PY 506 HW02

MB

1 September 2023

The goal of this assignment is to use the data provided (BE-per-nucleon data) and fit the semi-empirical binding energy formula to estimate the constants, given by

$$B = a_v A - a_s A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_{symm}(A-2Z)^2/A \tag{1}$$

Hints provided: (1) Fitting problems generally fall into two categories, linear and nonlinear. First, figure out what kind of problem this is. (2) There is data on both stable and unstable nuclides in the data file. (3) Columns are defined in the header of the data file.

Solution

As seen above, the semi-empirical binding energy (BE) formula predicts that the ground state BE is only dependent on the mass number A and independent of the atomic number Z. For code construction, Python's *scipy* package took implementation for its optimization and curve fit capabilities. We first read the data file into our platform and transferred the data into separate columns. This way any column could represent any particular axis for plotting. We then defined variables for the fitting parameters after the curve fit and produced the following graph:

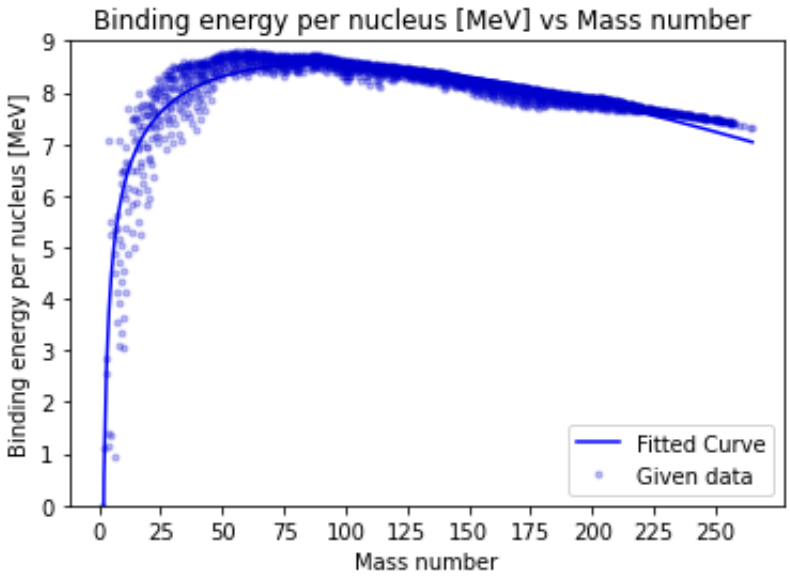

Figure 1: Caption

With the fitting parameters for the plot, which read: $a_v = 1.000020250263179$, $a_s = 2.1743294815216476$, $a_C = -0.21376731445574987$, and $a_{symm} = 257396.21191049257$.

Analysis

As seen above, the graph starts to rise linearly in agreement with the volume term where $B \propto A$, and then weakly falls as a parabola. This parabolic drop agrees with the remainder correction terms for the binding energy formula. Around the tail-end of the graph's curve data, the curve-fit drops away from the trend of the last few data points. We predict this occurs since the semi-empirical BE formula does not account for the protons and neutrons preference to live in pairs. In that account, we would've added to the formula a δ term such that

$$\delta = \begin{cases} +a_\rho A^{-3/4}, & Z \text{ and } N \text{ even} \\ -a_\rho A^{-3/4}, & Z \text{ and } N \text{ odd} \\ 0, & A \text{ odd} \end{cases} \tag{2}$$

Had the pairing factor δ been included, we would expect our curve fit to follow the provided data across all points. Nevertheless, the rest of the graph effectively follows the curve fit, successfully accounting for every other factor: drop volume, nucleons on surface seeing less neighbors, Coulomb repulsion, and symmetry.

Looking at the parameter values after curve-fitting, the highest parameter value is the symmetry term and the lowest parameter value is the Coulomb repulsion term. This is expected since the symmetry contains the highest power of A as A^2 and the Coulomb term contains the lowest power of A as $A^{-1/3}$. Lastly, the parameter fit values could appear well off from the parameters of a better fit for the data provided. For our code, the number of iterations for the curve fit was limited and thus reduced the precision of the results.

Conclusion

According to our code, the semi-empirical binding energy formula appears to be correctly curve-fitted. Extraneous solutions exist for curve-fitting problems (e.g. min-max) that, unsurprisingly, provide variations in curvature for resulting curve fits. Nevertheless, each approximation approach is expected to agree with the provided data points, as does our solution.