

# Estimating ladder fuel contributions to crown fire initiation

## Abstract

Predicting the onset of conifer crown fires is an uncertain process. Ladder fuels (LF), biomass that act as a bridge between surface and canopy fuel layers, are suspected of being disproportionately important in crown fire initiation, but are difficult to quantify and incorporate into prediction systems. In this study, we show how a simple rearranging of C.E. Van Wagner's classic model (Can. J. For Res. 1977, v7: 23-34) reframes crown fire initiation as a critical fuel consumption threshold-driven process. A new equation is presented for comparing fuel consumption at different vertical positions, which allows a rescaling of LF consumption down to the ground level and corresponding boost in equivalent mass. To test the theoretical model, we used several sources of evidence, primarily from Canadian experimental fires. Incorporating LF into empirical crown fire models improved model predictions, and best results were obtained with a LF multiplier of [??2-3??; draft results]. This suggests that LF consumption is two to three times as influential per unit mass as conifer surface fuel consumption, even accounting for vertical position. The resulting models will be useful for improve fire behaviour predictions as well as for improving the design of hazard reduction treatments.

## Introduction

Conifer forests cover most of the circumboreal region of the world and crown fires represent a significant fire-related hazard in these stands (Kneeshaw, Bergeron, and Kuuluvainen 2011; Brian J. Stocks 1991). As noted by Kiil et al. (1976) and quoted by Cruz et al. (2005), surface fires rarely spread  $> 6$  m/min under closed conifer stands, as higher spread rates are usually associated with some degree of crowning or torching. Once flames reach the upper reaches of the canopy, they can be directly influenced by open air winds in the lower atmosphere. The fully developed crowning process has long been known to be associated with a dramatic increase in fire spread rate, due to several processes including flame tilt, increased lee-side wind exposure and vigorous ember spotting (Taylor et al. 2004; M. E. Alexander and Cruz 2016).

Stand structure is well-known to influence crown fire occurrence and tendency. The element in fuel complexes that enables crown fires in most tall conifer stands are ladder fuels. This term refers to any easily-ignited fuels in the mid-canopy space, meaning fine-textured and flammable dead or live biomass above the surface fuel complex but below the live canopy base. In Canada, J.G. Wright first noted that 'crown fires are not likely to occur unless there is a large volume of fuel under the trees' (Wright 1932). Van Wagner (Van Wagner 1977), in describing his classic crown fire initiation model based on empirical data and physical convection theory, noted that such 'bridge fuels' consisted of 'combustible matter such as loose bark, dead lower branches, lichen, small conifers, etc.' located above the surface fuel complex, remarking that such fuels needed to be 'present in sufficient

quantity to intensify the surface fire appreciably as well as to extend the flame height'. While the paper describing Van Wagner's model has been cited become a classic (> 1500 citations as of 2023), methods for quantifying the effects of ladder fuels are still lacking 45 years later.

Van Wagner's model was originally formulated to solve for critical surface intensity ( $I_0$ ) as follows:

$$I_0 = (chz)^{1.5}$$

[1],

where  $h$  is heat of ignition,  $z$  is live crown base height (LCBH), and  $c$  is a dimensional constant derived empirically. Most modelling systems have combined  $I_0$  with Byram's (1959) fireline intensity equation in order to solve for critical rate of spread. This combination has allowed for linkages with Rothermel's (1972) semi-physical surface rate of spread model (e.g. Scott and Reinhardt 2001; Andrews 2014), or with the empirical ROS functions from the FBP System fuel types (Forestry Canada Fire Danger Group 1992). Surprisingly, a simple algebraic rearranging of the terms of Van Wagner's model allows for a useful and powerful estimator applicable to ladder fuels of varying heights.

## Objective

The objective of this study was to describe a simple theoretical framework for analyzing ladder fuel influence in crown fire initiation, based on a rearrangement of Van Wagner's (1977) equation. Secondary objectives include calibrations and evaluations of the theoretical equation, as well as refitting empirical crown fire models for use with measured or estimated ladder fuel loadings and structure.

## Theoretical model: scaling ladder fuel consumption to the surface

Ladder fuels are defined by their fine texture and intermediate vertical position, by definition (Van Wagner 1977; Kilgore and Sando 1975). They exist in conifer stands between the surface fuels, at  $z = 0$ , and canopy fuels, which for our purposes begin at the live canopy base height (LCBH). When connected to the surface fuels and ignited by a proximate source (i.e. an advancing fireline), the heat flux contribution of these fuels (Melnik et al. 2022) is much closer to the canopy base, and therefore much more likely to contribute to canopy ignition compared to similar fuel quantities at the surface.

The structure of the Van Wagner (Van Wagner 1977) CFI model permits a vertical rescaling that relies on an implied equivalency between SFC and the inverse of  $z$  (LCBH). When we replace  $I_0$  (eq. 1) with Byram's (1959) fireline intensity, we get the following equation, a much-used transformation (e.g. FCFDG 1992; Scott and Reinhardt 2001):

$$H \cdot SFC \cdot ROS = (c \cdot h)^{1.5} z^{1.5}$$

[2]

where  $H$  is the heat of combustion,  $SFC$  is surface fuel consumption,  $ROS$  is rate of forward spread, and the right side is as in Equation 1.

This relationship has often been evaluated in terms of identifying critical ROS for crown fire initiation (Van Wagner 1977; Martin E. Alexander 1988). However, if we assume that surface ROS is insensitive to small variations in LCBH, it should also hold for comparing a range of SFC values. Under surface fire conditions near the crown fire initiation threshold, critical SFC could then be defined as follows:

$$SFC_0 = \frac{(ch)^{1.5} z^{1.5}}{h \cdot ROS}$$

[3]

Critical SFC for crown fire initiation can then be compared between LCBH levels, e.g.  $z_1$  and  $z_2$ .

$$\frac{SFC_2}{SFC_1} = \frac{\left[ \frac{(ch)^{1.5} z_2^{1.5}}{h \cdot ROS} \right]}{\left[ \frac{(ch)^{1.5} z_1^{1.5}}{h \cdot ROS} \right]}$$

[4]

This also assumes no change in ROS with varying LCBH. Holding all other terms constant, we can simplify this to yield a basic relationship between SFC and LCBH ( $z$ ) at different heights:

$$SFC_2 = \left( \frac{z_2}{z_1} \right)^{1.5} \cdot SFC_1$$

[5].

A practical example helps illustrate the logic of comparing  $SFC_0$  at two different  $z$  values (eq. 3). Using Van Wagner's own empirical heat of ignition function (Van Wagner 1968), at 90% foliar moisture content (FMC) and LCBH=2 m, Equation 1 suggests  $I_0$  of about 417 kW/m, the intensity of a low to moderate intensity surface fire. At some moderate surface ROS value, e.g. 3 m/min, Equation 3 predicts  $SFC_0$  of 0.463 kg m<sup>-2</sup>. Using eq. 5, increasing LCBH to 5 m (holding other inputs constant) would raise  $SFC_0$  to 1.83 kg m<sup>-2</sup>; thus, a 2.5 times increase in LCBH results in a fourfold increase in fuel consumption needed for crowning, as a consequence of the much higher surface fire intensity required.

In the case of the consumption of an elevated ladder fuel quantity  $FC_L$  positioned at height above ground  $C_L$  (such as the centroid of a sapling cohort beneath a live conifer canopy), finding the surface-equivalent  $FC$  value ( $FC_{SE}$ ) is then a matter of inflating its influence due to its elevated position. In this case,  $z$  remains the normal LCBH for the live canopy and  $z - C_L$  can also be considered the fuel strata gap between ladder fuel elements and the LCBH:

$$FC_{SE} = \left( \frac{z}{z - C_L} \right)^{1.5} \cdot FC_L, \quad z > C_L$$

[6]. Using Equation 6, it becomes possible to estimate the contribution of canopy fuel elements and compare them with the surface fuel complex in an additive manner.

We note immediately two fire-related qualifiers. First, this theoretical relationship does not account for the obvious structural differences between the surface and crown fuel complexes. In particular, surface fuelbeds are much more compact (higher bulk density and packing ratio) than crown fuel complexes, including ladder fuels, with a much more aeration-limited combustion environment (Rothermel 1972; Schwilk 2015; Keane 2015). The general FBP System approach to SFC, where all consumed surface and ground fuels are assumed to contribute to surface fire intensity (Van Wagner 1977; Forestry Canada Fire Danger Group 1992), may overemphasize dense duff fuels that burn in post-frontal combustion and ignores factors such as residence time (Nelson and Adkins 1988; Wotton et al. 2012). In contrast, fine canopy and ladder fuel strata are much more aerated (more porous) but also vertically and horizontally discontinuous. This difference could be addressed theoretically or, as in the present investigation, empirically.

Second, while equation 6 specifies that the ladder fuel height cannot exceed the LCBH, it is apparent that as the difference between them ( $z - C_L$ ) diminishes,  $FC_G$  can grow to extreme levels, eventually producing a division by zero problem at  $z = C_L$ . Thus,  $0.1 \text{ kg} \cdot \text{m}^{-2}$  of fuel consumption 1 m below a 5 m tall canopy base ( $z - C_L = 1$ ) scales to equate to ground-level SFC of  $1.12 \text{ kg m}^{-2}$ , but the same fuel consumption 0.2 m below the same canopy scales to  $FC_G = 12.5 \text{ kg m}^{-2}$ . This logic holds in a theoretical sense – a fire is much more likely to ignite the lower live canopy if the burning fuel is immediately beneath, and a 0.2 m gap is almost negligible at the fuel particle scale. However, this suggests that caution is needed to utilize equation 6 in a practical sense to avoid inflating the ladder fuel effect to illogical levels.

### Ladder fuel multiplier

One simple approach to the challenge of comparing high porosity ladder fuels with low porosity surface fuels is to evaluate the use of a multiplier. This would be, in the simplest sense, a value that could be used to inflate the value of  $FC_L$  in order to bring it closer to comparable SFC values. Thus, we might expect  $SFC = FC_{SE} \cdot M$  [7], with  $FC_{SE}$  being the surface-equivalent ladder fuel consumption value from eq. 6 and  $M$  being the appropriate multiplier for converting  $FC_L$  to  $FC_{SE}$ .

Measured and estimated SFC values in Canadian experimental fires ranged from 0.2 to  $3.8 \text{ kg m}^{-2}$  (Perrakis et al. 2023). The value of this multiplier,  $M$ , could range from 1 (no effect) to 10 or more in the event that ladder fuel biomass and arrangement appeared to have more than an order of magnitude greater effect than surface biomass on vertical heat flux and crown fire initiation.

The remainder of the study tests and discusses empirical evidence for equations 5 –7, using analyses on previously documented Canadian experimental fire data.

## Methods

Three separate validation methods were used to evaluate the theoretical ladder fuel scaling relationship and estimate the value and significance of the multiplier  $M$ : 1) an expert opinion assessment; 2) a crown fire threshold model between three treatment types; and 3) modifications of an empirical crown fire model using the Canadian experimental fire dataset. All of these relate to stand level estimates of fuel structure using mean values, which is simple and operationally practical, but may mask significant impacts of within-stand heterogeneity and spatial pattern of fuel arrangement (e.g. clumping, proximity to overstory trees).

### Study area: Sharpsand Creek, Ontario

The first two validation methods used data from the fire experiments at Sharpsand Creek, Ontario. This site and its observations have been analyzed in several studies but remain useful for the present purposes. As previously described, 27 plots in three separate treatment types were burned between 1974 and 1991 at this site north of Thessalon, Ontario. The original unaltered stand (immature: IM) consisted of naturally regenerated jack pine following a 1948 wildfire, with dense stocking and abundant ladder fuels (Walker and Stocks 1975; B. J. Stocks and Walker 1973). Results from the initial experimental fires (1975-1981) were published by Stocks (1987) and formed the empirical basis for much of the FBP System C-4 fuel type (Forestry Canada Fire Danger Group 1992). These observations were also analyzed separately by Van Wagner (Van Wagner 1993) and used in additional fire behaviour studies (Call and Albin 1997; Albin and Stocks 1986; Cruz, Alexander, and Wakimoto 2003). During the course of these experiments, it was discovered that 6 of the plots had been hand-thinned (TH) in 1960, removing most of the live stems and resulting in much less dense stands compared with the 'control' IM plots [B. J. Stocks (1987); B. J. Stocks, personal communication, Jan. 2020].

In the later fires (1988-1991), stand characteristics indicated significant structural changes underway during the intervening years. The dense IM plots were initially surveyed at  $>9000$  live stems  $\text{ha}^{-1}$  and  $> 10,000$  dead stems  $\text{ha}^{-1}$  [measured in 1973; B. J. Stocks (1987)]; when re-surveyed over a decade later, self-thinning in so-called semi-mature (SM) stands had reduced live and dead stem density by 53 % and 63 %, respectively, while total basal area increased by 37 %; average LCBH had also increased to 5.3 m, about 1 m higher than IM stands (McRae et al. 2017).

As discussed in a previous study (Perrakis et al. 2023), earlier studies handled these three stand types in various ways. With the IM stands included in the C-4 fuel type, the TH plots had been deemed more representative of mature pine stands and previously included in the FBP System C-3 fuel type, despite similar age, LCBH and site characteristics as the IM stands (Groot, Hanes, and Wang 2022). In later studies, the ladder fuel influence in IM stands was estimated as a modification of the LCBH based on the logic of reduced distance between surface fuel and canopy fuel and lower crowning thresholds; this modified LCBH measure was termed the Fuel Strata Gap (Cruz, Alexander, and Wakimoto 2004; Cruz et al. 2006; Perrakis et al. 2023).

## Crowning behaviour and ladder fuel consumption

There is a question of which plots experienced crown fire vs surface fire or transitional torching behaviour. In the original Sharpsand experiments (1974-1981; Stocks 1987; Cruz 1999), fire type was identified as 'surface' or 'crown'. In the latter experiments, however, fire type was identified as 'surface', 'some torching', 'torching', and 'crown' (McRae et al. 2017). Descriptions in the latter text make it clear that 'torching' fires experienced behaviour akin to passive crowning, while those described as 'some torching' appeared to be near the crowning threshold (i.e., ~5-15% crown fraction burned). Following discussions with experimental leaders (BJ Stocks, pers. comm. 2020), the two fires described as 'some torching' were split, with fire #16 (ISI=8.1, ROS=3.5 m/min) considered a vigorous surface fire, while fire #18 (ISI=9.4, ROS=5.1 m/min) was considered a marginal passive crown fire.

The most challenging value for the analysis is the fuel consumption of ladder fuels, as this is seldom measured (though sometimes estimated indirectly) in experimental studies. Stocks (1987) presented estimated dead roundwood fuel consumption for the original Sharpsand Creek experimental fires, and these values are used in the present analysis.

For the present calculations, ladder fuel influence was quantified using equation 6 to scale the estimated ladder fuel consumption (dead canopy fuel < 1 cm; Stocks 1987) into an equivalent SFC contribution value,  $SFC_L$ . The vertical position of fine standing ladder fuels was assumed to be the crown centroid of the mean dead snag crowns  $C_L = CBH + \frac{height - CBH}{2}$  (Perrakis et al. 2023) at the stand level, where CBH is the mean crown base height of the dead saplings.



Diagram (not to scale) showing approximate estimation of live crown base height ( $z$ ), dead ladder fuel centroid ( $C_L$ ), and fuel strata gap ( $z_L$ ) for immature jack pine stands (unthinned controls) at Sharpsand Creek.



SFC<sub>L</sub> was then added to the actual (measured) SFC for modelling. Additional tests were completed where the elevated fuel contribution was increased using a multiplier, discussed further below.

#### *Estimation 1 - expert opinion*

The first method involved expert opinion from previous studies. Cruz et al. (2004) noted that the abundant ladder fuels characterizing the IM stands, in the form of standing dead trees (including dead branches, bark flakes, and fine stemwood) amounted to a reduced FSG of 2 m, compared to approximately 4 m for the actual LCBH (or  $z$ ). We used those values as a first estimate along with equation 5 to investigate a potential value for multiplier  $M$ .

#### *Estimation 2 - analysis of crowning thresholds, Sharpsand Creek (ON)*

For the second estimation method, we used the Sharpsand Creek, Ontario experiments once again, including the full complement of immature, thinned, and semi-mature stands in our analysis (B. J. Stocks 1987; McRae et al. 2017; Perrakis et al. 2023). This site, with many fires in stands that differed primarily by age and treatment, provided the ideal test case for assessing threshold crown fire conditions and ladder fuel effects. We first used logistic regression to compare the crown fire thresholds between treatments. This was initially done using the initial spread index (ISI) from the FWI System (Van Wagner 1987); we also tested a stand-adjusted version of the ISI (ISI<sub>SA</sub>), which was simply the ISI equation's  $f(F)$  function fitted to the MC<sub>SA</sub>, the stand-adjusted dead fuel moisture content estimate (Wotton and Beverly 2007a), instead of the usual FPMC fuel moisture estimate.

We then evaluated the crowning thresholds in terms of critical FC to solve for the multiplier effect ( $M$ ). This involved using Byram's (1959) fireline intensity along with predicted ROS using fitted surface ROS models, and solving for  $M$  using the threshold values.

#### *Estimation 3 - empirical fitting using a crown fire model*

For the third and final method of examining the theoretical relationships, we returned to a recent crown fire analysis and attempted to refit the relationship between fire environment variables and crown fire occurrence. As in Perrakis et al. (2023), we used 10 m open wind speed, estimated litter and fine dead surface fuel moisture (as represented by either the FPMC or the stand-adjusted moisture content,  $mc_{SA}$ ). This time, rather than subjectively estimating the FSG adjustments to LCBH in certain stands, we used eq. 7 to estimate the ladder fuel contributions of standing dead fuels, based on the measured or estimated < 1 cm elevated roundwood (dead) consumption values. We tested only the best-performing model forms for this purpose, which had log-transformed SFC values and a 3/2 exponent for the LCBH term. The multiplier  $M$  in this case was held inside the log-transformed SFC term; thus, the respective coefficient was fitted to  $\log(SFC + FC_{SE} \cdot M)$ . The initial value of  $M$  in this fitted model was taken from the previous estimates, but a sensitivity analysis step also tested other values of  $M$ , from 1 to 20.

## Results

### *Expert opinion [delete??]*

The first estimation method for the multiplier  $M$  was based on solving eq. 5 using the expert opinion values given in Cruz et al. (Cruz, Alexander, and Wakimoto 2004; Cruz, Alexander, and Wakimoto 2005) to account for ladder fuels.

From eq. 5, we can easily rearrange as follows:

$$\frac{SFC_2}{SFC_1} = \left(\frac{z_2}{z_1}\right)^{1.5} = M$$

Starting with the approximate LCBH values originally reported for the Sharpsand IM stands (4 m), this value was reduced by 2 m to produce the FSG estimate. Therefore, using a slight modification of equation 5, we can easily  $z_2 = 4$  m and  $z_1 = 2$  m, producing our first estimate for  $M$ :

$$\begin{aligned} M &= \left(\frac{4}{2}\right)^{1.5} \\ M &= 2.828 \end{aligned}$$

[7]. Instead of those approximate values, we could also use values derived from the more precise estimates from original stand data (Walker and Stocks 1975). As previously described (Perrakis et al. 2023), detailed stand survey analysis produced mean treatment-level LCBH values of about 4.36 m in the IM plots (4.45 m in the TH plots). If we assume that the 2 m reduction in FSG due to ladder fuels holds true (Cruz, Alexander, and Wakimoto 2004), we get the following values for equation 6:

$$\begin{aligned} M &= \left(\frac{4.36}{2.36}\right)^{1.5} \\ M &= 2.511 \end{aligned}$$

[8]. The estimate for  $M$  based on this logic appears to be in the order of  $\sim 2.5 - 3.0$ .

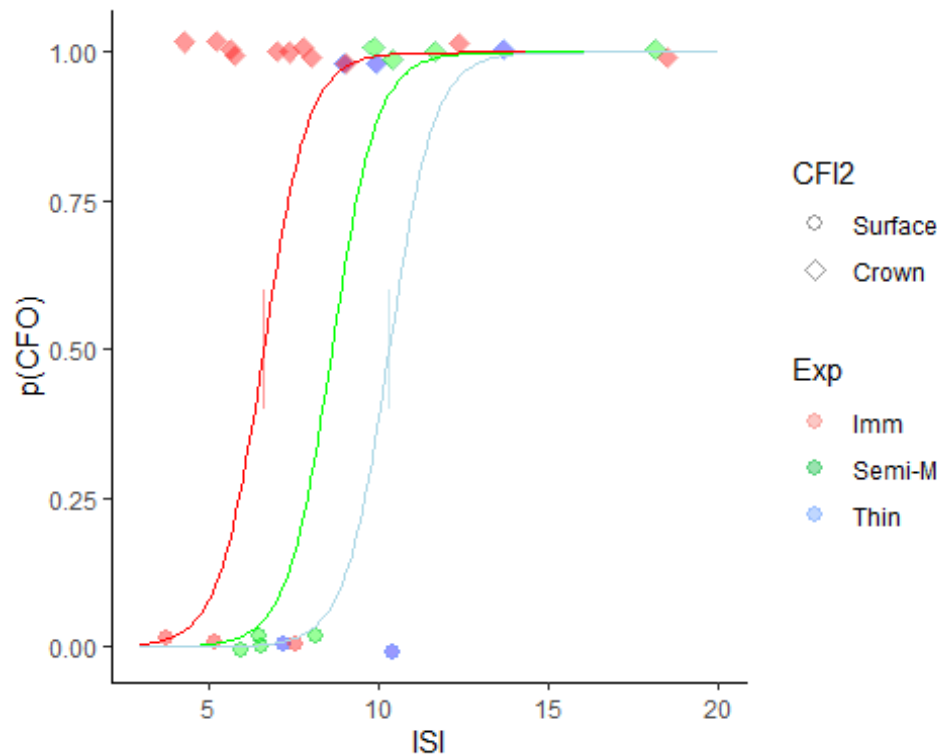
### *Crowning thresholds and critical FC, Sharpsand Creek*

For the second multiplier estimation method, we first tested the ISI crowning threshold between treatments (comparing IM, TH, and SM stands as treatments).

Based on the logistic regression model, ISI and treatment both appeared to be significant predictors of crown fire success at the  $\alpha=0.05$  level, with the Immature treatment as the base case. Note that a post-hoc Tukey multiple comparison test revealed that apparent difference between IM and TH treatments was marginal at best ( $p=0.102$ ), which is not unexpected with such small treatment group sizes; other pairwise contrasts were non-significant ( $p>0.2$  for all other comparisons). Nonetheless, at this point we are not testing the hypothesis of treatment effects, but rather identifying the most likely crowning threshold between the treatment groups. Thus, we used this model to identify the most likely crowning thresholds between treatments, in terms of ISI (Figure XX) or ISI<sub>SA</sub> (not

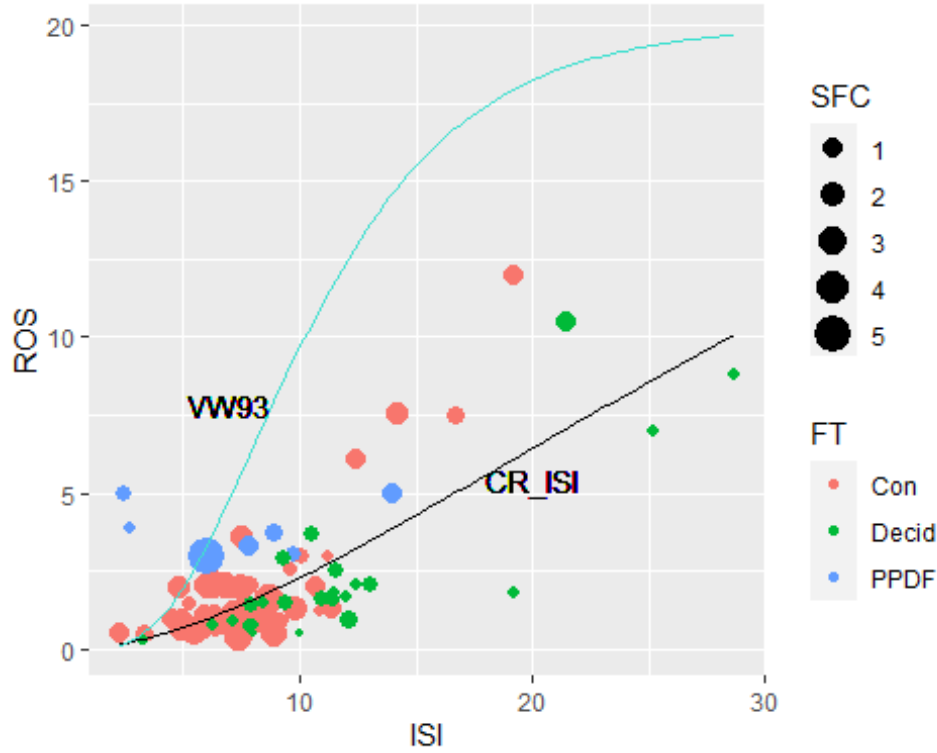


shown). The crown fire transition thresholds in the control and thinned stands were estimated by solving for  $p=0.5$  (50% probability).



Crowning thresholds were 6.61 and 10.31 ISI for immature controls and thinned stands, respectively. In terms of  $ISI_{SA}$ , the  $MC_{SA}$ -based spread index, the respective thresholds were 4.7 and 9.19 for unthinned controls and thinned stands. Given identical ignition patterns and stand origin, the difference between these values, therefore, represents the increased crowning threshold achieved by reducing ladder fuels, counteracted somewhat by reduced stand density. In the  $MC_{SA}$  (and  $ISI_{SA}$ ) estimates, stand density of 'dense' was used for the controls and 'moderate' for the thinned stands, resulting in slightly different fuel moisture estimates between stand types (Perrakis et al. 2023; Wotton and Beverly 2007b). The basic ISI function, in contrast, is not sensitive to stand density.

We then needed a model to predict surface ROS values near the crowning threshold in the Sharpsand stands. This resulted in the empirical analysis of Canadian surface fire ROS data described in Appendix 1. The best surface fire models were obtained using ISI (or  $ISI_{SA}$ ) alone or in combination with SFC. Notably, these models produce lower ROS predictions than Van Wagner's surface fire model (VW93), a presented 'to define the spread rate of all possible surface fires' in the Sharpsand stands (Van Wagner 1993). While the aggregated sROS predictions appear more scattered above ISI 12 or so, conditions favoring crown fire in many conifer stands, the observations are closer to the model under lower danger conditions. Based on the estimated crowning thresholds, representing ISI (or  $ISI_{SA}$ ) values of ~5-10, approximate surface fire spread rates of 0.7 – 2.5 m min<sup>-1</sup> are expected.



#### Byram's intensity and estimating $M$

We can finally use our scaled ladder fuel equation [7] along with Byram's equation (Byram 1959) and our empirical surface ROS models, and calculate the most likely  $M$  value. This calculation assumes that equivalent critical surface intensity values ( $I_0$ ) are required for crowning in the control and thinned stands, both with LCBH of 4.4 m.

In terms of ROS values at the crowning threshold ( $ROS_0$ ) in the Im and Th stands, the equivalency is expressed as follows:

$$\begin{aligned}
 I_{0.Im} &= I_{0.Th} \\
 h \cdot w_{Im} \cdot ROS_{Im} &= h \cdot w_{Th} \cdot ROS_{Th} \\
 SFC_{Im} \cdot ROS_{0.Im} &= SFC_{Th} \cdot ROS_{0.Th} \\
 (SFC + FC_{SE.Im} \cdot M) \cdot ROS_{0.Im} &= SFC_{Th} \cdot ROS_{0.Th}
 \end{aligned}$$

Based on [7],  $FC_{SE}$  is calculated using estimated ladder fuel consumption and structure. In this case, we calculate ladder fuel height ( $C_L$ ) using an estimated crown ratio of 0.5 and cohort heights from Walker and Stocks (1975) original stand data. Mean dead sapling height and < 1 cm dead roundwood ( $FC_L$ ) in control stands were 3.49 m and 0.159 kg m<sup>-2</sup>, resulting in mean  $C_L = 2.62$  m. Scaled surface-equivalent consumption values, calculated using [6], ranged from 0.291 to 1.227 kg m<sup>-2</sup>, with a mean of 0.658 kg m<sup>-2</sup>.

The final solution was calculated by solving for  $M$ :

$$(SFC + FC_{SE \cdot Im} \cdot M) \cdot ROS_{0 \cdot Im} = SFC \cdot ROS_{0 \cdot Th}$$

$$M = \frac{\frac{SFC \cdot ROS_{0 \cdot Th}}{ROS_{0 \cdot Im}} - SFC}{FC_{SE \cdot Im}}$$

This was expressed using overall mean SFC values from all IM and TH plots (1.34 kg m<sup>-2</sup>), as measured SFC values were not significantly different between treatments (p=0.077). Using the two spread rate models, this produced  $M = 2.2$  using the ISI-based sROS model and crowning thresholds, and  $M = 4.6$  using the ISI<sub>SA</sub>-based model and thresholds.

In addition to the sROS models, the sensitivity analysis compared using the dead ladder fuel midpoint (cohort height/2), the equivalent of a crown ratio of 1. This results in significantly lower FC<sub>SE</sub> values, due to the greater z<sub>L</sub> values.

Final estimates of M ranged from 2.2 to 8.7, as shown in the table below:

[finish table]

### Empirical crown fire analysis

Finally, we reanalyzed the experimental crown fire dataset using Monte Carlo cross-validation, comparing models with  $\log(SFC)$ ,  $\log(SFC + FC_{SE})$ , and  $\log(SFC + FC_{SE} \cdot M)$ . The starting value for M was 4, a rough mean of the previous estimates.

As show in the table, the cross-validation evidence supports the existence of  $M > 1$ . Both the FPMC- and MC<sub>SA</sub>-based models (7, 8) using M had higher mean accuracy and lower AIC than comparable models incorporating ladder fuels without a multiplier effect (5, 6). The best models were superior than either the base (LCBH only; 1, 2) or subjectively-assigned FSG (3, 4) models.

Table

##	Model	Accuracy	MCC	AIC
## 1	ws + I(FSG^1.5) + I(log(SFC)) + ws:MC.FPMC	0.8781	0.7547	46.19
## 2	ws + I(FSG^1.5) + I(log(SFC)) + ws:MC.SA	0.8635	0.7252	49.88
## 3	ws + I(FSG^1.5) + I(log(SFC)) + ws:MC.FPMC	0.8841	0.7671	43.27
## 4	ws + I(FSG^1.5) + I(log(SFC)) + ws:MC.SA	0.8879	0.7747	42.41
## 5	ws + I(FSG^1.5) + I(log(SFC2)) + ws:MC.FPMC	0.8950	0.7888	43.19
## 6	ws + I(FSG^1.5) + I(log(SFC2)) + ws:MC.SA	0.8742	0.7469	45.94
## 7	ws + I(FSG^1.5) + I(log(SFC3)) + ws:MC.FPMC	0.8963	0.7915	40.75
## 8	ws + I(FSG^1.5) + I(log(SFC3)) + ws:MC.SA	0.8898	0.7785	41.23
##	Dataset.reorder2. MCC.rank AIC.rank			
## 1	1.Base	6	7	
## 2	1.Base	8	8	
## 3	2.FSG Adj.	5	5	
## 4	2.FSG Adj.	4	3	
## 5	3.LF to SFC	2	4	
## 6	3.LF to SFC	7	6	
## 7	3.LF x M to SFC	1	1	
## 8	3.LF x M to SFC	3	2	

## Discussion

Aside from Van Wagner's equation, this is simply a slightly different representation of Thomas' (**thomas1963?**) equation for flame size under calm conditions.

- Figure showing more complex architecture; 2-story stands, using Eq 2 to estimate contribution of surface fuels to upper stratum crowning - Validation 4. Upper canopy crowning -Scale issues are obvious here. Following the logic and intent of Van Wagner's (1977) model, the scaling equation represents relationships between canopy strata logically in 1-dimensional space, assuming ladder fuels are positioned beneath overstory canopy fuels. They also may hold at the single tree level, where individual ladder fuel elements (saplings, bark flakes, arboreal lichen) are horizontally juxtaposed with overstory fuels. More detailed survey methods (e.g. Alexander et al. 2004; Reinhardt et al. 2006; Arkin et al. 2023) can describe the frequency distributions associated with vertical strata in a stand, while ecologists have described the spatial patterns of overstory trees in stands across various stand types (Kenkel et al. 1997; Larson and Churchill 2012). Future investigations will be needed on how these spatial distribution patterns affect ladder fuel influences near crown fire initiation thresholds.

-For the theoretical relationship to hold for actual ladder fuel elements (flammable shrubs, saplings, bark flakes, hanging lichens, etc.), it is imperative that they be connected to surface fuels, and in actual forest settings

-The concept of a multiplier for comparing elevated fuels to surface fuels is a similar topic to Van Wagner's [-vanwagner1977] 'c' parameter of 0.01, deduced from a comparison of only two experimental fires, as discussed by Alexander (**alexander1998?**). Densely-packed litter fuelbeds have lower spread rate than lower density fuelbeds with the same fuel load (**campbell-lochrie2021?**), although the relationship is likely more complex in canopy fuels (Schwilk 2015).

-As Andrews (**andrews2018?**) noted, Rothermel surface model only applicable to surface fuels, within ~6ft of the ground

-Thinning increases sub-canopy turbulence (**russell2018?**). See also Banerjee et al. (Banerjee 2020)

McAlpine and Xanthopoulos (McAlpine and Xanthopoulos 1989) also noted that C-6s appeared to overpredict sROS in needle fuelbed experiments. Although they explained it as a difficulty in estimating

Method could also be used to estimate influence of dead branch fuel below LCBH, bark flakes, arboreal lichens, or similar material.

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