PY 506 HW06

MB

19 October 2023

As part of the Brazil nuts analysis project, you will need to fit peaks in a γ-ray spectrum that have some underlying background. While there are several methods to achieving this, the most straight forward method is as follows: (i) fit a function to the background; (ii) sum up all of the counts in the peak region; (iii) subtract the modeled number of background counts.

The function for the background is a simple linear function:

$$f(E_i) = mE_i + b \tag{1}$$

where E_i is the energy for bin i. m and b are the slope and intercept of the background model. Your task is to handle the book keeping.

Once the background is fit, the total number of counts in the peak region can be found by simply summing each bin between two boundaries as shown in green. Finally, use the background model to predict the number of background counts, and subtract that from the total counts to obtain the net area of the peak. In the figure (i.e. figure of homework file), this corresponds to subtracting the green region in the trapezoid below the red line.

1. Write a function that extracts the total number of counts in a peak as outlined above. Your function should (i) read in the limits of the background and peak regions (6 inputs), (ii) print the net area of the peak, and (iii) produce a plot. Then use your function to analyze the peaks in bin 140, 285, and 520 in the data file provided.

Solution

The average background B_i in bin x_i can be linearly interpolated by utilizing the two background region location sx_{B1} and x_{B2}, and the mean background counts per bin B_{B1} and B_{B2}:

$$B_i = \left(\frac{B_{B_2} - B_{B1}}{x_{B2} - x_{B1}}\right)(x_i - x_{B1}) + B_{B1} \tag{2}$$

A method of count summation wherein the total number of counts in the peak region is comprised of n bins:

$$T = \sum_i^n T_i \tag{3}$$

where T_i represents the total number of counts in bin x_i. The estimated number of background counts from B_i is then

$$B = \sum_i^n B_i \tag{4}$$

The number of signal counts in the peak region is given by

$$S = \sum_{i}^{n} S_i = T - B \tag{5}$$

We may further assume the counting statistics follow a Poissonian distribution with standard deviation equal to the square root of the mean value, with no correlation between the peak's number of counts and the background regions. The uncertainty in the signal counts is

$$\sigma_S = \sqrt{\sigma_T^2 + \sigma_B^2} = \sqrt{T + B} \tag{6}$$

Where σ_T and σ_B are the standard deviations in the total and background counts, respectively. The signal centroid is estimable via the signal counts in each bin, $S_i = T_i - B_i$. Here the signal centroid is defined as the sample mean

$$\bar{x} = \frac{1}{S} \sum_{i}^{n} S_i x_i \tag{7}$$

Effectively, the peak analysis for bin 140 appears as

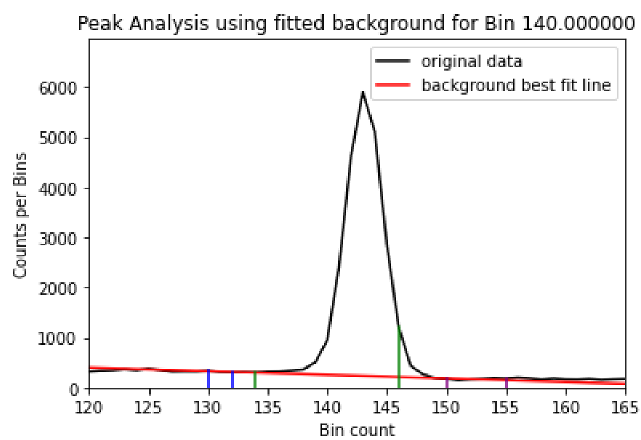

Figure 1: Peak Analysis using a fitted background - bin 140

Unsurprisingly, the background best fit line touches only the background lower left, lower right, upper left, upper right, center left, and center right bounds. As we can see, the peak in the counts per bins occurs at exactly bin 140.

Below is the peak analysis for bin 285 and 520:

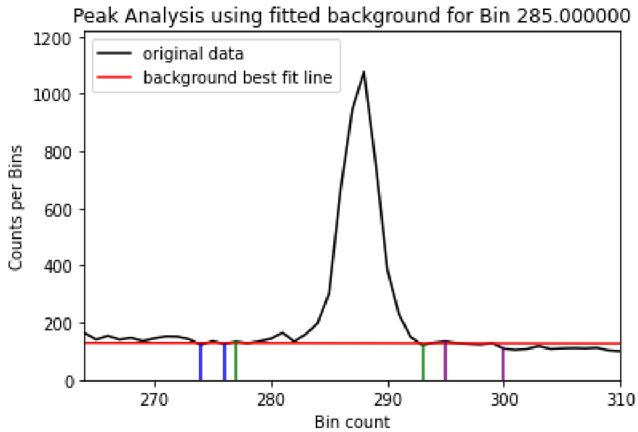

Figure 2: Peak Analysis using fitted background - bin 285

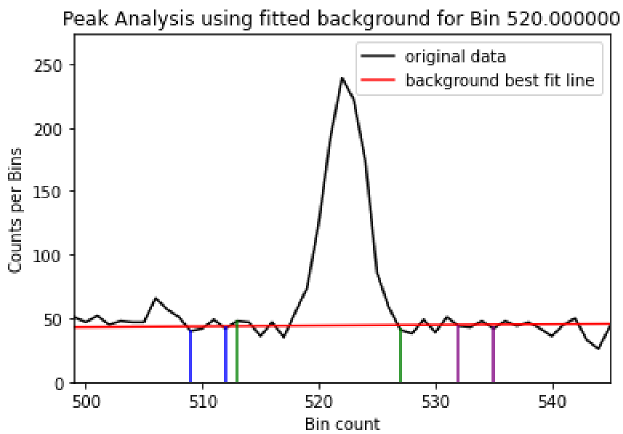

Figure 3: Peak Analysis using fitted background - bin 520

As we can see, analysis of the peaks in each bin results in best fit lines that each appear at a level significantly lower than the peak in the original data. This is expected since the function for the background is linear and thus must hit more points below the peak for any pattern. Nevertheless, reading in the limits of the background and peak regions to print the net area of the peaks and plotting the regions shows successful.

In conclusion, as counts accumulate in each peak, we are able to determine average locations by weighting the bin values x_i by the number of signal counts contained in each bin S_i.