CENG 371 - Scientific Computing
Fall 2022
Homework 4

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a Because of the random nature of approximation, especially the Gaussian Distribution Matrix Ω, I averaged out relative errors from multiple runs to get a better picture of the correlation between k and relative error, 50 runs in the case of cameraman.jpg and 10 runs in the case of fingerprint.jpg. Then I plotted the relative error of actual Single Value Decomposition (SVD)($actual$), relative error of a single run of SVD approximation($approx_1$) and average error of all SVD approximation runs($approx_{avg}$) in Figures 1 and 2.

From $approx_{avg}$ we can observe that as the k increases, the relative error of SVD approximation tends to shrink, mimicking the shape of the relative error of actual SVD, but due to the randomly introduced by Ω, increasing k does not guarantee a reduction in relative error, demonstrated by $approx_1$.

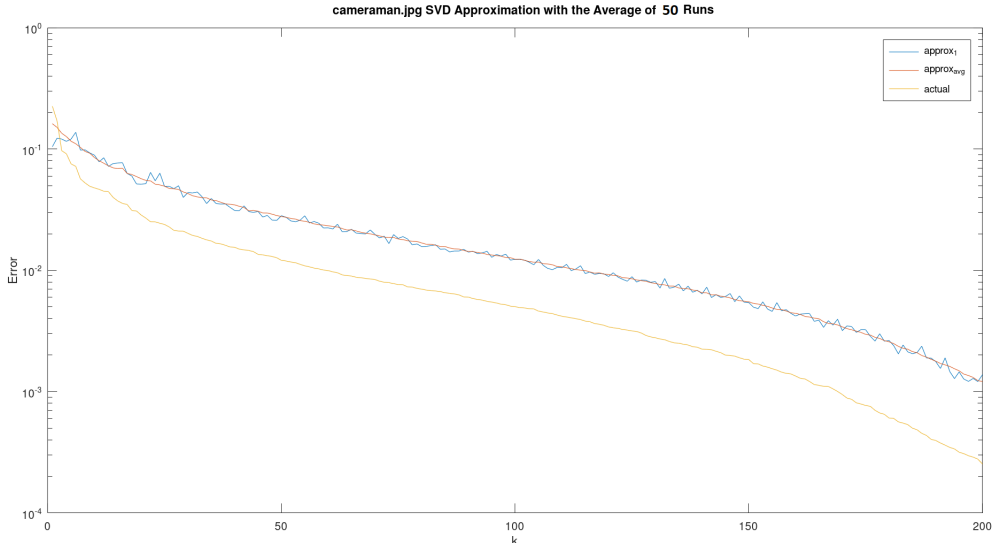

Figure 1: Relative Error Comparison of cameraman.jpg

b As seen in Figures 3 and 4, SVD approximation is about 100 times faster in the case of cameraman.jpg and about 10 time faster in the case of fingerprint.jpg. regardless of the k value. This is because the SVD decomposition is done on the smaller matrix $B = A\Omega$ where $B \in \mathcal{R}^{m \times k+p}, k \ll n$.

c When we compare the results of SVD in the Figures 5 and 6, for very small k values, like $k = 10$, in the approximate images, more detail is kept but the amount of detail kept is not uniform, probably because of the value of the Ω. For the greater k values, like $k = 100$ or $k = 200$, the difference between the quality of the images that was give by Figures 1 and 2 is not noticable to my eyes, especially for the case of $k = 200$. At the $k = 200$ point, the difference between the approximate SVD and the original image is so little that it is not noticeable for me.

d Easy answer for this question would by approximate SVD can be used instead of SVD in the applications where the accuracy is not a big concern or the computation time is a greater concern. But visual areas, like image processing, due to the limitation of human visual system, it can be used instead of full-rank SVD without creating a noticeable difference and greatly increasing computation times.

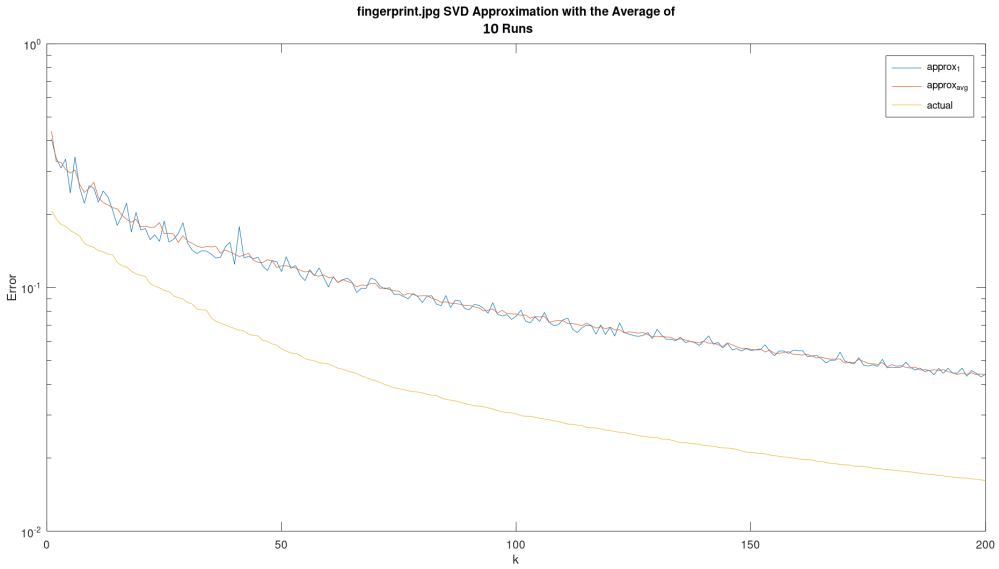

Figure 2: Relative Error Comparison of fingerprint.jpg

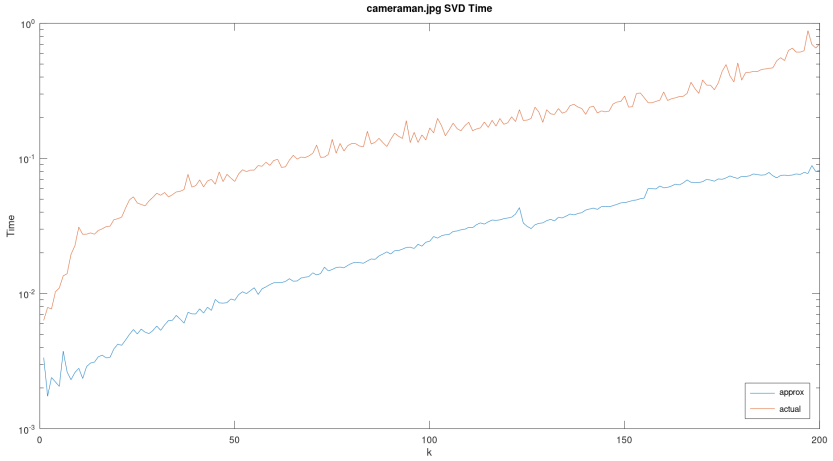

Figure 3: Time Comparison of cameraman.jpg

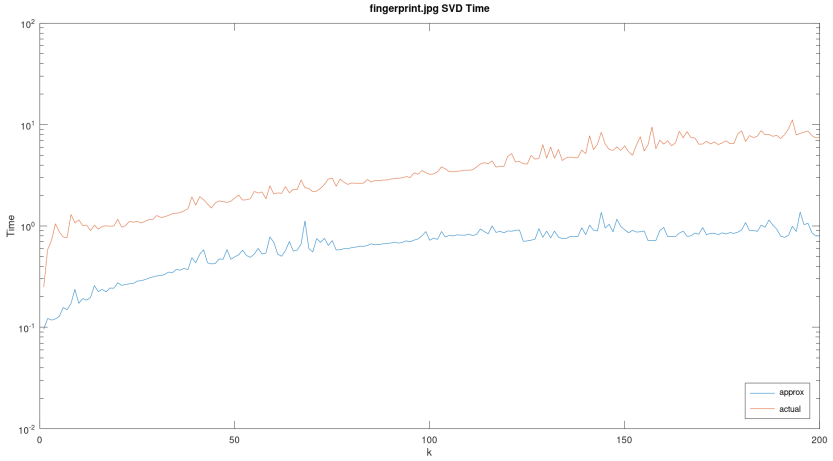

Figure 4: Time Comparison of fingerprint.jpg

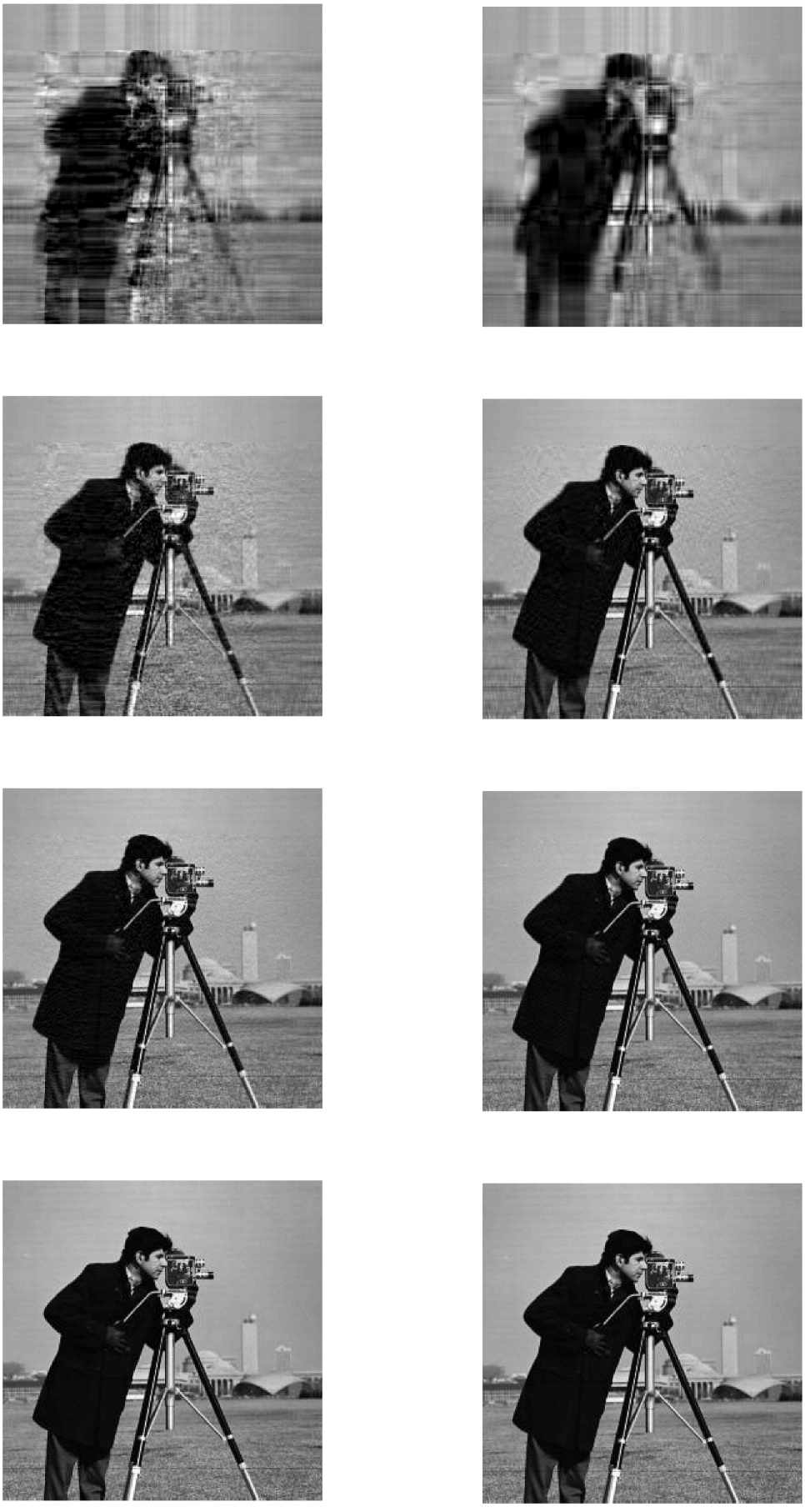

Figure 5: SVD Decomposition of cameraman.jpg for $k = 10, 50, 100, 200$, Approximate Result on the Left

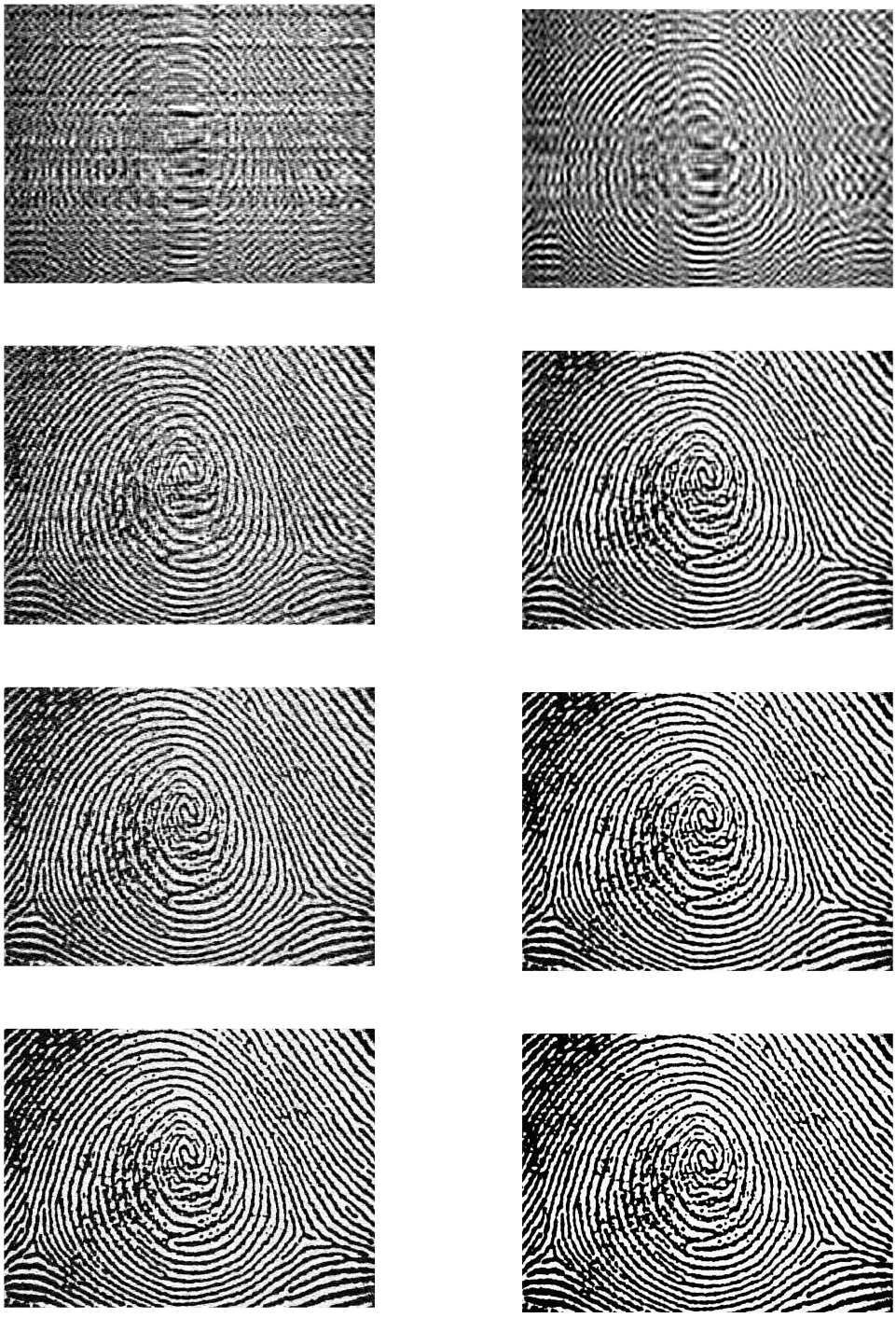

Figure 6: SVD Decomposition of fingerprint.jpg for $k = 10, 50, 100, 200$, Approximate Result on the Left