Criteria of a Good Estimator

Estimator:
Rule for calculating an estimate of a given quantity based on observed data.

They are used in two forms:

- Point Estimation
- Interval Estimation

Point Estimation: Single data is calculated to estimate the population parameter. The rule or formula that describes this calculation is called point estimator and resulting number is called as point estimate.

Interval Estimation: Based on sample data, **two numbers** are calculated to form an interval within the population range. Rule called as interval estimator and resulting value is called as interval estimate or confidence interval.

Properties of Good Estimators:

- Unbiasedness
- Consistency
- Efficiency
- Sufficiency

Unbiasedness:

- If the mean of estimated statistics is equal to that of population parameter.

Definition:

An estimator $T = t\ (x_1, x_2, x_3, x_4, \ldots, x_n)$ is said to be unbiased estimator of population parameter γ , if

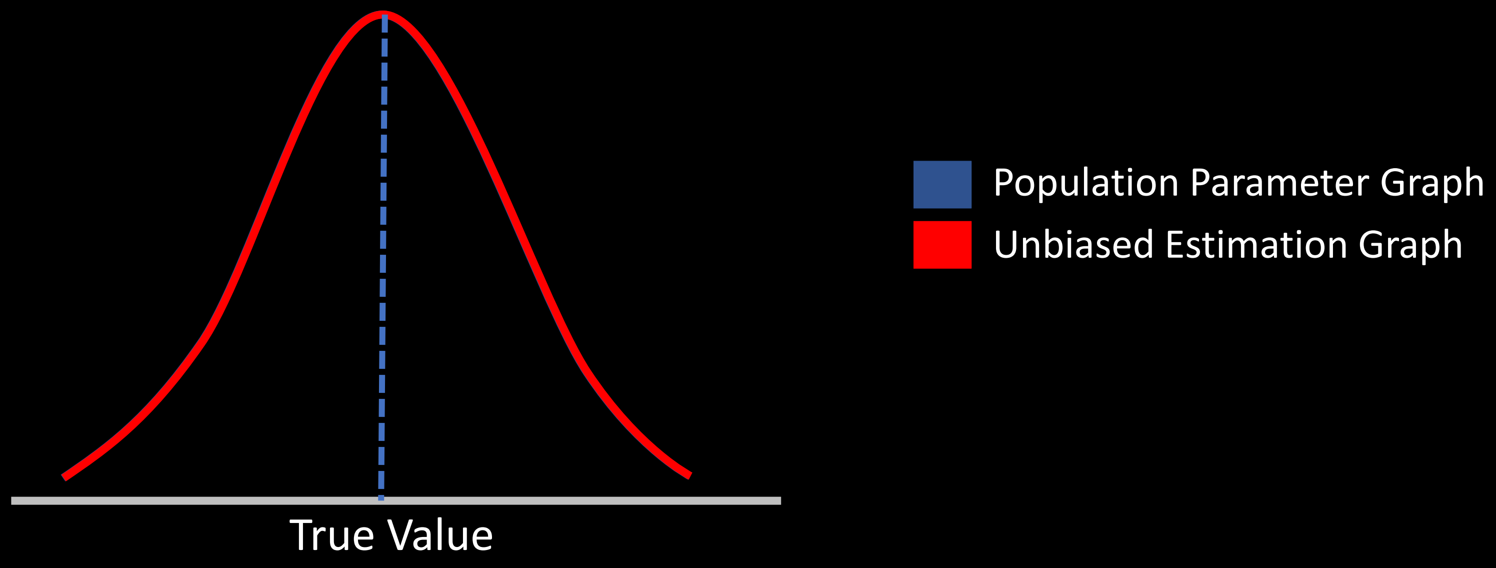
$$E(t) = \gamma$$

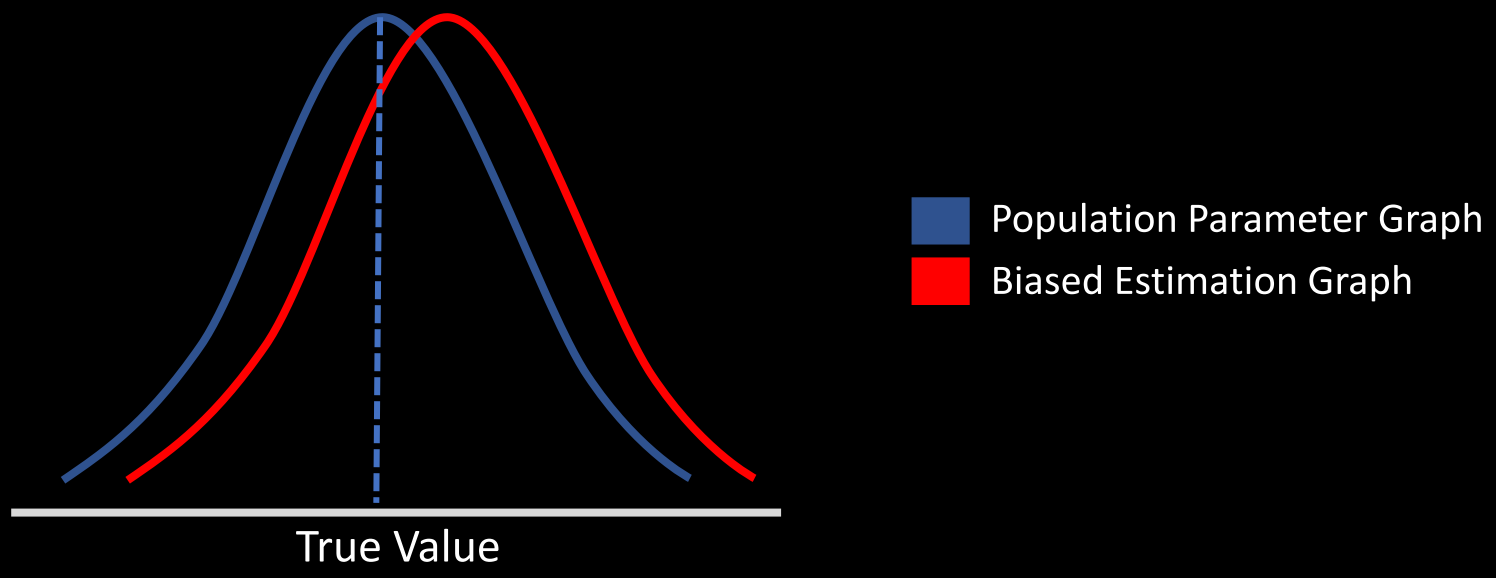
If $E(t) > \gamma$, T is said to be positively biased.

If $E(t) < \gamma$, T is said to be negatively biased.

Likewise amount of bias is: $E(t) - \gamma$

Learning From Graphs:





i.e. the sampling distribution of the point estimator should be centered over the true value of the parameter to be estimated.

There may exist more than one estimator which is unbiased for the same sample drawn from the population.

Example:

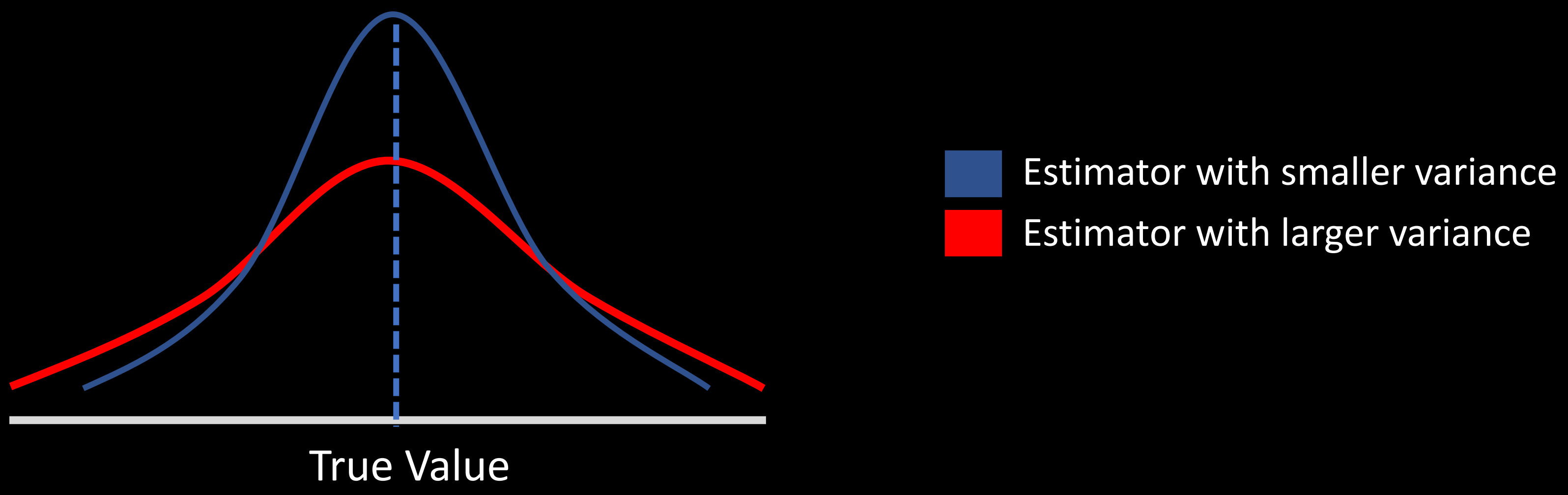
A random sample x_1, x_2, x_3, x_4, x_5 of size 5 is drawn from a normal distribution with unknown mean μ. The estimators:

$$T_1 = \frac{x_1+x_2+x_3+x_4+x_5}{5}; T_2 = \frac{2x_1+x_2+x_3}{4}$$

are unbiased estimator of μ if $E(T_1)$ and $E(T_2)$ both have the value μ.

Thus, there is a need to measure another parameter for a “Good” estimator, called as consistent estimator.

Second desirable characteristic of an estimator is that the spread (as measured by variance)of sampling distribution is small as possible.



Consistency:

An estimator $T_n = t\ (x_1, x_2, x_3, x_4, \dots, x_n)$ based on a random sample of size 'n' is said to be consistent estimator of γ If T_n converges to γ in probability i.e.,

$$T_n \xrightarrow{P} \gamma \ as\ n \to \infty$$

Remark :

The consistent estimator is a property concerning the behavior of the estimator for large value of sample size *n*. i.e., $n \to \infty$

Sufficient conditions for consistency:

Let $\{T_n\}$ be a sequence of estimators such that:

- $E(T_n) \to \gamma, as\ n \to \infty$
- $Var(T_n) \to 0, as\ n \to \infty$

Then T_n is a consistent estimator of γ.

Efficiency:

If we have **more than one consistent estimators** of a parameter, then efficiency is the criterion which enables us to choose between them by considering the variances of the sampling distributions of the estimators.

i.e., if T_1 and T_2 are consistent estimators of a parameter γ such that

$$Var(T_1) < Var(T_2), for\ all\ n$$

Then T_1 is said to be more efficient than T_2 .

If t is the most efficient estimator of a parameter with variance v and T_1 is any other estimator with variance v_1, then the efficiency (E) of T_1 is defined as:

$$E = \frac{v}{v_1}$$

Efficiency of any estimator cannot exceed Unity.

Sufficiency:
An estimator is said to be sufficient for a parameter θ, if it consists of all the information in the sample regarding the parameter.

Properties of Sufficient Estimators:

1. If a sufficient estimator exists for some parameter then, it is also the most efficient estimator.
2. It is always consistent
3. It may or may not be unbiased.