Lab Report 1: Design and Analysis of GCD Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. Theory:

The **Greatest Common Divisor (GCD)** of two integers a and b is the largest integer that divides both a and b without leaving a remainder. It is also known as the *highest common factor (HCF)*. The GCD has applications in number theory, cryptography, and computational mathematics.

Key Concepts:

- If $gcd(a, b) = 1$, then a and b are said to be ***coprime***.
- The Euclidean algorithm is one of the most efficient methods to compute the GCD by repeatedly applying division.

Properties of GCD:

1. $gcd(a, 0) = | a |$.
2. $gcd(a, b) = gcd(b, a \ mod \ b)$, where $\boldsymbol{mod}$ represents the remainder operation.
3. The algorithm terminates when the remainder becomes zero.

2. Algorithm:

Euclidean Algorithm for GCD:
The Euclidean algorithm computes the GCD by iteratively reducing the problem size using division with remainder. Below is the pseudocode for the algorithm:

```
   Algorithm: Euclidean_GCD ($a, b$)
   Input: Two positive integers $a$ and $b$ ($a \geq b > 0$)
   Output: The greatest common divisor of $a$ and $b$.
1. If b=0, return a as the GCD.
2. Otherwise:
     • Compute $r = a \ mod \ b$ (remainder when a is divided by b).
     • Replace $a$ with $b$, and $b$ with $r$.
     • Repeat Step 1 until $b = 0$.
3. Return $a$ as the GCD.
```

3. Time and Space Complexity Analysis:

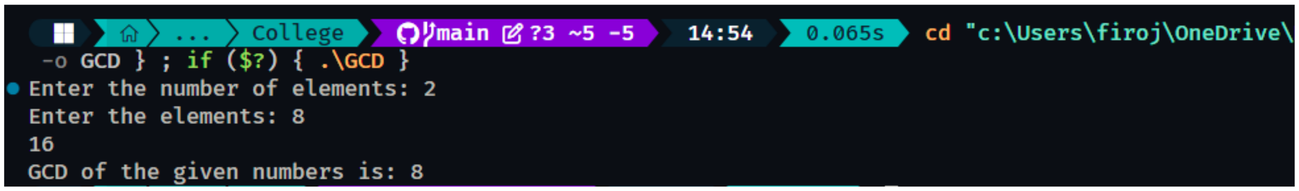
Time Complexity: $O(n)$

Space Complexity: $O(1)$

4. Code Implementation:

```cpp
/*Program that calculates the GCD of given N numbers*/
#include<iostream>
using namespace std;

// Function to calculate GCD of two numbers
int gcd(int a, int b) {
    if (b == 0)
        return a;
    return gcd(b, a % b);
}

// Function to calculate GCD of an array of numbers
int findGCD(int arr[], int n) {
    int result = arr[0];
    for (int i = 1; i < n; i++) {
        result = gcd(result, arr[i]);
        if(result == 1) {
           return 1;
        }
    }
    return result;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;
    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }
    cout << "GCD of the given numbers is: " << findGCD(arr, n) <<
endl;
    return 0;
}
```

5. **Output:**



```
  ... College  main ?3 ~5 -5  14:54  0.065s  cd "c:\Users\firoj\OneDrive\
 -o GCD } ; if ($?) { .\GCD }
Enter the number of elements: 2
Enter the elements: 8
16
GCD of the given numbers is: 8
```

Lab Report 2: Design and Analysis of Factorial Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   The factorial of a non-negative integer n, denoted as $n!$, is the product of all positive integers less than or equal to n. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. Factorial has applications in combinatorics, probability, and algorithm design. The base case is $0! = 1$ by definition. Factorial can be computed iteratively or recursively, with recursion being a common approach in programming.

2. **Key Concepts:**
   - Factorial grows extremely fast, leading to large numbers even for small inputs.
   - Recursive definition: $n! = n \times (n-1)!$, with base case $0! = 1$.
   - Iterative solutions avoid stack overflow for large n, unlike recursion.

3. **Properties of Factorial:**
   - $n! = n \times (n-1) \times (n-2) \times \ldots \times 1.$
   - Factorial is undefined for negative numbers.
   - For large n, factorial exceeds typical integer limits, requiring special handling (e.g., long or big integer types).

4. **Algorithm:**

   Recursive Algorithm for Factorial:
   The recursive factorial algorithm computes $n!$ by breaking it into smaller subproblems. Below is the algorithm:

```
Algorithm: Factorial ($n$)
Input: A non-negative integer $n$ ($n \geq 0$)
Output: The factorial of $n$
    1. If $n = 0$, return 1 as the factorial.
    2. Otherwise:
        • Compute $n \times$ Factorial$(n-1)$.
        • Return the result.
```

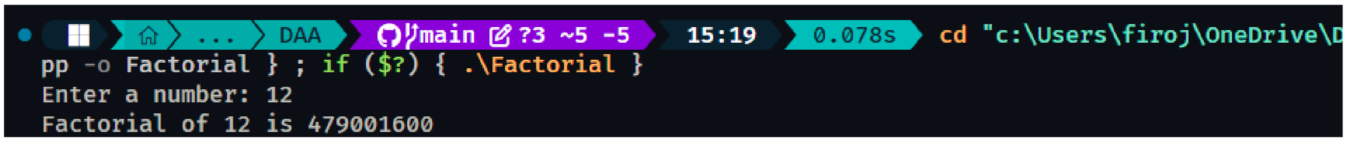
5. Time and Space Complexity Analysis:

Time Complexity: $O(n)$ – The algorithm makes n recursive calls or iterations.
Space Complexity: $O(n)$ – For the recursive version, the call stack uses space proportional to n. (Iterative version would be $O(1)$.)

6. Code Implementation:

```cpp
/*The program for factorial of given N numbers*/

#include<iostream>
using namespace std;

// Function for factorial
int factorial(int n) {
    if(n <= 1) return 1;
    else return n * factorial(n - 1);
}

int main() {
    int n;
    cout << "Enter a number: ";
    cin >> n;
    cout << "Factorial of " << n << " is " << factorial(n) << endl;
    return 0;
}
```

7. Output:



```
 ...  DAA   main ?3 ~5 -5   15:19   0.078s   cd "c:\Users\firoj\OneDrive\D
pp -o Factorial } ; if ($?) { .\Factorial }
Enter a number: 12
Factorial of 12 is 479001600
```

Lab Report 3: Design and Analysis of Fibonacci Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones, typically starting with 0 and 1. Formally, it is defined as: $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$. The sequence begins: $0, 1, 1, 2, 3, 5, 8, 13, 21$, and so on. The Fibonacci sequence has applications in mathematics, computer science, and nature (e.g., modeling growth patterns).
2. **Key Concepts:**
   - The sequence can be computed recursively or iteratively.
   - Recursive computation is intuitive but inefficient for large n due to exponential growth in calls.
   - Iterative computation is more efficient, using constant extra space and linear time.
3. **Properties of Fibonacci:**
   - $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$.
   - The ratio of consecutive Fibonacci numbers approximates the golden ratio (~ 1.618) as n increases.
   - Fibonacci numbers grow exponentially, requiring careful handling for large n (e.g., using long or big integer types).
4. **Algorithm:**
Iterative Algorithm for Fibonacci:
The iterative Fibonacci algorithm computes the n^{th} Fibonacci number by maintaining two variables and updating them in a loop. Below is the algorithm:

```
Algorithm: Fibonacci ($n$)
Input: A non-negative integer $n$ : $(n \geq 0)$
Output: The $n^{th}$ Fibonacci number
1. If $n = 0$, return 0.
2. If $n = 1$, return 1.
3. Otherwise:
        o Set $a = 0$ (first number), $b = 1$ (second number).
        o For $i = 2$ to $n$:
                ▪ Compute $c = a + b$.
                ▪ Update $a = b, b = c$.
        o Return $b$ as the $n^{th}$ Fibonacci number.
```

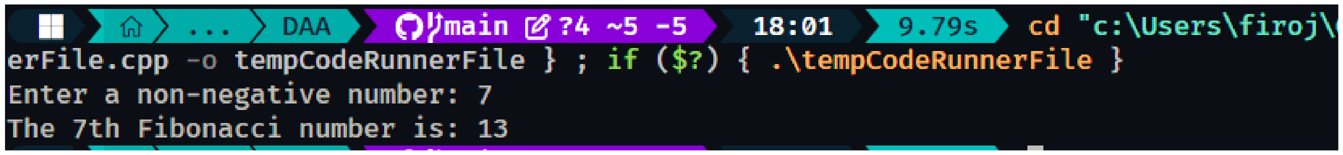
5. Time and Space Complexity Analysis:

Time Complexity: $O(n)$ – The algorithm performs $n-1$ iterations to compute the n^{th} number.
Space Complexity: $O(1)$ – Only a constant amount of extra space is used (variables a, b, and c).

6. Code Implementation:

```cpp
/*Program that calculates the nth Fibonacci number*/
#include<iostream>
using namespace std;

// Function to calculate the nth Fibonacci number
int fibonacci(int n) {
    if (n == 0)
        return 0;
    if (n == 1)
        return 1;
    int a = 0, b = 1, c;
    for (int i = 2; i <= n; i++) {
        c = a + b;
        a = b;
        b = c;
    }
    return b;
}

int main() {
    int n;
    cout << "Enter a non-negative number: ";
    cin >> n;
    if (n < 0) {
        cout << "Fibonacci is not defined for negative numbers." <<
endl;
    } else {
        cout << "The " << n << "th Fibonacci number is: " <<
fibonacci(n) << endl;
    }
    return 0;
}
```

7. Output:



```
 ... DAA  main ?4 ~5 -5  18:01  9.79s  cd "c:\Users\firoj\
erFile.cpp -o tempCodeRunnerFile } ; if ($?) { .\tempCodeRunnerFile }
Enter a non-negative number: 7
The 7th Fibonacci number is: 13
```

Lab Report 4: Design and Analysis of Linear Search Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. Theory:

Linear search, also known as sequential search, is a simple algorithm used to find the position of a target element within a list or array. It works by examining each element in the array sequentially until the target is found or the end of the array is reached. Linear search is straightforward and does not require the data to be sorted, making it applicable to a wide range of problems.

2. Key Concepts:

- Linear search checks each element one by one, starting from the first index.
- It returns the index of the target element if found, or a sentinel value (e.g., -1) if not found.
- The algorithm's simplicity makes it useful for small datasets or unsorted arrays.

3. Properties of Linear Search:

- Works on both sorted and unsorted arrays.
- The search terminates early if the target is found; otherwise, it examines the entire array.
- Performance degrades as the size of the array increases.

4. Algorithm:

Linear Search Algorithm:

The linear search algorithm iterates through the array, comparing each element with the target value. Below is the pseudocode for the algorithm:

```
Algorithm: Linear_Search (arr, n, target)
Input: An array arr of size $n$, and a target value target
Output: The index of target in arr, or $-1$ if not found
1. For i = 0 to n − 1:
       o If arr[i] = target, return i as the index.
2. If no match is found after the loop, return $-1$.
```

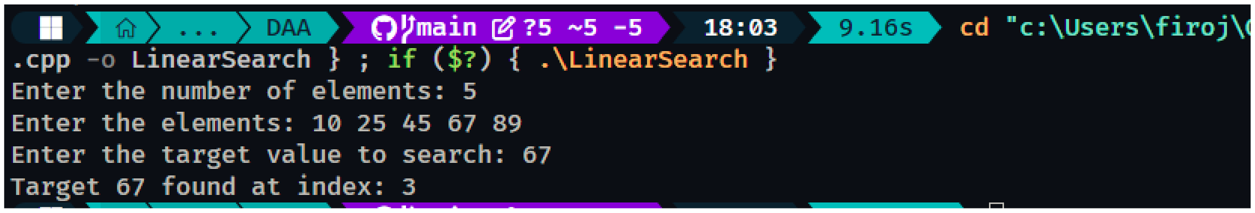
5. Time and Space Complexity Analysis:

- **Time Complexity:**
  - **Best Case:** $O(1)$ – Target is found at the first position.
  - **Worst Case:** $O(n)$ – Target is at the last position or not present.
  - **Average Case:** $O(n)$ – On average, half the array is searched.
- **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (e.g., loop variable).

6. Code Implementation:

```
/*Program that performs linear search on an array*/
#include<iostream>
using namespace std;

// Function to perform linear search
int linearSearch(int arr[], int n, int target) {
    for (int i = 0; i < n; i++) {
        if (arr[i] == target)
            return i; // Return index if target is found
    }
    return -1; // Return -1 if target is not found
}

int main() {
    int n, target;
    cout << "Enter the number of elements: ";
    cin >> n;
    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }
    cout << "Enter the target value to search: ";
    cin >> target;
    int result = linearSearch(arr, n, target);
    if (result == -1) {
        cout << "Target " << target << " not found in the array." <<
endl;
    } else {
        cout << "Target " << target << " found at index: " << result
<< endl;
    }
    return 0;
}
```

8. Output:



```
 ... DAA  main ?5 ~5 -5  18:03  9.16s  cd "c:\Users\firoj\
.cpp -o LinearSearch } ; if ($?) { .\LinearSearch }
Enter the number of elements: 5
Enter the elements: 10 25 45 67 89
Enter the target value to search: 67
Target 67 found at index: 3
```

Lab Report 5: Design and Analysis of Bubble Sort Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Bubble sort is a simple sorting algorithm that repeatedly steps through a list, compares adjacent elements, and swaps them if they are in the wrong order. The process continues until no more swaps are needed, indicating that the list is sorted. It is called "bubble sort" because smaller elements "bubble" to the top (beginning) of the list in each iteration. Though inefficient for large datasets, it is easy to understand and implement.

2. **Key Concepts:**
   - Bubble sort works by comparing and swapping adjacent elements in multiple passes.
   - It can be optimized by checking if any swaps occurred in a pass; if not, the array is already sorted.
   - It is a stable sorting algorithm, preserving the relative order of equal elements.

3. **Properties of Bubble Sort:**
   - Best case occurs when the array is already sorted.
   - Worst case occurs when the array is sorted in reverse order.
   - The algorithm always performs pairwise comparisons, even if unnecessary swaps are avoided with optimization.

4. **Algorithm:**
   Bubble Sort Algorithm:
   The bubble sort algorithm iterates through the array, swapping adjacent elements if they are out of order, and repeats until the array is fully sorted. Below is the algorithm:

```
Algorithm: Bubble_Sort (arr, n)
Input: An array arr of size n
Output: The array arr sorted in ascending order
  1. For i = 0 to n-1:
       o  Set swapped = false.
       o  For j = 0 to n-i-1:
            ▪  If arr[j] > arr[j+1]:
                 ▪  Swap arr[j] and arr[j+1].
                 ▪  Set swapped = true.
       o  If swapped = false, break (array is sorted).
  2. Return the sorted array.
```

5. Time and Space Complexity:

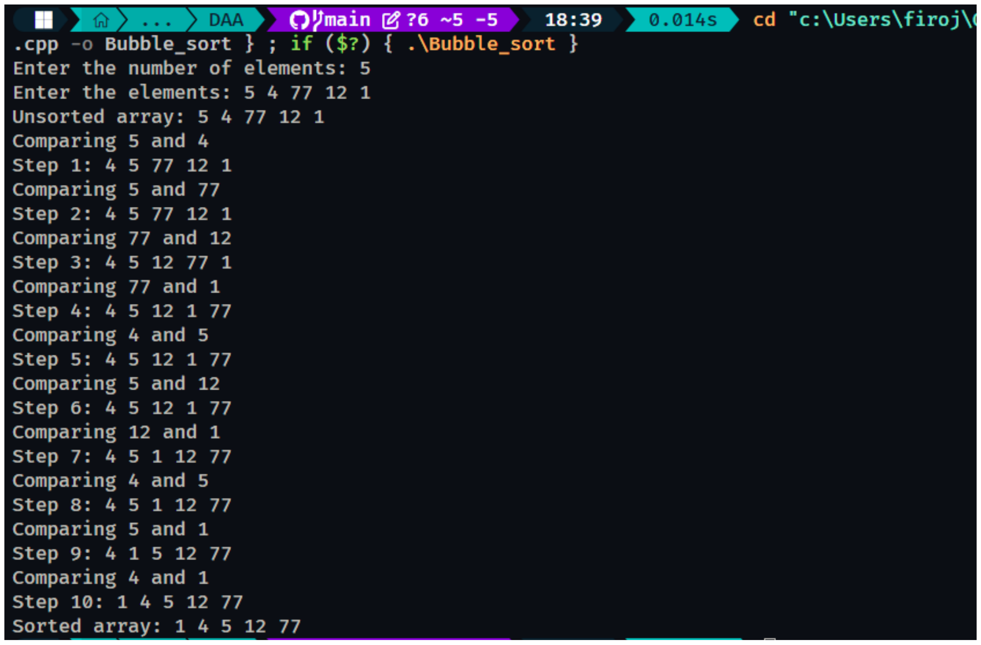
- **Time Complexity:**
  - **Best Case:** $O(n)$ – Array is already sorted, and optimization detects no swaps in the first pass.
  - **Worst Case:** $O(n^2)$ – Array is reverse sorted, requiring maximum comparisons and swaps.
  - **Average Case:** $O(n^2)$ – Randomly ordered array requires roughly $n^2/4$ comparisons and swaps.
- **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (e.g., for temporary swaps).

6. Code Implementation

```
#include<iostream>
using namespace std;

// Function to perform bubble sort and show the process
void bubbleSort(int arr[], int n) {
    int step = 1; // Initialize step counter
    // Traverse through all array elements
    for (int i = 0; i < n-1; i++) {
        // Last i elements are already in place
        for (int j = 0; j < n-i-1; j++) {
            // Print the elements being compared
            cout << "Comparing " << arr[j] << " and " << arr[j+1] <<
endl;
            // Swap if the element found is greater than the next
element
            if (arr[j] > arr[j+1]) {
                int temp = arr[j];
                arr[j] = arr[j+1];
                arr[j+1] = temp;
            }
            // Print array after each swap
            cout << "Step " << step << ": ";
            for (int k = 0; k < n; k++)
                cout << arr[k] << " ";
            cout << endl;
            step++; // Increment step counter
        }
    }
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
```

```cpp
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;

    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform bubble sort
    bubbleSort(arr, n);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. Output:



```
 ⌂ ... DAA  main ?6 ~5 -5  18:39  0.014s  cd "c:\Users\firoj\
.cpp -o Bubble_sort } ; if ($?) { .\Bubble_sort }
Enter the number of elements: 5
Enter the elements: 5 4 77 12 1
Unsorted array: 5 4 77 12 1
Comparing 5 and 4
Step 1: 4 5 77 12 1
Comparing 5 and 77
Step 2: 4 5 77 12 1
Comparing 77 and 12
Step 3: 4 5 12 77 1
Comparing 77 and 1
Step 4: 4 5 12 1 77
Comparing 4 and 5
Step 5: 4 5 12 1 77
Comparing 5 and 12
Step 6: 4 5 12 1 77
Comparing 12 and 1
Step 7: 4 5 1 12 77
Comparing 4 and 5
Step 8: 4 5 1 12 77
Comparing 5 and 1
Step 9: 4 1 5 12 77
Comparing 4 and 1
Step 10: 1 4 5 12 77
Sorted array: 1 4 5 12 77
```

Lab Report 6: Design and Analysis of Quick Sort Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Quick Sort is an efficient, comparison-based sorting algorithm that uses a divide-and-conquer strategy. It selects a "pivot" element from the array and partitions the other elements into two sub-arrays: those less than the pivot and those greater than the pivot. The sub-arrays are then recursively sorted. Quick Sort is widely used due to its average-case performance and in-place sorting capability, making it memory-efficient.
2. **Key Concepts:**
   - The choice of pivot affects performance (e.g., first element, last element, random, or median).
   - Partitioning rearranges the array so that the pivot is in its final sorted position.
   - Quick Sort is not stable, meaning it may change the relative order of equal elements.
3. **Properties of Quick Sort:**
   - Best case occurs when the pivot consistently divides the array into roughly equal halves.
   - Worst case occurs with already sorted or reverse-sorted arrays and a poor pivot choice (e.g., first or last element).
   - It performs in-place sorting, requiring minimal extra memory beyond the recursion stack.
4. **Algorithm:**
   Quick Sort Algorithm:
   The Quick Sort algorithm recursively partitions the array around a pivot. Below is the actual implementation in C++:

```
Algorithm: Quick_Sort (arr, low, high)
Input: An array arr, and indices low and high defining the sub-array to sort
Output: The array arr sorted in ascending order
    1. If low < high:
         • Call partition(arr, low, high) to get the pivot index pi.
         • Call Quick_Sort(arr, low, $p_{i-1}$) to sort the left sub-array.
         • Call Quick_Sort(arr, $p_{i+1}$, high) to sort the right sub-array.
    2. Return the sorted array.
```

```
Algorithm: Partition (arr, low, high)
Input: An array arr, and indices low and high defining the sub-array
Output: The index of the pivot in its final sorted position
    1. Set pivot = arr[high] (choose last element as pivot).
    2. Set i = low-1 (index of smaller element).
    3. For j = low to high-1:
         o If arr[j] <= pivot:
```

        ▪ Increment i.
        ▪ Swap arr[i] and arr[j].
4. Swap arr[i+1] and arr[high] (place pivot in its final position).
5. Return i+1 as the pivot index.

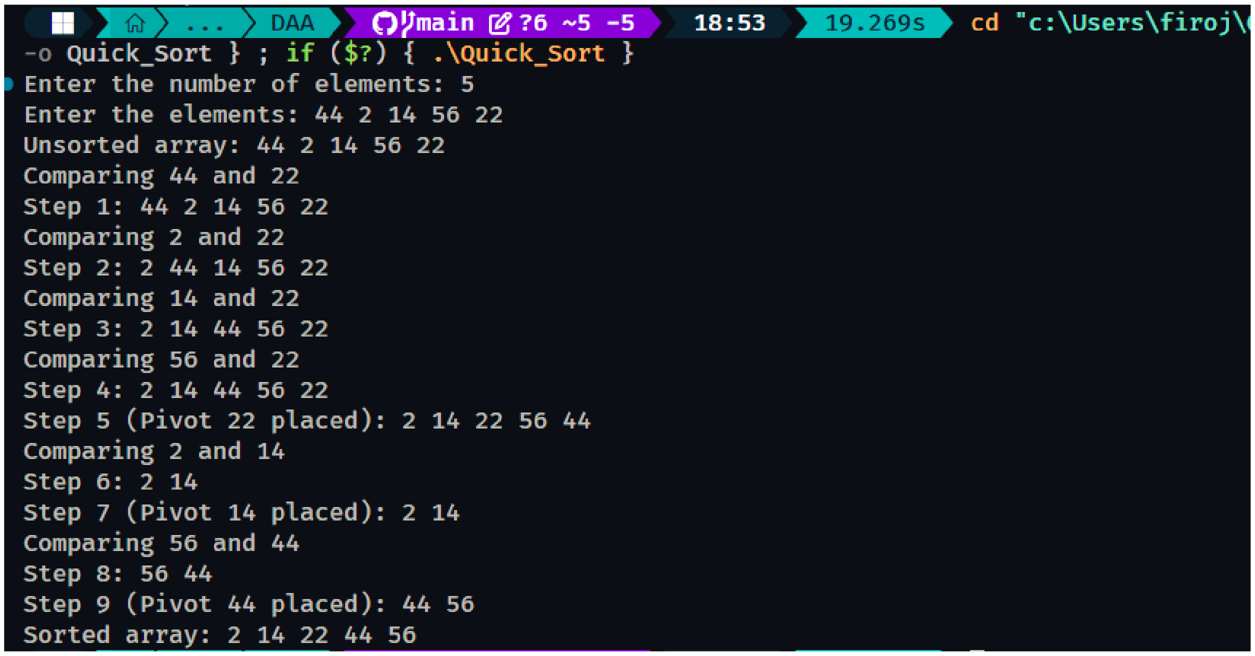
5. **Time and Space Complexity:**
   - **Time Complexity:**
     - **Best Case:** $O(n \log n)$ – Pivot splits the array into two roughly equal halves.
     - **Worst Case:** $O(n^2)$ – Pivot is the smallest or largest element, leading to unbalanced partitions.
     - **Average Case:** $O(n \log n)$ – Random pivot selection typically yields balanced partitions.
   - **Space Complexity:**
     - $O(\log n)$ – Due to the recursion stack in the average and best cases.
     - $O(n)$ – In the worst case, the recursion stack depth equals the array size.

6. **Code Implementation:**

```
#include<iostream>
using namespace std;

// Function to partition the array and show the process
int partition(int arr[], int low, int high, int& step) {
    int pivot = arr[high]; // Choose last element as pivot
    int i = low - 1;        // Index of smaller element
    for (int j = low; j < high; j++) {
        // Print the elements being compared
        cout << "Comparing " << arr[j] << " and " << pivot << endl;
        if (arr[j] <= pivot) {
            i++;
            int temp = arr[i];
            arr[i] = arr[j];
            arr[j] = temp;
        }
        // Print array after each comparison
        cout << "Step " << step << ": ";
        for (int k = low; k <= high; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++; // Increment step counter
    }
    int temp = arr[i + 1];
    arr[i + 1] = arr[high];
    arr[high] = temp;
    // Print array after placing pivot
    cout << "Step " << step << " (Pivot " << pivot << " placed): ";
```

```cpp
    for (int k = low; k <= high; k++)
        cout << arr[k] << " ";
    cout << endl;
    step++;
    return i + 1;
}

// Function to perform Quick Sort and show the process
void quickSort(int arr[], int low, int high, int& step) {
    if (low < high) {
        int pi = partition(arr, low, high, step);
        quickSort(arr, low, pi - 1, step);
        quickSort(arr, pi + 1, high, step);
    }
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;

    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform Quick Sort
    int step = 1; // Initialize step counter
    quickSort(arr, 0, n-1, step);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. **Output:**



```
 ⌂  ...  DAA  main  ?6 ~5 -5  18:53  19.269s  cd "c:\Users\firoj\
-o Quick_Sort } ; if ($?) { .\Quick_Sort }
Enter the number of elements: 5
Enter the elements: 44 2 14 56 22
Unsorted array: 44 2 14 56 22
Comparing 44 and 22
Step 1: 44 2 14 56 22
Comparing 2 and 22
Step 2: 2 44 14 56 22
Comparing 14 and 22
Step 3: 2 14 44 56 22
Comparing 56 and 22
Step 4: 2 14 44 56 22
Step 5 (Pivot 22 placed): 2 14 22 56 44
Comparing 2 and 14
Step 6: 2 14
Step 7 (Pivot 14 placed): 2 14
Comparing 56 and 44
Step 8: 56 44
Step 9 (Pivot 44 placed): 44 56
Sorted array: 2 14 22 44 56
```

Lab Report 7: Design and Analysis of Selection Sort Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Selection Sort is a simple comparison-based sorting algorithm that divides the input array into two parts: a sorted portion (initially empty) and an unsorted portion. In each iteration, it finds the minimum element in the unsorted portion and swaps it with the first element of the unsorted portion, effectively growing the sorted portion. It is inefficient for large datasets but easy to implement and understand.

2. **Key Concepts:**
   - Selection Sort repeatedly selects the smallest (or largest) element from the unsorted section.
   - It performs swaps only once per iteration, unlike Bubble Sort's frequent swaps.
   - It is an in-place algorithm but not stable, as it may change the relative order of equal elements.

3. **Properties of Selection Sort:**
   - The number of comparisons is always the same, regardless of the input order.
   - Best case occurs when the array is already sorted (fewer swaps), but comparisons remain unchanged.
   - Worst case occurs when the array is reverse sorted, maximizing swaps.

4. **Algorithm:**

```
Algorithm: Selection_Sort (arr, n)
Input: An array arr of size n
Output: The array arr sorted in ascending order
    1. For i = 0 to $n - 1$:
        • Set min_idx = i (assume the current position holds the minimum).
        • For j = $i + 1$ to $n - 1$:
            ▪ If arr[j] < arr[min_idx]:
                ▪ Set min_idx = j.
        • If min_idx $\neq$ i:
            ▪ Swap arr[i] and arr[min_idx].
    2. Return the sorted array.
```

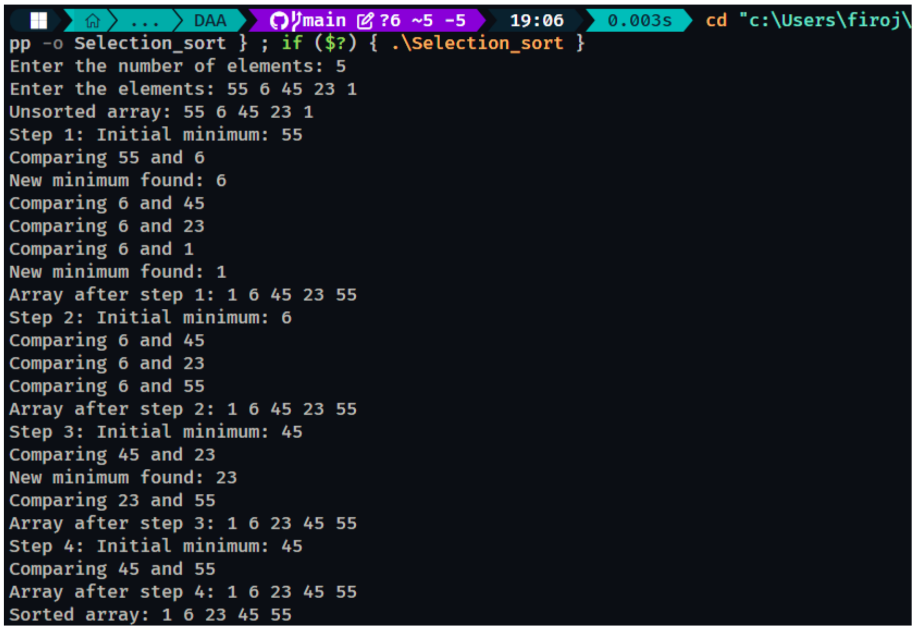
5. **Time and Space Complexity Analysis:**
   - **Time Complexity:**
     - **Best Case:** $O(n^2)$ – Even if sorted, it performs all comparisons.
     - **Worst Case:** $O(n^2)$ – Comparisons and swaps are maximized.
     - **Average Case:** $O(n^2)$ – Roughly $n^2/2$ comparisons regardless of input.
   - **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (e.g., for temporary variables).

6. **Code Implementation:**

```
#include<iostream>
using namespace std;

// Function to perform selection sort and show the process
void selectionSort(int arr[], int n) {
    int step = 1;
    // Traverse through all array elements
    for (int i = 0; i < n-1; i++) {
        // Find the minimum element in unsorted array
        int min_idx = i;
        cout << "Step " << step << ": Initial minimum: " <<
arr[min_idx] << endl;
        for (int j = i+1; j < n; j++) {
            // Print the elements being compared
            cout << "Comparing " << arr[min_idx] << " and " << arr[j]
<< endl;
            if (arr[j] < arr[min_idx]) {
                min_idx = j;
                cout << "New minimum found: " << arr[min_idx] << endl;
            }
        }
        // Swap the found minimum element with the first element
        int temp = arr[min_idx];
        arr[min_idx] = arr[i];
        arr[i] = temp;

        // Print array after each swap
        cout << "Array after step " << step << ": ";
        for (int k = 0; k < n; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++; // Increment step counter
    }
}

// Function to print an array
void printArray(int arr[], int size) {
```

```cpp
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;

    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform selection sort
    selectionSort(arr, n);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. Output:



```
 ⌂ ... DAA  main ?6 ~5 -5  19:06  0.003s  cd "c:\Users\firoj\
pp -o Selection_sort } ; if ($?) { .\Selection_sort }
Enter the number of elements: 5
Enter the elements: 55 6 45 23 1
Unsorted array: 55 6 45 23 1
Step 1: Initial minimum: 55
Comparing 55 and 6
New minimum found: 6
Comparing 6 and 45
Comparing 6 and 23
Comparing 6 and 1
New minimum found: 1
Array after step 1: 1 6 45 23 55
Step 2: Initial minimum: 6
Comparing 6 and 45
Comparing 6 and 23
Comparing 6 and 55
Array after step 2: 1 6 45 23 55
Step 3: Initial minimum: 45
Comparing 45 and 23
New minimum found: 23
Comparing 23 and 55
Array after step 3: 1 6 23 45 55
Step 4: Initial minimum: 45
Comparing 45 and 55
Array after step 4: 1 6 23 45 55
Sorted array: 1 6 23 45 55
```

Lab Report 8: Design and Analysis of Insertion Sort Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Insertion Sort is a simple comparison-based sorting algorithm that builds a sorted portion of the array one element at a time. It takes each element from the unsorted portion and inserts it into its correct position within the sorted portion by shifting larger elements to the right. It mimics how people sort playing cards and is efficient for small datasets or nearly sorted arrays.

2. **Key Concepts:**
   - Insertion Sort maintains a sorted sub-array that grows with each iteration.
   - It shifts elements rather than swapping them, reducing write operations compared to other algorithms like Bubble Sort.
   - It is stable (preserves the relative order of equal elements) and adaptive (faster for partially sorted data).

3. **Properties of Insertion Sort:**
   - Best case occurs when the array is already sorted, requiring minimal shifts.
   - Worst case occurs when the array is reverse sorted, maximizing comparisons and shifts.
   - It works well for online sorting, where data arrives incrementally.

4. **Algorithm:**

```
Algorithm: Insertion_Sort (arr, n)
Input: An array arr of size n
Output: The array arr sorted in ascending order
    1. For i = 1 to n − 1:
            • Set key = arr[i] (element to insert).
            • Set j = i − 1 (last index of sorted portion).
            • While j ≥ 0 and arr[j] > key:
                    ▪ Shift arr[j] to arr[j+1].
                    ▪ Decrement j.
            • Place key at arr[j+1].
    2. Return the sorted array.
```

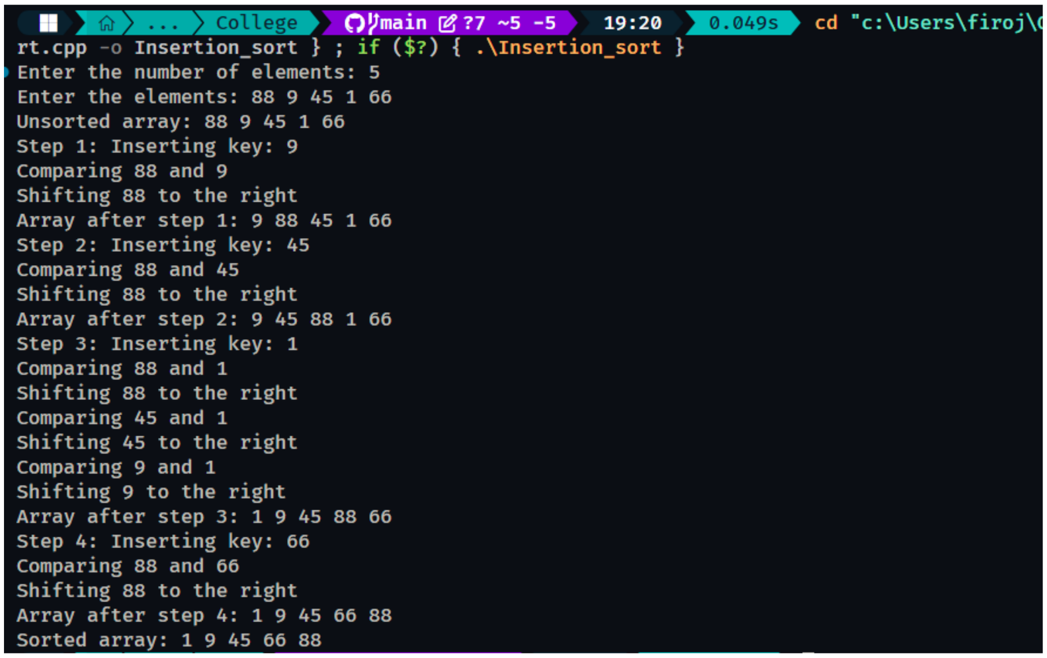
5. Time and Space Complexity Analysis:

- **Time Complexity:**
    - **Best Case:** $O(n)$ – Array is already sorted, requiring only one comparison per element.
    - **Worst Case:** $O(n^2)$ – Array is reverse sorted, requiring maximum comparisons and shifts.
    - **Average Case:** $O(n^2)$ – Roughly $n^2/4$ comparisons and shifts for random input.
- **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (e.g., for key and index variables).

6. Code Implementation:

```
#include<iostream>
using namespace std;

// Function to perform Insertion Sort and show the process
void insertionSort(int arr[], int n) {
    int step = 1; // Initialize step counter
    // Traverse through all array elements starting from the second
element
    for (int i = 1; i < n; i++) {
        int key = arr[i]; // Element to insert
        int j = i - 1;    // Last index of sorted portion
        cout << "Step " << step << ": Inserting key: " << key << endl;

        // Shift elements that are greater than key
        while (j >= 0 && arr[j] > key) {
            cout << "Comparing " << arr[j] << " and " << key << endl;
            cout << "Shifting " << arr[j] << " to the right" << endl;
            arr[j + 1] = arr[j];
            j--;
        }
        arr[j + 1] = key; // Place key in its correct position

        // Print array after each insertion
        cout << "Array after step " << step << ": ";
        for (int k = 0; k < n; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++; // Increment step counter
    }
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;

    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform Insertion Sort
    insertionSort(arr, n);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. Output:



```
 ... College  main ?7 ~5 -5  19:20  0.049s  cd "c:\Users\firoj\
rt.cpp -o Insertion_sort } ; if ($?) { .\Insertion_sort }
Enter the number of elements: 5
Enter the elements: 88 9 45 1 66
Unsorted array: 88 9 45 1 66
Step 1: Inserting key: 9
Comparing 88 and 9
Shifting 88 to the right
Array after step 1: 9 88 45 1 66
Step 2: Inserting key: 45
Comparing 88 and 45
Shifting 88 to the right
Array after step 2: 9 45 88 1 66
Step 3: Inserting key: 1
Comparing 88 and 1
Shifting 88 to the right
Comparing 45 and 1
Shifting 45 to the right
Comparing 9 and 1
Shifting 9 to the right
Array after step 3: 1 9 45 88 66
Step 4: Inserting key: 66
Comparing 88 and 66
Shifting 88 to the right
Array after step 4: 1 9 45 66 88
Sorted array: 1 9 45 66 88
```

Lab Report 9: Design and Analysis of Merge Sort Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Merge Sort is an efficient, comparison-based sorting algorithm that uses a divide-and-conquer approach. It divides the array into two halves, recursively sorts each half, and then merges the sorted halves back together. Merge Sort is stable (preserves the relative order of equal elements) and guarantees consistent performance regardless of the input order, making it suitable for large datasets.

2. **Key Concepts:**
   - The algorithm splits the array until sub-arrays are of size 1 (trivially sorted).
   - Merging combines two sorted sub-arrays into a single sorted array by comparing elements.
   - It requires additional space for temporary arrays during merging, unlike in-place algorithms like Quick Sort.

3. **Properties of Merge Sort:**
   - Performance is consistent across best, worst, and average cases due to balanced splitting.
   - It is not an in-place algorithm, requiring $O(n)$ extra space for merging.
   - It is widely used in external sorting (e.g., sorting data too large to fit in memory).

4. **Algorithm:**

```
Algorithm: Merge_Sort (arr, left, right)
Input: An array arr, and indices left and right defining the sub-array to sort
Output: The array arr sorted in ascending order
    1. If left < right:
          • Compute mid = (left + right) / 2.
          • Call Merge_Sort(arr, left, mid) to sort the left half.
          • Call Merge_Sort(arr, mid+1, right) to sort the right half.
          • Call Merge(arr, left, mid, right) to merge the sorted halves.
    2. Return the sorted array.
```

```
Algorithm: Merge (arr, left, mid, right)
Input: An array arr, and indices left, mid, and right defining the sub-arrays
Output: The merged sub-array in sorted order
  1. Create temporary arrays L (left to mid) and R (mid+1 to right).
  2. Set i = 0 (index for L), j = 0 (index for R), k = left (index for arr).
  3. While i < size of L and j < size of R:
       o If L[i] ≤ R[j]:
           ▪ Set arr[k] = L[i] and increment i.
       o Else:
           ▪ Set arr[k] = R[j] and increment j.
       o Increment k.
  4. Copy remaining elements of L (if any) to arr.
  5. Copy remaining elements of R (if any) to arr.
```

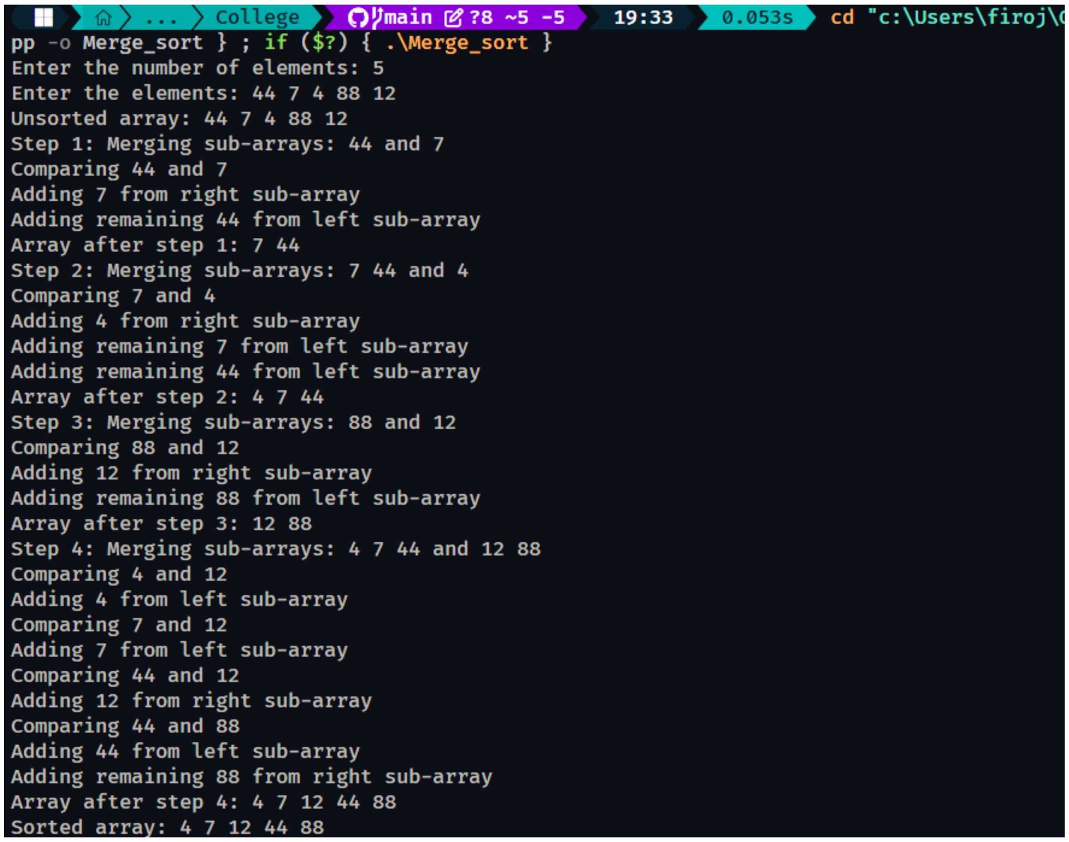
5. Time and Space Complexity Analysis:

- **Time Complexity:**
  - **Best Case:** $O(n\ log\ n)$ – Array is split into halves and merged consistently.
  - **Worst Case:** $O(n\ log\ n)$ – Same as best case, due to guaranteed balanced splits.
  - **Average Case:** $O(n\ log\ n)$ – Consistent performance across all inputs.
- **Space Complexity:** $O(n)$ – Extra space is needed for temporary arrays during merging.

6. Code Implementation:

```
/*Program that performs Merge Sort on an array with detailed step-by-
step output*/
#include<iostream>
using namespace std;

// Function to merge two sub-arrays and show the process
void merge(int arr[], int left, int mid, int right, int& step) {
    int n1 = mid - left + 1; // Size of left sub-array
    int n2 = right - mid;    // Size of right sub-array
    int L[n1], R[n2];        // Temporary arrays

    // Copy data to temporary arrays
    for (int i = 0; i < n1; i++)
        L[i] = arr[left + i];
    for (int j = 0; j < n2; j++)
        R[j] = arr[mid + 1 + j];

    cout << "Step " << step << ": Merging sub-arrays: ";
    for (int i = 0; i < n1; i++) cout << L[i] << " ";
    cout << "and ";
    for (int j = 0; j < n2; j++) cout << R[j] << " ";
    cout << endl;

    int i = 0, j = 0, k = left; // Indices for L, R, and main array
    while (i < n1 && j < n2) {
        cout << "Comparing " << L[i] << " and " << R[j] << endl;
        if (L[i] <= R[j]) {
            arr[k] = L[i];
            cout << "Adding " << L[i] << " from left sub-array" <<
endl;
            i++;
        } else {
            arr[k] = R[j];
            cout << "Adding " << R[j] << " from right sub-array" <<
endl;
            j++;
        }
        k++;
    }

    // Copy remaining elements of L[] if any
    while (i < n1) {
        arr[k] = L[i];
        cout << "Adding remaining " << L[i] << " from left sub-array"
<< endl;
        i++;
        k++;
    }

    // Copy remaining elements of R[] if any
    while (j < n2) {
        arr[k] = R[j];
        cout << "Adding remaining " << R[j] << " from right sub-array"
<< endl;
        j++;
        k++;
    }

    // Print array after merging
    cout << "Array after step " << step << ": ";
    for (int x = left; x <= right; x++)
        cout << arr[x] << " ";
    cout << endl;
    step++; // Increment step counter
}

// Function to perform Merge Sort and show the process
void mergeSort(int arr[], int left, int right, int& step) {
    if (left < right) {
        int mid = left + (right - left) / 2; // Avoid overflow
        mergeSort(arr, left, mid, step);
        mergeSort(arr, mid + 1, right, step);
        merge(arr, left, mid, right, step);
    }
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;

    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform Merge Sort
    int step = 1; // Initialize step counter
    mergeSort(arr, 0, n-1, step);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. Output:



```
   ⌂  ...  College   main  ?8 ~5 -5   19:33   0.053s   cd "c:\Users\firoj\
pp -o Merge_sort } ; if ($?) { .\Merge_sort }
Enter the number of elements: 5
Enter the elements: 44 7 4 88 12
Unsorted array: 44 7 4 88 12
Step 1: Merging sub-arrays: 44 and 7
Comparing 44 and 7
Adding 7 from right sub-array
Adding remaining 44 from left sub-array
Array after step 1: 7 44
Step 2: Merging sub-arrays: 7 44 and 4
Comparing 7 and 4
Adding 4 from right sub-array
Adding remaining 7 from left sub-array
Adding remaining 44 from left sub-array
Array after step 2: 4 7 44
Step 3: Merging sub-arrays: 88 and 12
Comparing 88 and 12
Adding 12 from right sub-array
Adding remaining 88 from left sub-array
Array after step 3: 12 88
Step 4: Merging sub-arrays: 4 7 44 and 12 88
Comparing 4 and 12
Adding 4 from left sub-array
Comparing 7 and 12
Adding 7 from left sub-array
Comparing 44 and 12
Adding 12 from right sub-array
Comparing 44 and 88
Adding 44 from left sub-array
Adding remaining 88 from right sub-array
Array after step 4: 4 7 12 44 88
Sorted array: 4 7 12 44 88
```

Lab Report 10: Design and Analysis of Min-Max Sort Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Min-Max Sort is a comparison-based sorting algorithm that improves upon Selection Sort by finding both the minimum and maximum elements in each pass through the unsorted portion of the array. It places the minimum at the left end and the maximum at the right end, reducing the unsorted portion from both sides simultaneously. This bidirectional approach can reduce the number of passes compared to standard Selection Sort, though it still involves extensive comparisons.
2. **Key Concepts:**
   - Min-Max Sort works by identifying the smallest and largest elements in each iteration.
   - It swaps these elements to their correct positions at the boundaries of the unsorted portion.
   - Like Selection Sort, it is in-place but not stable, as it may alter the relative order of equal elements.
3. **Properties of Min-Max Sort:**
   - Best case occurs when the array is already sorted, minimizing swaps.
   - Worst case occurs with a random or reverse-sorted array, requiring full comparisons and swaps.
   - It reduces the number of passes to roughly half that of Selection Sort by sorting from both ends.
4. **Algorithm:**

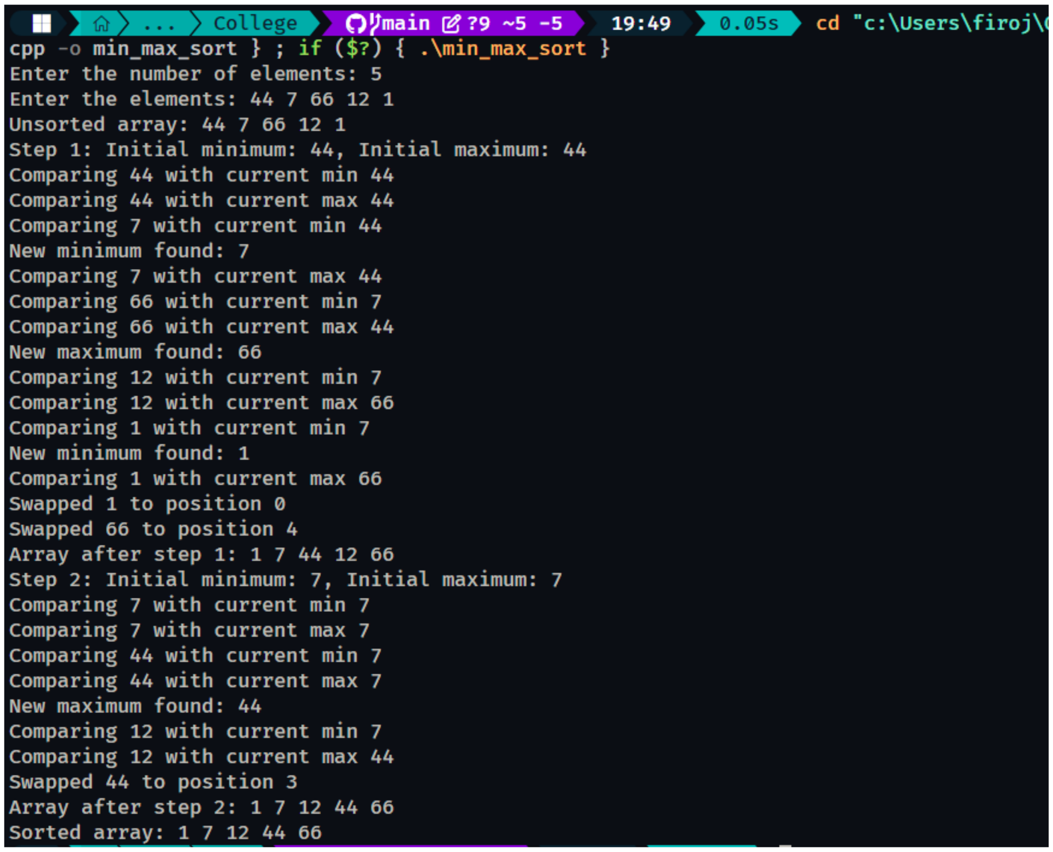
```
Algorithm: MinMax_Sort (arr, n)
Input: An array arr of size n
Output: The array arr sorted in ascending order
  1. For i = 0 to n/2:
       • Set min_idx = i and max_idx = i (initialize minimum and maximum indices).
       • For j = i to n-1-i:
           ▪ If arr[j] < arr[min_idx]:
               ▪ Set min_idx = j.
           ▪ If arr[j] > arr[max_idx]:
               ▪ Set max_idx = j.
       • If min_idx ≠ i:
           ▪ Swap arr[i] and arr[min_idx].
       • If max_idx = i (after min swap):
           ▪ Update max_idx to point to the new maximum.
       • If max_idx ≠ n-1-i:
           ▪ Swap arr[n-1-i] and arr[max_idx].
  2. Return the sorted array.
```

5. **Time and Space Complexity Analysis:**
   - **Time Complexity:**
     - **Best Case:** $O(n^2)$ – Comparisons are still performed, though swaps may be minimized.
     - **Worst Case:** $O(n^2)$ – Full comparisons and swaps for random or reverse-sorted input.
     - **Average Case:** $O(n^2)$ – Roughly $n^2/4$ comparisons, slightly better than Selection Sort due to dual sorting.
   - **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (e.g., for indices and temporary swaps).

6. **Code Implementation:**

```
#include<iostream>
using namespace std;

// Function to perform Min-Max Sort and show the process
void minMaxSort(int arr[], int n) {
    int step = 1; // Initialize step counter
    // Traverse through half the array since we sort from both ends
    for (int i = 0; i < n/2; i++) {
        int min_idx = i, max_idx = i;
        cout << "Step " << step << ": Initial minimum: " <<
arr[min_idx] << ", Initial maximum: " << arr[max_idx] << endl;

        // Find min and max in the unsorted portion
        for (int j = i; j < n-i; j++) {
            cout << "Comparing " << arr[j] << " with current min " <<
arr[min_idx] << endl;
            if (arr[j] < arr[min_idx]) {
                min_idx = j;
                cout << "New minimum found: " << arr[min_idx] << endl;
            }
            cout << "Comparing " << arr[j] << " with current max " <<
arr[max_idx] << endl;
            if (arr[j] > arr[max_idx]) {
                max_idx = j;
                cout << "New maximum found: " << arr[max_idx] << endl;
            }
        }

        // Swap minimum to the left end
        if (min_idx != i) {
            int temp = arr[i];
            arr[i] = arr[min_idx];
            arr[min_idx] = temp;
            cout << "Swapped " << arr[i] << " to position " << i <<
endl;
        }

        // Adjust max_idx if it was affected by the min swap
        if (max_idx == i) max_idx = min_idx;

        // Swap maximum to the right end
        if (max_idx != n-1-i) {
            int temp = arr[n-1-i];
            arr[n-1-i] = arr[max_idx];
            arr[max_idx] = temp;
            cout << "Swapped " << arr[n-1-i] << " to position " << n-
1-i << endl;
        }

        // Print array after each pass
        cout << "Array after step " << step << ": ";
        for (int k = 0; k < n; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++; // Increment step counter
    }
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;

    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform Min-Max Sort
    minMaxSort(arr, n);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. Output:



```
  ⌂  ...  College   main  ?9 ~5 -5   19:49   0.05s   cd "c:\Users\firoj\
cpp -o min_max_sort } ; if ($?) { .\min_max_sort }
Enter the number of elements: 5
Enter the elements: 44 7 66 12 1
Unsorted array: 44 7 66 12 1
Step 1: Initial minimum: 44, Initial maximum: 44
Comparing 44 with current min 44
Comparing 44 with current max 44
Comparing 7 with current min 44
New minimum found: 7
Comparing 7 with current max 44
Comparing 66 with current min 7
Comparing 66 with current max 44
New maximum found: 66
Comparing 12 with current min 7
Comparing 12 with current max 66
Comparing 1 with current min 7
New minimum found: 1
Comparing 1 with current max 66
Swapped 1 to position 0
Swapped 66 to position 4
Array after step 1: 1 7 44 12 66
Step 2: Initial minimum: 7, Initial maximum: 7
Comparing 7 with current min 7
Comparing 7 with current max 7
Comparing 44 with current min 7
Comparing 44 with current max 7
New maximum found: 44
Comparing 12 with current min 7
Comparing 12 with current max 44
Swapped 44 to position 3
Array after step 2: 1 7 12 44 66
Sorted array: 1 7 12 44 66
```

Lab Report 11: Design and Analysis of Binary Search Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   Binary Search is an efficient algorithm for finding a target element in a sorted array. It works by repeatedly dividing the search range in half, comparing the target with the middle element, and then narrowing the search to the left or right half based on the comparison. Binary Search is significantly faster than Linear Search for large datasets but requires the array to be sorted beforehand.

2. **Key Concepts:**
   - Binary Search relies on the array being sorted in ascending order.
   - It eliminates half of the remaining elements with each comparison, making it logarithmic in complexity.
   - It can be implemented iteratively or recursively; the iterative version is often preferred for simplicity and space efficiency.

3. **Properties of Binary Search:**
   - Best case occurs when the target is the middle element of the initial range.
   - Worst case occurs when the target is at the edges or not present, requiring maximum divisions.
   - It is not suitable for unsorted arrays or dynamic data without preprocessing.

4. **Algorithm:**

Algorithm: Binary_Search (arr, n, target)

Input: A sorted array arr of size n, and a target value target
Output: The index of target in arr, or -1 if not found

```
1. Set left = 0 and right = n-1 (initial search range).
2. While left ≤ right:
       • Compute mid = (left + right) / 2.
       • If arr[mid] = target:
             ▪ Return mid.
       • If arr[mid] > target:
             ▪ Set right = mid - 1 (search left half).
       • Else:
             ▪ Set left = mid + 1 (search right half).
3. Return -1 (target not found).
```

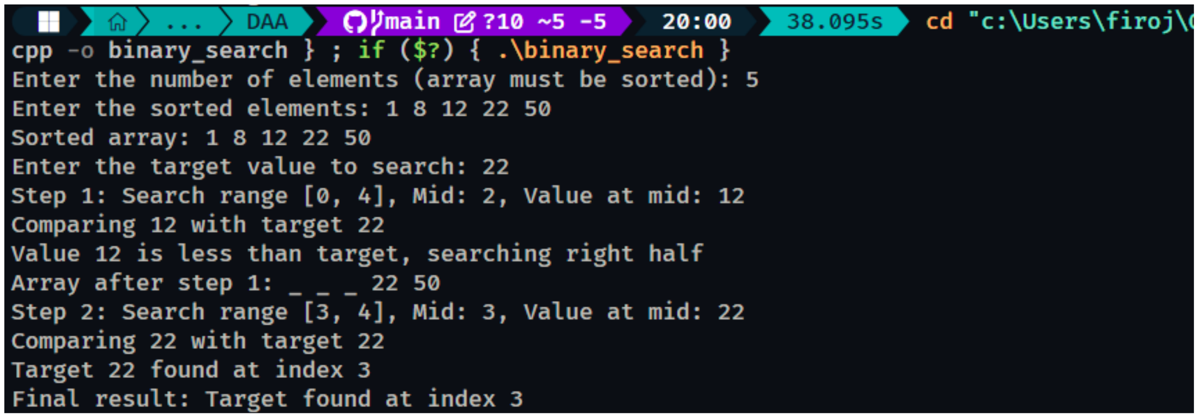
5. **Time and Space Complexity Analysis:**
   - **Time Complexity:**
     - **Best Case:** $O(1)$ – Target is found at the first middle element.
     - **Worst Case:** $O(log\ n)$ – Search range halves until the target is found or exhausted.
     - **Average Case:** $O(log\ n)$ – Roughly $log_2(n)$ comparisons for a random target.
   - **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (e.g., for indices).

6. **Code Implementation:**

```
#include<iostream>
using namespace std;

// Function to perform Binary Search and show the process
int binarySearch(int arr[], int n, int target) {
    int step = 1; // Initialize step counter
    int left = 0, right = n - 1;

    while (left <= right) {
        int mid = left + (right - left) / 2; // Avoid overflow
        cout << "Step " << step << ": Search range [" << left << ", " 
<< right << "], Mid: " << mid
             << ", Value at mid: " << arr[mid] << endl;

        // Print comparison
        cout << "Comparing " << arr[mid] << " with target " << target 
<< endl;

        if (arr[mid] == target) {
            cout << "Target " << target << " found at index " << mid 
<< endl;
            return mid;
        }
        else if (arr[mid] > target) {
            cout << "Value " << arr[mid] << " is greater than target, 
searching left half" << endl;
            right = mid - 1;
        }
        else {
            cout << "Value " << arr[mid] << " is less than target, 
searching right half" << endl;
            left = mid + 1;
        }

        // Print array with current search range
        cout << "Array after step " << step << ": ";
```

```cpp
        for (int i = 0; i < n; i++) {
            if (i >= left && i <= right)
                cout << arr[i] << " "; // Highlight current range
            else
                cout << "_ ";          // Indicate out-of-range
elements
        }
        cout << endl;
        step++; // Increment step counter
    }

    cout << "Target " << target << " not found in the array" << endl;
    return -1;
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n, target;
    cout << "Enter the number of elements (array must be sorted): ";
    cin >> n;

    int arr[n];
    cout << "Enter the sorted elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print the array
    cout << "Sorted array: ";
    printArray(arr, n);

    cout << "Enter the target value to search: ";
    cin >> target;

    // Perform Binary Search
    int result = binarySearch(arr, n, target);

    if (result != -1)
        cout << "Final result: Target found at index " << result <<
endl;
    else
        cout << "Final result: Target not found" << endl;

    return 0;
}
```

7. **Output:**



```
DAA  main ?10 ~5 -5  20:00  38.095s  cd "c:\Users\firoj\
cpp -o binary_search } ; if ($?) { .\binary_search }
Enter the number of elements (array must be sorted): 5
Enter the sorted elements: 1 8 12 22 50
Sorted array: 1 8 12 22 50
Enter the target value to search: 22
Step 1: Search range [0, 4], Mid: 2, Value at mid: 12
Comparing 12 with target 22
Value 12 is less than target, searching right half
Array after step 1: _ _ _ 22 50
Step 2: Search range [3, 4], Mid: 3, Value at mid: 22
Comparing 22 with target 22
Target 22 found at index 3
Final result: Target found at index 3
```

Lab Report 12: Design and Analysis of Fractional Knapsack Algorithm

Date: April 01, 2025 **Day:** Tuesday

1. **Theory:**
   The Fractional Knapsack problem involves maximizing the total value of items placed in a knapsack of limited capacity, where fractions of items can be taken. It is solved using a greedy approach: items are sorted by their value-to-weight ratio in descending order, and the knapsack is filled by taking as much as possible of the highest-ratio items first. This contrasts with the 0/1 Knapsack problem, which requires dynamic programming due to its discrete nature.

2. **Key Concepts:**
   - The algorithm prioritizes items based on their value per unit weight (value/weight ratio).
   - It allows partial inclusion of items, making it greedy and optimal for this specific problem.
   - Sorting by ratio ensures the maximum possible value is achieved within the capacity constraint.
3. **Properties of Fractional Knapsack:**
   - Best case occurs when all items can be fully included without exceeding capacity.
   - Worst case involves taking fractions of multiple items, but the greedy choice remains optimal.
   - It assumes items can be divided continuously (e.g., like grains or liquids), unlike discrete objects.
4. **Algorithm:**

```
Algorithm: Fractional_Knapsack (items, n, capacity)
Input: An array items of size n (each with value and weight), and knapsack capacity capacity
Output: The maximum value achievable
  1. For each item in items:
        • Compute ratio = value / weight.
  2. Sort items by ratio in descending order.
  3. Set current_weight = 0 and total_value = 0.
  4. For i = 0 to n − 1:
        • If current_weight + items[i].weight ≤ capacity:
              ▪ Add items[i].value to total_value.
              ▪ Add items[i].weight to current_weight.
        • Else:
              ▪ Compute fraction = (capacity − current_weight) / items[i].weight.
              ▪ Add fraction * items[i].value to total_value.
              ▪ Break (knapsack is full).
  5. Return total_value.
```

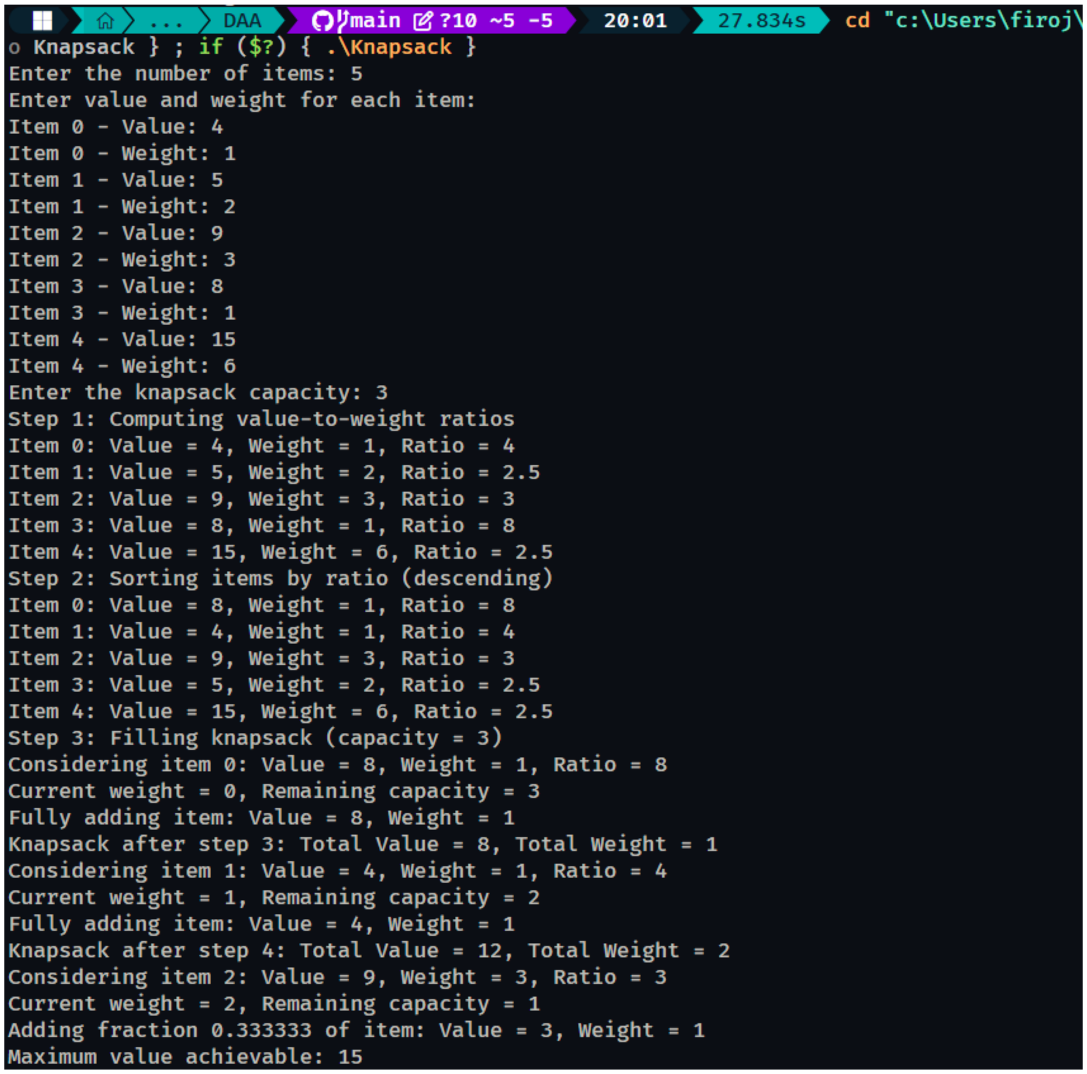
5. **Time and Space Complexity Analysis:**
   - **Time Complexity:**
     - $O(n\ log\ n)$ – Dominated by the sorting step; the greedy selection is O(n).
     - **Best/Worst/Average Cases** are all $O(n\ log\ n)$ due to consistent sorting overhead.
   - **Space Complexity:** $O(1)$ – Only a constant amount of extra space is used (excluding input storage), assuming in-place sorting.

6. **Code Implementation:**

```
#include<iostream>
#include<algorithm> // For sort
using namespace std;

// Structure to represent an item
struct Item {
    int value, weight;
    double ratio; // Value-to-weight ratio
};

// Function to compare items by ratio for sorting
bool compare(Item a, Item b) {
    return a.ratio > b.ratio; // Descending order
}

// Function to solve Fractional Knapsack and show the process
double fractionalKnapsack(Item arr[], int n, int capacity) {
    int step = 1; // Initialize step counter
    double total_value = 0.0;
    int current_weight = 0;

    // Step 1: Compute value-to-weight ratios
    cout << "Step " << step << ": Computing value-to-weight ratios" <<
endl;
    for (int i = 0; i < n; i++) {
        arr[i].ratio = (double)arr[i].value / arr[i].weight;
        cout << "Item " << i << ": Value = " << arr[i].value << ",
Weight = " << arr[i].weight
             << ", Ratio = " << arr[i].ratio << endl;
    }
    step++;

    // Step 2: Sort items by ratio
    cout << "Step " << step << ": Sorting items by ratio (descending)"
<< endl;
    sort(arr, arr + n, compare);
    for (int i = 0; i < n; i++) {
        cout << "Item " << i << ": Value = " << arr[i].value << ",
Weight = " << arr[i].weight
```

```
             << ", Ratio = " << arr[i].ratio << endl;
    }
    step++;

    // Step 3: Fill the knapsack
    cout << "Step " << step << ": Filling knapsack (capacity = " <<
capacity << ")" << endl;
    for (int i = 0; i < n; i++) {
        cout << "Considering item " << i << ": Value = " <<
arr[i].value << ", Weight = " << arr[i].weight
             << ", Ratio = " << arr[i].ratio << endl;
        cout << "Current weight = " << current_weight << ", Remaining
capacity = " << capacity - current_weight << endl;

        if (current_weight + arr[i].weight <= capacity) {
            current_weight += arr[i].weight;
            total_value += arr[i].value;
            cout << "Fully adding item: Value = " << arr[i].value <<
", Weight = " << arr[i].weight << endl;
        } else {
            double fraction = (double)(capacity - current_weight) /
arr[i].weight;
            total_value += arr[i].value * fraction;
            current_weight = capacity; // Knapsack is full
            cout << "Adding fraction " << fraction << " of item: Value
= " << arr[i].value * fraction
                 << ", Weight = " << capacity - current_weight +
arr[i].weight * fraction << endl;
            break; // No more capacity
        }

        cout << "Knapsack after step " << step << ": Total Value = "
<< total_value
             << ", Total Weight = " << current_weight << endl;
        step++;
    }

    return total_value;
}

int main() {
    int n, capacity;
    cout << "Enter the number of items: ";
    cin >> n;

    Item arr[n];
    cout << "Enter value and weight for each item:" << endl;
    for (int i = 0; i < n; i++) {
        cout << "Item " << i << " - Value: ";
        cin >> arr[i].value;
        cout << "Item " << i << " - Weight: ";
        cin >> arr[i].weight;
    }

    cout << "Enter the knapsack capacity: ";
    cin >> capacity;

    // Perform Fractional Knapsack
    double max_value = fractionalKnapsack(arr, n, capacity);

    // Print final result
    cout << "Maximum value achievable: " << max_value << endl;

    return 0;
}
```

7. **Output:**



```
 ⌂ ... DAA  main ?10 ~5 -5  20:01  27.834s  cd "c:\Users\firoj\
o Knapsack } ; if ($?) { .\Knapsack }
Enter the number of items: 5
Enter value and weight for each item:
Item 0 - Value: 4
Item 0 - Weight: 1
Item 1 - Value: 5
Item 1 - Weight: 2
Item 2 - Value: 9
Item 2 - Weight: 3
Item 3 - Value: 8
Item 3 - Weight: 1
Item 4 - Value: 15
Item 4 - Weight: 6
Enter the knapsack capacity: 3
Step 1: Computing value-to-weight ratios
Item 0: Value = 4, Weight = 1, Ratio = 4
Item 1: Value = 5, Weight = 2, Ratio = 2.5
Item 2: Value = 9, Weight = 3, Ratio = 3
Item 3: Value = 8, Weight = 1, Ratio = 8
Item 4: Value = 15, Weight = 6, Ratio = 2.5
Step 2: Sorting items by ratio (descending)
Item 0: Value = 8, Weight = 1, Ratio = 8
Item 1: Value = 4, Weight = 1, Ratio = 4
Item 2: Value = 9, Weight = 3, Ratio = 3
Item 3: Value = 5, Weight = 2, Ratio = 2.5
Item 4: Value = 15, Weight = 6, Ratio = 2.5
Step 3: Filling knapsack (capacity = 3)
Considering item 0: Value = 8, Weight = 1, Ratio = 8
Current weight = 0, Remaining capacity = 3
Fully adding item: Value = 8, Weight = 1
Knapsack after step 3: Total Value = 8, Total Weight = 1
Considering item 1: Value = 4, Weight = 1, Ratio = 4
Current weight = 1, Remaining capacity = 2
Fully adding item: Value = 4, Weight = 1
Knapsack after step 4: Total Value = 12, Total Weight = 2
Considering item 2: Value = 9, Weight = 3, Ratio = 3
Current weight = 2, Remaining capacity = 1
Adding fraction 0.333333 of item: Value = 3, Weight = 1
Maximum value achievable: 15
```

Lab Report 13: Design and Analysis of Job Sequencing with Deadline Algorithm

Date: April 01, 2025 **Day:** Tuesday

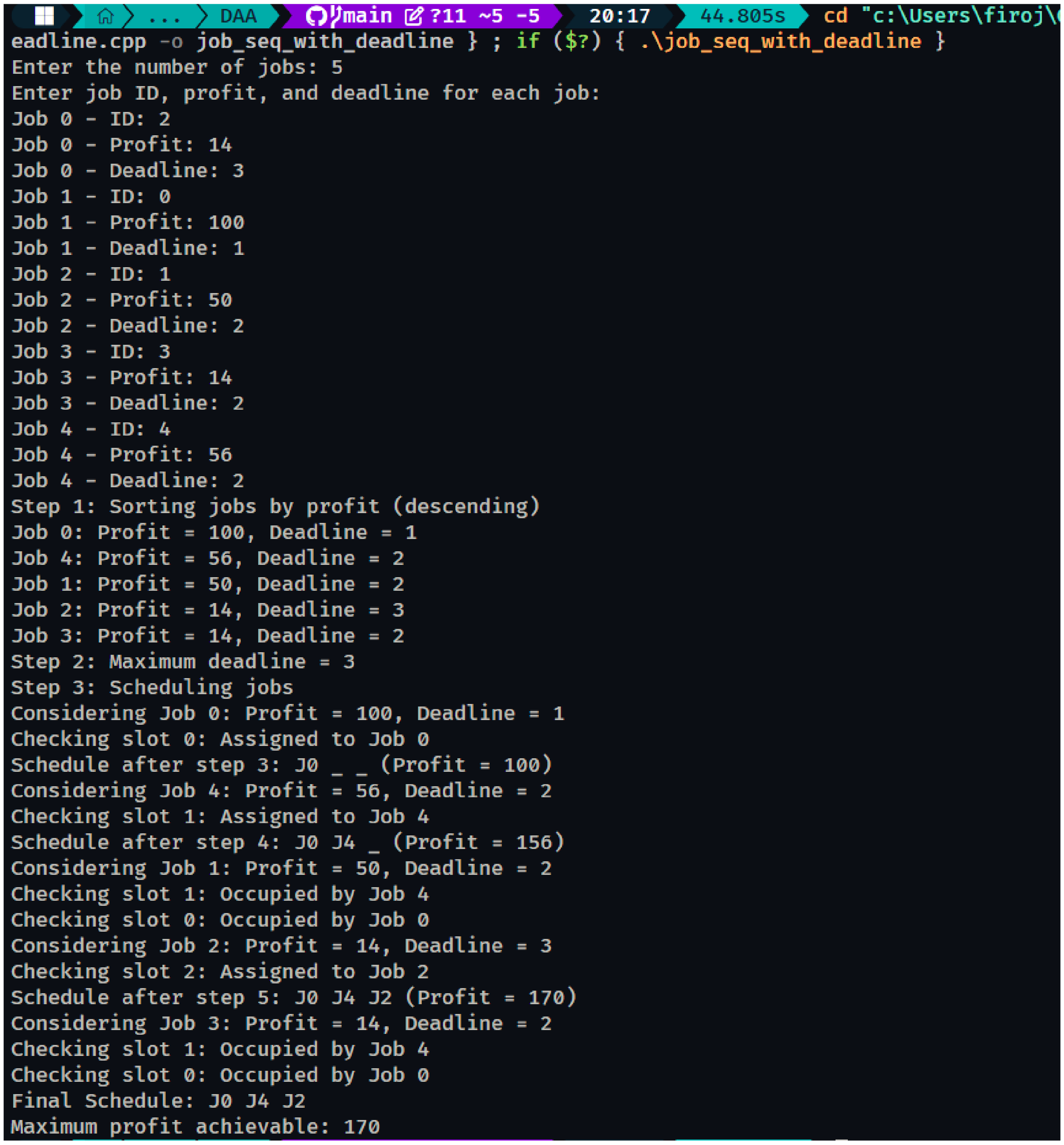
1. **Theory:**
   The Job Sequencing with Deadlines problem involves scheduling a set of jobs, each with a profit and a deadline, to maximize total profit. The algorithm assumes each job takes one unit of time, and a job must be completed by its deadline to earn its profit. A greedy approach is used: jobs are sorted by profit in descending order, and each job is scheduled in the latest possible time slot before its deadline, ensuring maximum utilization of available time.
2. **Key Concepts:**
   - Jobs are prioritized based on profit, favoring higher-profit jobs first.
   - Time slots are filled from the latest possible position backward to accommodate as many jobs as possible.
   - The algorithm assumes deadlines are given as integers representing the maximum time units allowed (e.g., deadline 2 means the job must be completed by time 2).
3. **Properties of Job Sequencing with Deadlines:**
   - Best case occurs when all jobs can be scheduled within their deadlines.
   - Worst case occurs when deadlines are tight, limiting the number of schedulable jobs.
   - The greedy choice of highest profit ensures an optimal solution for this problem.
4. **Algorithm:**

```
Algorithm: Job_Sequencing (jobs, n)
Input: An array jobs of size n (each with profit and deadline)
Output: A sequence of jobs maximizing total profit
    1. Sort jobs by profit in descending order.
    2. Find max_deadline (the maximum deadline among all jobs).
    3. Initialize an array slots of size max_deadline with -1 (empty slots).
    4. Set total_profit = 0.
    5. For i = 0 to n − 1:
          • For j = min(max_deadline, jobs[i].deadline) - 1 down to 0:
                ▪ If slots[j] = -1 (slot is free):
                      ▪ Set slots[j] = i (assign job to slot).
                      ▪ Add jobs[i].profit to total_profit.
                      ▪ Break (job scheduled).
    6. Return slots and total_profit.
```

5. **Time and Space Complexity Analysis:**
   - **Time Complexity:**
     - $O(n\, log\, n + n * d)$ – Sorting takes $O(n\, log\, n)$, and scheduling takes $O(n * d)$ where d is the maximum deadline.
     - **Best/Worst/Average Cases** depend on d, but sorting dominates unless d is very large.
   - **Space Complexity:** $O(d)$ – Extra space for the slots array, where d is the maximum deadline.
6. **Code Implementation:**

```
#include<iostream>
#include<algorithm> // For sort
using namespace std;

// Structure to represent a job
struct Job {
    int id, profit, deadline;
};

// Function to compare jobs by profit for sorting
bool compare(Job a, Job b) {
    return a.profit > b.profit; // Descending order
}

// Function to solve Job Sequencing and show the process
void jobSequencing(Job arr[], int n) {
    int step = 1; // Initialize step counter
    int total_profit = 0;

    // Step 1: Sort jobs by profit
    cout << "Step " << step << ": Sorting jobs by profit (descending)"
<< endl;
    sort(arr, arr + n, compare);
    for (int i = 0; i < n; i++) {
        cout << "Job " << arr[i].id << ": Profit = " << arr[i].profit
             << ", Deadline = " << arr[i].deadline << endl;
    }
    step++;

    // Step 2: Find maximum deadline
    int max_deadline = 0;
    for (int i = 0; i < n; i++) {
        if (arr[i].deadline > max_deadline)
            max_deadline = arr[i].deadline;
    }
    cout << "Step " << step << ": Maximum deadline = " << max_deadline
<< endl;
    step++;
    // Step 3: Initialize slots
    int slots[max_deadline];
    for (int i = 0; i < max_deadline; i++)
        slots[i] = -1; // -1 indicates empty slot

    // Step 4: Schedule jobs
    cout << "Step " << step << ": Scheduling jobs" << endl;
    for (int i = 0; i < n; i++) {
        cout << "Considering Job " << arr[i].id << ": Profit = " <<
arr[i].profit
             << ", Deadline = " << arr[i].deadline << endl;

        // Try to schedule job in the latest possible slot
        for (int j = min(max_deadline, arr[i].deadline) - 1; j >= 0;
j--) {
            cout << "Checking slot " << j << ": ";
            if (slots[j] == -1) {
                slots[j] = arr[i].id;
                total_profit += arr[i].profit;
                cout << "Assigned to Job " << arr[i].id << endl;

                // Print current schedule
                cout << "Schedule after step " << step << ": ";
                for (int k = 0; k < max_deadline; k++) {
                    if (slots[k] == -1) cout << "_ ";
                    else cout << "J" << slots[k] << " ";
                }
                cout << "(Profit = " << total_profit << ")" << endl;
                step++;
                break;
            } else {
                cout << "Occupied by Job " << slots[j] << endl;
            }
        }
    }

    // Print final schedule
    cout << "Final Schedule: ";
    for (int i = 0; i < max_deadline; i++) {
        if (slots[i] != -1)
            cout << "J" << slots[i] << " ";
    }
    cout << endl << "Maximum profit achievable: " << total_profit <<
endl;
}

int main() {
    int n;
    cout << "Enter the number of jobs: ";
    cin >> n;

    Job arr[n];
    cout << "Enter job ID, profit, and deadline for each job:" <<
endl;
    for (int i = 0; i < n; i++) {
        cout << "Job " << i << " - ID: ";
        cin >> arr[i].id;
        cout << "Job " << i << " - Profit: ";
        cin >> arr[i].profit;
        cout << "Job " << i << " - Deadline: ";
        cin >> arr[i].deadline;
    }

    // Perform Job Sequencing
    jobSequencing(arr, n);

    return 0;
}
```

7. Output:



```
 ⌂ ...  DAA  main ?11 ~5 -5  20:17  44.805s  cd "c:\Users\firoj\
eadline.cpp -o job_seq_with_deadline } ; if ($?) { .\job_seq_with_deadline }
Enter the number of jobs: 5
Enter job ID, profit, and deadline for each job:
Job 0 - ID: 2
Job 0 - Profit: 14
Job 0 - Deadline: 3
Job 1 - ID: 0
Job 1 - Profit: 100
Job 1 - Deadline: 1
Job 2 - ID: 1
Job 2 - Profit: 50
Job 2 - Deadline: 2
Job 3 - ID: 3
Job 3 - Profit: 14
Job 3 - Deadline: 2
Job 4 - ID: 4
Job 4 - Profit: 56
Job 4 - Deadline: 2
Step 1: Sorting jobs by profit (descending)
Job 0: Profit = 100, Deadline = 1
Job 4: Profit = 56, Deadline = 2
Job 1: Profit = 50, Deadline = 2
Job 2: Profit = 14, Deadline = 3
Job 3: Profit = 14, Deadline = 2
Step 2: Maximum deadline = 3
Step 3: Scheduling jobs
Considering Job 0: Profit = 100, Deadline = 1
Checking slot 0: Assigned to Job 0
Schedule after step 3: J0 _ _ (Profit = 100)
Considering Job 4: Profit = 56, Deadline = 2
Checking slot 1: Assigned to Job 4
Schedule after step 4: J0 J4 _ (Profit = 156)
Considering Job 1: Profit = 50, Deadline = 2
Checking slot 1: Occupied by Job 4
Checking slot 0: Occupied by Job 0
Considering Job 2: Profit = 14, Deadline = 3
Checking slot 2: Assigned to Job 2
Schedule after step 5: J0 J4 J2 (Profit = 170)
Considering Job 3: Profit = 14, Deadline = 2
Checking slot 1: Occupied by Job 4
Checking slot 0: Occupied by Job 0
Final Schedule: J0 J4 J2
Maximum profit achievable: 170
```

Lab Report 14: Design and Analysis of Heap Sort Algorithm

Date: April 06, 2025 **Day:** Sunday

1. Theory:

Heap Sort is an efficient sorting algorithm that leverages a binary max-heap (or min-heap) to sort an array. It first builds a max-heap from the array, where the largest element is at the root. Then, it repeatedly extracts the maximum element (root), places it at the end of the array, and reduces the heap size, restoring the heap property each time. This process sorts the array in ascending order. Heap Sort is in-place and has consistent performance, making it reliable for various input sizes.

2. Key Concepts:

- A max-heap ensures the parent node is greater than or equal to its children.
- The algorithm has two phases: building the heap and extracting elements to sort.
- It is not stable (may change the relative order of equal elements) but is in-place, requiring no extra array storage beyond the heap structure.

3. Properties of Heap Sort:

- Best, worst, and average cases have the same time complexity due to the heap-building and extraction process.
- It performs well for large datasets but is slower than Quick Sort in practice due to poor cache locality.
- The heapify operation ensures the heap property is maintained after each swap.

4. Algorithm:

```
Algorithm: Heap_Sort (arr, n)
Input: An array arr of size $n$
Output: The array arr sorted in ascending order
    1. For i = $n/2 - 1$ down to 0:
        o Call Heapify(arr, n, i) to build a max-heap.
    2. For i = $n - 1$ down to 1:
        o Swap arr[0] (maximum) with arr[i] (move to sorted portion).
        o Call Heapify(arr, i, 0) to restore heap property on reduced heap.
    3. Return the sorted array.
```

```
Algorithm: Heapify (arr, n, i)
Input: An array arr, size $n$, and index $i$ to heapify
Output: A max-heap rooted at $i$
    1. Set largest = $i$ (root of subtree).
    2. Set left = $2i + 1$* and right = $2i + 2$* (children indices).
    3. If left < n* and *arr[left] > arr[largest]:
        o Set largest = left.
    4. If right < n* and *arr[right] > arr[largest]:
        o Set largest = right.
    5. If largest ≠ i:
        o Swap arr[i] and arr[largest].
        o Call Heapify(arr, n, largest) recursively.
```

5. Time and Space Complexity Analysis:

- **Time Complexity:**

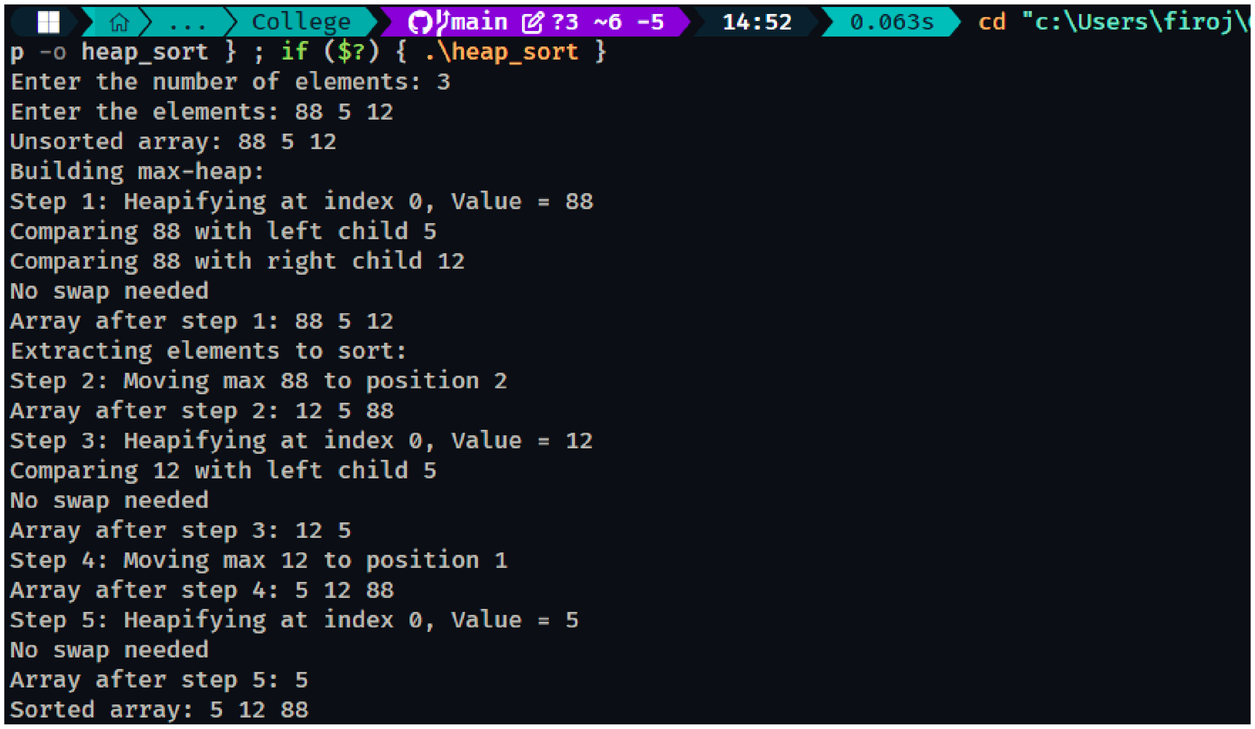
- **Best Case:** $O(n\ log\ n)$ – Building the heap is $O(n)$, and n extractions are $O(log\ n)$ each.
- **Worst Case:** $O(n\ log\ n)$ – Same as best case, as heap operations are consistent.
- **Average Case:** $O(n\ log\ n)$ – Uniform performance across inputs.

- **Space Complexity:** $O(1)$ – In-place sorting, with only recursive stack space ($O(log\ n)$ in the worst case, but iterative versions can reduce this to $O(1)$).

6. Code Implementation:

```
#include<iostream>
using namespace std;

// Function to heapify a subtree rooted at index i
void heapify(int arr[], int n, int i, int& step) {
    int largest = i;         // Initialize largest as root
    int left = 2 * i + 1;    // Left child
    int right = 2 * i + 2;   // Right child

    cout << "Step " << step << ": Heapifying at index " << i << ", Value = " << arr[i] <<
endl;

    // Compare with left child
    if (left < n) {
        cout << "Comparing " << arr[i] << " with left child " << arr[left] << endl;
        if (arr[left] > arr[largest]) {
            largest = left;
            cout << "Left child " << arr[left] << " is larger" << endl;
        }
    }

    // Compare with right child
    if (right < n) {
        cout << "Comparing " << arr[largest] << " with right child " << arr[right] << endl;
        if (arr[right] > arr[largest]) {
            largest = right;
            cout << "Right child " << arr[right] << " is larger" << endl;
        }
    }

    // If largest is not root
    if (largest != i) {
        swap(arr[i], arr[largest]);
        cout << "Swapped " << arr[i] << " with " << arr[largest] << endl;

        // Print array after swap
        cout << "Array after step " << step << ": ";
        for (int k = 0; k < n; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++;

        // Recursively heapify the affected subtree
        heapify(arr, n, largest, step);
    } else {
        cout << "No swap needed" << endl;
        cout << "Array after step " << step << ": ";
        for (int k = 0; k < n; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++;
    }
}

// Function to perform Heap Sort and show the process
void heapSort(int arr[], int n) {
    int step = 1; // Initialize step counter

    // Step 1: Build max-heap
    cout << "Building max-heap:" << endl;
    for (int i = n / 2 - 1; i >= 0; i--) {
        heapify(arr, n, i, step);
    }

    // Step 2: Extract elements from heap
    cout << "Extracting elements to sort:" << endl;
    for (int i = n - 1; i > 0; i--) {
        cout << "Step " << step << ": Moving max " << arr[0] << " to position " << i <<
endl;
        swap(arr[0], arr[i]);

        // Print array after swap
        cout << "Array after step " << step << ": ";
        for (int k = 0; k < n; k++)
            cout << arr[k] << " ";
        cout << endl;
        step++;

        // Heapify the reduced heap
        heapify(arr, i, 0, step);
    }
}

// Function to print an array
void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int n;
    cout << "Enter the number of elements: ";
    cin >> n;
    int arr[n];
    cout << "Enter the elements: ";
    for (int i = 0; i < n; i++) {
        cin >> arr[i];
    }

    // Print unsorted array
    cout << "Unsorted array: ";
    printArray(arr, n);

    // Perform Heap Sort
    heapSort(arr, n);

    // Print sorted array
    cout << "Sorted array: ";
    printArray(arr, n);

    return 0;
}
```

7. Output:



```
 ⌂  ...  College   main  ?3 ~6 -5   14:52   0.063s   cd "c:\Users\firoj\
p -o heap_sort } ; if ($?) { .\heap_sort }
Enter the number of elements: 3
Enter the elements: 88 5 12
Unsorted array: 88 5 12
Building max-heap:
Step 1: Heapifying at index 0, Value = 88
Comparing 88 with left child 5
Comparing 88 with right child 12
No swap needed
Array after step 1: 88 5 12
Extracting elements to sort:
Step 2: Moving max 88 to position 2
Array after step 2: 12 5 88
Step 3: Heapifying at index 0, Value = 12
Comparing 12 with left child 5
No swap needed
Array after step 3: 12 5
Step 4: Moving max 12 to position 1
Array after step 4: 5 12 88
Step 5: Heapifying at index 0, Value = 5
No swap needed
Array after step 5: 5
Sorted array: 5 12 88
```

Lab Report 15: Design and Analysis of Kruskal's Algorithm

Date: April 06, 2025 **Day:** Sunday

1. **Theory:**
   Kruskal's Algorithm is a greedy algorithm that finds the Minimum Spanning Tree (MST) of a connected, undirected graph with weighted edges. The MST is a subset of edges that connects all vertices with the minimum total edge weight and no cycles. The algorithm sorts all edges by weight, then iteratively adds the smallest edge to the MST if it doesn't form a cycle, using a disjoint-set data structure to detect cycles efficiently.

2. **Key Concepts:**
   - Edges are processed in non-decreasing order of weight.
   - A disjoint-set (union-find) structure tracks connected components to avoid cycles.
   - The algorithm ensures the MST spans all vertices with the least possible total weight.

3. **Properties of Kruskal's Algorithm:**
   - It works on connected graphs and produces a unique MST if all edge weights are distinct.
   - Best case occurs with already sorted edges or a sparse graph; worst case involves dense graphs.
   - It's optimal for finding the MST in terms of total weight.

4. **Algorithm:**

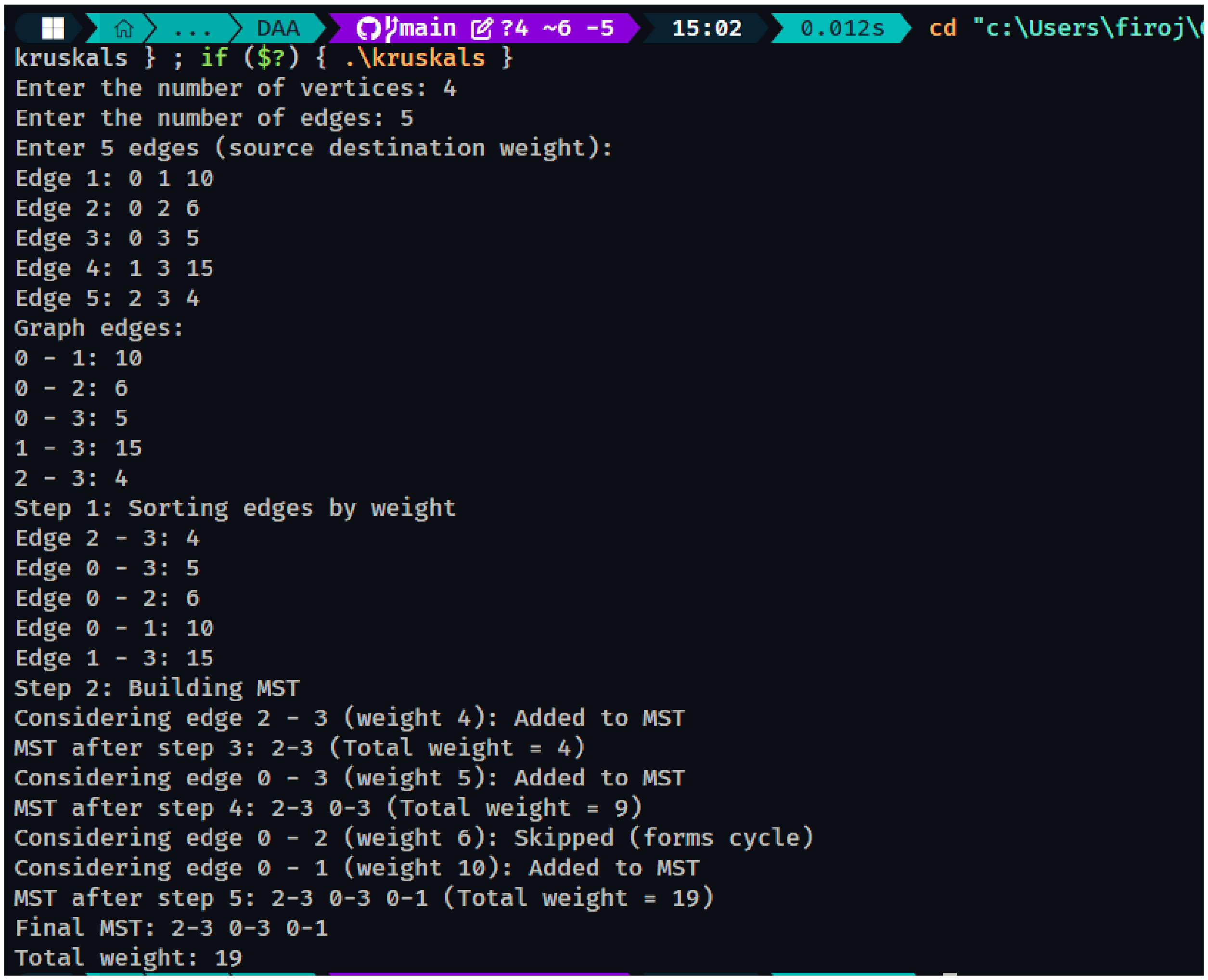
```
Algorithm: Kruskal_MST (graph, V, E)
Input: A graph with V vertices and E edges (each with source, destination, weight)
Output: The Minimum Spanning Tree edges and total weight
    1. Sort all edges in graph by weight in ascending order.
    2. Initialize a disjoint-set for V vertices (each vertex in its own set).
    3. Set mst_edges = empty and mst_weight = 0.
    4. For each edge (u, v, w) in sorted order:
           o  If find(u) ≠ find(v) (no cycle formed):
                  ▪  Add edge (u, v) to mst_edges.
                  ▪  Add w to mst_weight.
                  ▪  Union u and v in the disjoint-set.
    5. Return mst_edges and mst_weight.
```

5. **Time and Space Complexity Analysis:**
   - **Time Complexity:**
     - $O(E \log E)$ or $O(E \log V)$ – Sorting edges takes $O(E \log E)$, and union-find operations take $O(\alpha(V))$ per edge, where α is the inverse Ackermann function *(nearly constant)*.
     - **Best/Worst/Average Cases** are dominated by sorting, so $O(E \log E)$.
   - **Space Complexity:** $O(V + E)$ – Space for the edge list and disjoint-set structure.
6. **Code Implementation:**

```
#include<iostream>
#include<algorithm> // For sort
using namespace std;

// Structure to represent an edge
struct Edge {
    int src, dest, weight;
};

// Structure for disjoint-set
struct DisjointSet {
    int *parent, *rank;
    int n;

    DisjointSet(int n) {
        this->n = n;
        parent = new int[n];
        rank = new int[n];
        for (int i = 0; i < n; i++) {
            parent[i] = i; // Each vertex is its own parent
            rank[i] = 0;
        }
    }

    int find(int u) {
        if (parent[u] != u)
            parent[u] = find(parent[u]); // Path compression
        return parent[u];
    }

    void unionSet(int u, int v) {
        int pu = find(u), pv = find(v);
        if (pu == pv) return;
        if (rank[pu] < rank[pv])
            parent[pu] = pv;
        else if (rank[pu] > rank[pv])
            parent[pv] = pu;
        else {
            parent[pv] = pu;
            rank[pu]++;
        }
    }
};

// Function to compare edges by weight
bool compare(Edge a, Edge b) {
    return a.weight < b.weight;
}

// Function to perform Kruskal's Algorithm with concise output
```

```
void kruskalMST(Edge edges[], int V, int E) {
    int step = 1;

    // Step 1: Sort edges by weight
    cout << "Step " << step++ << ": Sorting edges by weight" << endl;
    sort(edges, edges + E, compare);
    for (int i = 0; i < E; i++)
        cout << "Edge " << edges[i].src << " - " << edges[i].dest << ": " <<
edges[i].weight << endl;

    // Step 2: Initialize disjoint-set and MST
    DisjointSet ds(V);
    Edge mst[V - 1]; // MST will have V-1 edges
    int mst_weight = 0, mst_index = 0;

    // Step 3: Process edges
    cout << "Step " << step++ << ": Building MST" << endl;
    for (int i = 0 ;i < E && mst_index < V - 1; i++) {
        int u = edges[i].src, v = edges[i].dest, w = edges[i].weight;
        cout << "Considering edge " << u << " - " << v << " (weight " << w << "): ";

        if (ds.find(u) != ds.find(v)) {
            mst[mst_index++] = edges[i];
            mst_weight += w;
            ds.unionSet(u, v);
            cout << "Added to MST" << endl;
            cout << "MST after step " << step++ << ": ";
            for (int j = 0; j < mst_index; j++)
                cout << mst[j].src << "-" << mst[j].dest << " ";
            cout << "(Total weight = " << mst_weight << ")" << endl;
        } else {
            cout << "Skipped (forms cycle)" << endl;
        }
    }

    // Print final MST
    cout << "Final MST: ";
    for (int i = 0; i < V - 1; i++)
        cout << mst[i].src << "-" << mst[i].dest << " ";
    cout << endl << "Total weight: " << mst_weight << endl;
}

int main() {
    int V, E;
    cout << "Enter the number of vertices: ";
    cin >> V;
    cout << "Enter the number of edges: ";
    cin >> E;

    Edge edges[E];
    cout << "Enter " << E << " edges (source destination weight):" << endl;
    for (int i = 0; i < E; i++) {
        cout << "Edge " << i + 1 << ": ";
        cin >> edges[i].src >> edges[i].dest >> edges[i].weight;
    }

    cout << "Graph edges:" << endl;
    for (int i = 0; i < E; i++)
        cout << edges[i].src << " - " << edges[i].dest << ": " << edges[i].weight << endl;

    // Perform Kruskal's Algorithm
    kruskalMST(edges, V, E);

    return 0;
}
```

7. **Output:**



```
 ⌂ ... DAA  main ?4 ~6 -5  15:02  0.012s  cd "c:\Users\firoj\
kruskals } ; if ($?) { .\kruskals }
Enter the number of vertices: 4
Enter the number of edges: 5
Enter 5 edges (source destination weight):
Edge 1: 0 1 10
Edge 2: 0 2 6
Edge 3: 0 3 5
Edge 4: 1 3 15
Edge 5: 2 3 4
Graph edges:
0 - 1: 10
0 - 2: 6
0 - 3: 5
1 - 3: 15
2 - 3: 4
Step 1: Sorting edges by weight
Edge 2 - 3: 4
Edge 0 - 3: 5
Edge 0 - 2: 6
Edge 0 - 1: 10
Edge 1 - 3: 15
Step 2: Building MST
Considering edge 2 - 3 (weight 4): Added to MST
MST after step 3: 2-3 (Total weight = 4)
Considering edge 0 - 3 (weight 5): Added to MST
MST after step 4: 2-3 0-3 (Total weight = 9)
Considering edge 0 - 2 (weight 6): Skipped (forms cycle)
Considering edge 0 - 1 (weight 10): Added to MST
MST after step 5: 2-3 0-3 0-1 (Total weight = 19)
Final MST: 2-3 0-3 0-1
Total weight: 19
```

Lab Report 16: Design and Analysis of Prims' Algorithm

Date: April 06, 2025 **Day:** Sunday

1. Theory:

Prim's Algorithm is a greedy algorithm that constructs the Minimum Spanning Tree (MST) of a connected, undirected graph with weighted edges. Starting from an arbitrary vertex, it incrementally adds the edge with the smallest weight that connects a visited vertex to an unvisited one, ensuring no cycles are formed. The result is a tree that spans all vertices with the minimum total edge weight.

2. Key Concepts:

- It grows the MST one edge at a time, always choosing the minimum-weight edge to an unvisited vertex.
- A visited set (or key array) tracks which vertices are included, avoiding cycles implicitly.
- It's particularly efficient for dense graphs when implemented with a priority queue (though we'll use a simple array-based approach here for clarity).

3. Properties of Prim's Algorithm:

- It produces a unique MST if all edge weights are distinct and the graph is connected.
- Best case occurs with a sparse graph or low-degree vertices; worst case involves dense graphs.
- It's optimal for finding the MST and is often used in network optimization (e.g., laying pipelines).

4. Algorithm:

```
Algorithm: Prim_MST (graph, V)
Input: A graph with V vertices represented as an adjacency matrix
Output: The Minimum Spanning Tree edges and total weight
    1. Initialize key[V] with infinity (minimum weight to include each vertex), parent[V] with -1 (to
       store MST edges), and visited[V] with false.
    2. Set key[0] = 0 (start from vertex 0).
    3. For count = 0 to V−1:
           o Find the unvisited vertex u with the minimum key[u].
           o Mark visited[u] = true.
           o For each vertex v adjacent to u:
                 ▪ If v is unvisited and graph[u][v] < key[v]:
                       ▪ Update key[v] = graph[u][v].
                       ▪ Set parent[v] = u.
    4. Return parent array (MST edges) and compute total weight from key.
```

5. Time and Space Complexity Analysis:

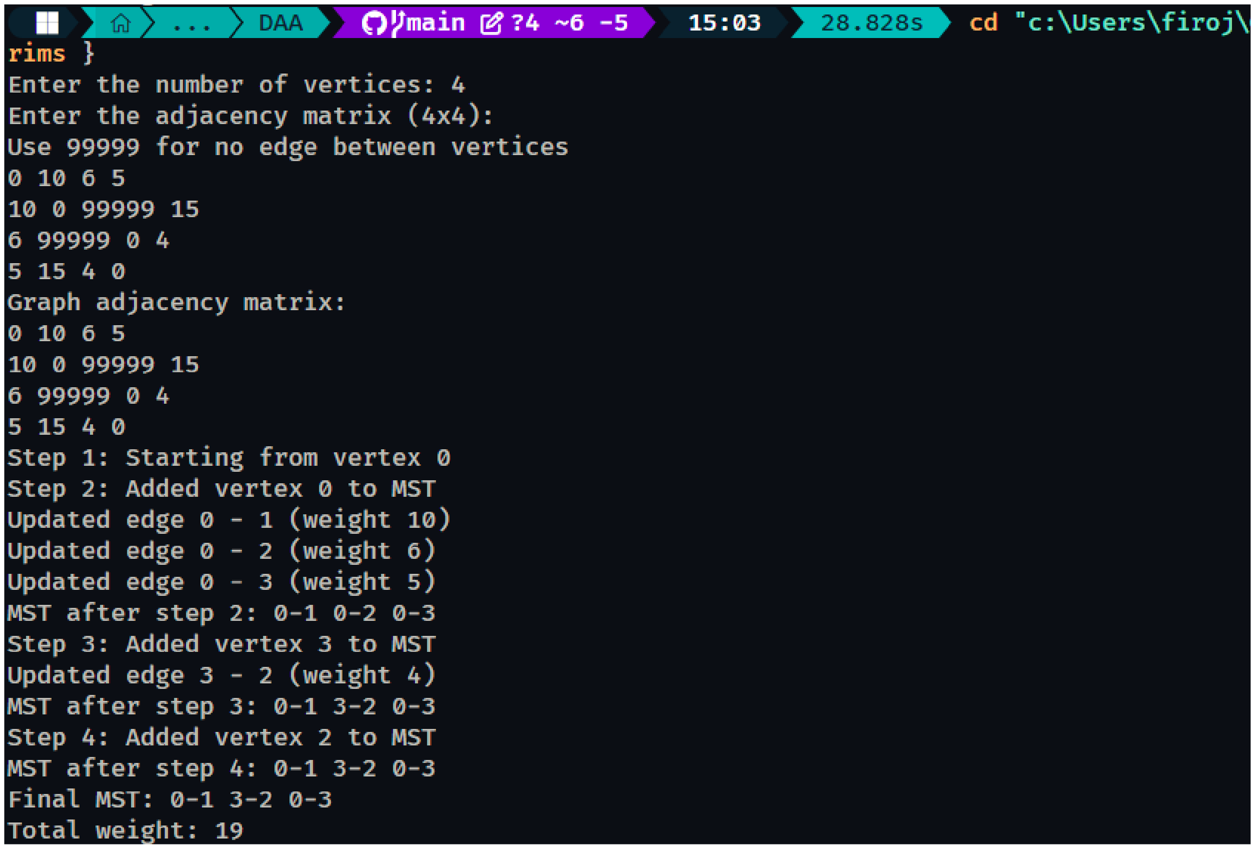
- **Time Complexity:**
    - $O(V^2)$ – Using an adjacency matrix and linear search for the minimum key (as implemented here).
    - Can be improved to $O(E \log V)$ with a priority queue, but this version keeps it simple.
- **Space Complexity:** $O(V)$ – Space for key, parent, and visited arrays.

6. Code Implementation:

```
#include<iostream>
#include<algorithm> // For sort
using namespace std;

// Structure to represent an edge
struct Edge {
    int src, dest, weight;
};

// Structure for disjoint-set
struct DisjointSet {
    int *parent, *rank;
    int n;

    DisjointSet(int n) {
        this->n = n;
        parent = new int[n];
        rank = new int[n];
        for (int i = 0; i < n; i++) {
            parent[i] = i; // Each vertex is its own parent
            rank[i] = 0;
        }
    }

    int find(int u) {
        if (parent[u] != u)
            parent[u] = find(parent[u]); // Path compression
        return parent[u];
    }

    void unionSet(int u, int v) {
        int pu = find(u), pv = find(v);
        if (pu == pv) return;
        if (rank[pu] < rank[pv])
            parent[pu] = pv;
        else if (rank[pu] > rank[pv])
            parent[pv] = pu;
        else {
            parent[pv] = pu;
            rank[pu]++;
        }
    }
};

// Function to compare edges by weight
bool compare(Edge a, Edge b) {
    return a.weight < b.weight;
}

// Function to perform Kruskal's Algorithm with concise output
```

```
void kruskalMST(Edge edges[], int V, int E) {
    int step = 1;

    // Step 1: Sort edges by weight
    cout << "Step " << step++ << ": Sorting edges by weight" << endl;
    sort(edges, edges + E, compare);
    for (int i = 0; i < E; i++)
        cout << "Edge " << edges[i].src << " - " << edges[i].dest << ": " <<
edges[i].weight << endl;

    // Step 2: Initialize disjoint-set and MST
    DisjointSet ds(V);
    Edge mst[V - 1]; // MST will have V-1 edges
    int mst_weight = 0, mst_index = 0;

    // Step 3: Process edges
    cout << "Step " << step++ << ": Building MST" << endl;
    for (int i = 0 ;i < E && mst_index < V - 1; i++) {
        int u = edges[i].src, v = edges[i].dest, w = edges[i].weight;
        cout << "Considering edge " << u << " - " << v << " (weight " << w << "): ";

        if (ds.find(u) != ds.find(v)) {
            mst[mst_index++] = edges[i];
            mst_weight += w;
            ds.unionSet(u, v);
            cout << "Added to MST" << endl;
            cout << "MST after step " << step++ << ": ";
            for (int j = 0; j < mst_index; j++)
                cout << mst[j].src << "-" << mst[j].dest << " ";
            cout << "(Total weight = " << mst_weight << ")" << endl;
        } else {
            cout << "Skipped (forms cycle)" << endl;
        }
    }

    // Print final MST
    cout << "Final MST: ";
    for (int i = 0; i < V - 1; i++)
        cout << mst[i].src << "-" << mst[i].dest << " ";
    cout << endl << "Total weight: " << mst_weight << endl;
}

int main() {
    int V, E;
    cout << "Enter the number of vertices: ";
    cin >> V;
    cout << "Enter the number of edges: ";
    cin >> E;

    Edge edges[E];
    cout << "Enter " << E << " edges (source destination weight):" << endl;
    for (int i = 0; i < E; i++) {
        cout << "Edge " << i + 1 << ": ";
        cin >> edges[i].src >> edges[i].dest >> edges[i].weight;
    }

    cout << "Graph edges:" << endl;
    for (int i = 0; i < E; i++)
        cout << edges[i].src << " - " << edges[i].dest << ": " << edges[i].weight << endl;

    // Perform Kruskal's Algorithm
    kruskalMST(edges, V, E);

    return 0;
}
```

7. **Output:**



```
 ⌂ > ... > DAA  main ?4 ~6 -5  15:03  28.828s  cd "c:\Users\firoj\
rims }
Enter the number of vertices: 4
Enter the adjacency matrix (4x4):
Use 99999 for no edge between vertices
0 10 6 5
10 0 99999 15
6 99999 0 4
5 15 4 0
Graph adjacency matrix:
0 10 6 5
10 0 99999 15
6 99999 0 4
5 15 4 0
Step 1: Starting from vertex 0
Step 2: Added vertex 0 to MST
Updated edge 0 - 1 (weight 10)
Updated edge 0 - 2 (weight 6)
Updated edge 0 - 3 (weight 5)
MST after step 2: 0-1 0-2 0-3
Step 3: Added vertex 3 to MST
Updated edge 3 - 2 (weight 4)
MST after step 3: 0-1 3-2 0-3
Step 4: Added vertex 2 to MST
MST after step 4: 0-1 3-2 0-3
Final MST: 0-1 3-2 0-3
Total weight: 19
```