UNIVERSITY OF STIRLING

PDM9L4
DATA SKILLS

WORKBOOK 3 (of 3)
PATH 4

Computing Science & Mathematics

Academic Year 16/17

7 Substitution and definite integrals.

7.1 Method 1.

In this method we treat the integral as an indefinte integral by removing the limits. We integrate by substitution, re-substitute, then re-instate the limits and evaluate.

Example.

Evaluate the following integral.

$$\int_1^2 3x^2(x^3+1)^{1/2}\ dx.$$

Solution.
Let

$$I=\int 3x^2(x^3+1)^{1/2}\ dx.$$

Let $u=x^3+1$. Then $du=3x^2\ dx$ and

$$I=\int u^{1/2}\ du=\frac{2u^{3/2}}{3}+c.$$

Rewrite the integral in terms of x:

$$I=\frac{2}{3}(x^3+1)^{3/2}+c.$$

Replace limits and evaluate. Hence

$$\begin{aligned}
\int_1^2 3x^2(x^3+1)^{1/2}\ dx &= \frac{2}{3}\left[(x^3+1)^{3/2}\right]_1^2 \\
&= \frac{2}{3}\left((2^3+1)^{3/2}-(1^3+1)^{3/2}\right) \\
&= \frac{2}{3}\left(9^{3/2}-2^{3/2}\right) \\
&= \frac{2}{3}\left(\sqrt{9}^3-\sqrt{2}^3\right) \\
&= \frac{2}{3}\left(27-2\sqrt{2}\right).
\end{aligned}$$

Do not forget the constant of integration for the indefinite integral.
You must NEVER write the integral in such a way that the limits are in terms of x but the integrand is in terms of u since this would be nonsense.

7.2 Method 2.

In this method we do not need to remove the limits. Instead we transform the limits at the same time.

Example.

Evaluate the following integral.

$$\int_1^2 3x^2(x^3+1)^{1/2}\ dx.$$

Solution.
Let

$$I = \int_1^2 3x^2(x^3+1)^{1/2}\ dx.$$

Let $u = x^3+1$. Then $du = 3x^2\ dx$.

Change limits: $\begin{array}{ll} \text{when } x=2, & u = 2^3+1 = 9, \\ \text{when } x=1, & u = 2. \end{array}$

Now

$$I = \int_2^9 u^{1/2} = \left[\frac{2u^{3/2}}{3}\right]_2^9 = \frac{2}{3}\left(9^{3/2} - 2^{3/2}\right).$$

Again, NEVER write the integral in terms of u whilst leaving the limits in terms of x. To do so is a strong indication that you haven't a clue what's going on!

You must decide whch method you prefer. It is best to be able to use both methods since one method may seem easier than the other for some particular integrals. Both methods require the same amount of work; the second method perhaps requires slightly less writing.

Examples.

1. Evaluate the following integral

$$\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} 2x\cos(\pi x^2)\ dx.$$

Solution.

Let

$$I = \int 2x\cos(\pi x^2)\ dx.$$

Let $u = \pi x^2$. Then $du = 2\pi x\ dx$, and

$$I = \frac{1}{\pi}\int \cos u\ du = \frac{1}{\pi}\sin u = \frac{1}{\pi}\sin(\pi x^2) + c.$$

Thus

$$\begin{aligned}
\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} 2x\cos(\pi x^2)\ dx &= \frac{1}{\pi}\Big[\sin(\pi x^2)\Big]_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \\
&= \frac{1}{\pi}\left(\sin\tfrac{\pi}{4} - \sin\tfrac{\pi}{2}\right) \\
&= \frac{1}{\pi}\left(\tfrac{1}{\sqrt{2}} - 1\right) \\
&= \frac{1-\sqrt{2}}{\pi\sqrt{2}}.
\end{aligned}$$

2. Evaluate the following integral

$$\int_0^{\ln 4} e^{2x}\ dx.$$

Solution.

Let

$$I = \int_0^{\ln 4} e^{2x}\ dx.$$

Let $u = 2x$. Then $du = 2\ dx$.

Change limits: $\begin{array}{ll} \text{when } x = \ln 4, & u = 2\ln 4 = \ln 4^2 = \ln 16, \\ \text{when } x = 0, & u = 0. \end{array}$

Then

$$I = \frac{1}{2}\int_0^{\ln 16} e^u\ du = \frac{1}{2}\ e^u\Big]_0^{\ln 16} = \frac{1}{2}\left(e^{\ln 16} - e^0\right) = \frac{1}{2}(16-1) = \frac{15}{2}.$$

3. Evaluate the following integral

$$\int_{-1}^{2} 3x^2\sin x^3\ dx.$$

Solution.

Let

$$I = \int_{-1}^{2} 3x^2\sin x^3\ dx.$$

Let $u = x^3$. Then $du = 3x^2\ dx$.

Change limits: $\begin{array}{ll} \text{when } x = 2, & u = 2^3 = 8, \\ \text{when } x = -1, & u = (-1)^3 = -1. \end{array}$

Then

$$\begin{aligned} I &= \int_{-1}^{8} \sin u \; du = (-\cos u)\Big]_{-1}^{8} \\ &= -\cos 8 + \cos(-1) \\ &= -\cos 8 + \cos 1 \quad [\text{since } \cos(-1) = \cos 1] \\ &= -(-0.1455) + 0.540 = 0.3942. \end{aligned}$$

Find $\cos 8$ and $\cos 1$ with your calculator; remember it must be in radian mode.

4. Evaluate the following integral

$$\int_{-2}^{1} \frac{-5t^5}{2-t^3} \; dt$$

Solution.

Let

$$I = \int \frac{-5t^5}{2-t^3} \; dt.$$

Let $u = 2 - t^3$. Then $du = -3t^2 \; dt$ and $t^3 = 2 - u$. Thus

$$I = \frac{5}{3}\int \frac{2-u}{u} \; du = \frac{5}{3}\int \frac{2}{u} - 1 \; du = \frac{10}{3}\ln u - u + c$$

and so

$$\begin{aligned} \int_{-2}^{1} \frac{-5t^5}{2-t^3} \; dt &= \frac{10}{3}\Big[\ln(2-t^3) - (2-t^3)\Big]_{-2}^{1} \\ &= \frac{10}{3}((\ln 1 - 1) - (\ln 10 - 10)) \\ &= \frac{10}{3}(0 - 1 - \ln 10 + 10) \\ &= \frac{10}{3}(9 - \ln 10) = 30 - \tfrac{10}{3}\ln 10. \end{aligned}$$

7.2.1 Exercise.

Evaluate the following integrals using the method of substitution or an appropriate standard integral. Use you calculator only when absolutely necessary.

1. (i) $\displaystyle\int_{-2}^{1}(x^3-2x^2+6)^3(3x^2-4x)\ dx$

   (ii) $\displaystyle\int_{1}^{-2}(x^3-2x^2+6)^3(3x^2-4x)\ dx$

   What does swopping the order of the limits do to the value of the integral?

2. (i) $\displaystyle\int_{5}^{10}\frac{2}{\sqrt{x-1}}\ dx$

   (ii) Show that $\displaystyle\int_{5}^{10}\frac{2}{x-1}\ dx=4\ln\left(\tfrac{3}{2}\right)$

3. (i) $\displaystyle\int_{\sqrt{3}}^{2\sqrt{2}}\frac{x}{\sqrt{x^2+1}}\ dx$

   (ii) $\displaystyle\int_{0}^{1}\frac{x}{\sqrt{x^2+1}}\ dx$

4. $\displaystyle\int_{2.1}^{4.5}\frac{x}{x^2+3}\ dx$

5. $\displaystyle\int_{1.2}^{2.9}\frac{x^3}{(x^2+2)^{1/2}}\ dx$

6. (i) $\displaystyle\int_{\pi}^{2\pi}\sin\tfrac{x}{2}+\cos\tfrac{x}{2}\ dx$

   (ii) $\displaystyle\int_{\pi/2}^{\pi/3}\sin\tfrac{x}{2}+\cos\tfrac{x}{2}\ dx$

7. $\displaystyle\int_{0}^{\pi/3}\frac{\sin\theta}{\cos^3\theta}\ d\theta$ [use the substitution $u=\cos\theta$]

   The variable θ (theta) is often used with trigonmetric functions.

8. (i) $\displaystyle\int_{\pi/4}^{\pi/3}\tan\theta\sec^2\theta\ d\theta$ [use the substitution $u=\tan\theta$]

   (ii) $\displaystyle\int_{0}^{\pi/6}\tan\theta\sec^2\theta\ d\theta$

9. (i) $\displaystyle\int_1^{\sqrt{2}} 2x\sin\pi x^2 \; dx$

(ii) $\displaystyle\int_{1/\sqrt{3}}^{1/2} 2x\sin\pi x^2 \; dx$

10. $\displaystyle\int_{-2}^{2} e^{at} \; dt$

11. $\displaystyle\int_{\ln 2}^{\ln 3} \frac{e^x}{e^x - 1} \; dx$ [remember $\exp(\ln x) = x$]

12. $\displaystyle\int_0^{-\ln 3} \frac{1}{e^t} \; dt$

13. $\displaystyle\int_{\ln 4}^{0} e^{-2x} \; dx$

14. $\displaystyle\int_{-1}^{2} \left(x^2 + \frac{2}{x^4}\right)\left(x^3 - \frac{2}{x^3}\right)^{-1} \; dx$ [re-write the integrand]

15. and finally, something a little trickier.

(i) $\displaystyle\int_1^{2} \exp(\ln 3^x) \; dx$ [use the power rule for logs]

(ii) $\displaystyle\int_1^{2} 3^x \; dx$ [use previous result]

8 Integration by parts.

8.1 Indefinite integrals.

For this you need to learn a formula.

$$\boxed{\int u \; dv = uv - \int v \; du}$$

where u and v are both functions of the variable of integration (usually x).
This method is useful when the integrand is a product of functions.

Example.
Let

$$\boxed{I = \int x^2 e^x \; dx.}$$

This integral has the form

$$I = \int u \; dv$$

and the first thing we have to decide is which bit is the u and which bit is the dv. The next step is to differentiate the u to get du, and integrate the dv to get v and we shoud bear this in mind when making our choice.

For dv choose a bit that *doesn't get more complicated after integrating.*

If we integrate x^2 the power of x will increase and it will get more complicated, but if we integrate e^x we just get e^x so the obvious choice here is to let $dv = e^x \; dx$. This leaves us with x^2 for the u.

$$\boxed{u = x^2} \qquad \boxed{dv = e^x \; dx}$$

Now we differentiate both sides of $u = x^2$ to get $du = 2x \; dx$, and integrate both sides of $dv = e^x \; dx$ to get

$$\int dv = \int e^x \; dx.$$

Now, $\int \; dv = \int 1 \; dv = v$ but it is easier to think of this as "the integral of dv is v". Hence we get $v = e^x$.

$$\boxed{du = 2x \; dx} \qquad \boxed{v = e^x}$$

Now we use the formula

$$I = \int u \; dv = uv - \int v \; du,$$

i.e.

$$\boxed{I = uv - \int v \; du}$$

(to remember the integral of $v\ du$ think "visual display unit")
and we put the pieces together to get

$$I = x^2e^x - \int (e^x)(2x\ dx)$$

or as usually written

$$\boxed{I = x^2e^x - 2\int xe^x\ dx.}$$

This still leaves us with an integral we cannot do, $(\int xe^x\ dx)$, but it is a simpler one and we just apply *integration by parts* to this integral. Note that if we write $I_2 = (\int xe^x\ dx)$ then we have

$$I = x^2e^x - 2I_2.$$

Let

$$\boxed{I_2 = \int xe^x\ dx}$$

$$\boxed{u = x} \qquad \boxed{dv = e^x\ dx}$$

$$\boxed{du = dx} \qquad \boxed{v = e^x}$$

$$\boxed{I = uv - \int v\ du}$$

$$\boxed{I_2 = xe^x - \int e^x\ dx.}$$

Now we have an integral we can do and we have

$$\boxed{I_2 = xe^x - e^x.}$$

Substituting this answer back into our previous result we get

$$\boxed{I = x^2e^x - 2\,(xe^x - e^x)}$$

and all that remains is to tidy up and add the constant of integration.

$$\boxed{I = (x^2 - 2x + 2)e^x + c.}$$

Take care with the minus signs!

Comments.

- Make sure your u and dv use *all* of the integral.
- Good choices for dv are exponentials, sines and cosines.
- Notice that you do not re-use the dv.
- If the choice for u and dv is not clear, just pick one and try it. If you find your integration is getting more complicated, abandon it and start again with a different dv.
- Sometimes either part of the integrand will do for dv
- If you have to apply integration by parts repeatedly, be consistent in the type of function you choose for dv.
- You can re-use previous results.

Examples.

1. Let

$$I = \int x \sin x \ dx$$

Let	$u = x$	and	$dv = \sin x \ dx.$
Then	$du = dx$	and	$v = -\cos x.$

Using

$$I = uv - \int v \ du,$$

$$\begin{aligned} I &= (x)(-\cos x) - \int (-\cos x) \ dx \\ &= -x\cos x + \int \cos x \ dx \\ &= -x\cos x + \sin x \\ &= \sin x - x\cos x + c. \end{aligned}$$

2. Evaluate the integral $\int e^x \sin x \ dx$.

Solution.

Let

$$I = \int e^x \sin x \ dx.$$

Let	$u = \sin x$	and	$dv = e^x \ dx.$
Then	$du = \cos x \ dx$	and	$v = e^x.$

Using

$$I = uv - \int v \; du,$$

$$I \;=\; e^x \sin x - \int e^x \cos x \; dx$$

Again this leaves us with an integral to solve.

Let

$$I_2 = \int e^x \cos x \; dx$$

so that

$$I = e^x \sin x - I_2 \qquad (1)$$

and solve I_2 using integration by parts.

Choose the same type of function for the dv as last time.

Let	$u = \cos x$	and	$dv = e^x \; dx.$
Then	$du = -\sin x \; dx$	and	$v = e^x.$

Using

$$I = uv - \int v \; du,$$

$$\begin{aligned} I_2 &= e^x \cos x - \int e^x(-\sin x) \; dx \\ &= e^x \cos x + \int e^x \sin x \; dx \end{aligned}$$

This does not seem to be getting any easier, in fact we seem to be going round in circles because the integral we started with has turned up again, but that is a good thing!

Using the labels for the integrals what we actually have here is

$$I_2 = e^x \cos x + I. \qquad (2)$$

Putting equations (1) and (2) together we obtain

$$\begin{aligned} I &= e^x \sin x - (e^x \cos x + I) \\ &= e^x \sin x - e^x \cos x - I \end{aligned}$$

Taking the minus I over the other side then dividing by 2 we obtain

$$I = \tfrac{1}{2}(\sin x - \cos x)e^x + c.$$

The choice for u and dv was quite arbitrary in this example; it would work just as well using sine/cosine for dv.

3. For some integrals you will also need the substitution method.

Let

$$I = \int x \sin 2x \ dx.$$

Let	$u = x$	and	$dv = \sin 2x \ dx.$
Then	$du = dx$	and	$v = -\frac{1}{2}\cos 2x.$

Using

$$I = uv - \int v \ du,$$

$$\begin{aligned} I &= -\tfrac{x}{2}\cos x + \tfrac{1}{2}\int \cos 2x \ dx \\ &= -\tfrac{x}{2}\cos 2x + \tfrac{1}{4}\sin 2x + c. \end{aligned}$$

4. You may have noticed that we have not yet integrated the log function, although ln has appeared in some answers. The integral $\int \ln x \ dx$ looks deceptively simple but it needs to be done by parts.

Let

$$I = \int \ln x \ dx$$

We do not know how to integrate $\ln x$ and so there is no point letting $dv = \ln x$. Instead,

let	$u = \ln x$	and	$dv = dx.$
Then	$du = \frac{1}{x} \ dx$	and	$v = x.$

Using

$$I = uv - \int v \ du,$$

$$\int \ln x \ dx = x\ln x - \int x \times \tfrac{1}{x} \ dx = x \ln x - \int 1 \ dx = x \ln x - x + c.$$

This is another standard integral.

$$\boxed{\int \ln x \ dx = x \ln x - x + c}$$

5. Sometimes you just have to get there.......

$$I = \int (x^3+1)\sin x \; dx$$

$$\begin{array}{ll} u = x^3+1 & dv = \sin x \; dx \\ du = 3x^2 \; dx & v = -\cos x \end{array}$$

$$I = (x^3+1)\sin x + 3\int x^2 \cos x \; dx = (x^3+1)\sin x + 3I_2$$

$$\begin{array}{ll} u = x^2 & dv = \cos x \; dx \\ du = 2x \; dx & v = \sin x \end{array}$$

$$I_2 = x^2 \sin x - 2\int x \sin x \; dx = x^2 \sin x - 2I_3$$

$$\begin{array}{ll} u = x & dv = \sin x \; dx \\ du = dx & v = -\cos x \end{array}$$

$$I_3 = -x\cos x + \int \cos x \; dx = -x\cos x + \sin x$$

and back again.......

$$I_2 = x^2 \sin x - 2(-x\cos x + \sin x) = x^2 \sin x + 2x\cos x - 2\sin x$$

$$I = (x^3+1)\sin x + 3(x^2 \sin x + 2x\cos x - 2\sin x)$$

$$= (x^3+3x^2-5)\sin x + 6x\cos x + c.$$

8.1.1 Exercise.

Use integration by parts to solve the following integrals.

1. $\int x\, e^{2x}\ dx$

2. $\int x\ \cos x\ dx$

3. $\int xe^{-x}\ dx$

4. $\int x\cos 2x\ dx$

5. $\int \sin x \cos x\ dx$

6. $\int e^{x}\cos 3x\ dx$

7. $\int xe^{5x}\ dx$

8. $\int \sin^2 x\ dx$ [first by parts and then by using a trigonometric identity]

9. $\int e^{2x}\cos 2x\ dx$

10. $\int x^2 e^{-x}\ dx$

11. $\int x\ln x\ dx$

12. $\int \ln x^2\ dx$ [think about it!]

13. $\int e^{-x}\cos 3x\ dx$

14. $\int x^2 e^{2x}\ dx$

15. $\int e^{3x}(x^2+2x)\ dx$

16. $\int \sqrt{x}\ \ln x\ dx$

17. $\int x\sec^2 x\ dx$ [by parts and then you may need substitution]

18. $\int \sin 3x \cos 5x\ dx$ [this is not the easiest way to do this]

8.2 Parts and definite integrals.

Evaluating definite integrals by parts is accomplished in an obvious fashion. As before it is the integral evaluated at the upper limit *minus* the integral evaluated at the lower limit. Unless there has been a substitution there is NO NEED TO CHANGE LIMITS.

Thus

$$\int_b^a u\ dv = uv\Big|_b^a - \int_b^a v\ du.$$

Notice that you must evaluated *all* of the integral at both limits, including the uv bit.

8.3 Notation: optional reading.

We have seen that the thing to be evaluated at the limits is usually put inside a matching pair of square brackets and that sometimes the left square bracket is omitted. Here we are using a simple straight line instead. This is quite useful notation and appears in different contexts. For example $\left.\dfrac{dy}{dx}\right|_{x=2}$ means evaluate $\frac{dy}{dx}$ at $x = 2$ and so you would substitute $x = 2$ into whatever expression you had for $\frac{dy}{dx}$.

Example.

Evaluate the definite integral $\displaystyle\int_{\pi/3}^{\pi} x\sin x\ dx$ using integration by parts.

Solution, version 1.

Let

$$I = \int_{\pi/3}^{\pi} x\sin x\ dx.$$

Let $u = x$ and $dv = \sin x\ dx$.

Then $du = dx$ and $v = -\cos x$.

Using

$$I = uv - \int v\ du,$$

$$\begin{aligned}
I &= -x\cos x\Big|_{\pi/3}^{\pi} + \int_{\pi/3}^{\pi} \cos x\ dx \\
&= (-x\cos x + \sin x)\Big|_{\pi/3}^{\pi} \\
&= (-\pi\cos\pi + \sin\pi) - (-\tfrac{\pi}{3}\cos\tfrac{\pi}{3} + \sin\tfrac{\pi}{3}) \\
&= -\pi(-1) + 0 + \frac{\pi}{3}\times\frac{1}{2} - \frac{\sqrt{3}}{2} \\
&= \pi + \frac{\pi}{6} - \frac{\sqrt{3}}{2} \\
&= \frac{7\pi}{6} - \frac{\sqrt{3}}{2}.
\end{aligned}$$

In this version of the solution, the entire integral is solved before evaluating at the limits. You may prefer to evaluate at the limits bit by bit as in the following version. Here we will also use the fact that changing the order of the limits changes the sign of the integral.

Solution, version 2.

As before,

$$\begin{aligned}
I &= -x\cos x\Big|_{\pi/3}^{\pi} + \int_{\pi/3}^{\pi} \cos x \; dx \\
&= x\cos x\Big|_{\pi}^{\pi/3} + \sin x\Big|_{\pi/3}^{\pi} \\
&= \frac{\pi}{3}\cos\tfrac{\pi}{3} - \pi\cos\pi + \sin\pi - \sin\tfrac{\pi}{3} \\
&= \frac{\pi}{3}\times\frac{1}{2} - \pi(-1) + 0 - \frac{\sqrt{3}}{2} \\
&= \frac{7\pi}{6} - \frac{\sqrt{3}}{2}
\end{aligned}$$

Another way would be to treat the integral as an indefinite integral, solve it, and put the limits in at the end. This is almost the same as the first version but is laid out slightly differently.

Solution, version 3.

Let

$$I = \int x\sin x \; dx.$$

Let $u = x$ and $dv = \sin x \; dx.$

Then $du = dx$ and $v = -\cos x.$

Thus

$$\begin{aligned}
I &= -x\cos x + \int \cos x \; dx \\
&= -x\cos x + \sin x.
\end{aligned}$$

Hence

$$I\Big|_{\pi/3}^{\pi} = (-x\cos x + \sin x)\Big|_{\pi/3}^{\pi}$$

and complete as before.
Whichever way do it you must be very careful with the minus signs.
You should also stick to using the paired square brackets for the limits if you prefer them.

Examples.

1. Use integration by parts to evaluate $\int_1^{\ln 2} x\, e^x \; dx$.

   Solution.

   Let $I = \int_1^{\ln 2} x\, e^x \; dx$.

   Let $\quad u = x \quad$ and $\quad dv = e^x \; dx$.
   Then $\; du = dx \quad$ and $\quad v = e^x$.

$$\text{Thus} \quad I = xe^x \Big|_1^{\ln 2} - \int_1^{\ln 2} e^x$$

$$= \Big[xe^x - e^x\Big]_1^{\ln 2}$$

$$= \left(\ln 2^{\ln 2} - e^{\ln 2}\right) - (e - e)$$

$$= 2\ln 2 - 2 \qquad [\text{since } \mathrm{e}^{\ln 2} = \exp(\ln 2) = 2]$$

2. Use integration by parts to evaluate $\int_{1/6}^{1} x\cos 2\pi x \; dx$.

   Solution.

   Let $I = \int_{1/6}^{1} x\cos 2\pi x \; dx$.

   Let $\quad u = x \quad$ and $\quad dv = \cos 2\pi x \; dx$.
   Then $\; du = dx \quad$ and $\quad v = \frac{1}{2\pi}(\sin 2\pi x)$.

$$\text{Thus} \quad I = \frac{x}{2\pi}(\sin 2\pi x)\Big|_{1/6}^{1} - \frac{1}{2\pi}\int_{1/6}^{1} \sin 2\pi x \; dx$$

$$= 0 - \frac{1}{12\pi} \times \frac{\sqrt{3}}{2} + \left(\frac{1}{2\pi}.\frac{1}{2\pi}\cos 2\pi x\right)\Big|_{1/6}^{1}$$

$$= \frac{-\sqrt{3}}{24\pi} + \frac{1}{4\pi^2}\left(1 - \frac{1}{2}\right)$$

$$= \frac{1}{8\pi^2} - \frac{1}{8\sqrt{3}\pi}$$

$$= \frac{1}{8\pi}\left(\frac{1}{\pi} - \frac{1}{\sqrt{3}}\right).$$

8.3.1 Exercise.

Use integration by parts to evaluate the following definite integrals.

1. $\int_0^{\pi} x \cos x \ dx$

2. $\int_{-\pi/2}^{\pi/4} x \sin 2x \ dx$

3. $\int_1^2 x^2 e^{x/2} \ dx$

4. $\int_0^1 x e^{3x} \ dx$

5. $\int_{\ln 1}^{\ln 3} x e^{3x} \ dx$

6. $\int_{-1}^2 x^2 \cos \pi x \ dx$ [persevere, most of it vanishes]

7. $\int_{-1}^2 x^2 (e^x - 1) \ dx$ [split up the integral]

8. $\int_4^9 \sqrt{x} \ln x^2 \ dx$ [use power rule for logs]

You are now ready for quiz 3.

Thank you.