MATU9D2 Practical Statistics
Practical 2: Hand Calculations: Solutions

1. $X \sim N(\mu, \sigma^2)$ $\quad Z \sim N(0,1)$ $\quad Z = \frac{X-\mu}{\sigma}$ (Z score)

$X \sim N(32, 16)$ $\quad \mu = 32 \quad \sigma = 4$

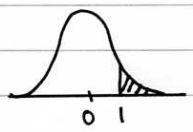
(a) $X = 35$ $\quad Z = \frac{35-32}{4} = \frac{3}{4} = 0.75$

(b) $X = 27$ $\quad Z = \frac{27-32}{4} = \frac{-5}{4} = -1.25$

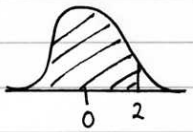
(c) $X = 22.7$ $\quad Z = \frac{22.7-32}{4} = \frac{-9.3}{4} = -2.325$

(d) $X = 40.5$ $\quad Z = \frac{40.5-32}{4} = \frac{8.5}{4} = 2.125$

(e) $X = 30$ $\quad Z = \frac{30-32}{4} = \frac{-2}{4} = -0.5$

2. (a) 

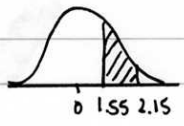
$P(Z > 1) = 1 - P(Z < 1)$
$= 1 - 0.8413 = 0.1587$ //

(b) 

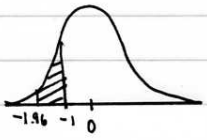
$P(Z < 2) = 0.9772$ //

(c) 

$P(Z > -0.85) = 1 - P(Z < -0.85)$
$= 1 - 0.1977 = 0.8023$ //

(d) 

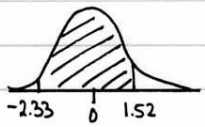
$P(1.55 < Z < 2.15) = P(Z < 2.15) - P(Z < 1.55)$
$= 0.9842 - 0.9394$
$= 0.0448$ //

(e) 

$P(-1.96 < Z < -1) = P(Z < -1) - P(Z < -1.96)$

$= 0.1587 - 0.025$

$= 0.1337$ //

(f) 

$P(-2.33 < Z < 1.52) = P(Z < 1.52) - P(Z < -2.33)$

$= 0.9357 - 0.0099$

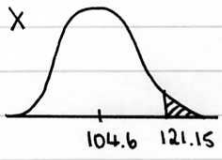
$= 0.9258$ //

3. let X be amount on invoices $\mu = 104.6$ $\sigma = 9.75$

$X \sim N(104.6, 9.75^2)$

(a) $P(X > 121.15) = P(Z > 1.70) = 1 - P(Z < 1.70) = 1 - 0.9554$

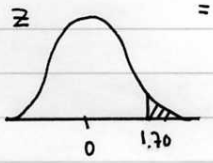
$= 0.0446$



$Z = \frac{X - \mu}{\sigma}$

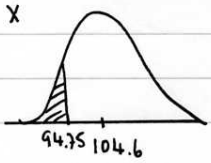
$= \frac{121.15 - 104.6}{9.75}$

$= 1.70$



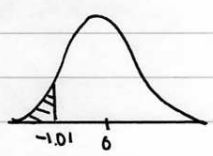
ie 4.46% are for amounts greater than £121.15

(b) $P(X < 94.75) = P(Z < -1.01) = 0.1562$



$Z = \frac{94.75 - 104.6}{9.75}$

$= -1.01$

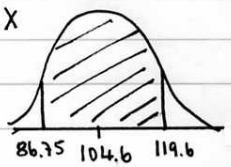


ie 15.6% are for amounts less than £94.75

(c) $P(86.75 < X < 119.60) = P(-1.83 < Z < 1.54) = P(Z < 1.54) - P(Z < -1.83)$

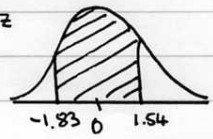
$= 0.9382 - 0.0336$

$= 0.9046$



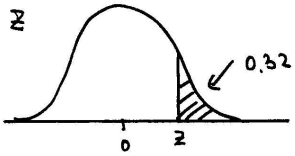
$Z_1 = -1.83$

$Z_2 = 1.54$



ie 90.5% are for amounts between £86.75 & £119.60

(d) $P(Z > z) = 0.32$ ie. $P(Z < z) = 1 - 0.32 = 0.68$



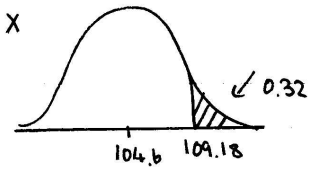
ie. $z = 0.47$

$$z = \frac{X - \mu}{\sigma}$$

$$0.47 = \frac{X - 104.6}{9.75}$$

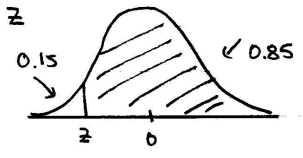
$$X = 104.6 + 0.47 \times 9.75$$

$$= 109.18$$



ie. 32% are for more than £109.18

(e) $P(Z > z) = 0.85$ ie $P(Z < z) = 1 - 0.85 = 0.15$



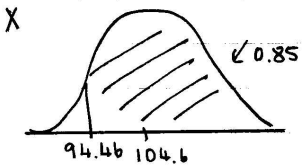
ie $z = -1.04$

$$z = \frac{X - \mu}{\sigma}$$

$$-1.04 = \frac{X - 104.6}{9.75}$$

$$X = 104.6 - 1.04 \times 9.75$$

$$= 94.46$$



ie. 85% are for more than £94.46

4. Using Binomial Distribution

$p = 0.3$ = probability of success = probability of a woman

$n = 15$ = no. of trials = members of the committee

Let X = number of women on the committee

then $X \sim Bi(15, 0.3)$

(a) $P(3 \text{ or fewer are women}) = P(X \leq 3)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.0047 + 0.0305 + 0.0916 + 0.1700$ (from tables)

$= 0.2968$

ie. 0.297 chance of 3 or fewer women on this committee

(b) P(10 or more are men) = P(5 or fewer are women)

$= P(X \leq 5) = P(X \leq 3) + P(X=4) + P(X=5)$

$= 0.2968 + 0.2186 + 0.2061$

$= 0.7215$

ie 0.722 chance of 10 or more men on this committee

5. Using Binomial $p = 0.11$ = probability of success
= probability of being a "high earner"

$n = 1500$ = no. of trials = number in sample

Let X = number of high earners

$X \sim Bi(1500, 0.11)$

(a) Mean $\mu = np = 1500 \times 0.11 = 165$

Standard Deviation $\sigma = \sqrt{np(1-p)} = \sqrt{1500 \times 0.11 \times (1-0.11)}$

$= 12.12$

(b) Using Normal Approximation, $X \sim N(165, 12.12^2)$

$P(X < 135) = P(X < -2.48) = 0.0066$

$Z = \frac{X-\mu}{\sigma} = \frac{135-165}{12.12} = -2.48$

ie probability of having 135 or fewer high earners in this sample of 1500 is 0.007

ie very small - if 0.11 is the probability of being a high earner