Lecture 4: Bayesian Networks 2

Jingpeng Li

Content

- How to build a Bayesian Network?
  - Example: Glandular fever
  - Step 1: Collect information
  - Step 2: Convert information into a Bayesian network
- Using the Bayesian network
  - 3 examples of questions/ information we may wish to determine
- Types of nodes
  - Parent node
  - Child node
- Bayesian network models

Example: glandular fever

- Suppose we know that on average, 1% of the population have had glandular fever (GF).
- (In probability terms p(had_GF) = 0.01)
- Suppose we have a test for having had glandular fever such that:
  - For a person who has **had GF** the test would give a positive result with probability 0.977
  - For a person who has **not had GF** the test would give a negative result with probability 0.926

Q: How could this information be represented as a BN
Q: How could the BN be used to find out new information?

3

How to build a Bayesian network?

Step 1: collect information

- List information we are given
- Determine information we can deduce from the information we are given

Step 2: convert information into a BN

- Determine the nodes
- Determine Relationships between Nodes
- Convert information into a Network

4

The GF network… what we know, and what we can deduce

P(had GF) = 0.01

Looking at the different events that can occur, we have:
A person has had GF
A person has not had GF
There are no other possibilities.

Therefore, we can calculate p(not had GF) as 0.99

So, information so far:

P(had GF) = 0.01
P(not had GF) = 0.99

The GF network… what we know, and what we can deduce

Information about the test.

Statement 1: "For a person who has **had GF** the test would give a positive result with probability 0.977"

The statement suggests that p(person has had GF) = 1 (i.e. in this sentence it is **known** that the person has had GF)

p(+ve test | person has had GF) = 0.977
"The probability of a +ve test given that a person has had GF is 0.977"

Can we deduce any further information from this?
The other situation that can occur is: a person who has had GF but they receive a –*ve* test result.

This can be expressed as:
p(-ve test | person has had GF) = 1 – p(+ve test | person has had GF)]
= 0.023

The GF network… what we know, and what we can deduce

Further information we have about the test.

Statement 2: "For a person who has **not had** GF the test would give a negative result with probability 0.926"

The statement suggests that p(person has not had GF) = 1(i.e. in this sentence it is known that the person has not had GF)

We know, p(-ve test | person has not had GF) = 0.926

"The probability of a -ve test given that a person has not had GF is 0.926"

Can we deduce any further information from this?

The other situation that can occur is: a person who has not had GF but receives a +*ve* test result.

This can be expressed as:

p(+ve test | person has not had GF) = 1 – p(-ve test | person has not had GF)

= 0.074

7

What we know

This gives us the following information:

P(had GF) = 0.01

P(not had GF) = 0.99

P(+ve test | had GF) = 0.977

P(-ve test | had GF) = 0.023

P(+ve test | not had GF) = 0.074

P(-ve test | not had GF) = 0.926

8

Step 2: Convert information into a BN

1. Determine Nodes
   - GF node
   - Test Result Node

2. Determine Relationships between Nodes

   GF node influences state of test result node

3. Establish values for Conditional Probability Tables (CPTs)

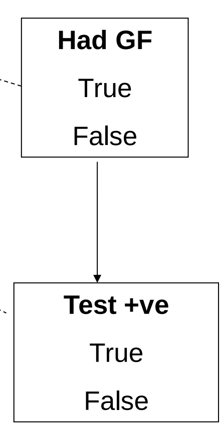
	Test True (+ve)	Test False (–ve)
Had GF True	0.977 (97.7%)	0.023 (2.3%)
Had GF False	0.074 (7.4%)	0.926 (92.6%)

[Rows should sum to 1 (probs), 100 (%’s)]

Step 2: Convert information into a BBN

True	0.01 (1%)
False	0.99 (99%)

	Test True +ve	Test False –ve
Had GF True	0.977 (97.7%)	0.023 (2.3%)
Had GF False	0.074 (7.4%)	0.926 (92.6%)



Using the Bayesian network

- Having constructed our information in the form of a BN, we can now use the network to determine new information.

- Examples of information we may wish to determine:

1. Given a person has had GF, what is the probability of a negative test result?
2. What is the probability of a +ve test result?
3. Given a positive test, what is the probability that the person has had GF?

11

Answering the questions

Q1: Given a person has had GF, what is the probability of a negative test?

Formulated in terms of probability, we wish to find out:

P(-ve test | has had GF)

We already have this probability from the conditional probability table.

P(-ve test | has had GF) = 0.023

12

Answering the questions

Q2: What is the probability of a +ve test result?

Well, we need to think of all the possible situations that could happen which would lead to a +ve test result.

Situation 1. +ve test result and had GF

Situation 2. +ve test result and not had GF

Expressed in probability terms:

Situation 1: p(+ve test result ∩ had GF)

Situation 2: p(+ve test result ∩ not had GF)

We don't know the above information, BUT we can now use what we know of conditional probability to calculate this…

13

Question 2

Reminder of conditional probability rule:

P(A ∩ B) = P(B|A) P(A)

So,

p(+ve test result ∩ had GF) = p(had GF | +ve test) * <u>p(+ve test)</u>

p(+ve test result ∩ not had GF) = p(not had GF | +ve test) * <u>p(+ve test)</u>

Looking over the equations, it is evident that we're still using information we don't know <u>p(+ve test</u>) so we won't be able to calculate an answer.

Don't despair, as remember from earlier lecture notes that

P(A ∩ B) = P(B ∩ A)

That is,

P(+ve test result ∩ had GF) = p(had GF ∩ +ve test result)

and

p(+ve test result ∩ not had GF) = p(not had GF ∩ +ve test result)

14

Question 2

Let's try it this way round…

p(had GF ∩ +ve test result) = p(+ve test | had GF) * p(had GF)
p(not had GF ∩ +ve test result) = p(+ve test | not had GF) * p(not had GF)

Now we have all the information we need:

p(had GF ∩ +ve test result) = 0.977 * 0.01 = 0.00977
p(not had GF ∩ +ve test result) = 0.074 * 0.99 = 0.07326

- Now we have values for the probabilities of all situations that would lead us to getting a +ve test.
- Now what…

Question 2

We're wanting to find p(+ve test), we now know:
P(+ve test ∩ had GF)
P(+ve test ∩ not had GF)

As either of these situations could lead to the test being +ve, we use our OR probability operator and simply add the 2 probabilities together.

So,
P(+ve test) = p(+ve test ∩ had GF) + p(+ve test ∩ not had GF)
= 0.00977 + 0.07326
= 0.08303

Question 3

Q3: Given a positive test, what is the probability that the person has had GF?

In probability notation: P(has had GF | +ve test)

Again, this isn't a piece of information we know, so we must use the network and our knowledge of Bayes' theorem to calculate the answer.

Reminder of Bayes' Theorem:

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

17

Question 3

P(has had GF | +ve test) (what we want to know)
Using Bayes' theorem:

$$P(\text{has had GF} \mid \text{+ve test}) = \frac{P(\text{+ve test} \mid \text{has had GF}) * P(\text{has had GF})}{P(\text{+ve test})}$$

$$= (0.977 * 0.01) / P(\text{+ve test})$$

We've just calculated p(+ve test) for question 2, so we can use it!

$$P(\text{has had GF} \mid \text{+ve test}) = \frac{P(\text{+ve test} \mid \text{has had GF}) * P(\text{has had GF})}{P(\text{+ve test})}$$

$$= (0.977 * 0.01) / 0.08303 = 0.118$$

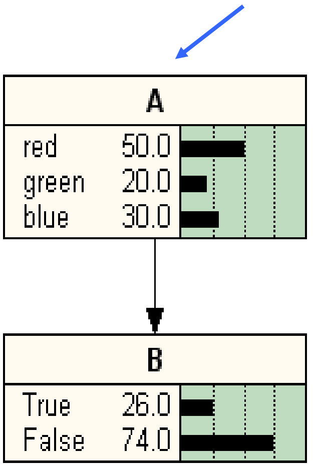
18

Types of nodes: parent node

Parent Node

Parent node

- Represents a node with no arrows leading into it
- Probability information is given (or can be deduced)
- Probabilities of different states node can have must sum to 1 (or if representing as percentages, percentages must add to 100)



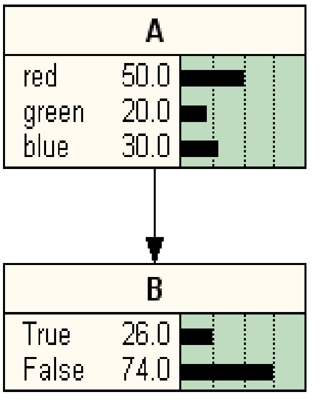
19

Types of nodes: child node

Child node

- Represents a node with **at least one** arrow leading into it
- Probability information dictated by the values of the parent(s) node feeding into the child node
- Probabilities of different states node can have must sum to 1 (or if representing as percentages, percentages must add to 100)

Child Node



20

Bayesian Network models

To date we have addressed:

- A BN model with parent and child node and calculations we can perform

Further models we will address:

- A BN model with a hierarchy of 3 nodes
- A BN model with 2 parent nodes and 1 child node

21