2.2.1

1) $y' = 2 \cdot 6x + 2$
$= 12x + 2$

2) $y' = -1x^{-2}$

3) $y' = 4\frac{1}{2}x^3 + 2 \cdot 9x$
$= 2x^3 + 18x$

4) $y' = -2x^{-3} + 2x$

5) $y' = 6x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$
$= \frac{1}{2}6x^{-\frac{1}{2}} - \frac{1}{2}2x^{-\frac{3}{2}}$
$= 3x^{-\frac{1}{2}} - 1x^{-\frac{3}{2}}$

6) $y' = 2ax$

7) $y' = 1x^{-2} - 2x^{-3}$
$= -1x^{-2} + 4x^{-3}$

8) $y' = \frac{1}{2}x^{5/3} - \frac{1}{2}x^{-3/2}$
$y' = \frac{5}{3} \cdot \frac{1}{2}x^{2/3} + \frac{3}{2} \cdot \frac{1}{2}x^{-\frac{5}{2}}$
$= \frac{5}{6}x^{2/3} + \frac{3}{4}x^{-\frac{5}{2}}$

9) $y = 3 \cdot 2x^{-3} + x - 4$
$= 6x^{-3} + x - 4$
$y' = -3 \cdot 6x^{-4} + 1$
$= -18x^{-4} + 1$

10) $y' = -4ax^{-5} + \frac{3}{2} \cdot \frac{2}{3}x^{\frac{1}{2}}$
$= -4ax^{-5} + x^{\frac{1}{2}}$

11) $y' = \sqrt{2} \cdot \frac{1}{2}x^{\frac{1}{2}}$
$y' = \frac{1}{2}x^{-\frac{1}{2}}\sqrt{2}$

12) $y' = \sqrt{2}x' = \sqrt{2}$

13) $y' = \frac{1}{4} \cdot 12x^{-\frac{3}{4}} + \frac{7}{5} \cdot 8x^{\frac{5}{2}}$
$= 3x^{-\frac{3}{4}} + 28x^{\frac{5}{2}}$

14) $y' = 0 \cdot ax + b^2$

15) $y = (1+2x)(1+2x)$
$= 1 + 2x + 2x + 4x^2$
$= 1 + 4x + 2x^2$
$y' = 4 + 8x$

16) $y = (x^{\frac{1}{3}} + 3x^{-\frac{1}{2}})(x^{\frac{1}{3}} + 3x^{-\frac{1}{2}})$
$= x + 3 + 3 + 9x^{-1}$
$= x + 6 + 9x^{-1}$
$y' = 1 - 9x^{-2}$

17) $y = (2 - 3x^2 + 4x - 6x^3)$
$y' = -6x + 4 - 18x^2$

2.3.1

1) i) $t' = 3t^2 - \frac{1}{2} 2t^{-\frac{1}{2}}$

$= 3t^2 - 1\, t^{-\frac{1}{2}}$

ii) $t' = -\frac{1}{2} t^{-\frac{3}{2}} + a$

2) i) $\frac{dz}{dx} = 3x^2$ $\qquad \frac{dz}{dy} = \frac{1}{2} y^{-\frac{1}{2}}$

ii) $\frac{dz}{dx} = 2ax + y$ $\qquad \frac{dz}{dy} = x$

iii) $\frac{dz}{dx} = 2xy + 3x^2 y + a$

$\frac{dz}{dy} = 2xy + x^3$.

2.6

1) $y=(x-2)(x^2+3)$ — Product

$y' = (x-2)\cdot(x^2+3)' + (x^2+3)\cdot(x-2)'$

$= (x-2)(2x) + (x^2+3)\cdot 1$

$= 2x^2 - 4x + x^2 + 3$

$= 3x^2 - 4x + 3$

2) $y = (\sqrt{x}-1)(x^2-2)$

$= (x^{\frac{1}{2}}-1)(x^2-2)' + (x^2-2)\cdot(x^{\frac{1}{2}}-1)'$ — P

$= (x^{\frac{1}{2}}-1)(2x) + (x^2-2)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$= 2x^{\frac{3}{2}} - 2x + \frac{1}{2}x^{\frac{3}{2}} - x^{-\frac{1}{2}}$

$= \frac{5}{2}x^{\frac{3}{2}} - 2x - x^{-\frac{1}{2}}$

3) $y=(x^{5/3}+x^2)^4$ — Chain

$y = 4(x^{5/3}+x^2)^3 \cdot (x^{5/3}+x^2)'$

$= 4(x^{5/3}+x^2)^3 \cdot \left(\frac{5}{3}x^{\frac{2}{3}} + 2x\right)$

4) $y = (x^2+2)^{-1}$ — Chain

$= -1(x^2+2)^{-2} \cdot (x^2+2)'$

$= -1(x^2+2)^{-2} \cdot 2x$

$= -2x(x^2+2)^{-2}$

5) $\frac{(0)}{(1)}\frac{1}{x^2+2} = \frac{(x^2+2)\cdot(1)' - (1)(x^2+2)'}{(x^2+2)^2}$ — Quotient

$= \frac{(x^2+2) - 2x}{(x^2+2)^2}$

6) $\frac{a-\sqrt{x}}{x} = \frac{(x)(a-x^{\frac{1}{2}})' - (a-x^{1/2})(x)'}{x^2}$ — Q

$= \frac{(x)\left(-\frac{1}{2}x^{-\frac{1}{2}}\right) - (a-x^{\frac{1}{2}})(1)}{x^2}$

$= -\frac{1}{2}x^{\frac{1}{2}} - a + x^{\frac{1}{2}}$

$= \frac{1}{2}x^{\frac{1}{2}} - a$

7) $y = \left(\frac{3}{2}x^2 + 4x\right)(2x+1)$

$y' = \left(\frac{3}{2}x^2 + 4x\right)(2x+1)' + (2x+1)\left(\frac{3}{2}x^2 + 4x\right)'$

$2\left(\frac{3}{2}x^2 + 4x\right) + (2x+1)(3x+4)$

$2\left(\frac{3}{2}x^2 + 4x\right) + 6x^2 + 8x + 3x + 4$

$3x^2 + 8x + 6x^2 + 11x + 4$

$9x^2 + 19x + 4$

8) ~~$y = (x+1)^3 (x-1)' + (x-1)(x+1)^3{}'$~~

~~$= (x+2)^3$~~

8) $y = (x+1)^3 (x-1)$

Product

$= (x+1)^3 (x-1)' + (x-1)(x+1)^3{}'$

$= (x+1)^3 \cdot 1 + (x-1)\, 3(x+1)^2 \cdot (x+1)'$

$= (x+1)^3 + (x-1) \cdot 3(x+1)^2 \cdot 1$

$= 3(x+1)^2 \cdot (x-1) + (x+1)^3$

Chain

9) $\frac{(x+1)^3}{x^2-1}$

$y' = \frac{(x^2-1)\cdot(x+1)^3{}' - (x+1)^3 \cdot (x^2-1)'}{(x^2-1)^2}$

$= \frac{(x^2-1)\cdot 3(x+1)^2 \cdot (x+1)' - (x+1)^3 \cdot 2x \cdot 1}{(x^2-1)^2}$

$= \frac{(x^2-1)\cdot 3(x+1)^2 \cdot 1 - (x+1)^3 \cdot 2x}{(x^2-1)^2}$

25) $(2-9x)^2 (1+4x^2)$

$y' = (2-9x)^2 (1+4x^2)' + (1+4x^2)(2-9x)^2{}'$

$= (2-9x)^2 \, 8x + (1+4x^2)\, 2(2-9x)\,(2-9x)'$

$= (2-9x)^2 \, 8x + (1+4x^2)(4-18x)(-9)$

26) $\dfrac{x^2+a}{x^{-2}-a} = \dfrac{(x^{-2}-a)(x^2+a)' - (x^2+a)(x^{-2}-a)'}{(x^{-2}-a)^2}$

$= \dfrac{(x^{-2}-a)\,2x - (x^2+a)(-2x^{-3})}{(x^{-2}-a)^2}$

27) $y = (x^3+ax)^{1/3}{}'$

$y' = \frac{1}{3}(x^3+ax)^{-\frac{2}{3}} \cdot (x^3+ax)'$

$= \frac{1}{3}(x^3+ax)^{-\frac{2}{3}} \, (3x^2+a)$

3.4.1

1) $y = 2\sin x + \cos x$

$y' = 2\cos x - \sin x$

2) $y = \cos x + x^2$

$y' = -\sin x + 2x$

3) $y = (1+x)e^x$

$y' = (1+x)e^{x'} + e^x (1+x)'$

$= (1+x)e^x + e^x$

4) $y = \dfrac{1-\sin x}{1+x} = \dfrac{(1+x)(1-\sin x)' - (1-\sin x)(1+x)'}{(1+x)^2}$

$y' = \dfrac{(1+x)(\cos x) - (1-\sin x)}{(1+x)^2}$

5) $y = \dfrac{1-\cos x}{1-x} = \dfrac{(1-x)(1-\cos x)' - (1-\cos x)(1-x)'}{(1-x)^2}$

$y' = \dfrac{(1-x)(\sin x) - 1 + \cos x(-1)}{(1-x)^2}$

6) $y = (1-x)\cos x = (1-x)(\cos x)' + (\cos x)(1-x)'$

$y' = (1-x) - \sin x + \cos x(-1)$

$= (1-x) - \sin x - \cos x$

7) $y = x\ln x + \ln x$

$y' = \frac{1}{x} + \left((x)(\ln x)' + (\ln x)(x)'\right)$

$= \frac{1}{x} + \left(x \cdot \frac{1}{x} + \ln x\right)$

$= 1 + \frac{1}{x} + \ln x$

8) $y = \frac{\ln x}{\sin x} = \frac{(\sin x)(\ln x)' - (\ln x)(\sin x)'}{(\sin x)^2}$

$= \frac{\sin x\left(\frac{1}{x}\right) - \ln x\ (\cos x)}{\sin^2 x}$

X 9) $y = \frac{(x+1)^3}{(x^2-1)}$

$y' = \frac{(x^2-1)(x+1)^{3'} - (x+1)^3 (x^2-1)'}{(x^2-1)^2}$

$= \frac{(x^2-1)\ 3(x+1)^2 - (x+1)^3\ 2x}{(x^2-1)^2}$

9) $y = \sin x \ln x + x \cos x$

$= (\sin x)(\ln x)' + (\ln x)(\sin x)' + (x)(\cos x)' + (\cos x)(x)'$

$= (\sin x)\frac{1}{x} + (\ln x)(\cos x) + (x)(-\sin x) + (\cos x)$

$= \frac{1}{x}\sin x + \ln x \cos x + -x \sin x + \cos x$

10) ~~$y' =$~~

3.5.1

1) $y = \sin x^3$

$y' = \sin x^{3'} \cdot x^{3'}$

$y' = \cos x^3 \cdot 3x^2$

2) $y = \cos(2x^2+1)$

$= \cos(2x^2+1)' \cdot (2x^2+1)'$

$= -\sin(2x^2+1)\ 4x$

3) $y = \cos 3x$

$= (\cos 3x)' (3x)'$

$= -3\sin(3x)$

11) $y = x(e^x)^2$

$y' = (x)'(e^x)^2 + (e^x)^{2'}(x)'$

$= (x)'(e^x)^{2'} + (e^x)^2 (x)'$

$= (x)'\ 2(e^x) + (e^x)' + (e^x)^2$

$= x\ 2e^x + e^x + (ex)^2$

$y = x(e^x)^2$

$= x(e^{2x})$

$= (x)'(e^{2x})' + (e^{2x})(x)'$

$= (x)'(e^{2x})(2x)' + (e^{2x})$

$= x\ e^{2x}\ 2 + e^{2x}$

15) $y = (\cos x)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(\cos x)^{-\frac{3}{2}} \cdot (-\sin x)$

$= \frac{1}{2}\sin x (\cos x)^{-\frac{3}{2}}$

21) $y = \sin \pi x$

$y' = (\sin \pi x)' \, (\pi x)'$

$= (\cos \pi x) \, \pi$

~~23) $y = \ln(e^{2x})$~~

~~$y' = (\ln)'(e^{2x})' + (e^{2x})(\ln)$~~

~~$y' = (\ln e^{2x})' \, (e^{2x})'$~~

39) $y = \dfrac{\sin 2t}{\cos t^2} = \dfrac{\cos(t^2)\,(\sin 2t)' - (\sin 2t)\,(\cos(t^2))'}{(\cos t^2)^2}$

$= \dfrac{\cos(t^2)\,(\sin 2t)'\,(2t)' - (\sin 2t)\,(\cos(t^2))'\,(t^2)'}{(\cos(t)^2)^2}$

$= \dfrac{(\cos(t^2)\,(\cos 2t)\,2 - (\sin 2t)\,(-\sin t^2)\,(2t)}{(\cos(t)^2)^2}$

42) $y = \dfrac{\cos 2x}{\sin 2x} =$

$= \dfrac{(\sin(2x))\,(\cos(2x))' - (\cos(2x))\,(\sin(2x))'}{(\sin 2x)^2}$

$= \dfrac{\sin(2x)\,(\cos(2x))'\,(2x)' - (\cos(2x))\,(\sin(2x))'\,(2x)'}{(\sin 2x)^2}$

$= \dfrac{(\sin(2x)\,(-\sin(2x))\,2 - (\cos(2x))\,(\cos(2x))\,2}{(\sin(2x))^2}$

X $y = \dfrac{\cos 2x}{\sin 2x} = \dfrac{\cos^2 x - \sin^2 x}{2 \sin \cos x}$