Bayesian Networks - Practical 2

This practical will help you follow the workings of a Bayesian network and give you some practice at using Bayes rule.

If you didn't finish the practical last time, please do that first. Make sure you have built the final network properly. This week, you will need the networks you built last time.

Part 1. Start with the first model, which calculates the probability of the suits of two cards. Start an excel spreadsheet to keep your workings in, because you are going to re-create the network in Excel. First, build the table of prior probabilities for the first card in a spreadsheet. Then, in the same sheet, build a table of conditional probabilities of the second card, given the first. It should look something like this:

Card 1	Prob		Card 2	Heart	Spade	Diamond	Club	
Heart		Card 1						Sum
Spade		Heart						1
Diamond		Spade						1
Club		Diamond						1
		Club						1
Sum	=sum(..)							

You, of course, will fill in the missing values! The sum values are to check that you have the right answers. They should sum the values in the row to their left, or the column above and should, of course, add up to 1. You should get excel to calculate the values for you by entering =x/y in the cell, where x is the number of cards that match each value, and y is the number of cards to pick from (52 or 51).

Once you have the values typed in, you can use Bayes rule to implement the reverse calculations:

1. Given that the second card is a Heart, what is the probability that the first one was also a Heart?
2. Given that the second card is a Heart, what is the probability that the first one was a Spade?

Compare the answer you get in the spreadsheet with what Genie tells you is the answer.

Bayes rule is: $P(B|A) = \dfrac{P(A|B) \, P(B)}{P(A)}$

You will find P(B) from the first table in your spreadsheet and P(A|B) from the second one. You can calculate P(A) either from the table or using what you already know. P(A) is the probability of getting a given suit (say a Heart) on the second card.

The long hand way to work out the probability of having Heart as second card is:
P(Heart2 | Heart1)*P(Heart1)+

P(Heart2 | Club1)*P(Club1)+
P(Heart2 | Diamond1)*P(Diamond1)+
P(Heart2 | Spade1)*P(Spade1)

Work that value out in the spreadsheet and see if it matches your common sense answer (if you see only one card, is the probability of it being Heart different if it is the first one or the second one drawn?).

Now you have all you need to answer questions 1 and 2 above.

That's an easy one, because P(A)=P(B).

Part 2. Try another example about cards. This time, build an excel version of the probabilities of picking a Red or Black card from a full deck connected to the conditional probabilities of that card being one of the possible suits, given the colour of the card. Don't worry about the value of the card, just imagine it is from a *full deck* (so it is more simple than the example from last week).

What is the probability of the card bing a diamond, given that it is red?
What is the probability of the card being black, given that it is a diamond? When you have given your answer, show how it follows from the values you have in your table and the Bayes theorem, if needed.

Part 3. Now, build an excel version of the probabilities of picking a Red or Black card from a full deck connected to the conditional probabilities of getting each of the possible suits, given the colour of the ***first*** card. Don't worry about the value of the card, just imagine it is from a *full deck* (so it is more simple than the example from last week).

Once out have completed the table above, try to work out the probability of the first card being red given that the second card is a diamond. Does the answer match what you would expect?