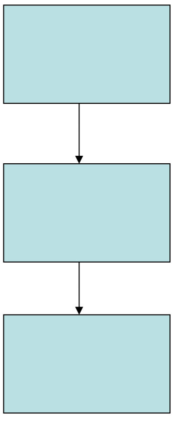
Lecture 5: Bayesian Networks 3

Jingpeng Li

Content

- Bayesian network models
  - Previous lectures: one parent and one child node
  - This lecture:
    - A hierarchy of 3 nodes
    - 2 parent nodes and 1 child node
- Things to remember about calculations in BNs
- Closing and perspectives
  - A recapitulation of Bayesian networks
  - Learning the structure of Bayesian networks

A BN model with a hierarchy of 3 nodes



3

Example: glandular fever

- Suppose we know that on average, 1% of the population have had glandular fever (GF).
- In probability terms p(had_GF) = 0.01
- Suppose we have a test for having had glandular fever such that:
  - For a person who has had GF the test would give a positive result with probability 0.977
  - For a person who has not had GF the test would give a negative result with probability 0.926

Q: How could this information be represented as a BN

Q: How could the BN be used to find out new information?

4

Extension to the glandular fever model

True	0.01 (1%)
False	0.99 (99%)

Had GFNode

True

False

	Test +ve True+	Test –ve False
Had GF True	0.977 (97.7%)	0.023 (2.3%)
Had GF False	0.074 (7.4%)	0.926 (92.6%)

Test +ve Node

True

False

Facts we deduced from the last time…

p(-ve test | has had GF) = 0.023

p(+ve test) = 0.08303

p(had GF | +ve test) = 0.118

5

Extension to the Glandular Fever Model

- Suppose we are informed that the school nurse sends home 80% of students that have a positive GF test.
- She also sends home 5% of students for other medical reasons (i.e. students that have not had a positive GF test).

Questions:

1. How do we incorporate this new information into our network?

2. What is the probability of being sent home?

3. Given that a child is sent home, what is the probability of them having had a negative test?

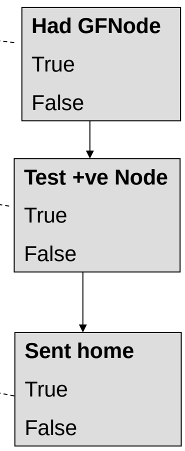
6

How do we incorporate this new information into our network?

True	0.01 (1%)
False	0.99 (99%)

	Test +ve True+	Test –ve False
Had GF True	0.977 (97.7%)	0.023 (2.3%)
Had GF False	0.074 (7.4%)	0.926 (92.6%)

	Sent home True	Sent home False
Test +ve True	0.8 (80%)	0.2 (20%)
Test +ve False	0.05 (5%)	0.95 (95%)



What is the probability of being sent home?

We need to add up all the combinations of scenarios which would lead us to being sent home…

1. Sending home due to +ve test
2. Sending home due to another reason

in mathematical terms…

p(home) = p(home ∩ +ve Test) + p(home ∩ -ve Test)
= [p(home|+ve Test) * p(+ve Test)] + [p(home|-ve Test) * p(-ve Test)]
= 0.8 * p(+ve Test) + 0.05 * p(-ve Test)

We need to calculate p(+ve test) and p(-ve test)

p(Test+) = p(Test+|GF) * p(GF) + p(Test+| not had GF) * p(not had GF)

We calculated this before so we can use this information. Otherwise, our first step would be to calc p(test+)…

What is the probability of being sent home?

From earlier lecture notes…

p(+ve test) = p(+ve test ∩ had GF) + p(+ve test ∩ not had GF)
= 0.00977 + 0.07326
= 0.08303

From this, we can also deduce that P(-ve test) = 1 -0.08303 = 0.91697

p(home) = 0.8 * p(+veTest) + 0.05 * p(-ve Test)
= 0.8 * 0.08303 + 0.05 * 0.91697
= 0.066424 +0.0458485
= 0.1122725 = 0.112 (3 d.p)

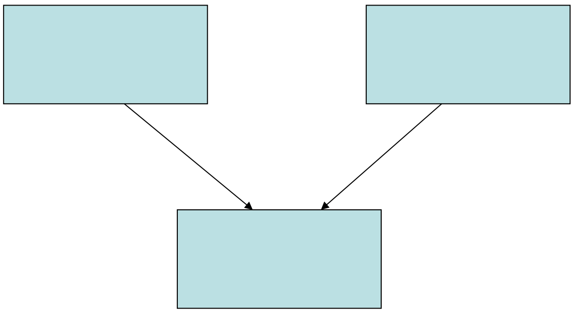
Given that a child is sent home, what is the probability of them having had a negative test?

We've already done a similar calculation to this before:

$$p(\text{-ve test} \mid \text{home}) = \frac{p(\text{home} \mid \text{-ve test}) * p(\text{-ve test})}{p(\text{home})}$$

= (0.05 * 0.91697) / 0.1122725
= 0.0458485 / 0.1122725
= 0.408 (3 d.p.)

2 parents, 1 child model



11

2 parents, 1 child model: Diane's shopping model

- Diane does her shopping each week. The bill for her shop is sometimes under £30, sometimes £30 or over. Two factors influence the cost of her shopping: whether she takes her 2 year old son with her, and whether she takes her 40 year old husband with her.
  - If we know Diane has gone shopping by herself, the likelihood that the bill will be less than £30 is 90%.
  - If we know that Diane took only her son, the likelihood that the bill will be less than £30 is 80%.
  - If we know that Diane was accompanied only by her husband, the likelihood that the bill will be less than £30 is 70%.
  - Given we know both son and husband accompanied Diane to the shops, then the likelihood that the bill is under £30 reduces to 60%.
- 60% of the time Diane is accompanies by her son. 50% of the time Diane's husband accompanies her to the shops.

12

2 parents, 1 child model: Diane's shopping model

Questions

1. What is the probability of the bill being under £30?

2. Given that the bill is under £30, what is the probability that Diane's husband (with or without her son) accompanied her to the shops?

13

2 parents, 1 child model: Diane's shopping model

Firstly, let's list what we know, and what we can deduce…

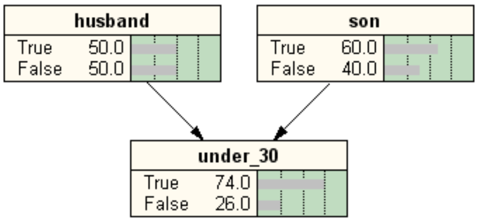
p(under_30 | (no husband ∩ no son)) = 0.9
p(under_30 | (no husband ∩ son)) = 0.8
p(under_30 | (husband ∩ no son)) = 0.7
p(under_30 | (husband ∩ son)) = 0.6
p(son) = 0.6
p(husband) = 0.5

What we can deduce….

p(not_under_30 | (no husband ∩ no son)) = 0.1
p(not_under_30 | (no husband ∩ son)) = 0.2
p(not_under_30 | (husband ∩ no son)) = 0.3
p(not_under_30 | (husband ∩ son)) = 0.4
p(no_son) = 0.4
p(no_husband) = 0.5

14

2 parents, 1 child model: Diane's shopping model



CPT		True	False
Hub T	Son T	60%	40%
Hub T	Son F	70%	30%
Hub F	Son T	80%	20%
Hub F	Son F	90%	10%

What is the probability of the bill being under £30?

Firstly, list situations that lead to bill being under £30

p(under_30) = p(under_30 | no husband ∩ no son) p(no husband ∩ no son) +
p(under_30 | no husband ∩ son) p(no husband ∩ son) +
p(under_30 | husband ∩ no son) p(husband ∩ no son) +
p(under_30 | husband ∩ son) p(husband ∩ son)

What do we know about the presence of Diane's husband and the presence of her son?

Are these 2 events dependent on each other in any way? No…

These events are independent [Independent events are where the occurrence of one event does not impact on the occurrence of another event.. See 1st set of lecture notes on probabilities for more info.]

This will influence how we calculate the probability of both events occurring.

Independent events A and B, p(A ∩ B) = p(A) * p(B)

What is the probability of the bill being under £30?

p(no husband ∩ no son) = p(no husband) * p(no son) = 0.5 * 0.4 = 0.2

p(no husband ∩ son) = p(no husband) * p(son) = 0.5 * 0.6 = 0.3

p(husband ∩ no son) = p(husband) * p(no son) = 0.5 * 0.4 = 0.2

p(husband ∩ son) = p(husband) * p(son) =0.5 * 0.6 = 0.3

p(under_30)
= p(under_30 | no husband ∩ no son) * p(no husband ∩ no son) +
p(under_30 | no husband ∩ son) * p(no husband ∩ son) +
p(under_30 | husband ∩ no son) * p(husband ∩ no son) +
p(under_30 | husband ∩ son) * p(husband ∩ son)

p(under_30) = (0.9 * 0.2) + (0.8 * 0.3) + (0.7 * 0.2) + (0.6 * 0.3) = 0.74

17

Given that the bill is under £30, what is the probability that Diane's husband (with or without her son) accompanied her to the shops?

2 scenarios:
p(husband | under £30)
= p(husband ∩ son | under £30) + p(husband ∩ not son | under £30)

p(husband ∩ son | under £30) = { using Bayes')

$$= \frac{p(\text{under £30} \mid p(\text{husband} \cap \text{son})) * p(\text{husband} \cap \text{son})}{p(\text{under £30})}$$

= (0.6 * 0.3) / 0.74 = 0.243^r (r = recurring)

p(husband ∩ not son | under £30) = {using Bayes'}

$$= \frac{p(\text{under £30} \mid p(\text{husband} \cap \text{not son})) * p(\text{husband} \cap \text{not son}))}{p(\text{under £30})}$$

= (0.7 * 0.2) / 0.74 = 0.189^r

p(husband | under £30) = 0.243^r + 0.189^r = 0.432^r (3 d.p.)

18

Things to remember about calculations in BNs

1. Know parent information, want to find out child node information:

   Use conditional probability

2. Know child information, want to find out parent node information:

   Use Bayes' Theorem

3. If have more than 1 parent to a node, remember parents are independent.

   Thus p(parent A ∩ parent B) = P(parent A) * p(parent B)

19

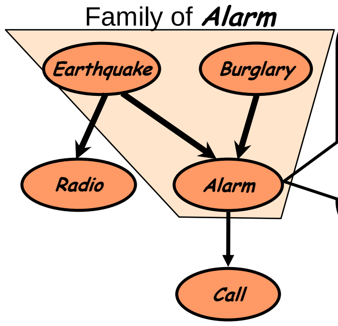
Bayesian Networks

Compact representation of probability distributions via conditional independence

Qualitative part:
Directed acyclic graph (DAG)

- Nodes - random variables
- Edges - direct influence

Quantitative part:
Set of conditional probability distributions



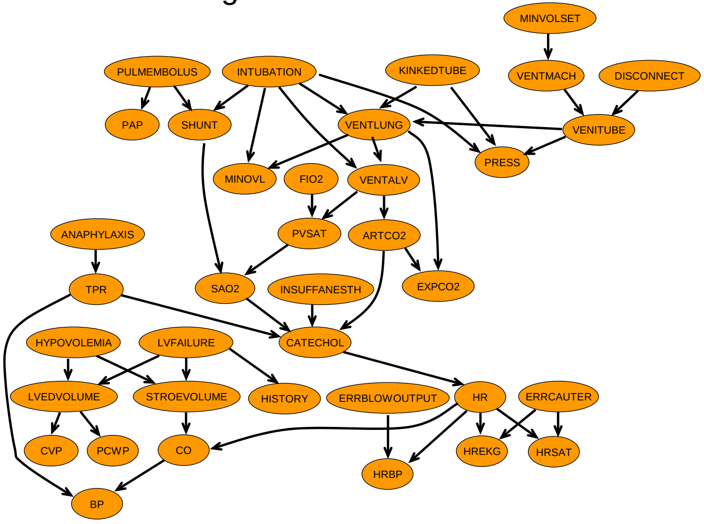
E	B	P(A | E,B)	
e	b	0.9	0.1
e	$\bar{b}$	0.2	0.8
$\bar{e}$	b	0.9	0.1
$\bar{e}$	$\bar{b}$	0.01	0.99

$$P(B,E,A,C,R) = P(B)P(E)P(A \mid B,E)P(R \mid E)P(C \mid A)$$

20

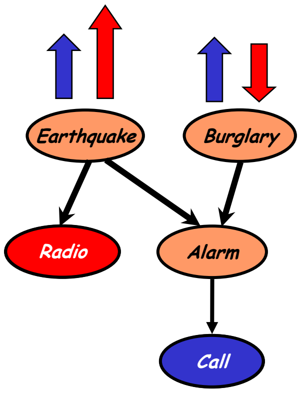
Example: "ICU Alarm" network

Domain: Monitoring Intensive-Care Patients



Inference

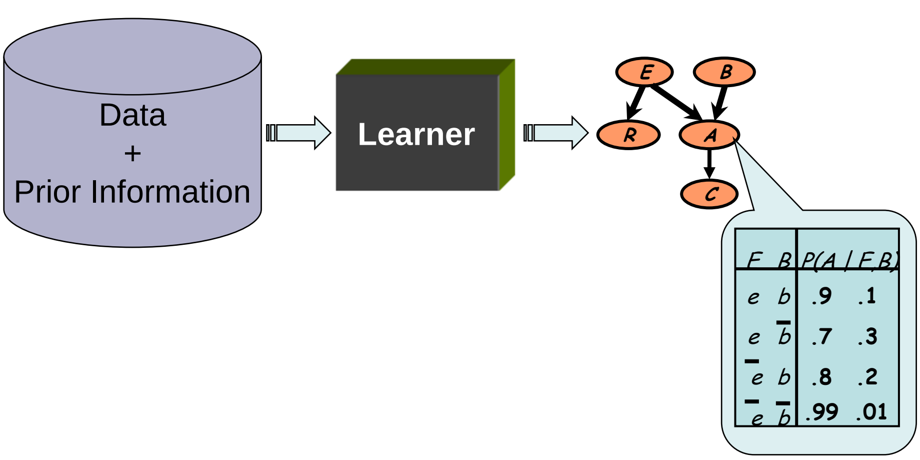
- **Posterior probabilities**
  - Probability of any event given any evidence
- Most likely explanation
  - Scenario that explains evidence
- Rational decision making
  - Maximize expected utility
  - Value of Information
- Effect of intervention



Why learn Bayesian networks?

- Conditional independencies & graphical language capture structure of many real-world distributions
- Graph structure provides much insight into domain
  - Allows “knowledge discovery”
- Learned model can be used for many tasks
- Supports all the features of probabilistic learning
  - Model selection criteria
  - Dealing with missing data & hidden variables

Learning Bayesian networks



Ways to learn BN structure

From an expert

Knowledge acquisition bottleneck

- Knowledge acquisition is an expensive process
- Often we don't have an expert

From data

Data is cheap

- Amount of available information growing rapidly
- Learning allows us to construct models from raw data

Methods for learning the structure of a BN

- Learning from experience: counting-learning algorithm
- Learning using trees
- Heuristic methods
- Bayesian Inference
- Scoring methods

.. Many more