MATU9D2 : Practical Statistics Spring 2017
Practical 6 : Hand Calculations : Solutions

Question 1

<u>One Sample t-test</u> (since one sample & sd estimated from data

$H_0: \mu = 224$ $H_1: \mu \neq 224$ μ = mean dimension

<u>Significance Level</u> 0.05

<u>Test Statistic</u> $t = \frac{\bar{x} - 224}{s/\sqrt{n}} \sim t(n-1)$ under H_0

<u>Observed Test Statistic</u>

$n = 16$ $\Sigma x = 3585.02$ $\Sigma x^2 = 803274.0454$

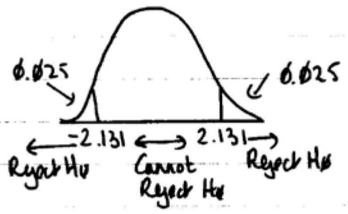
$$\bar{x} = \frac{\Sigma x}{n} = \frac{3585.02}{16} = 224.06375 = 224.064 //$$

$$s = \sqrt{\frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]} = \sqrt{\frac{1}{15}\left[803274.0454 - \frac{3585.02^2}{16}\right]}$$

$$= \sqrt{0.06803} = 0.2608 //$$

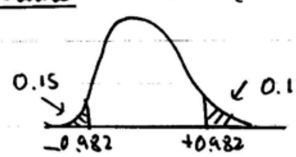
$$t = \frac{224.064 - 224}{0.2608/\sqrt{16}} = \frac{0.064}{0.0652} = 0.982 //$$

<u>Rejection Region</u> Sig level 0.05 ; 2 tailed ; $t(n-1)$



Critical Value = $t(15; 0.025)$
= 2.131

<u>p value</u> $= 2 \times P(t(15) > 0.982) = 2 \times 0.15 = 0.3 //$



← from tables $t(15)$

<u>Conclusion</u>

Observed Test Statistic of 0.982 is not in the Rejection Region & $p > 0.05$ so insufficient evidence to reject H_0 in favour of H_1 at 5% level.

ie insufficient evidence to reject that mean diameter is 224mm

(ii) $H_0: \mu = 224 \quad H_1: \mu \neq 224$

μ = mean dimension

(One Sample t-interval since sd not known & estimated from sample)

95% CI for μ

$\bar{x} \pm t(n-1; 0.025) s/\sqrt{n}$

$n = 16 \quad \bar{x} = 224.064$

$s = 0.2608$

$t(15; 0.025) = 2.131$

$224.064 \pm 2.131 \times \frac{0.2608}{\sqrt{16}}$

$224.064 \pm 2.131 \times 0.0652$

ie 224.064 ± 0.139

ie $(223.925, 224.203)$

Question 2

Unpaired t-test (assuming equal variance)

$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$

μ_1 = mean weight loss (A)

μ_2 = mean weight loss (B)

Significance level 0.05

Test Statistic $\quad t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t(n_1+n_2-2)$ under H_0

Observed Test Statistic

$$Cleaner\ A \quad n_1 = 10 \quad \bar{x}_1 = 10.36 \quad s_1 = 0.756$$

$$Cleaner\ B \quad n_2 = 8 \quad \bar{x}_2 = 9.50 \quad s_2 = 0.676$$

See below for full working for means and sd's.

It should always be included.

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{5.1438 + 3.1988}{16}$$

$$= 0.5214$$

$$t = \frac{10.36 - 9.50}{\sqrt{0.5214\left(\frac{1}{10} + \frac{1}{8}\right)}} = \frac{0.86}{0.3425} = 2.511$$

FULL WORKING FOR MEANS & SD's FOR QUESTION 2

$$\overline{x_1} = \frac{\sum x}{n} = \frac{103.6}{10} = 10.36$$

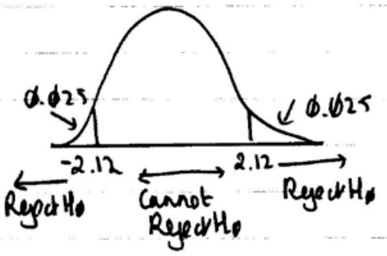
$$s_1 = \sqrt{\frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]} = \sqrt{\frac{1}{9}\left[1078.44 - \frac{103.6^2}{10}\right]} = \sqrt{\frac{5.144}{9}} = 0.756$$

$$\overline{x_2} = \frac{\sum x}{n} = \frac{76.00}{8} = 9.50$$

$$s_2 = \sqrt{\frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]} = \sqrt{\frac{1}{7}\left[725.50 - \frac{76.00^2}{8}\right]} = \sqrt{\frac{3.20}{7}} = 0.676$$

<u>Rejection Region</u> Sig Level $\phi.\phi 5$; 2 tailed ; $t(n_1+n_2-2)$

Critical Value = $t(16;\phi.\phi 25) = 2.12$



<u>p value</u> $= 2 \times P(t(16) > 2.51) = 2 \times 0.01 = 0.02$ //

$\rightarrow P(t(16) > 2.583) = 0.01$

<u>Conclusion</u> Observed Test Statistic is in the Rejection Region & $p < 0.05$ so can reject H_ϕ in favour of H_1 at 5% level. ie conclude sufficient evidence of a difference in mean weight loss on the two cleaners.

(iii) 95% CI for $\mu_1 - \mu_2$

$$\bar{x}_1 - \bar{x}_2 \pm t(n_1+n_2-2;\phi.\phi 25)\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$t(16;\phi.\phi 25) = 2.12$

$$(10.36 - 9.50) \pm 2.12 \times \sqrt{0.5214\left(\frac{1}{10}+\frac{1}{8}\right)}$$

$$0.86 \pm 2.12 \times 0.3425$$

$$0.86 \pm 0.7261$$

(0.134, 1.586)

Question 3 Paired t-test (2 Sample Paired Data)

$H_0 : \mu_d = 0 \qquad H_1 : \mu_d \neq 0 \qquad \mu_d$ = mean diff Before − After

Significance Level 0.05

Test Statistic $t = \bar{x}_d / (s_d/\sqrt{n}) \sim t(n-1)$ under H_0

Observed Test Statistic

Patient	After	Before	Diff
1	120	122	+2
2	124	127	+3
3	130	129	−1
4	118	120	+2
5	140	145	+5
6	128	129	+1
7	140	138	−2
8	135	132	−3
9	126	127	+1
10	130	129	−1
11	126	131	+5
12	127	131	+4

$\sum x_d = 16$

$\sum x_d^2 = 100$

$\bar{x}_d = 1.333$

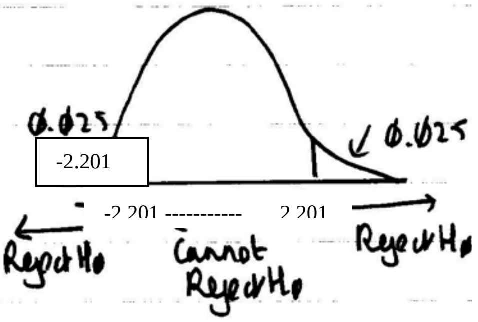
$s_d = 2.674$

$n = 12$

$$t = \frac{1.333}{2.674/\sqrt{12}} = 1.7269$$

Rejection Region 2 tailed; 0. 05; t(11)

t(11; 0.025) = 2.201



p value = 2 x P(t(11) >1.7269) =
2 x 0.06 = 0.13

Conclusion Observed Test Statistic outside the rejection region & p > 0.05 so insufficient evidence to reject H_0 in favour of H_1 at 5% level. ie Insufficient evidence of a difference in diastolic blood pressure on average.

$$95\%\ CI\ for\ \mu_d \quad \overline{x_d} \pm t(n-1; 0.026)\frac{s_d}{\sqrt{n}}$$

$$1.333 \pm 2.201 \times \frac{2.674}{\sqrt{12}}$$

$$1.333 \pm 1.699$$

$$(-0.366, 3.302)$$

95% CI for the difference includes zero so we cannot reject H_o in favour of H_1 at 5% level. So no statistically significant difference at 5% level.