1 $(\lambda x.\, x\, y)\{y := x\}$

2 $(\lambda x.\, y\, x)\{x := y\}$

3 $(\lambda x.\, x\, y)\{y := \lambda x.\, x\}$

4 $(\lambda x.\, x\,(\lambda z.\, z\, y))\{y := x\}$

5 $(\lambda x.\, x\,(\lambda z.\, z\, y))\{x := y\}$

① $\lambda x.\, x\, y \xrightarrow{\alpha} \lambda z.\, z\, y \xrightarrow{\{\}} \lambda z.\, z\, x$

② $(\lambda x.\, y\, x)\{x := y\} \xrightarrow{\alpha} (\lambda z.\, y\, z)\{x := y\} \xrightarrow{\{\}} \lambda z.\, y\, z$

$\xrightarrow{\{\}} \lambda x.\, y\, x$

③ $(\lambda x.\, x\, y)\{y := \lambda x.\, x\} \xrightarrow{\{\}} \lambda x.\, x\,(\lambda x.\, x)$

④ $(\lambda x.\, x\,(\lambda z.\, z\, y))\{y := x\}$

$\downarrow \alpha$

$(\lambda w.\, w\,(\lambda z.\, z\, y))\{y := x\}$

$\downarrow \{\}$

$(\lambda w.\, w\,(\lambda z.\, z\, x))$

⑤ $(\lambda x.\, x\,(\lambda z.\, z\, y))\{x := y\} \xrightarrow{\{\}} (\lambda x.\, x\,(\lambda z.\, z\, y))$

$flip = \lambda f.\, \lambda x.\, \lambda y.\, f\, y\, x$

$g = \lambda a.\, \lambda b.\, c$

$flip\ g \xrightarrow{\{f := g\}} (\lambda f.\, \lambda x.\, \lambda y.\, f\, y\, x)\, g$

$\rightarrow \lambda x.\, \lambda y.\, g\, y\, x$

$= \lambda x.\, \lambda y.\, ((\lambda a.\, \lambda b.\, c)\, y\, x)$

$\rightarrow \lambda x.\, \lambda y.\, (((\lambda b.\, c)\{a := y\})\, x)$

$\rightarrow \lambda x.\, \lambda y.\, (c\{a := y\}\{b := x\})$

$$flip = \lambda f.\ \lambda x.\ \lambda y.\ f\ y\ x$$

$$(\lambda f.\lambda x.\lambda y.\ f\ y\ x)\ \overline{(\lambda k.\ \lambda x.\ k)}$$

$$\to \lambda x.\ \lambda y.\ \underline{(\lambda k.\ \lambda x.\ k)\ \overline{y}}\ x$$

$$\to \lambda x.\ \lambda y.\ \underline{(\lambda x.\ y)\ x}$$

$$\to \lambda x.\ \lambda y.\ y$$

$$\equiv_\alpha \lambda x.\lambda k.\ k \qquad\qquad \lambda k.\lambda x.\ k$$

$$\Big(\big(flip\ \overline{(\lambda f.\ \lambda x.\ f\ x)}\big)\ w\Big)\ (\lambda x.\ x)$$

$$\to \Big(\big((\lambda x.\ \lambda y.\ \underline{(\lambda f.\ \lambda x.\ f\ x)\ \overline{y}}\ x)\ w\big)\ (\lambda x.\ x)\Big)$$

$$\to \Big(\big((\lambda x.\ \lambda y.\ \underline{(\lambda x.\ y\ x)\ x})\ w\big)\ (\lambda x.\ x)\Big)$$

$$\to \Big(\big(\underline{(\lambda x.\ \lambda y.\ y\ x)\ \overline{w}}\big)\ (\lambda x.\ x)\Big)$$

$$\to \underline{(\lambda y.\ y\ w)\ \overline{(\lambda x.\ x)}}$$

$$\to \underline{(\lambda x.\ x)\ \overline{w}}$$

$$\to w$$

$$flip = \lambda f.\ \lambda x.\ \lambda y.\ f\ y\ x$$

somma

$$\underline{flip\ \overline{(\lambda m.\ \lambda n.\ \lambda z.\ \lambda s.\ m\ (n\ z\ s)\ s)}}$$

$$\to \lambda x.\ \lambda y.\ \underline{(\lambda m.\ \lambda n.\ \lambda z.\ \lambda s.\ m\ (n\ z\ s)\ s)\ \overline{y}\ x}$$

$$\to \lambda x.\ \lambda y.\ \underline{(\lambda n.\ \lambda z.\ \lambda s.\ y\ (n\ z\ s)\ s)\ \overline{x}}$$

$$\to \lambda x.\ \lambda y.\ (\lambda z.\ \lambda s.\ y\ (x\ z\ s)\ s)$$

$$\equiv_\alpha \lambda m.\ \lambda n.\ \lambda z.\ \lambda s.\ n\ (m\ z\ s)\ s)$$

Strategie di riduzione

call - by - value
"valuto prima i parametri"

$$\frac{e_2 \to e_2'}{(\lambda x.e_1)\,e_2 \to (\lambda x.e_1)\,e_2'}$$

$$\frac{v \not\to}{(\lambda x.e_1)\,v \longrightarrow e_1\{x := v\}}$$

call - by - name
"applico le funzioni a parametri non valutati"

$$\frac{}{(\lambda x.e_1)\,e_2 \longrightarrow e_1\{x := e_2\}}$$

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① (by-value) $(\lambda x.x)\,\underline{\Omega} \to (\lambda x.x)\,\underline{\Omega} \to \dots$

(by-name) $\underline{(\lambda x.x)\,\Omega} \to \underline{\Omega} \to \underline{\Omega} \to \dots$

② (by-value) $(\lambda x.k)\,\underline{\Omega} \to (\lambda x.k)\,\underline{\Omega} \to \dots$

(by-name) $\underline{(\lambda x.k)\,\Omega} \to k \not\to$

③ (by-value) $(\lambda x.xy)\,\underline{\Omega} \to (\lambda x.xy)\,\underline{\Omega} \to \dots$

(by-name) $\underline{(\lambda x.xy)\,\Omega} \to \underline{\Omega}\,y \to \underline{\Omega}\,y \to \dots$

④ (by-value) $(\lambda x.x\,\Omega) \not\to$

(by-name) $(\lambda x.x\,\Omega) \not\to$

⑤ (by-value) $(\lambda x.yx)\,\underline{\Omega} \to (\lambda x.yx)\,\underline{\Omega} \to \dots$

(by-name) $\underline{(\lambda x.yx)\,\Omega} \to y\,\Omega \not\to$