UNIVERSITY OF
STIRLING

PDMU9L4
DATA SKILLS

WORKBOOK 3 (of 3)
PATH 3

Computing Science & Mathematics
Faculty of Natural Sciences

Academic Year 16/17

PDMU9L4: DATA SKILLS
PATH 3:
KEY MATHEMATICAL SKILLS

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Topic 9: Algebra Rules, OK!

In a previous sheet we solved a variety of problems which involved lines. We solved these problems both graphically and algebraically. In t he latter method we had to introduce a set of rules by which to manipulate the algebraic equations to reveal the required solution. Before looking again at these rules let's look at a 'cruder' method for solving algebraic equations. For example:

To find the value of x to satisfy the equation:

$$3*x+4=10 \qquad (1)$$

we could do it by 'trial and error' or more optimistically 'trial and improvement'. Let's guess the answer and see whether we are correct:

Guess: $x=1$ then LHS $=3*0+4=4$ RHS $=10$
(where LHS = left hand side; RHS = right hand side).

So $x=0$ is not the answer because if it was LHS = RHS. So let us try another guess.

Guess: $x=1$ then LHS $=3*1+4=6$ RHS $=10$

So $x=1$ is not the answer.

Guess: $x=2$ then LHS $=3*2+4=10$ RHS $=10$

So we have the solution!

This is a bit 'hit and miss' so it is useful. One method is to manipulate the equation into a simpler form so that it is much easier to deduce the solution. The strategy is to get all the x bits on the left hand side and all the non x bits on the right hand side. To do this we have to 'drag' 4 to the other side of the equation. To do this formally we *subtract* 4 from *both* sides of the equation:

$$3*x+4-4=10-4$$

But $3*x+0=6$

i.e. $3*x=6 \qquad (2)$

Why do we subtract? Subtraction is the 'inverse' operation to addition. To reverse the effect of adding 4 we have to subtract 4, and do this to both sides of the equation so tht the equation stays an equation.

To solve the simpler equation (2) we have to find a number x which when tripled equals 6. This is clearly 2. We can obtain this solution formally by undoing the effect of multiplying x by 3, i.e. dividing (both sides of the equation) by 3, the 'inverse' operation.

$$\frac{x*3}{3} = \frac{6}{3} = 2 \quad \text{i.e. } x = 2$$

Equations (1) and (2) are *linear* equations and their manipulation is relatively straightforward – we add, subtract, multiply and divide by appropriate numbers to get x on its own and t herefore to have a solution.

As an application consider the following example concerning Ohm's law:

The voltage across a resistor R carrying current I in an electrical circuit is given by:

$$V = I * R \qquad (3)$$

If the voltage is 15.6 volts and the resistor is 214 ohms then this equation for current I takes the form:

$$15.6 = 214 * I$$

To find current I divide both sides of the equation by 214:

$$I = 15.6/214 = 0.073 \text{ (amps)}$$

In general, if we divide both sides of (3) by R we obtain:

$$I = V/R.$$

Nonlinear equations are more difficult to handle and there are special techniques based on the structural properties of the equations and the special 'inverse' operations that will unwrap the variable whose value is to be found. In the previous sheet we considered equations such as:

$$(3.25)^n = 16.45$$

To unwrap n we have to apply the operation which is the inverse of taking powers, namely taking lots. So what we did was to log both sides of the equation:

$$\log((3.25)^n) = \log(16.45)$$

Using the structural property:

$$\log(a^n) = n\log(a) \qquad \text{for } a = 3.25$$

we have: $n\log(3.25) = \log(16.45)$

i.e. $0.512 * n = 1.216$

i.e. $n = 2.38.$

Let's look at some other examples:

Example 1
What is the radius r of the sphere with a volume (V) of 15.6 cubic metres?

The formula for the volume of a sphere is:

$$V = \frac{4}{3} * \pi * r^3 = 4.189 * r^3$$

Volume of Sphere

i.e. $15.6 = 4.189 * r^3$

i.e. $r^3 = 3.724$

on dividing by 4.189. To unwrap r we have to find an operation which will undo the operation of taking the third power. The required operation is taking the cube root, i.e. taking the 1/3 = 0.3333 power. So:

$$r = (3.724)^{0.333} = 1.55$$

$r^n = x$
$r = x^{\frac{1}{n}}$

One of the nice things about mathematics is that you can usually check a solution to see whether it is correct. Let's use this radius to evaluate the volume to see whether it really is 15.6. so:

$$V = 4.189 * (1.55)^3 = 4.189 * 3.724 = 15.6 \text{ !!!}$$

Example 2
In sound, the stimulus S and response R are related by the formula

$$R = \log(S)$$

when S, R are in suitable units. If the response is $R = 2.71$ then what is the stimulus S? This means we have to solve the equation:

$$2.71 = \log(S).$$

to solve this equation we have to unwrap S by finding the inverse operation to the log. This is the antilog, namely taking powers of 10.

$$10^{2.71} = 10^{\log(S)} = S.$$

So: $S = 512.9$.

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Topic 9: Algebra Rules, OK! – Worksheet 9/1

1. Solve the following equations:

(i) $4*x+3=11$ for x $4x+3=11$ $\frac{4x}{4}=\frac{8}{4}$ $x=2$

(ii) $2*x-5=7$ for x $2x-5=7$ $\frac{2x}{2}=\frac{12}{2}$ $x=6$

(iii) $4-2*x=2$ for x $4-2x=2$ $\frac{-2x}{-2}=\frac{-2}{-2}$ $x=1$

(iv) $5*x-2=3*x+8$ for x $5x-2=3x+8$ $5x-3x=10$ $\frac{2x}{2}=\frac{10}{2}$

(v) $14-3*x=6+x$ for x $14-3x=6+x$ $-3x-x=-8$ $\frac{-4x}{-4}=\frac{-8}{-4}$

Answers:

(i) 2 ✓ (ii) 6 ✓ (iii) 1 ✓

(iv) 5 ✓ (v) 2 ✓

2. Solve the following problems:

(i) Find the radius of the circle with area $A = 7.32$ (square centimetres) using the formula

$$A = \pi * r^2$$

$\frac{7.32}{\pi}=\frac{\pi \cdot r^2}{\pi}$ $2.33=r^2$ $r^2=2.33^{\frac{1}{2}}$ $r=1.53$

(ii) The formula for the area A of the curved surface of a symmetric cone with height h and base radius r is given by

$$A = \pi * r * (r^2 + h^2)^{0.5}$$

If $A = 8.123$ (square metres) and $r = 1.246$ (metres) then what is the height h?

$8.123=\pi r(r^2+h^2)^{0.5}$

$\frac{8.123}{3.914}=(r^2+h^2)^{0.5}$

$(2.075)^2=r^2+h^2$

$4.3062-1.5525=h^2$

$2.7537=h^2$

$h=\sqrt{2.7537}$

$h=1.6594$

(i) 1.53cm ✓ (ii) 1.6594 m ✓

3. (i) If the body heat H generated by an animal is 4123 (cals/day) then what is likely to be its body weight W? Use the formula:

$$H = 60 * W^{0.79}$$

(ii) If the response R is related to stimulus S by the formula:

$$R = 1.87 * S^{0.56}$$

then find the stimulus S that will yield a response of $R = 5.921$.

(i) 213.80 ✓ (ii) 7.76 ✓

4. A kettle that has just boiled cools according to the law:

$$T = 21 + 100 * 10^{-0.45t}$$

where T is the temperature. How long will it take for the temperature to fall to 45 degrees? (i.e. find t to satisfy this equation.)

Answer: ✓ $t = 1.377$

$T = 45$

$45 = 21 + 100 \cdot 10^{-0.45t}$

$45 - 21 = 100 \cdot 10^{-0.45t}$

$\log(24) = \log(100 \cdot 10^{-0.45t})$

$\log(24) = \log(100) + \log(10^{-0.45t})$

$\frac{\log(24) - \log(100)}{\log(10)} = \frac{-0.45t \log(10)}{\log(10)}$

$\frac{-0.62}{-0.45} = \frac{-0.45t}{-0.45}$

$t = 1.377$

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Topic 10: Orders of Magnitude!

It is claimed that many students are using their calculators 'mechanically' without, for example, checking that their answers make sense. So this worksheet is about how best to spot check these answers. First some worked examples. In these questions use your calculator only to check the answers.

Example 1
Which of the following numbers is closest to 36×51 ?

2000 ☑ 350 ☐ 15000 ☐ 90 ☐ *Tick one!*

Check with your calculator. one way to answer the question is to round up or down the factors in the product to the nearest 10, i.e. 40×50, giving the first of the four choices listed.

Example 2
Work out which of the following numbers

10.1 ☑ 1.9 ☑ 98.6 ☐ 0.31 ☐ *Tick one!*

is closest to $\dfrac{581.2 \times 0.23}{61.1}$.

One way to answer this question is again to round off the numbers to make the calculation much easier. 581.2 is roughly 600 while 61.1 is roughly 60. So 600/60 is 10, then multiply by0.23 to give roughly 2. So the first number 1.9 seems to be the closest. Check with your calculator.

Example 3
If the area A of a circle is 63.7 square metres then roughly calculate the radius r from the formula:

$$A = \pi * r^2$$

radius [4.5] metres

To answer this question, π is roughly 3. Divide by π i.e. 3 and the LHS becomes roughly 60/3 = 20. What number squared equals 20? Well, 4 squared is 16, while 5 squared is 25, so the answer is somewhere in between – say 4.5.

Example 4
Which of the following numbers is closest to $6^{3.8}$?

221 ☐ 1000 ☑ 5000 ☐ 45612 ☐
It is almost the fourth power i.e. $64 = 6 \times 6 \times 6 \times 6 = 1300$. So it will be just below this.
Answer: second choice 1000.

Example 5
Which of the following numbers is closest to $(6.32)^{-1.8}$

40 ☐ 4.2 ☐ 0.38 ☐ 0.031 ☑ 0.0039 ☐

Round off 6.32 to 6 and -1.8 to -2 , which is the reciprocal of 6 squred which is 36, i.e. roughly 40. If it were l/4, i.e. one quarter, this would be 0.25 in decimals. So 1/40 is 0.025 dividing by the extra 10. So the nearest number in the list is 0.031, i.e. the fourth number in the list.

Example 6
Which of the following numbers is closest to $0.51^{2.5}$

2.5 ☐ 5.2 ☐ 0.17 ☑ 0.05 ☐

As you raise the power of a number between 0 and 1 it will get smaller, but how small? It will be greater than 0.5 cubed which is 0.125. So closest is the third number 0.17.

Example 7
Which of the following numbers is closest to $1.54^{-0.8}$

1.5 ☐ 5.2 ☐ 0.7 ☑ 0.01 ☐

Answer will be close to the reciprocal of 1.5 = 3/2, i.e. 2/3 = 0.7.

Example 8
If your reaction time to a dangerous situation on the road is 3 seconds and you are travelling at 70mph, how far would you have travelled before you apply the brakes?

1.5 feet ☐ 40 yards ☐ 3 miles ☐ 350 feet ☑

[Note: roughly 5000 feet in a mile; roughly 1800yards in a mile.]

60×60 seconds in an hour, so number of miles travelled is 70/3600, which is roughly 70/3600, i.e. 2/100 = 0.02 of a mile which is about 36 yards or 100 feet. So in 3 seconds you travel roughly 300 feet (answer #4).

Example 9
If it costs 10^{-4} pence to heat 1 cubic centimetre of water how much would it cost to take a bath:

£1.50 ☑ £0.15 ☑ £100.43 ☐ £9.54 ☐

Say the bath is 2 metres × 1 metre × 1 metre i.e. 2×10^6 cubic centimetres then the cost is $2 \times 10^6 \times 10^{-4}$ = £2 (so the closest answer is £1.50).

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Topic 10: Orders of Magnitude! – Worksheet 10/1

Do not use your calculator in this worksheet – except after completing it, to check your answers.

1. Which of the following numbers is the product $3.72 * 41.89$ closest to? (Tick one.)

3500 ☐ 160 ☑ 44 ☐ 6.7 ☐

2. Which of the following numbers is closest to the expression $\frac{42.6+371.8}{5.8\times 8.2^2}$?

1.0 ☑ 7.2 ☐ 57.6 ☐ 281.8 ☐

3. (a) Which of the following numbers is closest to the expression $7.21\times(0.481)^{2.27}$?

30 ☐ 1.5 ☑ 2800 ☐ 398 ☐

(b) Repeat part (a) for $\frac{(9.21)^{1.02}}{(10.75)^{3.98}}\times 125.63$

868 ☐ 150 ☐ 1.34 ☐ 0.1 ☑

(c) Repeat part (a) for $\frac{432.7\times(3.87)^{-0.45}}{27.1\times 2.91+(0.21)^{-3.1}}$.

0.001 ☐ 63.2 ☐ 0.91 ☑ 281.8 ☐

4. If a car is travelling at 50 kilometres each hour, how far will it travel in:

15 mins [12.5Km] 400 mins [333.3Km] 2 days [1200Km] 100 secs [1388m]

5. Roughly find the radius r of a sphere (in centimetres) if its volume V is:

845 cc [5.96] 84.5 cc [2.72] 8.45 cc [1.26]

Sphere volume is : $V=\frac{4}{3}\pi r^3$