Middle East Technical University — Department of Computer Engineering

CENG371

Scientific Computing

Fall 2022-2023

Homework 4

Due: January 23rd, 2022

Question 1 (30 points)

Implement the randomized low-rank approximation algorithm using $p = 5$ as your safety parameter.
(**filename:** `approximate_svd.m` **input:** matrix A, rank k, **returns:** u_k, σ_k, v_k)

Question 2 (70 points)

For this question you will work with the image files `cameraman.jpg` and `fingerprint.jpg`. Let u_k, σ_k, v_k be the matrices returned by `approximate_svd`, u'_k, σ'_k, v'_k be the matrices returned by `svds`, and U, Σ, V be the matrices returned by `svd`. Let k run from 1 to the rank of the matrices representing the images.

a) Plot the relative errors $\dfrac{\|u_k\,\sigma_k\,v_k^T - U\Sigma V^T\|_2}{\|U\Sigma V^T\|2}$ and $\dfrac{\|u'_k\,\sigma'_k\,v'^T_k - U\Sigma V^T\|_2}{\|U\Sigma V^T\|2}$ versus k for both of the images. Discuss your observations.

b) Plot the run times of `approximate_svd` with `svds` versus k for both of the images. Discuss your observations.

c) So far you have reported and discussed quantitative results. Display the images resulting from low-rank approximations and make *qualitative* comparisons between $u_k\,\sigma_k\,v_k^T$, $u'_k\,\sigma'_k, v'^T_k$ and $U\,\Sigma\,V^T$. You are free to choose any k value(s) you think is helpful for your discussions.

d) Suggest use cases for `approximate_svd` based on your discussions.

Note: You will find the built-in MATLAB functions `imread`, `im2double`, `imshow`, `imsave`, `svd`, and `svds` useful. Consult to the help pages to learn more about these functions.

Regulations

1. While discussing your findings make sure that you reflect **your own reasoning** in a clean and concise manner. Give proper references to the sources in your report if you used any besides the lecture notes.
2. Your submission should include a single PDF and your `.m` files.
3. Submissions will be done via odtuclass.
4. **Late Submission: Not accepted**.