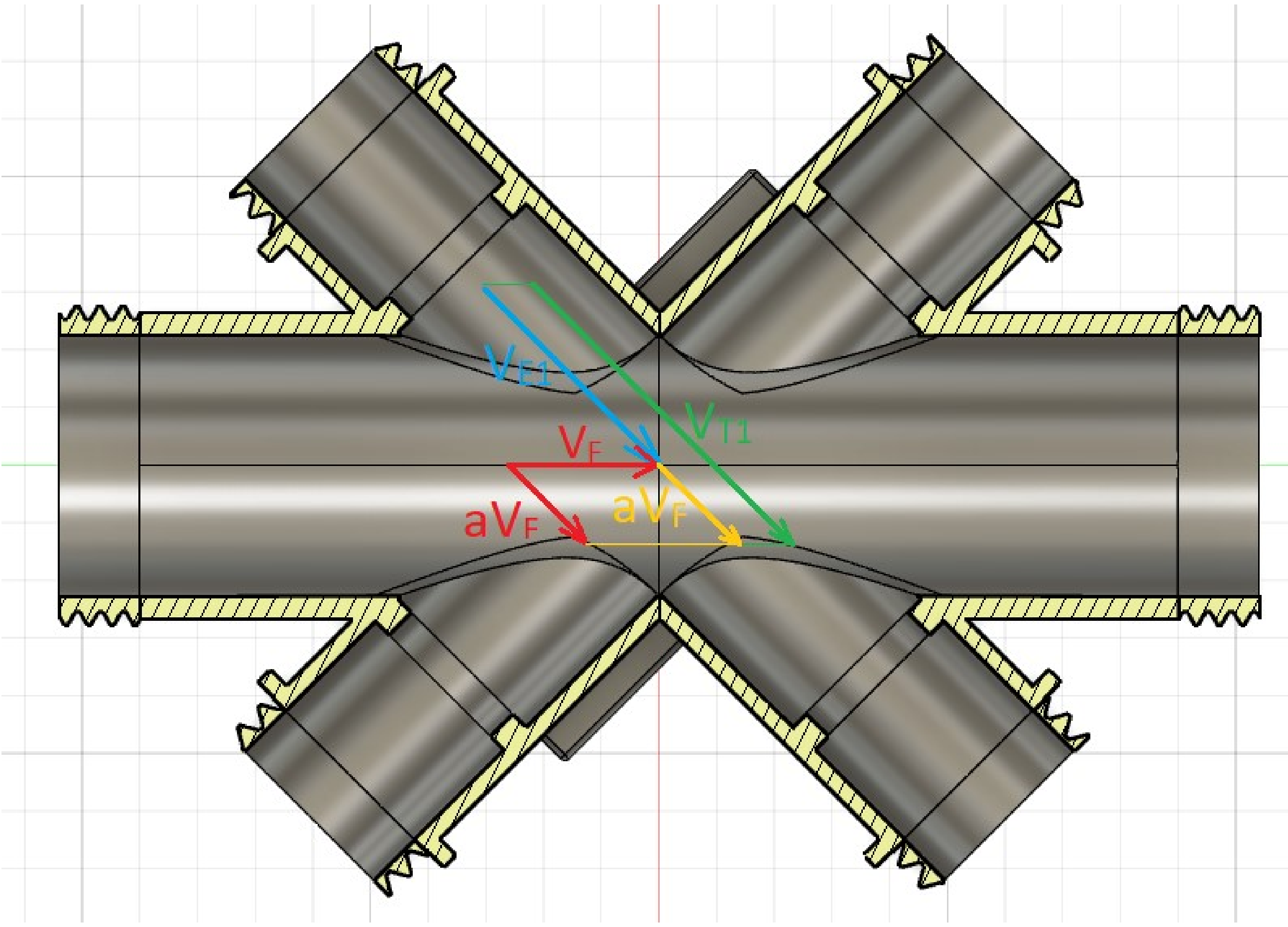
**Calculations for an ultrasound flow rate sensor**




***Syllabus***

| | |
|---|---|
| $V_{T1}$: | Final velocity of tone 1 |
| $V_{T2}$: | Final velocity of tone 2 |
| $V_{S1}$: | Source velocity of tone 1 |
| $V_{S2}$: | Source velocity of tone 2 |

$V_{E1}$ e $V_{E2}$ depend on the pressure within the flow but they will disappear over calculus, thus we don't really need to know their actual value.

| | |
|---|---|
| $V_F$: | Air flow velocity |
| $t_1$: | Travel time of tone 1 |
| $t_2$: | Travel time of tone 2 |
| L: | transmitter-receiver distance (inferred from $t_1$ or $t_2$ measured in quiet state) |

***Calculus***

$$V_{T1} = V_{E1} + aV_F \qquad (1)$$

$$V_{T2} = V_{E2} - aV_F \qquad (2)$$

Since the air blows onto the acoustic wave from a flank, with an angle (α), both $V_{E1}$ and $V_{E2}$ will be increased by a component of $V_F$, $aV_F$, named this way because its magnitude is a percentage (a) of $V_F$. The difference between the higher magnitude ($V_{T1}$) and the lower ($V_{T2}$) is almost twice $aV_F$. And since sensors actually allow us measure these values, the difference can be calculated easily. From what was said here, and considering we are interested in the difference as an absolute value without sign, you extrapolate that:

$$aV_F = \frac{|V_{T1} - V_{T2}|}{2} \qquad (3)$$

So far, (a) has been referred to as a generic percentage. But in order for the calculus to go on, you've to find out its content and write it explicitly. If we were given the number (x) and were we said that it is the value of $aV_F$ and were the angle (α) 45 deg, finding the whole $V_F$ would only require the trigonometric rule to switch from the side to the diagonal of a square, that is multiplying x by $\sqrt{2}$. We would have the following situation:

$$V_F = \sqrt{2} \cdot x \qquad (4)$$

$$V_F = \sqrt{2} \cdot aV_F \qquad (5)$$

Instinctively, we would realize that in order for proposition (5) to be true, $\sqrt{2} \cdot a$ must return 1 and therefore a = $1/\sqrt{2}$. But $1/\sqrt{2}$ is sin(45°) or cos(45°), that is the sine or the cosine of our angle. For, we choose a = cos(α).

Going back to proposition (3) and replacing with the result we found and showing (L)s and (t)s contained in ($V_T$)s, this is what we find:

$$\cos(\alpha) \cdot V_F = \frac{1}{2}|V_{T1} - V_{T2}| \qquad (6)$$

$$\cos(\alpha) \cdot V_F = \frac{1}{2}\left|\frac{L}{t_1} - \frac{L}{t_2}\right| \qquad (7)$$

$$V_F = \frac{1}{2}\frac{1}{\cos(\alpha)}L\left|\frac{1}{t_1} - \frac{1}{t_2}\right| \qquad (8)$$

$$V_F = \frac{L}{2\cos(\alpha)} \cdot \frac{|t_2 - t_1|}{t_1 \cdot t_2} \qquad (9)$$

Now the trouble is that, in the real world, L measures the distance between transmitter and receiver, which changes slightly in each couple, no matters how much effort we make to keep it the same everywhere. Nor can we consider an average value of it (what would produce great inaccuracy). We are compelled to consider both distances individually. For, let's discard the generic relation (9), go back to step (7) and let's solve for the real world:

$$\cos(\alpha) \cdot V_F = \frac{1}{2}\left|\frac{L_1}{t_1} - \frac{L_2}{t_2}\right| \quad (7a)$$

$$V_F = \frac{1}{2}\frac{1}{\cos(\alpha)}\left|\frac{L_1}{t_1} - \frac{L_2}{t_2}\right| \quad (8a)$$

$$V_F = \frac{1}{2\cos(\alpha)} \cdot \frac{|L_1 t_2 - L_2 t_1|}{t_1 \cdot t_2} \quad (9a)$$

Flow is meant here as the volume of air passing across a section of pipe over the time unit. To reckon it, you also need to measure the surface (S) of the opening of the air pipe and multiply it by the distance (d) that the air covers over the time unit. If time units are seconds, we'll have:

$$d(m) = V_F(m/s) \cdot 1(s) = V_F(m)$$

$$\boldsymbol{\dot{F}(m^3/s)} = \frac{S(m^2) \cdot V_F(m)}{1(s)} = \boldsymbol{S \cdot V_F(m^3/s)}$$

That we can turn into either L/m or $dm^3/s$ through the proper equivalence.

If we also know the density (ρ) of the gas, we can reckon the thrust of the air flow. To begin you need to estimate the flow rate (the mass in grams of the air flowing through the section over the time unit):

$$\dot{R}(g) = \dot{F}(m^3/s) \cdot \rho(kg/m^3)$$

The result you need to multiply by velocity, because thrust, as a phisical dimension, isn't other but momentum (quantity of motion) Q = m·v. Our mass (m) is the flow rate (R), while our velocity keeps being ($V_F$):

$$Q\left(\frac{kg \cdot m}{s^2}\right) = \dot{F}(m^3/s) \cdot \rho(kg/m^3) \cdot V_F(m/s)$$

$$\boldsymbol{Q(N) = S \cdot V_F^2 \cdot \rho\left(\frac{kg \cdot m}{s^2}\right)}$$