

# Chapter 11: Vector Spaces

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# 1 Introduction

## 1.1 General approach

Studying Vector spaces will allow us to notice general theorems that can be applied to many mathematical structures.

## 1.2 Definition

A vector space is a set whose elements, often called vectors, may be added together and multiplied ("scaled") by numbers called scalars.

## 1.3 Notation

A  $\mathbb{K}$  – *VectorSpace* is non-empty set  $E$  that has :

- An internal law, which is an map of  $E \times E$  :

$$\begin{aligned} E \times E &\longrightarrow E \\ (u, v) &\longmapsto u + v \end{aligned}$$

## 1.4 Properties

# 2 Vector Sub-Spaces

## 2.1 Definition

## 2.2 Properties

# 3 Relations between $V_s$ and $V_{ss}$

## 3.1 Linear Combinations

## 3.2 $V_{ss}$ Intersection

## 3.3 $V_{ss}$ Sum

## 3.4 Generated $V_{ss}$

# 4 Families

## 4.1 Free Family

## 4.2 Spanning Family

# 5 Basis

# 6 Dimension of a $V_{ss}$

# 7 Proofs

## 7.1 Intersection of linear subspaces

## 7.2 Sum of linear subspaces

## 7.3 Spanned linear subspaces

## 7.4 Basis

## 7.5 Existence of a basis in a finite dimension