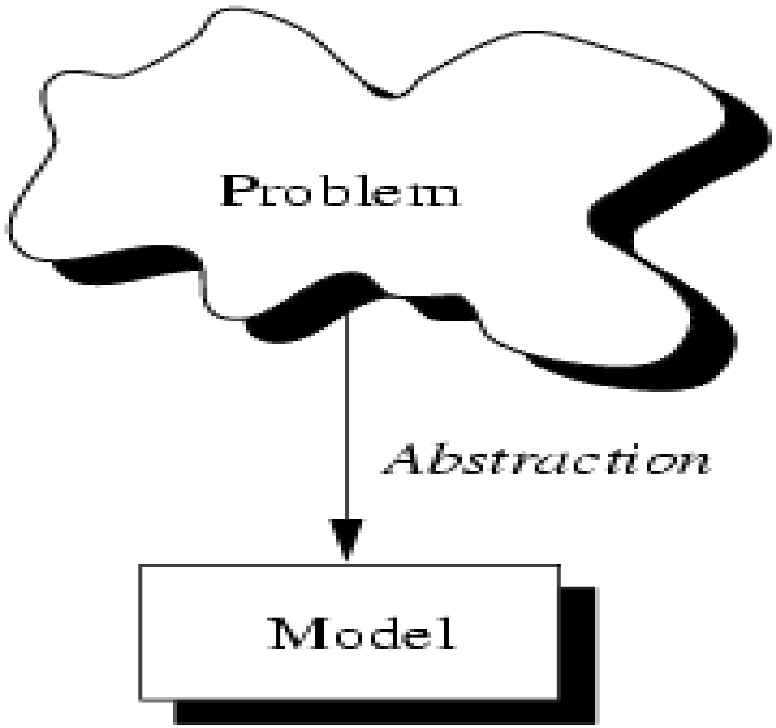
Introduction…

Data structures are used to model the world or part of the world. How?

1. The value held by a data structure represents some specific characteristic of the world.
2. The characteristic being modeled restricts the possible values held by a data structure and the operations to be performed on the data structure.

- The first step to solve the problem is obtaining ones own **abstract view**, or **model**, of the problem.
- This process of modeling is called **abstraction.**

Introduction…



- The model defines an abstract view to the problem.
- The model should only focus on problem related stuff

Abstraction

- Is a process of classifying characteristics as **relevant** and **irrelevant** for the particular purpose at hand and ignoring the **irrelevant** ones.

Example: model students of A University.

- **Relevant:**

  Char Name[15];

  Char ID[11];

  Char Dept[20];

  int Age, year;

- **Non relevant**

  float hieght, weight;

Abstraction…

- Using the model, a programmer tries to define the **properties** of the problem.
- These properties include
  - ✓ The **data** which are affected and
  - ✓ The **operations** that are involved in the problem
- An entity with the properties just described is called an **abstract data type (ADT).**

Abstract Data Types

- Consists of data to be stored and operations supported on them.
- Is a specification that describes a data set and the operation on that data.

- The ADT specifies:
  - ✓ What data is stored.
  - ✓ What operations can be done on the data.
- Does not specify how to store or how to implement the operation.
- Is independent of any programming language

Abstract Data Types…

<u>Example</u>: ADT employees of an organization:

- ✓ This ADT stores employees with their relevant attributes and discarding irrelevant attributes.

Relevant:- Name, ID, Sex, Age, Salary, Dept, Address

Non Relevant :- weight, color, height

- ✓ This ADT supports **hiring, firing, retiring,** … operations.

Data Structure

- In Contrast a data structure is a language construct that the programmer has defined in order to implement an *abstract data type*.
- What is the purpose of data structures in programs?
  – Data structures are used to model a problem.

Example:

```
struct Student_Record
{
    char name[20];
    char ID_NO[10];
    char Department[10];
    int age;
};
```

Data Structure…

- Attributes of each variable:
  - **Name:** Textual label.
  - **Address**: Location in memory.
  - **Scope**: Visibility in statements of a program.
  - **Type**: Set of values that can be stored + set of operations that can be performed.
  - **Size**: The amount of storage required to represent the variable.
  - **Life time**: The time interval during execution of a program while the variable exists.

Algorithm

- Is a concise specification of an operation for solving a problem.
- Is a well-defined computational procedure that takes some value or a set of values as input and produces some value or a set of values as output.

Inputs → Algorithm → Outputs

- An algorithm is a specification of a behavioral process. It consists of a finite set of instructions that govern behavior step-by-step.
- Is part of what constitutes a data structure

Algorithm…

- Data structures model the static part of the world. They are unchanging while the world is changing.
- In order to model the dynamic part of the world we need to work with algorithms.
- Algorithms are the dynamic part of a program's world model.
- An **algorithm** transforms **data structures** from one state to another state.

Algorithm…

- What is the purpose of algorithms in programs?
  - Take values as input.

  Example: cin>>age;
  - Change the values held by data structures.

  Example: age=age+1;
  - Change the organization of the data structure:

  Example:

  Sort students by name
  - Produce outputs:

  Example: Display student's information

Algorithm…

- The quality of a data structure is related to its ability to successfully model the characteristics of the world (problem).
- Similarly, the quality of an algorithm is related to its ability to successfully simulate the changes in the world.
- However, the quality of data structure and algorithms is determined by their ability to work together well.
- Generally speaking, correct data structures lead to simple and efficient algorithms.
- And correct algorithms lead to accurate and efficient data structures.

Properties of Algorithms

1. Finiteness:

- Algorithm must complete after a finite number of steps.
- Algorithm should have a finite number of steps.

Example of Finite Steps

```
int i=0;
while(i<10)
{
  cout<< i;
  i++;
}
```

Example of Infinite Steps

```
while(true)
{
  cout<<"Hello";
}
```

2. Definiteness (Absence of ambiguity):

- Each step must be clearly defined, having one and only one interpretation.
- At each point in computation, one should be able to tell exactly what happens next.

3. Sequential:

- Each step must have a uniquely defined preceding and succeeding step.
- The first step (start step) and last step (halt step) must be clearly noted.

4. Feasibility:

- It must be possible to perform each instruction.
- Each instruction should have possibility to be executed.

a)
```
for(int i=0; i<0; i++)
   {
     cout<< i;  // there is no possibility that this
   }            //statement to be executed.
```

b)
```
if(5>7)
{
   cout<<"hello"; // not executed.
}
```

5. Correctness:

- It must compute correct answer for all possible legal inputs.
- The output should be as expected and required and correct.

6. Language Independence:

- It must not depend on any one programming language.

7. Completeness:

- It must solve the problem completely.

8. Effectiveness:

- Doing the right thing. It should yield the correct result all the time for all of the possible cases.

9. Efficiency:

- It must solve with the least amount of computational resources such as time and space.
- Producing an output as per the requirement within the given **resources (constraints).**

<u>**Example:**</u> Write a program that takes a number and displays the square of the number.

1)
```
int x;
cin>>x;
cout<<x*x;
```

2)
```
int x,y;
cin>>x;
y=x*x;
cout<<y;
```

Example:

Write a program that takes two numbers and displays the sum of the two.

Program a

```
cin>>a;
cin>>b;
sum= a+b;
cout<<sum;
```

Program b

```
cin>>a;
cin>>b;
a= a+b;
cout<<a;
```

Program c (the most efficient)

```
cin>>a;
cin>>b;
cout<<a+b;
```

All are **effective** but with **different efficiencies**.

10. Input/output:

- There must be a specified number of input values, and one or more result values.
- Zero or more inputs and one or more outputs.

11. Precision:

- The result should always be the same if the algorithm is given identical input.

12. Simplicity:

- A good general rule is that each step should carry out one logical step.
  - What is simple to one processor may not be simple to another.

13. Levels of abstraction:

- Used to organize the ideas expressed in algorithms.
  - Used to hide the details of a given activity and refer to just a name for those details.
  - The simple (detailed) instructions are hidden inside modules.
  - Well-designed algorithms are organized in terms of levels of abstraction.

Algorithm Analysis

- Algorithm analysis refers to the process of determining how much **computing time** and **storage** that algorithms will require.
- In other words, it's a process of predicting the **resource requirement** of algorithms in a given environment.
- In order to solve a problem, there are many possible algorithms.
- One has to be able to choose the best algorithm for the problem at hand using some scientific method.

- To classify some data structures and algorithms as good:
  - we need precise ways of analyzing them in terms of resource requirement.

The main resources are:

- **Running Time**
- **Memory Usage**
- **Communication Bandwidth**

Note: ***Running time*** is the most important since computational time is the most precious resource in most problem domains.

There are *two* approaches to measure the efficiency of algorithms: Empirical and Theoretical

1. Empirical

- based on the total running time of the program.
- Uses actual system clock time.

Example:

t1= initial time before the program starts

```
for(int i=0; i<=10; i++)
    cout<<i;
```

t2 = final time after the execution of the program is finished

Running time taken by the above algorithm or

Total Time = t2-t1;

- It is difficult to determine efficiency of algorithms using this approach, because clock-time can vary based on many factors.

For example:

a) **Processor speed of the computer**

1.78GHz	2.12GHz
10s	<10s

b) **Current processor load**

- Only the work 10s
- With printing 15s
- With printing & browsing the internet >15s

c) **Specific data for a particular run of the program**

- Input size, Input properties

```
t1
    for(int i=0; i<=n; i++)
        cout<<i;
t2
T=t2-t1;
```

For n=100, T>=0.5s

n=1000, T>0.5s

d) **Operating System**

- Multitasking Vs Single tasking
- Internal structure

2. Theoretical

- Determining the quantity of resources required using mathematical concept.
- Analyze an algorithm according to the **number of basic operations (time units)** required, rather than according to an absolute amount of time involved.

We use theoretical approach to determine the efficiency of algorithm because:

- The number of operation will not vary under different conditions.
- It helps us to have a meaningful measure that permits comparison of algorithms independent of operating platform.
- It helps to determine the **complexity of algorithm.**

Complexity Analysis

Complexity Analysis is the systematic study of the cost of computation, measured either in:

- Time units
- Operations performed, or
- The amount of storage space required.

Two important ways to characterize the effectiveness of an algorithm are its:

Space Complexity and

Time Complexity.

Time Complexity:

- Determine the approximate amount of time (number of operations) required to solve a problem of size n.
  - The limiting behavior of time complexity as size increases is called the **Asymptotic Time Complexity.**

Space Complexity:

- Determine the approximate memory required to solve a problem of size n.
  - The limiting behavior of space complexity as size increases is called the **Asymptotic Space Complexity.**

- **Asymptotic Complexity** of an algorithm determines the size of problems that can be solved by the algorithm.
- Factors affecting the running time of a program:
  - CPU type (80286, 80386, 80486, Pentium I---IV)
  - Memory used, Computer used, Programming Language - C (fastest), C++ (faster), Java (fast)
    C is relatively faster than Java, because C is relatively nearer to Machine language. So, Java takes relatively larger amount of time for interpreting or translation to machine code.
  - Algorithm used and Input size

Note: Important factors for this course are

Input size and Algorithm used.

Complexity analysis involves two distinct phases:

- **Algorithm Analysis**: Analysis of the algorithm or data structure to produce a function **T(n)** that describes the algorithm in terms of the operations performed in order to measure the complexity of the algorithm.

Example: Suppose we have hardware capable of executing 10^6 instructions per second. How long would it take to execute an algorithm whose complexity function is $T(n)=2n^2$ on an input size of $n=10^8$?

Solution: $T(n)= 2n^2=2(10^8)^2 = 2*10^{16}$

Running time$=T(10^8)/10^6=2*10^{16}/10^6=2*10^{10}$ seconds.

Order of Magnitude Analysis: Analysis of the function *T (n)* to determine the general complexity category to which it belongs.

- There is no generally accepted set of rules for algorithm analysis.
- However, an exact count of operations is commonly used.
- To count the number of operations we can use the following Analysis Rule.

Analysis Rules:

1. Assume an arbitrary time unit.
2. Execution of one of the following operations takes time 1 unit:
   - Assignment Operation, <u>Example:</u> i=0;
   - Single Input/Output Operation

     <u>Example:</u> cin>>a;

     cout<<“hello”;
   - Single Boolean Operations, <u>Example:</u> i>=10
   - Single Arithmetic Operations, <u>Example:</u> a+b;
   - Function Return, <u>Example:</u> return sum;
3. Running time of a selection statement (if, switch) is the time for the condition evaluation plus the maximum of the running times for the individual clauses in the selection.

Example:

```
int x;
int sum=0;
if(a>b)
 {
    sum= a+b;
    cout<<sum;
 }
else
 {
   cout<<b;
 }
```

$T(n) = 1 + 1 + \max(3,1) = 5$

4. **Loop statements:**

- The running time for the statements inside the loop * number of iterations + time for setup(1) + time for checking (number of iteration + 1) + time for update (number of iteration)
- The total running time of statements inside a group of nested loops is the running time of the statements * the product of the sizes of all the loops.
- For nested loops, analyze inside out.
- Always assume that the loop executes the maximum number of iterations possible. (Why?)
    - Because we are interested in the worst case complexity.

5. Function call:

1 for setup + the time for any parameter calculations + the time required for the execution of the function body.

Examples:

1)

```
int k=0,n;
cout<<"Enter an integer";
cin>>n
for(int i=0;i<n; i++)
        k++;
```

T(n)= 3+1+n+1+n+n=3n+5

2)
```
int i=0;
while(i<n)
{
   cout<<i;
   i++;
 }
  int j=1;
  while(j<=10)
{
  cout<<j;
  j++;
}
```

$T(n) = 1+n+1+n+n+1+11+2(10)$

$= 3n+34$

3)

```
int k=0;
for(int i=1 ; i<=n; i++)
  for( int j=1; j<=n; j++)
      k++;
```

$$T(n)=1+1+(n+1)+n+n(1+(n+1)+n+n)$$

$$= 2n+3+n(3n+2)$$

$$= 2n+3+3n^2+2n$$

$$= 3n^2+4n+3$$

4). int sum=0;

```
for(i=1;i<=n;i++))
    sum=sum+i;
```

T(n)=1+1+(n+1)+n+(1+1)n

=3+4n=O(n)

5). int counter(){

```
int a=0;
cout<<"Enter a number";
cin>>n;
for(i=0;i<n;i++)
   a=a+1;
 return 0;     }
```

T(n)=1+1+1+(1+n+1+n)+2n+1

=4n+6=O(n)

6). void func( ){

```
int x=0;  int i=0;  int j=1;
cout<<"Enter a number";
cin>>n;
while(i<n){
  i=i+1;
}
  while(j<n){
      j=j+1;
  }
}
```

$T(n)=1+1+1+1+1+n+1+2n+n+2(n-1)$

$= 6+4n+2n-2$

$=4+6n=O(n)$

7). int sum(int n){

```
int sum(int n){
        int s=0;
        for(int i=1;i<=n;i++)
            s=s+(i*i*i);
    return s;
    }
```

$$T(n)=1+(1+n+1+n+5n)+1$$
$$=7n+4=O(n)$$

8). int sum=0;

```
int sum=0;
    for(i=0;i<n;i++)
        for(j=0;j<n;j++)
            sum++;
```

$$T(n)=1+1+(n+1)+n+n*(1+(n+1)+n+n)$$
$$=3+2n+n^2+2n+2n^2$$
$$=3+2n+3n^2+2n$$
$$=3n^2+4n+3=O(n^2)$$

Formal Approach to Analysis

- In the above examples we have seen that analyzing Loop statements is so complex.
- It can be simplified by using some formal approach in which case we can ignore **initializations, loop controls**, and **updates**.

Simple Loops: Formally

- **For loop** can be translated to a summation.
- The index and bounds of the summation are the same as the index and bounds of the **for** loop.

- Suppose we count the number of additions that are done. There is 1 addition per iteration of the loop, hence n additions in total.

```
for (int i = 1; i <= N; i++) {
    sum = sum+i;
}
```

$$\sum_{i=1}^{N} 1 = N$$

Nested Loops: Formally

- Nested **for** loops translate into multiple summations, one for each **For loop**.

```
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= M; j++) {
        sum = sum+i+j;
    }
}
```

$$\sum_{i=1}^{N}\sum_{j=1}^{M} 2 = \sum_{i=1}^{N} 2M = 2MN$$

Consecutive Statements: Formally

- Add the running times of the separate blocks of your code.

```
for (int i = 1; i <= N; i++) {
    sum = sum+i;
}
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        sum = sum+i+j;
    }
}
```

$$\left[\sum_{i=1}^{N} 1\right] + \left[\sum_{i=1}^{N}\sum_{j=1}^{N} 2\right] = N + 2N^2$$

Conditionals: (Formally take maximum)

Example:

```
if (test == 1) {
    for (int i = 1; i <= N; i++) {
        sum = sum+i;
}}
else for (int i = 1; i <= N; i++) {
        for (int j = 1; j <= N; j++) {
            sum = sum+i+j;
}}
```

$$\max\left(\sum_{i=1}^{N} 1, \sum_{i=1}^{N}\sum_{j=1}^{N} 2\right) =$$

$$\max\left(N, 2N^2\right) = 2N^2$$

Recursive: Formally

Usually difficult to analyze.

Example: Factorial

```
long factorial(int n){
    if(n<=1)
        return 1;
    else
        return n*factorial(n-1);
}
```

T(n)=1+T(n-1)=2+T(n-2)=3+T(n-3)=-------

=n-1 (counting the number of multiplication)

Categories of Algorithm Analysis

- Algorithms may be examined under different situations to correctly determine their efficiency for accurate comparison.

Best Case Analysis:

- Assumes the input data are arranged in the most advantageous order for the algorithm.
- Takes the smallest possible set of inputs.
- Causes execution of the fewest number of statements.

Computes the **lower bound of T(n)**, where T(n) is the complexity function.

Examples:

For **sorting algorithm**

- If the list is already sorted (data are arranged in the required order).

For **searching algorithm**

- If the desired item is located at first accessed position.

Worst Case Analysis:

- Assumes the input data are arranged in the most disadvantageous order for the algorithm.
- Takes the worst possible set of inputs.
- Causes execution of the largest number of statements(operations).
- Computes the upper bound of T(n) where T(n) is the complexity function.

Example:

While sorting, if the list is in opposite order.

While searching, if the desired item is located at the last position or is missing.

Worst Case Analysis:

Worst case analysis is the most common analysis because:

- It provides the upper bound for all input (even for bad ones).
- Average case analysis is often difficult to determine and define.
- If situations are in their best case, no need to develop algorithms because data arrangements are in the best situation.
- Best case analysis can not be used to estimate complexity.
- We are interested in the worst case time since it provides a bound for all input-this is called the "Big-Oh" estimate.

Average Case Analysis:

- Determine the average of the running time overall permutation of input data.
- Takes an average set of inputs.
- It also assumes random input size.
- It causes average number of executions.
- Computes the **optimal bound** of T(n) where T(n) is the complexity function.
- Sometimes average cases are as bad as worst cases and as good as best cases.

<u>Examples:</u>

For sorting algorithms

- While sorting, considering any arrangement (order of input data).

For searching algorithms

- While searching, if the desired item is located at any location or is missing.

The study of algorithms includes:

- How to Design algorithms (Describing algorithms)
- How to Analyze algorithms (In terms of time and memory space)
- How to validate algorithms (for any input)
- How to express algorithms (Using programming language)
- How to test a program (debugging and maintaining)

But, in this course more focus will be given to Design and Analysis of algorithms.

Order of Magnitude

- Refers to the rate at which the storage or time grows as a function of problem size.
- It is expressed in terms of its relationship to some known functions.
- This type of analysis is called **Asymptotic analysis.**

Asymptotic Notations

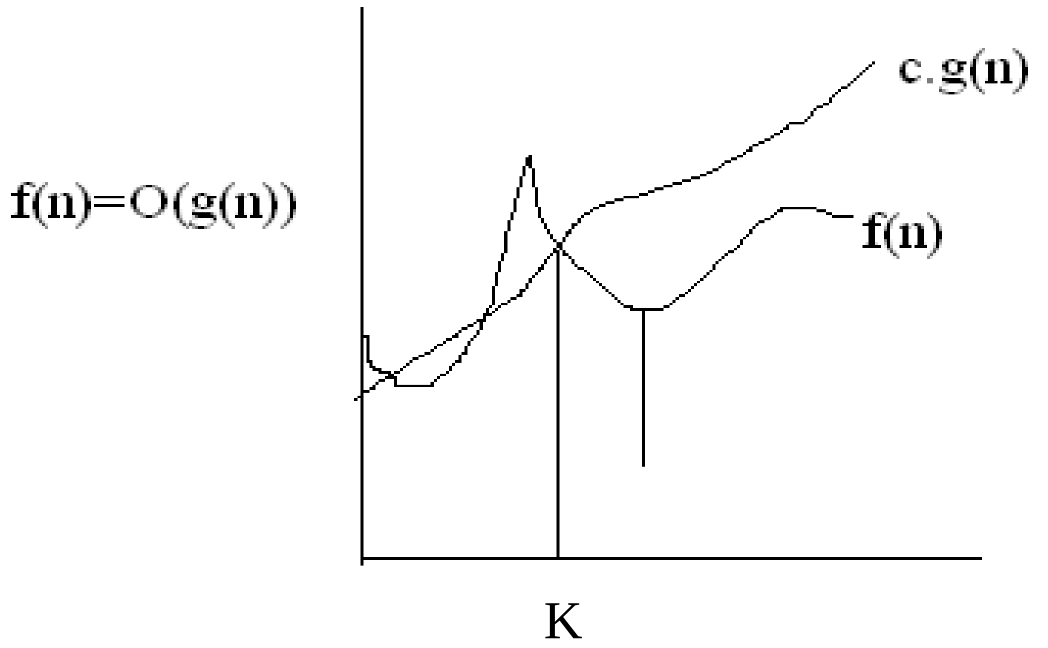
- Asymptotic Analysis is concerned with how the running time of an algorithm increases with the size of the input **in the limit**, as the size of the input increases **without bound!**
- Asymptotic Analysis makes use of **O (Big-Oh) , Ω (Big-Omega), θ (Theta), o (little-o), ω (little-omega)** - notations in performance analysis and characterizing the complexity of an algorithm.
- Note: The complexity of an algorithm is a numerical function of the size of the problem (instance or input size).

Types of Asymptotic Notations

1. Big-Oh Notation

Definition: We say **f(n)=O(g(n))**, if there are positive constants **k** and **c**, such that to the right of **k**, the value of **f(n)** always lies on or below **c.g(n)**.

- As n increases **f(n)** grows no faster than **g(n).**
- It's only concerned with what happens for very large values of n.
- Describes the worst case analysis.
- Gives an upper bound for a function to within a constant factor.



- O-Notations are used to represent the amount of time an algorithm takes on the worst possible set of inputs, "Worst-Case"

Question-1

$f(n)=10n+5$ and **$g(n)=n$.**

Show that **$f(n)$ is $O(g(n))$.** To show that $f(n)$ is $O(g(n))$, we must show that there exist constants c and k such that $f(n)<=c.g(n)$ for all $n>=k$.

$10n+5<=c.n$ ➔ for all $n>=k$

let $c=15$, then show that $10n+5<=15n$

$5<=5n$ or $1<=n$

So, $f(n)=10n+5<=15.g(n)$ for all $n>=1$

($c=15$, $k=1$), there exist two constants that satisfy the above constraints.

Question-2

$f(n)=3n^2+4n+1$.

Show that **$f(n)=O(n^2)$.**

$3n^2 <= 3n^2$ for all $n>=1$

$4n<=4n^2$ for all $n>=1$ and

$1<=n^2$ for all $n>=1$

$3n^2+4n+1<=3n^2+4n^2+n^2$ for all $n>=1$

$<=8n^2$ for all $n>=1$

So, we have shown that $f(n)<=8n^2$ for all $n>=1$.

Therefore, $f(n)$ is $O(n^2)$, $(c=8, k=1)$, there exist two constants that satisfy the constraints.

2. **Big-Omega (Ω)-Notation** (Lower bound)

- Definition: We write **f(n)= Ω(g(n))** if there are positive constants **k** and **c** such that to the right of **k** the value of **f(n)** always lies on or above **c.g(n).**
- As n increases **f(n)** grows no slower than **g(n).**
- Describes the best case analysis.
- Used to represent the amount of time the algorithm takes on the smallest possible set of inputs-"Best case".

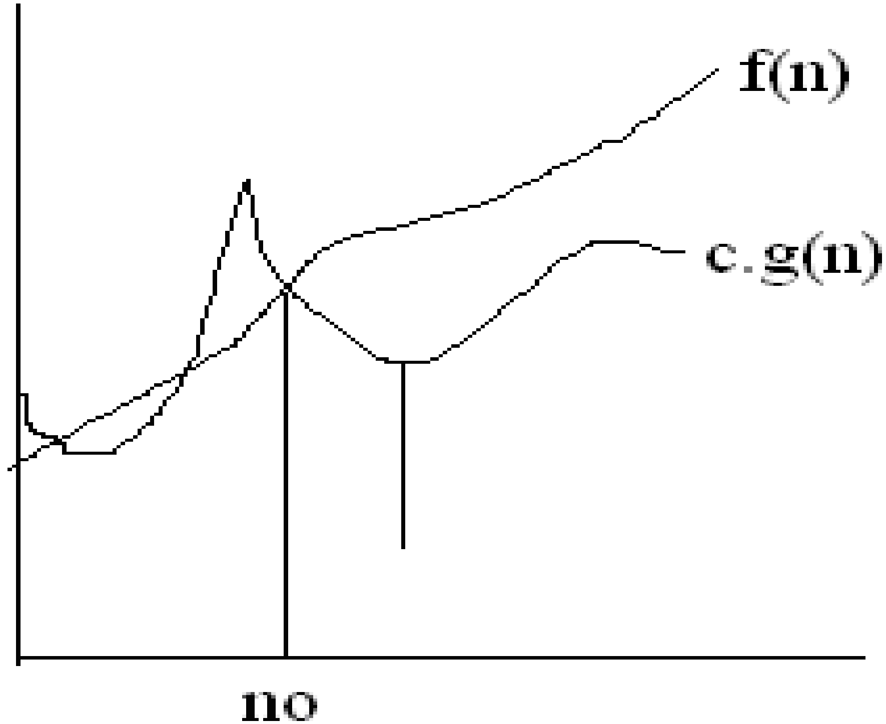
Example:

Find **g(n)** such that **f(n) = Ω(g(n))** for **f(n)=3n+5, g(n) = √n, c=1, k=1.**

f(n)=3n+5=Ω(√n)

Big-Omega (Ω)-Notation (Lower bound)

$f(n) = \Omega(g(n))$



3. Theta Notation (θ-Notation) (Optimal bound)

- Definition: We say **f(n)= θ(g(n))** if there exist positive constants n_o, **c1** and **c2** such that to the right of n_o, the value of **f(n)** always lies between **c1.g(n)** and **c2.g(n)** inclusive, i.e., **c2.g(n)<=f(n)<=c1.g(n),** for all $n>=n_o$.
- As n increases **f(n)** grows as fast as **g(n).**
- Describes the average case analysis.
- To represent the amount of time the algorithm takes on an average set of inputs- "Average case".

Example: Find **g(n)** such that **f(n) = Θ(g(n)) for**

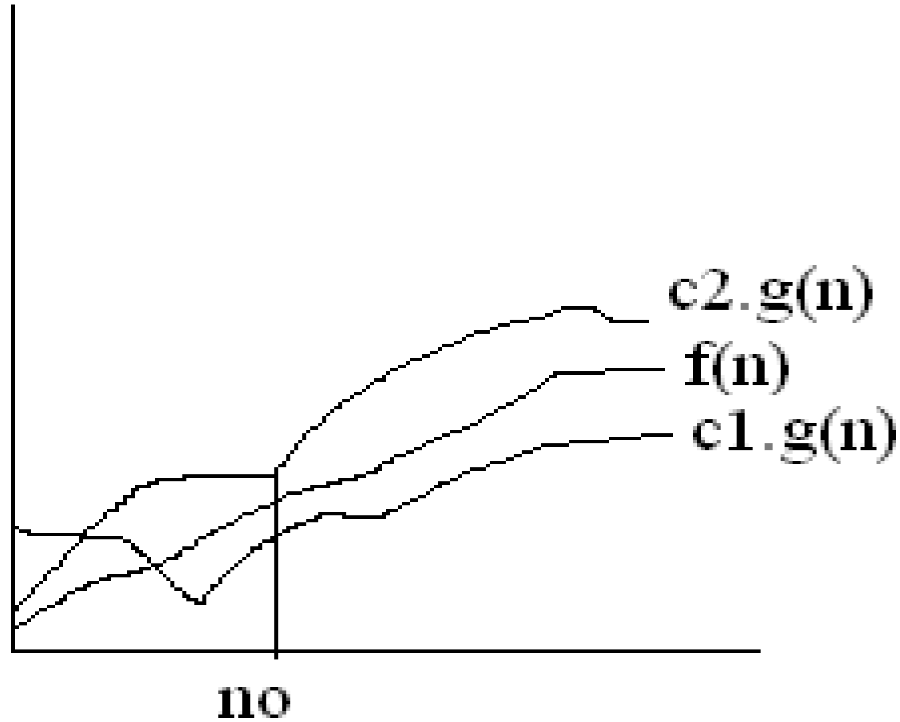
$f(n)=2n^2+3$

➔ $n^2 \leq 2n^2 \leq 3n^2$ ➔ $c1=1$, $c2=3$ and $n_o=1$

➔ $f(n) = \Theta(g(n))$.

Theta Notation (θ-Notation) (Optimal bound)

$f(n) = \theta(g(n))$



4. Little-oh (small-oh) Notation

- Definition: We say **f(n)=o(g(n)),** if there are positive constants n_o and **c** such that to the right of n_o, the value of **f(n)** lies below **c.g(n).**
- As n increases, **g(n)** grows strictly faster than **f(n).**
- Describes the worst case analysis.
- Denotes an upper bound that is not asymptotically tight.
- Big O-Notation denotes an upper bound that may or may not be asymptotically tight.

Example:

Find **g(n)** such that **f(n) = o(g(n))** for **f(n)** = n^2

$n^2 < 2n^2$, for all $n > 1$, ➔ $k=1$, $c=2$, $g(n)=n^2$

$n^2 < n^3$, $g(n) = n^3$, $f(n)=o(n^3)$

$n^2 < n^4$, $g(n) = n^4$, $f(n)=o(n^4)$

5. Little-Omega (ω) notation

- Definition: We write **$f(n)=\omega(g(n))$**, if there are positive constants **n_o** and **c** such that to the right of **n_o**, the value of **$f(n)$** always lies above **$c.g(n)$**.
- As n increases **$f(n)$** grows strictly faster than **$g(n)$**.
- Describes the best case analysis.
- Denotes a lower bound that is not asymptotically tight.
- Big **Ω**-Notation denotes a lower bound that may or may not be asymptotically tight.

Example: Find **$g(n)$** such that **$f(n)=\omega(g(n))$** for **$f(n)=n^2+3$**

$g(n)=n$, Since **$n^2 > n$, c**$=1$, **k**$=2$.

$g(n)=\sqrt{n}$, Since $n^2 > \sqrt{n}$, $c=1$, $k=2$, can also be solution.

Rules to estimate Big Oh of a given function

- Pick the highest order.
- Ignore the coefficient.

Example:

1. $T(n)=3n + 5 \rightarrow O(n)$
2. $T(n)=3n^2+4n+2 \rightarrow O(n^2)$

Some known functions encountered when analyzing algorithms. (Complexity category for Big-Oh).

See next slide →

Rule 1:

If $T1(n)=O(f(n))$ and $T2(n)=O(g(n))$, then

a) $T1(n)+T2(n)=max(O(f(n)),O(g(n)))$,

b) $T1(n)*T2(n)=O(f(n)*g(n))$

Rule 2:

If $T(n)$ is a polynomial of degree k, then $T(n)=\theta(n^k)$.

Rule 3:

$\log {}^{n}{}_{k}=O(n)$ for any constant k. This tells us that logarithms grow very slowly.

- We can always determine the relative growth rates of two functions f(n) and g(n) by computing

  lim n → infinity f(n)/g(n).

- The limit can have four possible values.

- The limit is 0: This means that $f(n)=o(g(n))$.
- The limit is $c \neq 0$: This means that $f(n)=\theta(g(n))$.
- The limit is infinity: This means that $g(n)=O(f(n))$.
- The limit oscillates: This means that there is no relation between $f(n)$ and $g(n)$.

Example:

- n^3 grows faster than n^2, so we can say that $n^2=O(n^3)$ or $n^3=\Omega(n^2)$.
- $f(n)=n^2$ and $g(n)=2n^2$ grow at the same rate, so both $f(n)=O(g(n))$ and $f(n)=\Omega(g(n))$ are true.
- If $f(n)=2n^2$, $f(n)=O(n^4)$, $f(n)=O(n^3)$, and **$f(n)=O(n^2)$** are all correct, but the last option is the best answer.

$T(n)$	Complexity Category functions $F(n)$	Big-O
c, c is constant	1	$C=O(1)$
$10\log n + 5$	$\log n$	$T(n)=O(\log n)$
$\sqrt{n} +2$	$\sqrt{n}$	$T(n)=O(\sqrt{n})$
$5n+3$	n	$T(n)=O(n)$
$3n\log n+5n+2$	$n\log n$	$T(n)=O(n\log n)$
$10n^2 +n\log n+1$	n^2	$T(n)=O(n^2)$
$5n^3 + 2n^2 + 5$	n^3	$T(n)=O(n^3)$
2^n+n^5+n+1	2^n	$T(n)=O(2^n)$
$7n!+2^n+n^2+1$	$n!$	$T(n)=O(n!)$
$8n^n+2^n +n^2 +3$	n^n	$T(n)=O(n^n)$

Complexity category	Big-Oh	Example
Constant	$T(n)=O(1)$-constant growth	Determining if a number is even or odd.
Logarithmic	$T(n)=O(\log n)$	Finding an item in a sorted array with a binary search. $10\log n+5$
Linear	$T(n)=O(n)$	Finding an item in an unsorted list, adding two n-digit numbers. $5/3n+10$
Loglinear	$T(n)=O(n\log n)$	Merge sort, heap sort $3n\log n+5n+2$
Quadratic	$T(n)=O(n^2)$	Adding two nXn matrices, shell sort, insertion sort, multiplying two n-digit numbers by a simple algorithm. $11/7n^2+2n+5$
Cubic	$T(n)=O(n^3)$	Multiplying two nXn matrices by simple algorithm. $3n^3+4n^2+5n+7$
Exponential	$T(n)=O(c^n)$, $c>1$	2^n+n^2+n+1
Factorial	$T(n)=O(n!)$	$7n!+n^2+1$
Double exponential	$T(n)=O(c1^{c2n})$, c1 and c2>1	$2^{2n}+2^n+n^2+1$
n^n	$T(n)=O(n^n)$	$8n^n+2^n+n^2+3$

- ➔ Arrangement of common functions by growth rate.
- ➔ List of typical growth rates.

Function	Name
c	Constant
log N	Logarithmic
$\log^2 N$	Log-squared
N	Linear
N log N	Log-Linear
N^2	Quadratic
N^3	Cubic
2^N	Exponential

- The order of the body statements of a given algorithm is very important in determining Big-Oh of the algorithm.

Example: Find Big-Oh of the following algorithm.

1. for(int i=1;i<=n; i++)

   sum=sum + i;

   $T(n)=2*n=2n=O(n)$.

2. for(int i=1; i<=n; i++)

   for(int j=1; j<=n; j++)

   k++;

   $T(n)=1*n*n=n^2 = O(n^2)$.