Further Issues Using OLS with Time Series Data

Lecture 10/A

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Analyzing Time Series: Further Issues Using OLS

- Unfortunately, the usual inference methods – *t* and *F* statistics, confidence intervals – are much more fragile for TS applications than for CS applications.
- In the CS case, with even moderate sample sizes, we can entirely dispense with the normality assumption in the CS case and use *t* and *F* statistics as approximately valid.
- We also know how to adjust our statistics for heteroskedasticity of unknown form.
- When we drop normality for TS applications, we must impose assumptions on the underlying time series processes for $\{x_t\}$ and $\{u_t\}$.
- Large-sample analysis is much harder with TS data is that LLN and CLT do not always hold for the usual statistics. More easy with CS data and SRS.

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- **The assumptions used so far seem to be too restricitive**
  - Strict exogeneity, homoscedasticity, and no serial correlation are very demanding requirements, especially in the time series context
  - Statistical inference rests on the validity of the normality assumption
  - Much weaker assumptions are needed if the sample size is large
  - A key requirement for large sample analysis of time series is that the time series in question are stationary and weakly dependent
- **<u>Stationary time series</u>**
  - Loosely speaking, a time series is stationary if its stochastic properties and its temporal dependence structure do not change over time

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- **Stationary stochastic processes**

  A stochastic process $\{x_t : t = 1, 2, \ldots\}$ is stationary, if for every collection of indices $1 \leq t_1 \leq t_2 \leq \cdots \leq t_m$ the joint distribution of $(x_{t_1}, x_{t_2}, \ldots, x_{t_m})$ is the same as that of $(x_{t_1+h}, x_{t_2+h}, \ldots, x_{t_m+h})$ for all integers $h \geq 1$.

- **Covariance stationary processes**

  A stochastic process $\{x_t : t = 1, 2, \ldots\}$ is covariance stationary, if its expected value, its variance, and its covariances are constant over time: 1) $E(x_t) = \mu$, 2) $Var(x_t) = \sigma^2$, and 3) $Cov(x_t, x_{t+h}) = f(h)$.

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- **Weakly dependent time series**

  A stochastic process $\{x_t : t = 1, 2, \ldots\}$ is weakly dependent, if x_t is "almost independent" of x_{t+h} if h grows to infinity (for all t).

- **Discussion of the weak dependence property**
  - An implication of weak dependence is that the correlation between x_t and x_{t+h} must converge to zero if h grows to infinity
  - For the LLN and the CLT to hold, the individual observations must not be too strongly related to each other; in particular their relation must become weaker (and this fast enough) the farther they are apart
  - Note that a series may be nonstationary but weakly dependent

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- **<u>Examples for weakly dependent time series</u>**
- **Moving average process of order one (MA(1))**

$$x_t = e_t + \alpha_1 e_{t-1}$$

The process is a short moving average of an i.i.d. series e_t

The process is weakly dependent because observations that are more than one time period apart have nothing in common and are therefore uncorrelated.

- **Autoregressive process of order one (AR(1))**

$$y_t = \rho_1 y_{t-1} + e_t$$

The process carries over to a certain extent the value of the previous period (plus random shocks from an i.i.d. series e_t)

$$\Rightarrow \quad Corr(y_t, y_{t+h}) = \rho_1^h$$

If the stability condition $|\rho_1| < 1$ holds, the process is weakly dependent because serial correlation converges to zero as the distance between observations grows to infinity.

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- <u>**Asymptotic properties of OLS**</u>
- **Assumption TS.1' (Linear in parameters)**
  - Same as assumption TS.1 but now the dependent and independent variables are assumed to be <u>stationary</u> and <u>weakly dependent</u>
- **Assumption TS.2' (No perfect collinearity)**
  - Same as assumption TS.2
- **Assumption TS.3' (Zero conditional mean)**
  - Now the explanatory variables are assumed to be only contemporaneously exogenous rather than strictly exogenous, i.e.

$$E(u_t|\mathbf{x}_t) = 0$$

The explanatory variables <u>of the same period</u> are uninformative about the mean of the error term

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- **Theorem 11.1 (Consistency of OLS)**

$$TS.1'\text{–}TS.3' \quad \Rightarrow \quad plim\ \widehat{\beta}_j = \beta_j, \quad j = 0, 1, \ldots, k$$

Important note: For consistency it would even suffice to assume that the explanatory variables are merely contemporaneously *uncorrelated* with the error term.

- **Why is it important to relax the strict exogeneity assumption?**
  - Strict exogeneity is a serious restriction beause it rules out all kinds of dynamic relationships between explanatory variables and the error term
  - In particular, it rules out feedback from the dep. var. on future values of the explanat. variables (which is very common in economic contexts)
  - Strict exogeneity precludes the use of lagged dep. var. as regressors

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- **Why do lagged dependent variables violate strict exogeneity?**

$$y_t = \beta_0 + \beta_1 y_{t-1} + u_t$$

This is the simplest possible regression model with a lagged dependent variable

<u>Contemporanous exogeneity:</u> $E(u_t | y_{t-1}) = 0$

<u>Strict exogeneity:</u> $E(u_t | y_0, y_1, \ldots, y_{n-1}) = 0$

Strict exogeneity would imply that the error term is uncorre-lated with all y_t, t=1, ... , n-1

This leads to a contradiction because:

$$Cov(y_t, u_t) = \beta_1 Cov(y_{t-1}, u_t) + Var(u_t) > 0$$

- **OLS estimation in the presence of lagged dependent variables**
  - Under contemporaneous exogeneity, OLS is consistent but biased

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- **Assumption TS.4‘ (Homoscedasticity)**

$$Var(u_t|\mathbf{x}_t) = Var(u_t) = \sigma^2$$

The errors are contemporaneously homoscedastic

- **Assumption TS.5‘ (No serial correlation)**

$$Corr(u_t, u_s|\mathbf{x}_t, \mathbf{x}_s) = 0, \;\; t \neq s$$

Conditional on the explanatory variables in periods t and s, the errors are uncorrelated

- <u>**Theorem 11.2 (Asymptotic normality of OLS)**</u>
  - Under assumptions TS.1‘ – TS.5‘, the OLS estimators are asymptotically normally distributed. Further, the usual OLS standard errors, t-statistics and F-statistics are asymptotically valid.

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- **Using trend-stationary series in regression analysis**
  - Time series with deterministic time trends are nonstationary
  - If they are stationary around the trend and in addition weakly dependent, they are called trend-stationary processes
  - Trend-stationary processes also satisfy assumption TS.1‘
- <u>**Using highly persistent time series in regression analysis**</u>
  - Unfortunately many economic time series violate weak dependence
  - In this case OLS methods are generally invalid (Spurious Regression)
  - In some cases transformations to weak dependence are possible

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- **Random walks**

$$\boxed{y_t = y_{t-1} + e_t}$$

The random walk is called random walk because it wanders from the previous position y_{t-1} by an i.i.d. random amount e_t

$$\Rightarrow \; y_t = (y_{t-2}+e_{t-1})+e_t = \ldots = e_t+e_{t-1}+\ldots+e_1+y_0$$

The value today is the accumulation of all past shocks plus an initial value. This is the reason why the random walk is highly persistent: The effect of a shock will be contained in the series forever.

$$E(y_t) = E(y_0)$$

$$Var(y_t) = \sigma_e^2 t$$

$$Corr(y_t, y_{t+h}) = \sqrt{t/(t+h)}$$

The random walk is <u>not covariance stationary</u> because its variance and its covariance depend on time.

It is also <u>not weakly dependent</u> because the correlation between observations vanishes very slowly and this depends on how large t is.

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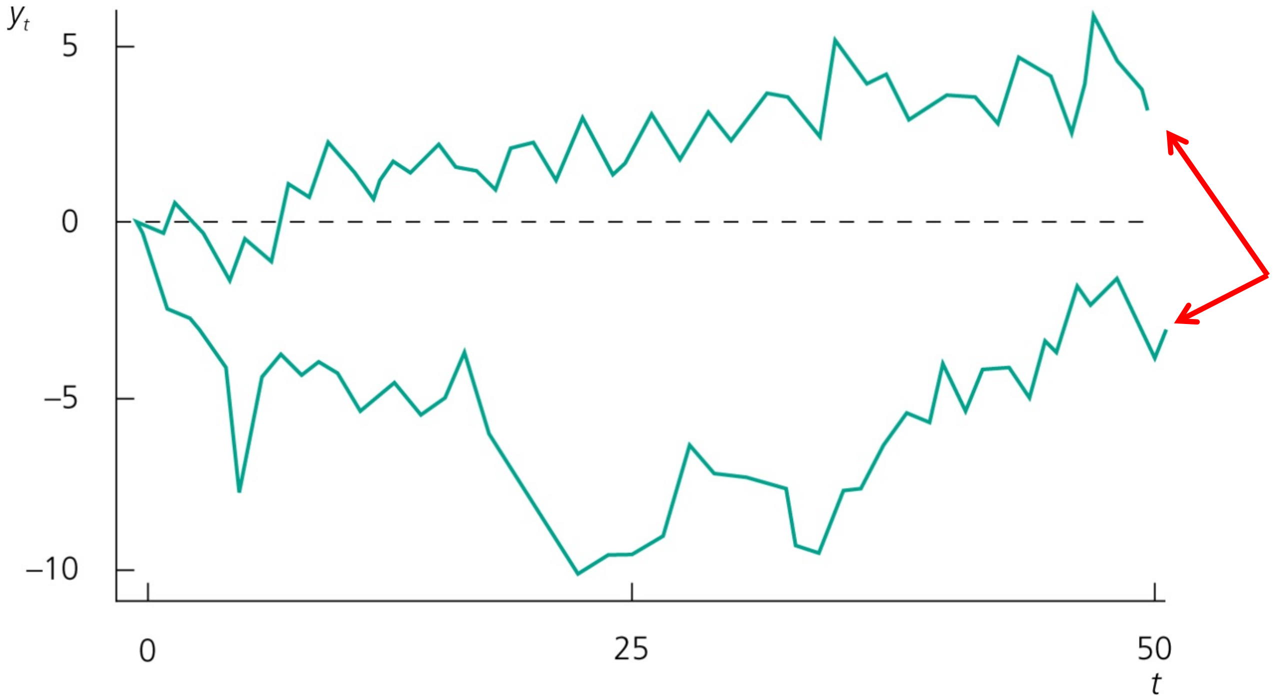
- **Examples for random walk realizations**



The random walks wander around with no clear direction

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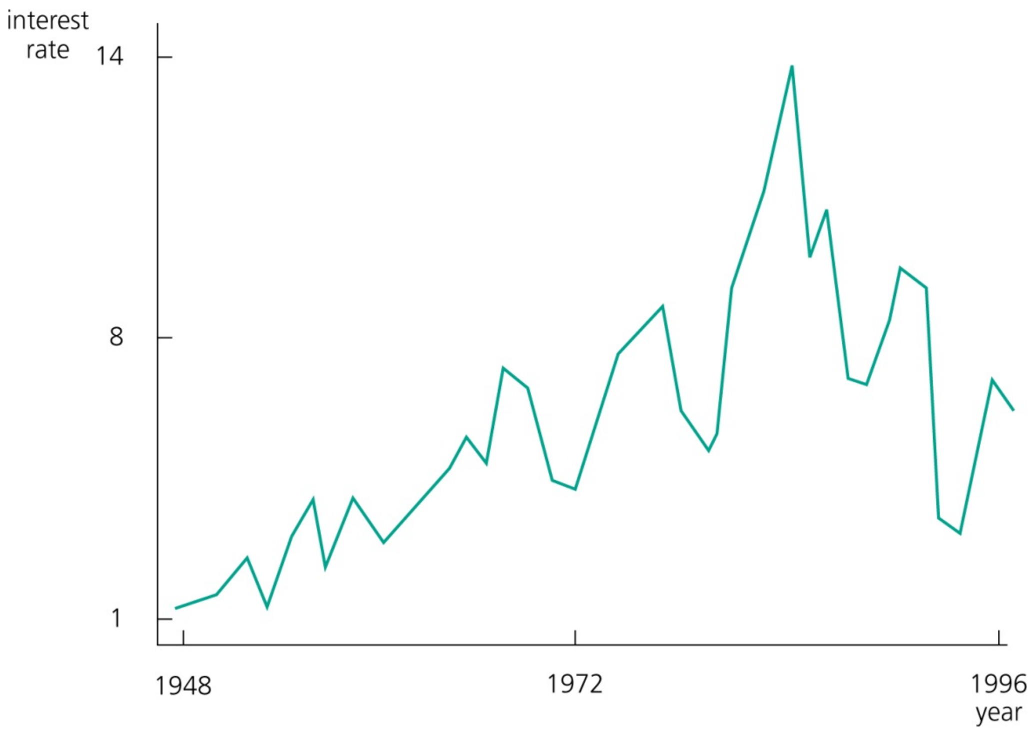
- **Three-month T-bill rate as a possible example for a random walk**



A random walk is a special case of a unit root process.

Unit root processes are defined as the random walk but e_t may be an arbitrary weakly dependent process.

From an economic point of view it is important to know whether a time series is highly persistent. In highly persistent time series, shocks or policy changes have lasting/permanent effects, in weakly dependent processes their effects are transitory.

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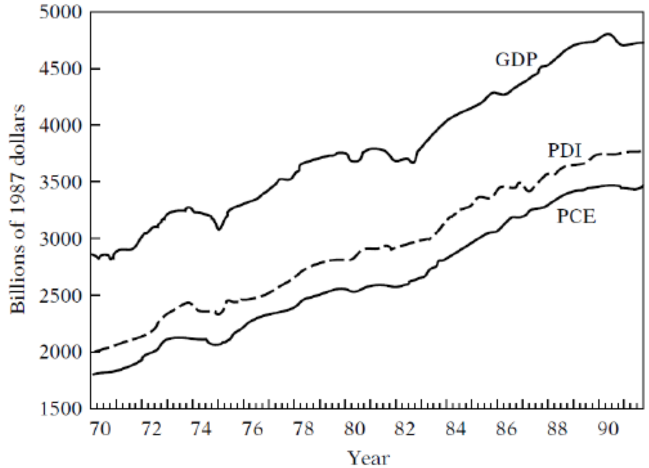
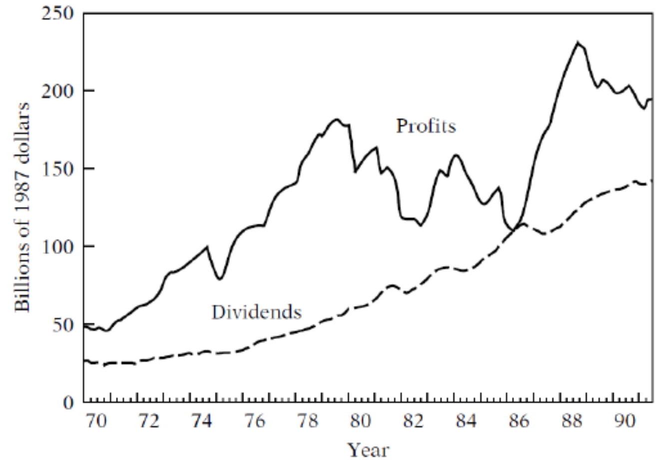
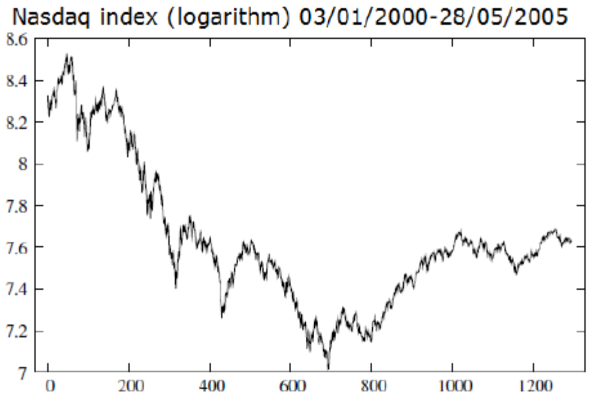
U.S. Economic Time Series (Examples)







Billions of 1987 U.S. Dollars

Quarterly period 1970-1991 (88 obs)

GDP: Gross Domestic Product

PDI: Personal Disposable Income

PCE: Personal Consumption Expenditure

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- **<u>Transformations on highly persistent time series</u>**
- **Order of integration**
  - Weakly dependent time series are integrated of order zero (= I(0))
  - If a time series has to be <u>differenced</u> one time in order to obtain a weakly dependent series, it is called integrated of order one (= I(1))
- **Examples for I(1) processes**

$$y_t = y_{t-1} + e_t \;\Rightarrow\; \Delta y_t = y_t - y_{t-1} = e_t$$

$$\log(y_t) = \log(y_{t-1}) + e_t \;\Rightarrow\; \Delta \log(y_t) = e_t$$

After differencing, the resulting series are weakly dependent (because e_t is weakly dependent).

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Reading

- Chapter 11
- Sections 11.1-11.3

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