INTRODUCTION TO DATA ANALYTICS

Linear Regression

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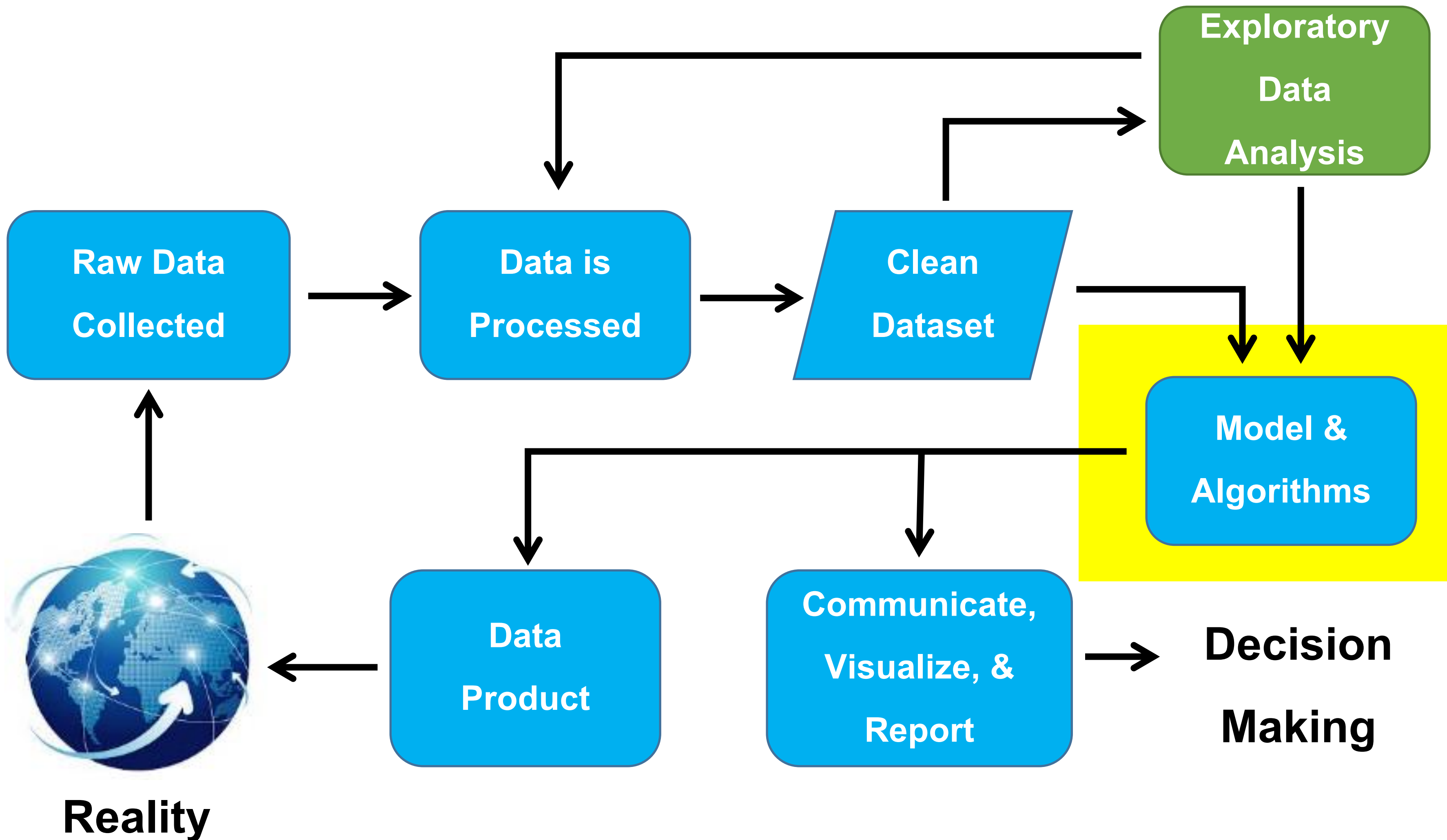
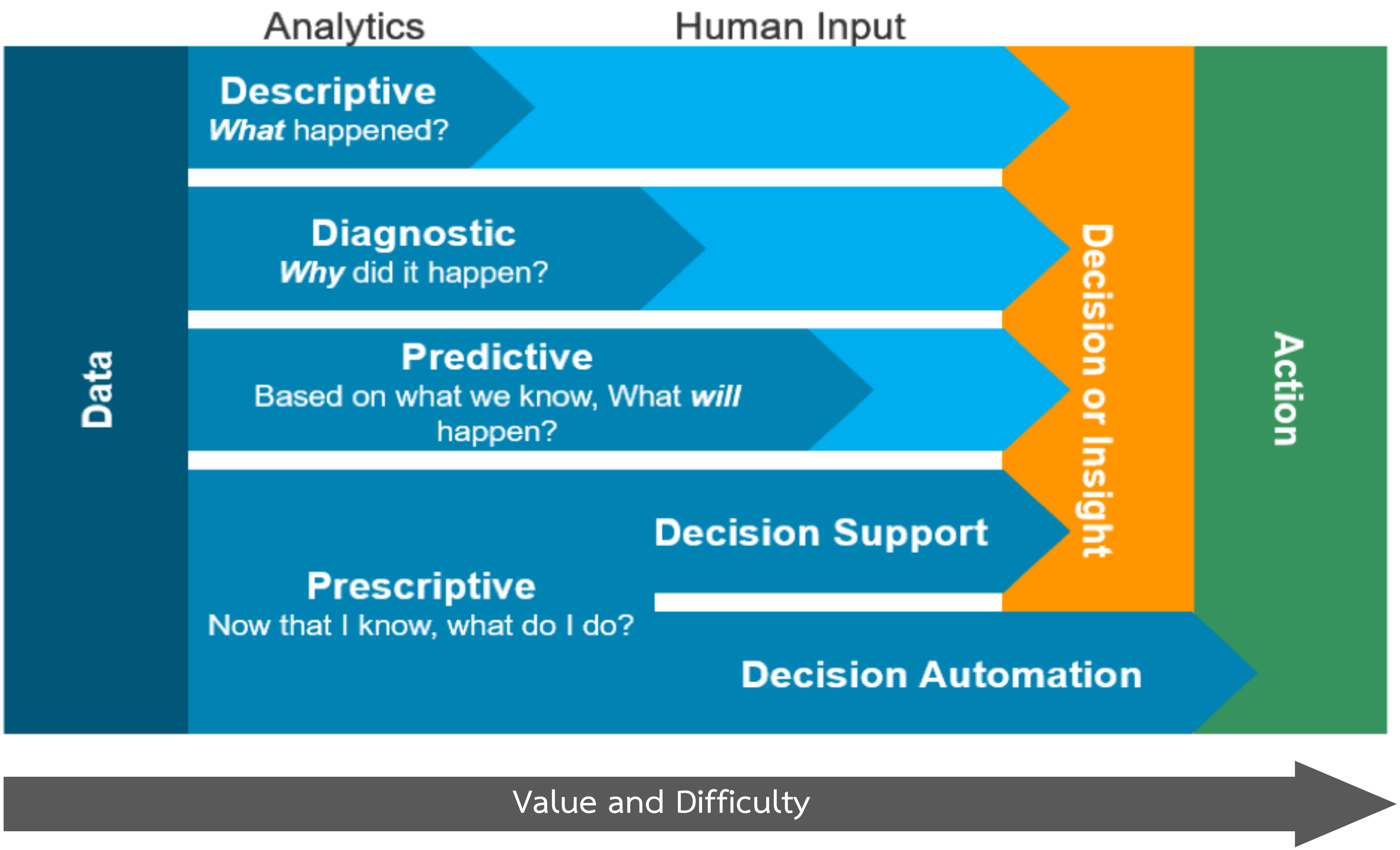
Agenda

- Linear Regression
- Evaluation Methods
- Feature Selection
- Cross Validation
- Feature Scaling

Data Science Process



Data Analytics



Ref:
- Four types of analytics capability (Gartner, 2014)
- (image) https://www.healthcatalyst.com/closed-loop-analytics-method-healthcare-data-insights

Machine Learning

Supervised Learning

Develop predictive model based on both input and output data

Regression

- Linear Regression
- Polynomial Regression

Classification

- Decision Tree
- Logistic Regression
- Neural Network
- etc.

Unsupervised Learning

Develop predictive model based on both input and output data

Clustering

- K-Means
- DB-SCAN
- etc.

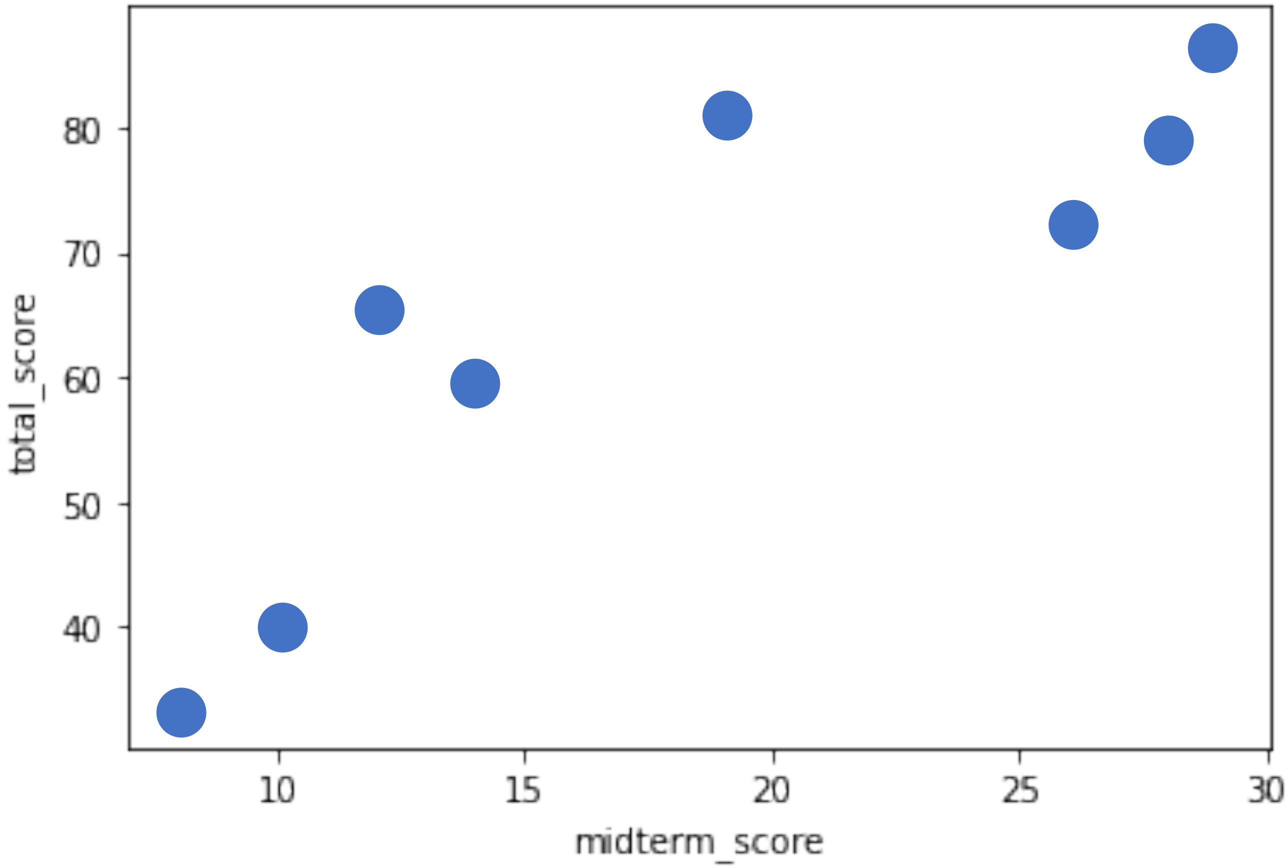
Linear Regression

Course Score

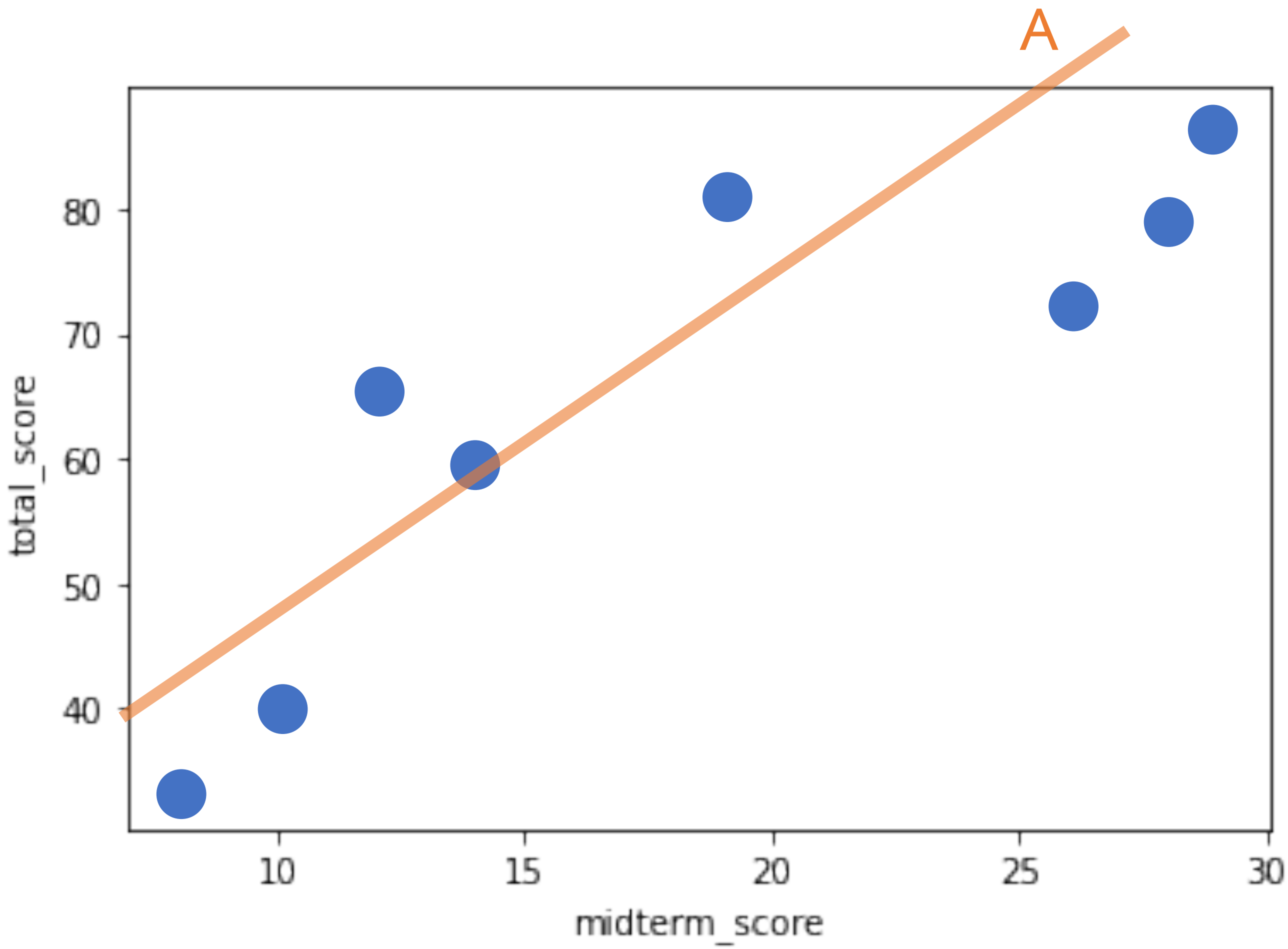
midterm_score	total_score
10	40
8	33
14	59
12	65
19	81
26	72
28	79
29	87

(assumed data)

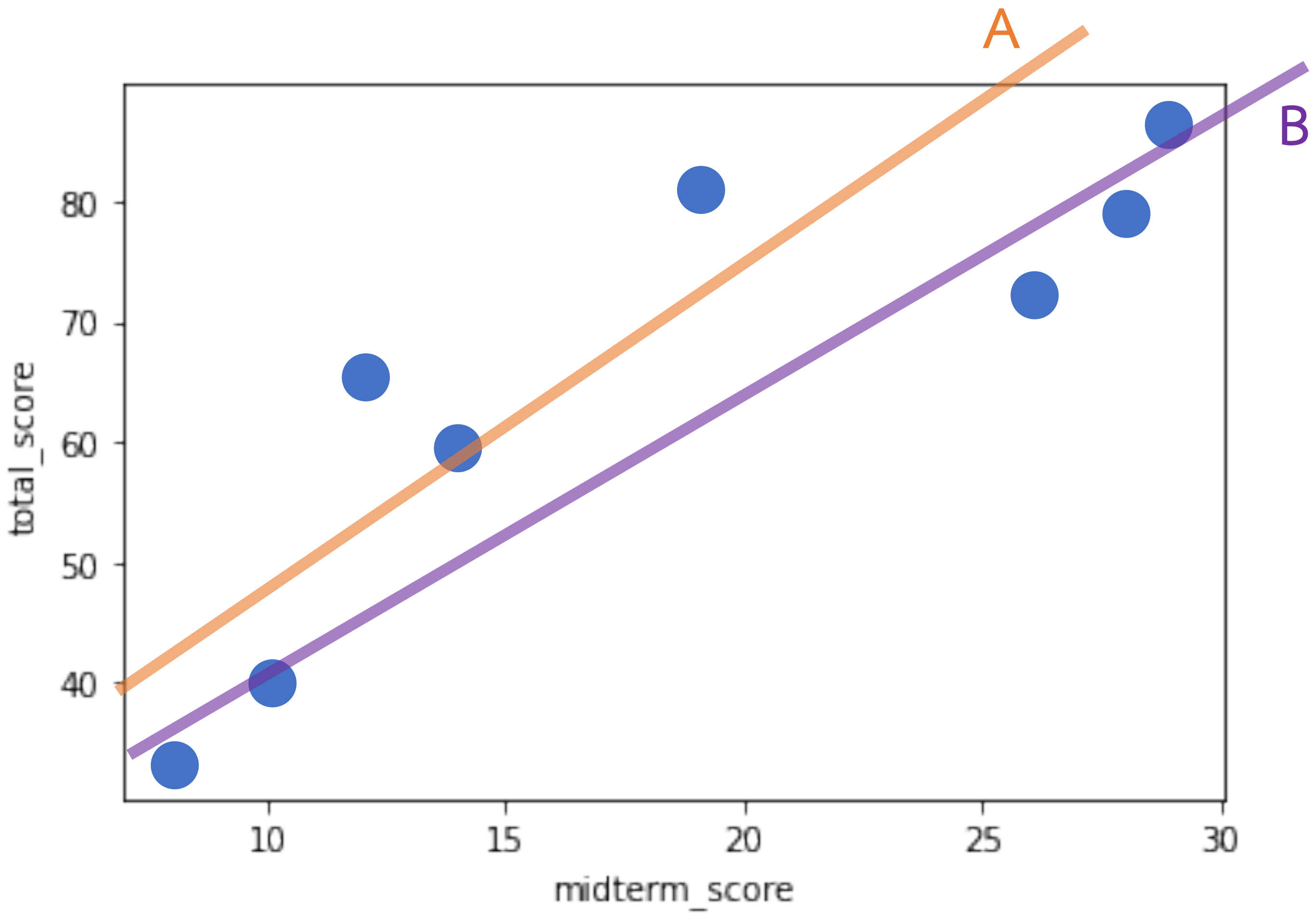
Prediction?



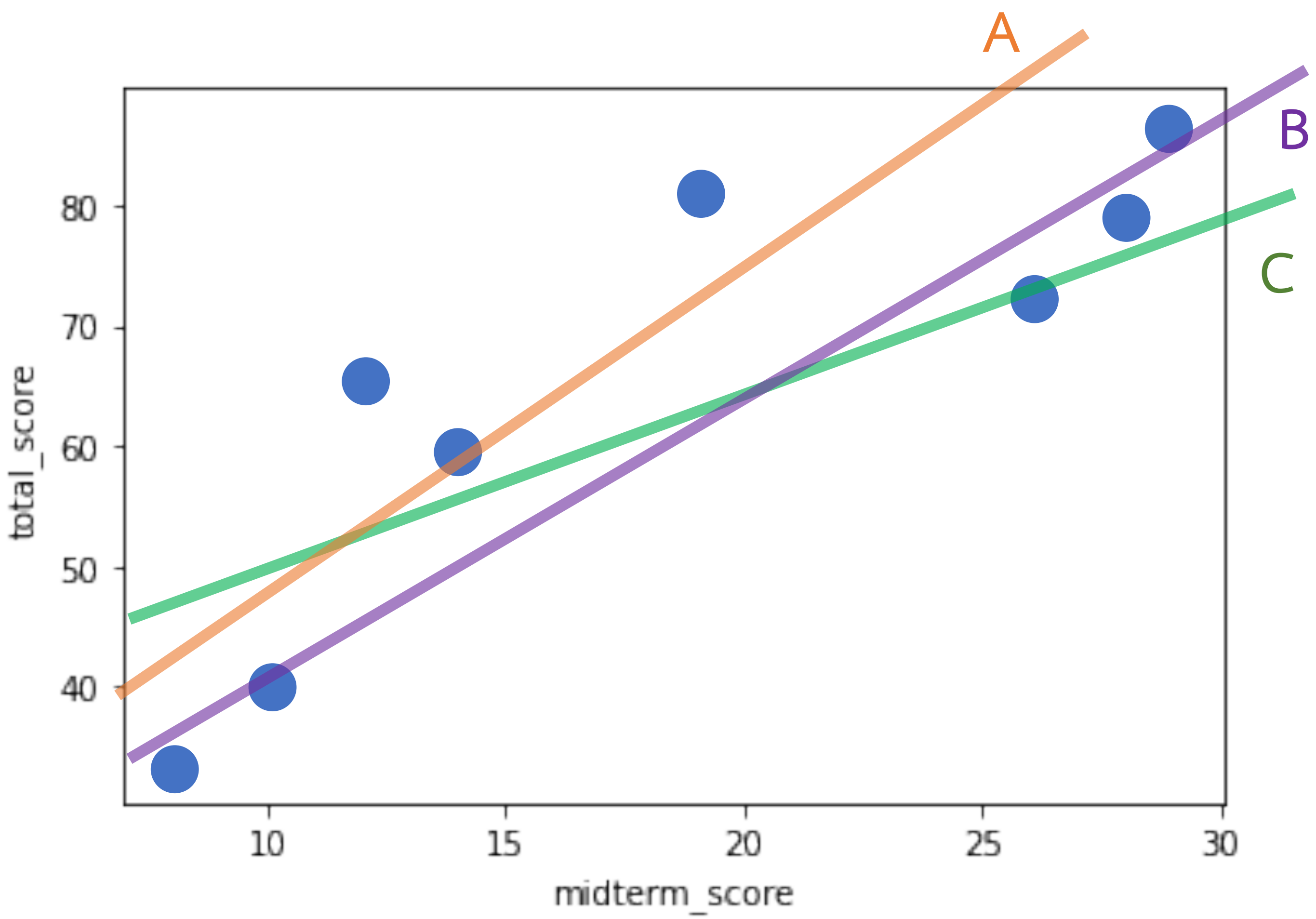
Prediction?



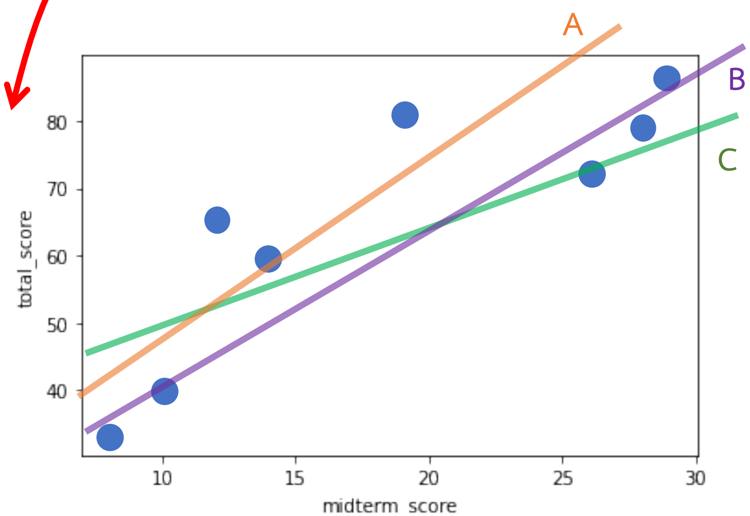
Prediction?



Prediction?



Linear Equation



$$y = \overset{?}{a}x + \overset{?}{c}$$

Ref: • (image) http://www.chegg.com/homework-help/questions-and-answers/orientation-meeting-new-students-college-president-wants-emphasize-importance-getting-good-q12615281

Simple Linear Regression

$$\hat{Y}_i = b_0 + b_1 X_i$$

Simple Linear Regression

$$\hat{Y}_i = b_0 + b_1 X_i$$

(actual values)

total_score
40
33
59
65
81
72
79
87

total_score

midterm_score
10
8
14
12
19
26
28
29

Ref:

Course Score

midterm_score	hour_study	total_score
10	3	40
8	2	33
14	5	59
12	5	65
19	7	81
26	8	72
28	8	79
29	9	87

(assumed data)

Multiple Linear Regression

$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + \cdots$$

Ref:

Multiple Linear Regression

$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + \cdots$$

actual	total_score	midterm_score	hour_study
40		10	3
33		8	2
59		14	5
65		12	5
81		19	7
72		26	8
79		28	8
87		29	9

Ref:

Coefficients

$$\hat{Y}_i = 19.8 - 1.8 * midterm\_score + 13.3 * hour\_study$$

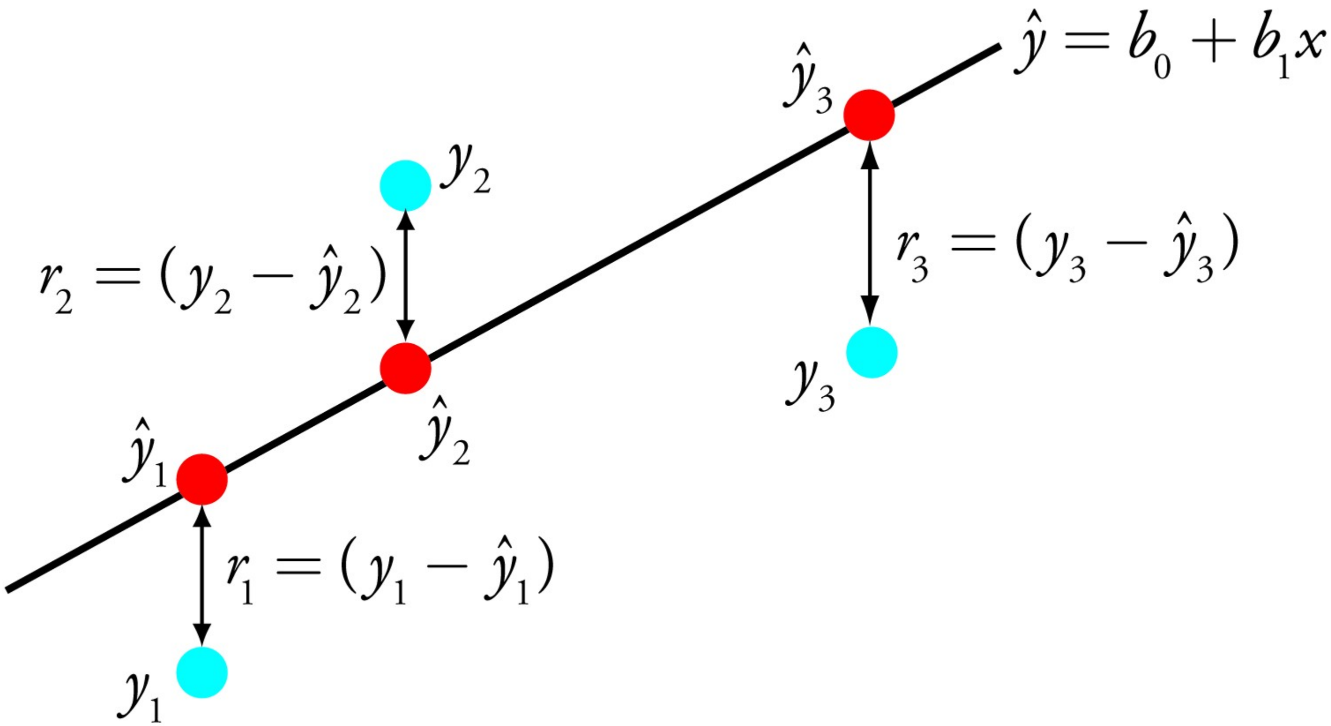
Python

```python
from sklearn import linear_model
X = df[["midterm_score","hour_study"]]
y = df["total_score"]
lm = linear_model.LinearRegression()
lm.fit(X, y)
print("intercept=", lm.intercept_)
print("coef=", lm.coef_)
```

```
intercept= 19.79667010461179 2
coef= [-1.84647119 13.34492412]
```

Evaluation Methods

Residual



Ref: • (image) https://towardsdatascience.com/simple-linear-regression-2421076a5892

Root Mean Squared Error (RMSE)

predicted

actual

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}{N}}$$

number of samples

RMSE

$$\hat{Y}_i = 19.8 - 1.8 * mid + 13.3 * hour$$

mid	hour	y_i
10	3	40
8	2	33
14	5	59
12	5	65
19	7	81
26	8	72
28	8	79
29	9	87

RMSE

$$\hat{Y}_i = 19.8 - 1.8 * mid + 13.3 * hour$$

mid	hour	$\boldsymbol{y_i}$	$\boldsymbol{\hat{y}_i}$
10	3	40	41.4
8	2	33	31.7
14	5	59	60.7
12	5	65	64.4
19	7	81	78.1
26	8	72	78.5
28	8	79	74.9
29	9	87	86.4

RMSE

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}{N}}$$

$$\hat{Y}_i = 19.8 - 1.8 * mid + 13.3 * hour$$

mid	hour	y_i	$\hat{y}_i$	$\hat{y}_i - y_i$	$(\hat{y}_i - y_i)^2$
10	3	40	41.4	1.4	1.9
8	2	33	31.7	-1.3	1.7
14	5	59	60.7	1.7	2.8
12	5	65	64.4	-0.6	0.4
19	7	81	78.1	-2.9	8.2
26	8	72	78.5	6.5	42.9
28	8	79	74.9	-4.1	17.2
29	9	87	86.4	-0.6	0.4

RMSE

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(\hat{y}_i - y_i)^2}{N}}$$

$$\hat{Y}_i = 19.8 - 1.8 * mid + 13.3 * hour$$

mid	hour	y_i	$\hat{y}_i$	$\hat{y}_i - y_i$	$(\hat{y}_i - y_i)^2$
10	3	40	41.4	1.4	1.9
8	2	33	31.7	-1.3	1.7
14	5	59	60.7	1.7	2.8
12	5	65	64.4	-0.6	0.4
19	7	81	78.1	-2.9	8.2
26	8	72	78.5	6.5	42.9
28	8	79	74.9	-4.1	17.2
29	9	87	86.4	-0.6	0.4
				Sum	**75.4**
				Mean	**9.4**
				RMSE	3.1

Other Evaluation Methods

Mean squared error	$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n} e_t^2$
Root mean squared error	$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$
Mean absolute error	$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} \lvert e_t \rvert$
Mean absolute percentage error	$\text{MAPE} = \frac{100\%}{n}\sum_{t=1}^{n} \left\lvert \frac{e_t}{y_t} \right\rvert$

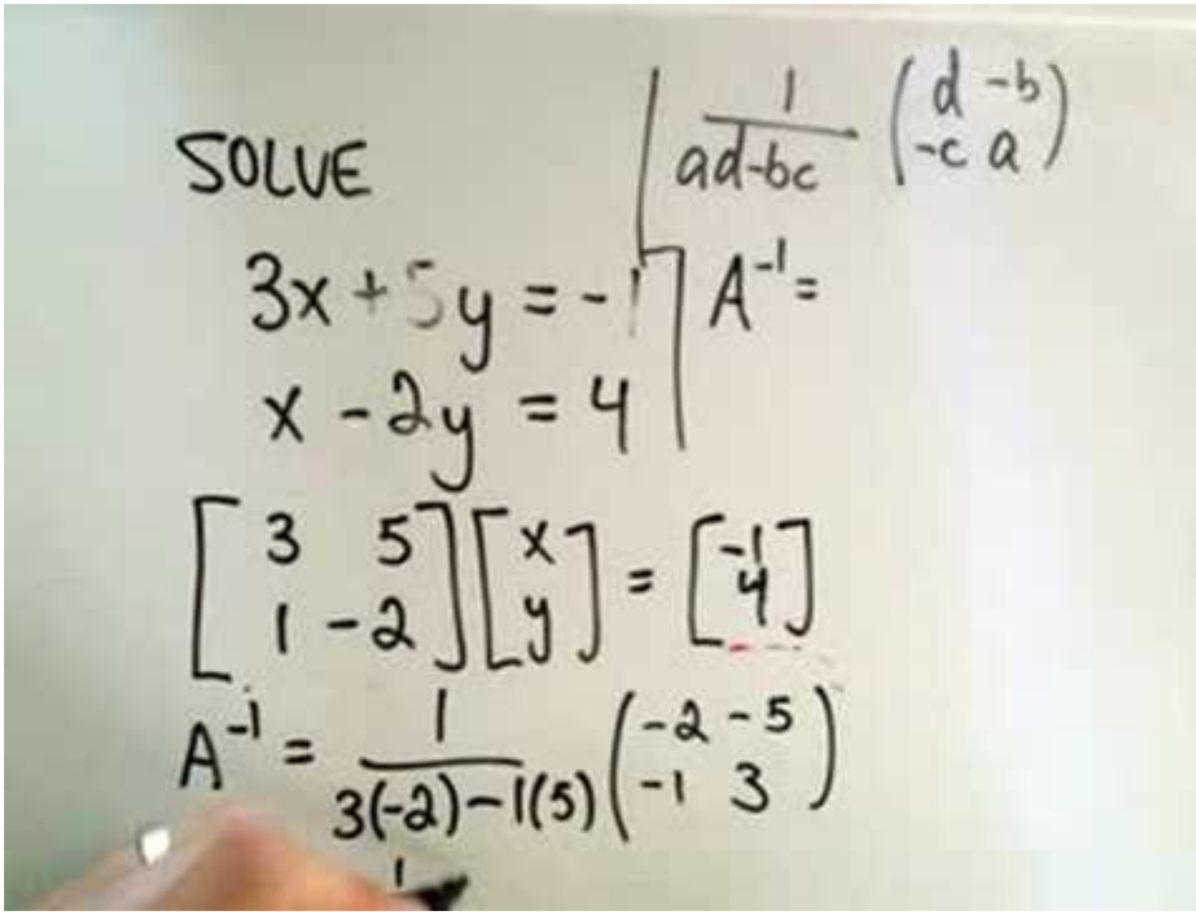
Ref: • (image) https://tex.stackexchange.com/questions/170372/increase-vertical-space-in-table

Python

1. from sklearn import linear_model
2. from sklearn.metrics import mean_squared_error
3. from math import sqrt
4. X = df[“midterm_score”, “hour_study”]
5. y = df[“total_score”]
6. lm = linear_model.LinearRegression()
7. lm.fit (X,y)
8. y_predicted = lm.predict(X)
9. rmse = mean_squared_error(y, y_predicted, squared=False)

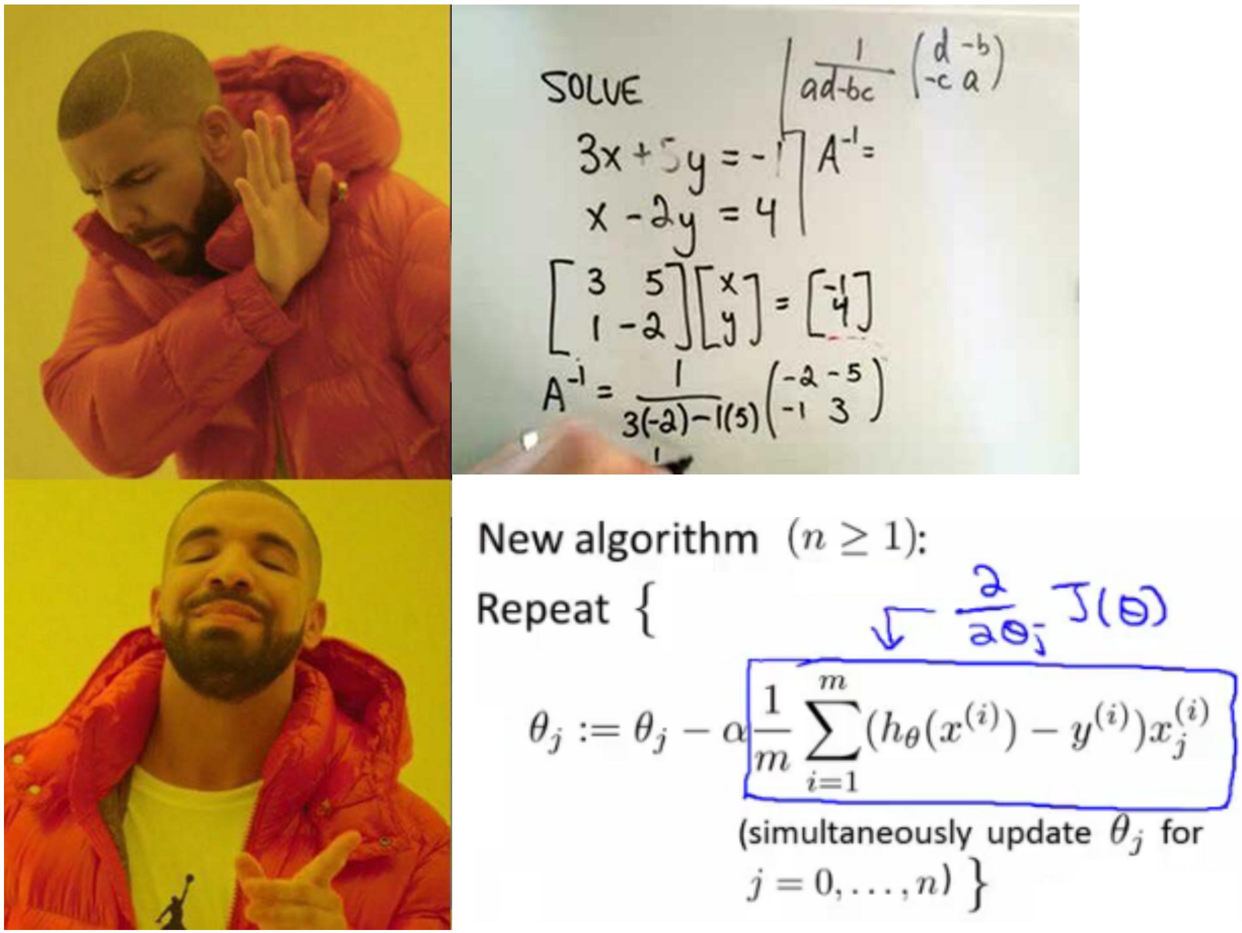
Learning

Solve by Matrix



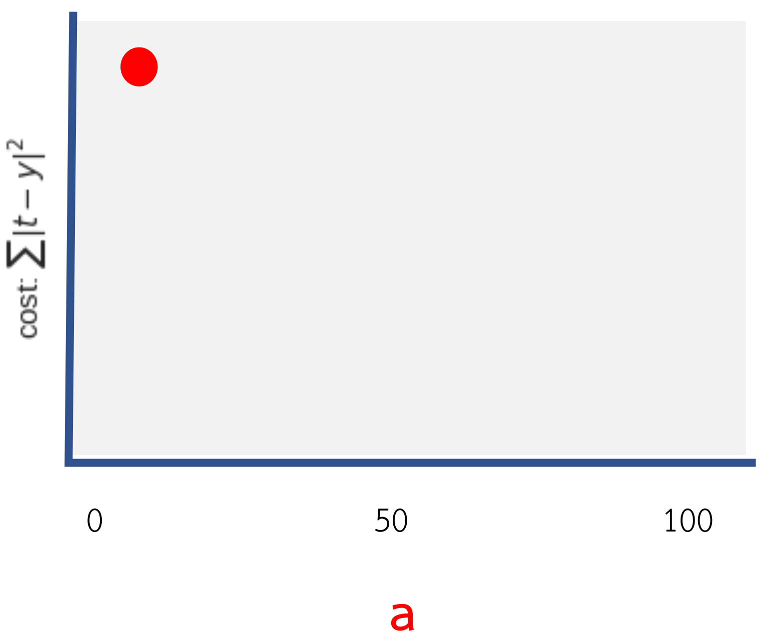
Ref: • https://www.youtube.com/watch?v=PTN8rganLWo

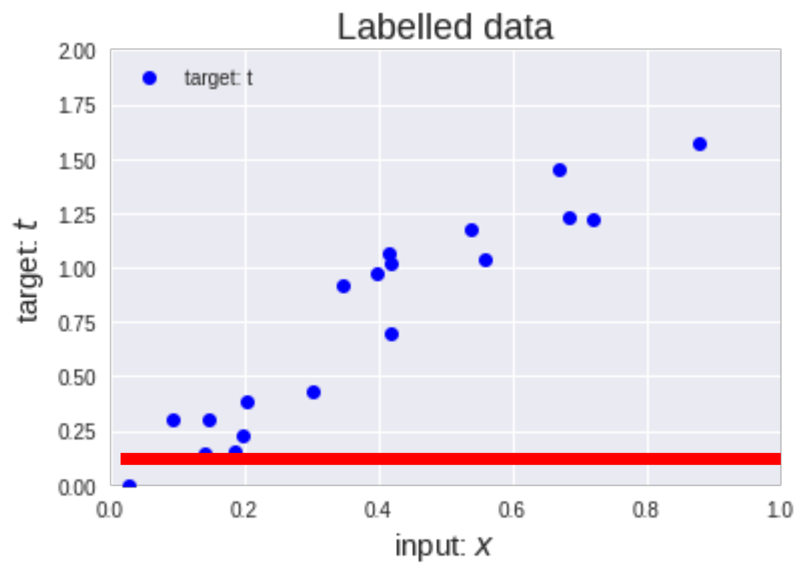
Solve by ………


SOLVE
$3x + 5y = -1$
$x - 2y = 4$
$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
$A^{-1} =$
$\begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$
$A^{-1} = \frac{1}{3(-2)-1(5)}\begin{pmatrix} -2 & -5 \\ -1 & 3 \end{pmatrix}$
New algorithm $(n \geq 1)$:
Repeat {
$\frac{\partial}{\partial \theta_j} J(\theta)$
$\theta_j := \theta_j - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}}$
(simultaneously update θ_j for $j = 0, \ldots, n$) }

Learning based on Gradient Descent

$y = ax + c$

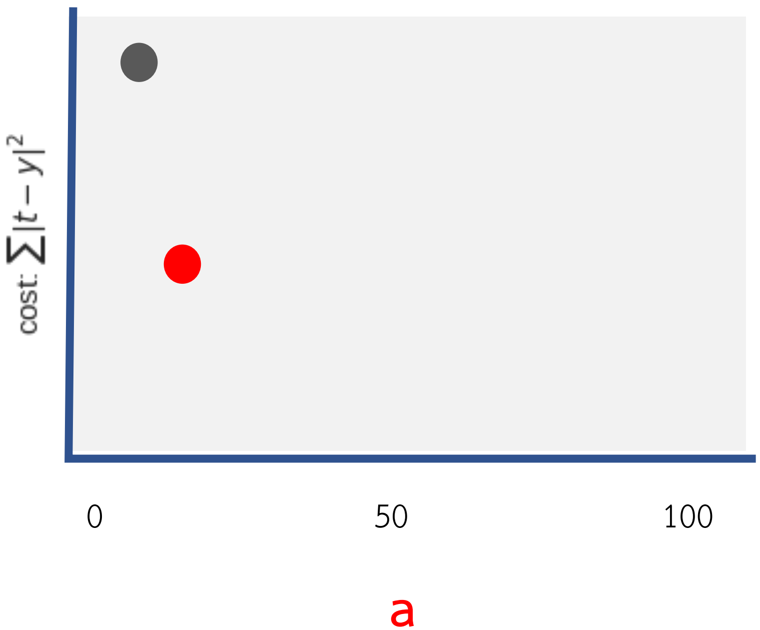


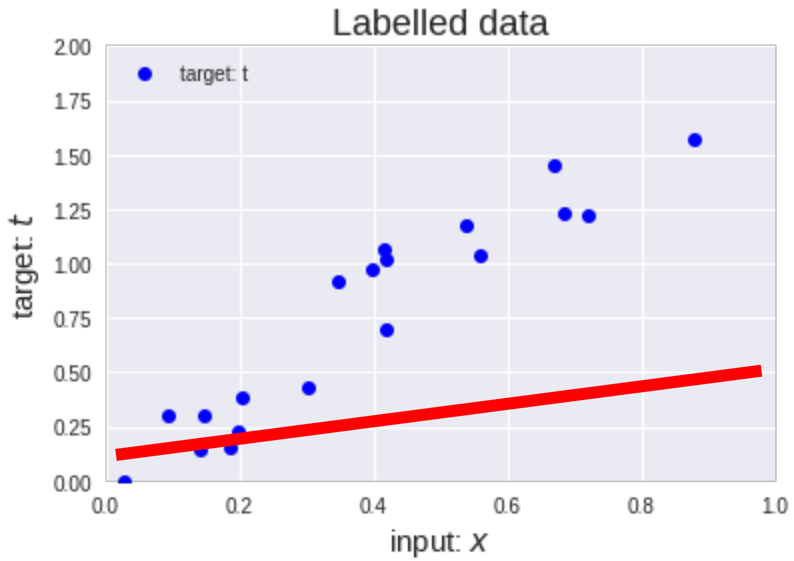


Ref: • (image) https://medium.com/onfido-tech/machine-learning-101-be2e0a86c96a

Learning

$y = ax + c$





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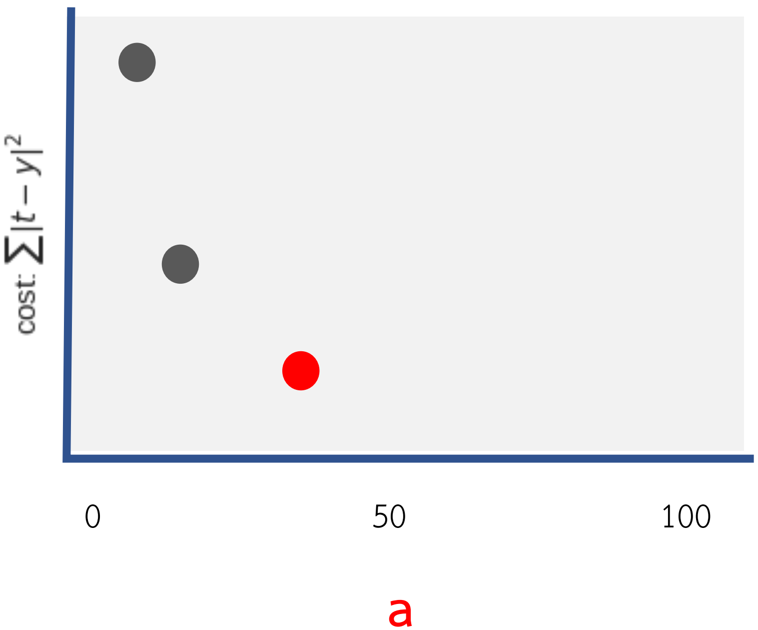

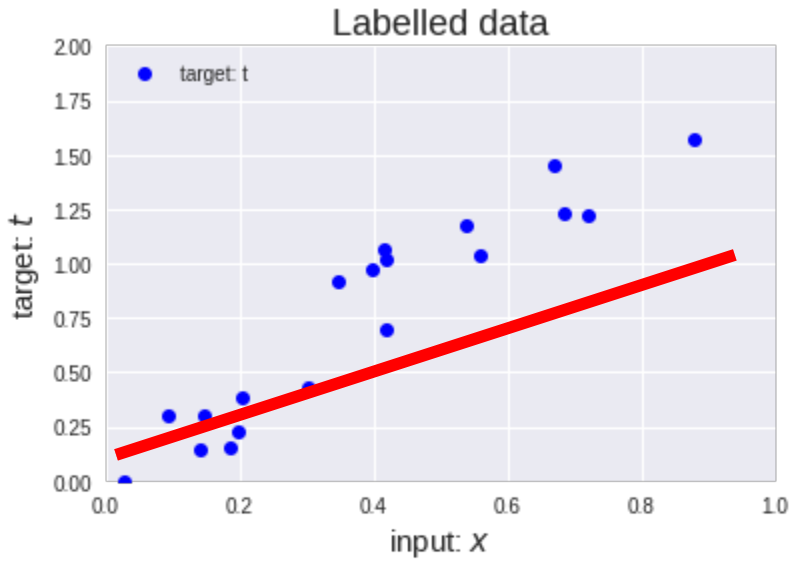

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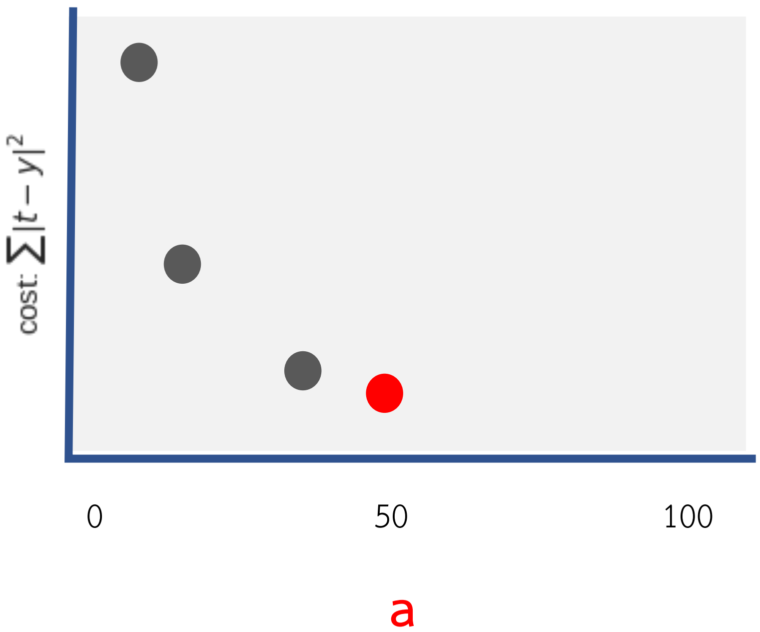

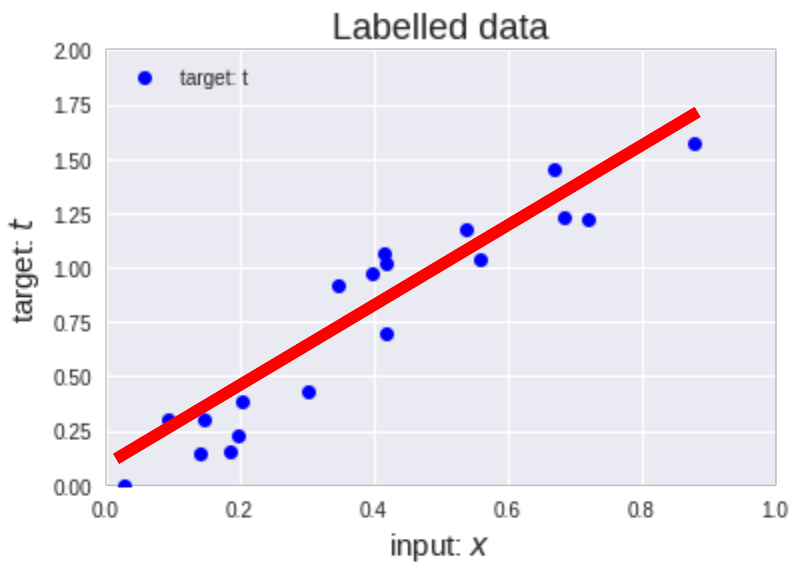

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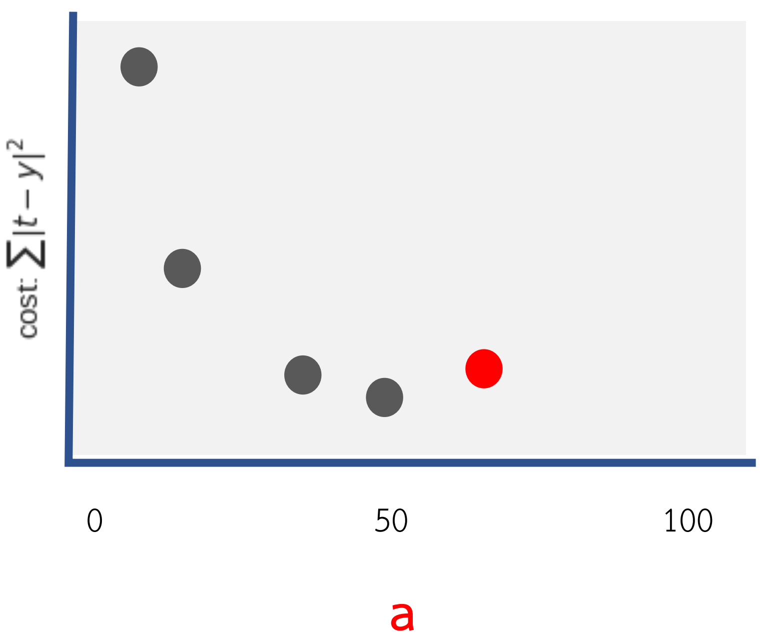

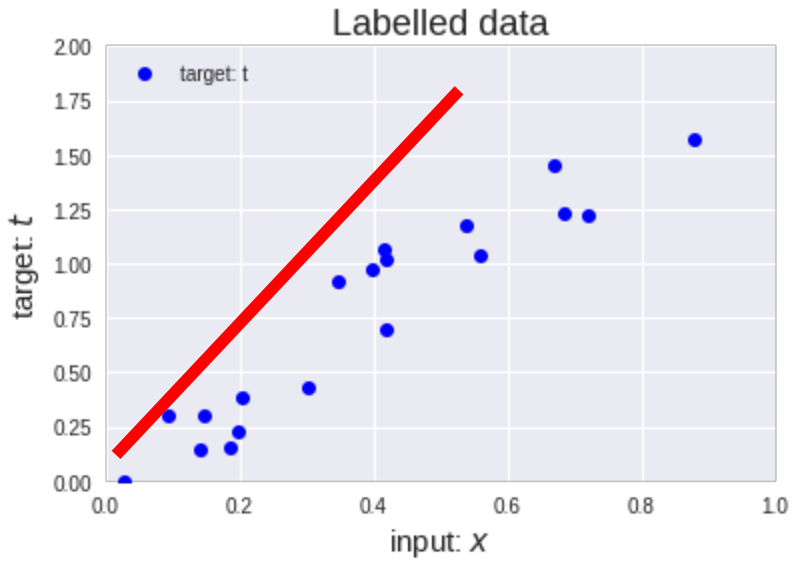

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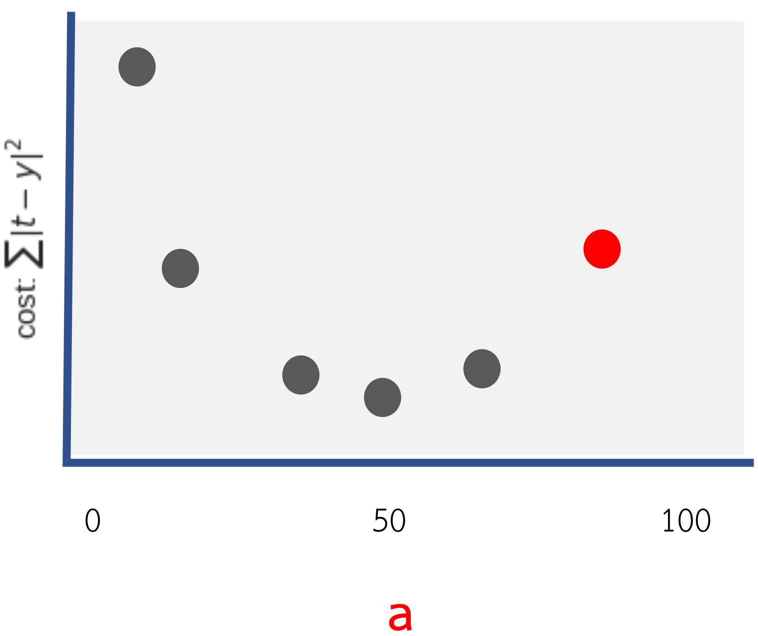

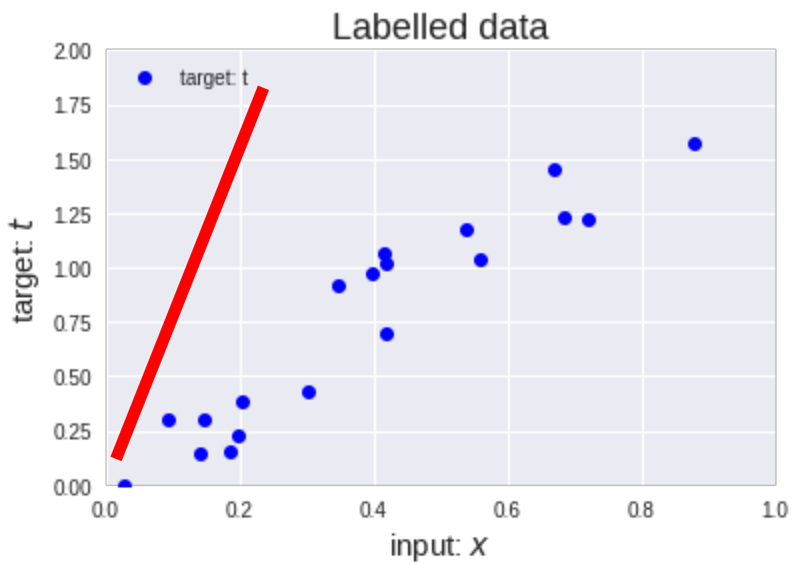

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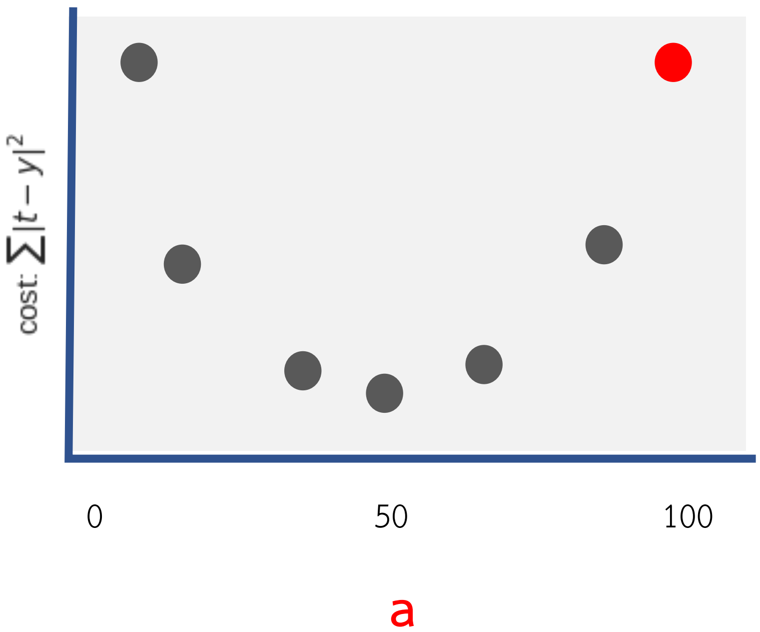
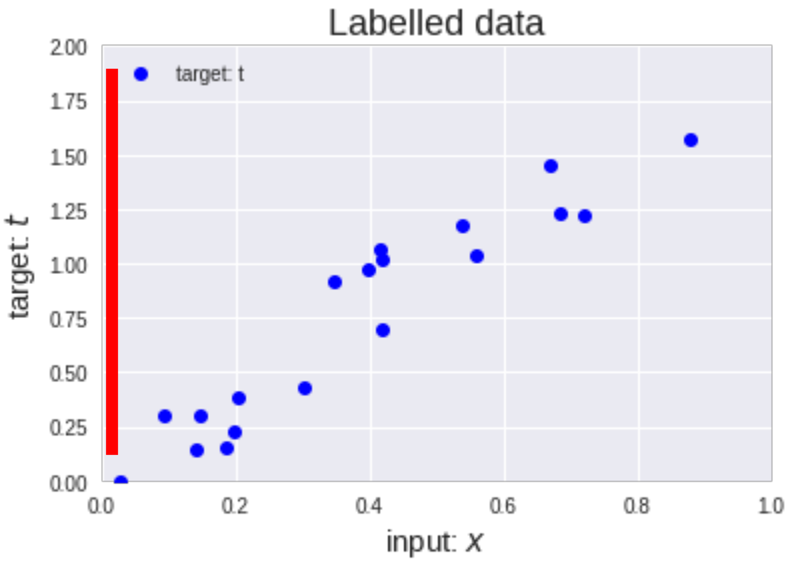
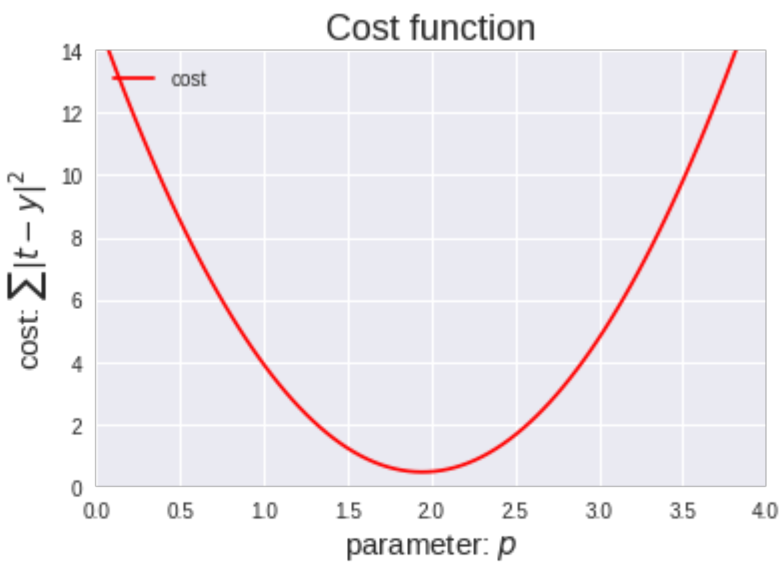
Learning

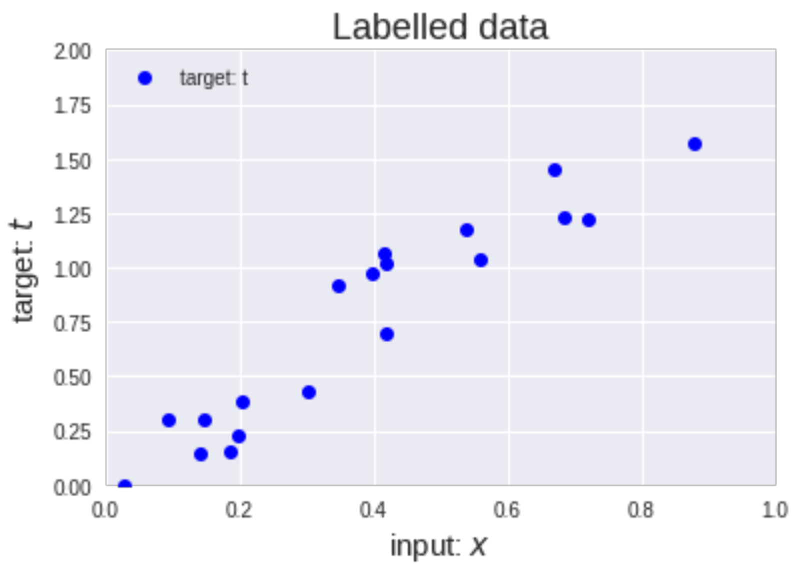
$$y = ax + c$$





Learning





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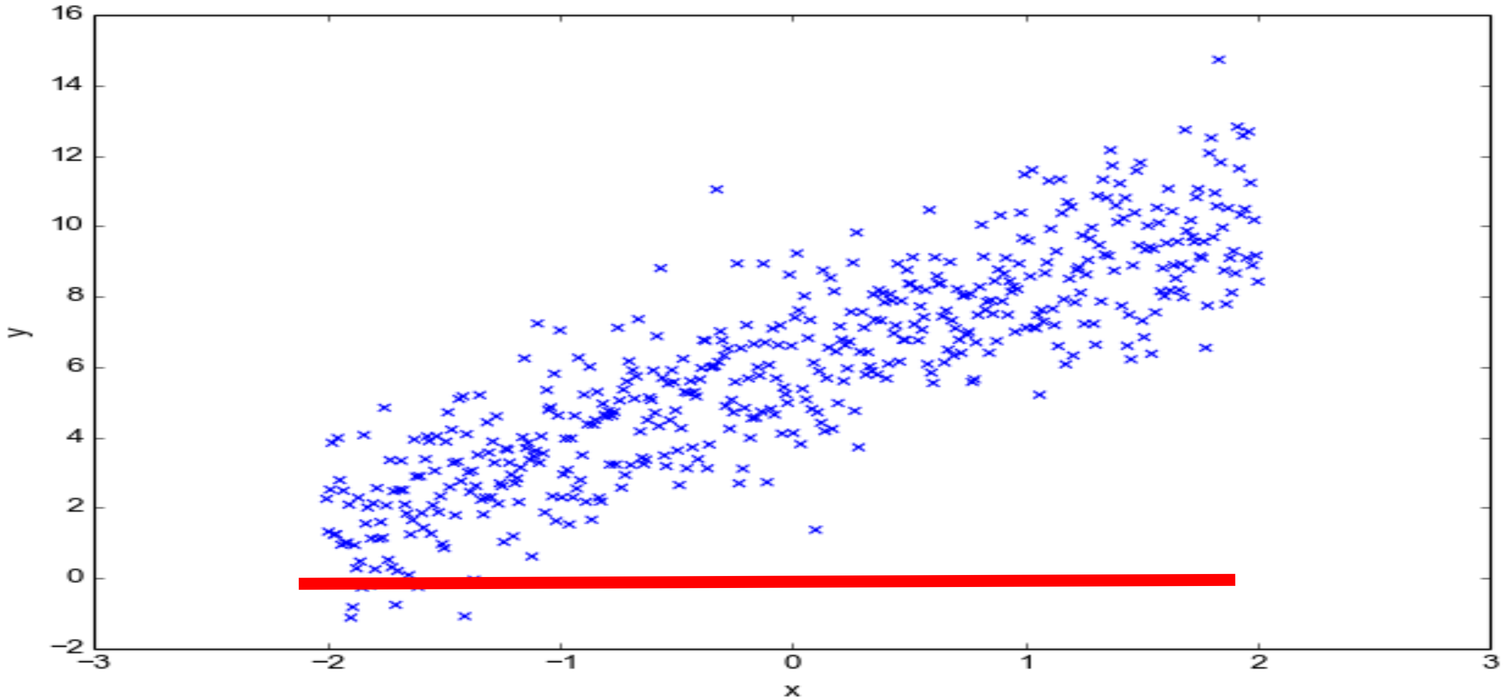
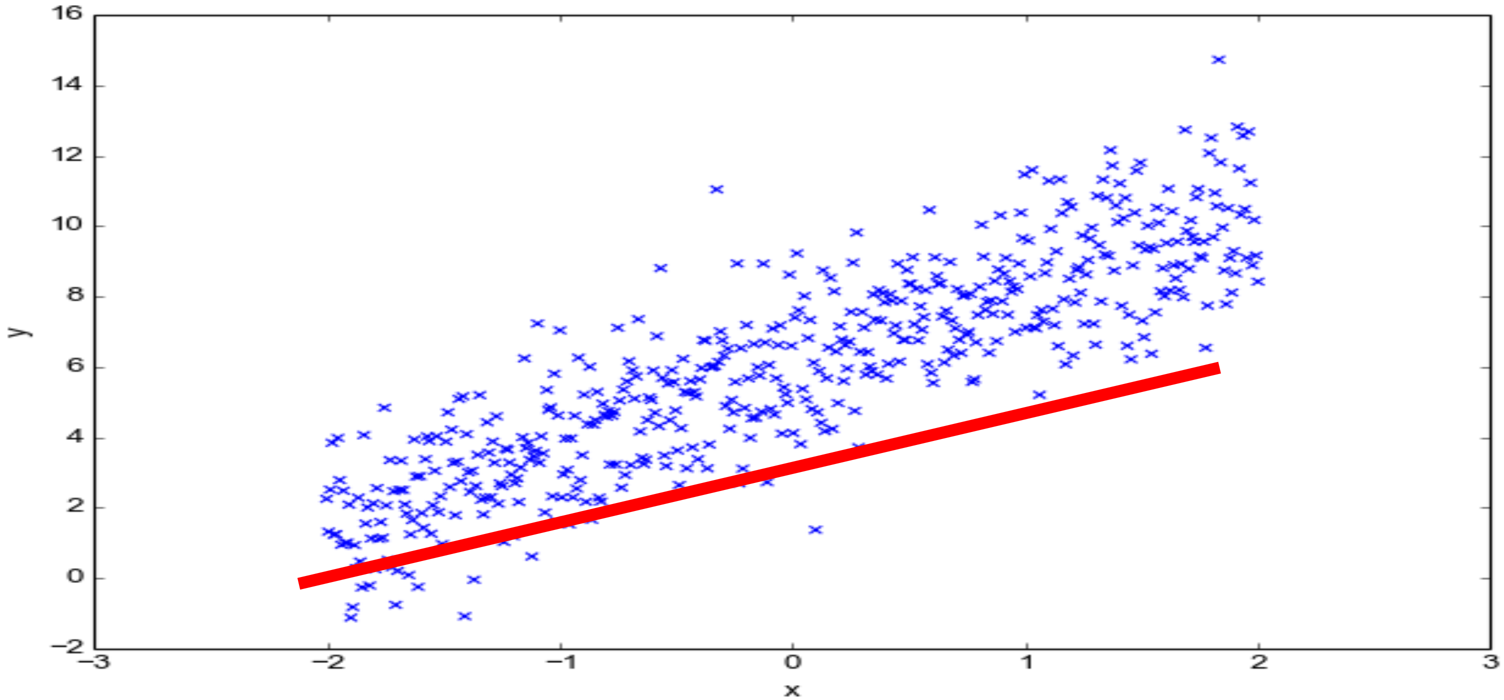


n-th round

Learning



n-th round

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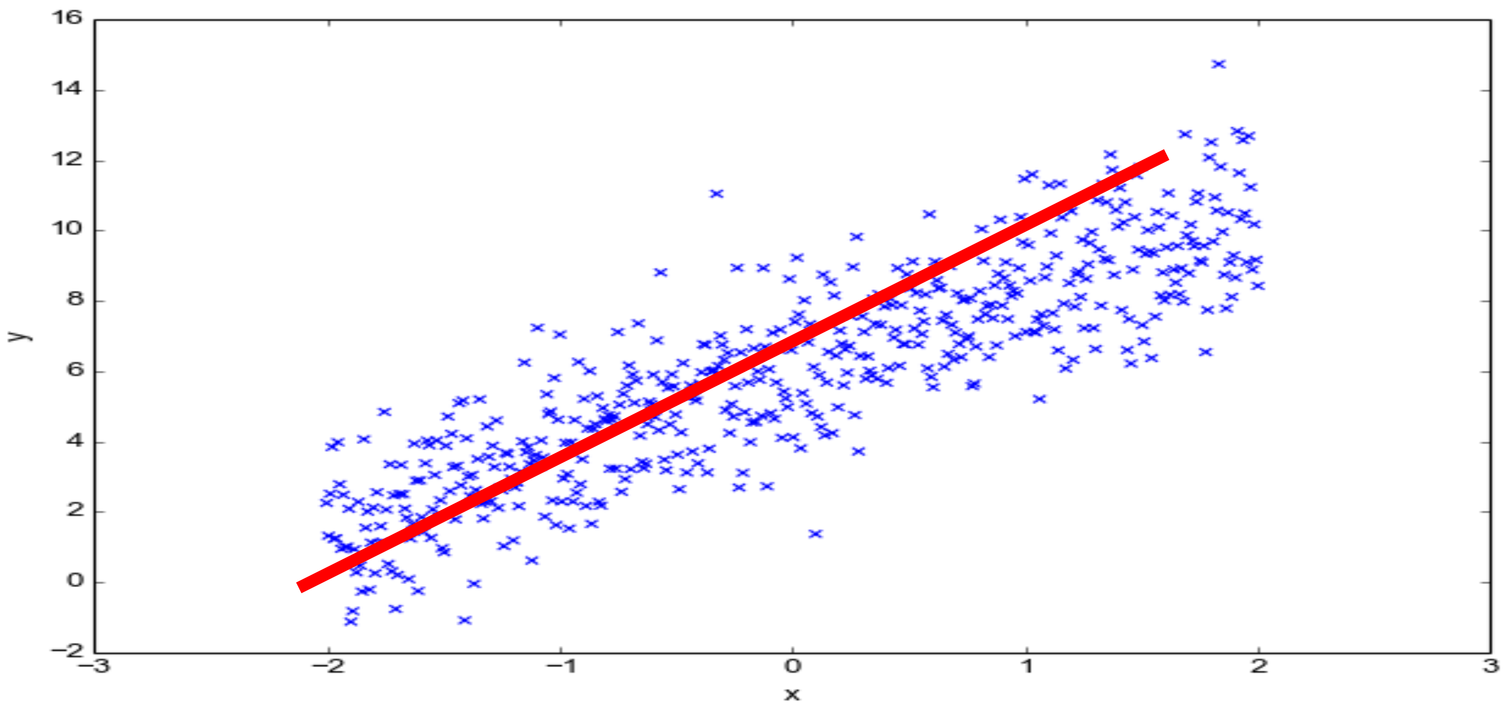
Learning



n-th round

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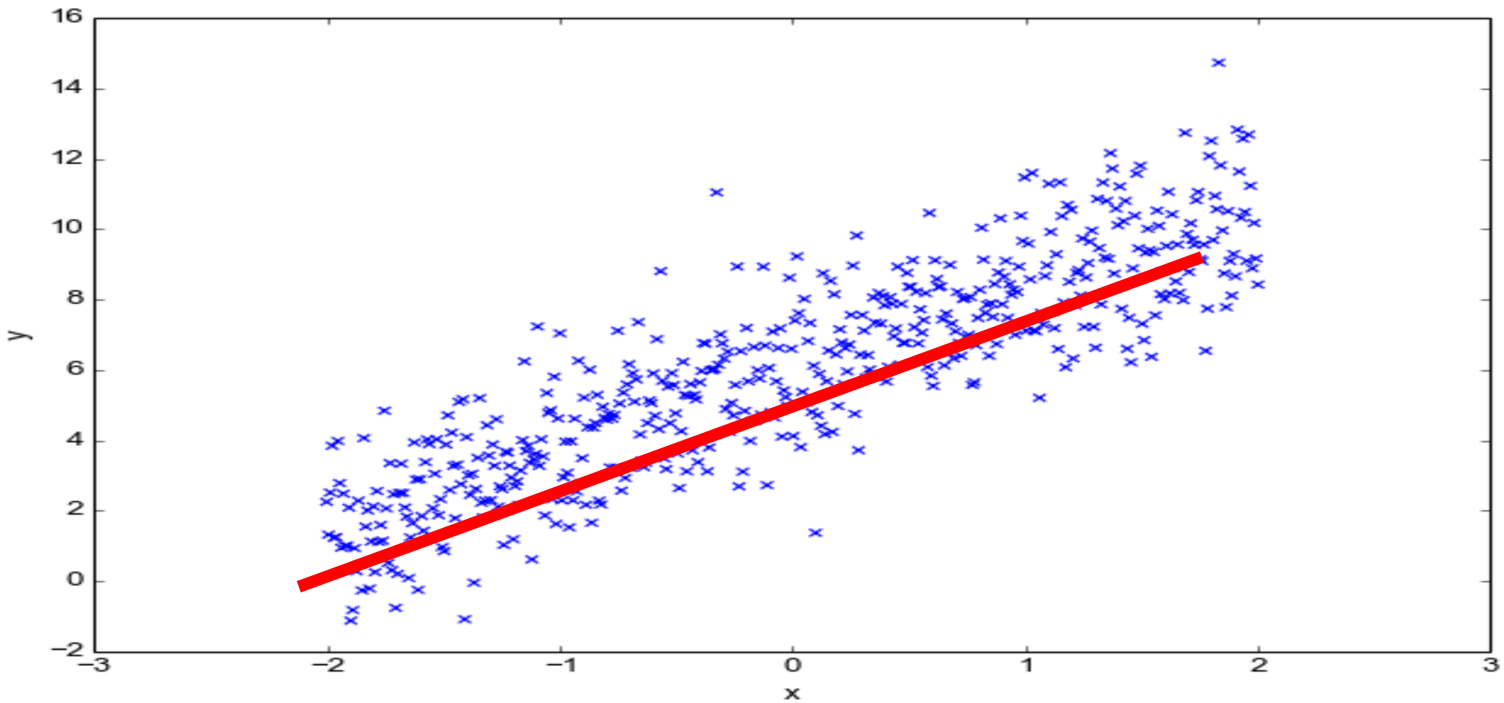
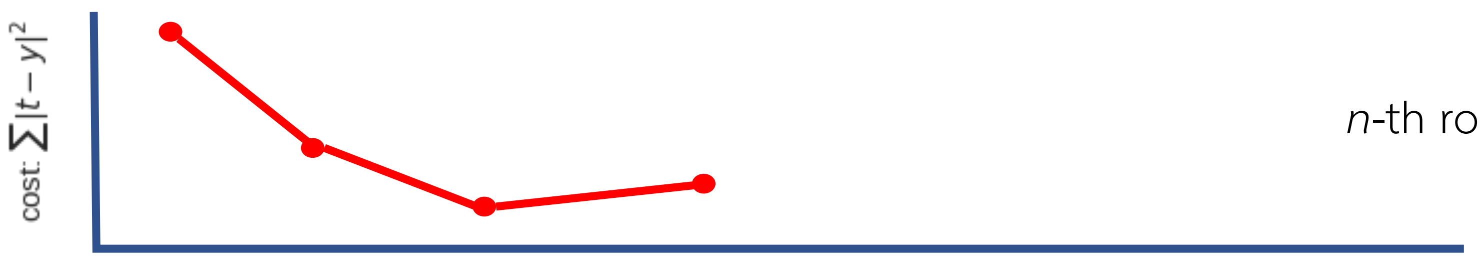
Learning





n-th round

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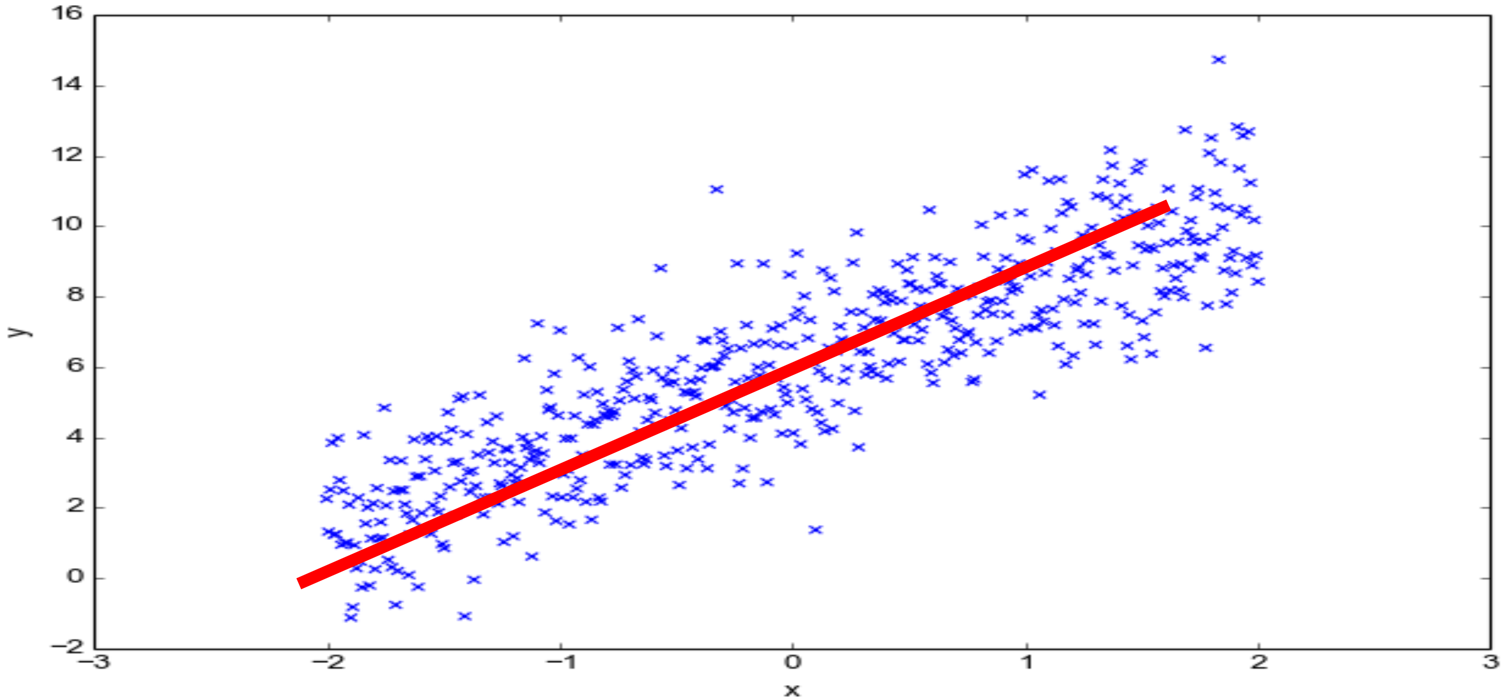
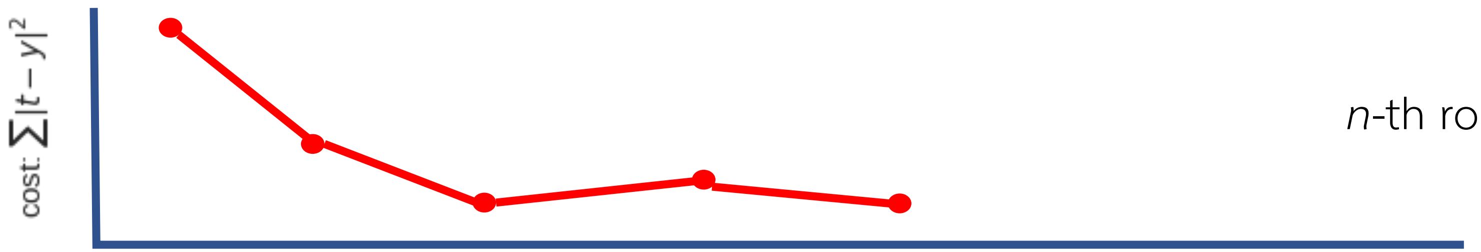
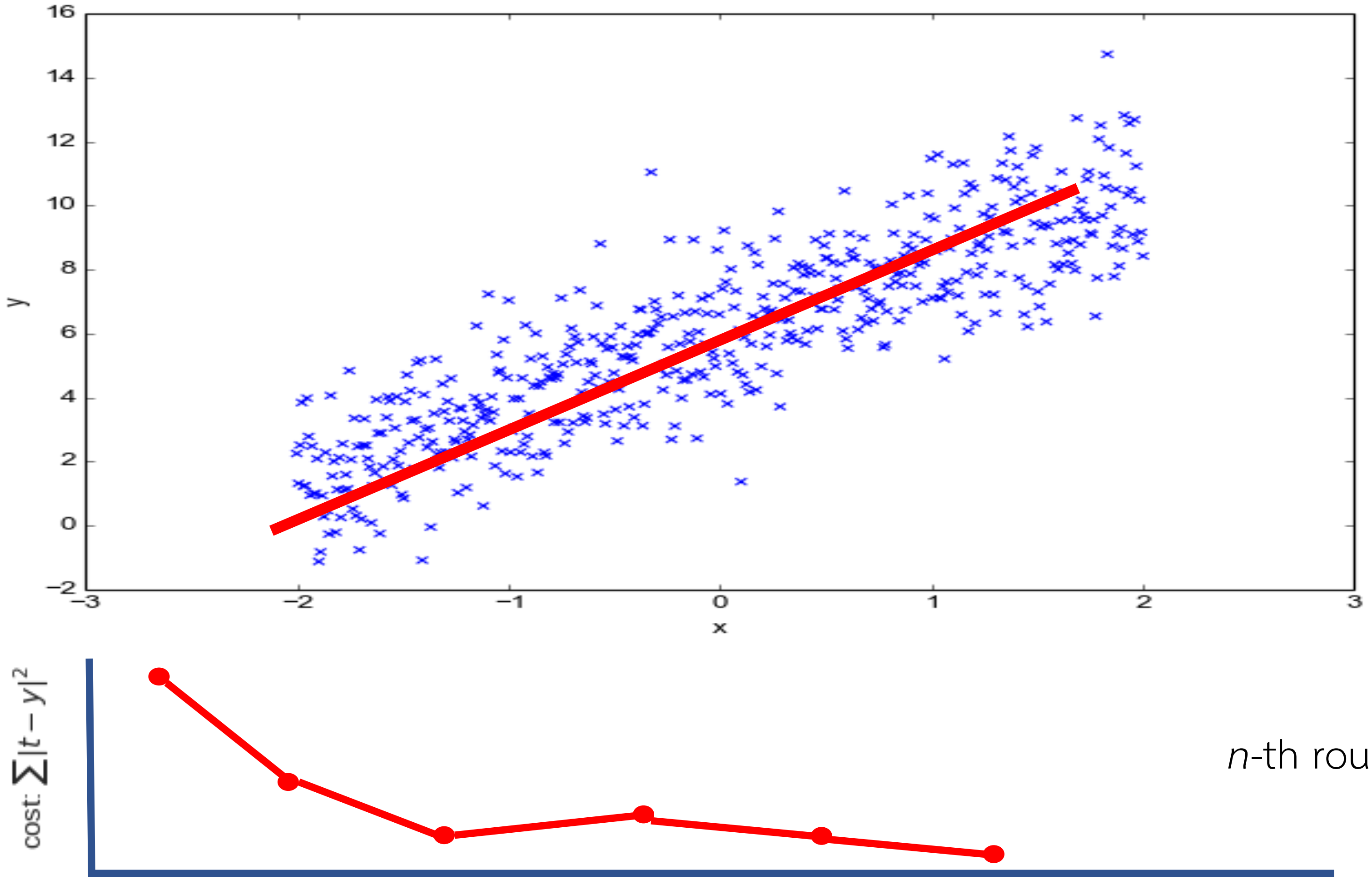
Learning





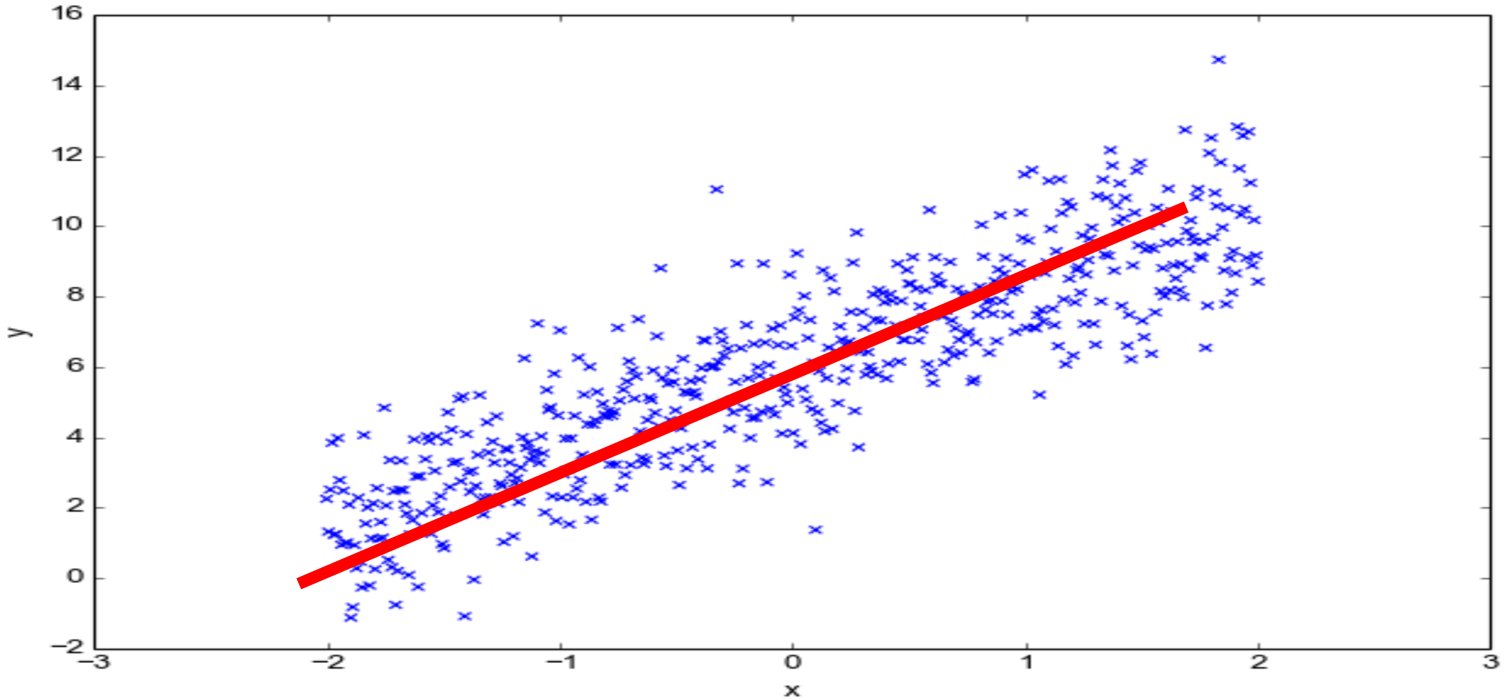
n-th round

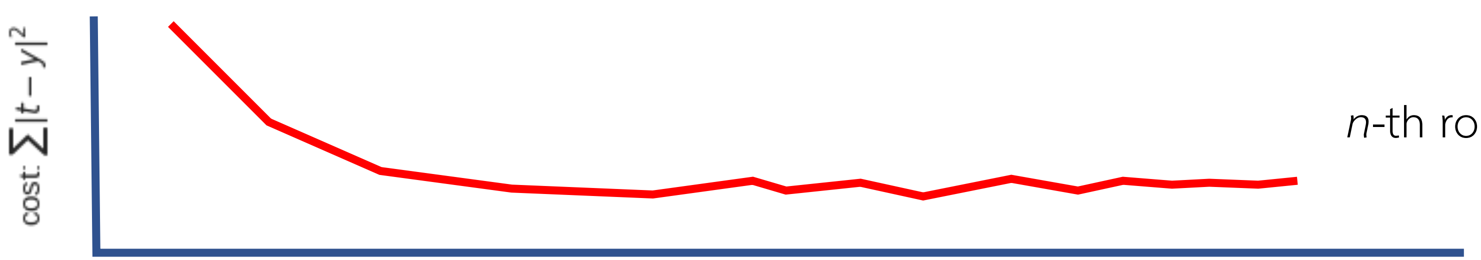
Learning



n-th round

Learning





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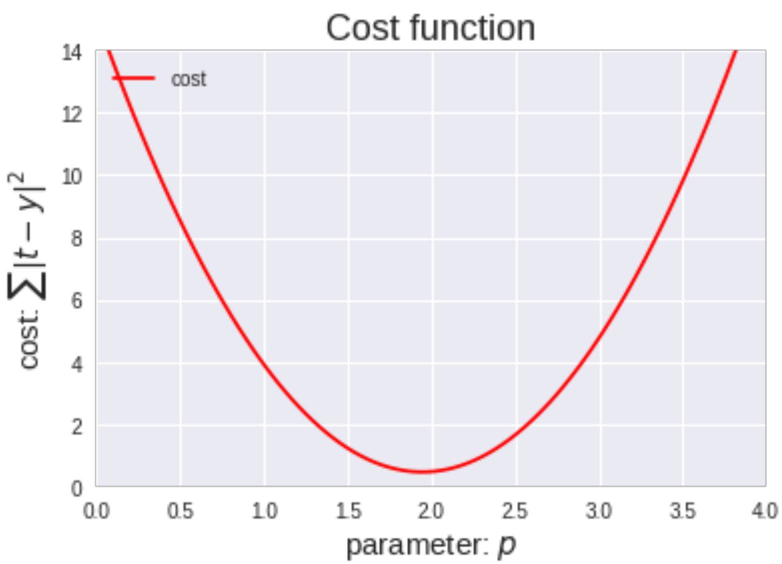

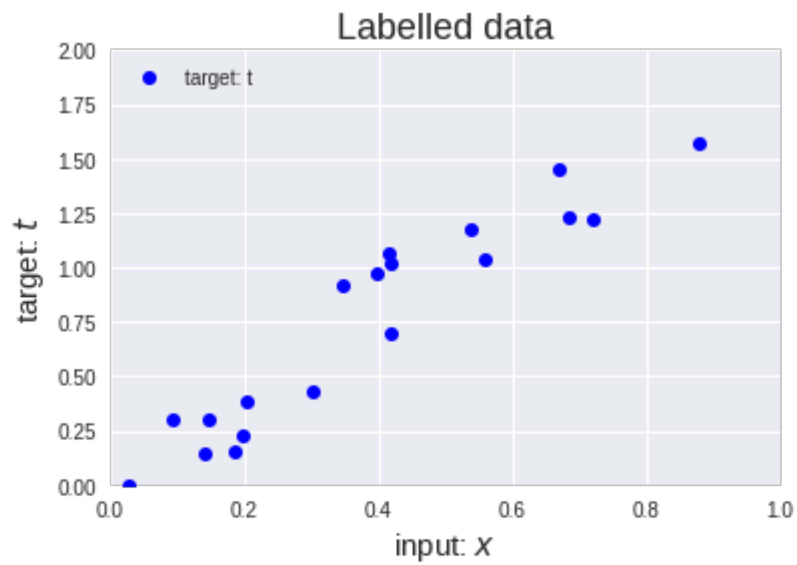

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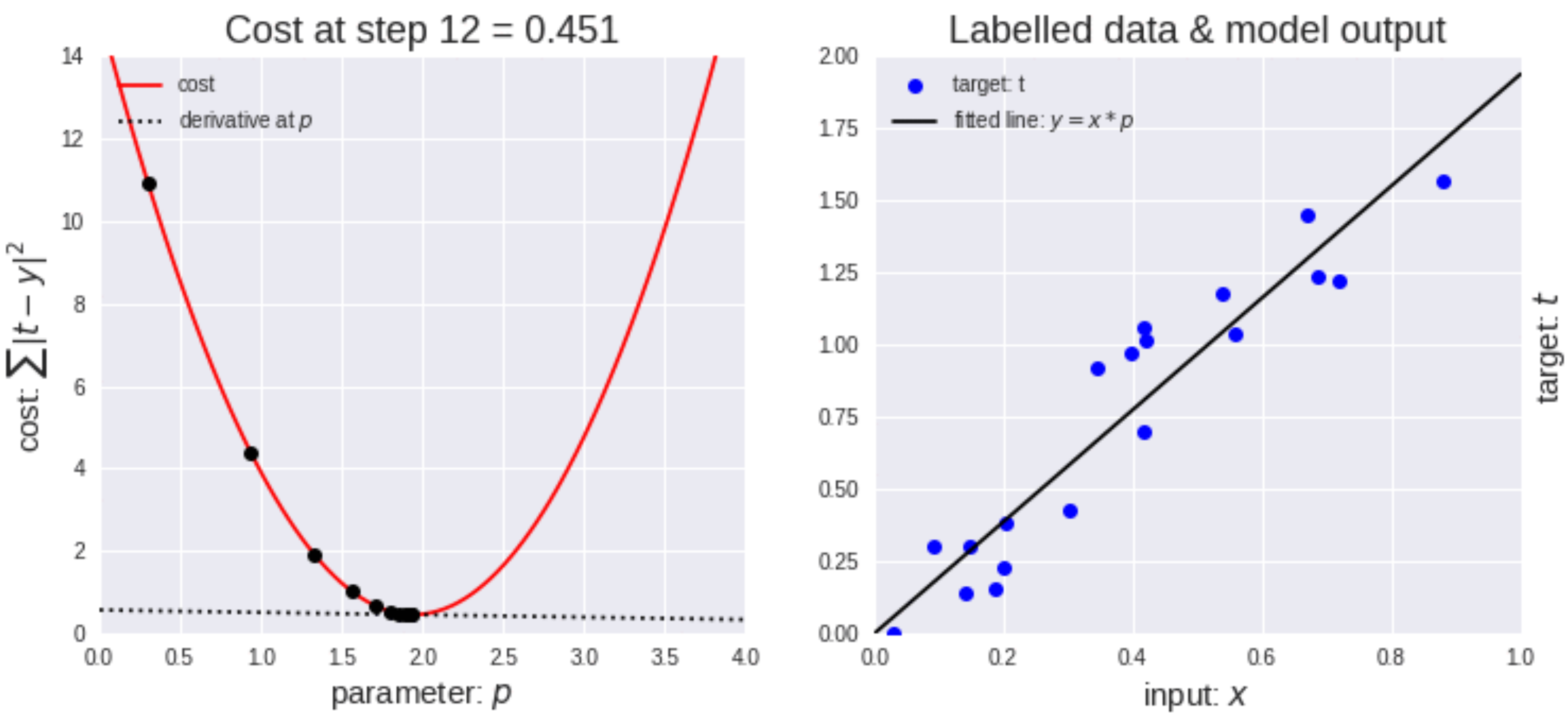

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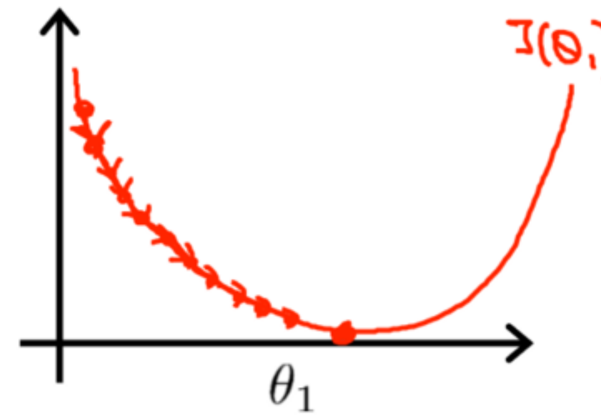
Gradient Descent for Linear Regression

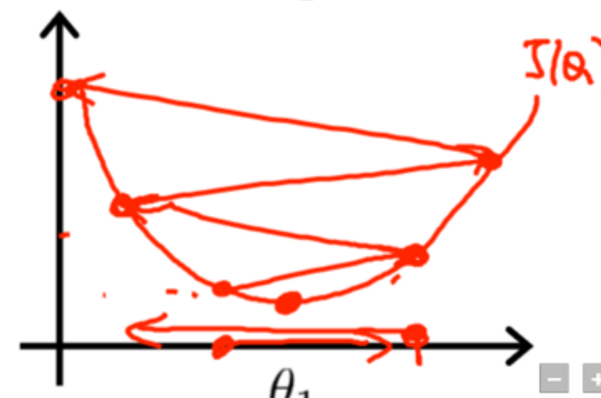
Andrew Ng

$$\boxed{\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)}$$

If α is too small, gradient descent can be slow.

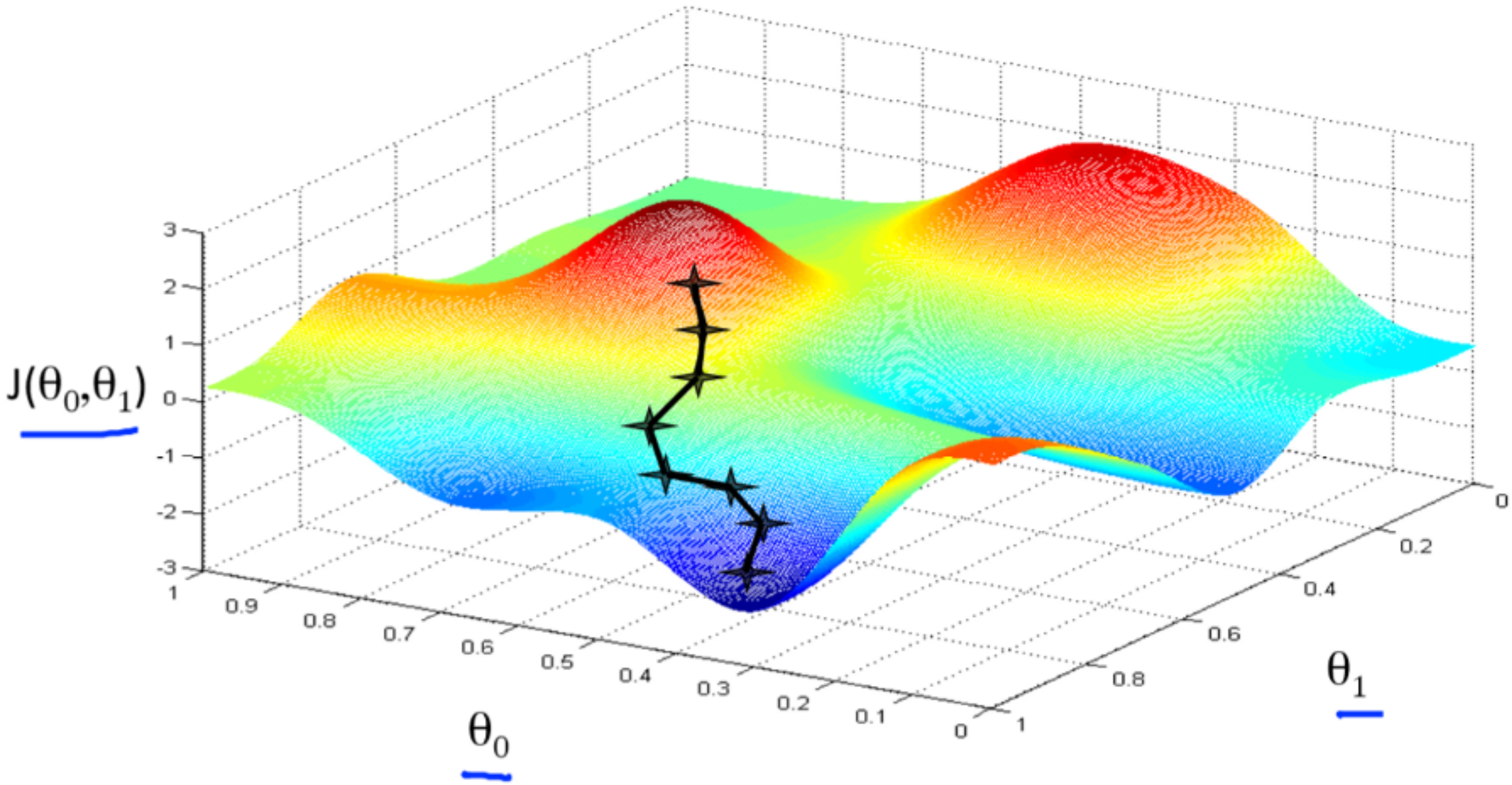


If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



Ref: • screenshot from Andrew Ng video tutor

Gradient Descent for Linear Regression



Feature Selection

Boston Housing

CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV	CAT. MEDV
0.00632	18	2.31	0	0.538	6.575	65.2	4.09	1	296	15.3	396.9	4.98	24	0
0.02731	0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.9	9.14	21.6	0
0.02729	0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7	1
0.03237	0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4	1
0.06905	0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.9	5.33	36.2	1
0.02985	0	2.18	0	0.458	6.43	58.7	6.0622	3	222	18.7	394.12	5.21	28.7	0
0.08829	12.5	7.87	0	0.524	6.012	66.6	5.5605	5	311	15.2	395.6	12.43	22.9	0
0.14455	12.5	7.87	0	0.524	6.172	96.1	5.9505	5	311	15.2	396.9	19.15	27.1	0
0.21124	12.5	7.87	0	0.524	5.631	100	6.0821	5	311	15.2	386.63	29.93	16.5	0
0.17004	12.5	7.87	0	0.524	6.004	85.9	6.5921	5	311	15.2	386.71	17.1	18.9	0
0.22489	12.5	7.87	0	0.524	6.377	94.3	6.3467	5	311	15.2	392.52	20.45	15	0
0.11747	12.5	7.87	0	0.524	6.009	82.9	6.2267	5	311	15.2	396.9	13.27	18.9	0
0.09378	12.5	7.87	0	0.524	5.889	39	5.4509	5	311	15.2	390.5	15.71	21.7	0
0.62976	0	8.14	0	0.538	5.949	61.8	4.7075	4	307	21	396.9	8.26	20.4	0
0.63796	0	8.14	0	0.538	6.096	84.5	4.4619	4	307	21	380.02	10.26	18.2	0

Ref: • https://www.solver.com/multiple-linear-regression-example

Boston Housing

CRIM	Per capita crime rate by town
ZN	Proportion of residential land zoned for lots over 25,000 sq.ft.
INDUS	Proportion of non-retail business acres per town
CHAS	Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
NOX	Nitric oxides concentration (parts per 10 million)
RM	Average number of rooms per dwelling
AGE	Proportion of owner-occupied units built prior to 1940
DIS	Weighted distances to five Boston employment centers
RAD	Index of accessibility to radial highways
TAX	Full-value property-tax rate per $10,000
PTRATIO	Pupil-teacher ratio by town
B	1000(Bk - 0.63)^2 where Bk is the proportion of African-Americans by town
LSTAT	% Lower status of the population
MEDV	Median value of owner-occupied homes in $1000's

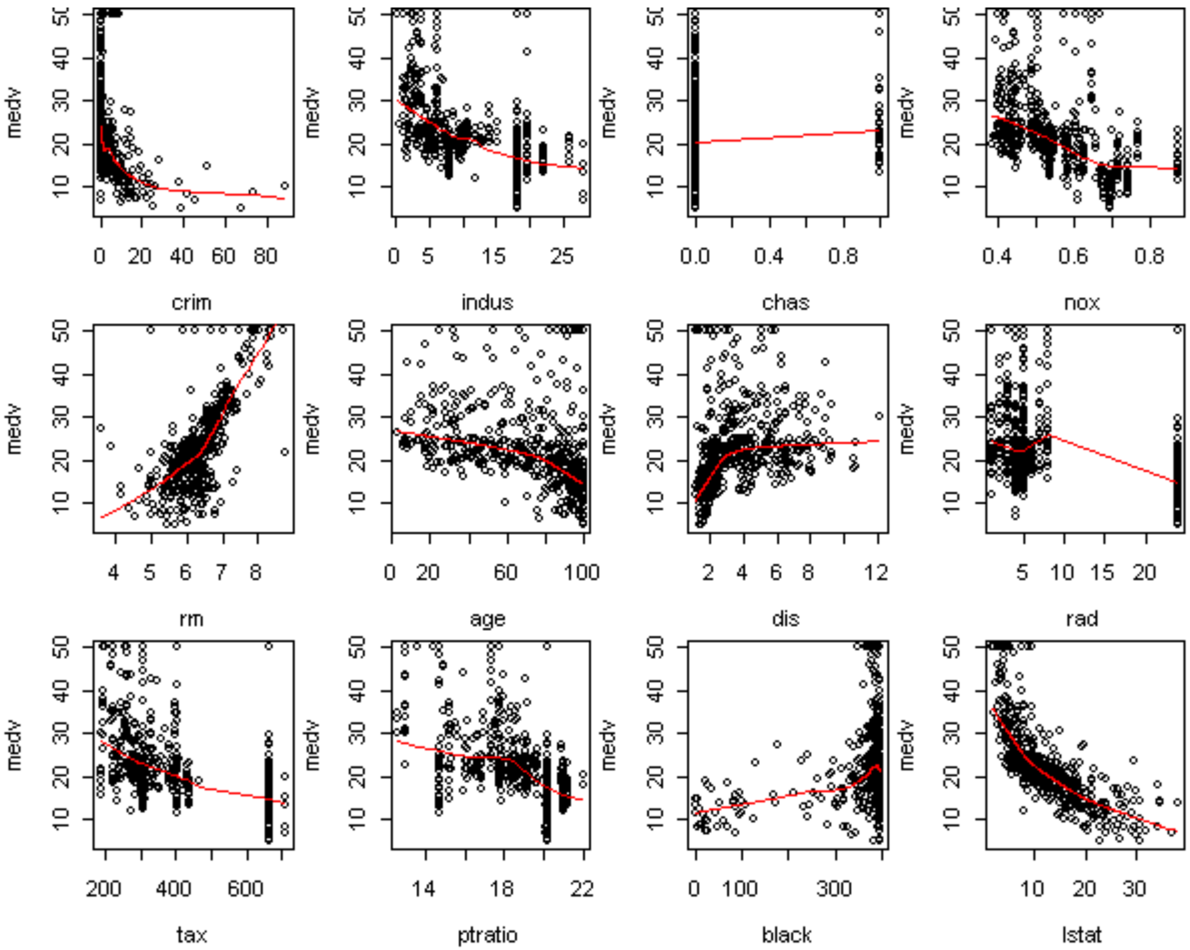
X

y

Ref: • https://www.solver.com/multiple-linear-regression-example

Correlations

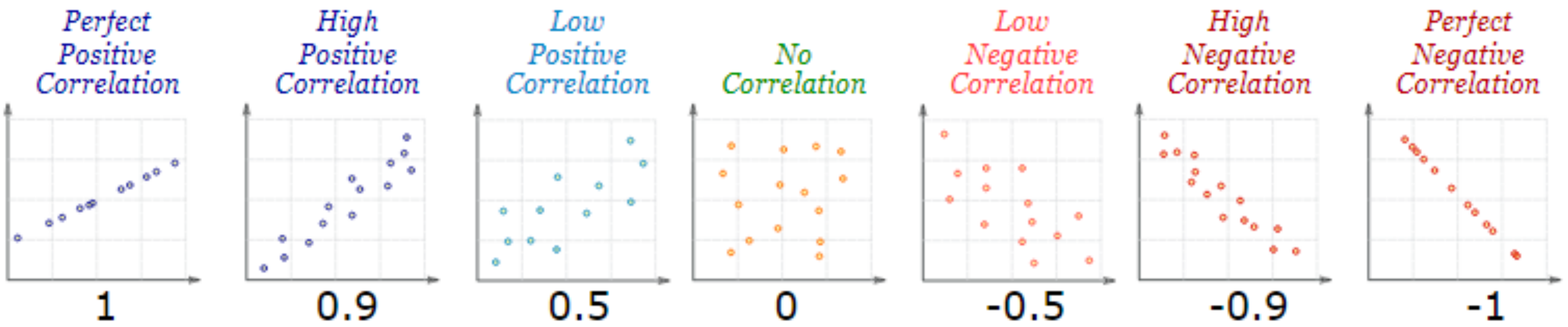
- Content



Ref: • (image) http://alumni.media.mit.edu/~tpminka/courses/36-350.2001/lectures/day30/

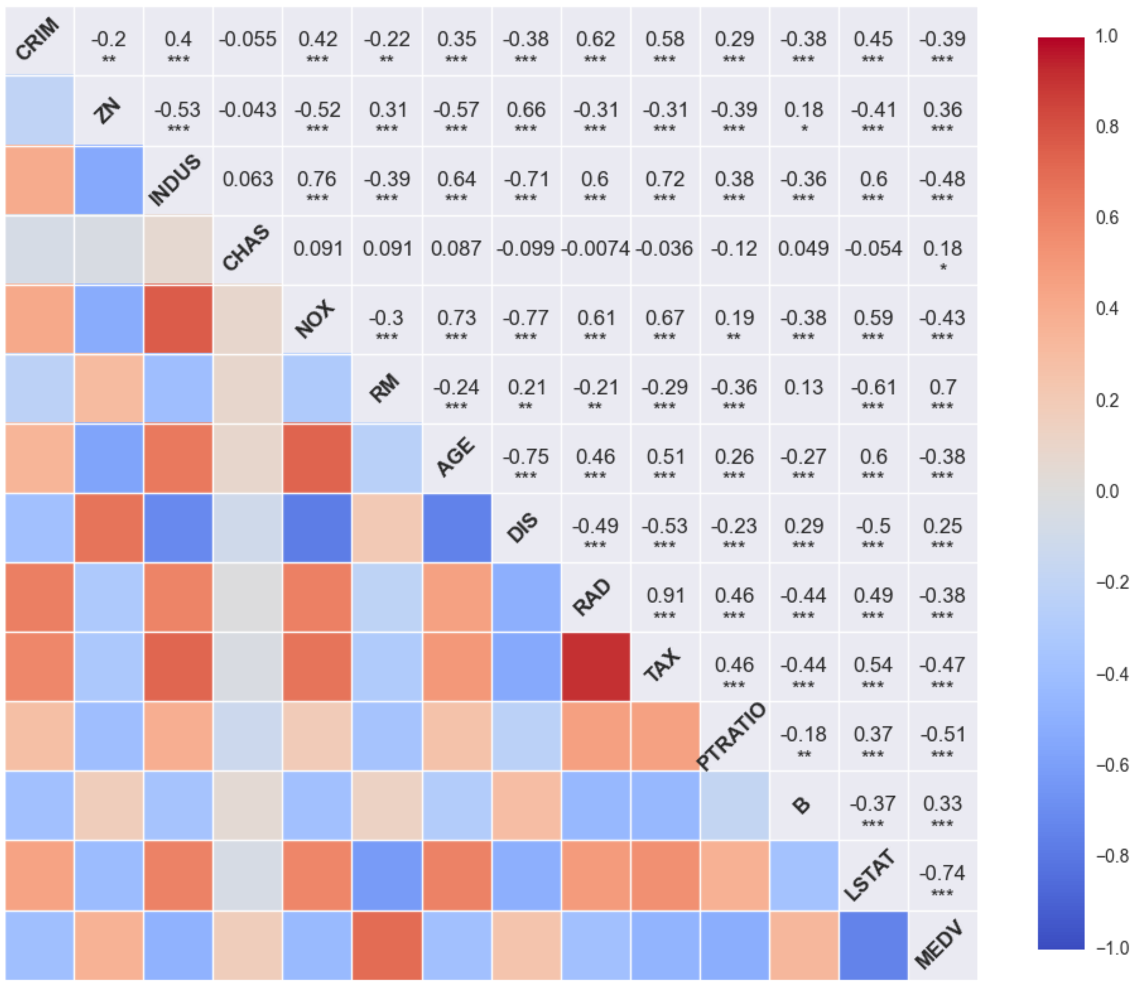
Pearson Correlation

- When two sets of data are strongly linked together, we say they have a High Correlation.
  - Correlation is Positive when the values increase together, and
  - Correlation is Negative when one value decreases as the other increases



Ref: • https://www.mathsisfun.com/data/correlation.html

Pearson Correlations



Ref: • (image) http://www.neural.cz/dataset-exploration-boston-house-pricing.html

Cross Validation

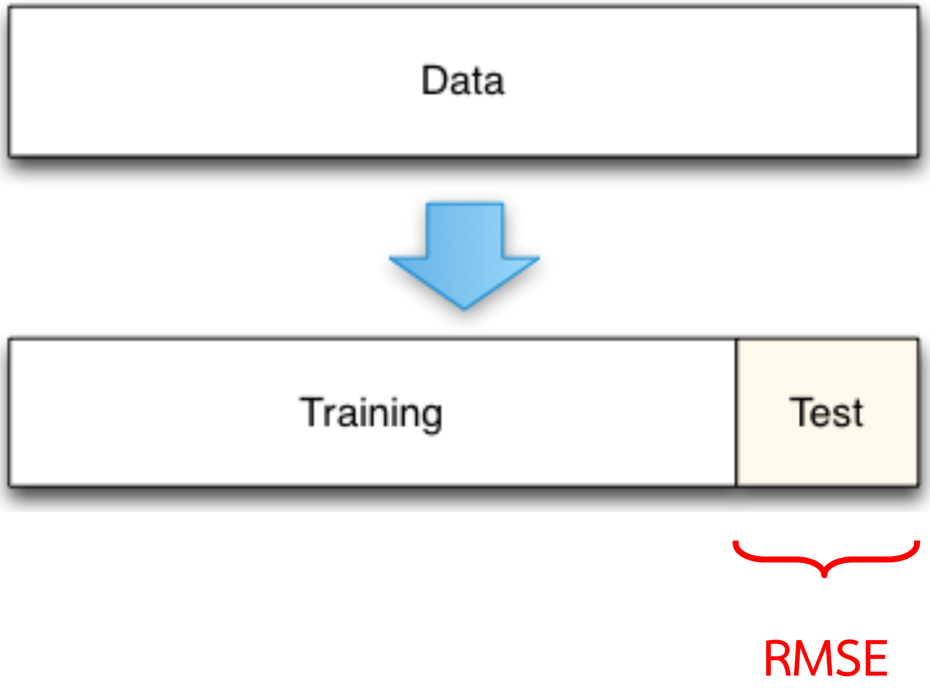
Rotation Estimation

How accurately a predictive model will perform in practice?

- Cross-validation is a technique to evaluate predictive models by partitioning the original sample into a training set to train the model, and a test set to evaluate it.
- In k-fold cross-validation, the original sample is randomly partitioned into k equal size subsamples. Of the k subsamples, a single subsample is retained as the validation data for testing the model, and the remaining k-1 subsamples are used as training data. The cross-validation process is then repeated k times (the folds), with each of the k subsamples used exactly once as the validation data. The k results from the folds can then be averaged (or otherwise combined) to produce a single estimation. The advantage of this method is that all observations are used for both training and validation, and each observation is used for validation exactly once.

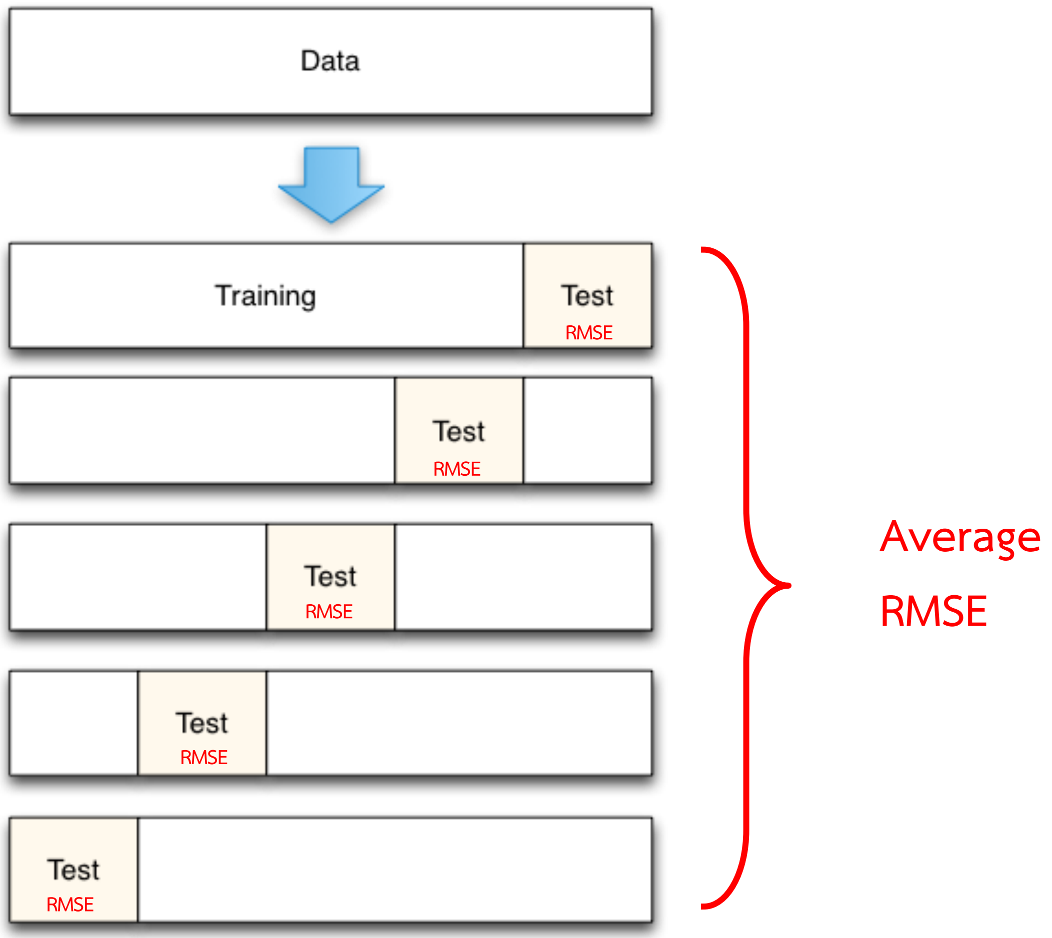
Ref: • (content) https://www.openml.org/a/estimation-procedures/1

Train-Test Split



Ref: • (image) http://blog.kaggle.com/2015/06/29/scikit-learn-video-7-optimizing-your-model-with-cross-validation/

K-Fold Cross-Validation



Ref: • (image) http://blog.kaggle.com/2015/06/29/scikit-learn-video-7-optimizing-your-model-with-cross-validation/

Python Code

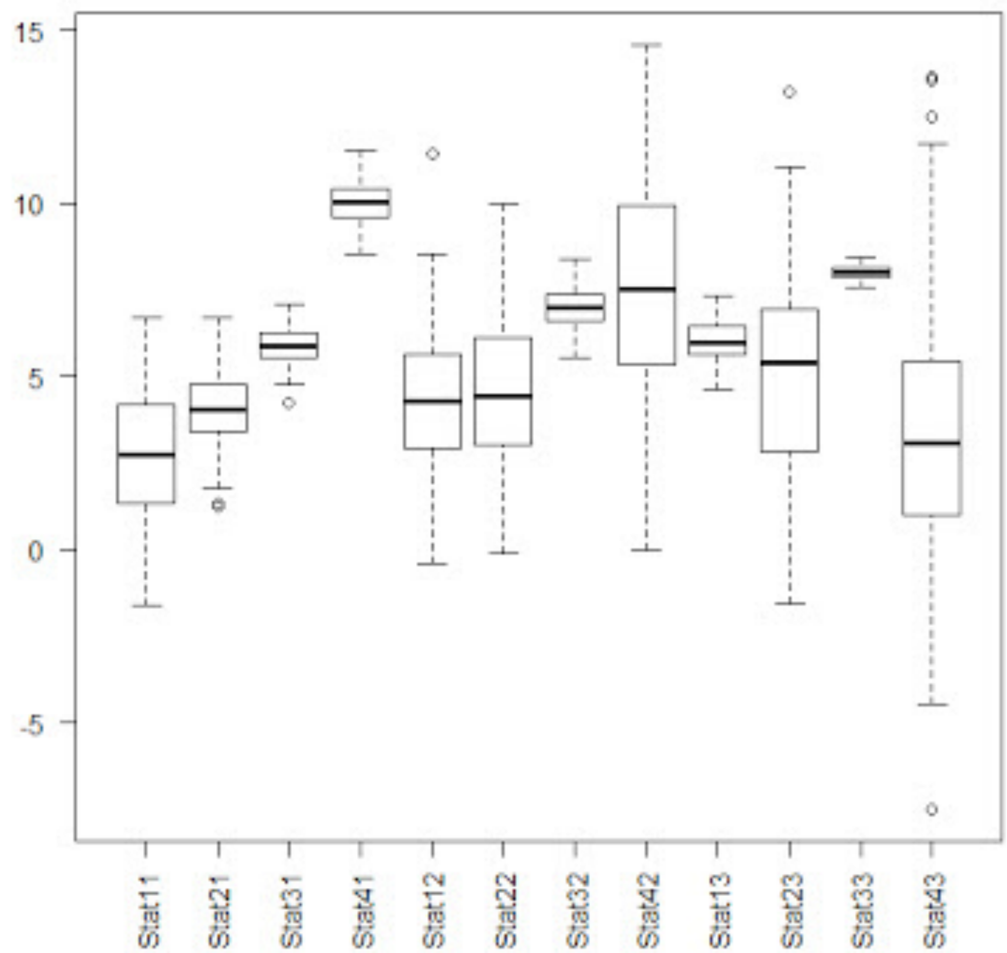
```
1.  from sklearn import neighbors, datasets, preprocessing
2.  from sklearn.model_selection import train_test_split
3.  iris = datasets.load_iris()
4.  X, y = iris.data[:, :2], iris.target
5.  X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=33)
6.  lm = linear_model.LinearRegression()
7.  lm.fit (X_train, y_train)
8.  y_predicted = lm.predict(X_test)
9.  rmse = mean_squared_error(y_test, y_predicted, squared=False)
```

Ref: • https://www.datacamp.com/community/blog/scikit-learn-cheat-sheet

Feature Scaling

Feature Scaling

- Feature scaling is a method used to standardize the range of independent variables or features of data. In data processing, it is also known as data normalization and is generally performed during the data preprocessing step.



Ref:
- (content) https://en.wikipedia.org/wiki/Feature_scaling
- (image) https://www.r-bloggers.com/box-plot-with-r-tutorial/

Rescaling

- The simplest method is rescaling the range of features to scale the range in [0, 1] or [−1, 1]. Selecting the target range depends on the nature of the data. The general formula is given as:

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

- where x is an original value x' is the normalized value. For example, suppose that we have the students' weight data, and the students' weights span . To rescale this data, we first subtract 160 from each student's weight and divide the result by 40 (the difference between the maximum and minimum weights).

Ref: • https://en.wikipedia.org/wiki/Feature_scaling

Mean Normalization

$$x' = \frac{x - \text{mean}(x)}{\max(x) - \min(x)}$$

- where x is an original value, x' is the normalized value.

Ref: • https://en.wikipedia.org/wiki/Feature_scaling

Standardization

- In machine learning, we can handle various types of data, e.g. audio signals and pixel values for image data, and this data can include multiple dimensions. Feature standardization makes the values of each feature in the data have zero-mean (when subtracting the mean in the numerator) and unit-variance.

$$x' = \frac{x - \bar{x}}{\sigma}$$

- Where x is the original feature vector, x-bar is the mean of that feature vector, and sigma is its standard deviation.

Ref: • https://en.wikipedia.org/wiki/Feature_scaling

Scaling to unit length

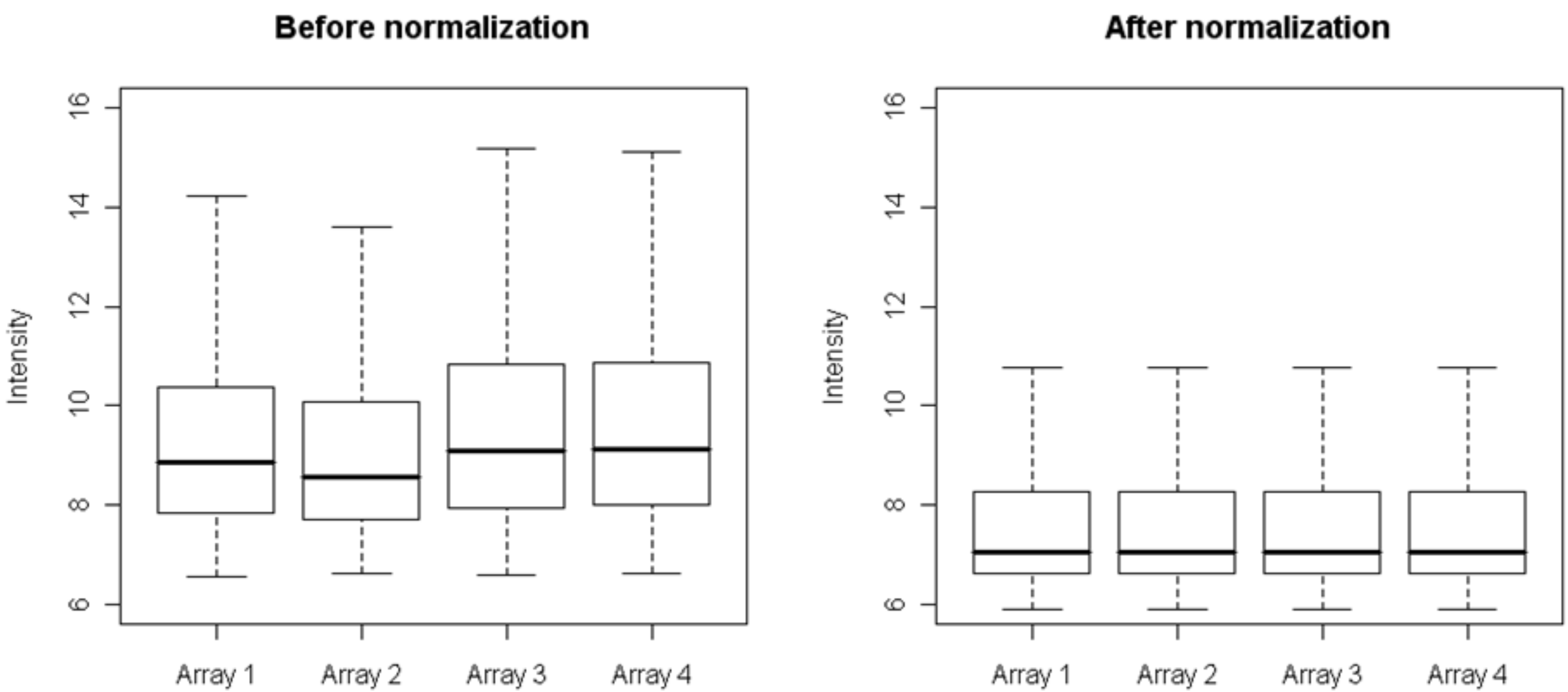
- Another option that is widely used in machine-learning is to scale the components of a feature vector such that the complete vector has length one. This usually means dividing each component by the Euclidean length of the vector:

$$x' = \frac{x}{||x||}$$

- In some applications (e.g. Histogram features) it can be more practical to use the L1 norm (i.e. Manhattan Distance, City-Block Length or Taxicab Geometry) of the feature vector. This is especially important if in the following learning steps the Scalar Metric is used as a distance measure.

Ref: • https://en.wikipedia.org/wiki/Feature_scaling

Box Plot



Ref: • (image) http://telemedicina.med.muni.cz/genomic-proteomic-analysis/index-en.php?pg=E-learning--2-data-analysis--2-2-analysis-of-genomic-data--2-2-1-DNA-microarrays-1--2-2-1-6-normalization

Python Code

```python
from sklearn import neighbors, datasets, preprocessing
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score
iris = datasets.load_iris()
X, y = iris.data[:, :2], iris.target
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=33)
scaler = preprocessing.StandardScaler().fit( X_train )
X_train = scaler.transform(X_train)
X_test = scaler.transform(X_test)
# Do Prediction
```

Ref: • https://www.datacamp.com/community/blog/scikit-learn-cheat-sheet

Summary

Steps to Create a Linear Regression Model

1. Import a Clean Dataset
2. Preprocess the Dataset
3. Explore the Dataset
4. Select some Features
5. Adjust some Parameters of the **Linear Regression**
6. **Build and Test a Model**
   - 6.1 Split the Dataset into Training and Test Sets
   - 6.2 Transform the Training Set (if any)
   - 6.3 Build a Model using the **Linear Regression** from a Training Set
   - 6.4 Transform the Test Set (if any)
   - 6.5 Test the Model against a Test Set

   K-Fold Cross-Validation
7. Evaluate this Technique
8. Build a Final Model

Data is a precious thing and will last longer than the systems themselves.

Tim Berners-Lee

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