MFFM ECON5022

Lecture 1: Introduction to Financial Time Series

Dr. Minjoo Kim

Economics
University of Glasgow

Financial Econometrics

- Estimating the parameters of well-defined probability models that describe the behavior of financial time series.
- Testing hypotheses how financial markets (or systems) generate the financial time series like stocks, bonds, foreign exchange rates.
- Forecasting the future values of financial time series.

What is a time series?

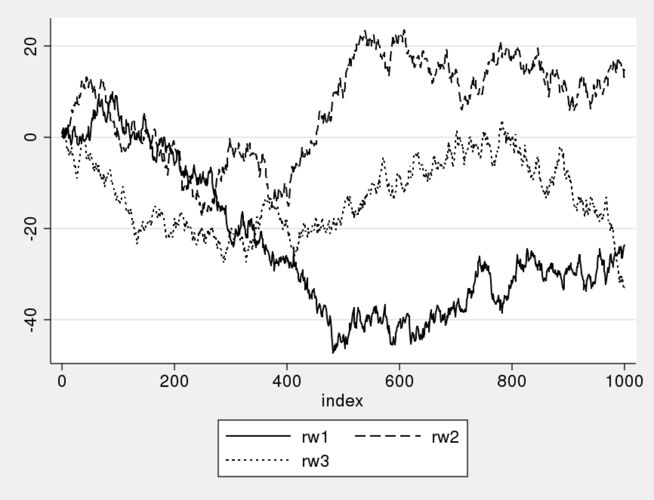


Figure: Simulated random walk processes

- A random variable which evolves in time.
- A variable which is dynamic.
- A variable which fluctuates over time.
- A variable which admits different values at different points in time.

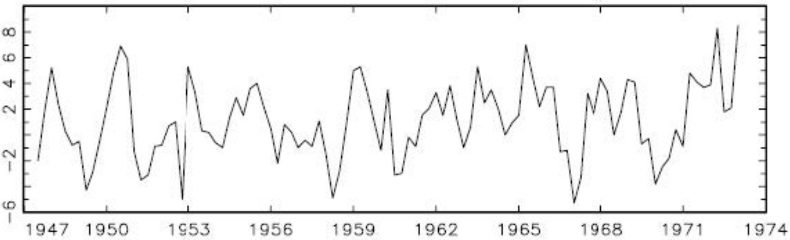
For that reason, we will denote a time series with a subscript t to denote time periods, i.e: X_t, ϵ_t, Y_t, where $t = 1, \ldots, T$, or $t = 1980, \ldots, 2000$ or $t = 01/01/1965, \ldots, 31/12/2000$.

How do we measure a time series?

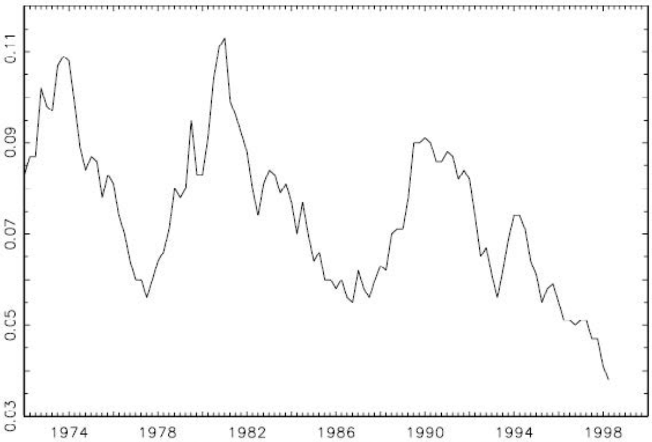
- Time series are (usually, but not necessarily always) measured at regular intervals, e.g. every day, every Friday of the week, every 1st of the month, every 17th of the month.
- We can collect data from national statistical offices, central banks, commercial banks, and databases: Bloomberg, IMF Financial Statistics, OECD database.

Examples of time series

- Economics - e.g., monthly unemployment rate, hospital admissions, etc.
- Finance - e.g., daily exchange rate, share price, etc.
- Environmental - e.g., daily rainfall, air quality readings.
- Medicine - e.g., ECG brain wave activity every 2-8 secs.



(a) quarterly changes in U.S. fixed investment



(b) quarterly German long-term interest rate (Umlaufsrendite)

How do we characterise a time series?

- Consider we collect a dataset like stock returns,

$$\{x_1, x_2, \ldots, x_T\}.$$

- Then we may need to get some insight on the statistical properties characterizing it. The usual approach is to see its distributional properties described by **mean**, **variance**, **skewness** and **kurtosis**.

- **Mean**:
$$\mu_x := E\left[X_t\right]. \tag{1}$$
The sample counterpart of mean is given as
$$\hat{\mu}_x = \frac{1}{T}\sum_{t=1}^{T} x_t. \tag{2}$$
- **Variance**:
$$\sigma_x^2 := Var\left(X_t\right) = E\left[\left(X_t - \mu_x\right)^2\right]. \tag{3}$$
The sample counterpart of variance is given as
$$\hat{\sigma}_x = \frac{1}{T-1}\sum_{t=1}^{T}\left(x_t - \hat{\mu}_x\right)^2. \tag{4}$$

- **Skewness**:

$$S_x := Skew\left(X_t\right) = E\left[\left(\frac{X_t - \mu_x}{\sigma_x}\right)^3\right]. \tag{5}$$

The sample counterpart of skewness is given as

$$\hat{S}_x = \frac{1}{T-1}\sum_{t=1}^{T}\left(\frac{x_t - \hat{\mu}_x}{\hat{\sigma}_x}\right)^3. \tag{6}$$

- **Kurtosis**:

$$K_x := Kurt\left(X_t\right) = E\left[\left(\frac{X_t - \mu_x}{\sigma_x}\right)^4\right]. \tag{7}$$

The sample counterpart of kurtosis is given as

$$\hat{K}_x = \frac{1}{T-1}\sum_{t=1}^{T}\left(\frac{x_t - \hat{\mu}_x}{\hat{\sigma}_x}\right)^4. \tag{8}$$

Autocorrelation

Autocorrelation function (acf)

- Time series variables have the property that a value at any point in time t can be correlated with past or future values. Thus we define the notion of autocorrelation, i.e. how a time-series is correlated with its past or future values.
- The correlation between current and past values is a function of the distance between the two time realizations, k:

$$\rho_k = \frac{Cov\left(X_t, X_{t-k}\right)}{\sqrt{Var\left(X_t\right)}\sqrt{Var\left(X_{t-k}\right)}}. \tag{9}$$

- The sample counterpart of autocorrelation is given as

$$\hat{\rho}_k = \frac{\frac{1}{T-k}\sum_{t=k+1}^{T}\left(x_t - \hat{\mu}_x\right)\left(x_{t-k} - \hat{\mu}_x\right)}{\frac{1}{T-1}\sum_{t=1}^{T}\left(x_t - \hat{\mu}_x\right)^2} \tag{10}$$

under the stationarity condition.

Autocorrelation

Partial autocorrelation function (pacf)

- Measures the correlation between an observation k periods ago and the current observation, after controlling for observations at intermediate lags (i.e. all lags $< k$). So the partial autocorrelation measures the correlation between X_t and X_{t-k} after removing the effects of $X_{t-1}, X_{t-2}, \dots, X_{t-k+1}$.

$$\tau_k = \frac{Cov\left(X_t, X_{t-k} | X_{t-1}, \dots, X_{t-k+1}\right)}{\sqrt{Var\left(X_t | X_{t-1}, \dots, X_{t-k+1}\right) Var\left(X_{t-k} | X_{t-1}, \dots, X_{t-k+1}\right)}}. \tag{11}$$

  - At lag 1, the $ACF = PACF$ always.
  - At lag 2, $\tau_2 = (\rho_2 - \rho_1^2)(1 - \rho_1^2)$.
  - For lags 3+, the formulae are more complex.

- From an empirical point of view, pacf measures the linear relationship between t and $t-k$. We can estimate it by the following regression:

$$X_t = \beta_0 + \beta_1 X_{t-1} + \cdots + \beta_{t-k+1} X_{t-k+1} X_{t-k+1} + \tau_k X_{t-k} + \epsilon_t. \tag{12}$$

Clarification

What is x_t and x_{t-k} in the definition above?

- Usually we assume a x_t which takes values $\{x_1, x_2, \ldots, x_{T-1}, x_T\}$, so that x_{t-k} will take values $\{x_{1-k}, x_{2-k}, \ldots, x_{T-k-1}, x_{T-k}\}$, for any $k > 0$.
- The notation x_0 or x_{-100} is valid to use, since we assume (at least theoretical statisticians do...) that: A time series x_t is the realization of an unobserved stochastic process X_t which started in the (in)finite past.
- Values x_{-h} do exist, however we do not observe them in our sample (you might gather data for U.S. inflation between 1960-2012, but inflation did exist before 1960!).
- Since the history of x_t for $t < 1$ is unknown (because a sample was not available), we usually refer to x_0 as the "initial condition" of x_t i.e. the point it time where x_t initiated/started.

Stationarity

A time series is called stationary if it has invariable first (mean) and second (variance) moments. (**weak stationarity**)

Example: According to the above it should hold that

$$E\left[X_t\right] = E\left[X_{t-k}\right] \tag{13}$$

$$Var\left(X_t\right) = Var\left(X_{t-k}\right) \tag{14}$$

A **more realistic example**: Given $\{X_t\}_{t=1}^{T} := \{X_1, X_2, \ldots, X_{T-1}, X_T\}$ and any random samples, for instance $\{X_{t_1}\}_{t_1=1}^{s} := \{X_1, X_2, \ldots, X_{s-1}, X_s\}$ and $\{X_{t_2}\}_{t_2=s+1}^{T} := \{X_{s+1}, X_{s+2}, \ldots, X_{T-1}, X_T\}$ for any $1 < s < T$. Then the stationarity implies that

$$E\left[X_{t_1}\right] = E\left[X_{t_2}\right] \tag{15}$$

$$Var\left(X_{t_1}\right) = Var\left(X_{t_2}\right) \tag{16}$$

$$\tag{17}$$

Are economic variables stationary?

In most cases economic variables are not stationary.

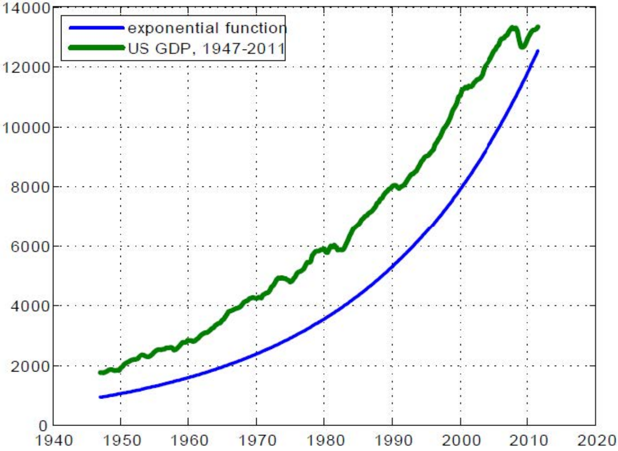
Economic and financial time series can be decomposed into the following components:

$$X_t = T_t \times S_t \times C_t \times E_t \tag{18}$$

where

- Trend (T_t): (Deterministic) long term movements in the mean;
- Seasonal effects (S_t): Cyclical fluctuations related to the calendar;
- Cycles (C_t): Other cyclical fluctuations (such as a business cycles);
- Residuals (E_t): Other random or systematic fluctuations.

An example: U.S. GDP



- We see that U.S. GDP has an upward trend. This trend follows closely the exponential function, hence we talk about "exponential trend":

$$T_t = \exp(\alpha t).$$

- There may be also cycles in this series (the upward and downward movements in the curve):

$$C_t = \exp(\beta \cos(\omega t)).$$

- There is no seasonality in this particular series (GDP in the figure has been seasonally adjusted by the source, the Federal Reserve Bank of St. Louis).

Are the upward, exponential trend, and the cycles evident in the GDP series, connected somehow to its nonstationary features?

Variable with trend

- Assume for now that GDP has only an exponential trend, plus random fluctuations:

$$X_t = \exp(\alpha t) \times E_t.$$

- Take a logarithms of GDP:

$$x_t = \alpha t + \epsilon_t$$

where $x_t = \ln X_t$ and $\epsilon_t = \ln E_t$.

Even if ϵ_t is stationary, i.e. $E[\epsilon_t] = c$, x_t is not.

- Assume $T = 10$, and take two subsamples of 5 observations each. Then we have:

$$E[x_{t_1}] = E\left[\sum_{t=1}^{5} \alpha t + \epsilon_t\right] = \alpha(1 + 2 + \cdots + 5) + c$$

$$E[x_{t_2}] = E\left[\sum_{t=6}^{10} \alpha t + \epsilon_t\right] = \alpha(6 + 7 + \cdots + 10) + c$$

- Hence, it holds that

$$E[x_{t_1}] \neq E[x_{t_2}] \text{ for } \alpha \neq 0.$$

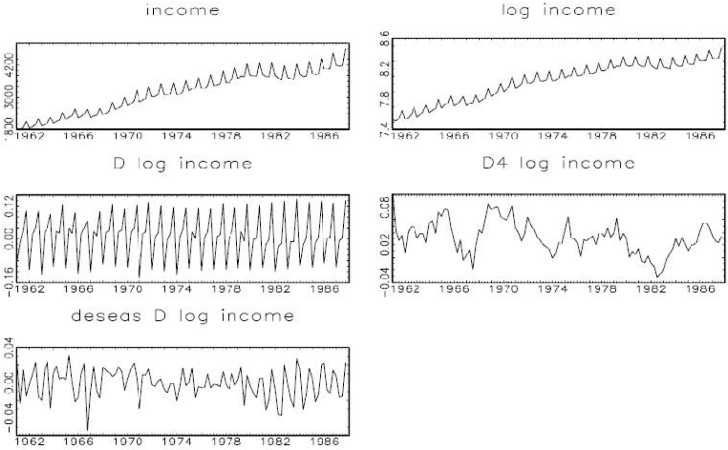
Stationarity transformations

- We see that variables like GDP are nonstationary. However, for reasons we are going to see later, it is preferable to work with stationary variables.
- In many cases, we cannot model at all highly nonstationary variables. Hence, we need to transform our variable to a new one which is stationary, i.e. $Y_t = f(X_t)$.
- For example, take a logarithms of X_t in (16), $Y_t = f(X_t) = \ln X_t$:

$$Y_t = \tau_t + s_t + c_t + \epsilon_t \tag{19}$$

where $\tau_t = \ln T_t$, $s_t = \ln S_t$, $c_t = \ln C_t$ and $\epsilon_t = \ln E_t$.

An example: German Disposable Income, 1961-1987



- The level of the income series has an upward trend over time (time scale is on the horizontal axis).
- There is also a strong seasonal pattern. This can be climatic (in the summer more consumption of holidays in Greece is demanded by Germans), or just periodic (more consumption during Christmas, hence less disposable income at that period)

Log transformation

- The first transformation is to take natural logarithms, ln (note: this is valid only for positive valued time series).
- Since the trend in the income is linear (not like the exponential trend of U.S. GDP), this transformation does nothing to the series.
- Seasonality, which can lead to spurious results, is not affected by taking logarithms.

First difference

- Taking first differences of the logarithms, $Y_t = \ln X_t - \ln X_{t-1}$, we can see that we have created a stationary time series (see how all values fluctuate around the mean, which is close to 0).
- However, seasonality is still a problem. The "peaks" in the series are due to seasonal effects, and not because of the structure of the time series.

- Put differently: disposable income increases the exact same quarter every single year just because Christmas is over, not because the economy has actually grown?
- What if we take the annual (4 quarter) differences of the logarithms, $Y_t = \ln X_t - \ln X_{t-4}$?
- Then seasonality will be removed, AND we will have a stationary series. The upward trend is removed, the mean and variance look constant over time. The mean is around 0.02, and the series fluctuates? normally? within the bounds (-0.03, 0.07).

Seasonality adjusted dataset

- The last graph shows what happen if we remove seasonality first, and then take first log-differences.
- There are many ways to remove seasonality (none is “perfect”), but these will not be discussed in this class.
- Usually all the data you get from statistical agencies, or Bloomberg etc, are deseasonalised for you.

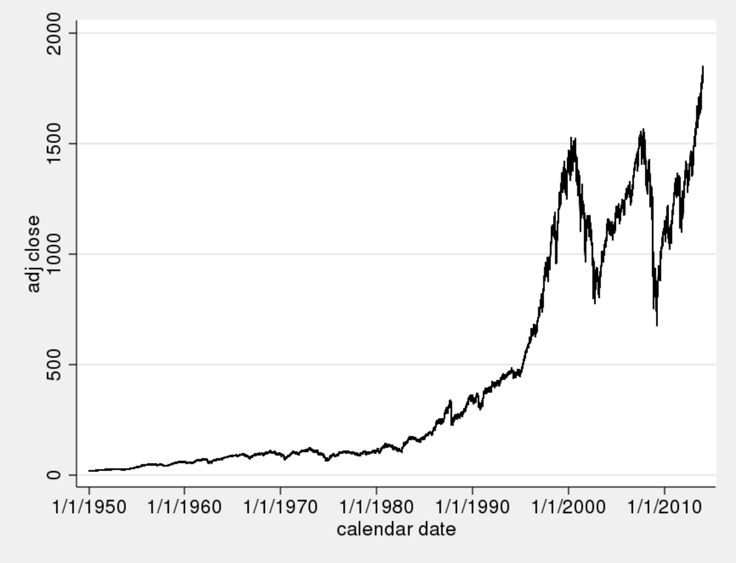
A few helpful transformations

Popular stationarity transformations.

1. First difference: $Y_t = X_t - X_{t-1}$.
2. Second difference: $Y_t = X_t - X_{t-2}$.
3. Logarithm: $Y_t = \ln X_t$.
4. First log-difference: $Y_t = \ln X_t - \ln X_{t-1}$.
5. Second log-difference: $Y_t = \ln X_t - \ln X_{t-2}$.
6. Annual log-difference, where $h = 12$ for monthly, and $h = 4$ for quarterly data: $Y_t = \ln X_t - \ln X_{t-h}$.

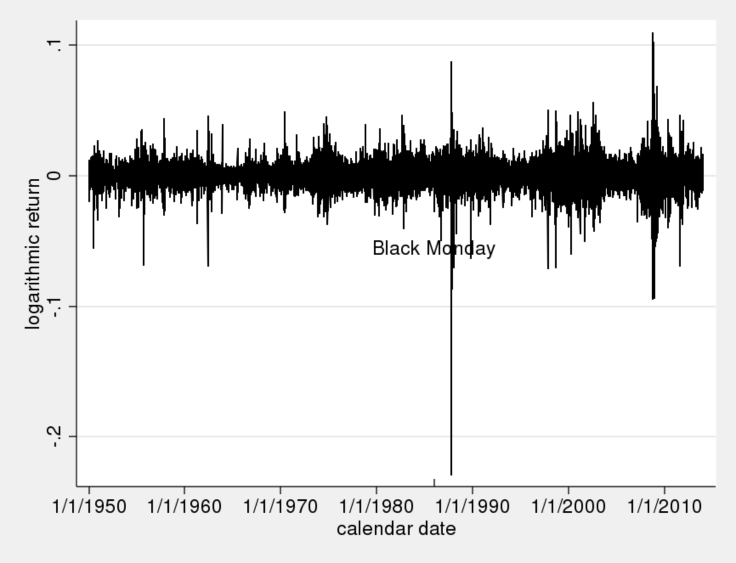
Note that there are several other transformations which are based on more advanced time-series theory, that will not be covered in this introductory course.

Analysing stock prices

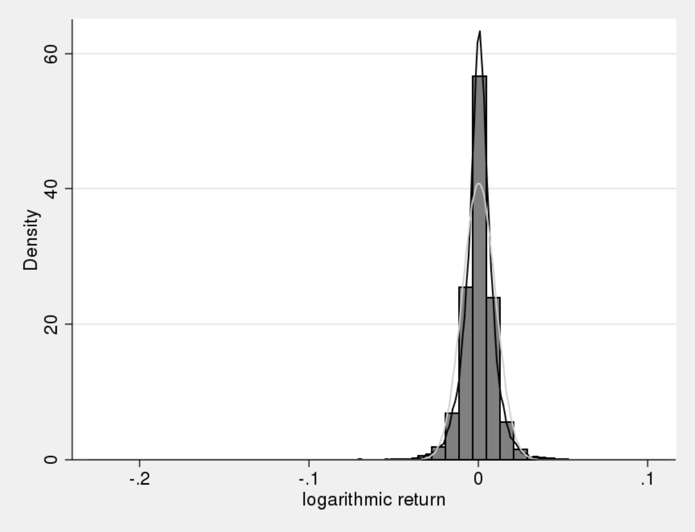


You can easily see that the price series is not stationary!

- Take the mean in sample 0 – 1000, then 1001 – 2000, then 2001 – 3000 and so on, or any other arbitrary choice of subsamples. The means (and variances) in these subsamples differ substantially.
- The series has an upward trend, especially during the first 10,500 observations.
- Take a transformation of stock prices which will create a stationary variable that we can easily work with.
- For that reason we will work with stock returns which is the first log-difference of stock price.

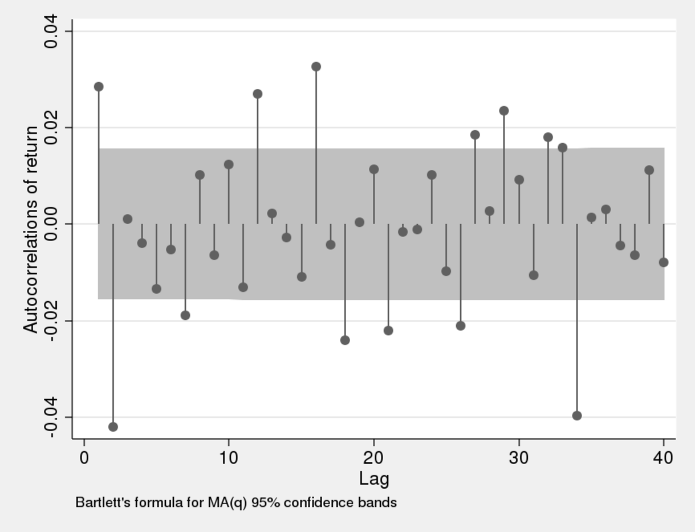

logarithmic return
.1
0
-.1
-.2
Black Monday
1/1/1950
1/1/1960
1/1/1970
1/1/1980
1/1/1990
1/1/2000
1/1/2010
calendar date

- We see from the graph of the series that the series has NO trend, and that the mean is constant (the series fluctuates around its mean which is close to 0).
- However, the variance is not constant. There are periods with very extreme negative returns (stock market crashes) and a few periods with extreme positive returns.



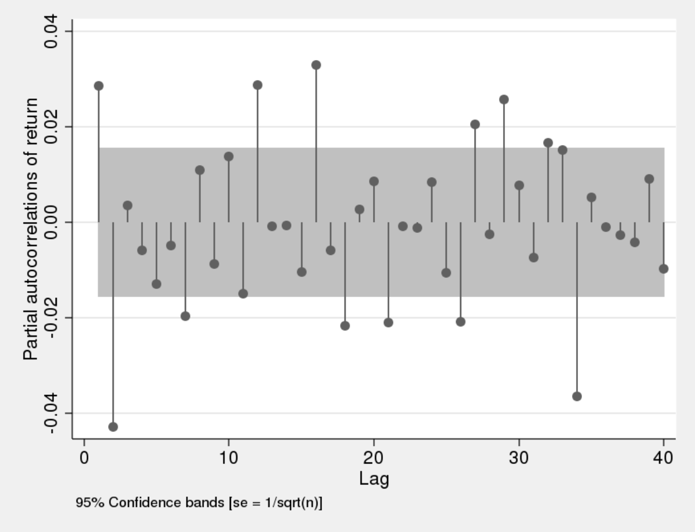
Source: Boffelli and Urga (2016)

Figure: Histogram of daily S&P returns



Source: Boffelli and Urga (2016)

Figure: Autocorrelogram for daily S&P returns



Source: Boffelli and Urga (2016)

Figure: Partial autocorrelogram for daily S&P returns

Which transformation to use?

- *One-period simple or arithmetic returns*

$$R_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}$$

where P_t is (ex-dividend) asset price, D_t is dividends.

- *Continuously compounded or logarithmic returns*

$$r_t = \ln(1 + R_t) = \ln \frac{P_t + D_t}{P_{t-1}}$$

Which transformation to use?

- Advantage of simple returns: return of portfolio is weighted sum of returns on individual assets:

$$R_t = \sum_{i=1}^{N} w_i R_{i,t}$$

- Advantage of continuously compounded returns: multi-period return is sum of single-period returns:

$$r_{t-k+1,t} = r_{t-k+1} + r_{t-k+2} + \cdots + r_t$$

while $R_{t-k+1,t} = (1 + R_{t-k+1})(1 + R_{t-k+2}) \cdots (1 + R_t) - 1$.

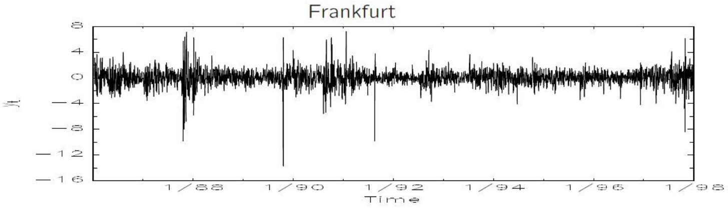
Usually, financial econometricians work with logarithmic returns because they have better properties with respect to arithmetic returns.

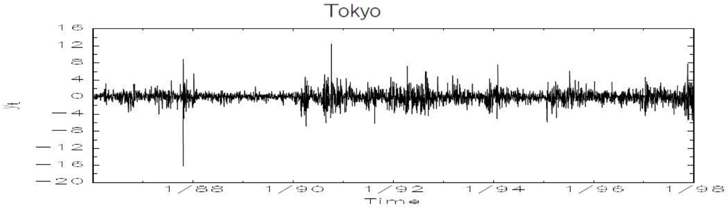
1. By assuming that returns are normally distributed, prices will by definition follow the log-normal distribution. The log-normal distribution ensures that prices are positive.
2. Logarithmic returns are usually adopted to derive closed-form solutions for pricing formulas.

Properties of high-frequency stock returns

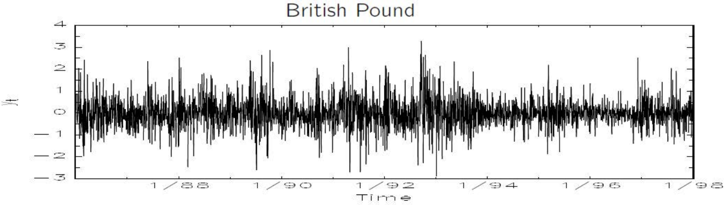
- **Leptokurtic**: Large (and small) returns occur more often than expected under normality, i.e. Distribution of returns is fat-tailed (and peaked).
- **Negative skewness**: Large negative stock returns occur more often than large positive ones.
- **Volatility clustering**: Large returns tend to occur in clusters. Periods of high volatility alternate with more tranquil periods.
- **Leverage effect**: Large volatility often follows large negative returns.

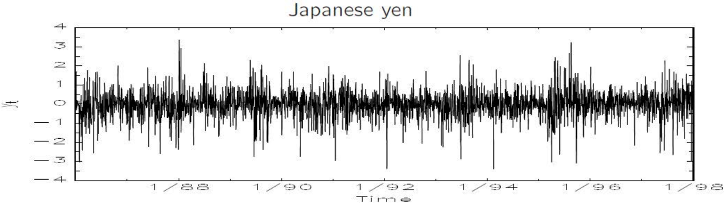
Daily stock index returns, 1986-1997



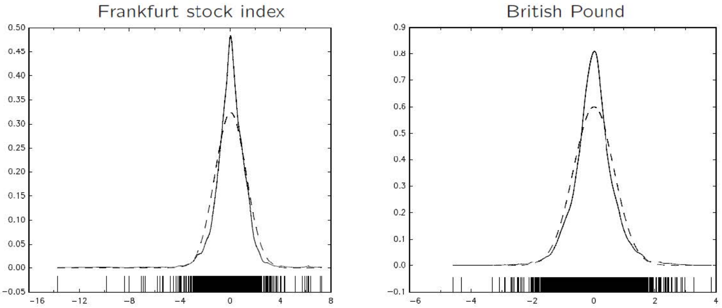


Daily exchange rate returns, 1986-1997





Distribution of daily returns, 1986-1997



Summary statistics for daily stock returns

Stock market	Mean	Med	Var	Skew	Kurt
Amsterdam	0.038	0.029	1.279	−0.693	19.795
Frankfurt	0.035	0.026	1.520	−0.946	15.066
Hong Kong	0.057	0.022	2.867	−5.003	119.241
London	0.041	0.027	0.845	−1.590	27.408
New York	0.049	0.038	0.987	−4.299	99.680
Paris	0.026	0.000	1.437	−0.529	10.560
Singapore	0.019	0.000	1.021	−0.247	28.146
Tokyo	0.005	0.000	1.842	−0.213	14.798

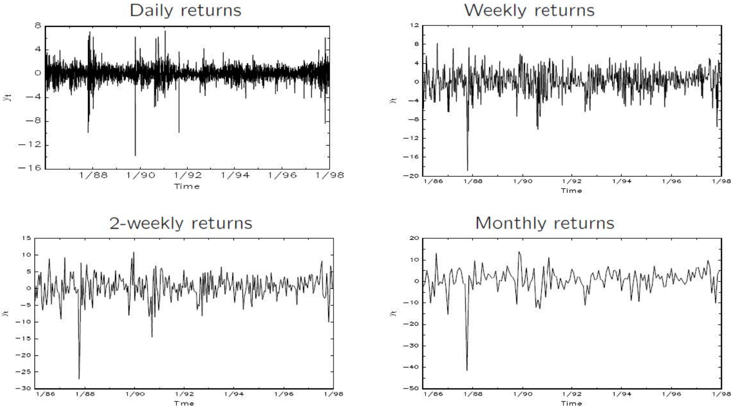
Summary statistics for daily stock returns, January 1, 1986 - December 31, 1997

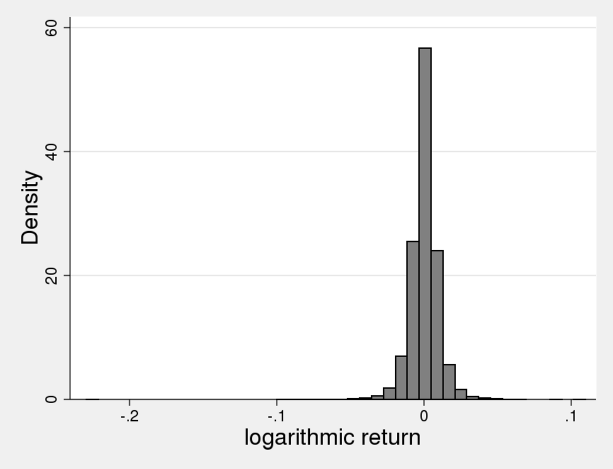
Summary statistics for daily exchange rate returns

Currency	Mean	Med	Var	Skew	Kurt
Australian dollar	0.012	−0.012	0.377	1.893	35.076
British Pound	0.006	0.000	0.442	0.058	5.932
Canadian dollar	0.006	0.000	0.076	0.101	6.578
Dutch guilder	−0.000	0.012	0.464	−0.143	4.971
French franc	0.008	0.016	0.457	0.054	6.638
German DMark	−0.001	0.017	0.475	−0.136	4.921
Japanese yen	−0.016	0.006	0.478	−0.541	6.898
Swiss franc	−0.003	0.020	0.582	−0.188	4.557

Summary statistics for daily exchange rate returns, January 1, 1986 - December 31, 1997

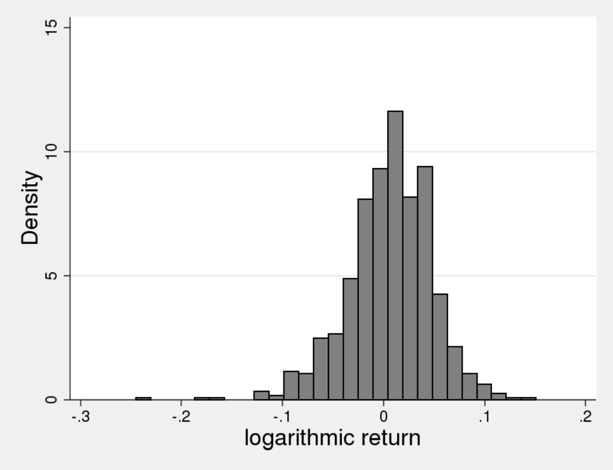
Data frequency matters





Source: Boffelli and Urga (2016)

Figure: Histogram of daily S&P returns



Source: Boffelli and Urga (2016)

Figure: Histogram of monthly S&P returns

- The daily distribution is much longer than that of the monthly one due to the events of Black Monday.
- The daily distribution is much more peaked than the monthly one. The monthly one looks close to the normal distribution.
- The higher the frequency of sampled data, the more deviated from normality.