University of Stirling
Computing Science and Mathematics
Faculty of Natural Sciences

MATU9D2 : PRACTICAL STATISTICS

Chapter 8 Analysis of Variance

Before the next topic - ANALYSIS OF VARIANCE - is introduced some definitions which will be necessary are presented.

EXPERIMENTAL UNITS - the items in an experiment

e.g.

RESPONSE the numerical result observed for a particular experimental unit

e.g.

FACTOR - a factor is a variable which is believed to affect the outcome of the experiment and is therefore under investigation

e.g.

LEVELS - in an experiment each factor takes two or more levels

e.g.

GROUPING FACTOR - a factor which classifies the experimental units into groups (between group factor)

e.g.

WITHIN FACTOR - a factor for which each experimental unit is measured at each level (within factor or trial factor)

e.g.

Analysis of Variance

The aims, ideas and assumptions underlying the general technique known as Analysis of Variance will be introduced by the simplest case i.e.

One Way Analysis of Variance (One Way ANOVA)

One Way ANOVA for independent samples is the most commonly used method for examining differences between group means. It is the natural extension of the two sample (unpaired) t test.

The technique is based upon the analysis of factors known or unknown which account for the variability in the data.

The variability in the data can be divided in two parts :

a) Explained (or Systematic) Variation

“ any natural or man-made influences that cause events to happen in a certain predictable way are systematic”

b) Unexplained (or Error or Residual) Variation

“fluctuation or varying of measures due to chance”

In general terms, ANOVA involves further partitioning and subsequent analysis of those systematic and error variances. Differences between groups are tested by calculating the statistic F which compares the variability between group means with the variability between individual values within a group.

Assumptions

1. The data are Normally distributed
2. The samples have common variance
3. The observations within each sample are a random sample and are independent of any other observation.

Situation

I independent samples; question of interest : comparison of the population means
J observations per group (i.e. this is the simplest case , with equal numbers of observations per group)

Hypotheses

$$H_0 \quad : \quad \mu_1 \quad = \quad \mu_2 \quad = \quad \square\square \quad = \quad \mu_I$$

i.e. The population means are equal.

H_1 : The population means are not all equal

i.e. There is at least one difference

Analysis

The total variance in this independent, randomised design can be partitioned as follows :

TOTAL VARIATION	=	BETWEEN GROUP VARIATION	+	WITHIN GROUP VARIATION

where

BETWEEN GROUP VARIATION : variances between group means caused by the independent variable

WITHIN GROUP VARIATION : variances due to uncontrolled factors and experimental unit differences.

TEST STATISTIC : $$F_{BET} = \frac{Between\ \ Group\ Variance}{Within\ \ Group\ Variance}$$

Calculations

$$Total\ \ Variation \quad = \quad SS_{TOT} \quad = \quad \sum_i \sum_j \left(y_{ij} - \bar{y}_{..}\right)^2 \quad = \quad \sum_i \sum_j y_{ij}^2 \quad - \quad \frac{\left(\sum_i \sum_j y_{ij}\right)^2}{IJ}$$

$$Between\ \ Group\ Variation \quad = \quad SS_{BG} \quad = \quad J \sum_i \left(\bar{y}_{i.} - \bar{y}_{..}\right)^2$$

$$Within\ Group\ Variation \quad = \quad SS_{WG} \quad = \quad \sum_i \sum_j \left(y_{ij} - \bar{y}_{i\,.}\right)^2$$

where

$$y_{ij} \quad = \quad jth\ response\ within\ the\ ith\ group \quad (j = 1, ...J)$$

$$\bar{y}_{i.} \quad = \quad \frac{\sum_j y_{ij}}{J} \quad = \quad mean\ of\ the\ ith\ group \quad (i = 1, ...I)$$

$$\bar{y}_{..} \quad = \quad \frac{\sum_i \sum_j y_{ij}}{IJ} \quad = \quad overall\ mean$$

Note $\quad SS_{TOT} \quad = \quad SS_{BG} \quad + \quad SS_{WG}$

It is easiest to calculate SS_{TOT} *and* SS_{BG} using the formulae above and to calculate SS_{WG}

as $\quad SS_{WG} \quad = \quad SS_{TOT} \quad - \quad SS_{BG}$

The test is then usually summarised in an ANOVA table :

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares	F
Between Groups	I - 1	SS_{BG}	$\frac{SS_{BG}}{I-1}$	$\frac{SS_{BG}/(I-1)}{SS_{WG}/[I(J-1)]}$
Within Groups	I (J - 1)	SS_{WG}	$\frac{SS_{WG}}{I(J-1)}$	
Total	IJ - 1	SS_{TOT}		

So F as calculated above is the **Observed Test Statistic**.

Rejection Region & Conclusion

For a significance level of 0.05; the critical value is c = F(I-1, IJ-I; 0.05).

i.e. if Observed F is greater than F(I-1, IJ-I; 0.05), reject H_o in favour of H_1 at 5% level and conclude that the means are not all equal.

i.e. if Observed F is less than F(I-1, IJ-I; 0.05), cannot reject H_o in favour of H_1 at 5% level and conclude that we have insufficient evidence to reject the hypothesis that the means are equal.

Comment

If there are more than 2 groups then when H_o is rejected we can conclude that there is a difference between the group means but we cannot identify where the differences lie.

The methods uses to identify where the differences lie is called **Multiple Comparisons**.

If we simply used standard t intervals (tests) for each difference then if each interval have confidence 1 - α, then the overall confidence of the set of intervals

i.e. the probability that every interval covers the true value, is far less than 1 - α (1 - α is usually 0.95 since α is usually 0.05).

Multiple Comparisons

There are several commonly used techniques - Scheffe, Bonferroni, Tukey, Dunnett, Least Squares Differences.

Scheffe - Pairwise Comparisons of Group Means

A set of simultaneous confidence intervals with at least 95% confidence are :

$$\left(\bar{y}_{i.} \quad - \quad \bar{y}_{j.}\right) \quad \pm \quad \sqrt{(I-1)F(I-1, IJ-I; 0.05)}\sqrt{\frac{SS_{WG}}{IJ-I}}\sqrt{\left(\frac{1}{J}+\frac{1}{J}\right)}$$

where $1 \leq \ i < j \leq I$

Bonferroni - Pairwise Comparisons of Group Means

A set of simultaneous confidence intervals with at least 95% confidence are :

$$\left(\bar{y}_{i.} \quad - \quad \bar{y}_{j.}\right) \quad \pm \quad t(IJ-I; 0.05/k)\sqrt{\frac{SS_{WG}}{IJ-I}}\sqrt{\left(\frac{1}{J}+\frac{1}{J}\right)}$$

where $1 \leq \ i < j \leq I$ and k is the number of differences

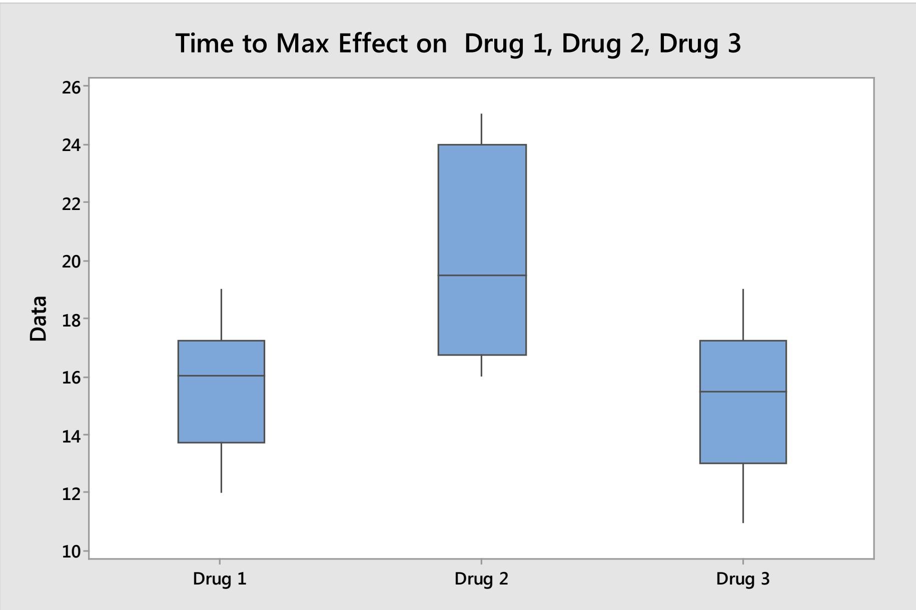
Example

We have collected data from three independent groups with 10 observations per group. The response measured is the time in hours until the maximum effect of three different drugs.

The question of interest : Is there a difference in the mean times to maximum effect on the three drugs?

Subject	DRUG 1	DRUG 2	DRUG 3
1	17	17	13
2	17	16	13
3	19	24	19
4	15	16	16
5	18	23	15
6	14	17	18
7	13	24	15
8	12	25	16
9	15	19	17
10	17	20	11

SOLUTION



One-way ANOVA: Drug 1, Drug 2, Drug 3

Method

Null hypothesis	All means are equal
Alternative hypothesis	At least one mean is different
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Factor	3	Drug 1, Drug 2, Drug 3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	141.9	70.933	8.82	0.001
Error	27	217.1	8.041		
Total	29	359.0			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.83562	39.52%	35.04%	25.33%

Means

Factor	N	Mean	StDev	95% CI
Drug 1	10	15.700	2.263	(13.860, 17.540)
Drug 2	10	20.10	3.60	(18.26, 21.94)
Drug 3	10	15.300	2.452	(13.460, 17.140)

Pooled StDev = 2.83562

Tukey Pairwise Comparisons

Grouping Information Using the Tukey Method and 95% Confidence

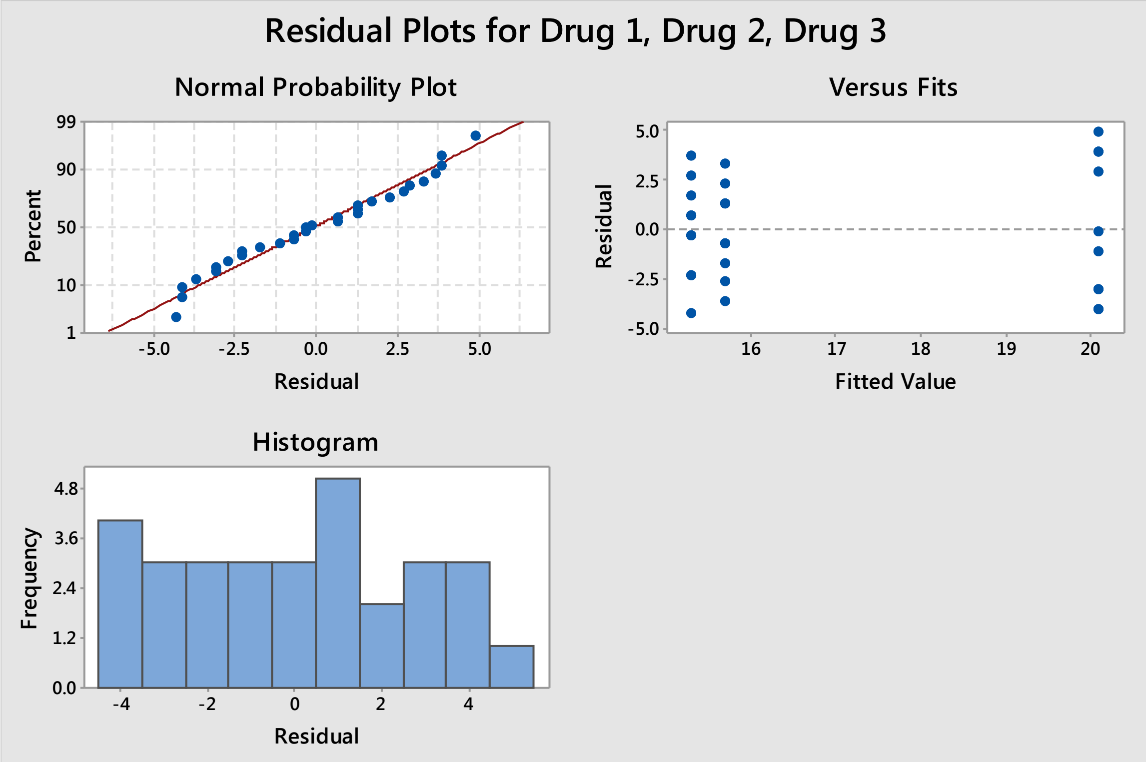
Factor	N	Mean	Grouping	
Drug 2	10	20.10	A	
Drug 1	10	15.700		B
Drug 3	10	15.300		B

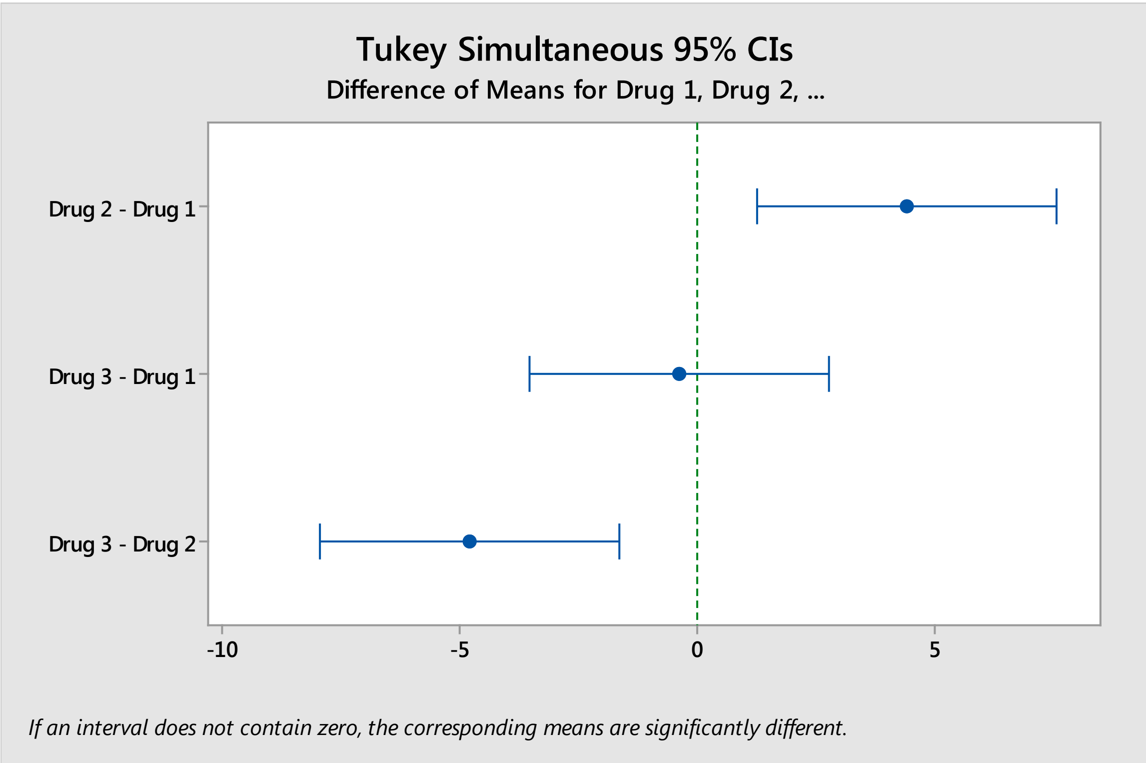
Means that do not share a letter are significantly different.

Tukey Simultaneous Tests for Differences of Means

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
Drug 2 - Drug 1	4.40	1.27	(1.25, 7.55)	3.47	0.005
Drug 3 - Drug 1	-0.40	1.27	(-3.55, 2.75)	-0.32	0.947
Drug 3 - Drug 2	-4.80	1.27	(-7.95, -1.65)	-3.79	0.002

Individual confidence level = 98.04%


Residual Plots for Drug 1, Drug 2, Drug 3
Normal Probability Plot
Percent
Residual
Versus Fits
Residual
Fitted Value
Histogram
Frequency
Residual


Tukey Simultaneous 95% CIs
Difference of Means for Drug 1, Drug 2, ...
Drug 2 - Drug 1
Drug 3 - Drug 1
Drug 3 - Drug 2
If an interval does not contain zero, the corresponding means are significantly different.

COMMENTS

The same analysis would be appropriate if there were different numbers per group and with any number of groups.

1. In fact, if I=2

   i.e. 2 groups the results are exactly equivalent to performing an Unpaired t test.

2. If there are unequal numbers of observations in the groups the only difference is that the calculations become more complex.

   i.e. I groups with n_i observations in the ith group.

Hypotheses

$$H_0 \quad : \quad \mu_1 \quad = \quad \mu_2 \quad = \quad \cdots \quad = \quad \mu_I$$

i.e. The population means are equal.

H_1 : The population means are not all equal

i.e. There is at least one difference

ANOVA Table

$$N \quad = \quad \sum_i n_i$$

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares	F
Between Groups	I - 1	$SS_{BG} \quad = \quad \sum_i^I n_i \left(\bar{y}_{i.} - \bar{y}_{..}\right)^2$	$\dfrac{SS_{BG}}{I-1}$	$\dfrac{SS_{BG}/(I-1)}{SS_{WG}/(N-I)}$
Within Groups	N - I	$SS_{BG} \quad = \quad \sum_i^I \sum_j^{ni} \left(y_{ij} - \bar{y}_{i.}\right)^2$	$\dfrac{SS_{WG}}{(N-I)}$	
Total	N - 1	$SS_{TOT} \quad = \quad \sum_i^I \sum_j^{ni} \left(y_{ij} - \bar{y}_{..}\right)^2$		

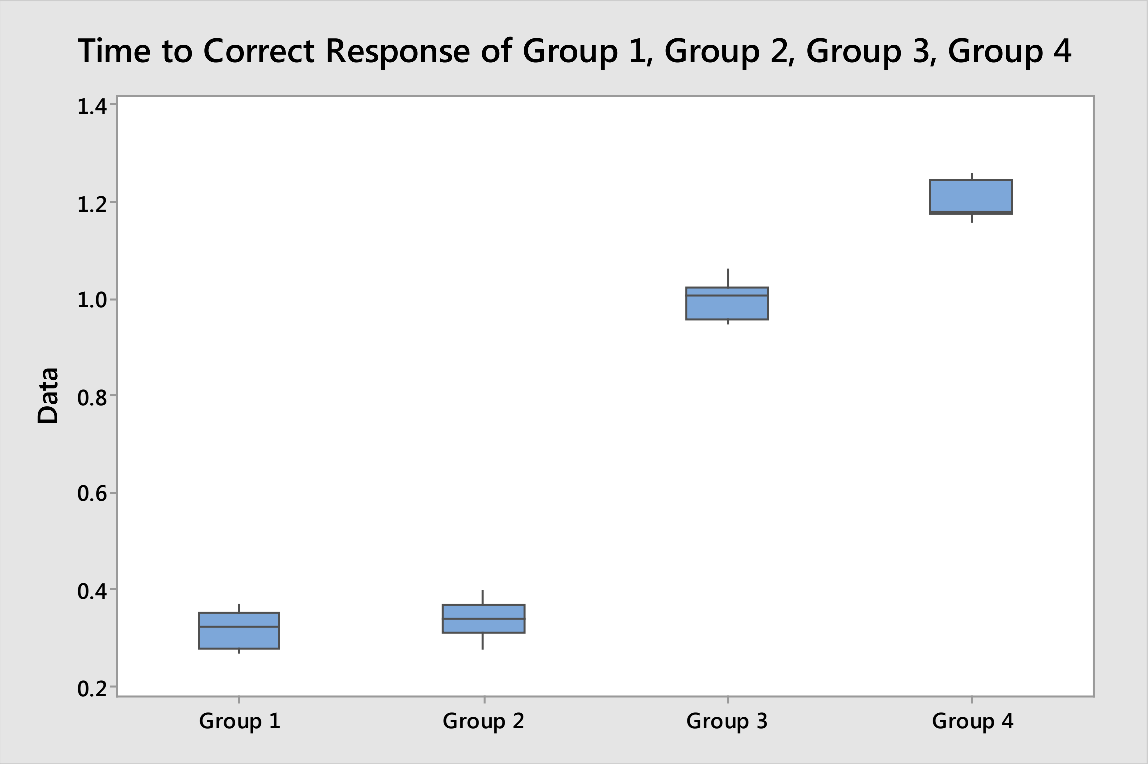
Note because of the complexity we will not be attempting to perform the calculations by hand for this situation when the number of observations in unequal.

Example : One Way ANOVA

A study was designed to investigate responses to conflicting information. A group of subjects were randomly assigned to one of four experiments (1) Shown a rectangle identify the colour; (2) Colour in text and identify the colour; (3) Mixture of noncorresponding words and colours and say the word; (4) Mixture of noncorresponding words and colours and identify the colour. The time required to give a correct responses was recorded. The data is given below. Are there any differences in the time to a correct response?

Group 1	Group 2	Group 3	Group 4
0.37	0.40	1.06	1.26
0.27	0.28	0.95	1.24
0.28	0.36	0.96	1.16
0.35	0.32	1.01	1.18
0.34	0.34	1.01	1.18
0.31	0.34	1.01	1.18

Solution



One-way ANOVA: Group 1, Group 2, Group 3, Group 4

Method

Null hypothesis	All means are equal
Alternative hypothesis	At least one mean is different
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Factor	4	Group 1, Group 2, Group 3, Group 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	3	3.67860	1.22620	766.38	0.000
Error	20	0.03200	0.00160		
Total	23	3.71060			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.04	99.14%	99.01%	98.76%

Means

Factor	N	Mean	StDev	95% CI
Group 1	6	0.3200	0.0400	(0.2859, 0.3541)
Group 2	6	0.3400	0.0400	(0.3059, 0.3741)
Group 3	6	1.0000	0.0400	(0.9659, 1.0341)
Group 4	6	1.2000	0.0400	(1.1659, 1.2341)

Pooled StDev = 0.04

Tukey Pairwise Comparisons

Grouping Information Using the Tukey Method and 95% Confidence

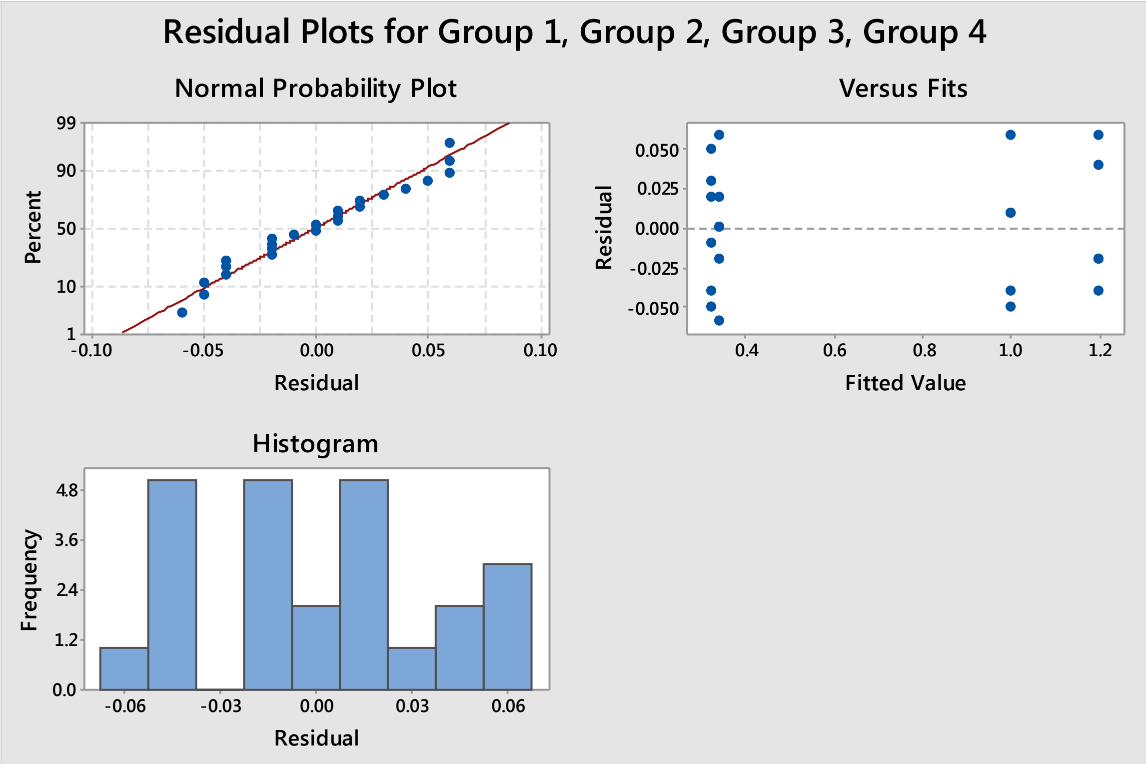
Factor	N	Mean	Grouping
Group 4	6	1.2000	A
Group 3	6	1.0000	B
Group 2	6	0.3400	C
Group 1	6	0.3200	C

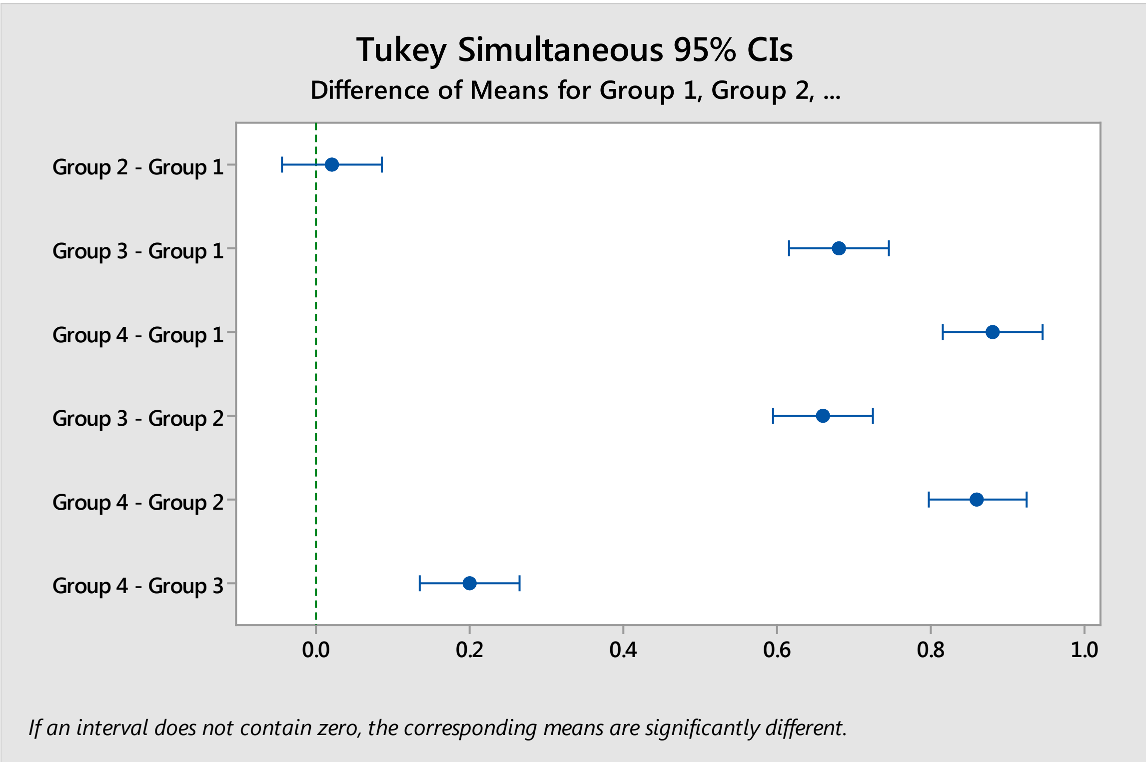
Means that do not share a letter are significantly different.

Tukey Simultaneous Tests for Differences of Means

Difference of Levels	Difference of Means	SE of Difference	95% CI	T-Value	Adjusted P-Value
Group 2 - Group 1	0.0200	0.0231	(-0.0447, 0.0847)	0.87	0.822
Group 3 - Group 1	0.6800	0.0231	(0.6153, 0.7447)	29.44	0.000
Group 4 - Group 1	0.8800	0.0231	(0.8153, 0.9447)	38.11	0.000
Group 3 - Group 2	0.6600	0.0231	(0.5953, 0.7247)	28.58	0.000
Group 4 - Group 2	0.8600	0.0231	(0.7953, 0.9247)	37.24	0.000
Group 4 - Group 3	0.2000	0.0231	(0.1353, 0.2647)	8.66	0.000

Individual confidence level = 98.89%


Residual Plots for Group 1, Group 2, Group 3, Group 4
Normal Probability Plot
Percent
99
90
50
10
1
-0.10
-0.05
0.00
0.05
0.10
Residual
Versus Fits
Residual
0.050
0.025
0.000
-0.025
-0.050
0.4
0.6
0.8
1.0
1.2
Fitted Value
Histogram
Frequency
4.8
3.6
2.4
1.2
0.0
-0.06
-0.03
0.00
0.03
0.06
Residual


Tukey Simultaneous 95% CIs
Difference of Means for Group 1, Group 2, ...
Group 2 - Group 1
Group 3 - Group 1
Group 4 - Group 1
Group 3 - Group 2
Group 4 - Group 2
Group 4 - Group 3
0.0
0.2
0.4
0.6
0.8
1.0
If an interval does not contain zero, the corresponding means are significantly different.

Alternative Views

Now that we have introduced the technique and demonstrated its implementation. It will be helpful to develop our ideas and understanding if we review some of the theoretical concepts.

We have seen that although the tests actually tested hypotheses about the means of data the method derived its name because it partitions the variability in the data and that the variability was determined by the Sum of Squares.

In the example (One Way ANOVA) the variability was divided into two parts i.e. Between Groups and Within Groups. The Hypotheses tested were :

Ho : The population means are equal.

H1 : The population means are not all equal.

The above could be re-written as :

$$H_0 \; : \; y_{ij} = \mu + \varepsilon_{ij} \qquad RSS_0 = \sum\sum\left(y_{ij} - \bar{y}_{..}\right)^2$$

$$H_1 \; : \; y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \qquad RSS_1 = \sum\sum\left(y_{ij} - \bar{y}_{i.}\right)^2$$

and j = 1, 2,.........J and i = 1, 2,......... I

where y_{ij} represents the jth response on the ith factor level

J represents the number of experimental units on level i

I represents the number of levels

μ represents the overall mean

α_i represents the deviation from the overall mean at level i

and ε_{ij} represents the random experimental error

With the addition of various numerical assumptions that summarise the assumptions of Normality and constant variance this is exactly as before.

The above states that we assume that the responses are generated by an underlying linear model i.e. the observed response is assumed to consist of an overall mean plus deviations due to the factors of the experiment (plus possibly their interactions - see later) plus a random error term.

We would then use an F test (General Linear Test) to test the models as follows :

$$F \quad = \quad \frac{\left(RSS_0 - RSS_1\right)/(I-1)}{RSS_1/(IJ-I)}$$

which is distributed as F(I-1, IJ - I) under Ho

which is exactly the same as

$$F \quad = \quad \frac{\left(SS_{TOT} \quad - SS_{WG}\right)/(I-1)}{SS_{WG}/(IJ-I)}$$

$$= \quad \frac{SS_{BG}/(I-1)}{SS_{WG}/(IJ-I)}$$

We will now consider the next most complicated case, the ideas behind which can be extended to accommodate many more factors.

Two Way Analysis of Variance (Two Way ANOVA)

This essentially means that we have two factors. Two examples of studies requiring this form of analysis are :

One factor - drug and the other factor - dose i.e. a comparison of responses of three drugs on three equivalent doses .

One factor - drug and the other factor - disease state i.e. a comparison of different drugs in different groups of subjects (healthy volunteers, patients with renal failure, patients on renal dialysis).

Under the general term two-way ANOVA there are three possible situations :

(a) equal numbers at each treatment combination (number > 1)

(b) unequal numbers at each treatment combination

(c) one experimental unit at each treatment combination.

The full model for situation (a) with two factors A and B is

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

and $i = 1, 2, \ldots\ldots\ldots I$, $j = 1, 2, \ldots\ldots\ldots J$ and $k = 1,2, \ldots\ldots\ldots\ldots K$

where	y_{ijk}	represents the kth response on the ith level of factor A and the jth level of factor B
	I	represents the number of levels of factor A
	J	represents the number of levels of factor B
	K	represents the number of experimental units for combination ij of factors A and B
	μ	represents the overall mean
	α_i	represents the effect of level i of factor A (MAIN EFFECT)
	β_i	represents the effect of level j of factor B (MAIN EFFECT)
	$(\alpha\beta)_{ij}$	represents the interaction between the ith level of factor A and the jth level of factor B (INTERACTION)
and	ε_{ijk}	represents the random experimental error

There are addition numerical assumptions to represent the assumptions of Normality and constant variance.

Two Way Analysis of Variance - Equal Numbers per Combination (> 1) (continued)

Questions posed by the analysis are

(I) Does factor A have a significant effect ?

i.e. Is the responses the same for each level of factor A ?

(II) Does factor B have a significant effect ?

i.e. Is the responses the same for each level of factor B ?

(III) Do factors A and B interact ?

i.e. Is the relative responses of factor A the same at each level of factor B ?

The above imply that the hypotheses tested are

$$(I) \qquad H_A \; : \; \alpha_i = 0 \qquad \forall i$$

$$(II) \qquad H_B \; : \; \beta_j = 0 \qquad \forall j$$

$$(III) \qquad H_{AB} \; : \; (\alpha\beta)_{ij} = 0 \qquad \forall i, j$$

Since this is an ANOVA the calculations involve subdividing the total variability in the data. In this case it is divided into four parts i.e.

$$\left.\begin{array}{ll} 1. & \text{Variation due to Factor A} \\ 2. & \text{Variation due to Factor B} \\ 3. & \text{Variation due to the Interaction of Factors A and B} \end{array}\right\} \text{Explained Variation}$$

4. The Residual / Error / Unexplained Variation

i.e. $SS_{TOT} = SS_A + SS_B + SS_I + SS_{RES}$

For information, the following are the formulae (We will not be calculating them by hand !!)

Let	I	=	no of levels of Factor A
	J	=	no of levels of Factor B
	K	=	no of experimental units for each combination
	y_{ijk}	=	jth observation for the ith level of Factor A and jth level of Factor B

	<u>Summary Statistics</u>	<u>Sums of Squares</u>
Overall	$\bar{y}_{...} = \dfrac{\sum\sum\sum y_{ijk}}{IJK}$	$SS_{TOT} = \sum\sum\sum\left(y_{ijk} - \bar{y}_{...}\right)^2$
Factor A	$\bar{y}_{i..} = \dfrac{\sum\sum y_{ijk}}{JK}$	$SS_A = \sum\sum\sum\left(\bar{y}_{i..} - \bar{y}_{...}\right)^2$
Factor B	$\bar{y}_{.j.} = \dfrac{\sum\sum y_{ijk}}{IK}$	$SS_B = \sum\sum\sum\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2$
Interaction	$\bar{y}_{ij.} = \dfrac{\sum y_{ijk}}{K}$	$SS_I = \sum\sum\sum\left(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}\right)^2$
Residual		$SS_{RES} = \sum\sum\sum\left(y_{ijk} - \bar{y}_{ij.}\right)^2$

In this case when the numbers in each combination are equal there is a unique ANOVA table as follows :

Source of Variation	df	SS	MS	F
Factor A	I - 1	SS_A	$\frac{SS_A}{I-1}$	$\frac{SS_A/(I-1)}{SS_{RES}/(IJK-I)}$
Factor B	J - 1	SS_B	$\frac{SS_B}{J-1}$	$\frac{SS_B/(J-1)}{SS_{RES}/(IJK-I)}$
Interaction	(I-1) (J-1)	SS_I	$\frac{SS_I}{(I-1)(J-1)}$	$\frac{SS_I/[(I-1)(J-1)]}{SS_{RES}/(IJK-I)}$
Residual	IJK -IJ	SS_{RES}	$\frac{SS_{RES}}{IJK-IJ}$	
Total	IJK - 1	SS_{TOT}		

The calculated F's are compared to the appropriate point of the F distribution and if F is greater than the tabulated value then the null hypothesis of no effect is rejected.

As with any analysis the assumptions should be checked.

If there is a significant result then the exact nature of the difference in effect should be investigated using multiple comparisons, as discussed previously.

Example

An study consists of measuring the fall in blood pressure on a random sample of men and women; with each patient randomly allocated to one of the three drugs being investigated. In this study there are 12 patients, 6 men and 6 women. Note that this is really far too small a sample size !!

The data are

		Drug		
		1	2	3
Gender	Male	45 55	30 40	10 10
	Female	65 55	20 15	5 10

The model is

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

and $i = 1, 2$, $j = 1, 2, 3$ and $k = 1,2$

and α_i represents the effect of gender i ; β_j the effect due to drug j and $(\alpha\beta)_{ij}$ represents the interaction.

The ANOVA table is

Analysis of Variance for BPChange

Source	DF	SS	MS	F	P
Gender	1	33.33	33.33	1.14	0.326
Drug	2	4362.50	2181.25	74.79	0.000
Gender*Drug	2	379.17	189.58	6.50	0.031
Error	6	175.00	29.17		
Total	11	4950.00			

Conclusions

The main gender effect is not significant at the 5% significance level while the main drug effect and the interaction are significant at the 5% significance level.

Note that when there is a significant interaction the interpretation of the main effects is difficult so it is on the follow up multiple comparisons for the interaction term that any interpretation is made.

To investigate the reasons for the above results the following plots were constructed.

ROWS: Gender   COLUMNS: Drug

	1	2	3	ALL
1	50.000	35.000	10.000	31.667
2	60.000	17.500	7.500	28.333
ALL	55.000	26.250	8.750	30.000

CELL CONTENTS -- BPChange:MEAN

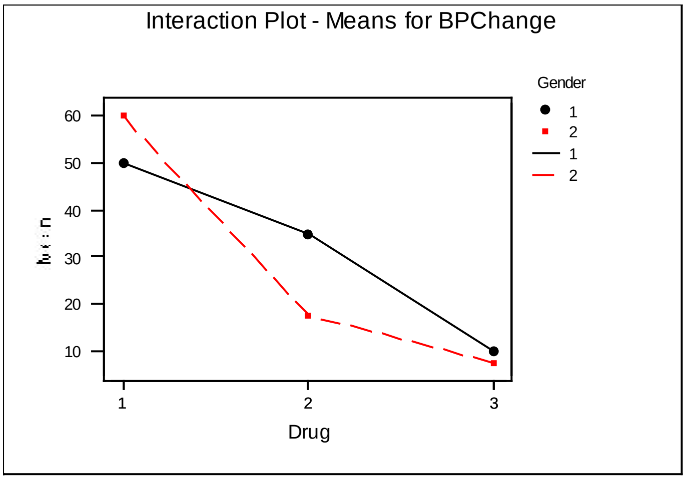


Figure 1

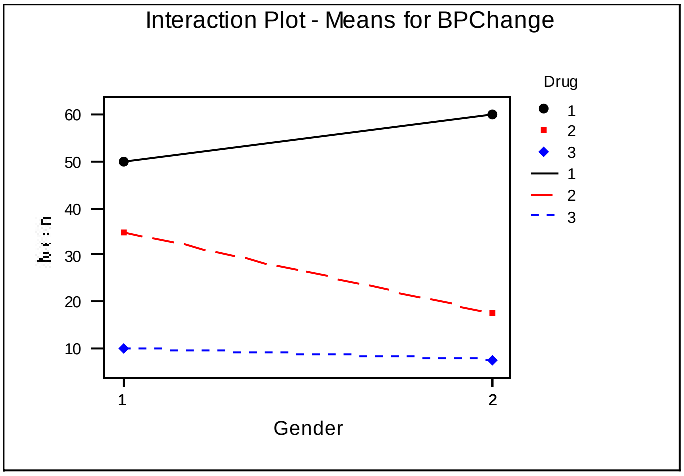


Figure 2

Figure 1 shows that the BP fall decreases between the Drugs

Figure 2 shows that the Gender effect is small and the fall on Drug 1 is greater with Females than for males but the fall is smaller for Females on the other two drugs. This accounts for the significant interaction term in the ANOVA table.

The discussion so far related to the situation where there are equal numbers at each treatment combination i.e. Situation (a). In Situations (b) and (c) the following points should be noted :

(b) When there are unequal numbers of observations per treatment combination there is no single partitioning of the total variation.

i.e. the conclusions may depend on the order in which the effects are added into the model.

(c) When we only have one observation per combination we cannot estimate the interaction term. So the experiment must be carefully designed to minimise any interactions and we can then analyse the data on the assumption that the interactions are all zero.

Situation (c) An Extension of Paired t test

We will re-examine an example we have tackled before in Practical Session 4 and then looke at an extension to more than 2 related samples.

In an experiment to detect any relationship between the heights of Aberdonian fathers and their eldest sons, eight pairs of fathers and sons were selected at random from the city population and their exact heights recorded, in inches, as follows :

Height of father	:	63.2	74.1	70.6	72.3	65.8	67.3	68.9	66.7
Height of son	:	65.7	67.0	69.5	70.5	64.4	68.3	71.4	63.5

Is there a significant difference in the heights of fathers and sons ?

One-Sample T: F-S

```
Test of μ = 0 vs ≠ 0

Variable  N  Mean  StDev  SE Mean       95% CI        T      P
F-S       8  1.07   3.19     1.13  (-1.59, 3.74)  0.95  0.373
```

Paired T-Test and CI: Height of father, Height of son

```
Paired T for Height of father - Height of son

                  N   Mean  StDev  SE Mean
Height of father  8  68.61   3.59     1.27
Height of son     8  67.54   2.88     1.02
Difference        8   1.07   3.19     1.13


95% CI for mean difference: (-1.59, 3.74)
T-Test of mean difference = 0 (vs ≠ 0): T-Value = 0.95  P-Value = 0.373
```

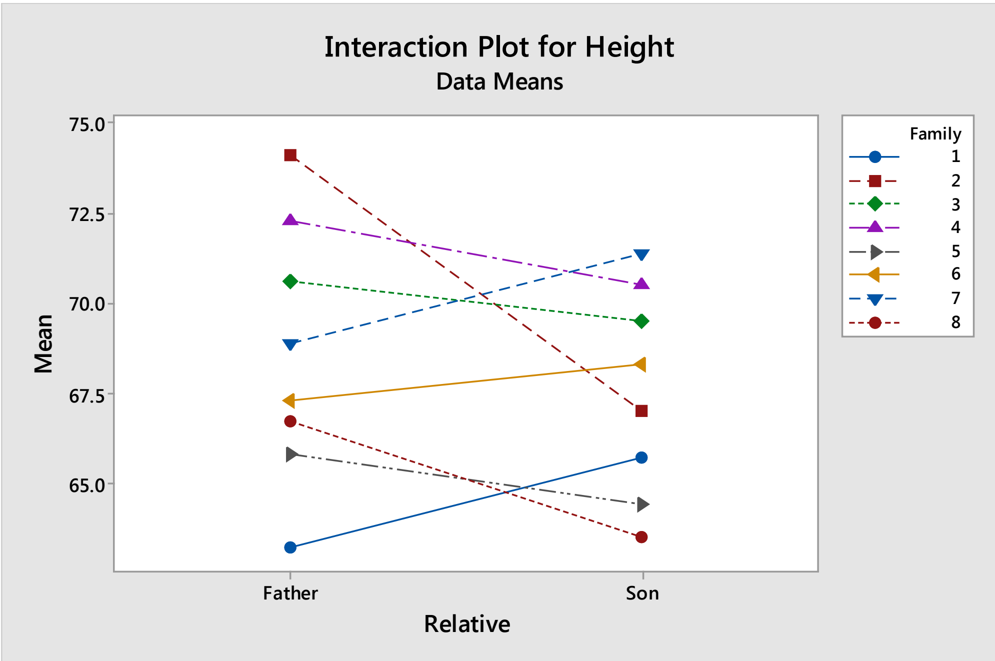
General Linear Model: Height versus Family, Relative

```
Factor    Type   Levels  Values
Family    Fixed       8  1, 2, 3, 4, 5, 6, 7, 8
Relative  Fixed       2  Father, Son

Analysis of Variance

Source      DF   Adj SS  Adj MS  F-Value  P-Value
  Family     7  112.610  16.087     3.16    0.076
  Relative   1    4.622   4.622     0.91    0.373
Error        7   35.657   5.094
Total       15  152.890


Model Summary
      S    R-sq  R-sq(adj)  R-sq(pred)
2.25697  76.68%     50.02%       0.00%
```



Example A random sample of **10 patients receive each of 3 drugs** in random order. The responses are given below. Is there a significant difference between the responses on the drugs?

Subj.	Drug1	Drug2	Drug3
1	12	13	12
2	12	13	11
3	12	14	13
4	13	12	11
5	13	14	12
6	14	13	14
7	13	14	12
8	14	15	12
9	13	13	14
10	14	15	14

1. **Incorrect analysis** - because related samples - One Way ANOVA

One-way ANOVA: Drug 1, Drug 2, Drug 3

Method

Null hypothesis	All means are equal
Alternative hypothesis	At least one mean is different
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	6.067	3.0333	3.04	0.064
Error	27	26.900	0.9963		
Total	29	32.967			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.998146	18.40%	12.36%	0.00%

Means

Factor	N	Mean	StDev	95% CI
Drug 1	10	13.000	0.816	(12.352, 13.648)
Drug 2	10	13.600	0.966	(12.952, 14.248)
Drug 3	10	12.500	1.179	(11.852, 13.148)

Pooled StDev = 0.998146

2. **Correct analysis** - because related samples

General Linear Model: Response versus Subject, Drug

```
Method

Factor coding  (-1, 0, +1)


Factor Information

Factor   Type   Levels  Values
Subject  Fixed      10  1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Drug     Fixed       3  Drug 1, Drug 2, Drug 3


Analysis of Variance

Source     DF  Adj SS  Adj MS  F-Value  P-Value
  Subject   9  15.633  1.7370     2.78    0.031
  Drug      2   6.067  3.0333     4.85    0.021
Error      18  11.267  0.6259
Total      29  32.967


Model Summary

       S    R-sq  R-sq(adj)  R-sq(pred)
0.791155  65.82%     44.94%       5.07%
```