

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \text{ (var)}$$

$$\frac{\Gamma \vdash e : \mathbf{Bool} \quad \Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 : \tau} \text{ (if)}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \text{fun } x : \tau_1 = e : \tau_1 \rightarrow \tau_2} \text{ (fun)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash (e_1 e_2) : \tau_2} \text{ (appl)}$$

① let  $f \times y =$

(match (x 10) with  
| true  $\rightarrow y$   
| false  $\rightarrow []$ )

let  $f = \text{fun } x \rightarrow (\text{fun } y \rightarrow ( \dots ))$

'a  $\rightarrow$  'b

$x : 'a$

$\emptyset, x : 'a \vdash \left( \text{fun } y \rightarrow \text{match } (x \ 10) \text{ with} \right. : 'b$   
| true  $\rightarrow y$   
| false  $\rightarrow []$

'b = 'c  $\rightarrow$  'd

$y : 'c$

$x : 'a, y : 'c \vdash \left( \text{match } (x \ 10) \text{ with} \right. : 'd$   
| true  $\rightarrow y$   
| false  $\rightarrow []$

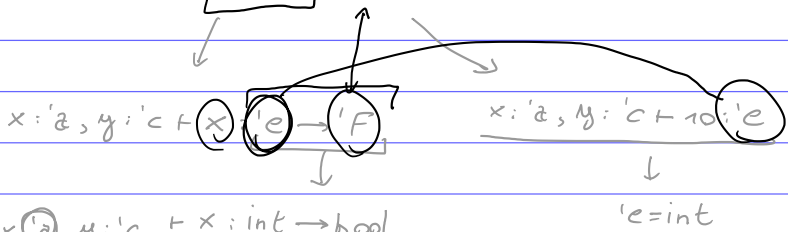
: bool

$x : 'a, y : 'c \vdash (x \ 10) : \text{bool}$  \*

$x : 'a, y : 'c \vdash y : 'd$  \*

$x : 'a, y : 'c \vdash [] : 'd$  \*

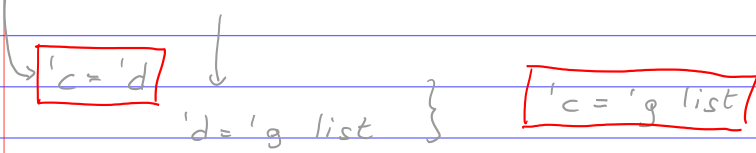
$x:'a, y:'c \vdash (x \ 10): 'F$  \*  $\vdash ? \text{bool}$



$'a = \text{int} \rightarrow \text{bool}$

$x:'a, y:'c \vdash y: 'd$  \*

$x:'a, y:'c \vdash []: 'd$  \*



$f: 'a \rightarrow 'b$  ??

$'b = 'c \rightarrow 'd$

$'a = \text{int} \rightarrow \text{bool}$

$'c = 'g \ \text{list}$

$'c = 'd$

$(\text{int} \rightarrow \text{bool}) \rightarrow 'a \ \text{list} \rightarrow 'a \ \text{list}$

$\Gamma \vdash f \ (1+2): 'a \ \text{list}$

$\Gamma \vdash 1+2: \text{int}$

$\Gamma \vdash f: \text{int} \rightarrow 'a \ \text{list}$   
(fun x => ...)

let  $h \times y =$

let  $(w, z) = y$  true in

if  $w$  then  $z :: x$  else  $x ; j$

let  $h = \text{fun } x \rightarrow (\text{fun } y \rightarrow (\dots))$   
 $\tau_2 = \tau_3 \rightarrow \tau_4$

$h : \tau_1 \rightarrow \tau_2$

$x : \tau_1$

$\tau_2 = \tau_3 \rightarrow \tau_4$

$y : \tau_3$

$(\text{let } (w, z) = y \text{ true in } \text{if } w \text{ then } z :: x \text{ else } x) : \tau_4$

$(w, z) \equiv y \text{ true}$   
 $\tau_5$   
 $\tau_5$

$\tau_6 \times \tau_7$

$w : \tau_6$   
 $z : \tau_7$

$y : \text{bool} \rightarrow \tau_6 \times \tau_7 = \tau_3$   
 $\uparrow$

if  $w$  then  $z :: x$  else  $x : \tau_4$   
 $w : \text{bool}$      $(z :: x) : \tau_6$      $x : \tau_4$

$w : \text{bool} = \tau_6$   
 $\uparrow$

$x : \tau_4 = \tau_1$

$(z :: x) : \tau_4$

$z : \tau_2$

$x : \tau_7 \text{ list} = \tau_4$

$\tau_4 = \tau_7 \text{ list} \quad \tau_3$

$z : \tau_2$   
 $x : \tau_7 \text{ list} = \tau_4 = \tau_1$   
 $\uparrow$

$h : 'a \text{ list} \rightarrow ((\text{bool} \rightarrow \text{bool} \times 'a) \rightarrow 'a \text{ list})$   
 $\tau_7$      $\tau_2$