

CENG371

Scientific Computing

Fall 2022-2023

Homework 3

Due: December 23rd, 2022, 23:55

Question 1 (45 points)

- a) (10 pts) Implement the power method.
(**filename:** `power_method.m`, **inputs:** matrix A and (optional) starting vector v , **returns:** the maximum eigenvalue and the corresponding eigenvector of A)
- b) (10 pts) Implement the shifted inverse power method.
(**filename:** `inverse_power.m`, **inputs:** matrix A and the shift value α , **returns:** the closest eigenvalue to α and the corresponding eigenvector)
- c) (10 pts) Find the largest and smallest –in magnitude– eigenvalues and corresponding eigenvectors of

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

- d) (15 pts) Find the largest eigenvalue and eigenvector of

$$B = \begin{bmatrix} 0.2 & 0.3 & -0.5 \\ 0.6 & -0.8 & 0.2 \\ -1.0 & 0.1 & 0.9 \end{bmatrix}$$

starting with $\mathbf{x} = [1 \ 1 \ 1]^T$ via pen and paper. Do the same using the power method. Reflect on your findings.

Question 2 (55 points)

One can use the power method to find the largest k eigenvalues of a matrix A in an iterative fashion by successively subtracting the value $\lambda_i \frac{v_i v_i^T}{v_i^T v_i}$ from the current matrix at each step.

- a) (10 pts) Show how the above idea works.
- b) (15 pts) Implement the above idea.
(**filename:** `power_k.m`, **inputs:** matrix A and k , **returns:** the k largest eigenvalues and corresponding eigenvectors of A .)
- c) (15 pts) Implement the subspace iterations.
(**filename:** `subspace_iteration.m`, **inputs:** matrix A and k , **returns:** the k largest eigenvalues and corresponding eigenvectors of A .)
(You can use the function `qr` provided by MATLAB.)
- d) (15 pts) Compare the performances of the above two methods on the matrix `can229` of University of Florida Sparse Matrix Collection.

Regulations

1. Make sure that you reflect **your own reasoning** in a clean and concise manner.
2. Your submission should include a single PDF and your `.m` files.
3. Submission will be done via odtuclass.
4. **Late Submission:** Accepted with a penalty of $-5 \times (\text{day})^2$.