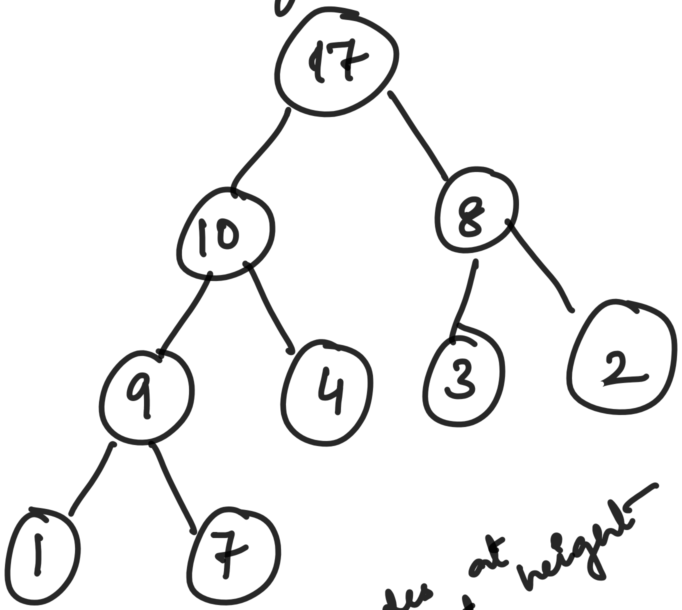
Monday, 27 January 2020 3:58 PM

Building an heap using insertion operation

Time Complexity :-



height

nodes at that height

$$S = h \times 2^{(h-1)} + (h-1)2^{h-2} + (h-2)2^{h-3} \cdots$$

$$\cdots 2 \times 4 + 1 \times 2 + 0 \times 1$$

height

(1)

h-1

$$S = 2^{\cdot\cdot}\left(n + \frac{n-1}{2} + \frac{n-2}{2^2} + \frac{n-3}{2^3} \cdots\cdots \frac{2}{2^{n-3}} + \frac{1}{2^{n-2}}\right)$$

2

$$2S = 2^{n-1}\left(2n + (n-1) + \left(\frac{n-2}{2}\right) + \left(\frac{n-3}{2^2}\right) \cdots\cdots \frac{2}{2^{n-4}} + \frac{1}{2^{n-3}}\right)$$

$$2S - S = 2^{n-1}\left(2n + (n-1-n) + \left(\frac{n-2}{2} - \frac{n-1}{2}\right) + \left(\frac{n-3}{2^2} - \frac{n-2}{2^2}\right) \cdots\cdots\right)$$

$$S = 2^{n-1}\left(2n + (-1) + (-1/2) + \left(-\frac{1}{2^2}\right) \cdots\cdots -\frac{1}{2^{n-3}} - \frac{1}{2^{n-2}}\right)$$

$$S = 2^{n-1}\left(2n - \left(1 + \frac{1}{2} + \frac{1}{2^2} \cdots\cdots \frac{1}{2^{n-3}} + \frac{1}{2^{n-2}}\right)\right)$$

$$S = 2^{h-1}\left(2h - \left[\frac{1 \times \left(1-\left(\frac{1}{2}\right)^{h-1}\right)}{\frac{1}{2}}\right]\right)$$

$$S = 2^{h-1}\left(2h - \left[\frac{(2^{h-1}-1)/2^{h-1}}{\frac{1}{2}}\right]\right)$$

$$S = 2^{h-1}\left(2h - \left(\frac{2^{h-1}-1}{2^{h-2}}\right)\right)$$

$$S = 2^{h-1}\left(\frac{\overbrace{2h \times 2^{h-2}}\; - 2^{h-1} + 1}{2^{h-2}}\right)$$

$$S = 2 \times \left(h \times 2^{h-1} - 2^{h-1} + 1\right)$$

$$S = 2 \times \left(2^{h-1}(h-1) + 1\right)$$

$$S = 2 \times \left(2^{\log_2 n - 1}(\log_2 n - 1) + 1\right)$$

$$h = \log_2 n$$

$$S = 2^{\log_2 n}(\log_2 n - 1) + 2$$

$$S = n(\log_2 n - 1) + 2$$

$$S = \underbrace{n\log_2 n - n + 2}_{\downarrow}$$

$$\underline{\underline{O(n\log n)}}$$