1-misol. Ushbu $x_n = \frac{n}{4+n^2} \quad (n=1,2,3,...)$
ketma-ketlikning chegaralanganligi isbotlansin.

Ravshanki, $\forall n \in N$ uchun

$$x_n = \frac{n}{4+n^2} > 0$$

bo‘ladi. Demak, qaralayotgan ketma-ketlik quyidan chegaralangan.

Ma'lumki,

$$0 \le (n-2)^2 = n^2 - 4n + 4$$

bo’lib,unda $4n \le 4 + n^2$ ya’ni,

$$\frac{n}{4+n^2} \le \frac{1}{4}$$

bo‘lishi kelib chiqadi.Bu esa berilgan ketma-ketlikning yuqoridan chegaralanganligini bildiradi.Demak, ketma-ketlik chegaralangan.

2-misol. Ushbu

$$x_n = c \quad (c \in R,\ n = 1, 2, 3, ...)$$

ketma-ketlikning limiti c ga teng bo‘ladi.

Haqiqatan ham, bu holda $\forall\ \varepsilon > 0$ ga ko’ra $n_0 = 1$ deyilsa, unda $\forall n > n_0$ uchun $|x_n - c| = 0 < \varepsilon$ bo’ladi.Demak, $\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} c = c$

3-misol. Ushbu $x_n = \frac{1}{n} \quad (n = 1,2,3,....)$
ketma-ketlikning limiti 0 ga teng bo‘lishi isbotlansin:

$$\lim_{n\to\infty} \frac{1}{n} = 0 .$$

Ravshanki,

$$\left|\frac{1}{n} - 0\right| = \frac{1}{n}$$

bo’lib, $\frac{1}{n} < \varepsilon \ (\varepsilon > 0)$ tengsizlik barcha $n > \frac{1}{\varepsilon}$ bo’lganda o’rinli.Bu holda

$$n_0 = \left[\frac{1}{\varepsilon}\right] + 1$$

deyilsa, $([a] - a$ sonidan katta bo‘lmagan uning butun qismi), unda $\forall n > n_0$ uchun

$$\left|\frac{1}{n} - 0\right| < \varepsilon$$

bo’ladi. Ta`rfga binoan

$$\lim_{n\to\infty}\frac{1}{n}=0.$$

4-misol. Aytaylik, $a\in R,\ |a|>1$ bo'lsin.U holda

$$\lim_{n\to\infty}\frac{1}{a^n}=0$$

bo'lishi isbotlansin.

$|a|=1+\delta$ deylik.Unda $\delta=|a|-1>0$ va Bernulli tengsizligiga ko‘ra

$$(1+\delta)^n\ge 1+n\delta>n\delta$$

bo'lib, $\forall n\in N$ da

$$\frac{1}{|a|^n}<\frac{1}{n\delta}$$

bo'ladi.Demak,

$$\left|\frac{1}{a^n}-0\right|=\frac{1}{a^n}<\varepsilon \qquad (\varepsilon>0)$$

tengsizlik barcha

$$n>\frac{1}{\varepsilon\delta}$$

bo'lganda o'rinli.Agar

$$n_0=\left[\frac{1}{\varepsilon\delta}\right]+1$$

deyilsa,Ravshanki, $\forall n>n_0$ uchun

$$\left|\frac{1}{a^n}-0\right|<\varepsilon$$

bo'ladi.Demak,

$$\lim_{n\to\infty}\frac{1}{a^n}=0$$

5-misol. Ushbu $x_n=\dfrac{n}{n+1}\ \ (n=1,2,3,...)$

ketma-ketlikning limiti 1 ga teng bo‘lishi isbotlansin.

Ixtiyoriy $\varepsilon>0$ son olamiz. So‘ng ushbu

$$|x_n-1|<\varepsilon$$

tengsizlikni qaraymiz. Ravshanki,

$$|x_n-1|=\left|\frac{n}{n+1}-1\right|=\frac{n}{n+1}$$

Unda yuqoridagi tengsizlik

$$\frac{n}{n+1}<\varepsilon$$

ko‘rinishga keladi. Keyingi tengsizlikdan

$$n>\frac{1}{\varepsilon}-1$$

bo‘lishi kelib chiqadi. Demak, limit ta'rifidagi $n_0\in N$ sifatida $n_0=\left[\frac{1}{\varepsilon}-1\right]+1$ olinsa ($\varepsilon>0$ ga ko’ra $n_0\in N$ topilib), $\forall n>n_0$ uchun $|x_n-1|<\varepsilon$ bo’ladi.Bu esa

$$\lim_{n\to\infty}\frac{n}{n+1}=1$$

bo‘lishini bildiradi.

6-misol. Faraz qilaylik, $a\in R,\ |a|>1$ va $\alpha\in R$ bo’lsin.U holda

$$\lim_{n\to\infty}\frac{n^\alpha}{a^n}=0$$

bo‘lishi isbotlansin.

Shunday natural k sonni olamizki $k\geq\alpha+1$ bo’lsin. Endi $|a|^{\frac{1}{k}}>1$ bo‘lishini e'tiborga olib, $|a|^{\frac{1}{k}}=1+\delta$, ya’ni $\delta=|a|^{\frac{1}{k}}-1>0$ ya'ni deymiz. Unda Bernulli tengsizligiga ko‘ra

$$|a|^{\frac{n}{k}}=(1+\delta)^n\geq1+n\delta>n\delta$$

bo’lib, $\forall\ n\in N$ da

$$\frac{n^{k-1}}{a^n}<\frac{1}{n\delta^k}$$

bo’ladi.Bu holda

$$n_0=\left[\frac{1}{\delta^k\cdot\varepsilon}\right]+1\quad(\varepsilon>0)$$

deyilsa, $\forall\ n>n_0$ uchun

$$\left|\frac{n^\alpha}{a^n}-0\right|=\frac{n^\alpha}{|a|^n}\leq\frac{n^{k-1}}{|n|^n}<\varepsilon$$

bo’ladi.Demak, $\lim\limits_{n\to\infty}\frac{n^\alpha}{a^n}=0$.

7-misol. Ushbu $\lim\limits_{n\to\infty}\frac{\lg n}{n}=0$

tenglik isbotlansin.

Ravshanki, $\forall\ \varepsilon > 0$ va $\forall\ n \in N$ uchun

$$0 \le \frac{\lg n}{n} < \varepsilon \ \Leftrightarrow\ \lg n < n\varepsilon \ \Leftrightarrow\ n < 10^{n\varepsilon} \ \Leftrightarrow\ \frac{n}{(10^{\varepsilon})^{n}} < 1$$

bo’ladi.Agar $10^{\varepsilon} > 1$ bo‘lishini e'tiborga olsak, 6-misolga ko‘ra

$$n \to \infty \text{ da } \frac{n}{\left(10^{\varepsilon}\right)^{n}} \to 0$$

ekanini topamiz. Unda ta'rifga ko‘ra 1 soni uchun

$$\exists\ n_0 \in N,\ \forall\ n > n_0 :\ \frac{n}{(10^{\varepsilon})^{n}} < 1$$

bo‘ladi. Shunday qilib, $\forall\ n > n_0$ uchun $\frac{\lg n}{n} < \varepsilon$ bo’ladi. Demak, $\lim\limits_{n\to\infty} \frac{\lg n}{n} = 0$.

8-misol. Ushbu $x_n = (-1)^n \qquad (n = 1, 2, 3, \ldots)$
ketma-ketlikning limiti mavjud emasligi isbotlansin.

Teskarisini faraz qilaylik a limitga ega bo‘lsin. Unda ta'rifga binoan,

$$\forall \varepsilon > 0,\quad \exists n_0 \in N,\quad \forall n > n_0 :\ |(-1)^n - a| < \varepsilon$$

bo‘ladi.

Ravshanki, n juft bo’lganda $|1 - a| < \varepsilon$, n toq bo’lganda $|(-1) - a| < \varepsilon$, ya’ni $|1 + a| < \varepsilon$ ya'ni bo‘ladi. Bu tengsizliklardan foydalanib topamiz:

$$2 = |(1-a) + (1+a)| \le |1 - a| + |1 - a| < 2\varepsilon .$$

Bu tengsizlik $\varepsilon > 1$ bo‘lgandagina o‘rinli. Bunday vaziyat $\varepsilon > 0$ sonining ixtiyoriy bo‘lishiga zid. Demak, ketma-ketlik limitga ega emas.