Around Quantum Decision Diagrams

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Research Track – CentraleSupélec

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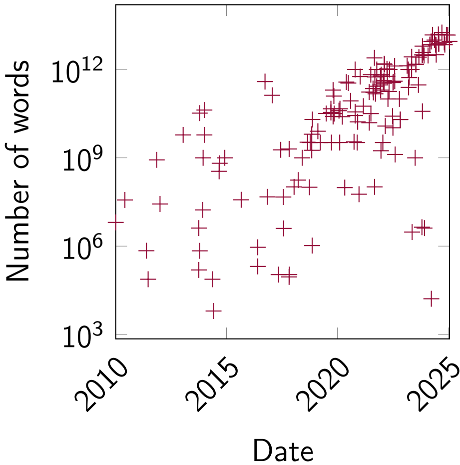
Databases grow rapidly in size

Classical algorithms are sometimes not efficient enough



Number of words used to train language models

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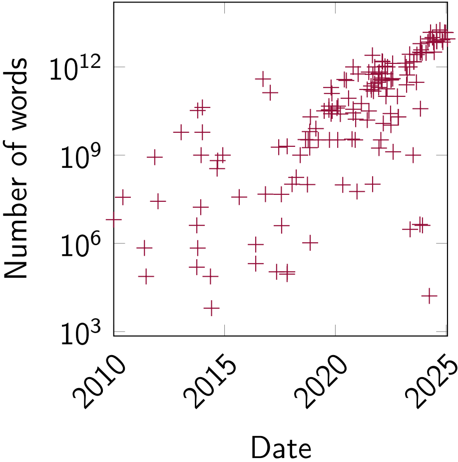

Quantum algorithms enable us to solve some probels faster

Number of words used to train language models

Quantum machines are still in development and will remain expensive

$$\Downarrow$$

There is a need for quantum algorithm **simulation** and verification tools

Simulations are very costly in terms of computation time

Grover	Classical	Quantum	Simulation
Complexity	N	$\sqrt{N}$	$N\sqrt{N}$

Simulation requires adapted **data structures**

State of the art

- Abstract interpretation
- Arithmetics of real intervals
- Quantum decision diagrams

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Solution : abstract additive decision diagrams

Abstract interpretation enables us to determine mathematical properties or to speed up calculations

Example : sign of an expression $e = (3 + 2) \times (-5)$

$$\begin{aligned} \mathrm{signe}(e) &= (\mathrm{signe}(3) + \mathrm{signe}(2)) \times \mathrm{signe}(-5) \\ &= (\oplus + \oplus) \times \ominus \\ &= \oplus \times \ominus \\ &= \ominus \end{aligned}$$

Abstract interpretation enables us to determine mathematical properties or to **speed up calculations**

It can be exact or **approximate**

Abstract interpretation can be applied to **real intervals**

$$
\begin{aligned}
[1,2] * [-1,1] &= [-2,2] \\
[1,2] + [-1,1] &= [0,3] \\
[1,2] \wedge [-1,1] &= [1,1]
\end{aligned}
$$

The result of the operation is **the smallest interval** containing all the possible 1-to-1 results

Una **Boolean function**

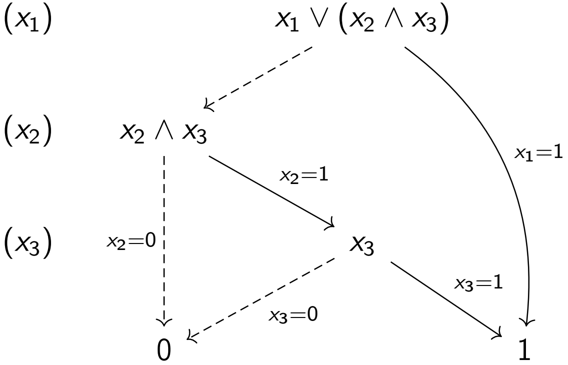
$$f : \{0,1\}^n \to \{0,1\}$$

can be represented by a **truth table**

x_1	x_2	x_3	$f(x_1, x_2)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

for $f(x_1, x_2) = x_1 \vee (x_2 \wedge x_3)$

Decision diagrams can represent Boolean functions



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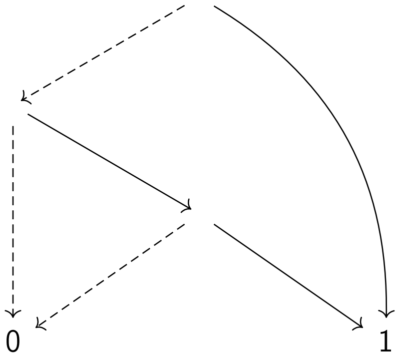

We make use of the internal **structure** of the function

A **quantum state** is a superposition of incompatible states

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \quad \text{(un qubit)}$$

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$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \quad \text{(un qubit)}$$

$$n \text{ qubits} \Rightarrow 2^n \text{ incompatible states}$$

The states are noted in the form of **vecteurs**

$$\alpha\,|01\rangle + \beta\,|10\rangle = \begin{pmatrix} \alpha \\ 0 \\ \beta \\ 0 \end{pmatrix}$$

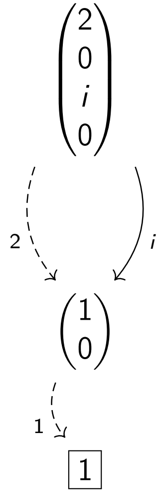
The usual representation is similar to **truth tables**

x_1	x_2	$\langle x_1 x_2 \vert \psi \rangle$
0	0	α
0	1	0
1	0	β
1	1	0

for $|\psi\rangle = \alpha\,|00\rangle + \beta\,|10\rangle$

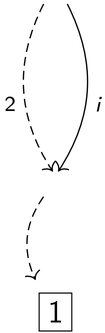
States can be represented by **quantum decision diagrams**

We make use of the internal **structure** of the state



States can be represented by **quantum decision diagrams**

In the worst case it is still space-**exponential** in n



Look back at the state of the art

- ✓ Abstract interpretation
- ✓ Arithmetics of real intervals
- ✓ Quantum decision diagrams

We will use these concepts together,
with an innovation : l'**additivity**

$$\begin{tikzcd} & \Psi \arrow[dl, dashed] \arrow[d, dashed] \arrow[dr] & \\ \phi_1 & \phi_2 & \psi \end{tikzcd} \quad \Leftrightarrow \quad \begin{tikzcd} & \Psi \arrow[dl, dashed] \arrow[dr] & \\ \phi_1 + \phi_2 & & \psi \end{tikzcd}$$

Look back at the state of the art

- ✓ Abstract interpretation
- ✓ Arithmetics of real intervals
- ✓ Quantum decision diagrams
- + Innovation : additivity

Solution : abstract additive decision diagrams

Objectives

- **Formal model** for abstract additive quantum decision diagrams
- **Implementation** of the model

Methodology

$$\begin{tikzcd}
{} \arrow[r] & \text{Documentation} \arrow[rr, "\text{usual cases}"] \arrow[dr, "\text{state of the art}"'] & & \text{Examples} \arrow[dl, bend left, "\text{model}"'] \arrow[dr, "\text{tests}"] & \\
& & \text{Theory} \arrow[ur, bend left] \arrow[rr, "\text{code}"] & & \text{Code}
\end{tikzcd}$$

GitHub for project management

- **Issues** for tasks and bugs
- Priorities, sizes, and deadlines
- Branches and **merge requests**

Model

- S6 Cartesian & polar intervals of $\mathbb{C}$
- S6 Diagrams
- S6 Local and global approximations
- S6 Forced merge
- S6 Redcution algorithms
- S7 Error
- S7 Gate application

Example : we consider the state $\begin{pmatrix} 10i+2 \\ 4i+1 \\ 2i \\ i \end{pmatrix}$

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There are **regularities**.

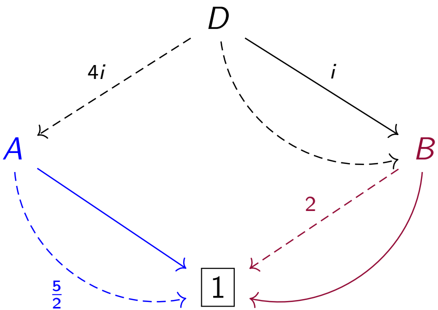
Example : we consider the state $\begin{pmatrix} \begin{pmatrix} 10i \\ 4i \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ i \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{pmatrix}$

There are **regularities**.

Example : we consider the state $\begin{pmatrix} 4i \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ i \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{pmatrix}$

There are **regularities**.

Example : we get the additive diagram



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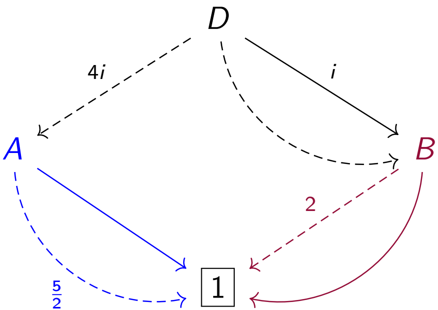
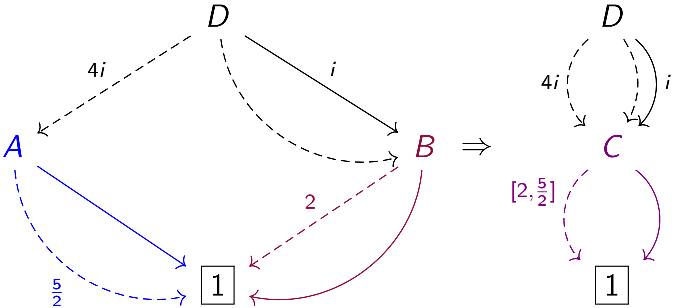


Let's **reduce** this diagram

We can force the merge of A and B



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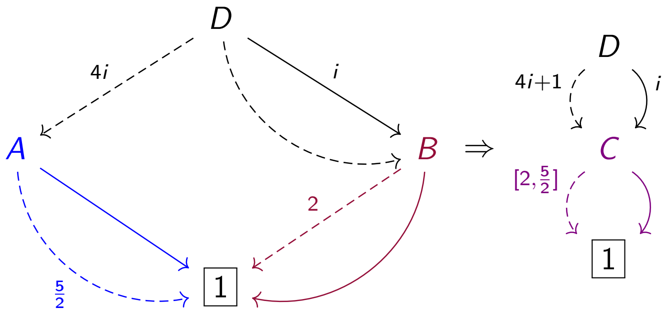

We can always reduce diagrams more

$$\Downarrow$$

Arbitrarily large space gain (up to exponential)

How to choose which diagrams to merge?

$$\Downarrow$$

Error : we can compute the error induced by a diagram

Quantum circuits are based on **gates**

They are represented by **matrices**

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Example : Hadamard

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

To apply a gate M to a diagram D

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We *want* $\mathcal{E}(M(D)) = M\ \mathcal{E}(D)$

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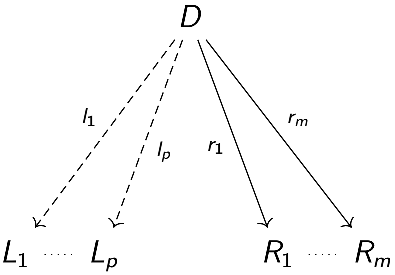
$$D = (\{(l_1, L_1), ..., (l_p, L_p)\}, \{(r_1, R_1), ..., (r_m, R_m)\})$$

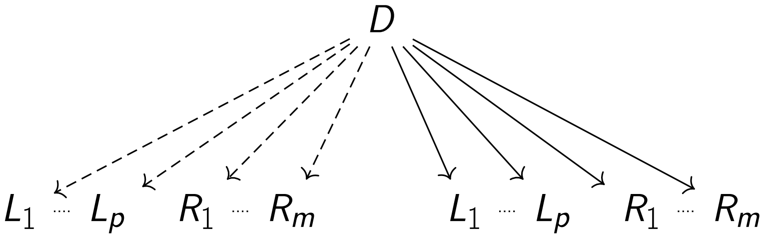


Before application of M

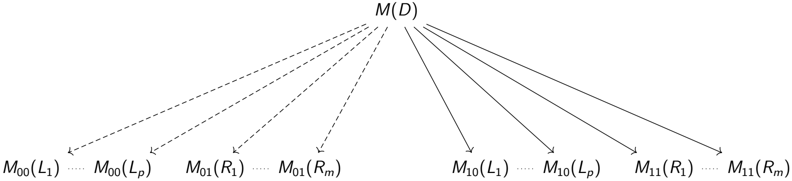
1.

$$\begin{tikzcd}
 & & & & & & M(D) \arrow[dllllll, dashed] \arrow[dllll, dashed] \arrow[dlll, dashed] \arrow[dl, dashed] \arrow[dr] \arrow[drrr] & & & \\
L_1 & \cdots & L_p & R_1 & \cdots & R_m & & R_1 & \cdots & R_m
\end{tikzcd}$$

2.



3.



$$M = \begin{pmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{pmatrix}$$

3.

$$
\begin{tikzcd}[column sep=small]
 & & & & & & M(D) \arrow[dllllll, dashed] \arrow[dllll, dashed] \arrow[dlll, dashed] \arrow[dl, dashed] \arrow[d] \arrow[drr] \arrow[drrr] \arrow[drrrrr] & & & & & \\
M_{00}(L_1) & \cdots & M_{00}(L_p) & M_{01}(R_1) & \cdots & M_{01}(R_m) & M_{10}(L_1) & \cdots & M_{10}(L_p) & M_{11}(R_1) & \cdots & M_{11}(R_m)
\end{tikzcd}
$$

$$\mathcal{E}(M(D)) = \begin{pmatrix} \sum l_i M_{00}\mathcal{E}(L_i) + \sum r_j M_{01}\mathcal{E}(R_j) \\ \sum l_i M_{10}\mathcal{E}(L_i) + \sum r_j M_{11}\mathcal{E}(R_j) \end{pmatrix} = M\ \mathcal{E}(D)$$

Implementation

- S6 Cartesian & polar intervals of $\mathbb{C}$
- S6 Diagrams : building, evaluation
- S6 Forced merge
- S6 Redcution algorithms
- S7 Random diagrams
- S7 Error
- S7 Gate application
- S7 QASM

Future developments

- **Implementation**
  - Graphical interface
  - Benchmarks
- **Tweaks**
  - Error function
  - Reduction algorithms
- New concepts
  - Tree automata
  - Local Invertible Map Decision Diagrams (LIMDD)

Project framework
Future training

- **Supervisor** : Renaud Vilmart
- **Team** : QuaCS
- **Laboratory** : Laboratoire Méthodes Formelles

LMF Laboratoire Méthodes Formelles

QuaCS

Gap year *Digital Tech Year*

- Semester at Paris Digital Lab
- Various tech projects in teams
- 6-month internship in a company or lab, in France or abroad

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After the gap year

- S8 at CentraleSupélec
- S8 Pro (internship)
- Academic S8 abroad

Majors / mentions

- **Computer science and digital technology**
  - Software engineering
  - Computer systems architecture
- **Physics and nanotechnologies**
  - Quantum engineering

Other training programs

- ARTeQ (ENS Paris-Saclay)
- QMI M2 (Télécom Paris, among others)

Conclusion

Questions

Implementation

- **Code** (4.9k lines)
  - Langage C++
  - LLVM / Clang
  - Ninja
  - CMake
- **Tests**
  - Google Test
  - GitHub Actions

Tools

- **Version control**
  - Git
  - GitHub
- **Documentation**
  Doxygen

$$\rho(\boxed{1}) = \{1\}$$

$$\varepsilon(\boxed{1}) = \{0\})$$

$$\forall G, D \in \mathscr{P}_f(\mathcal{A}_0 \times \mathcal{D}_n), \rho((G, D)) = \left(\sum_{(l,L) \in G} l\rho(L) \right) \bigsqcup \left(\sum_{(r,R) \in D} r\rho(I$$

$$\forall G, D \in \mathscr{P}_f(\mathcal{A}_0 \times \mathcal{D}_n),$$

$$\varepsilon((G, D)) = \left(\sum_{(l,L) \in G} l \max |\rho(L) \ominus \varepsilon(L)| + \varepsilon(L) \right) \bigsqcup \left(\sum_{(r,R) \in D} r \max |$$