Chapter 13: Matrix

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1 General approach

1.1 Definition

1.1.1 Definition of a matrix

We call matrix of n rows and p columns any mapping in the following form:

$$\begin{aligned} [\![1,n]\!] \times [\![1,p]\!] &\to \mathbb{K} \\ i,j \quad &\quad a_{ij} \end{aligned}$$

We denote such maps as tables of n rows and p columns, and we write:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{np} \end{pmatrix}$$

$\forall (i,j) \in [\![1,n]\!] \times [\![1,p]\!]$, we call a_{ij} a coefficient of the matrix. In this case coefficient if i-th row and j-th column.

1.1.2 Notation

We denote $M_{np}(\mathbb{K})$ the set of matrix of n rows and p columns with coefficient from $\mathbb{K}$.

1.1.3 Examples

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \in M_{32}(\mathbb{R})$$

$$B = \begin{pmatrix} i \\ 1+i \\ 3 \end{pmatrix} \in M_{31}(\mathbb{C})$$

1.2 Particular matrices

Let $A \in M_{np}(\mathbb{K})$ then:

1.2.1 Null matrix

1. $[\forall (i,j) \in [\![1,n]\!] \times [\![1,p]\!], a_{ij} = 0] \Rightarrow [A = 0_{np}]$ We say A is the null matrix $M_{np}(\mathbb{K})$.

1.2.1.1 Example

$$A' = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \in M_{32}(\mathbb{R})$$

1.2.2 Column matrix

2. $B \in M_{np}(\mathbb{K})$ and $p = 1 \Rightarrow$ B is a column matrix of n rows

1.2.2.1 Example

$$B' = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \in M_{31}(\mathbb{R})$$

1.2.3 Row matrix

3. $B \in M_{np}(\mathbb{K})$ and $n = 1 \Rightarrow$ C is a row matrix of p columns

1.2.3.1 Example

$$C' = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \in M_{13}(\mathbb{R})$$

1.2.4 Square matrix

We call square matrix any matrix with same number of rows and columns. We denote $M_n(\mathbb{K})$ the set of square matrix of n rows and columns with coefficient from $\mathbb{K}$.

4. $D \in M_{np}(\mathbb{K})$ and $n = p \Rightarrow$ D is a square matrix denote $M_n(\mathbb{K})$

1.2.4.1 Example

$$D' = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \in M_3(\mathbb{R})$$

1.2.5 Diagonal matrix

5. $\forall E \in M_n(\mathbb{R})$, if $\forall (i,j) \in [\![1,n]\!]^2, i \neq j \Rightarrow a_{ij} = 0$ then we say E is a diagonal matrix

1.2.5.1 Example

$$E' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \in M_2(\mathbb{R})$$

$$E'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \in M_3(\mathbb{R})$$

1.2.6 Triangular matrix

6. $\forall F \in M_n(\mathbb{R})$, if $\forall (i,j) \in [\![1,n]\!]^2, i > j \Rightarrow a_{ij} = 0$ then we say F is a lower triangular matrix

7. $\forall G \in M_n(\mathbb{R})$, if $\forall (i,j) \in [\![1,n]\!]^2, i < j \Rightarrow a_{ij} = 0$ then we say G is a upper triangular matrix

1.2.6.1 Example

$$F' = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} \in M_2(\mathbb{R})$$

$$G' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \in M_2(\mathbb{R})$$

1.3 Transposed matrix

1.3.1 Definition

Let $A \in M_{np}(\mathbb{K})$. We call transposed matrix of A (or A transpose) a matrix B from $M_{pn}(\mathbb{K})$ such as:

$$\forall (i,j) \in [\![1,n]\!] \times [\![1,p]\!], a_{ij} = b_{ji}$$

1.3.2 Notation

We denote B as ${}^tA$

1.3.3 Example

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \in M_{23}(\mathbb{R})$$

$${}^tA = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \in M_{32}(\mathbb{R})$$

1.4 Symmetric matrix

1.4.1 Symmetric

If ${}^tA = A$ then we say A is symmetric

1.4.1.1 Example

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} = {}^tA = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} \in M_3(\mathbb{R})$$

1.4.2 Anti-Symmetric

If ${}^tA = -A$ then we say A is Anti-symmetric

1.4.2.1 Example

$$A = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & -5 \\ -3 & 5 & 0 \end{pmatrix} = {}^tA = \begin{pmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{pmatrix} \in M_3(\mathbb{R})$$