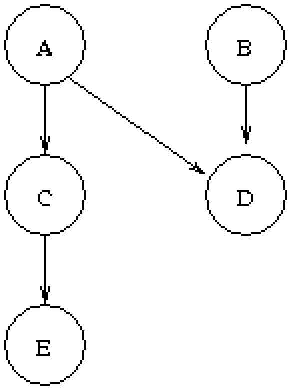
CSCU9T6 Bayesian Networks – Tutorial (Solutions)

Consider the following Bayesian network:



Thus, the independence expressed in this Bayesian net are that
A and B are (absolutely) independent.
C is independent of B given A.
D is independent of C given A and B.
E is independent of A, B, and D given C.

Suppose that the net further records the following probabilities:

Prob(A=T) = 0.3
Prob(B=T) = 0.6
Prob(C=T|A=T) = 0.8
Prob(C=T|A=F) = 0.4
Prob(D=T|A=T,B=T) = 0.7
Prob(D=T|A=T,B=F) = 0.8
Prob(D=T|A=F,B=T) = 0.1
Prob(D=T|A=F,B=F) = 0.2
Prob(E=T|C=T) = 0.7
Prob(E=T|C=F) = 0.2

The computations:

1) Prob(D=T)

P(D=T) =

P(D=T,A=T,B=T) + P(D=T,A=T,B=F) + P(D=T,A=F,B=T) + P(D=T,A=F,B=F) =

P(D=T|A=T,B=T) P(A=T,B=T) + P(D=T|A=T,B=F) P(A=T,B=F) +
P(D=T|A=F,B=T) P(A=F,B=T) + P(D=T|A=F,B=F) P(A=F,B=F) =
(since A and B are independent absolutely)

P(D=T|A=T,B=T) P(A=T) P(B=T) + P(D=T|A=T,B=F) P(A=T) P(B=F) +
P(D=T|A=F,B=T) P(A=F) P(B=T) + P(D=T|A=F,B=F) P(A=F) P(B=F) =

0.7*0.3*0.6 + 0.8*0.3*0.4 + 0.1*0.7*0.6 + 0.2*0.7*0.4 = 0.32

2) Prob(D=F,C=T)

$P(D=F,C=T) =$

$P(D=F,C=T,A=T,B=T) + P(D=F,C=T,A=T,B=F) +$
$P(D=F,C=T,A=F,B=T) + P(D=F,C=T,A=F,B=F) =$

$P(D=F,C=T|A=T,B=T)\ P(A=T,B=T) + P(D=F,C=T|A=T,B=F)\ P(A=T,B=F) +$
$P(D=F,C=T|A=F,B=T)\ P(A=F,B=T) + P(D=F,C=T|A=F,B=F)\ P(A=F,B=F) =$
(since C and D are independent given A and B)

$P(D=F|A=T,B=T)\ P(C=T|A=T,B=T)\ P(A=T,B=T) + P(D=F|A=T,B=F)\ P(C=T|A=T,B=F)\ P(A=T,B=F) +$
$P(D=F|A=F,B=T)\ P(C=T|A=F,B=T)\ P(A=F,B=T) + P(D=F|A=F,B=F)\ P(C=T|A=F,B=F)\ P(A=F,B=F) =$
(since C is independent of B given A and A and B are independent absolutely)

$P(D=F|A=T,B=T)\ P(C=T|A=T)\ P(A=T)\ P(B=T) + P(D=F|A=T,B=F)\ P(C=T|A=T)\ P(A=T)\ P(B=F) +$
$P(D=F|A=F,B=T)\ P(C=T|A=F)\ P(A=F)\ P(B=T) + P(D=F|A=F,B=F)\ P(C=T|A=F)\ P(A=F)\ P(B=F) =$

$0.3*0.8*0.3*0.6 + 0.2*0.8*0.3*0.4 + 0.9*0.4*0.7*0.6 + 0.8*0.4*0.7*0.4 = 0.3032$

3) Prob(A=T|C=T)

$P(A=T|C=T) = P(C=T|A=T)P(A=T) / P(C=T)$.

Now $P(C=T) = P(C=T,A=T) + P(C=T,A=F) =$
$P(C=T|A=T)P(A=T) + P(C=T|A=F)P(A=F) =$
$0.8*0.3+ 0.4*0.7 = 0.52$

So $P(C=T|A=T)P(A=T) / P(C=T) = 0.8*0.3/0.52= 0.46$.

4) Prob(A=T|D=F)

$P(A=T|D=F) =$

$P(D=F|A=T)\ P(A=T)/P(D=F)$.

Now $P(D=F) = 1-P(D=T) = 0.68$ from the first question above.

$P(D=F|A=T) = P(D=F,B=T|A=T) + P(D=F,B=F|A=T) =$

$P(D=F|B=T,A=T)\ P(B=T|A=T) + P(D=F|B=F,A=T)\ P(B=F|A=T) =$
(since B is independent of A)

$P(D=F|B=T,A=T)\ P(B=T) + P(D=F|B=F,A=T)\ P(B=F) = 0.3*0.6 + 0.2*0.4 = 0.26$.
So $P(A=T|D=F) = P(D=F|A=T)\ P(A=T)/P(D=F) = 0.26 * 0.3 / 0.68 = 0.115$

5) Prob(A=T,D=T|B=F)

$P(A=T,D=T|B=F) =$

$P(D=T|A=T,B=F)\ P(A=T|B=F) =$
(since A and B are independent)

P(D=T|A=T,B=F) P(A=T) = 0.8*0.3 = 0.24.

6) Prob(C=T | A=F, E=T)

Prob(C=T | A=F, E=T) = (By Bayes' law)

Prob(E=T|C=T,A=F) * Prob(C=T|A=F) / Prob(E=T|A=F) = (since E is independent of A given C)

Prob(E=T|C=T) * Prob(C=T|A=F) / Prob(E=T|A=F).

Now Prob(E=T|A=F) = Prob(E=T,C=T|A=F) + Prob(E=T,C=F|A=F) =

Prob(E=T|C=T,A=F) Prob(C=T|A=F) + Prob(E=T|C=F,A=F) Prob(C=F|A=F) = (since E is independent of A given C)

Prob(E=T|C=T) * Prob(C=T|A=F) + Prob(E=T|C=F) * Prob(C=F|A=F).

So we have
Prob(C=T | A=F, E=T) =
Prob(E=T|C=T) * Prob(C=T|A=F) / (Prob(E=T|C=T) * Prob(C=T|A=F) + Prob(E=T|C=F) * Prob(C=F|A=F)) =

0.7*0.4 / (0.7 * 0.4 + 0.2 * 0.6) = 0.7