CSCU9YM: Modelling for Complex Systems

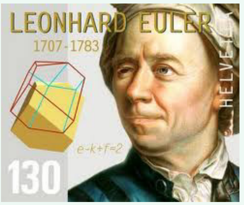
Lecture 5: Networks

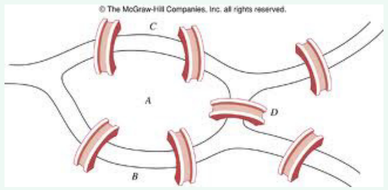
Introduction to Complex Networks

- We do not cover the mathematical study of networks in this module (though some students may have taken courses in graph theory).
  - A very interesting and highly relevant topic, but outside our scope
- Instead we take a very brief and informal look at some types of network that have been found useful in the study of complex systems.
- And we look at algorithms used for creating these networks to use in simulations.
- Outline:
  - Euler and the origins of graph / network theory
  - Basic concepts
  - Random graphs, Erdős-Rényi graphs
  - Small world networks, Watts-Strogatz model
  - Scale-free networks, preferential attachment
  - References

Euler and graph theory

- Euler: and the bridges of Konigsberg problem:

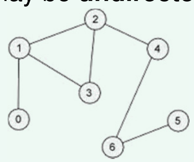


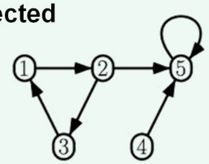


- Euler solved this problem and invented graph theory
- Contributions made by many other mathematicians: Hamilton, Kirchhoff, Cayley, Kuratowski,...

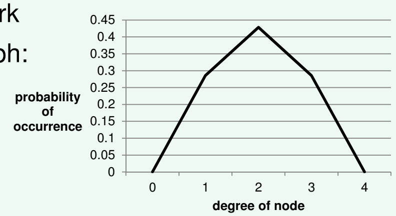
Graph / Network Theory: basic concepts

- **node** (or **vertex**)
- connected to each other by **edges** or **arcs**
- edges may be **undirected** or **directed**





- the **degree** of a node is the number of edges connected to that node
- the **degree distribution** of a network is the probability distribution of these degrees over the whole network
- Degree distribution of above left graph:



Random graphs

- A random graph is a set of nodes with randomly chosen links between them.
- Typically, we have parameters *n* (number of nodes) and *v* (number of edges) and create the graph by choosing *v* of the possible edges.
- Alternatively, we can fix the number of nodes *n* and the probability *p* that there is an edge between any given pair of nodes.
- A graph constructed in this way is called an *Erdős-Rényi* random graph.

- Used as early models of social networks

- But criticised as unrealistic

Random graphs: giant component

- A key result in the study of random graphs is the emergence of a *giant component*

- Probability *p* is the probability that two nodes are connected.
  - When *p* is very small the network is made up of many disjoint components
  - If we increase *p* the components get bigger (and there are fewer of them).
  - When *p* crosses a critical threshold p_c, there emerges a **giant component** which contains the majority of the nodes in the network

  - the critical threshold $p_c \sim (\ln N) / N$, where *N* is the number of nodes

- See Code Examples/Random Network Example and Networks/Giant Component in the NetLogo Models Library

It's a small world after all..

- Stanley Milgram (1933 – 1984), famous for some classic (and notorious) experiments in psychology

  - Obedience experiment (not really relevant, but fascinating!)

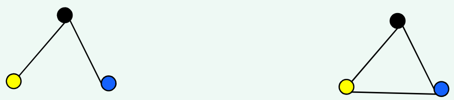
  - Small world experiment
    - » People in Omaha, Nebraska and Wichita, Kansas asked to send a letter to an unknown person in Boston, Massachussetts, over 1000 miles away
    - » Senders asked to forward the letter to a person they considered closer to the target person
    - » Out of 296 letters sent, 64 made it to the target
    - » The average path length was just 5.2 steps

Small world networks

- The *distance between two nodes* on a network is the length of the shortest path between those nodes.
- The *diameter* of a network is the greatest distance between any pair of nodes
- A *small-world network* is one which has a diameter that is very small in comparison to the number of nodes.
- It has been claimed that many real-world networks are small-world networks
  - "six degrees of separation"
  - "six degrees of Kevin Bacon"
  - Erdős numbers

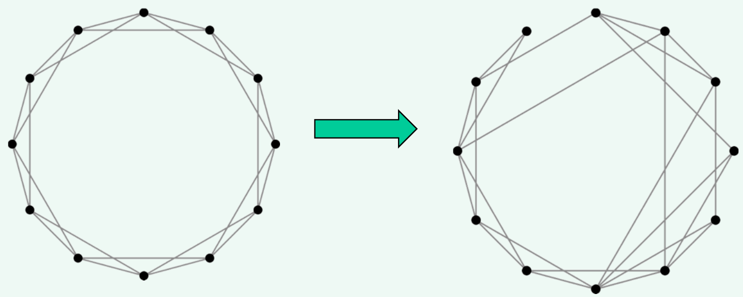
Clustering

- Many real world networks have a high *clustering coefficient*.
  - clustering coefficient of a single node:
    - » The proportion of *potential* links between neighbours that are actually present in the network (see below)
  - Clustering coefficient of a network
    - » The average clustering coefficient over all nodes



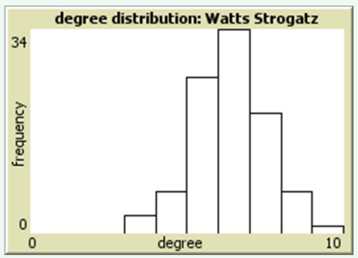
Watts-Strogatz Small World networks

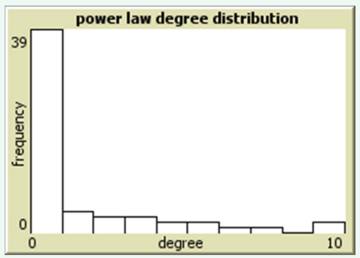
- The Watt-Strogatz algorithm produces a small world network with a high clustering coefficient.
- Algorithm: start with a **ring lattice** network, then randomly rewire some nodes (shortcuts) with probability p
- [demo] NetLogo / Models Library / Networks / Small Worlds



Hubs in real-world networks

- It has been found that many real world networks have **hubs**: a few very highly connected nodes, linking a much larger number of nodes with low degree.
- The degree distribution follows a power-law (see below)
- Random graphs and Watts-Strogatz graphs instead have a bell-curve shaped degree distribution.

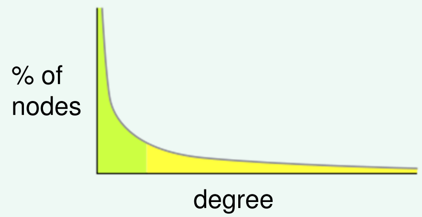




Scale-free networks

- Many real networks have a power-law degree distribution:

  $P(k) \sim k^{-\gamma}$

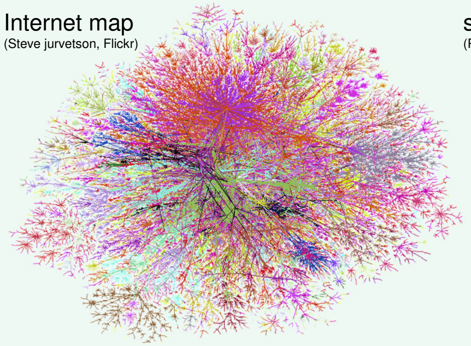


- Power-law distributions are often likened to the Pareto principle:
  - "80% of the wealth is owned by 20% of the population"
  - N% of the connections belong to n% of the nodes (where N >> n).
- Networks which have a power-law degree distribution are called **scale-free** networks.

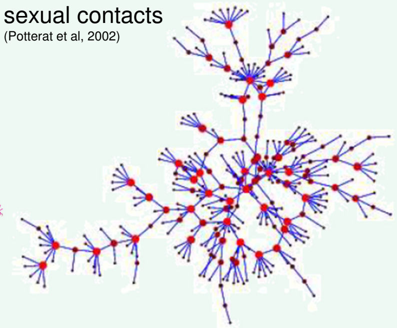
Scale free networks

- Barabási (1999) analyzed the topology of part of the World Wide Web and found it to be a scale-free network.
  - Most nodes have few links. A few “hubs” have most of the links.
  - Since then, many other social and biological networks have been found to be scale-free networks.

Internet map
(Steve jurvetson, Flickr)



sexual contacts
(Potterat et al, 2002)



Preferential attachment

- Barabási proposed a mechanism called ***preferential attachment*** to explain how scale-freee networks arise.
  - Preferential attachment: when a new node joins the network, it is more likely to connect to an existing node with many connections than to a node with few connections.
  - This is the basis of an algorithm for creating scale free networks.
  - “The rich get richer” phenomenon
- You can see this algorithm in action in the NetLogo Models Library / Sample Models / Networks / Preferential Attachment
- [Demo]

Using Network Models

- Scale free networks are not a panacea for modelling real-world networks. There are some properties (like clustering) that are better captured by Watts-Strogatz graphs.
- Networks can be constructed in NetLogo by using links. This is the subject of the fifth practical.
- We have not said much in the lecture about the dynamics of networks: how the network structure affects the processes and interactions that take place among the nodes connected by that network.
  - This is a huge topic.
  - We touch upon it in the practical when we look at how the structure of a contact network affects the spread of an epidemic.
  - See also the paper by Newman (next slide).

References

Railsback and Grimm have little to say about complex networks. Try these references instead:

- The Mathematics Illuminated website, which has a very accessible introduction to network theory. See:
  - http://www.learner.org/courses/mathilluminated/units/11/
  - http://www.learner.org/courses/mathilluminated/interactives/network/
- *Complex and Adaptive Dynamical Systems: a primer*, Claudius Gros, 2011, e-book available from the library
- *Networks, Crowds, and Markets: Reasoning about a Highly Connected World*, Easley and Kleinberg, 2010, free draft available at `http://www.cs.cornell.edu/home/kleinber/networks-book/`
- The Structure and Function of Complex Networks, M.E.J. Newman, 2003, `http://arxiv.org/pdf/cond-mat/0303516v1.pdf`

End of lecture