GATE WALLAH

TOPICWISE

GATE PREVIOUS YEAR QUESTIONS

2008 – 2023

ENGINEERING MATHEMATICS & GENERAL APTITUDE

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PREFACE

A highly skilled professional team of GATE Wallah works arduously to ensure that the students receive the best content for their **GATE** exam.

A plethora of GATE Study Material is available in the market but GATE Wallah professionals at PW are continuously working to provide supreme quality study material for the GATE students.

From the beginning, the content team comprising Subject Matter Experts, Content Creators, Reviewers, DTP operators, Proofreaders, and others is involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

GATE Wallah Faculties have adopted a novel style of presenting the content in easy-to-understand language and have provided the content team with expert guidance and supervision throughout the creation and curation of this book.

PW's GATE Wallah strongly believes in conceptual and fun-based learning. GATE Wallah provides highly exam-oriented content to bring quality and clarity to the students.

This book adopts a multi-faceted approach to mastering and understanding the concepts by having a rich diversity of questions asked in the examination and equipping the students with the knowledge for this highly competitive exam.

The main objective of this book is to provide an edge to your preparation with high-quality content & video solutions.

BOOK FEATURES

This book, especially designed for GATE aspirants, contains

- A topic-wise set of questions from the past 16 years (2008-2023)
- Embedded QR codes for Video Solutions (by expert faculties)
- Complete explanations to help you ace your exam
- Detailed Chapter Wise Analysis of PYQs
- Covers Engineering Mathematics and Aptitude based PYQs from the 5 branches:
    - Mechanical Engineering
    - Civil Engineering
    - Electrical Engineering
    - Electronics & Communication Engineering
    - Computer Science & Information Technology

SPECIAL ABOUT THIS BOOK

We take pride in providing high-quality study material for the **GATE** exam. Our team at **GATE WALLAH** has meticulously prepared each solution to ensure they are error-free and easy to understand.

The **GATE** exam pattern has shifted in recent years, with an increased emphasis on topics that previously received fewer questions. At **GATE WALLAH**, we understand the importance of identifying these crucial topics, and that's why we've provided a chapter-wise analysis for the last 16 years. Our analysis will help you understand the significance of each chapter and focus your preparation accordingly. We believe that this will be a valuable resource in your journey to success in the **GATE** exam.

To ensure that you focus on the most relevant questions, we have arranged the questions in this book topic-wise and year-wise in descending order. This means that the questions from the most recent years are emphasized, as we believe they are the most relevant for your **GATE** exam preparation. By studying these questions, you'll gain a deeper understanding of the exam pattern and the type of questions asked in recent years, which will help you perform better in the upcoming **GATE** exam.

We've designed the text solutions in this book to be in sync with our video solutions recorded by expert faculties, making it easier for you to understand the concepts. We understand that every student has a unique learning style, and that's why we've included both text and video solutions to cater to your needs. By using our comprehensive guide and video solutions together, you'll gain a better understanding of the concepts and be better prepared to tackle the **GATE** exam.

Steps to Open Video solutions through mobile:

(1) Scan the given embedded QR Code for a particular solution.

(2) Visit the link generated & you'll be redirect to the video solution.

GATE

ENGINEERING MATHEMATICS & GENERAL APTITUDE

CONTENTS

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Engineering Mathematics

Engineering Mathematics

(Mechanical Engineering)

Syllabus

- **Linear Algebra:** Matrix algebra, systems of linear equations, eigenvalues and eigenvectors.
- **Calculus:** Functions of single variable, limit, continuity and differentiability, mean value theorems, indeterminate forms; evaluation of definite and improper integrals; double and triple integrals; partial derivatives, total derivative, Taylor series (in one and two variables), maxima and minima, Fourier series; gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, applications of Gauss, Stokes and Green's theorems.
- **Differential Equations:** First order equations (linear and nonlinear); higher order linear differential equations with constant coefficients; Euler-Cauchy equation; initial and boundary value problems; Laplace transforms; solutions of heat, wave and Laplace's equations.
- **Complex Variables:** Analytic functions; Cauchy-Riemann equations; Cauchy's integral theorem and integral formula; Taylor and Laurent series.
- **Probability and Statistics:** Definitions of probability, sampling theorems, conditional probability; mean, median, mode and standard deviation; random variables, binomial, Poisson and normal distributions.
- **Numerical Methods:** Numerical solutions of linear and non-linear algebraic equations; integration by trapezoidal and Simpson's rules; single and multi-step methods for differential equations.

ME Chapter wise Weightage Analysis (Marks)

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6	Ch.7	Ch.8	Ch.9
2008	5	10	1	3	3	2	0	0	0
2009	1	6	3	2	1	2	0	1	0
2010	2	3	2	1	2	1	0	2	2
2011	3	3	2	2	0	1	0	0	2
2012	4	4	2	2	1	0	0	2	0
2013	1	4	3	4	0	0	0	2	1
2014 (P1)	4	3	2	0	3	1	0	0	0
2014 (P2)	3	4	1	2	1	0	0	0	2
2014 (P3)	0	1	4	3	0	2	0	1	2
2014 (P4)	0	2	3	1	0	3	0	0	4
2015 (P1)	1	5	3	2	0	1	0	0	1
2015 (P2)	1	3	3	2	1	0	0	1	2
2015 (P3)	3	3	1	0	1	0	0	2	3
2016 (P1)	1	2	1	2	0	3	0	1	3
2016 (P2)	1	3	2	0	0	3	0	1	3
2016 (P3)	3	5	3	1	0	0	0	0	1
2017 (P1)	3	3	1	2	2	0	0	0	2
2017 (P2)	3	2	2	2	1	2	0	1	0
2018 (P1)	1	3	4	0	0	1	0	2	2
2018 (P2)	1	0	2	4	3	2	1	0	0
2019 (P1)	3	3	3	1	0	2	0	0	1
2019 (P2)	1	2	3	3	3	1	0	0	0
2020 (P1)	1	3	3	0	1	3	0	1	2
2020 (P2)	2	1	3	0	2	2	0	1	2
2021 (P1)	2	3	1	1	0	2	0	1	1
2021 (P2)	3	2	1	2	0	1	0	1	3
2022 (P1)	3	2	2	0	2	2	1	1	**0**
2022 (P2)	4	2	0	2	1	2	1	0	1
2023	0	4	1	2	1	1	0	2	2

Engineering Mathematics

(Civil Engineering)

Syllabus

- **Linear Algebra:** Matrix algebra, systems of linear equations; Eigen values and Eigen vectors.
- **Calculus:** Functions of single variable; Limit, continuity and differentiability; Mean value theorems, local maxima and minima; Taylor series; Evaluation of definite and indefinite integrals, application of definite integral to obtain area and volume; Partial derivatives; Total derivative; Gradient, Divergence and Curl, Vector identities; Directional derivatives; Line, Surface and Volume integrals.
- **Ordinary Differential Equation (ODF):** First order (linear and non-linear) equations; higher order linear equations with constant coefficients; Euler-Cauchy equations; initial and boundary value problems.
- **Partial Differential Equation (PDF):** Fourier series; separation of variables; solutions of one-dimensional diffusion equation; first and second order one-dimensional wave equation and two-dimensional Laplace equation.
- **Probability and Statistics:** Sampling theorems; Conditional probability; Descriptive statistics - Mean, median, mode and standard deviation; Random Variables - Discrete and Continuous, Poisson and Normal Distribution; Linear regression.
- **Numerical Methods:** Error analysis. Numerical solutions of linear and non-linear algebraic equations; Newton's and Lagrange polynomials; numerical differentiation; Integration by trapezoidal and Simpson's rule; Single and multi-step methods for first order differential equations.

CE Chapter wise Weightage Analysis (Marks)

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6	Ch.7	Ch.8	Ch.9
2008	5	4	6	3	2	0	0	0	0
2009	1	0	2	2	3	3	0	2	2
2010	2	7	1	3	0	0	0	0	2
2011	1	4	1	2	2	2	0	0	1
2012	2	1	3	2	2	0	0	0	3
2013	1	2	2	2	0	0	0	0	2
2014 (P1)	2	4	3	2	0	0	0	0	0
2014 (P2)	3	2	2	1	0	1	0	0	0
2015 (P1)	3	0	1	2	2	2	0	0	3
2015 (P2)	3	2	2	2	0	1	0	0	3
2016 (P1)	1	4	3	4	0	0	0	0	1
2016 (P2)	2	8	4	0	0	0	1	0	0
2017 (P1)	3	2	3	3	0	0	0	0	2
2017 (P2)	3	5	1	2	1	0	0	0	0
2018 (P1)	2	3	0	4	0	0	0	0	2
2018 (P2)	4	0	3	1	2	0	0	2	1
2019 (P1)	0	6	0	2	0	0	0	0	2
2019 (P2)	3	1	2	2	2	0	0	1	1
2020 (P1)	2	2	0	3	2	0	0	0	3
2020 (P2)	2	1	1	4	0	0	2	0	1
2021 (P1)	2	2	1	2	0	0	0	0	4
2021 (P2)	4	2	0	2	3	0	0	0	2
2022 (P1)	1	5	2	1	0	0	1	0	3
2022 (P2)	4	3	2	2	1	0	0	0	**0**
2023 (P1)	3	3	1	1	0	0	1	0	4
2023 (P2)	5	0	2	5	1	0	0	0	**0**

Engineering Mathematics

Syllabus

(Electrical Engineering)

- **Linear Algebra:** Matrix Algebra, Systems of linear equations, Eigenvalues, Eigenvectors.
- **Calculus:** Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series, Vector identities, Directional derivatives, Line integral, Surface integral, Volume integral, Stokes's theorem, Gauss's theorem, Divergence theorem, Green's theorem.
- **Differential Equations:** First order equations (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's equation, Euler's equation, Initial and boundary value problems, Partial Differential Equations, Method of separation of variables.
- **Complex Variables:** Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, Taylor series, Laurent series, Residue theorem, Solution integrals.
- **Probability and Statistics:** Sampling theorems, Conditional probability, Mean, Median, Mode, Standard Deviation, Random variables, Discrete and Continuous distributions, Poisson distribution, Normal distribution, Binomial distribution, Correlation analysis, Regression analysis.

EE Chapter wise Weightage Analysis (Marks)

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6	Ch.7	Ch.8	Ch.9
2008	6	2	1	2	0	0	0	2	6
2009	1	4	2	0	2	0	0	2	1
2010	4	3	2	2	1	0	0	0	4
2011	2	3	0	1	2	1	0	2	2
2012	2	2	3	3	2	2	1	0	2
2013	3	2	1	0	2	3	0	2	3
2014 (P1)	3	3	2	1	2	1	1	0	3
2014 (P2)	1	5	3	1	0	1	0	0	1
2014 (P3)	1	0	3	0	2	2	0	2	1
2015 (P1)	3	1	3	2	1	0	0	0	3
2015 (P2)	1	2	2	2	0	1	0	0	1
2016 (P1)	5	3	2	1	0	1	0	0	5
2016 (P2)	3	2	2	0	1	3	3	0	3
2017 (P1)	1	3	0	2	0	3	0	2	1
2017 (P2)	2	3	4	0	0	2	0	2	2
2018	3	4	0	0	3	3	0	0	3
2019	4	0	1	1	3	3	1	0	4
2020	2	4	0	1	4	1	0	0	2
2021	3	4	2	0	2	1	0	0	3
2022	5	2	2	0	6	0	0	0	5
2023	1	3	4	0	3	0	0	0	1

Engineering Mathematics

(Electronic and Communication Engineering)

Syllabus

- **Linear Algebra:** Vector space, basis, linear dependence and independence, matrix algebra, Eigen values and eigen vectors, rank, solution of linear equations- existence and uniqueness.
- **Calculus:** Mean value theorems, theorems of integral calculus, evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, line, surface and volume integrals, Taylor series.
- **Differential Equations:** First order equations (linear and nonlinear), higher order linear differential equations, Cauchy's and Euler's equations, methods of solution using variation of parameters, complementary function and particular integral, partial differential equations, variable separable method, initial and boundary value problems.
- **Vector Analysis:** Vectors in plane and space, vector operations, gradient, divergence and curl, Gauss's, Green's and Stokes' theorems.
- **Complex Analysis:** Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, sequences, series, convergence tests, Taylor and Laurent series, residue theorem.
- **Probability and Statistics:** Mean, median, mode, standard deviation, combinatorial probability, probability distributions, binomial distribution, Poisson distribution, exponential distribution, normal distribution, joint and conditional probability.

ECE Chapter wise Weightage Analysis (Marks)

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6	Ch.7	Ch.8	Ch.9
2008	4	4	0	1	4	3	2	4	4
2009	2	2	1	3	2	1	0	2	2
2010	1	2	2	1	2	2	2	1	2
2011	2	0	2	1	0	1	2	2	0
2012	2	2	2	1	0	2	0	2	2
2013	3	1	0	0	0	0	0	3	1
2014 (P1)	4	4	1	0	0	1	0	4	4
2014 (P2)	5	2	0	1	2	2	0	5	2
2014 (P3)	2	4	3	1	0	0	1	2	4
2014 (P4)	0	3	4	3	2	0	0	0	3
2015 (P1)	2	5	1	2	0	1	0	2	5
2015 (P2)	1	1	2	3	0	2	0	1	1
2015 (P3)	1	1	2	2	0	1	2	1	1
2016 (P1)	3	3	1	1	2	2	0	3	3
2016 (P2)	1	2	0	2	3	1	0	1	2
2016 (P3)	3	3	1	3	0	3	0	3	3
2017 (P1)	2	3	1	1	1	0	1	2	3
2017 (P2)	1	2	1	1	2	2	0	1	2
2018	2	4	1	4	0	2	0	2	4
2019	1	3	1	3	2	2	0	1	3
2020	3	1	0	3	3	0	0	3	1
2021	3	0	2	1	1	2	0	3	**0**
2022	3	4	1	2	3	1	0	3	4
2023	4	0	0	0	5	2	0	4	**0**

Engineering Mathematics

(Computer Science & IT)

Syllabus

- **Linear Algebra:** Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
- **Calculus:** Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.
- **Probability and Statistics:** Random variables. Uniform, normal, exponential, Poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

CSE & IT Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3
2008	3	4	6
2009	0	2	0
2010	2	1	4
2011	2	2	6
2012	1	1	3
2013	0	1	1
2014 (P1)	3	3	1
2014 (P2)	2	0	0
2014 (P3)	1	2	2
2015 (P1)	3	2	0
2015 (P2)	1	2	0
2015 (P3)	1	4	0
2016 (P1)	1	1	3
2016 (P2)	2	0	1
2017 (P1)	3	2	2
2017 (P2)	3	1	5
2018	2	1	1
2019	3	1	4
2020	0	1	2
2021 (P1)	1	2	3
2021 (P2)	1	1	1
2022	3	1	0
2023	2	2	0

CHAPTER

1

LINEAR ALGEBRA

Algebra of Matrices

1. [MCQ] [GATE-CE-2023:2M]

Cholesky decomposition is carried out on the following square matrix $[A]$.

$$[A]=\begin{bmatrix}8 & -5\\ -5 & a_{22}\end{bmatrix}$$

Let l_{ij} and a_{ij} be the $(i,j)^{th}$ elements of matrices $[L]$ and $[A]$, respectively. If the element l_{22} of the decomposed lower triangular matrix $[L]$ is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22}?

(a) 5 (b) 7

(c) 9 (d) 11

2. [MCQ] [GATE-EE-2023:1M]

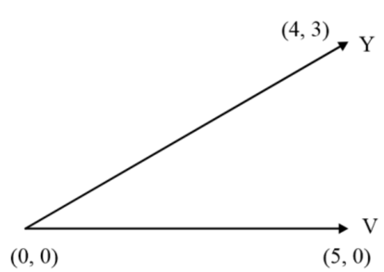
For a given vector $w = [1\ 2\ 3]^T$, the vector normal to the plane defined by $w^Tx = 1$ is

(a) $[-2\ -2\ 2]^T$ (b) $[3\ 0\ -1]^T$

(c) $[3\ \ 2\ \ 1]^T$ (d) $[1\ \ 2\ \ 3]^T$

3. [MCQ] [GATE-EE-2023:1M]

In the figure, the vectors u and v are related as: Au = v by a transformation matrix A. The correct choice of A is



(a) $\begin{bmatrix}\frac{4}{5} & \frac{3}{5}\\ -\frac{3}{5} & \frac{4}{5}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{4}{5} & -\frac{3}{5}\\ \frac{3}{5} & \frac{4}{5}\end{bmatrix}$

(c) $\begin{bmatrix}\frac{4}{5} & \frac{3}{5}\\ \frac{3}{5} & \frac{4}{5}\end{bmatrix}$ (d) $\begin{bmatrix}\frac{4}{5} & \frac{-3}{5}\\ \frac{3}{5} & -\frac{4}{5}\end{bmatrix}$

4. [MCQ] [GATE-CS-2023:2M]

Let $A=\begin{bmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 2 & 3\\ 3 & 4 & 1 & 2\\ 2 & 3 & 4 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 4 & 1 & 2\\ 4 & 1 & 2 & 3\\ 1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{bmatrix}$

Let det(A) and det(B) denote the determinants of the matrices A and B, respectively. Which one of the options given below is TRUE?

(a) det(A) = det(B)

(b) det(B) = −det(A)

(c) det(A) = 0

(d) det(AB) = det(A)+det(B)

5. [MCQ] [GATE-CS-2022:1M]

Consider the following two statements with respect to the matrices $A_{m\times n}$, $B_{n\times m}$, $C_{n\times n}$ and $D_{n\times n}$.

Statement 1: tr(AB) = tr(BA)

Statement 2: tr(CD) = tr(DC)

Where tr() represents the trace of a matrix. Which one of the following holds?

(a) Statement 1 is correct and Statement 2 is wrong.

(b) Statement 1 is wrong and Statement 2 is correct.

(c) Both Statement 1 and Statement 2 are correct.

(d) Both Statement 1 and Statement 2 are wrong.

6. [MCQ] [GATE-CE-2022:2M]

The Cartesian coordinates of a point P in a right handed coordinate system are (1, 1, 1). The transformed coordinates of P due to a 45° clockwise rotation of the coordinate system about the positive x-axis are

(a) $(1, 0, \sqrt{2})$ (b) $(1, 0, -\sqrt{2})$

(c) $(-1, 0, \sqrt{2})$ (d) $(-1, 0, -\sqrt{2})$

7. [NAT] [GATE-CE-2022:1M]

The components of pure shear strain in a sheared material are given in the matrix form $E = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Here, Trace (E) = 0, Given, P = Trace (E^8) and Q = Trace (E^{11})

The numerical value of (P + Q) is ________ (in integer)

8. [MCQ] [GATE-ME-2022: 1M]

If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is ______.

(a) 8 (b) 5

(c) –0.4 (d) $\frac{1+\sqrt{1561}}{12}$

9. [MCQ] [GATE-CE-2021:2M]

If $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $Q^T P^T$ is

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

10. [MCQ] [GATE-CE-2021:1M]

If A is a square matrix then orthogonality property mandates

(a) $AA^T = I$ (b) $AA^T = 0$

(c) $AA^T = A^{-1}$ (d) $AA^T = A^2$

11. [MCQ] [GATE-ME-2020: 1M]

Multiplication of real valued square matrices of same dimension is

(a) associative

(b) commutative

(c) always positive definite

(d) not always possible to compute

12. [MCQ] [GATE-ME-2020: 1M]

A matrix P is decomposed into its symmetric part S and skew symmetric part V. If

$$S = \begin{pmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{pmatrix}, V = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{pmatrix}$$

then matrix P is

(a) $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} -2 & 9/2 & -1 \\ -1 & 81/4 & 11 \\ -2 & 45/2 & 73/4 \end{pmatrix}$

13. [MCQ] [GATE-EE-2020: 2M]

The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is_________.

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(a) 5 (b) 6

(c) 8 (d) 9

14. [MCQ] [GATE-CE-2019:2M]

The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is.

(a) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -\frac{4}{5} & -\frac{9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & -\frac{1}{5} & -\frac{6}{5} \end{bmatrix}$ (d) $\begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

15. [MCQ] [GATE-CS-2019:1M]

Let X be a square matrix. Consider the following two statements on X.

I:X is invertiable.

II: Determinant of X is non-zero.

Which one of the following is TRUE?

(a) I implies II; II does not imply I.
(b) II implies I: I does not imply II.
(c) I does not imply II; II does not imply I.
(d) I and II are equivalent statements.

16. [MCQ] [GATE-EE-2019: 2M]

Consider a 2 × 2 matrix M = [v_1, v_2], where, v_1 and v_2 are the column vectors. Suppose $M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$, where u_1^T and u_2^T are the row vectors. Consider the following statements:

Statement 1: $u_1^T v_1 = 1$ and $u_2^T v_2 = 1$

Statement 2: $u_1^T v_2 = 0$ and $u_2^T v_1 = 0$

Which of the following option is correct?

(a) Statement 1 is true and statement 2 is false
(b) Statement 2 is true and statement 1 is false
(c) Both the statements are true
(d) Both the statements are false

17. [MCQ] [GATE-CE-2018:1M]

For the given orthogonal matrix Q = $\begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$ The inverse is

(a) $\begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{3}{7} & -\frac{2}{7} & -\frac{6}{7} \\ \frac{6}{7} & -\frac{3}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{6}{7} & \frac{3}{7} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{3}{7} & \frac{6}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{3}{7} & -\frac{6}{7} \\ -\frac{6}{7} & -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$

18. [MCQ] [GATE-CE-2018:1M]

Which one of the following matrices is singular

(a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

19. [NAT] [GATE-ME-2018: 1M]

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is ______ (Correct to two decimal places).

20. [MCQ] [GATE-CE-2017:2M]

If $A = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix}$,AB^T is equal to

(a) $\begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 40 \\ 42 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 43 & 27 \\ 34 & 50 \end{bmatrix}$ (d) $\begin{bmatrix} 38 & 32 \\ 28 & 56 \end{bmatrix}$

21. [MCQ] [GATE-CE-2017:1M]

The matrix P is the inverse of a matrix Q. If I denotes the identify matrix, which one of the following option is correct?

(a) PQ = I but QP ≠ I (b) QP = I but PQ ≠ I
(c) PQ = I and QP = I (d) PQ – QP = I

22. [NAT] **[GATE-ME-2017: 2M]**

Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigenvectors corresponding to eigenvalues λ_1 and λ_2 are $x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix}$ and $x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$, respectively.

The value of $x_1^T x_2$ is ______.

23. [MCQ] **[GATE-ME-2017: 2M]**

Consider the matrix

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Which one of the following statements about P is INCORRECT?

(a) Determinant of P is equal to 1.
(b) P is orthogonal.
(c) Inverse of P is equal to its transpose.
(d) All eigenvalues of P are real numbers.

24. [NAT] **[GATE-EE-2017: 1M]**

Let x and y be integers satisfying the following equations

$2x^2 + y^2 = 34$
$x + 2y = 11$

The value of $(x + y)$ is ______________.

25. [MCQ] **[GATE-ME-2016: 1M]**

A real square matrix A is called skew-symmetric if

(a) $A^T = A$ (b) $A^T = A^{-1}$
(c) $A^T = -A$ (d) $A^T = A + A^{-1}$

26. [NAT] **[GATE-EC-2016: 2M]**

The matrix $\begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det (A) = 100 and trace (A) = 14. The value of |a-b| is ____

27. [MCQ] **[GATE-EC-2016:1M]**

Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(a) $M^{4k} + 1$ (b) M^{4k+2}
(c) M^{4k+3} (d) M^{4k}

28. [NAT] **[GATE-EC-2016: 2M]**

A sequence x[n] is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ for } n \geq 2.$$

The initial conditions are x[0] = 1, x[1] = 1 and x[n] = 0 for n < 0. The value of x[12] is ______

29. [MCQ] **[GATE-EE-2016: 2M]**

Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$. Then S is

(a) a circle of radius $\sqrt{10}$
(b) a circle of radius $\frac{1}{\sqrt{10}}$
(c) An ellipse with major axis along $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
(d) An ellipse with major axis along $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

30. [NAT] **[GATE-CS-2016:1M]**

Suppose that the eigenvalues of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is __________.

31. [NAT] **[GATE-CS-2015:1M]**

In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$ if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ________.

32. [MCQ] [GATE-ME-2015: 1M]

If any two columns of a determinant $P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$ are interchanged, which one of the following statements regarding the value of the determinant is **CORRECT**?

(a) Absolute value remains unchanged but sign will change.
(b) Both absolute value and sign will change.
(c) Absolute value will change but sign will not change.
(d) Both absolute value and sign will remain unchanged.

33. [MSQ] [GATE-CE-2015:2M]

P and Q are two square matrices of the same order. Which of the following statements(s) is/are correct?

(a) If P and Q are invertible then $[PQ]^{-1} = Q^{-1}P^{-1}$
(b) If P and Q are invertible then $[QP]^{-1} = P^{-1}Q^{-1}$
(c) If P and Q are invertible then $[PQ]^{-1} = P^{-1}Q^{-1}$
(d) If P and Q are not invertible then $[PQ]^{-1} = Q^{-1}P^{-1}$

34. [MCQ] [GATE-ME-2015: 2M]

For a given matrix $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$, where $i = \sqrt{-1}$, the inverse of matrix P is

(a) $\frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$ (b) $\frac{1}{25}\begin{bmatrix} i & 4-3i \\ 4+3i & -i \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ (d) $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

35. [MCQ] [GATE-ME-2014: 1M]

Given that the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is –12, the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is

(a) –96 (b) –24
(c) 24 (d) 96

36. [MCQ] [GATE-ME-2014: 1M]

Which one of the following equations is a correct identity for arbitrary 3 × 3 real matrices P, Q and R?

(a) $P(Q + R) = PQ + RP$
(b) $(P - Q)^2 = P^2 - 2PQ + Q^2$
(c) $\det(P + Q) = \det P + \det Q$
(d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$

37. NAT [GATE-EC-2014:2M]

The maximum value of the determinant among all 2 × 2 real symmetric matrices with trace 14 is________

38. NAT [GATE-EC-2014:2M]

Consider the matrix

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Which is obtained by reversing the order of the columns of the identity matrix I_6.

Let $P = I_6 + \alpha J_6$, where a is a non-negative real number. The value of α for which det (P) = 0 is __________.

39. [MCQ] [GATE-EC-2014:1M]

For matrices of same dimension M, N and scalar c, which one of these properties DOES NOT ALWAYS hold?

(a) $(M^T)^T = M$
(b) $(cM)^T = c(M)^T$
(c) $(M + N)^T = M^T + N^T$
(d) $MN = NM$

40. [NAT] [GATE-EC-2014: 1M]

The determinant of matrix A is 5 and the determinant of matrix B is 40. The determinant of matrix AB is.

41. [MCQ] [GATE-CS-2014: 1M]

Let the function

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin(\pi/6) & \cos(\pi/6) & \tan(\pi/6) \\ \sin(\pi/3) & \cos(\pi/3) & \tan(\pi/3) \end{vmatrix}$$

Where $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ. Which of the following statements is/are TRUE?

I. There exists $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that $f'(\theta) = 0$.

II. There exists $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that $f'(\theta) \neq 0$.

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

42. [NAT] [GATE-CE-2014:1M]

Given the matrices $J = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and $K = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ the product $K^T JK$ is _______?

43. [MCQ] [GATE-CE-2014:2M]

With reference to the conventional Cartesian (x, y) coordinate system the vertices of a triangle have the following coordinates; $(x_1, y_1) = (1, 0)$; $(x_2, y_2) = (2, 2)$; $(x_3, y_3) = (4, 3)$. The area of the triangle is equal to

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$
(c) $\frac{4}{5}$ (d) $\frac{5}{2}$

44. [NAT] [GATE-CE-2014:1M]

The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is _______?

45. [MCQ] [GATE-CE-2014:2M]

The square matrix B is skew-symmetric if

(a) $B^T = -B$ (b) $B^T = B$
(c) $B^{-1} = B$ (d) $B^{-1} = B^T$

46. [MCQ] [GATE- EC-2013: 2M]

Let A be an m×n matrix and B an n×m matrix. It is given that determinant $(I_m + AB)$ = determinant $(I_n + BA)$. Where I_k is the $k \times k$ identity matrix, Using the above property, the determinant of the matrix given below is

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(a) 2 (b) 5
(c) 8 (d) 16

47. [NAT] [GATE-CE-2013:1M]

There are three matrixes P(4 × 2), Q(2 × 4) and R(4 × 1). The minimum of multiplication required to compute the matrix PQR is _______?

48. [MCQ] [GATE-EE-2012: 2M]

Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the value of A^3 is

(a) 15A + 12I (b) 19A + 30I
(c) 17A + 15I (d) 17A + 21I

49. [MCQ] [GATE-CE-2011:1M]

[A] is square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and difference of these matrices are defined as $[S] = [A] + [A]^T$ and $[D] = [A] - [A]^T$, respectively. Which of the following statements is TRUE?

(a) Both [S] and [D] are symmetric
(b) Both [S] and [D] are skew-symmetric
(c) [S] is skew-symmetric and [D] is symmetric
(d) [S] is symmetric and [D] is skew-symmetric

50. [MCQ] [GATE-EE-2011: 2M]

The two vectors $[1 1 1]$ and $[1\ a\ a^2]$ where $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ and are

(a) orthonormal (b) orthogonal
(c) parallel (d) collinear

51. [MCQ] [GATE-EE-2011: 2M]

The matrix $[A]=\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix}$ is decomposed into a product of a lower triangular matrix [L] and an upper triangular matrix [U]. The properly decomposed [L] and [U] matrices respectively are

(a) $\begin{bmatrix}1 & 0\\ 4 & -1\end{bmatrix}$ and $\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}$

(b) $\begin{bmatrix}2 & 0\\ 4 & -1\end{bmatrix}$ and $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}$ and $\begin{bmatrix}2 & 1\\ 0 & -1\end{bmatrix}$

(d) $\begin{bmatrix}2 & 0\\ 4 & -3\end{bmatrix}$ and $\begin{bmatrix}1 & 0.5\\ 0 & 1\end{bmatrix}$

52. [MCQ] [GATE-CE-2011:2M]

The Eigen values of the matrix $\begin{bmatrix}9 & 5\\ 5 & 8\end{bmatrix}$ are

(a) – 2.42 and 6.86 (b) 3.48 and 13.53
(c) 4.70 and 6.86 (d) 6.86 and 9.50

53. [MCQ] [GATE-ME-2009: 1M]

For a matrix $[M]=\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by

(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$
(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

54. [MCQ] [GATE-CE-2009:2M]

The inverse of the matrix $\begin{bmatrix}3+2i & i\\ -i & 3-2i\end{bmatrix}$ is,

(a) $\frac{1}{12}\begin{bmatrix}3+2i & -i\\ i & 3-2i\end{bmatrix}$ (b) $\frac{1}{12}\begin{bmatrix}3-2i & -i\\ i & 3+2i\end{bmatrix}$

(c) $\frac{1}{14}\begin{bmatrix}3+2i & -i\\ i & 3-2i\end{bmatrix}$ (d) $\frac{1}{14}\begin{bmatrix}3+2i & -i\\ i & 3+2i\end{bmatrix}$

55. [MCQ] [GATE-CE-2008:1M]

The product of matrices $(PQ)^{-1}P$ is

(a) P^{-1} (b) Q^{-1}
(c) $P^{-1}Q^{-1}P$ (d) PQP^{-1}

56. [MCQ] [GATE-EE-2008: 2M]

Let P be a 2 × 2 real orthogonal matrix and $\vec{x}$ is a real vector $[x_1, x_2]^T$ with length $\|\vec{x}\| = (x_1^2 + x_2^2)^{\frac{1}{2}}$. Then, which one of the following statements is correct?

(a) $\|P\vec{x}\| \le \|\vec{x}\|$ where at least one vector satisfies $\|P\vec{x}\| < \|\vec{x}\|$

(b) $\|P\vec{x}\| = \|\vec{x}\|$ for all vectors $\vec{x}$

(c) $\|P\vec{x}\| \ge \|\vec{x}\|$ where at least one vector satisfies $\|P\vec{x}\| > \|\vec{x}\|$

(d) No relationship can be established between $\|\vec{x}\|$ and $\|P\vec{x}\|$

57. [MCQ] [GATE-EE-2008: 2M]

A is m × n full rank matrix with m > n and I is an identity matrix. Let matrix $A' = (A^TA)^{-1}A^T$. Then which one of the following statements is FALSE?

(a) AA'A = A (b) $(AA')^2 = AA^1$
(c) A'A = I (d) AA'A = A'

Rank of Matrices

58. [NAT] [GATE-CS-2021: 1M]

Suppose that P is a 4 × 5 matrix such that every solution of the equation Px = 0 is a scalar multiple of $[2\ 5\ 4\ 3\ 1]^T$. The rank of P is ____________.

59. [MCQ] [GATE-CE-2021:1M]

The rank of matrix $\begin{bmatrix}1 & 2 & 2 & 3\\ 3 & 4 & 2 & 5\\ 5 & 6 & 2 & 7\\ 7 & 8 & 2 & 9\end{bmatrix}$ is

(a) 1 (b) 2
(c) 3 (d) 4

60. [MCQ] **[GATE-CE-2021:1M]**

The rank of matrix $\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 4

61. [NAT] **[GATE-EE-2019: 1M]**

The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, is ______.

62. [MCQ] **[GATE-EE-2018: 1M]**

Let A be a 4×3 real matrix with rank 2, which one of the following statements is True?

(a) Rank of $A^T A$ is less than 2.

(b) Rank of $A^T A$ is equal to 2.

(c) Rank of $A^T A$ is greater than 2.

(d) Rank of $A^T A$ can be any number between 1 and 3

63. [MCQ] **[GATE-ME-2018: 1M]**

The rank of the matrix $\begin{pmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{pmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 4

64. [MCQ] **[GATE-CE-2018:2M]**

The rank of the following matrix is $\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$

(a) 1 (b) 2

(c) 3 (d) 4

65. [MCQ] **[GATE-CE-2017: 1M]**

The rank of the matrix $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is

(a) 0 (b) 1

(c) 2 (d) 3

66. [NAT] **[GATE-CS-2017:1M]**

Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$. Then the rank of P + Q is______

67. [NAT] **[GATE-EC-2017:1M]**

The rank of the matrix $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ is _____.

68. [NAT] **[GATE-EC-2015: 1M]**

The value of P such that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}$ is

69. [MCQ] **[GATE-CE-2015:1M]**

Let A = $[a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = ij$. The rank of A is

(a) 0 (b) 1

(c) n–1 (d) n

70. [MCQ] **[GATE-EE-2014: 1M]**

Two matrices A and B are given below:

$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$; $B = \begin{bmatrix} p^2 + q^2 & pr + qs \\ pr + qs & r^2 + s^2 \end{bmatrix}$

If the rank of matrix A is N, then the rank of matrix B is

(a) N/2 (b) N – 1

(c) N (d) 2N

71. [NAT] **[GATE-CE-2014:2M]**

The rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is _______?

72. [MCQ] [GATE-EE-2008: 1M]

If the rank of a (5 × 6) matrix Q is 4, then which one of the following statements is correct?

(a) Q will have four linearly independent rows and four linearly independent columns.
(b) Q will have four linearly independent rows and five linearly independent columns
(c) QQ^T will be invertible
(d) Q^TQ will be invertible

System of Equations

73. [MSQ] [GATE-CE-2023:2M]

For the matrix

$$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Which of the following statements is/are TRUE?

(a) $[A]\{x\}=\{b\}$ has a unique solution
(b) $[A]\{x\}=\{b\}$ does not have a unique solution
(c) $[A]$ has three linearly independent eigenvectors
(d) $[A]$ is a positive definite matrix

74. [MCQ] [GATE-EC-2022:1M]

Consider a system of linear equations Ax = *b*, where

$$\begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

This system of equations admits ______ .

(a) a unique solution for *x*
(b) infinitely many solutions for *x*
(c) no solutions for *x*
(d) exactly two solutions for *x*

75. [MCQ] [GATE-CS-2022:2M]

Consider solving the following system of simultaneous equations using LU decomposition.

$x_1 + x_2 - 2x_3 = 4$
$x_1 + 3x_2 - x_3 = 7$
$2x_1 + x_2 - 5x_3 = 7$

where L and U are denoted as $\begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$,

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?

(a) $L_{32} = 2$, $U_{33} = -1/2$, $x_1 = -1$
(b) $L_{32} = -1/2$, $U_{33} = 2$, $x_1 = 0$
(c) $L_{32} = 2$, $U_{33} = 2$, $x_1 = -1$
(d) $L_{32} = -1/2$, $U_{33} = -1/2$, $x_1 = 0$

76. [MSQ] [GATE-ME-2022: 2M]

The system of linear equations in real (x, y) given by

$(x\,y)\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0 \quad 0)$ involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

(a) x = 2, y = –2
(b) x = –1, y = 4
(c) x = 1, y = 1
(d) x = 4, y = –2

77. [MSQ] [GATE-ME-2022: 1M]

A is a 3 × 5 real matrix of rank 2. For the set of homogeneous equations Ax = O, where O is a zero vector and x is a vector of unknown variables, which of the following is/are true?

(a) The given set of equations will have a unique solution.
(b) The given set of equations will be satisfied by a zero vector of appropriate size.
(c) The given set of equations will have infinitely many solutions.
(d) The given set of equations will have many but a finite number of solutions.

78. [NAT] [GATE-CE-2020:2M]

Consider the system of equations $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & -3 \\ 4 & 4 & -6 \\ 2 & 5 & 2 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.

The value of x_3 (round off to the nearest integer), is _______.

79. [MCQ] [GATE-EC-2020:2M]

Consider the following system of linear equations.

$x_1 + 2x_2 = b_1$; $2x_1 + 4x_2 = b_2$;
$3x_1 + 7x_2 = b_3$; $3x_1 + 9x_2 = b_4$

Which one of the following conditions ensures that a solution exists for the above system?

(a) $b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$
(b) $b_3 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$
(c) $b_2 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$
(d) $b_3 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

80. [MCQ] [GATE-ME-2019: 2M]

The set of equations

$x + y + z = 1$
$ax - ay + 3z = 5$
$5x - 3y + az = 6$

has infinite solutions, if a =

(a) –3 (b) 3
(c) 4 (d) – 4

81. [NAT] [GATE-EC-2018:1M]

Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real value of k for which the equation Ax = 0 has infinitely many solutions is_____.

82. [MCQ] [GATE-CS-2017: 1M]

Let $c_1, \ldots c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} a_i x_i = 0$ where a_i are column vectors in R^n.

Consider the set of linear equations

$Ax = b$

where $A = [a_1 \ldots . a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

(a) a unique solution at $x = J_n$ where J_n denotes a n-dimensional vector of all 1
(b) no solution
(c) infinitely many solutions
(d) finitely many solutions

83. [MCQ] [GATE-CE-2017:2M]

Consider the following simultaneous equations (with c_1 and c_2 being constants)

$3x_1 + 2x_2 = c_1$

$4x_1 + x_2 = c_2$

The characteristics equation for these simultaneous equation is

(a) $\lambda^2 - 4\lambda - 5 = 0$ (b) $\lambda^2 - 4\lambda + 5 = 0$
(c) $\lambda^2 + 4\lambda - 5 = 0$ (d) $\lambda^2 + 4\lambda + 5 = 0$

84. [MCQ] [GATE-ME-2016: 1M]

The solution to the system of equations

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 2 \\ -30 \end{Bmatrix}$$

is

(a) 6, 2 (b) –6, 2
(c) –6, –2 (d) 6, –2

85. [MCQ] [GATE-CS-2016:1M]

Consider the systems, each consisting of m linear equations in n variables.

I. If m < n, then all such systems have a solution
II. If m > n, then none of these systems has a solution
III. If m = n, then there exists a system which has a solution

Which one of the following is CORRECT?

(a) I, II and III are true
(b) Only II and III are true
(c) Only III is true
(d) None of them is true

86. [MCQ] [GATE-CE-2016:1M]

Consider the following linear system

$x + 2y - 3z = a$
$2x + 3y + 3z = b$
$5x + 9y - 6z = c$

This system is consistent if a, b and c satisfy the equation

(a) $7a - b - c = 0$ (b) $3a + b - c = 0$
(c) $3a - b + c = 0$ (d) $7a - b + c = 0$

87. [NAT] [GATE-EC-2015: 1M]

Consider a system of linear equations:

$x - 2y + 3z = -1$

$x - 3y + 4z = 1$, and

$-2x + 4y - 6z = k$

The value of k for which the system has infinitely - many solutions is _________.

88. [NAT] [GATE-CE-2015:1M]

For what value of P the following set of equations will have no solutions

$2x + 3y = 5$

$3x + py = 10$

89. [MCQ] [GATE-EC-2014:2M]

The system of linear equations

$$\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix} \text{ has}$$

(a) a unique solution

(b) infinitely many solutions

(c) no solution

(d) exactly two solutions

90. [MCQ] [GATE-EE-2014: 1M]

Given a system of equation:

$x + 2y + 2z = b_1$

$5x + y + 3z = b_2$

Which of the following is true regarding its solutions

(a) The system has a unique solution for any given b_1 and b_2

(b) The system will have infinitely many solutions for any given b_1 and b_2

(c) Whether or not a solution exists depends on the given b_1 and b_2

(d) The system would have no solution for any value of b_1 and b_2

91. [MCQ] [GATE-ME-2014: 1M]

The matrix form of the linear system $\frac{dx}{dt} = 3x - 5y$ and $\frac{dy}{dt} = 4x + 8y$ is

(a) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(b) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & 8 \\ 4 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(c) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(d) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & 8 \\ 3 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

92. [MCQ] [GATE-CE-2014:2M]

The following simultaneous equations

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + kz = 6$

will NOT have a unique solution for k equal to

(a) 0 (b) 5

(c) 6 (d) 7

93. [NAT] [GATE-CS-2014:1M]

Consider the following system of equations:

$3x + 2y = 1$

$4x + 7z = 1$

$x + y + z = 3$

$x - 2y + 7z = 0$

The number of solutions for this system is _______.

94. [MCQ] [GATE-ME-2012: 2M]

$x + 2y + z = 4$

$2x + y + 2z = 5$

$x - y + z = 1$

The system of algebric equations given above has

(a) A unique solution of $x = 1$, $y = 1$ and $z = 1$

(b) Only the two solutions of ($x = 1$, $y = 1$, $z = 1$) and ($x = 2$, $y = 1$, $z = 0$)

(c) Infinite number of solutions

(d) No feasible solution

95. [MCQ] [GATE-EC-2011:2M]

The system of equations

$$x + y + z = 6$$
$$x + 4y + 6z = 20$$
$$x + 4y + \lambda z = \mu$$

has No solution for values of λ and μ given by

(a) $\lambda = 6, \mu = 20$ (b) $\lambda = 6, \mu \neq 20$

(c) $\lambda \neq 6, \mu = 20$ (d) $\lambda \neq 6, \mu \neq 20$

96. [MCQ] [GATE-ME-2011: 2M]

Consider the following system of equations

$$2x_1 + x_2 + x_3 = 0$$
$$x_2 - x_3 = 0$$
$$x_1 + x_2 = 0$$

The system has

(a) A unique solution

(b) No solution

(c) Infinite number of solutions

(d) Five solutions

97. [MCQ] [GATE-EE-2010: 2M]

For the set of equations

$x_1 + 2x_2 + x_3 + 4x_4 = 2$

$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6$

The following statement is true:

(a) Only the trivial solutions $x_1 = x_2 = x_3 = x_4 = 0$

(b) There are no solutions

(c) A unique non-trivial solution exists

(d) Multiple non-trivial solutions exist.

98. [MCQ] [GATE-CS-2008:1M]

The following system of equations

$x_1 + x_2 + 2x_3 = 1$

$x_1 + 2x_2 + 3x_3 = 2$

$x_1 + 4x_2 + \alpha x_3 = 4$

Has a unique solution. The only possible value(s) for α is/are

(a) 0 (b) either 0 or 1

(c) One of 0, 1 or –1 (d) any real number

99. [MCQ] [GATE-EC-2008:1M]

The system of linear equations

$$4x + 2y = 7$$
$$2x + y = 6$$

has

(a) a unique solution

(b) no solution

(c) an infinite number of solution

(d) exactly two distinct solutions

100.[MCQ] [GATE-ME-2008: 2M]

For what value of a, if any, will the following system of equations in x, y and z have a solution?

$$2x + 3y = 4$$
$$x + y + z = 4$$
$$x + 2y - z = a$$

(a) Any real number

(b) 0

(c) 1

(d) There is no such value

Eigen Values and Eigen Vectors

101.[MSQ] [GATE-CE-2023:2M]

If M is an arbitrary real n × n matrix, then which of the following matrices will have non-negative eigenvalues?

(a) M^2 (b) MM^T

(c) M^TM (d) $(M^T)^2$

102. [MCQ] [GATE-EC-2023: 1M]

Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{v_k \mid 1 \leq k \leq n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B = P^{-1}AP$, respectively, are

(a) $\{\lambda_k \det(A) \mid 1 \leq k \leq n\}$ and $\{Pv_k \mid 1 \leq k \leq n\}$

(b) $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{v_k \mid 1 \leq k \leq n\}$

(c) $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{Pv_k \mid 1 \leq k \leq n\}$

(d) $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{P^{-1}v_k \mid 1 \leq k \leq n\}$

103.[MSQ] [GATE-CE-2023:2M]

For the matrix

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

which of the following statements is/are TRUE?

(a) The eigenvalues of $[A]^T$ are same as the eigenvalues of $[A]$

(b) The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$

(c) The eigenvectors of $[A]^T$ are same as the eigenvectors of $[A]$

(d) The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of $[A]$

104.[MSQ] [GATE-CE-2022:1M]

Ley y be a non-zero vector of the size 2022 × 1. Which of the following statement(s) is/are TRUE?

(a) yy^T is symmetric matrix

(b) yy^T is invertible

(c) yy^T has a rank of 2022

(d) y^Ty is an eigen value of yy^T

105.[MSQ] [GATE-CE-2022:1M]

The matrix M is defines as $M = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ and has eigenvalues 5 and – 2. The matrix Q is formed as $Q = M^3 - 4M^2 - 2M$

Which of the following is/are the eigenvalue(s) of matrix Q?

(a) 15 (b) 28

(c) –20 (d) –30

106.[NAT] [GATE-ME-2022: 2M]

If the sum and product of eigenvalues of a 2 × 2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and –1 respectively, then |p| is ___ (in integer).

107.[MCQ] [GATE-EE-2022: 2M]

Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix. Then (c + d) is equal to

(a) 5 (b) 17

(c) –6 (d) 11

108.[MCQ] [GATE-EE-2022: 2M]

e^A denotes the exponential of a square matrix A. Suppose λ is an eigenvalue and v is the corresponding eigen-vector of matrix A.

Consider the following two statements:

Statement 1: e^{λ} is an eigenvalue of e^A

Statement 2: v is an eigen-vector of e^A.

Which one of the following options is correct?

(a) Statement 1 is true and statement 2 is false

(b) Statement 1 is false and statement 2 is true

(c) Both the statement are correct

(d) Both the statements are false.

109.[MCQ] [GATE-EE-2022: 1M]

Consider a 3 × 3 matrix A whose (i, j)-th element, $a_{i,j} = (i - j)^3$. Then the matrix A will be

(a) symmetric (b) skew-symmetric

(c) unitary (d) null

110.[MCQ] [GATE-ME-2021: 2M]

Consider a vector p in 2-dimensional space. Let its direction (counter clockwise angle with the positive x-axis) be θ. Let p be an eigenvector of a 2 × 2 matrix A with corresponding eigenvalue λ, λ > 0. If we denote the magnitude of a vector v by ||v||, identify the VALID statement regarding p′, where p′= Ap.

(a) Direction of P′ = λθ, || p′ || = || p ||

(b) Direction of P′ = θ, || p′ || = λ || p ||

(c) Direction of P′ = λθ, || p′ || = λ || p ||

(d) Direction of P′ = θ, || p′ || = || p || / λ

111.[NAT] [GATE-ME-2021: 2M]

Consider an n × n matrix A and a non-zero n × 1 vector P. Their product AP = α^2P, where $\alpha \notin R$ and $\alpha \notin \{-1, 0, 1\}$. Based on the given information, the eigenvalue of A^2 is:

(a) α (b) α^2
(c) $\sqrt{\alpha}$ (d) α^4

112.[NAT] [GATE-EE-2021: 2M]

Let A be a 10 × 10 matrix such that A^5 is a null matrix, and let I be the 10 × 10 identity matrix. The determinant of A + I is _______________.

113.[MCQ] [GATE-EE-2021: 1M]

Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

(a) 1 and 1 (b) 1 and –1
(c) j and –j (d) pq and –pq

114.[NAT] [GATE-CS-2021:1M]

Consider the following matrix.

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The largest eigen value of the above matrix is ______.

115.[MCQ] [GATE-CE-2021:1M]

The smallest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 2 & -2 \\ -1 & 6 \end{bmatrix}$ respectively are

(a) 1.55 and $\begin{Bmatrix} 2.00 \\ -0.45 \end{Bmatrix}$ (b) 1.55 and $\begin{Bmatrix} 2.00 \\ 0.45 \end{Bmatrix}$

(c) 2.00 and $\begin{Bmatrix} 1.00 \\ 1.00 \end{Bmatrix}$ (d) 1.55 and $\begin{Bmatrix} -2.55 \\ -0.45 \end{Bmatrix}$

116.[MCQ] [GATE-CE-2020:2M]

A 4 × 4 matrix [P] is given below

$$[P] = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

The eigen value of [P] are

(a) 0, 3, 6, 6 (b) 1, 2, 3, 4
(c) 3, 4, 5, 7 (d) 1, 2, 5, 7

117.[NAT] [GATE-ME-2020: 1M]

Let **I** be a 100 dimensional identity matrix and **E** be the set of its distinct (no value appears more than once in **E**) real eigenvalues. The number of elements in **E** is ____

118.[NAT] [GATE-EC-2019:1M]

The number of distinct eigenvalues of the matrix.

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to _______.

119.[MCQ] [GATE-ME-2019: 1M]

Consider the matrix $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. The number of distinct eigen values of P is

(a) 0 (b) 1
(c) 2 (d) 3

120.[MCQ] [GATE-ME-2019: 1M]

In matrix equation [A] {X} = {R},

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{X\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}.$$

One of the eigenvalues of matrix [A] is

(a) 4 (b) 8
(c) 15 (d) 16

121.[MCQ] [GATE-EE-2019: 1M]

M is a 2 × 2 matrix with eigenvalues 4 and 9.
The eigenvalues of M^2 are

(a) 4 and 9 (b) 2 and 3
(c) –2 and –3 (d) 16 and 81

122.[NAT] [GATE-CS-2019: 2M]

Considering the following matrix

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of R is_____.

123.[NAT] [GATE-EE-2018: 2M]

Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3 × 3 identity matrix. The determinant of B is _______ (up to 1 decimal place).

124.[MCQ] [GATE-CE-2018:2M]

The matrix $\begin{bmatrix} 2 & -4 \\ 4 & -2 \end{bmatrix}$ has

(a) Real eigenvalues and eigenvectors

(b) Real eigenvalue but complex eigenvectors

(c) Complex eigenvalues but real eigenvectors

(d) Complex eigenvalues and eigenvectors

125.[NAT] [GATE-CS-2018: 2M]

Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements.

I: P does not have an inverse

II: P has a repeated eigenvalue

III: P cannot be diagonalized

Which one of the following options is correct?

(a) Only I and III are necessarily true

(b) Only II is necessarily true

(c) Only I and II are necessarily true

(d) Only II and III are necessarily true

126.[NAT] [GATE-EE-2018: 1M]

Consider a non-singular 2 × 2 square matrix A. If trace(A) = 4 and trace $(A^2) = 5$, the determinant of the matrix A is ____ (up to 1 decimal place).

127. [MSQ] [GATE-EC-2018:1M]

Let M be a real 4 × 4 matrix. Consider the following statements:

S1: M has 4 linearly independent eigenvectors.

S2: M has 4 distinct eigenvalues.

S3: M is non-singular (invertible).

Which one among the following is TRUE?

(a) S1 implies S2 (b) S1 implies S3

(c) S2 implies S1 (d) S3 implies S2

128. [MCQ] [GATE-CS-2017: 2M]

Let A be n × n real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{i=1}^{n} A_{ij}^2 = 50$ Consider the following statements.

(I) One eigenvalue must be in [–5, 5]

(II) The eigenvalue with the largest magnitude must be strictly greater than 5

Which of the above statements about eigenvalues of A is/are necessarily CORRECT?

(a) Both (I) and (II) (b) (I) only

(c) (II) only (d) Neither (I) nor (II)

129.[NAT] [GATE-CS-2017:2M]

If the characteristic polynomial of a 3 × 3 matrix M over R (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30$, $a \in R$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is ________.

130.[MCQ] [GATE-CE-2017:2M]

Consider the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$ which one of the following statements is TRUE for the eigenvalues and eigen vectors of this matrix?

(a) Eigenvalue 3 has multiplicity of 2, and only one independent eigenvector exists.

(b) Eigenvalue 3 has a multiplicity of 2, and two independent eigenvector exists.

(c) Eigenvalue 3 has a multiplicity of 2, and no independent eigenvector exists.

(d) Eigenvalue are 3 and – 3 and two independent eigenvector exists.

131.[MCQ] [GATE-EC-2017: 1M]

Consider the 5 × 5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

It is given that A has only one real eigenvalue. Then the real eigenvalue of A is

(a) –2.5 (b) 0

(c) 15 (d) 25

132.[MCQ] [GATE-ME-2017: 1M]

The product of eigenvalues of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

(a) –6 (b) 2

(c) 6 (d) –2

133.[NAT] [GATE-ME-2017: 1M]

The determinant of a 2 × 2 matrix is 50. If one eigenvalue of the matrix is 10, the other eigenvalue is ______.

134.[MCQ] [GATE-EE-2017: 2M]

The eigen values of the matrix given below are:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

(a) (0, –1, –3) (b) (0, –2, –3)

(c) (0, 2, 3) (d) (0, 1, 3)

135.[MCQ] [GATE-EE-2017:1M]

The matrix $A = \begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct eigenvalues and one of its eigenvectors is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

Which one of the following can be another eigenvector of A?

(a) $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

136.[MCQ] [GATE-ME-2016: 1M]

The condition for which the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$$

are positive, is

(a) $k > 1/2$ (b) $k > -2$

(c) $k > 0$ (d) $k < -1/2$

137.[MCQ] [GATE-ME-2016: 2M]

The number of linearly independent eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is _____

138.[MCQ] [GATE-EC-2016:1M]

Consider a 2 × 2 square matrix $A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$ where x is unknown. If the eigenvalues of the matrix A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$, then x is equal to

(a) $+j\omega$ (b) $-j\omega$

(c) $+\omega$ (d) $-\omega$

139.[NAT] [GATE-EC-2016: 1M]

The value of x for which the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$ has zero as an eignevalues is_______

140.[MCQ] [GATE-CE-2016:1M]

If the entries in each column of a square matrix M add up to 1, then an eigen value of M is

(a) 4 (b) 3

(c) 2 (d) 1

141.[NAT] [GATE-CS-2016: 1M]

Two eigenvalues of a 3 × 3 real matrix P are $\left(2 + \sqrt{-1}\right)$ and 3. The determinant of P is ______.

142.[MCQ] [GATE-EE-2016: 2M]

Let the eigenvalues of a 2 × 2 matrix A be 1, –2 with eigenvectors x_1 and x_2 respectively. Then the eigenvalues and eigenvectors of the matrix $A^2 - 3A + 4I$ would, respectively, be

(a) 2, 14 ; x_1, x_2

(b) 2, 14 ; $x_1 + x_2$, $x_1 - x_2$

(c) 2, 0 ; x_1, x_2

(d) 2, 0 ; $x_1 + x_2$, $x_1 - x_2$

143.[MCQ] [GATE-EE-2016: 1M]

A 3 × 3 matrix P is such that, $P^3 = P$. Then the eigenvalues of P are

(a) 1, 1, –1

(b) 1, 05 + j0.866, 0.5 – j0.866

(c) 1, –0.5 + j0.868, –0.5 – 0.866

(d) 0, 1, –1

144.[NAT] [GATE-EE-2016: 1M]

Consider a 3 × 3 matrix with every element being equal to 1. Its only non-zero eigenvalue is ______.

145.[MCQ] [GATE-EE-2015: 1M]

The maximum value of ‘a’ such that the matrix $\begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$ has three linearly independent real eigenvectors is

(a) $\frac{2}{3\sqrt{3}}$

(b) $\frac{1}{3\sqrt{3}}$

(c) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$

(d) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

146.[MCQ] [GATE-EE-2015: 1M]

We have a set of 3 linear equations in 3 unknowns.

‘X ≡ Y' means X and Y are equivalent statements.

'X ≢ Y' means X and Y are not equivalent statements.

P. There is a unique solution.

Q. The equations are linearly independent.

R. All eigenvalues of the coefficient matrix are nonzero.

S. The determinant of the coefficient matrix is nonzero.

Which one of the following is TRUE?

(a) P ≡ Q ≡ R ≡ S

(b) P ≡ Q ≢ R ≡ S

(c) P ≡ Q ≢ R ≡ 0

(d) P ≢ Q ≢ R ≢ S

147.[NAT] [GATE-EE-2015: 1M]

If the sum of the diagonal elements of a 2 × 2 matrix is –6, then the maximum possible value of determinant of the matrix is ____.

148.[MCQ] [GATE-CS-2015:2M]

Consider the following 2 × 2 matrix A where two elements are unknown and are marked by a and b. The eigenvalues of this matrix are -1 and 7. What are the values of a and b?

$$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$$

(a) a = 6, b = 4

(b) a = 4, b = 6

(c) a = 3, b = 5

(d) a = 5, b = 3

149.[MCQ] [GATE-CE-2015:2M]

The smallest and largest Eigen value of the following matrix are

$$\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$$

(a) 1.5 and 2.5

(b) 0.5 and 2.5

(c) 1.0 and 3.0

(d) 1.0 and 2.0

150.[MCQ] [GATE-CE-2015:2M]

The two Eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have a ratio of 3 : 1 for p = 2. What is the another value of P if eigen value have the same ratio of 3 : 1 ?

(a) –2

(b) 1

(c) 7/3

(d) 14/3

151.[NAT] [GATE-CS-2015: 1M]

The largest of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is______.

152.[MCQ] [GATE-CS-2015:1M]

In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$ one of the eigenvalues is 1. The eigenvectors corresponding to the eigenvalues 1 are

(a) $\{\alpha(4, 2, 1) \mid \alpha \neq 0, \alpha \in R\}$
(b) $\{\alpha(-4, 2, 1) \mid \alpha \neq 0, \alpha \in R\}$
(c) $\{\alpha(\sqrt{2}, 0, 1) \mid \alpha \neq 0, \alpha \in R\}$
(d) $\{\alpha(-\sqrt{2}, 0, 1) \mid \alpha \neq 0, \alpha \in R\}$

153.[MCQ] [GATE-EC-2015:1M]

The value of x for which all the eigen-values of the matrix given below are real is $\begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$

(a) $5 + j$
(b) $5 - j$
(c) $1 - 5j$
(d) $1 + 5j$

154.[MCQ] [GATE-ME-2015: 1M]

At least one eigenvalue of a singular matrix is

(a) Positive
(b) Zero
(c) Negative
(d) Imaginary

155.[NAT] [GATE-ME-2015: 1M]

The lowest eigenvalue of the 2×2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is _____.

156.[MCQ] [GATE-ME-2014: 1M]

One of the eigenvectors of the matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$
(b) $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$
(c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$
(d) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

157.[MCQ] [GATE-ME-2014: 1M]

Consider a 3×3 real symmetric matrix S such that two of its eigenvalues are $a \neq 0, b \neq 0$ with respective eigenvectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$. If $a \neq b$, then $x_1 y_1 + x_2 y_2 + x_3 y_3$ equals

(a) a
(b) b
(c) ab
(d) 0

158.[MCQ] [GATE-EE-2014: 1M]

Which one of the following statements is true for all real symmetric matrices?

(a) All the eigenvalues are real
(b) All the eigenvalues are positive
(c) All the eigenvalues are distinct
(d) Sum of all the eigenvalues is zero

159.[NAT] [GATE-EE-2014: 2M]

A system matrix is given as follows:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}$$

The absolute value of the ratio of the maximum eigenvalue to the minimum eigenvalue is _______.

160.[NAT] [GATE-CS-2014:1M]

The value of the dot product of the eigenvectors corresponding to any pair of different eigenvalues of a 4-by-4 symmetric positive definite matrix is ________.

161.[NAT] [GATE-CS-2014:2M]

If the matrix A is such that $A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$

then the determinant of A is equal to ____________.

162.[MCQ] [GATE-CE-2014:1M]

The sum of Eigen values of matrix, [M] is where

$$[M] = \begin{bmatrix} 215 & 650 & 795 \\ 655 & 150 & 835 \\ 485 & 355 & 550 \end{bmatrix}$$

(a) 915
(b) 1355
(c) 1640
(d) 2180

163.[MCQ] **[GATE-EC-2014:2M]**

Which one of the following statements is NOT true for a square matrix A?

(a) "If A is upper triangular, the eigen values of A are the diagonal elements of it"

(b) "If A is real, symmetric, the eigen values of A are always real and positive"

(c) "If A is real, the eigen values of A and " A^T " are always the same."

(d) "If all the principal minors of A are positive, all the eigen values of A are also positive."

164.[MCQ] **[GATE-ME-2013: 1M]**

The eigenvalues of a symmetric matrix are all

(a) Complex with non-zero positive imaginary part.

(b) Complex with non-zero negative imaginary part.

(c) Real.

(d) Pure imaginary.

165.[MCQ] **[GATE-EE-2013: 2M]**

A matrix has eigenvalues –1 and –2. The corresponding eigen vectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively. The matrix is

(a) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

166.[MCQ] **[GATE-EE-2013: 1M]**

The equation $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has

(a) no solution

(b) only one solution $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) non-zero unique solution

(d) multiple solutions

167.[MCQ] **[GATE-EC-2013:1M]**

The minimum eigen value of the following matrix is

$$\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

(a) 0 (b) 1

(c) 2 (d) 3

168.[MCQ] **[GATE-ME-2012: 2M]**

For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, one of the normalized eigen vector is given as

(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

169.[MCQ] **[GATE-EE-2012: 2M]**

Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the value of A^3 is

(a) 15A + 12I (b) 19A + 30I

(c) 17A + 15I (d) 17A + 21I

170.[NAT] **[GATE-CS-2012:1M]**

Let A be the 2 × 2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$. Then the eigenvalues of the matrix A^{19} are

(a) 1024 and –1024

(b) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(c) $4\sqrt{2}$ and $-4\sqrt{2}$

(d) $512\sqrt{2}$ and $-512\sqrt{2}$

171.[MCQ] [GATE-CS-2011:2M]

Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

Which one of the following options provides the CORRECT values of the Eigen values of the matrix?

(a) 1, 4, 3 (b) 3, 7, 3
(c) 7, 3, 2 (d) 1, 2, 3

172.[MCQ] [GATE-ME-2011: 1M]

Eigenvalues of a real symmetric matrix are always

(a) Positive (b) Negative
(c) Real (d) Complex

173.[MCQ] [GATE-ME-2010: 2M]

One of the eigen vectors of the matrx $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is

(a) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (b) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$

(c) $\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

174.[MCQ] [GATE-EC-2010:1M]

The eigenvalues of a skew symmetric matrix are

(a) always zero
(b) Always pure imaginary
(c) Eithe zero or pure imaginary
(d) Always real

175.[MCQ] [GATE-CS-2010:2M]

Consider the following matrix $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$. If the Eigen values of A are 4 and 8, then

(a) x = 4, y = 10 (b) x = 5, y = 8
(c) x = 3, y = 9 (d) x = –4, y = 10

176.[MCQ] [GATE-EE-2010: 2M]

An eigenvector of $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} -1 & 1 & 1 \end{bmatrix}^T$ (b) $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T$

(c) $\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}^T$ (d) $\begin{bmatrix} 2 & 1 & -1 \end{bmatrix}^T$

177.[MCQ] [GATE-EE-2009: 1M]

The trace and determinant of a 2 × 2 matrix are known to be –2 and –35 respectively its eigenvalues are

(a) –30 and –5 (b) –37 and –1
(c) –7 and 5 (d) 17.5 and –2

178.[MCQ] [GATE-EC-2009:2M]

The eigen values of the following matrix are

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) 3, 3 + 5j, 6 – j (b) –6 + 5j, 3 + j, 3 – j
(c) 3 + j, 3 – j, 5 + j (d) 3, –1 + 3j, –1 – 3j

179.[MCQ] [GATE-CE-2008:1M]

The Eigen values of the matrix $[P] = \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$ are

(a) – 7 and 8 (b) – 6 and 5
(c) 3 and 4 (d) 1 and 2

180.[MCQ] [GATE-CS-2008:2M]

How many of the following matrices have an Eigen value 1?

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{and} \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

(a) One (b) Two
(c) Three (d) Four

181.[MCQ] [GATE-ME-2008: 1M]

The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigenvalue equal to 3.

The sum of the other two eigenvalue is

(a) p (b) p – 1
(c) p – 2 (d) p – 3

182.[MCQ] **[GATE-ME-2008: 2M]**

The eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is a + b?

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) 2

183.[MCQ] **[GATE-EC-2008: 1M]**

All the four entries of the 2 × 2 matrix $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ are nonzero, and one of its eigenvalues is zero. Which of the following statements is true?

(a) $P_{11}P_{22} - P_{12}P_{21} = 1$
(b) $P_{11}P_{22} - P_{12}P_{21} = -1$
(c) $P_{11}P_{22} - P_{12}P_{21} = 0$
(d) $P_{11}P_{22} + P_{12}P_{21} = 0$

Basis and Spaces

184.[MCQ] **[GATE-CE-2023:2M]**

Two vectors $[2\ 1\ 0\ 3]^T$ and $[1\ 0\ 1\ 2]^T$ belong to the null space of a 4×4 matrix of rank 2. Which one of the following vectors also belongs to the null space?

(a) $[1\ 1\ -1\ 1]^T$
(b) $[2\ 0\ 1\ 2]^T$
(c) $[0\ -2\ 1\ -1]^T$
(d) $[3\ 1\ 1\ 2]^T$

185.[MCQ] **[GATE-EC-2015:1M]**

$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

(a) $\sec^2 x$
(b) $\cos 4x$
(c) 1
(d) 0

186.[NAT] **[GATE-CS-2014:1M]**

If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is ______.

❑ ❑ ❑

ANSWER KEY

1.	(b)	**2.**	(d)	**3.**	(a)	**4.**	(b)
5.	(c)	**6.**	(a)	**7.**	(32)	**8.**	(a)
9.	(c)	**10.**	(a)	**11.**	(a)	**12.**	(b)
13.	(marks to all)	**14.**	(d)	**15.**	(d)	**16.**	(c)
17.	(c)	**18.**	(c)	**19.**	(0.25 to 0.25)	**20.**	(a)
21.	(c)	**22.**	(0 to 0)	**23.**	(d)	**24.**	(7)
25.	(c)	**26.**	(3)	**27.**	(c)	**28.**	(233 to 233)
29.	(d)	**30.**	(0.125)	**31.**	(5)	**32.**	(a)
33.	(a, b)	**34.**	(a)	**35.**	(a)	**36.**	(d)
37.	(49)	**38.**	(1)	**39.**	(d)	**40.**	(200)
41.	(c)	**42.**	(23 to 23)	**43.**	(a)	**44.**	(88 to 88)
45.	(a)	**46.**	(b)	**47.**	(16)	**48.**	(b)
49.	(d)	**50.**	(b)	**51.**	(d)	**52.**	(b)
53.	(a)	**54.**	(b)	**55.**	(b)	**56.**	(b)
57.	(d)	**58.**	(4)	**69.**	(b)	**60.**	(c)
61.	(3)	**62.**	(b)	**63.**	(b)	**64.**	(b)
65.	(c)	**66.**	(2)	**67.**	(4)	**68.**	(17)
69.	(b)	**70.**	(c)	**71.**	(2 to 2)	**72.**	(a)
73.	(b, c)	**74.**	(c)	**75.**	(d)	**76.**	(a, b)
77.	(b, c)	**78.**	(3 to 3)	**79.**	(a)	**80.**	(c)
81.	(2 to 2)	**82.**	(c)	**83.**	(a)	**84.**	(d)
85.	(c)	**86.**	(b)	**87.**	(2)	**88.**	(4.49 to 4.51)
89.	(b)	**90.**	(b)	**91.**	(a)	**92.**	(d)
93.	(1)	**94.**	(c)	**95.**	(b)	**96.**	(a)
97.	(d)	**98.**	(c)	**99.**	(b)	**100.**	(b)
101.	(b, c)	**102.**	(c)	**103.**	(a, b, d)	**104.**	(a, d)
105.	(a, c)	**106.**	(2 to 2)	**107.**	(a)	**108.**	(c)
109.	(b)	**110.**	(b)	**111.**	(d)	**112.**	(1)
113.	(b)	**114.**	(3 to 3)	**115.**	(b)	**116.**	(d)
117.	(1 to 1)	**118.**	(3 to 3)	**119.**	(b)	**120.**	(d)
121.	(d)	**122.**	(12 to 12)	**123.**	(1.0)	**124.**	(d)
125.	(d)	**126.**	(5.5 to 5.5)	**127.**	(c)	**128.**	(b)
129.	(5)	**130.**	(a)	**131.**	(c)	**132.**	(b)
133.	(5 to 5)	**134.**	(a)	**135.**	(c)	**136.**	(a)
137.	(2 to 2)	**138.**	(d)	**139.**	(1)	**140.**	(d)
141.	(15 to 15)	**142.**	(a)	**143.**	(d)	**144.**	(3)

145.	(b)	**146.**	(a)	**147.**	(9)	**148.**	(d)
149.	(d)	**150.**	(d)	**151.**	(6 to 6)	**152.**	(b)
153.	(b)	**154.**	(b)	**155.**	(2 to 2)	**156.**	(d)
157.	(d)	**158.**	(a)	**159.**	(0.33)	**160.**	(0 to 0)
161.	(0)	**162.**	(a)	**163.**	(b)	**164.**	(c)
165.	(d)	**166.**	(d)	**167.**	(a)	**168.**	(b)
169.	(b)	**170.**	(d)	**171.**	(a)	**172.**	(c)
173.	(b)	**174.**	(c)	**175.**	(d)	**176.**	(b)
177.	(c)	**178.**	(d)	**179.**	(b)	**180.**	(a)
181.	(c)	**182.**	(b)	**183.**	(c)	**184.**	(a)
185.	(c)	**186.**	(2)				

SOLUTIONS

1. **(b)**

As per Cholesky decomposition; $L L^T = A$

$$\begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} L_{11} & L_{21} \\ 0 & L_{22} \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & 0_{22} \end{bmatrix}$$

Let L be the lower triangular matrix $= \begin{bmatrix} L_{11} & O \\ L_{21} & L_{22} \end{bmatrix}$

$\Rightarrow \quad L_{11} = 2\sqrt{2} \qquad\qquad L_{21}{}^2 + L_{22}{}^2 a_{22}$

$\Rightarrow \quad L_{11} \quad L_{21} = -5 \left(-\dfrac{5}{2\sqrt{2}}\right)^2 + (1.968)^2 = a_{22}$

$L_{21} = -\dfrac{5}{2\sqrt{2}}$

$a_{22} = 6.99 \approx 7$

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2. **(d)**

$w^T x = 1$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

$$[1\ 2\ 3]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

$x + 2y + 3z = 1$

$f \to x + 2y + 3z - 1$

grad $f \to \dfrac{\partial f}{\partial x}\hat{i} + \dfrac{\partial f}{\partial y}\hat{j} + \dfrac{\partial f}{\partial z}\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k} \quad [1\ 2\ 3]^T$

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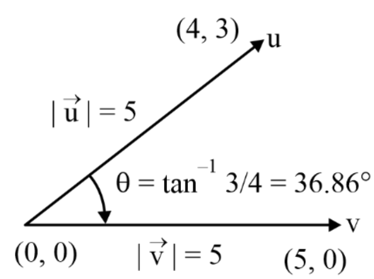
3. **(a)**



For anti-clockwise rotation by θ $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

But for clockwise rotation by θ $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$(\theta < 0)$

$$\therefore A = \begin{bmatrix} \cos 36.86° & \sin 36.86° \\ -\sin 36.86° & \cos 36.86° \end{bmatrix}$$

$$A = \begin{bmatrix} 4/5 & 3/5 \\ -3/5 & 4/5 \end{bmatrix}$$

$\therefore$ AU = v

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4. **(b)**

Given $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$

'B' is obtained by interchanging 1^{st} and 3^{rd} rows of 'A'.

if two rows (or) columns of a determinant are interchanged then determinant changes its sign.

$\Rightarrow$ det(B) = –det(A)

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5. **(c)**

Given,

order of 'A" is m × n; order of 'B' is 'n × m'

Order of 'C' is n × m; order of 'O' is 'n × m'

For any two matrices A and B, if both AB and BA exist, then tr(AB) = tr(BA)

$\therefore$ Both statements 1 and statements 2 are correct

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6. **(a)**

Let the coordinate of point P in transformed system is

(x', y', z') $(x, y, z) \xrightarrow[\text{about x-axis}]{45° \text{ rotation}} (x', y', z')$

The potential matrix for transformation about x-axis

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Since angle of rotation θ = 45°

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

The transformed coordinates of P is (1, 0, $\sqrt{2}$)

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7. **(32)**

Given $E = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$P1 Let eigen values of E be λ_1 and λ_2.

$\lambda_1 + \lambda_2 = 0$

$\lambda_1\lambda_2 = -2$

$\therefore \lambda_1 = \sqrt{2}; \lambda_2 = -\sqrt{2}$

$\therefore$ Eigen values of $E^8 = (\sqrt{2})^8$ and $(-\sqrt{2})^8$

= 16 and 16

$\therefore$ Eigen values of $E^{11} = (\sqrt{2})^{11}$ and $(-\sqrt{2})^{11}$

$= 32\sqrt{2}$ and $-32\sqrt{2}$

$\therefore$ P = Trance (E^8) = 16 + 16 = 32

$\therefore$ Q = Trance (E^{11}) = $32\sqrt{2} - 32\sqrt{2} = 0$

$\therefore$ P + Q = 32 + 0

P + Q =32

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8. (a)

Given:

Matrix $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is symmetric.

$\Rightarrow \quad 2k + 5 = 3k - 3 \qquad \Rightarrow \quad \boxed{k = 8}$

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9. (c)

$$Q^T P^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

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10. (a)

If $A\ A^T = I$ or $A^T = A^{-1}$ then the matrix is said to be orthogonal

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11. (a)

→ Matrix multiplication is associative.

→ Matrix multiplication NEED not be commutative.

→ A matrix 'A' is said to be definite if for any non-zero column vector 'z', $z^T Az$ is always definite.

In actual cases, $z^T Az$ need not be positive always.

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12. (b)

→ Every square matrix 'P' can be decomposed to symmetric and skew-symmetric parts.

$\Rightarrow \quad P = S + V$

$$\Rightarrow \quad P = \begin{bmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \boxed{P = \begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}}$$

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13. (marks to all)

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} 1$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{11}a_{12} & a_{11}a_{13} \\ a_{11}a_{12} & a_{12}^2 + a_{22}^2 & a_{12}a_{13} + a_{22}a_{23} \\ a_{11}a_{13} & a_{12}a_{13} + a_{22}a_{23} & a_{13}^2 + a_{23}^2 + a_{33}^2 \end{bmatrix}$$

$$\Rightarrow a_{11}^2 = 2 \Rightarrow \boxed{a_{11} = \sqrt{2}}$$

$$\Rightarrow a_{11}a_{12} = 3 \Rightarrow \sqrt{2}\ a_{12} = 3 \Rightarrow \boxed{a_{12} = 3/\sqrt{2}}$$

$$\Rightarrow a_{11}a_{13} = 3 \Rightarrow \sqrt{2}\ a_{13} = 3 \Rightarrow \boxed{a_{13} = 3/\sqrt{2}}$$

$$\Rightarrow a_{12}^2 + a_{12}^2 = 2 \Rightarrow \left(3/\sqrt{2}\right)^2 + a_{22}^2 = 2$$

$$\Rightarrow \boxed{a_{13} = \sqrt{-\frac{\sqrt{5}}{2}} = 1.58i}$$

$$\Rightarrow a_{11}a_{13} + a_{22}a_{23} = 1$$

$$\sqrt{2} \times \frac{3}{\sqrt{2}} + (1.58i)a_{23} = 1$$

$$a_{23} = 2.21i$$

$$\Rightarrow \quad a_{13}^2 + a_{23}^2 + a_{33}^2 = 7$$

$$\left(\frac{3}{\sqrt{2}}\right)^2 + (2.21i)^2 + a_{33}^2 = 7$$

$$\boxed{a_{33} = \sqrt{7.38}}$$

Now,

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 3/\sqrt{2} & 1.58i & 0 \\ 3/\sqrt{2} & 2.21i & \sqrt{7.38} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 3/\sqrt{2} & 1.58i & 0 \\ 3/\sqrt{2} & 2.21i & \sqrt{7.38} \end{bmatrix}^T$$

$[A = LL^T]$

$\therefore$ Number of purely real elements in matrix L = 7

Final answer is not matching with any option hence this question was **marks to all**.

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14. (d)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

We know, $A^{-1} = \dfrac{\text{adj } A}{|A|}$

Adj. (A) (Cofactor matrix)^T

$$\begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}^T = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$$

$|A| = 2(12 - 2) - 2(16 - 1) + 4(8 - 3) = 2 \times 10 - 3 \times 15 + 4 \times 5 = -5$

$$A^{-1} = \frac{\text{Adj}}{|A|} = -\frac{1}{5}\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix} = \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$$

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15. (d)

For the inverse of matrix to exist, the matrix should be non-singular

(i.e.) determinant is non-zero

$\therefore$ I and II are equivalent statements

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16. (c)

Let $M_{2\times2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$

$V_1 = \begin{bmatrix} a \\ c \end{bmatrix} \quad V_2 = \begin{bmatrix} b \\ d \end{bmatrix}$

$$\therefore \; M^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix}$$

$$U_1^T = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \end{bmatrix}, U_2^T = \begin{bmatrix} \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

Now,

$$U_1^T \cdot V_1 = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \frac{ad-bc}{ad-bc} = 1$$

$$U_2^T \cdot V_2 = \begin{bmatrix} \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \frac{ad-bc}{ad-bc} = 1$$

$\therefore \; U_1^T \cdot V_1 = 1$ and $U_2^T \cdot V_2 = 1$

Hence, statement 1 is true.

Now,

$$U_1^T \cdot V_2 = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \frac{bd-bd}{ad-bc} = 0$$

$$U_2^T \cdot V_1 = \begin{bmatrix} \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \frac{-ca+ca}{ad-bc} = 0$$

$\therefore \; U_1^T \cdot V_2 = 0$ and $\therefore \rho(A^TA) = 2$

Hence statement 2 is also true.

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17. (c)

Since matrix Q is orthogonal hence

$Q Q^T = I$

Pre multiply by Q^{-1}

$Q^{-1} Q Q^T = Q^{-1} I$

$Q^T = Q^{-1}$

$$\text{Hence } Q^{-1} = Q^T = \begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{bmatrix}$$

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18. (c)

If $|A| = 0$ (Singular matric)

Option a:- $\Delta = 6 - 5 = 1 \neq 0$

Option b:- $\Delta = 9 - 4 = 5 \neq 0$

Option c:- $\Delta = 12 - 12 = 0$ Hence it is singular

Option d:- $\Delta = 8 - 18 = 10 \neq 0$

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19. (0.25 to 0.25)

Given,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ 'A' is an upper triangular matrix.

For any upper triangular matrix, determinant is product of principal diagonal elements.

$\Rightarrow \quad |A| = (1)(4)(1) = 4$

Now $\quad |A^{-1}| = \frac{1}{|A|} = \frac{1}{4} = 0.25$

$\therefore \quad |A^{-1}| = 0.25$

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20. (a)

$$AB^T = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 7 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}}$$

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21. (c)

Given:-$P = Q^{-1}$

- Post multiply by Q
  $PQ = Q^{-1} Q \qquad [\because A^{-1} A = I]$
  $PQ = I$
- Pre multiply by Q
  $QP = Q Q^{-1} \qquad [\because AA^{-1} = I]$
  $QP = I$

Hence $PQ = I$ and $QP = I$

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22. (0 to 0)

Given: Matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$

'A' is symmetric matrix

If x_1, x_2 are eigen vectors of 'A' corresponding to λ_1 and λ_2, then

$\boxed{x_1^T \cdot x_2 = 0}$ (In general, $x_i^T \cdot x_j = 0 \ \forall \ i \neq j$)

OR

Given, $x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix}$; $\quad x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$

$$\Rightarrow \quad x_1^T \cdot x_2 = [70 \quad \lambda_1 - 50] \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$$

$= 70(\lambda_2 - 80) + 70(\lambda_1 - 50)$

$= 70(\lambda_1 + \lambda_2 - 130)$

$= 70$(Sum of diagonal elements $- 130$)

$= 70(130 - 130) = 0$

$\Rightarrow \quad \boxed{x_1^T \cdot x_2 = 0}$

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23. (d)

Given

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Rightarrow \quad |P| = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - 0\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} + \frac{1}{2} = 1$$

$P.P^T$ is given by

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Since $P.P^T = I \quad \Rightarrow$ 'P' is orthogonal $\quad \Rightarrow \quad P^{-1} = P^T$

An orthogonal matrix need not have all the eigen values being real.

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24. (7)

$$2x^2 + y^2 = 34 \qquad \text{.....(1)}$$

$$x + 2y = 11$$

$$y = \frac{11 - x}{2} \qquad \text{... (2)}$$

From value of y from equation (2) to equation (1),

$$2x^2 + \left(\frac{11 - x}{2}\right)^2 = 34$$

$$\Rightarrow \quad 8x^2 + 121 + x^2 - 22x = 136$$

$$\Rightarrow \quad 9x^2 - 22x - 15 = 0$$

$$\Rightarrow \quad x = \frac{-10}{18}, 3$$

Discarding $x = -10/18$ because it is not on integer.

Hence $x = 3$.

Put value of x in equation (2),

$$y = \frac{11 - x}{2} = \frac{11 - 3}{2} = 4$$

Hence,

$$x + y = 3 + 4$$

$$\Rightarrow \quad \boxed{x + y = 7}$$

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25. (c)

A square matrix 'A' is said to be Skew-symmetric if $\quad A = -A^T$

$\Rightarrow \quad A^T = -A$

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26. (3)

Let the given matrix is

$$A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}; \; |A| = 100, \; tr(A) = 14$$

$tr(A) = a + 5 + 2 + b = 14$

$\Rightarrow a + b = 7$

$$|A| = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix} =$$

$$|A| = a\begin{vmatrix} 5 & 1 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & b \end{vmatrix} + 3\begin{vmatrix} 2 & 5 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & b \end{vmatrix} - 7\begin{vmatrix} 2 & 5 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{vmatrix}$$

$= a(10b) = 10ab$

$\Rightarrow 10ab = 100 \Rightarrow ab = 10$

Since $(a - b)^2 = (a + b)^2 - 4\,ab$

$= (7)^2 - 40 = 9$

$\Rightarrow (a - b)^2 = 9$

$\Rightarrow |(a - b)| = 3$

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27. (c)

Given $M^4 = I \Rightarrow (M^4)^k = I^k = I \Rightarrow M^{4k} = I$

Multiplying both sides with M^{-1}

$\Rightarrow M^3 = M^{-1}$

$\Rightarrow M^{-1} = M^3 = M^3 \cdot I$

$\Rightarrow M^{-1} = M^3 \cdot M^{4k}$

$\Rightarrow M^{-1} = M^{4k+3}$

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28. (233 to 233)

Given that a sequence is defined by

x [n] and it is given by

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

And x [0] = 1; x [1] = 1

For n = 2, $\begin{bmatrix} x[2] \\ x[1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\therefore$ x [2] = 2

Given sequence is a Fibonacci sequence

$\therefore$ The given sequence is equal to

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …..

$\therefore$ x [12] = 233

$\boxed{\therefore\ 13^{th} \text{ term of the sequence} = x[12] = 233}$

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29. (d)

Given $\begin{pmatrix} a \\ b \end{pmatrix} = P \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

3x + y = a …..(i)

x + 3y = b ….(ii)

On squaring (i) and (ii) and then adding, we get

$a^2 + b^2 = (3x + y)^2 + (x + 3y)^2 = 1$

$9x^2 + y^2 + 6xy + x^2 + 9y^2 + 6xy = 1$

$10x^2 + 10y^2 + 12xy = 1$….(i)

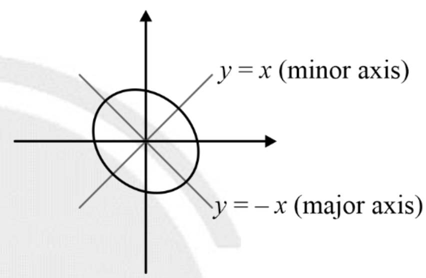
On comparing with std from $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\therefore$ a = 10, b = 10, h = 6

$h^2 - ab = 36 - 10\times10 = -64 < 0 \Rightarrow$ Ellipse

$\therefore$ Equation (i) represents an ellipse

With major axis along $y = -x \simeq \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and with minor axis along $y = x \simeq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.



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30. (0.125)

Given, eigen values of A = 1, 2, 4

$\Rightarrow |A|$ = product of eigen values = 1.2.4 = 8

Determinant of $(A^{-1})^T$

= Determinant of $(A^{-1}) = \frac{1}{|A|} = \frac{1}{8} = 0.125$

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31. (5)

Let, $A = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$

LU decomposition $\Rightarrow$ A = LU

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{12} & L_{22} \end{bmatrix}\begin{bmatrix} 1 & U_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{11}U_{12} \\ L_{12} & L_{12}U_{12} + L_{22} \end{bmatrix}$$

$L_{11} = 2$, $L_{12} = 4$, $U_{12} = 1$

$L_{12}U_{12} + L_{22} = 9$

$\Rightarrow L_{22} = 9 - (4 \times 1) = 5$

$\therefore L_{22} = 5$

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32. (a)

When two rows (or) two columns of a determinants are interchanged, then the absolute value of the determinant does not change but the sign of the value changes.

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33. (a, b)

If P and Q are invertible, then

$(PQ)^{-1} = Q^{-1}P^{-1}$ and $(QP)^{-1} = P^{-1}Q^{-1}$

$\rightarrow$ If P and Q are not invertible, then

$(PQ)^{-1} \neq Q^{-1}P^{-1}$

$(QP)^{-1} \neq P^{-1}Q^{-1}$

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34. (a)

For a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Rightarrow \quad P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix},$$

$$P^{-1} = \frac{1}{(4+3i)(4-3i)+i^2}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$$

$$\Rightarrow \quad \boxed{P^{-1} = \frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}}$$

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35. (a)

Given

$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix} = -12$$

$$\begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 2\times1 & 2\times3 & 2\times0 \\ 2\times2 & 2\times6 & 2\times4 \\ 2\times-1 & 2\times0 & 2\times2 \end{vmatrix}$$

$$= 2\times2\times2\begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix} = 8 \times (-12) = -96$$

$$\left\{\because \begin{vmatrix} k_1a_1 & k_1a_2 & k_1a_3 \\ k_2b_1 & k_2b_2 & k_2b_3 \\ k_3c_1 & k_3c_2 & k_3c_3 \end{vmatrix} = k_1 \cdot k_2 \cdot k_3 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\right\}$$

$$\therefore \quad \boxed{\begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix} = -96}$$

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36. (d)

(i) P (Q + R) = PQ + PR

PR need not be equal to RP since matrix multiplication is not commutative.

(ii) $(P - Q)^2 = (P - Q)(P - Q) = P^2 - QP - PQ + Q^2$

(PQ need not be equal to QP)

(iii) $|P + Q| \neq |P| + |Q|$ (Since $|P + Q| \leq |P| + |Q|$)

(iv) $(P + Q)^2 = (P + Q)(P + Q) = P^2 + QP + PQ + Q^2$

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37. (49)

Let the 2 × 2 symmetric matrix = $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

Given tr (A) = 14 $\Rightarrow$ a + c = 14 $\Rightarrow$ c = 14 – a

Maximum value of $|A| = ac - b^2$

For |A| to be maximum, $b^2 = 0$ ($\because b^2 \geq 0$)

$\therefore |A| = ac = a(14 - a)$

For |A| to be maximum,

$$\frac{d}{da}\{a(14-a)\}=0$$

$\Rightarrow$ 14 – 2a = 0

$\boxed{\Rightarrow a = 7}$

Considering $\left.\frac{d^2|A|}{da^2}\right|_{a=7} = -2 < 0$

$\therefore$ |A| is maximum at a = 7

$\Rightarrow$ c = 14 – a $\Rightarrow$ c = 7

$\boxed{\therefore |A|_{max} = ac = 7(7) = 49}$

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38. **(1)**

$$J_6 = \begin{bmatrix} 0&0&0&0&0&1\\ 0&0&0&0&1&0\\ 0&0&0&1&0&0\\ 0&0&1&0&0&0\\ 0&1&0&0&0&0\\ 1&0&0&0&0&0 \end{bmatrix}$$

Given $P = I_6 + \alpha J_6$

$$\Rightarrow P = \begin{bmatrix} 1&0&0&0&0&\alpha\\ 0&1&0&0&\alpha&0\\ 0&0&1&\alpha&0&0\\ 0&0&\alpha&1&0&0\\ 0&\alpha&0&0&1&0\\ \alpha&0&0&0&0&1 \end{bmatrix}$$

$R_6 \rightarrow R_6 - \alpha R_1$

$R_5 \rightarrow R_5 - \alpha R_2$

$R_4 \rightarrow R_4 - \alpha R_3$

$$\Rightarrow P \sim \begin{bmatrix} 1&0&0&0&0&\alpha\\ 0&1&0&0&\alpha&0\\ 0&0&1&\alpha&0&0\\ 0&0&0&1-\alpha^2&0&0\\ 0&0&0&0&1-\alpha^2&0\\ 0&0&0&0&0&1-\alpha^2 \end{bmatrix}$$

The matrix got converted to upper triangular matrix.

$\Rightarrow$ |P| = Product of Principal diagonal elements
$= (1-\alpha^2)^3 = 0$

$\Rightarrow 1-\alpha^2 = 0$

$\Rightarrow$ $\boxed{\alpha = +1}$ (Since α is non-negative)

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39. **(d)**

(i) $(M^T)^T = M$ (Always hold)

(ii) $(cM)^T = cM^T$ (Always hold)

(iii) $(M + N)^T = M^T + N^T$ (Always hold)

(iv) MN need not be equal to NM since matrix multiplication need not be commutative

$\boxed{\therefore \text{ option 4 does not Always hold}}$

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40. **(200)**

Given |A| = 5 and |B| = 40

$\therefore$ |AB| = |A| |B| = 5(40) = 200

$\therefore$ |AB| = 200

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41. **(c)**

Given function is

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \sqrt{3} \end{vmatrix}$$

$$\Rightarrow f'(\theta) = \begin{vmatrix} \cos\theta & -\sin\theta & \sec^2\theta \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & \sqrt{3} \end{vmatrix}$$

$= \quad k_1 \cdot \cos\theta + k_2 \cdot \sin\theta + k_3 \cdot \sec^2\theta$

Where

k_1, k_2 and k_3 are constants

By LaGrange's' mean value theorem

$\exists$ at least one $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that

$$f'(\theta) = \frac{f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)}{\frac{\pi}{3} - \frac{\pi}{6}}$$

Since $f\left(\frac{\pi}{3}\right) = f\left(\frac{\pi}{6}\right) = 0$ because two rows become identical for $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{6}$

$\therefore \quad \exists$ some 'θ' where $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that $f'(\theta) = 0$

Also, since $f'(\theta)$ is not an identity, $\exists$ some 'θ' where $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ such that $f'(\theta) \neq 0$

$\therefore$ Both statements are true.

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42. (23 to 23)

$$J = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix} k = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$K^T = [1 \ 2 \ -1]$

$$K^T \cdot JK = [1 \quad 2 \quad -1]\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = [6 \quad 8 \quad -1]$$

$$[1 \quad 2 \quad -1]\begin{bmatrix} 6 \\ 8 \\ -1 \end{bmatrix}$$

$= 6 + 16 + 1 = 23.$

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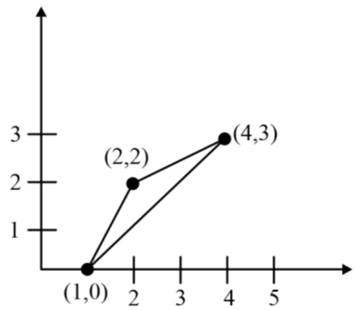
43. (a)



$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}[1(2-3) + 2(3-0) + 4(0-2)]$$

$$= \frac{1}{2}[-1 + 6 - 8] = \left|\frac{-3}{2}\right| = \frac{3}{2}$$

$$\boxed{\text{Area of}\Delta = \frac{3}{2}}$$

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44. (88 to 88)

$$\begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 0 & -6 & -8 \\ 3 & 0 & 1 & 2 \end{vmatrix}$$

Interchange column 1 and column 2 and expand about first column.

$$-\begin{vmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 0 \\ 0 & 2 & -6 & -8 \\ 0 & 3 & 1 & 2 \end{vmatrix} = -1\begin{vmatrix} 1 & 3 & 0 \\ 2 & -6 & -8 \\ 3 & 1 & 2 \end{vmatrix} = \{1(-12+8) - 3(4+24) + 0(2+18)\}$$

$$-\{-4 - 84 + 0\} = \boxed{88}$$

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45. (a)

A square matrix is defined as skew – Symmetric matrix if and only if $\boxed{A^T = -A}$

Ex: $A = \begin{bmatrix} o & h & g \\ -h & o & f \\ -g & -f & o \end{bmatrix}$

$$A^T = \begin{bmatrix} o & -h & -g \\ h & o & -f \\ g & f & o \end{bmatrix}$$

$$A^T = -\begin{bmatrix} o & h & g \\ -h & o & f \\ -g & -f & o \end{bmatrix}$$

$A^T = -A$

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46. (b)

Given matrix is (Say C) = $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

Converting 'C' to (I+AB) form

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Since both the matrices C and I+AB are same, the determinant is same.

Writing the matrix AB as Product of a 4x1 and 1x4 matrices,

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = [4]_{1x1}$$

$\therefore I_{1x1} + BA = [5] \Rightarrow |I_{1x1} + BA| = 5$

Since |c|=determinant ($I_4 + AB$) = determinant ($I_1 + BA$) = 5

By above property, determinant of given Matrix is 5.

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47. (16)

$P \rightarrow 4 \times 2$ ($m \times p$)

$Q \rightarrow 2 \times 2$ ($p \times n$)

$R \rightarrow 4 \times 1$ ($n \times p$)

Total number of multiplications = $m \cdot np$

$PQ \rightarrow 4 \times 2$	$QR \rightarrow 2 \times 4$
2×4	4×1
$= 4 \times 4 = 16$	$= 2 \times 1$
$m \times n$	$m \times n = 2$
$PQ = mnp = 16 \times 2 = 32$	$mnp = 2 \times 4 = 8$
$PQR \rightarrow 4 \times 4 = 4 \times 1$	$PQR = 4 \times 2 = 2 \times 1$
$= (4 \times 1)$	$= 4 \times 1 = 4$
$mnp = 4 \times 4 = 16$	$mnp = 2 \times 4 = 8$
$pmn + mnp$	$mnp + mnp$
$= 16 + 32 = 48$	$= 8 + 8 = 16$

Minimum number of multiplications = 16.

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48. (b)

Given matrix is $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$;

Characteristic equation of 'A' is given by

$$\begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad +(5+\lambda)\cdot(+\lambda) + 6 = 0$

$\Rightarrow \quad \lambda^2 + 5\lambda + 6 = 0$

$\therefore$ By cayley-Hamilton theorem,

$A^2 + 5A + 6I = 0 \quad \ldots(1)$

$\Rightarrow A^2 = -5A - 6I \quad \ldots(2)$

(1) $\Rightarrow A^2 + 5A + 6I = 0$

$\Rightarrow A(A^2 + 5A + 6I) = 0$

$\Rightarrow A^3 + 5A^2 + 6A = 0$

$\Rightarrow A^3 + 5(-5A - 6I) + 6A = 0$

$\Rightarrow \boxed{A^3 = 19A + 30I}$

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49. (d)

Since $(A + A^T)^T = A^T + (A^T)^T$

$= A^T + A \quad [\therefore (A^T)^T = A]$

$A + A^T = (A + A^T)^T$

$[S] = [S]^T$ hence [S]is symmetric.

- Since $(A - A^T)^T = A^T - (A^T)^T$

  $= A^T - A \quad [\therefore (A^T)^T = A]$

  $-(A - A^T) = (A - A^T)^T$

  $-[D] = [D]^T$ hence [D]is skew symmetric.

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50. (b)

We know cube roots of unity are 1, w, w^2.

$1,\ w = -\frac{1}{2} + \frac{\sqrt{3}}{2} j,\ w^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} j$

$\vec{x} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\ \vec{x}_2 = \begin{bmatrix} 1 & a & a^2 \end{bmatrix}$

$$\vec{x}_1 \cdot \vec{x}_2^{\,T} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$$

$= \begin{bmatrix} 1 + a + a^2 \end{bmatrix}$

$\vec{x}_1 \cdot \vec{x}_2^{\,T} = 0$

$\therefore$ x_1 and x_2 are orthogonal vectors.

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51. (d)

$A = L\,U$

$$\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} 1 & U_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{11}L_{12} \\ L_{21} & L_{21}L_{12} + L_{22} \end{bmatrix}$$

$\therefore L_{11} = 2$

$\therefore L_{21} = 4$

$L_{11}\ U_{12} = 1$

$\therefore U_{12} = 0.5$

$L_{21} = U_{12} + L_{21} = -1$

$4 \times 0.5 + L_{22} = -1$

$\therefore L_{22} = -3$

$$\therefore L = \begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$

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52. (b)

Let $A = \begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$

The characteristic equation $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 9 - \lambda & 5 \\ 5 & 8 - \lambda \end{vmatrix} = 0$$

$(9 - \lambda)(8 - \lambda) - 5 \times 5 = 0$

$\lambda^2 - 17\lambda + 47 = 0$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \times 1 \times 47}}{2 \times 1}$$

$\lambda = 3.48,\ 13.53$

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53. (a)

Given:

$$\text{Matrix } M = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} \text{ and } M^T = M^{-1}$$

$\Rightarrow$ 'M' is orthogonal matrix $\quad (\because MM^T = MM^{-1} = I)$

$\Rightarrow \quad |M| = \pm 1$

Let $|M| = 1$

$$\Rightarrow \quad \frac{9}{25} - \frac{4x}{5} = 1$$

$$\Rightarrow \quad \frac{4x}{5} = \frac{-16}{25} \Rightarrow x = \frac{-4}{5}$$

(Or) Let $|M| = -1$

$$\Rightarrow \quad \frac{9}{25} - \frac{4x}{5} = -1$$

$$\Rightarrow \quad \frac{4x}{5} = \frac{34}{25} \Rightarrow x = \frac{17}{10} \text{ (Not in given options)}$$

$\therefore \quad \boxed{x = -4/5}$

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54. (b)

We know that;

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Alternatively; $A^{-1} = \dfrac{\text{adiA}}{|A|}$

$$\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix} = \frac{1}{(3+2i)(3-2i)+i^2} \begin{bmatrix} 3-2i & -i \\ +i & 3+2i \end{bmatrix}$$

$$= \boxed{\frac{1}{12} \begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}}$$

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55. (b)

$(PQ)^{-1}$ P

$[Q^{-1} P^{-1}]$ P

Q^{-1} $(P^{-1}$ P)

Q^{-1} (I)

Q^{-1}

$\{\therefore (AB)^{-1} = B^{-1} A^{-1}\}$

$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

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56. (b)

We know $PP^T = I$ ($\because$ P is orthogonal)

Let $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is an orthogonal matrix

$$P \cdot x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Px = x

$\therefore$ $||px|| = ||x|| = \left(x_1^2 + x_2^2\right)^{1/2}$ for all vectors $\vec{x}$

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57. (d)

Given

$$A^1 = (A^T \cdot A) \cdot AT$$

$$A^1 = A^T \cdot (AT)^{-1} \cdot AT \quad \because (AB)^{-1} = B^{-1}A^{-1}$$

$$A \cdot A^1 = (A^T)^{-1} \cdot A^T$$

From equation

$\boxed{A \cdot A^1 = I}$ ...(*i*) | $(A \cdot A^1)(A \cdot A^1) = A \cdot A^1$

$\boxed{A \cdot A^1 A = I}$ ...(ii) | $(A \cdot A^1)^2 = A \cdot A^1$

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58. (4)

Given P is a 4 × 5 matrix.

Every solution of Px = 0 is a scalar multiple of $[2\ 5\ 4\ 3\ 1]^T$.

$\Rightarrow$ $[0\ 0\ 0\ 0\ 0]^T$ is the only independent (trivial) solution of Px = 0

$\therefore$ Number of linearly independent solutions = 1

$\therefore$ Number of unknowns – ρ(P) = 1

$\Rightarrow 5 - \rho(P) = 1$

$\Rightarrow \rho(P) = 4$

$\therefore$ Rank of matrix ‘P’ is 4.

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59. (b)

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix} \xrightarrow[R_4 \to R_4 - 4R_1]{\substack{R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 5R_1}} \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & -4 & -8 & -8 \\ 0 & -6 & -12 & -12 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - 3R_2]{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No of non-zero rows in echelon form =2

Hence Rank of A = 2

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60. (c)

$$\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_1} \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 - \frac{R_2}{2}} \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3/2 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_4} \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows in echelon form = 3

$\therefore$ Rank of A = 3

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61. (3)

$|M| = 0 - 1\,(0 - 1) + 1\,(1 - 0) = 2$

Since $|M| \neq 0 \ \therefore\ \rho(M) = 3$

$\because$ Rank = order of largest non-zero minor.

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62. (b)

We know, $\rho(A) = \rho(A^T A)$

$\therefore \rho(A^T A) = 2$

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63. (b)

Given

$$\begin{pmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{pmatrix} = A \text{ (Say)}$$

Converting ‘A’ to Row-echelon form

$R_2 \to 4R_2 - R_1$ and $R_3 \to 4R_3 + 7R_1$

$$\Rightarrow \quad A \sim \begin{bmatrix} -4 & 1 & -1 \\ 0 & -5 & -3 \\ 0 & -5 & -3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\Rightarrow \quad A \sim \begin{bmatrix} -4 & 1 & -1 \\ 0 & -5 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

Since number of non-zero rows of 'A' in Row-echelon form is 2,

$$\boxed{\text{Rank of A} = \rho(A) = 2}$$

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64. (b)

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 4 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - \frac{3}{2}R_2} \begin{bmatrix} 4 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows = 2

Hence Rank = 2

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65. (c)

Given:

$$\text{Matrix } M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$$

Converting M to row echelon form,

$R_2 \rightarrow 5R_2 - R_1$

$R_3 \rightarrow 5R_3 - 3R_1$

$$\Rightarrow \quad M \sim \begin{bmatrix} 5 & 10 & 10 \\ 0 & -10 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The above matrix is in row-echelon form

Number of non-zero rows = 2, $\therefore \rho(M) = \mathbf{2}$

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66. (2)

$$\text{Given } P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix};\ Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

$$\Rightarrow P + Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

$$\therefore\ |P+Q| = 0\begin{vmatrix} 9 & 10 \\ 8 & 8 \end{vmatrix} + 1\begin{vmatrix} 8 & 10 \\ 8 & 8 \end{vmatrix} - 2\begin{vmatrix} 8 & 9 \\ 8 & 8 \end{vmatrix}$$

$= 0 + 1(-16) - 2(-8) = 0$

$\therefore\ |P + Q| = 0$

There exist a 2×2 non-zero minor of P + Q

$$\begin{vmatrix} 8 & 9 \\ 8 & 8 \end{vmatrix} \neq 0$$

$\therefore$ Rank of P + Q = 2

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67. (4)

Let the given matrix is

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Converting to row-echleon form

$R_4 \leftrightarrow R_2$

$$\Rightarrow A \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_5 \rightarrow R_1 + R_2 + R_3 + R_4 + R_5$

$$\Rightarrow A \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$; $R_4 \to R_4 + R_3$

$$\Rightarrow A \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\Rightarrow A \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above matrix is in Row- echelon form.

∴ Number of non-zero rows of 'A' in echleon form = 4

$\Rightarrow \rho(A) = 4$

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68. **(17)**

Let the given matrix be

$$A = \begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix} \text{ and eigen vector is } x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

For an eigen values λ, $AX = \lambda X$

$$\therefore \begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 12 \\ p+7 \\ 36 \end{bmatrix} = \begin{bmatrix} \lambda \\ 2\lambda \\ 3\lambda \end{bmatrix}$$

$\Rightarrow \lambda = 12$ and $p + 7 = 2\lambda \Rightarrow p + 7 = 24 \Rightarrow P = 17$

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69. **(b)**

Since $a_{ij} = ij$

Hence the matrix will be

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots\ldots \\ a_{21} & a_{22} & a_{23} & \ldots\ldots \\ a_{31} & a_{32} & a_{33} & \ldots\ldots \\ \vdots & \vdots & \vdots & \ldots\ldots \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & \ldots \\ 2 & 4 & 6 & \ldots\ldots \\ 3 & 6 & 9 & \ldots\ldots \\ \vdots & \vdots & \vdots & \ldots\ldots \end{bmatrix}$$

Rank of A = 1

Because each row will be scalar multiple of first row, hence we will get only one non-zero row in Echelon form of A.

Alternatively;

Rank of A =1

Because all the minors of order greater than 1 will be 0.

Hence $\boxed{\text{Rank of A = 1}}$

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70. **(c)**

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \therefore A^T = \begin{bmatrix} p & r \\ q & s \end{bmatrix}$$

$$AA^T = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} p & r \\ q & s \end{bmatrix} = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix} = B$$

We know, $\rho(A) = \rho(AA^T)$

$\therefore \rho(A) = \rho(B) = N$

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71. **(2 to 2)**

$$\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_1 + R_2} \begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No of non-zero rows = Rank = 2

Determinant of sub-matrix $\begin{bmatrix} 6 & 0 \\ -2 & 14 \end{bmatrix} \neq 0$

$\therefore$ Rank is 2.

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72. (a)

Given $\rho(Q) = 4$

→ Rank = Row Rank = No. of linearly independent rows

→ Rank = Column rank = No. of Linearly independent columns.

$\therefore$ option (a) is correct

$$Q_{5\times6}\ Q^T_{6\times5} = QQ^T_{5\times5}$$

$$Q^T_{6\times5}\ Q_{5\times6} = Q^TQ_{6\times6}$$

We know, $\rho(Q) = \rho(QQ^T) = \rho(Q^TQ) = 4$

$\therefore$ $|QQ^T| = |Q^TQ| = 0$

$\therefore$ QQ^T and Q^TQ are not invertible

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73. (b, c)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

For eigen values; $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\{(2-\lambda)(1-\lambda)-1\}+1\{-(1-\lambda)\} = 0$$

$$(1-\lambda)\{(2-\lambda)(1-\lambda)-1-1\} = 0$$

$$(1-\lambda)\lambda(\lambda-3) = 0$$

$\lambda = 0, 1, 3$

Since there are three eigen values so number of linearly independent eigen vector are 3.

Since; product or eigen values = $|A| = 0$

Since $|A| = 0$ hence its Rank(A) < 3

$\therefore$ Rank (A) < number of unknowns

$\Rightarrow$ [A]{x} = [B] does not have a unique solution. For the positive definite matrix, all the eigen values must be positive. But here one eigen value is $\lambda = 0$ so A is not a possible definite matrix.

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74. (c)

Given, system is

$$\begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Let, x_1, x_2, x_3 be the unknowns.

$$\Rightarrow \quad A\times = b \Rightarrow \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad x_1 - \sqrt{2}\,x_2 + 3x_3 = 1 \qquad \ldots(1)$$

$$-x_1 + \sqrt{2}\,x_2 - 3x_3 = 3 \qquad \ldots(2)$$

From equation (1) and (2),

1 = –3 (False)

($\because$ LHS of both (1) and (2) differ by '–' sign)

$\therefore$ Given system has no solution.

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75. (d)

Given system of equation is

$x_1 + x_2 - 2x_3 = 4$; $x_1 + 3x_2 - x_3 = 7$; $2x_1 + x_2 - 5x_3 = 7$

The given system can be written as

$$\begin{bmatrix}1 & 1 & -2\\1 & 3 & -1\\2 & 1 & -5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\7\\7\end{bmatrix} \Rightarrow A \times B \text{ and } A = LU$$

Considering (LU = A)

$$LU = \begin{bmatrix}1 & 0 & 0\\L_{21} & 1 & 0\\L_{31} & L_{32} & 1\end{bmatrix}\begin{bmatrix}U_{11} & U_{12} & U_{13}\\0 & U_{22} & U_{23}\\0 & 0 & U_{33}\end{bmatrix}$$

$$LU = \begin{bmatrix}U_{11} & U_{12} & U_{13}\\L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23}\\U_{11}L_{31} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33}\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & -2\\1 & 3 & -1\\2 & 1 & -5\end{bmatrix}$$

By equating the corresponding elements

$U_{11} = 1$; $U_{12} = 1$, $U_{13} = -2$

$L_{21}U_{11} = 1 \Rightarrow L_{21} = 1$

$L_{21}U_{12} + U_{22} = 3 \Rightarrow (1 \times 1) + U_{22} = 3 \Rightarrow U_{22} = 2$

$L_{21}U_{13} + U_{23} = -1 \Rightarrow (1 \times -2 + U_{23}) = -1 \Rightarrow U_{23} = 1$

$U_{11}L_{31} = 2 \Rightarrow L_{31} = 2$

$L_{31}U_{12} + L_{32}U_{22} = 1 \Rightarrow (2 \times 1) + L_{32} \times 2 = 1 \Rightarrow L_{32} = -1/2$

$L_{31}U_{13} + L_{32}U_{23} + U_{33} = -5$

$$2 \times -2 + \left(-\frac{1}{2} \times 1\right) + U_{33} = -5$$

$$\Rightarrow U_{33} = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$\Rightarrow U_{33} = -\frac{1}{2}$$

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76. (a, b)

Given

$$[xy]\begin{bmatrix}2 & 5-2\alpha\\\alpha & 1\end{bmatrix} = [0 \quad 0]$$

For the system to have non-trivial solution,

$$\begin{vmatrix}2 & 5-2\alpha\\\alpha & 1\end{vmatrix} = 0$$

$\Rightarrow 2 - \alpha(5 - 2\alpha) = 0 \qquad \Rightarrow 2 - 5\alpha + 2\alpha^2 = 0$

$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0 \qquad \Rightarrow \alpha = 2, \frac{1}{2}$

For $\alpha = 2$: system is $\qquad 2x + 2y = 0$

$x + y = 0$

$\therefore$ (2, –2) is a solution.

For $\alpha = \frac{1}{2}$, $2x + \frac{y}{2} = 0 \qquad \Rightarrow 4x + y = 0$

$\therefore$ (–1, 4) is a solution.

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77. (b, c)

Given

System is Ax = 0

A is of order 3 × 5 and $\rho(A) = 2$

For the system

Ax = 0 if $\rho(A) = < n$ (Number of unknowns),

Along with the trivial solution (zero-vector), the system has infinitely many solutions.

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78. (3 to 3)

$$(A/B) = \left[\begin{array}{ccc|c}1 & 3 & 2 & 1\\2 & 2 & -3 & 1\\4 & 4 & -6 & 2\\2 & 5 & 2 & 1\end{array}\right] \xrightarrow[R_4 \to R_4 - 2R_1]{\substack{R_2 \to R_2 - 2R_1\\R_3 \to R_3 - 4R_1}} \left[\begin{array}{ccc|c}1 & 3 & 2 & 1\\0 & -4 & -7 & -1\\0 & -8 & -14 & -2\\0 & -1 & -2 & -1\end{array}\right]$$

$$\to \left[\begin{array}{ccc|c}1 & 3 & 2 & 1\\0 & -1 & -2 & -1\\0 & -4 & -7 & -1\\0 & -8 & -14 & -2\end{array}\right] \xrightarrow{R_4 \to R_4 - 2R_3} \left[\begin{array}{ccc|c}1 & 3 & 2 & 1\\0 & -1 & -2 & -1\\0 & -4 & -7 & -1\\0 & 0 & 0 & 0\end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

After converting to echelon form, equations have reduced to

$$\Rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \\ 0 \end{bmatrix}$$

From 3rd $x_3 = 3$ equation of system.

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79. (a)

Given: System is

$x_1 + 2x_2 = b_1$; $\quad 2x_1 + 4x_2 = b_2$

$3x_1 + 7x_2 = b_3$; $\quad 3x_1 + 9x_2 = b_4$

The above system can be written in matrix form as

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 3 & 7 & 0 & 0 \\ 3 & 9 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

AX = B (say)

$\therefore \quad x_1 + 2x_2 = b_1$

$\Rightarrow \quad 2x_1 + 4x_2 = 2b_1 = b_2 \quad \therefore b_2 = 2b_1$

For the system to have a solution,

$\rho(A|B) = 2$ = number of unknowns

All the 4×4 and 3×3 minors of [A|B] should be 0.

The 3 × 3 minor of [A|B] invloving b_1, b_3, b_4 is

$$\begin{vmatrix} 1 & 2 & b_1 \\ 3 & 7 & b_3 \\ 3 & 9 & b_4 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(7b_4 - 9b_3) - 2(3b_4 - 3b_3) + b_1(27 - 21) = 0$

$\Rightarrow \quad b_4 - 3b_3 + 6b_1 = 0 \Rightarrow 6b_1 - 3b_3 + b_4 = 4$

$\therefore$ For the system to have a solution,

$b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

Scan for Video solution

80. (c)

Given

System is $\quad x + y + z = 1$

$ax - ay + 3z = 5$

$5x - 3y + az = 6$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ a & -a & 3 \\ 5 & -3 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix} \qquad \text{(Say Ax = B)}$$

$$\Rightarrow [A \mid B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & -a & 3 & 5 \\ 5 & -3 & a & 6 \end{bmatrix}$$

Converting to row echelon form

$R_2 \to R_2 - aR_1$ and $R_3 \to R_3 - 5R_1$

$$\Rightarrow [A \mid B] \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2a & 3-a & 5-a \\ 0 & -8 & a-5 & 1 \end{bmatrix}$$

$R_3 \to R_3 a - 4R_2$

$$\Rightarrow [A \mid B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2a & 3-a & 5-a \\ 0 & 0 & a^2 - 5a - 12 + 4a & a - 20 + 4a \end{bmatrix}$$

For the system to have infinite solutions,

$a^2 - a - 12 = 0,\ 5a - 20 = 0$

$\Rightarrow \boxed{\therefore\ a = 4 \text{ for inifinitely many solutions}}$

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81. (2 to 2)

Given:

$$A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}; \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For the system Ax = 0 to have infinitely many solutions,

$|A| = 0$

$\Rightarrow \quad k(k^2) - 2k(k^2 - k) = 0 \quad \Rightarrow -k^3 + 2k^2 = 0$

$\Rightarrow \quad -k^2(k - 2) = 0 \quad \Rightarrow \quad k = 0,0,2$

$\therefore$ No of distinct real value of k = 2.

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82. (c)

Given a_i are column vectors in R^n.

$\sum_{i=1}^{n} c_i a_i = 0$ for $c_1, c_2, \ldots\ldots c_n$ not all zero.

$\Rightarrow$ $a_1, a_2, \ldots\ldots, a_n$ are linearly dependent.

$\therefore$ $|A| = |a_1\ a_2\ a_3 \ldots\ldots a_n| = 0$

$\Rightarrow$ $\rho(A) < n$

$\therefore$ The system has infinitely many solutions.

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83. (a)

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{bmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{bmatrix} = 0$$

$(3-\lambda)(1-\lambda) - 8 = 0$

$\lambda^2 - 4\lambda + 3 - 8 = 0$

$\lambda^2 - 4\lambda - 5 = 0$

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84. (d)

Given

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 2 \\ -30 \end{Bmatrix}$$

$\Rightarrow$ $2x + 5y = 2 \quad \Rightarrow 4x + 10y = 4 \quad \ldots (1)$

$\Rightarrow$ $-4x + 3y = -30 \quad \Rightarrow -4x + 3y = -30 \quad \ldots (2)$

$(1) + (2) \Rightarrow \quad 13y = -26$

$\Rightarrow \quad y = -2 \Rightarrow x = 6$

$\therefore \quad \boxed{\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 6 \\ -2 \end{Bmatrix}}$

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85. (c)

Given system has 'm' linear equation in n variables

(i) For m <n (or) m > n (or) m = n, we can have systems with no solution, infinitely many solutions (or) single solution depending on Homogeneous or Non -Homogeneous system.

$\therefore$ First two statement (I) and (II) are generalized statements which need not be true.

(ii) $\exists$ a system which has a solution for m = n which is correct.

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86. (b)

For system to be consistent;

Rank (A) Rank (A/B)

Augmented matrix for the system of equations

$$(A/B) = \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & 0 & 0 & c-5a-(b-2a) \end{array}\right] \xleftarrow[R_3 \to R_3 - R_2]{}$$

Rank(A) = 2 Hence Rank(A/B) should be 2 for this to happen $C - 5a - b + 2a = 0$

$3a + b - c = 0$

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87. (2)

Given system is $\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -2 & 4 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$

AX = B

Augmented matrix = $\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -2 & 4 & -6 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 + 2R_1$

$$\Rightarrow [A/B] \sim \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & k-2 \end{bmatrix}$$

If $\rho(A) = \rho(A{:}\ B) < n$ for

Infinitely many solutions

$\therefore\ k - 2 = 0$

$\Rightarrow k = 2$

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88. (4.49 to 4.51)

For no solution the given lines must be parallel and hence their slope should be equal.

$$-\frac{2}{3} = -\frac{3}{P}$$

$$\boxed{P = \frac{9}{2} = 4.5}$$

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89. (b)

Given system is

$$\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix}$$

Let Ax = B

$\Rightarrow$ Augmented matrix; [A|B]

$$\Rightarrow [A\,|\,B] = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & -4 \\ 1 & 2 & 5 & 14 \end{bmatrix}$$

$R_2 \to 2R_2 - 3R_1$

$R_3 \to 2R_3 - R_1$

$$\Rightarrow [A\,|\,B] \sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & -3 & -7 & -23 \\ 0 & 3 & 7 & 23 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\Rightarrow [A\,|\,B] \sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & -3 & -7 & -23 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow\ \rho(A) = \rho(A\,|\,B) = 2 < 3$ (No. of unknowns)

$\boxed{\therefore \text{ The system has infinitely many solutions}}$

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90. (b)

Augmented matrix (A: B) = $\left[\begin{array}{ccc|c} 1 & 2 & 2 & b_1 \\ 5 & 1 & 3 & b_2 \end{array}\right]$

$$\xrightarrow{R_2 \to R_2 - 5R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & -9 & -7 & b_2 - 5b_1 \end{array}\right]$$

$\rho(A) = \rho(A/B) = 2$ < No. of unknown (3)

$\therefore$ Infinite solution exists.

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91. (a)

Given system of equation is

$$\frac{dx}{dt} = 3x - 5y \text{ and } \frac{dy}{dt} = 4x + 8y$$

$$\Rightarrow \begin{bmatrix} \dfrac{dx}{dt} \\ \dfrac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$\Rightarrow \boxed{\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}}$$

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92. (d)

The augmented matrix for the given system is

$$(A/B)=\left[\begin{array}{ccc|c}1&1&1&3\\1&2&3&4\\1&4&k&6\end{array}\right]$$

Convert it to row echelon from which will reduce it in to upper triangular matrix to investigate its rank.

$$\left[\begin{array}{ccc|c}1&1&1&3\\1&2&3&4\\1&4&k&6\end{array}\right]\xrightarrow[R_3\to R_3-R_1]{R_2\to R_2-R_1}\left[\begin{array}{ccc|c}1&1&1&3\\0&1&2&1\\0&3&k-1&3\end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$(A/B)=\left[\begin{array}{ccc|c}1&1&1&3\\0&1&2&1\\0&0&k-7&0\end{array}\right]$$

- For unique solution; Rank (A) = Rank (A/B) = no. of unknowns and for that $k \neq 7$

  $\Rightarrow$ Rank (A) = 3

  $\Rightarrow$ Rank (A/B) = 3
- If k = 7; Rank (A) = Rank (A/B) = 2 < no of unknowns (3). This is the condition for infinite solution and hence unique solution is not possible

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93. (1)

Given system is

$3x + 2y = 1;\ 4x + 7z = 1$

$x + y + z = 3;\ x - 2y + 7z = 0$

The matrix form of given system,

$$Ax = B \Rightarrow \begin{bmatrix}3&2&0&0\\4&0&7&0\\1&1&1&0\\1&-2&7&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\w\end{bmatrix}=\begin{bmatrix}1\\1\\3\\0\end{bmatrix}$$

$\Rightarrow$ Augmented matrix is

$$[A/B]=\left[\begin{array}{cccc|c}3&2&0&0&1\\4&0&7&0&1\\1&1&1&0&3\\1&-2&7&0&0\end{array}\right]$$

$R_2 \to 3R_2 - 4R_1$

$R_3 \to 3R_3 - R_1$

$R_4 \to 3R_4 - R_1$

$$\Rightarrow [A/B]\sim\left[\begin{array}{cccc|c}3&2&0&0&1\\0&-8&21&0&-1\\0&-1&3&0&8\\0&-8&21&0&-1\end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\Rightarrow [A/B]\sim\left[\begin{array}{cccc|c}3&2&0&0&1\\0&-1&3&0&8\\0&-8&21&0&-1\\0&-8&21&0&-1\end{array}\right]$$

$R_3 \leftrightarrow R_3 - 8R_2$

$$\therefore [A/B]\sim\left[\begin{array}{cccc|c}3&2&0&0&1\\0&-1&3&0&8\\0&0&-3&0&-65\\0&0&0&0&0\end{array}\right]$$

$\rho(A/B) = 3 = \rho(A)$ = Number of unknowns

$\therefore$ The system has unique solution.

$\therefore$ Number of solutions = 1

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94. (c)

Given: The system is

$x + 2y + z = 4$

$2x + y + 2z = 5$

$x - y + z = 1$

$$\Rightarrow \underbrace{\begin{bmatrix}1&2&1\\2&1&2\\1&-1&1\end{bmatrix}}_{(\text{say } A)}\underbrace{\begin{bmatrix}x\\y\\z\end{bmatrix}}_{X}=\underbrace{\begin{bmatrix}4\\5\\1\end{bmatrix}}_{B}$$

For the system Ax = B to have a solution

$\rho(A) = \rho(A/B)$

A ~ A/B

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 1 & 2 & 5 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

Converting to Row-echelon form

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & -3 & 0 & -3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$\Rightarrow$ $\rho(A) = \rho(A/B) = 2 < 3$

$\therefore$ Given system has infinitely many solutions.

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95. **(b)**

Given: System of equations

$$x + y + z = 6$$
$$x + 4y + 6z = 20$$
$$x + 4y + \lambda z = \mu$$

$$\Rightarrow \quad AX = B \to \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 6 \\ 1 & 4 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 6 \\ 20 \\ \mu \end{bmatrix}$$

Augmented matrix [A|B] is

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{bmatrix}$$

Converting to row echelon form

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda-1 & \mu-6 \end{bmatrix}$$

Now, $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda-1 & \mu-6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & \lambda-6 & \mu-20 \end{bmatrix}$$

For the system to have no solution, $\rho[A] \neq \rho[A|B]$

$\Rightarrow$ $\lambda - 6 = 0$ and $\mu - 20 \neq 0$

$\therefore$ $\boxed{\lambda = 6 \text{ and } \mu \neq 20}$

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96. **(a)**

Given: system is

$$2x_1 + x_2 + x_3 = 0$$
$$x_2 - x_3 = 0$$
$$x_1 + x_2 = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to \text{Homogeneous system}$$

For this system to have non-trivial (or) infinity many solutions,

$$\begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ $2(1) - 1(1) + 1(-1) = 0$ (True)

$\boxed{\therefore \text{ Given system has infinitely many solutions.}}$

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97. **(d)**

Augmented matrix

$$(A : B) = \left[\begin{array}{cccc|c} 1 & 2 & 1 & 4 & 2 \\ 3 & 6 & 3 & 12 & 6 \end{array}\right]$$

$$\underline{R_2 \to R_2 - 3R_1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 4 & 2 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ ρ (A) = ρ (A| B) = 1 < Number of unknowns (4)

$\therefore$ Infinite solutions exist.

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98. (c)

Given System is

$x_1 + x_2 + 2x_3 = 1$

$x_1 + 2x_2 + 3x_3 = 2$

$x_1 + 4x_2 + \alpha x_3 = 4$

If the system has unique soltuion, $|A| \neq 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & \propto \end{vmatrix} \neq 0$$

$\Rightarrow (2\propto -12) - (\propto -3) + 2(4-2) \neq 0$

$\Rightarrow 2\propto -12 - \propto +3 + 4 \neq 0$

$\Rightarrow \propto -5 \neq 0$

$\Rightarrow \propto \neq 5$

$\therefore$ Best option is (c).

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99. (b)

Given: equations are

$4x + 2y = 7$, $\qquad 2x + y = 6$

$\Rightarrow 2(2x + y) = 7 \qquad \Rightarrow 2x + y = 7/2$

The given equations can be written as

$2x + y = 7/2$ and $2x + y = 6$

$\because 7/2 = 6$ (Not possible), the system has no solution.

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100. (b)

Given: The system of equations

$$2x + 3y = 4$$

$$x + y + z = 4$$

$$x + 2y - z = a$$

$$\Rightarrow \underbrace{\begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}}_{(\text{say}\mathbf{A})} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{(\mathbf{X})} = \underbrace{\begin{bmatrix} 4 \\ 4 \\ a \end{bmatrix}}_{(\mathbf{B})}$$

For the system AX = B, to have a solution,

Rank $\rho(A) =$ Rank $\rho(A/B)$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 0 & 4 \\ 1 & 1 & 1 & 4 \\ 1 & 2 & -1 & a \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$ and $R_3 \to 2R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 0 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 0 & 4 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & -2 & 2a-4 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 0 & 4 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 0 & 2a \end{bmatrix}$$

$\Rightarrow \rho(A) = 2$

For $\rho(A/B) = 2$, All elements in 3rd row of matrix (A/B) should be zero.

$\Rightarrow 2a = 0 \qquad \Rightarrow \boxed{a = 0}$

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101. (b, c)

We know,

$Mx = \lambda x$

$\therefore$ λ is eigen value of M.

Multiply by M; $\quad M^2x = \lambda Mx$

$M^2x = \lambda(\lambda x)$

$M^2x = \lambda^2 x$

$\therefore$ λ^2 is eigen value of M^2

Since M and M^T have same eigen values,

$\therefore$ We can write $M^Tx = \lambda x$

Multiply by M^T; $(M^T)^2 x = \lambda M^T x$

$(M^T)^2 x = \lambda (\lambda x)$

$(MT)^2 x = \lambda^2 x$

$\therefore$ λ^2 is eigen value of $(M^T)^2$

$MM^T x = \lambda Mx$

$MM^T x = \lambda(\lambda x)$

$MM^T x = \lambda^2 x$

$\therefore$ λ^2 is eigen value of MM^T.

$M^TM x = \lambda M^T x$

$M^TM\,x = \lambda(\lambda x)$

$M^TM\,x = \lambda^2 x$

$\therefore$ λ^2 is eigen value of M^TM

Since MM^T and M^TM is always symmetric matrix.

Eigen values of symmetric matrices are always real. Hence λ^2 will non-negative iff λ is real

Hence option b and c are correct.

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102. (c)

Given that $\{\lambda_k | 1 \le k \le n\}$ and $\{v_k | 1 \le k \le n\}$ are eigen values and eigen vectors of 'B' respectively.

$B = P^{-1}AP$

$\Rightarrow$ 'A' is the matrix obtained from diagonalization of 'B'.

$\therefore$ B and A have some eigen values.

Let 'v' be the eigen vector of 'B' for an eigen values λ.

$\Rightarrow BV = \lambda_v$

Multiplying with 'P'

$\Rightarrow (AP)V = \lambda(PV)$

$\Rightarrow A(PV)\ \lambda(PV)$

i.e. eigen vector of 'A' is PV

$\therefore$ Eigen vectors of 'A' are $\{Pv_k \,|1 \le v \le n\}$

$\therefore$ Eigen values and eigen ectors of 'A' are

$\{\lambda\,|1 \le v \le n\}$ and $\{PV_v\,|1 \le v \le n\}$

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103. (a, b, d)

Using standard properties of eigen values and eigen vectors and vectors:

- Eigen values of A and A^T are same
- Eigen values of A^{-1} is reciprocal of eigen values of A.
- Eigen vectors of A^{-1} are same as that eigen vectors of A.

Row space and column space for any matrix are not same.

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104. (a, d)

Let $y = \begin{bmatrix}1\\1\\1\end{bmatrix}_{3\times1}$ Hence $y_T = \begin{bmatrix}1 & 1 & 1\end{bmatrix}_{1\times3}$

Then $yy^T = \begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 1\end{bmatrix} = \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}_{3\times3}$

Then $Y^Ty = \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = [3]_{1\times1}$

- Rank(y) = Rank (y^T) = Rank (yy^T) = Rank (y^Ty) = 1

$\therefore$ Since $yy^T = (yy^T)$ hence yy^T is symmetric.

$\therefore$ Since determinate of yyT is zero hence its inverse does not exits.

- Eigen values for yy^T are 0,03

  Hence y^Ty is an eigen value for yy^T i.e.3
- Rank of $y\,y^T$ is 1

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105. (a, c)

Since M has $\lambda_1 = 5$ and $\lambda_2 = -2$

Hence matrix $Q = M^3 - 4M^2 - 2M$ will have its eigen values as

$\rightarrow$ 1st eigen value =

$\lambda_1^3 - 4\lambda_1^2 - 2\lambda_1 = 5^3 - 4(5)^2 - 2(5) = 15$

$\rightarrow$ 2nd eigen value =

$\lambda_2^3 - 4\lambda_2^2 - 2\lambda_2 = (-2)^3 - 4(-2)^2 - 2(-2) = -20$

using eigen value properties.

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106. (2 to 2)

Let λ_1, λ_2 are Eigen values of $\begin{bmatrix}3 & p\\ p & q\end{bmatrix}$=A (say)

Given $\lambda_1 + \lambda_2 = 4$ and $\lambda_1.\ \lambda_2 = -1$

$\Rightarrow$ $\lambda_1 + \lambda_2 = t_r(A) = 3 + q = 4 \Rightarrow q = 1$

$\lambda_1 . \lambda_2 = |A| = -1$

$\Rightarrow \quad 3q - p^2 = -1 \quad \Rightarrow \quad p^2 = 3q + 1$

$\Rightarrow \quad p^2 = 3(1) + 1 \Rightarrow \quad p^2 = 4$

$\Rightarrow \quad |p| = 2$

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107. (a)

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & -2 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)[(4-\lambda)(1-\lambda)+2] = 0$$

$$(1-\lambda)(\lambda^2 - 5\lambda + 6) = 0$$

$$-\lambda^3 + 5\lambda^2 - 6\lambda + \lambda^2 - 5\lambda + 6 = 0$$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ (Characteristic equation)

By Cayley Hamilton theorem,

$A^3 - 6A^2 + 11A - 6I = 0$

$A^2 - 6A + 11I - 6A^{-1} = 0$

$6A^{-1} = A^2 - 6A + 11I$ [Multiply by A^{-1}]

$6A^{-1} = A^2 + cA + dI$

$\therefore$ On comparing; $c = -6$

$d = 11$

$\therefore$ $c + d = -6 + 11 = 5$

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108. (c)

We know; $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ..$

$e^x V = V + V_x + \frac{Vx^2}{2!} + \frac{Vx^3}{3!} + ...$ [Multiply by V]

$e^A V = V + vA + v\frac{A^2}{2!} + \frac{vA^3}{3!} + ...$ [$x \to A$]

$$e^A V = V\left[1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + ...\right] \qquad [A \to \lambda]$$

$e^A V = e^\lambda V$

$\therefore$ e^λ is an eigen value of e^A.

V is an eigen vector of e^A.

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109. (b)

$a_{ij} = (i - j)^3$

$$A = \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & -1 \\ 8 & 1 & 0 \end{bmatrix} \quad a_{ij} = -a_{ji}$$

$$A^T = \begin{bmatrix} 0 & 1 & 8 \\ -1 & 0 & 1 \\ -8 & -1 & 0 \end{bmatrix} = -A$$

$\therefore$ A is skew-symmetric.

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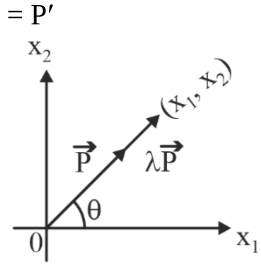
110. (b)

Given

'P' is a two-dimensional vector and also Eigen vector of 'A' corresponding to λ.

$\Rightarrow$ $AP = \lambda P = P'$



So, since Eigen value is just a scalar, P' and P will be in the same direction.

$\Rightarrow$ Direction of $P' = \theta$ and $||P'|| = \lambda.||P||$

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111. (d)

Given

For a non-zero column vector 'P'

$AP = \lambda P$ where λ is an Eigen value.

Also, $AP = \alpha^2 P$

$\Rightarrow \lambda = \alpha^2$ is Eigen value of 'A'.

$\therefore$ Eigen value of $A^2 = \lambda^2 = (\alpha^2)^2$

$\therefore$ Eigen value of $A^2 = \alpha^4$

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112. (1)

$A \rightarrow 10 \times 10$ matrix

$A^5 = 0$

$\therefore A^{10} = A^5 \cdot A^5 = 0 \cdot 0 = 0$

$\therefore$ Eigen values of A^{10} are all 0.

Let eigen values of A are $\lambda_1, \lambda_2 \ldots \lambda_{10}$.

Then eigen values of A^{10} are $\lambda_1^{10}, \lambda_2^{10}, \lambda_3^{10}, \ldots\ldots\ldots \lambda_{10}^{10}$

$\therefore \lambda_1^{10} = \lambda_2^{10} = \ldots\ldots = 0$

$\therefore \lambda_1, \lambda_2 \ldots \lambda_{10} = 0$

$\therefore$ Eigen values of $A + I = \lambda_1 + 1, \lambda_2 + 1, \ldots \lambda_{10} + 1$.

$= 1, 1, \ldots\ldots\ldots$ 10 times

$\therefore$ Determinate of A+ I = |A + I | = Product of eigen values

$= 1 \times 1 \times \ldots\ldots 10$ times

$|A+I| = \boxed{1}$

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113. (b)

Let $A = \begin{bmatrix} p & q \\ q & -p \end{bmatrix}$

$|A - \lambda I| = 0 \Rightarrow \left| \begin{bmatrix} p & q \\ q & -p \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$

$\Rightarrow \begin{vmatrix} p-\lambda & q \\ q & -p-\lambda \end{vmatrix} = 0$

$\Rightarrow (p - \lambda)(-p - \lambda) - q^2 = 0$

$-p2 - \lambda p + \lambda p + \lambda^2 - q^2 = 0$

$\lambda^2 - (p^2 + q^2) = 0 \quad [\because p^2 + q^2 = 1]$

$\lambda^2 - 1 = 0$

$\boxed{\lambda = \pm 1}$

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114. (3 to 3)

Given matrix is

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

If sum of all the elements of a row (or) column is constant (say k) for all the rows (or) columns, then 'k' is the largest eigen value of the matrix.

For given matrix, sum of all the elements of a row (or) column = 3 (constant for any row or column)

$\therefore$ Largest eigen value of given matrix = 3

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115. (b)

Let $A = \begin{bmatrix} 2 & -2 \\ -1 & 6 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$= \begin{bmatrix} 2-\lambda & -2 \\ -1 & 6-\lambda \end{bmatrix} = 0$

$= (2 - \lambda)(6 - \lambda) - 2 = 0$

$= \lambda^2 - 8\lambda + 10 = 0$

$\Rightarrow \lambda = 4 \pm \sqrt{6}$

Hence smallest eigen value is $4 - \sqrt{6}$

For $\lambda = 4 - \sqrt{6}$ find eigen vector; (A - λI) X = 0

$$\begin{bmatrix} 2-\lambda & -2 \\ -1 & 6-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-\left(4-\sqrt{6}\right) & -2 \\ -1 & 6-\left(4-\sqrt{6}\right) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left(-2+\sqrt{6}\right)x_1 - 2x_2 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2+\sqrt{6} \end{bmatrix} = \begin{bmatrix} 2 \\ 0.45 \end{bmatrix}$$

Hence smallest eigen value is 1.55 its corresponding eigen vector is $\begin{bmatrix} 2 \\ 0.45 \end{bmatrix}$

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116. (d)

Sum of eigen values = Trace of matrix

Trace(p) = 0 + 3 + 6 + 6 = 15

Product of eigen values = Determinant of matrix

$$= -(-2)\begin{bmatrix} 1 & 3 & 0 \\ 0 & 6 & 1 \\ 0 & 1 & 6 \end{bmatrix} = 2 \times 35 = 70$$

Only **option (c)** is correct because sum of eigen values = 15 & product of eigen values = 70

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117. (1 to 1)

Given

'I' is an identity matrix of dimensions 100 × 100

→ All the eigen values of an identity matrix are '1'.

∴ The number of distinct eigen values of 'A' is 1.

∴ E = {1}

⇒ Number of elements in E = 1

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118. (3 to 3)

Given:

$$\text{Matrix A} = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

'A' is an upper triangular matrix

∴ Eigen values of 'A' are principal diagonal elements of 'A' i.e., 2, 1, 3, and 2.

∴ Number of distinct eigen values = 3

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119. (b)

Given

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ (Upper triangular matrix)}$$

Eigen values of upper triangular matrix are diagonal elements.

∴ Eigen values of P are 1, 1, 1

∴ Number of distinct Eigen values of P = 1

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120. (d)

Given

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{X\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}$$

For 'X' to be an Eigen vector of 'A', $AX = \lambda X$

$$\Rightarrow \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \lambda \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 32 \\ 16 \\ 64 \end{bmatrix}$$

$$\Rightarrow \lambda \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = 16 \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \Rightarrow \lambda = 16$$

$\therefore$ One of the Eigen values = 16

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121. (d)

$M \rightarrow \lambda_1, \lambda_2$

$\Rightarrow M^2 \rightarrow \lambda_1^2, \lambda_2^2$

Eigen values of M are 4 and 9.

$\therefore$ Eigen values of M^2 are 4^2 and 9^2

$\Rightarrow$ 16 and 81.

[By property of eigen values]

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122. (12 to 12)

Given $R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$

Absolute value of product of eiger values = modulus of $|R|$

$\therefore \det(R) = R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$

$R_2 \Rightarrow R_2 - R_1$
$R_3 \Rightarrow R_3 - R_1$
$R_4 \Rightarrow R_4 - R_1$

$$\Rightarrow |R| = \begin{vmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 2 & 12 & 56 \\ 0 & 3 & 21 & 117 \end{vmatrix}$$

Since $|R| = |R^T| = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 2 & 3 \\ 4 & 5 & 12 & 21 \\ 8 & 19 & 56 & 117 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 5 & 12 & 21 \\ 19 & 56 & 117 \end{vmatrix}$$

$= 1((12 \times 117) - (21 \times 56)) - 2((5 \times 117) - (19 \times 21)) + 3((5 \times 56) - (19 \times 12)) = 12$

$\therefore$ Absolute value of product of eigen values $= \text{mod}(|R|) = 12$

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123. (1.0)

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1-\lambda)(-2-\lambda)(2-\lambda) = 0$

$\lambda = 1, 2, -2$

$\therefore$ eigen value of $B = A^3 - A^2 - 4A + 5I$

For $\lambda = 1, \ \lambda_1 = 1^3 - 1^2 - 4.1 + 5.1 = 1$

For $\lambda = 2, \lambda_2 = 2^3 - 2^2 - 4.2 + 5.1 = 1$

For $\lambda = -2, \lambda_3 = (-2)^3 - (-2)^2 - 4(-2) + 5.1 = 1$

$\therefore \quad |B| = \lambda_1.\lambda_2.\lambda_3$

$= 1 \times 1 \times 1 = 1$

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124. (d)

Let $A = \begin{bmatrix} 2 & -4 \\ 4 & -2 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -4 \\ 4 & -2-\lambda \end{vmatrix} = 0$$

$(2-\lambda)(-2-\lambda) + 16 = 0$

$\lambda^2 - 2\lambda + \lambda - 4 + 16 = 0$

$\lambda^2 = 12 = 0$

$\lambda + \pm 2\sqrt{3}$; (Hence complex eigen values) for 1st eigen vector, put $\lambda = 2\sqrt{3}i$ $(A - \lambda I)\, x = 0$

$$\begin{bmatrix} 2-\lambda & -4 \\ 4 & -2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-2\sqrt{3}i & -4 \\ 4 & -2-2\sqrt{3}i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(2-\sqrt{3}i\right)x_1 - 4x_2 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2-2\sqrt{3}i \end{bmatrix}$$

For 2nd eigen vector, put $\lambda = -2\sqrt{3}i$

$$\begin{bmatrix} 2+2\sqrt{3}i & -4 \\ 4 & 02+2\sqrt{3}i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(2\_2\sqrt{3}i\right)x_1 - 4x_2 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2+2\sqrt{3}i \end{bmatrix}$$

$\therefore$ The eigen values and eigen vector are complex.

NOTE: If eigen value is real the eigen vector is also real, If eigen value is complex then eigen vector is also complex.

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125. (d)

The condition of diagonalization of matrix is that all its eigen vectors should be linearly Independent. (The eigen values may not be distinct).

Linearly dependent eigen vectors are due to the repeated eigen Values.

So, statements (II) and statement (III) are necessarily true.

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126. (5.5 to 5.5)

Let eigen values of A $\rightarrow \lambda_1, \lambda_2$

Then eigen values of $A^2 \rightarrow \lambda_1^2, \lambda_2^2$

Given $T_r(A) = \lambda_1 + \lambda_2 = 4$ ----(1)

$T_r(A^2) = \lambda_1^2 + \lambda_2^2 = 5$ ----(2)

We know, $(\lambda_1 + \lambda_2)^2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2$

$4^2 = 5 + 2\lambda_1\lambda_2$

$\therefore \lambda_1\lambda_2 = 11/2$

$\therefore$ Determinant of $A = \lambda_1\lambda_2 = 5.5$

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127. (c)

Eigen vectors of distinct eigen values are always linearly independent

$\therefore$ If eigen values of a matrix are distinct then it implies that eigen vectors are linearly independent

$\therefore$ S2 implies S1

Note: Invertible (Non-singularity) depends on non-zero Eigen values.

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128. (b)

Given 'A' is a square matrix.

$\Rightarrow$ $A = A^T$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = 50$$

For any square matrix;

Sum of squares of all elements = trace of $\left(A \cdot A^T\right)$

$$\Rightarrow \quad \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = tr\left(A \cdot A^T\right) = tr\left(A \cdot A^T\right) = tr\left(A^2\right)$$

Given ρ (A) = 2

⇒ n – {Algebraic multiplicity of '0' as Eigen value} = 2

⇒ Algebraic multiplying of '0' as Eigen value= n – 2

∴ Let λ_1, λ_2, 0, 0, …… are Eigen values of 'A'.

⇒ λ_1^2, λ_2^2, 0, 0, ……, 0 are

⇒ $50 = \lambda_1^2 + \lambda_2^2$ …(1)

Let $|\lambda_1| = 5 \Rightarrow |\lambda_2| = 5$

$|\lambda_1| = 4 \Rightarrow |\lambda_2| \le 6$

So, for any combination, one Eigen value lies in [–5, 5].

(ii) The Eigen value with largest magnitude need not be greater than 5. Since $\lambda_1 = 5$ and $\lambda_2 = 5$ satisfy equation (i).

∴ Only statement I is true.

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129. (5)

Given characteristic polynomial of $M_{3\times3}$ is

$C(\lambda) = \lambda^3 - 4\lambda^2 + a\lambda + 30$

Let $\lambda_1, \lambda_2, \lambda_3$, are eigen values of M

$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 = 4$ and $\lambda_1, \lambda_2, \lambda_3 = -30$

Let $\lambda_1 = 2$

$\Rightarrow \lambda_2 + \lambda_3 = 2$ and $\lambda_2, \lambda_3, = -15$

On solving the above two equations, $\lambda_2 = 5, \lambda_3 = -3$

∴ Eigen values of M are 2, 5, – 3

∴ Absolute values of eigen values of M are 2, 5, 3

∴ Largest among the absolute Eigen values of M = 5

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130. (a)

Let $A\begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$

Hence characteristic equation is $|A - \lambda I| = 0$

$$|A - \lambda I| = \begin{vmatrix} 5-\lambda & -1 \\ 4 & 1-\lambda \end{vmatrix} = 0$$

$(5 - \lambda)(1 - \lambda) + 4 = 0$

$5 - 6\lambda + \lambda^2 + 4 = 0$

$\lambda^2 - 6\lambda + 9 = 0$

$(\lambda - 3)^2 = 0$

$\lambda = 3, 3$

Hence eigen value 3 has algebraic multiplicity of 2 for eigen vector $(A - \lambda I)x = 0$

$$\begin{bmatrix} 5-\lambda & -1 \\ 4 & 1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For $\lambda = 3$; $\begin{bmatrix} 5-3 & -1 \\ 4 & 1-3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x_1 - x_2 = 0$

Let $x_1 = k_1$ then $x_2 = 2k$

Hence eigen vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = k\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Eigen vector is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Hence only one independent eigen vector.

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131. (c)

Given:

$$\text{Matrix } A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

To calculate the eigen values of A, we need $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3 + C_4 + C_5$

$$\Rightarrow \begin{vmatrix} 15-\lambda & 2 & 3 & 4 & 5 \\ 15-\lambda & 1-\lambda & 2 & 3 & 4 \\ 15-\lambda & 5 & 1-\lambda & 2 & 3 \\ 15-\lambda & 4 & 5 & 1-\lambda & 2 \\ 15-\lambda & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(15 - \lambda)$ is a factor of above equation

$\Rightarrow$ $\lambda = 15$ is an eigen value of 'A'

$\therefore$ The one and only real values of 'A is 15.

Note: If sum of all the elements of a row (or) column is constant (k) for all the rows (or) columns of a matrix, then 'k' is an eigen values of the matrix.

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132. (b)

Given

$$\text{Matrix } P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

Product of Eigen values of a matrix is equal to its determinant.

$\Rightarrow$ $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |P| = 2(3 - 6) - 0(-4) + 1(8)$

$= -6 + 8 = 2$

$\therefore$ $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 2$

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133. (5 to 5)

Determinant of a given square matrix is equal to product of its eigen values.

For any 2 × 2 matrix 'A', $|A| = \lambda_1 \cdot \lambda_2$

$\Rightarrow$ $50 = (10)\,\lambda_2$

$\Rightarrow$ $\lambda_2 = 5$

$\therefore$ Other Eigen value = 5

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134. (a)

$|A - \lambda I| = 0$

$$\begin{vmatrix} 0-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ 0 & -3 & -4-\lambda \end{vmatrix} = 0$$

$-\lambda(\lambda(\lambda+4)+3) = 0$

$-\lambda[\lambda^2 + 4\lambda + 3] = 0$

$\lambda(\lambda+3)(\lambda+1) = 0$

$\therefore\ \lambda = 0, -1, -3$

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135. (c)

We know that $AX = \lambda X$

Option a)

$$\begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & -1 & 0 \\ 1/2 & 0 & 3/2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 0 \\ -3/2 \end{bmatrix} \neq \lambda X = \lambda \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

Option b)

$$\begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & -1 & 0 \\ 1/2 & 0 & 3/2 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3/2 \\ 0 \\ -1/2 \end{bmatrix} \neq \lambda X = \lambda \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

Option c)

$$\begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & -1 & 0 \\ 1/2 & 0 & 3/2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \lambda X = 1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$\therefore$ Option c) is correct.

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136. (a)

Given

Matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$

Let λ_1, λ_2 be the eigen values of 'A'.

Since the product of eigenvalues of a matrix is equal to its determinant.

$\therefore \quad \lambda_1 . \lambda_2 = |A| = 2k - 1$

If Eigen values are positive then $\lambda_1, \lambda_2 > 0$

$\Rightarrow \quad 2k - 1 > 0 \Rightarrow k > \dfrac{1}{2}$

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137. (2 to 2)

Given

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow$ 'A' is an upper Triangular matrix.

- Number of linearly independent Eigen vectors of a matrix is equal to number of distinct Eigen values of the Matrix.

  Since 'A' is an upper triangular matrix, the eigen values of 'A' are diagonal elements.

$\therefore$ The Eigen values of 'A' are 2, 2, 3.

$$\therefore \left\{\begin{matrix} \text{Number of distinct} \\ \text{Eigen values} \end{matrix}\right\} = \left\{\begin{matrix} \text{Number of linearly} \\ \text{independent Eigen vectors} = 2 \end{matrix}\right\}$$

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138. (d)

Given: $A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$ and eigen values of A are $\sigma + j\omega$ and $\sigma - j\omega$,

Product of eigen vaules of A = |A|

$\Rightarrow (\sigma + j\omega)(\sigma - j\omega) = \sigma^2 - x\omega$

$\Rightarrow \sigma^2 + \omega^2 = \sigma^2 - x\omega$

$\therefore x = -\omega$

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139. (1)

Given $A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$

For zero to be an eigen values, |A| = 0;

$$\Rightarrow \begin{vmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{vmatrix} = 0$$

$R_3 \rightarrow R_3 + 2R_1$

$$\Rightarrow \begin{vmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ 0 & 0 & -1+x \end{vmatrix} = 0$$

This determinant is zero if $-1 + x = 0$

$\Rightarrow x = 1$

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140. (d)

Consider a 2 ×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Hence characteristic equation will be $|M - \lambda I| = 0$

$$= \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$= (a-\lambda)(d-\lambda) - bc = 0$$

$= \quad \lambda^2 - (a + d)\lambda + (ad - bc) = 0$

$= \quad \lambda^2 - (a + d)\lambda + ad - (1 - d)(1 - a) = 0$

Now put $\lambda = 1$,

$1 - a - d + ad - 1 + a + d - ad = 0$

Hence $\lambda = 1$ satisfies the characteristic equation similarly put $\lambda = 2,3,4$ but this value will not satisfy characteristic equation for any value of a & d

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141. (15 to 15)

Let $\lambda_1, \lambda_2, \lambda_3$, are eigen values of 'P'

And $\lambda_1 = 2 + \sqrt{-1}$ and $\lambda_2 = 3$

Complex roots always occur as conjugate pairs.

$\Rightarrow \lambda_3 = 2 - \sqrt{-1}$

$\therefore |P| = \lambda_1, \lambda_2, \lambda_3,$

$= \left(2+\sqrt{-1}\right)\left(2-\sqrt{-1}\right)3 = 5(3) = 15$

$\therefore$ Determine of 'P' is 15.

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142. (a)

By property, if eigen values of A are λ_1, λ_2 then then the eigen values of A^m is λ_1^m, λ_2^m but eigen vector of A & A^m are same respectively.

Eigen values of A are 1 ,–2

$\therefore$ Eigen values of $A^2 - 3A + 4I$

$\Rightarrow \quad \lambda_1 = 1^2 - 3 \times 1 + 4\times1 = 2$

$\Rightarrow \quad \lambda_2 = (-2)^2 - 3(-2) + 4\times1 = 14$

& $\therefore$ using property eigen vector of $A^2 - 3A + 4I$ are x_1 and x_2.

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143. (d)

$p^3 = p$

According to cayley hamilton therorem every square matix satisfies its own characterstic equation.

$\therefore$ Characterstic equation matrix P is $\lambda^3 = \lambda$

If eigen values of P are λ_1, λ_2 and λ_3.

$\lambda^3 = \lambda$

$\lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda(\lambda+1)(\lambda-1) = 0$

$\boxed{\lambda = 0,\ 1, -1}$

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144. (3)

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\begin{bmatrix} 3-\lambda & 3-\lambda & 3-\lambda \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = 0$$

$$(3-\lambda)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = 0$$

$(3-\lambda)\,[(1-\lambda)^2-1]-1(1-\lambda-1)+1(1-(1-\lambda))]=0$

$(3-\lambda)\,[1+\lambda^2-2\lambda-1+\lambda+\lambda]=0$

$\lambda^2(3-\lambda)=0$

$\lambda = 0,0,3$

Hence only non-zero eigen value is 3.

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145. (b)

$$A = \begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$$

$$|A-\lambda I| = \begin{bmatrix} -3-\lambda & 0 & -2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda \end{bmatrix}$$

$= (-3-\lambda)\,[(-1-(-\lambda)(-2-\lambda)]-2a$

$= -(\lambda+3)\,[2+\lambda+2\lambda+\lambda^2]-2a$

$= -(\lambda+3)\,[\lambda^2+3\lambda+2]-2a$

$= -(\lambda^3+6\lambda^2+11\lambda+6)-2a = 0$

$a = -\frac{1}{2}(\lambda^3+6\lambda^2+11\lambda+6)$

$a = f(\lambda)$ Hence, to find maximum value of a we can use concept of maximum & maxima.

$$\frac{da}{d\lambda} = 0 \Rightarrow \frac{da}{d\lambda}\left(-\frac{1}{2}\left(\lambda^3+6\lambda^2+11\lambda+6\right)\right) = 0$$

$3\lambda^2+12\lambda+11=0$

$$\lambda = \frac{-12 \pm \sqrt{12^2-4\times3\times11}}{2\times3}$$

$$\lambda = -2 \pm \frac{1}{\sqrt{3}}$$

$\lambda = -22.577, -1.422$

We can write,

$$\frac{d^2a}{d\lambda^2} = \frac{d}{d\lambda}\left(-\frac{1}{2}\left(3\lambda^2+12\lambda+11\right)\right)$$

$$\frac{d^2a}{d\lambda^2} = -\frac{1}{2}\left(6\lambda+12\right)$$

For $\lambda = -2.577$

$$\frac{d^2a}{d\lambda^2} = -\frac{1}{2}(6\times-2.577+12) = 1.731 > 0$$

$\therefore$ At $\lambda = -2.577$, a , is minimum.

For $\lambda = 1.422$

$$\frac{d^2a}{d\lambda^2} = -\frac{1}{2}(6\times-1.422+12) = -1.734 < 0$$

$\therefore$ At $\lambda = -1.422$, a is maximum

$$\therefore a = -\frac{1}{2}\left(\lambda^3+6\lambda^2+11\lambda+6\right)$$

$= -\frac{1}{2}[(-1.422)^2+6(-1.442)^2+11(-1.422)+$

$= 0.1924$

$$= \boxed{\frac{1}{3\sqrt{3}}}$$

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146. (a)

Let $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$AX = B$

For unique solution $\delta(A) = \delta(A:B)$ = No of unknowns (3)

$\therefore$ if solutions are unique the all 3 equations are independent and if equation are independent

$\therefore |A| \neq 0$

Let eigen values of coefficient matrix A are λ_1, λ_2, λ_3

$\therefore$ Determinate = Product of eigen values of λ_1, λ_2, λ_3.

$\therefore$ If $|A| \neq 0$ then name of the eigen values are zero.

$\therefore$ $P \equiv Q \equiv R \equiv S$

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147. (9)

Let A be 2 × 2 matrix whose eigen values are λ_1 and λ_2.

Sum of eigen values of $\lambda_1 + \lambda_2 = -6$

We know, determinate = Product of eigen values

$D = \lambda_1 \lambda_2$

$D = \lambda_1(-6 - \lambda_1)$

$D = -6\lambda_1 - \lambda_1^2$

$$\frac{dD}{d\lambda_1} = -6 - 2\lambda_1 = 0$$

$\lambda_1 = -3$

$$\frac{d^2D}{d\lambda_1^2} = -6 < 0$$

$\therefore$ At $\lambda_1 = -3$, D is maximum.

$\therefore$ Maximum value of determinate

$D = -6\lambda_1 - \lambda_1^2$

$= -6(-3) - (-3)^2$

$= 18 - 9 = 9$

$\boxed{D_{max} = 9}$

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148. (d)

Given $A = \begin{bmatrix} 1 & 4 \\ b & a \end{bmatrix}$ and eigen value are –1 and 7

Sum of eigen values = trace (A)

$\Rightarrow 1 + a = -1 + 7 \Rightarrow a = 5$

Product of eigen values = $|A| = a - 4b = -7$

$\Rightarrow 4b = a + 7 = 12 \Rightarrow b = 3$

$\therefore a = 5, b = 3$

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149. (d)

The characteristic equation is $|A - \lambda I| = 0$

$$\left|\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}\right| = 0$$

$$\begin{vmatrix} 3-\lambda & -2 & 2 \\ 4 & -4-\lambda & 6 \\ 2 & -3 & 5-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\{(-4-\lambda)(5-\lambda)+18\} + 2(20-4\lambda-12)$$

$$+2\{-12-2(-4-\lambda)\} = 0$$

$$(3-\lambda)(-20+4\lambda-5\lambda+\lambda^2+18)+16-8\lambda-8+4\lambda = 0$$

$$(3-\lambda)(\lambda^2-\lambda-2)+8-4\lambda = 0$$

$$3\lambda^2 - 3\lambda - 6 - \lambda^3 + \lambda^2 + 2\lambda + 8 - 4\lambda = 0$$

$$-\lambda^3 + 4\lambda^2 - 5\lambda + 2 = 0$$

$$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$$

$$\lambda^3 - \lambda^2 - 3\lambda^2 + 3\lambda + 2\lambda - 2 = 0$$

$$\lambda^2(\lambda-1) - 3\lambda(\lambda-1) + 2(\lambda-1) = 0$$

$$(\lambda-1)(\lambda^2-3\lambda+2) = 9$$

$$\lambda = 1, 1, 2$$

Hence smallest eigen value = 1

& largest eigen value = 2

Scan for Video solution

150. (d)

$$A=\begin{bmatrix}2 & 1\\ 1 & p\end{bmatrix}$$

Let λ_1 and λ_2 be the eigen values of matrix A.

Sum of eigen values = $\lambda_1 + \lambda_2 = 2 + p$ …….(i)

Products of eigen value $\lambda_1\lambda_2 = 2p = 1$……..(ii)

Given $\lambda_1/\lambda_2 = 3 = \lambda_1 \Rightarrow 3\lambda_2$

From equation (i) $\Rightarrow 3\lambda_2 + \lambda_2 = 2 + P$

$$\lambda_2 = \frac{2+P}{4}$$

From equation (ii)

$$3\lambda_2^2 = 2p-1$$

$$3\left(\frac{2+p}{4}\right)^2 = 2p-1$$

$3(4 + p^2 + 4p) = 32p - 16$

$12 + 3p^2 + 12p = 32p - 16$

$3p^2 - 20p + 28 = 0$

$3p^2 - 6p - 14p + 28 = 0$

$3p(p - 2) - 14(p - 2) = 0$

$(p - 2)(3p - 14) = 0$

$$p = 2, \frac{14}{3}$$

Hence another value of $\boxed{p = \frac{14}{3}}$

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151. (6 to 6)

Given matrix is $\begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix}$ = A(say)

Let λ_1, λ_2, are eigen value of 'A'

$\Rightarrow \lambda_1 + \lambda_2 = \text{trace}(A) = 5$ …(1)

$\lambda_1, \lambda_2 = |A| = -6$

since $(\lambda_1 - \lambda_2)^2 = (\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2$

$\Rightarrow (\lambda_1 - \lambda_2)^2 = 5^2 - 4 \times -6$

$\Rightarrow (\lambda_1 - \lambda_2)^2 = 25 + 24 = 49$

$\Rightarrow \lambda_1 - \lambda_2 = 7$ …(2)

adding equation (1) and (2),

$\lambda_1 = 6$

$\lambda_2 = -1$

Largest eigen value = 6

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152. (b)

Let the given matrix be

$$A = \begin{bmatrix}1 & -1 & 2\\ 0 & 1 & 0\\ 1 & 2 & 1\end{bmatrix}$$

And $\lambda_1 = 1$ For $\lambda_1 = 1$

$AX = \lambda_1 X$

$\Rightarrow AX = X$

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 1 & 0\\ 1 & 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$\Rightarrow x_1 - x_2 + 2x_3 = x_1$

$\Rightarrow x_2 - 2x_3 = 0 \Rightarrow x_3 = \frac{x_2}{2}$

$\Rightarrow x_1 + 2x_2 + x_3 = x_3$

$\Rightarrow x_1 + 2x_2 = 0 \Rightarrow x_2 = -\frac{x_1}{2},\ x_3 = -\frac{x_1}{4}$

$$\therefore \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}x_1\\ \frac{-x_1}{2}\\ \frac{-x_1}{4}\end{bmatrix} = \frac{-x_1}{4}\begin{bmatrix}-4\\ 2\\ 1\end{bmatrix}$$

$$\therefore \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \alpha\left[-4,\ 2,\ 1\right]$$

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153. (b)

Given matrix is

Let $P = \begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$

All the eigen values of Hermitian matrix are always real.

$\therefore$ For 'P' to be Hermitian matrix, $x = 5 - j$

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154. (b)

For a singular matrix, Determinant (i.e., product of all Eigen Values) is zero.

$\Rightarrow$ $\boxed{\text{At least one eigen value should be zero.}}$

Scan for Video solution

155. (2 to 2)

Given

Matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = A$ (say)

Let λ_1, λ_2 are the eigenvalues of this matrix.

$\Rightarrow$ $\lambda_1 + \lambda_2 = t_r(A) = 7$

and $\lambda_1 . \lambda_2 = |A| = 10$

$\Rightarrow$ Eigen values are 2, 5.

$\therefore$ $\boxed{\text{Lowest Eigen value} = 2}$

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156. (d)

Let $A = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$

The Characteristic equation is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -5-\lambda & 2 \\ -9 & 6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow -(5+\lambda)(6-\lambda) + 18 = 0$$

$$\Rightarrow (\lambda+5)(\lambda-6) + 18 = 0$$

$$\Rightarrow \lambda^2 - \lambda - 12 = 0$$

$$\Rightarrow \lambda = 4, -3$$

For $\lambda = 4$, $\quad Ax = \lambda x$

$$\Rightarrow \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4x_1 \\ 4x_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5x_1 + 2x_2 \\ -9x_1 + 6x_2 \end{bmatrix} = \begin{bmatrix} 4x_1 \\ 4x_2 \end{bmatrix} \quad \Rightarrow 9x_1 = 2x_2$$

$$\therefore \text{ Eigen Vector} = \begin{bmatrix} x_1 \\ \frac{2}{9}x_1 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

For $\lambda = -3$, $\quad Ax = \lambda x$

$$\Rightarrow \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3x_1 \\ -3x_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5x_1 + 2x_2 \\ -9x_1 + 6x_2 \end{bmatrix} = \begin{bmatrix} -3x_1 \\ -3x_2 \end{bmatrix} \Rightarrow 2x_2 = 2x_1 \Rightarrow x_1 = x_2$$

$$\therefore \text{ Eigen vector} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\therefore \text{ One of the Eigen vectors} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

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157. (d)

Eigen vectors of a symmetric matrix are orthogonal to each other.

$\therefore$ $x_1 y_1 + x_2 y_2 + x_3 y_3 = [x_1\ x_2\ x_3]\ [y_1\ y_2\ y_3]^T = 0$

Scan for Video solution

158. (a)

Eigen value of real symmetric matrix, it may be distinct positive or negative.

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159. (0.33)

Let us find characterstic equation;

$$|A-\lambda I|=0 \Rightarrow \begin{vmatrix} 0-\lambda & 1 & -1 \\ -6 & -11-\lambda & 6 \\ -6 & -11 & 5-\lambda \end{vmatrix}=0$$

$$\Rightarrow \quad \lambda^3+6\lambda^2+11\lambda+6=0$$

$$(\lambda+1)(\lambda+2)(\lambda+3)=0$$

$$\lambda=-1,-2,-3$$

$$\lambda_{min}=-3\,,\ \lambda_{max}=-1$$

$$\left|\frac{\lambda_{max}}{\lambda_{min}}\right|=\left|\frac{-1}{-3}\right|=0.33$$

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160. (0 to 0)

The Eigen vectors corresponding to different Eigen values of a symmetric matrix are orthogonal to each others. (i.e., $X_i^T . X_j = 0\ \forall\ i \neq j$)

$\therefore$ Dot product of eigen vectors = 0

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161. (0)

Given $A=\begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}_{3\times 1} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}_{1\times 3}=\begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}_{3\times 3}$

Since R_1 and R_2 are proportional to each other, $|A| = 0$

$\therefore$ **$|A| = 0$**

Scan for Video solution

162. (a)

Sum of eigen values of a given matrix = Trace of that given matrix

= sum of principal diagonal elements

= 215 + 150 + 550

= 915

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163. (b)

For a Real symmetric matrix 'A'.

Eigen values are always Real but need not be positive.

All other given options are correct.

Scan for Video solution

164. (c)

The Eigen values of a symmetric matrix are always real numbers.

Scan for Video solution

165. (d)

We know AX – λX

Let A $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\rightarrow$ for $\lambda = -1 \rightarrow X = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$a - b = -1$…..(i)

$c - d = 1$…..(ii)

$\rightarrow$ for $\lambda = -2 \rightarrow X = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$a - 2b = -2$…..(iii)

$c - 2d = 4$…..(iv)

On solving (i) and (iii); $a = 0$ and $b = 1$

On solving (ii) and (iv); $c = -2$ and $d = -3$

$$\therefore\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}$$

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166. (d)

$2x_1 - 2x_2 = 0$

$x_1 - x_2 = 0$

$$A = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$|A| = 0 \ \therefore\ \rho(A) = 1$

Since $\rho(A) <$ Number of unknowns $\{x_1, x_2\}$

$1 < 2$

$\therefore$ Infinite solution exists.

Let, $x_1 = k$,

Then, $x_2 = k$

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167. (a)

Let the given matrix is

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

$C_1 \rightarrow C_1 + C_3$

$$\Rightarrow\ A \sim \begin{bmatrix} 5 & 5 & 2 \\ 12 & 12 & 7 \\ 7 & 7 & 5 \end{bmatrix}$$

Since two columns are identical,

$\Rightarrow |A| =$ Product of eigen values of $A = 0$

$\Rightarrow$ '0' is an eigen value of 'A'

$\boxed{\therefore \text{ From given options, '0' is the least Eigen value}}$

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168. (b)

Given: $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$.

$\Rightarrow$ Characteristic equation is $|A - \lambda I| = 0$.

$$\Rightarrow \begin{bmatrix} 5-\lambda & 3 \\ 1 & 3-\lambda \end{bmatrix} = 0$$

$\Rightarrow (\lambda - 5)(\lambda - 3) - 3 = 0$

$\Rightarrow \lambda^2 - 8\lambda + 12 = 0$

$\Rightarrow \lambda = 2, 6$

(i) Eigen vector of '2'

$$\Rightarrow \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 \\ 2x_2 \end{bmatrix}$$

$\Rightarrow 5x_1 + 3x_2 = 2x_1$

$\Rightarrow x_1 = -x_2$

$x_1 + 3x_2 = 2x_1 \Rightarrow x_1 = -x_2$

$\therefore$ Eigen vector $= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigen vector of 'A'.

$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$ is a normalized Eigen vector.

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169. (b)

We know characterstic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} -5-\lambda & -3 \\ 2 & 0-\lambda \end{vmatrix} = 0$$

$\lambda(\lambda + 5) + 6 = 0$

$\lambda^2 + 5\lambda + 6 = 0$

According to cayley Hamiltan Theorem,

$A^2 + 5A + 6I = 0$

$A^2 = -5A - 6I$

$A^2 \cdot A = (-5A - 6I)A$

$A^3 = -5A^2 - 6IA$

$= -5(-5A - 6I) - 6A$

$\boxed{A^3 = 19A + 30I}$

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170. (d)

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Let Eigen values of 'A' be λ_1 and λ_2.

$\Rightarrow \quad \lambda_1 + \lambda_2 = tr(A) = 1 - 1 = 0$

$\Rightarrow \quad \lambda_1 + \lambda_2 = 0$

$\Rightarrow \quad \lambda_1 = -\lambda_2$

$\lambda_1 . \lambda_2 = |A| = -2$

$\therefore \quad \lambda_1 = \sqrt{2}$ and $\lambda_2 = -\sqrt{2}$

Eigen value of A^{19} are

$$\lambda_1^{19}, \lambda_2^{19} = \left(\sqrt{2}\right)^{19}, \left(-\sqrt{2}\right)^{19} = 512\sqrt{2}, -512\sqrt{2}$$

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171. (a)

Let the given matrix is $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$

Since the given matrix is upper triangular matrix, Eigen values are Principal diagonal elements.

$\therefore$ Eigen values of given matrix are 1, 4, 3.

Scan for Video solution

172. (c)

- Eigen values of real symmetric matrix are always real. (Ex: Principal stresses of a stress tensor are always real.)
- Eigen values of skew symmetric matrix are either zero (or) purely imaginary.

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173. (b)

Given:

Matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

The characteristic equation of 'A' is $|A - \lambda I| = 0$

$\Rightarrow \quad (2-\lambda)(3-\lambda) - 2 = 0 \Rightarrow \quad \lambda^2 - 5\lambda + 6 - 2 = 0$

$\Rightarrow \quad \lambda^2 - 5\lambda + 4 = 0 \qquad \Rightarrow \quad \lambda = 1, 4$

Eigen vector for $\lambda = 1$: $\quad Ax = \lambda x$

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\Rightarrow \quad 2x_1 + 2x_2 = x_2 \qquad \Rightarrow x_2 = -2x_1$

and $\quad x_1 + 3x_2 = x_2 \qquad \Rightarrow \quad x_1 = -2x_2$

$\boxed{\therefore \begin{bmatrix} 2 \\ -1 \end{bmatrix} \text{is an Eigen vector}}$

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174. (c)

- Eigen vales of a symmetric matrix are always real.
- Eigen values of a skew-symmetric matrix are either zero (or) Puely Imaginay complex Number.

Scan for Video solution

175. (d)

Given $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$ and 4, 8 are Eigen values of A.

Since sum of eigen values = trace (A)

$\Rightarrow \quad 2 + y = 4 + 8$

$\Rightarrow \quad y = 10$

Product of Eigen Values = |A|

$\Rightarrow \quad \lambda_1 . \lambda_2 = 2y - 3x$

$\Rightarrow \quad 4 \times 8 = (2 \times 10) - 3x$

$\Rightarrow \quad 3x = -12$

$\Rightarrow \quad x = -4$

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176. (b)

We know $PX = \lambda X$

From option(a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = PX$

$$PX = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} \neq \lambda \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} (\therefore PX \neq \lambda X)$$

From option (b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = PX$

$$PX = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Since for $\lambda = 3$
$PX = \lambda X$ Hence option (b)
is correct

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177. (c)

Let the eigen of 2×2 matrix are λ_1 and λ_2.

Trace of matrix $= \lambda_1 + \lambda_2 = -2$ ….(i)

Determinant of matrix $= \lambda_1 \cdot \lambda_2 = -35$ ….(ii)

$$\lambda_1 + \frac{-35}{\lambda_1} = -2$$

$$\lambda_1^2 + 2\lambda_1 - 35 = 0$$

$$(\lambda_1 + 7)(\lambda_1 - 5) = 0$$

$$\lambda_1 = -7, \quad 5$$

$$\downarrow \quad \downarrow$$

$$\lambda_2 = \ 5, \quad -7$$

$\therefore$ Eigen values are– 7 and 5.

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178. (d)

Given:

Matrix $A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

Eigen values of 'A' are obtained by the equation $|A - \lambda I| = 0$. In this case the characteristic equation will be of 3rd degree.

All options involve complex roots of $|A - \lambda I| = 0$

We know complex roots occurs as conjugate pairs:

i.e Given matrix should have 1 real eigen value and two conjugates complex eigen values.

$\therefore$ Option 'D' is correct

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179. (b)

$$[P] = \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$$

Hence the characteristic equation for matrix P,

$$|P - \lambda I| = 0$$

$$\left|\begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & -5-\lambda \end{vmatrix} = 0$$

$(4–\lambda)\ (–5–\lambda) – 2 \times 5 = 0$

$–20 – 4\lambda + 5\lambda + \lambda^2 – 10 = 10$

$\lambda^2 + \lambda – 30 = 0$

$\lambda^2 – 5\lambda + 6\lambda – 30 = 0$

$1(\lambda – 5) + 6(\lambda – 5) = 0$

$(\lambda – 5)\ (\lambda + 6) = 0$

$\lambda = 5, –6$

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180. (a)

Given matrices are

(i) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow$ Diagonal matrix.

$\therefore$ Eigen values are 1, 0.

(ii) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow$ Upper triangular matrix.

$\therefore$ Eigen values are 0, 0.

(iii) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \rightarrow$ Let λ_1, λ_2 are Eigen values

$\Rightarrow$ $\lambda_1 + \lambda_2 = 2; \lambda_1.\lambda_2 = 2$

$\Rightarrow$ $\lambda_1 + \dfrac{2}{\lambda_1} = 2$

$\Rightarrow$ $\lambda^2 – 2\lambda + 2 = 0$

$\Rightarrow$ $(\lambda – 1)^2 + 1 = 0$

$\Rightarrow$ $\lambda = 1 + i, 1 – i$

(iv) $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \rightarrow$ Lowest triangular matrix

$\Rightarrow$ Eigen values are –1, –1.

$\therefore$ Number of matrices with eigen values '1' is 1.

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181. (c)

Given:

Matrix, $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$

Let $\lambda_1, \lambda_2, \lambda_3$ are eigenvalues of A

$\Rightarrow$ $\lambda_1 + \lambda_2 + \lambda_3 =$ trace (A)= Sum of diagonal element

Let $\lambda_1 = 3$,

$\Rightarrow 3 + \lambda_2 + \lambda_3 = 1 + 0 + p \Rightarrow \lambda_2 + \lambda_3 = p – 2$

$\therefore$ $\boxed{\text{Sum of other two eigen values} = p – 2}$

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182. (b)

Given:

Matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

$\because$ 'A' is an upper triangular matrix. Therefore, Eigen values of 'A' are principal diagonal elements.

$\therefore$ Let $\lambda_1 = 1$ & $\lambda_2 = 2$

For $\lambda_1 = 1$, let the eigen vector be $\begin{bmatrix} 1 \\ a \end{bmatrix}$

$\Rightarrow$ $A\begin{bmatrix} 1 \\ a \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ a \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ a \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 1+2a \\ 2a \end{bmatrix}\begin{bmatrix} 1 \\ a \end{bmatrix}$ $\Rightarrow$ $a = 0$

Let $\lambda_2 = 2$, let the Eigen vector be $\begin{bmatrix} 1 \\ b \end{bmatrix}$

$$\Rightarrow \quad A\begin{bmatrix}1\\b\end{bmatrix} = \lambda_2 \cdot \begin{bmatrix}1\\b\end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}\begin{bmatrix}1\\b\end{bmatrix} = 2\begin{bmatrix}1\\b\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}1+2b\\2b\end{bmatrix} = \begin{bmatrix}2\\2b\end{bmatrix} \quad \Rightarrow \quad 1 + 2b = 2 \Rightarrow b = 1/2$$

$$\therefore \quad \boxed{a + b = 0 + \frac{1}{2} = \frac{1}{2}}$$

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183. (c)

Given:

$$P = \begin{bmatrix}P_{11} & P_{12}\\P_{21} & P_{22}\end{bmatrix}$$ and 'P' has '0' as an eigen value.

$\Rightarrow$ Product of eigen value of P = |P| = 0

$\Rightarrow$ |P| = 0 $\quad \Rightarrow \quad P_{11}P_{22} - P_{12}P_{21} = 0$

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184. (a)

Rank $(A_{4\times4}) = 2$

Nullity (A) = Number of columns – Rank

= 4 –2 = 2

i.e., Null space of A will consist only two linear independent vectors which are given.

Eigen vectors of A are $\underset{C_1}{\begin{bmatrix}2\\1\\0\\3\end{bmatrix}}$ and $\underset{C_2}{\begin{bmatrix}1\\0\\1\\2\end{bmatrix}}$

As these are linearly independent eigen vectors so remaining eigen vectors of null space must be linearly dependent

Hence $C_1 - C_2 \begin{bmatrix}1\\1\\-1\\1\end{bmatrix} = \begin{bmatrix}1 & 1 & -1 & 1\end{bmatrix}^T$

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185. (c)

For a square matrix 'A'

$$\left|A^T \cdot A^{-1}\right| = \left|A^T\right| \cdot \left|A^{-1}\right|$$

$$= |A| \cdot \left|\frac{1}{A}\right| = 1$$

$(\because \left|A^T\right| = |A|$ & $\left|A^{-1}\right| = |A|^{-1})$

$$\boxed{\therefore \left|A^T \cdot A^{-1}\right| = 1}$$

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186. (2)

Given dim(v) = 6 and

$\text{Dim}(V_1) = \dim(V_2) = 4$

Since V_1, V_2 are subspaces of 'V'

$\text{Dim}(V) \geq \dim(V_1) + \dim(V_2) - \dim(V_1 \cap V_2)$

$\Rightarrow 6 \geq 4 + 4 - \dim(V_1 \cap V_2)$

$\Rightarrow -2 \geq -\dim(V_1 \cap V_2)$

$\Rightarrow \dim(V_1 \cap V_2) \geq 2$

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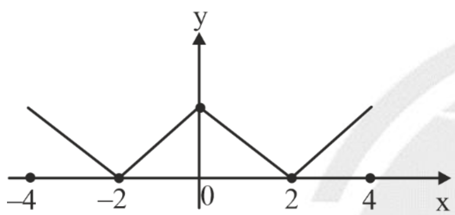
CHAPTER

2 BASIC CALCULUS

Differential Calculus

1. [MCQ] **[GATE-ME-2023: 1M]**

The figure shows the plot of a function over the interval [–4, 4], which one of the options given CORRECTLY identifies the function?



(a) $|2 - x|$ (b) $|2 - |x||$
(c) $|2 + |x||$ (d) $2 - |x|$

2. [MCQ] **[GATE-ME-2022: 1M]**

The limit $P = \lim_{x\to\pi}\left(\frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x}\right)$ has a finite value for a real α. The value of α and the corresponding limit p are

(a) $\alpha = -3\pi$, and $p = \pi$ (b) $\alpha = -2\pi$, and $p = 2\pi$
(c) $\alpha = \pi$, and $p = \pi$ (d) $\alpha = 2\pi$, and $p = 3\pi$

3. [NAT] **[GATE-CS-2022: 1M]**

The value of the following limit is ______________.

$\lim_{x\to 0+}\frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$.

4. [NAT] **[GATE-CS-2021: 2M]**

Consider the following Expression. $\lim_{x\to -3}\frac{\sqrt{2x + 22 - 4}}{x + 3}$ .The value of the above expression (rounded to 2 decimal place) is ________________

5. [MCQ] **[GATE-ME-2021: 1M]**

The value of $\lim_{x\to 0}\left(\frac{1 - \cos x}{x^2}\right)$ is

(a) 1/4 (b) 1/3
(c) 1/2 (d) 1

6. [MCQ] **[GATE-CE-2021-:1M]**

The value of $\lim_{x\to\infty}\frac{x\ln(x)}{1 + x^2}$ is

(a) 1.0 (b) 0.5
(c) ∞ (d) 0

7. [NAT] **[GATE-CE-2021:1M]**

Consider the limit: $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x - 1}\right)$.

The limit (correct up to one decimal place) is ____.

8. [MCQ] **[GATE-CE-2020:1M]**

The value of $\lim_{x\to\infty}\frac{\sqrt{9x^2 + 2020}}{x + 7}$ is

(a) 1 (b) 3
(c) $\frac{7}{9}$ (d) Indeterminable

9. [MCQ] **[GATE-CE-2020:1M]**

The true value of ln(2) is 0.69. If the value of ln(2) is obtained by linear interpolation between ln(1) and ln(6), the percentage of absolute error (round off the nearest integer), is

(a) 35 (b) 69
(c) 69 (d) 48

10. [MCQ] [GATE-CE-2020:1M]

The value of $\lim_{x\to\infty}\frac{x^2-5x+4}{4x^2+2x}$ is

(a) 0
(b) 1
(c) 1/4
(d) 1/2

11. [MCQ] [GATE-ME-2020: 1M]

The value of $\lim_{x\to 1}\left(\frac{1-e^{-c(1-x)}}{1-xe^{-c(1-x)}}\right)$ is

(a) c
(b) c + 1
(c) $\frac{c}{c+1}$
(d) $\frac{c+1}{c}$

12. [MCQ] [GATE-CE-2019:2M]

Consider the functions: $x=\psi\ln\phi$ and $y=\phi\ln\psi$. Which one of the following the correct expression for $\frac{\partial\psi}{\partial x}$

(a) $\frac{\ln\psi}{\ln\phi\ln\psi-1}$
(b) $\frac{x\ln\psi}{\ln\phi\ln\psi-1}$
(c) $\frac{\ln\phi}{\ln\phi\ln\psi-1}$
(d) $\frac{x\ln\phi}{\ln\phi\ln\psi-1}$

13. [MCQ] [GATE-CS-2019:1M]

Compute $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

(a) 1
(b) $\frac{53}{12}$
(c) $\frac{108}{7}$
(d) Limit does not exist

14. [MCQ] [GATE-CE-2019:1M]

The following inequality is true for all x close to 0.

$$2-\frac{x^2}{3}<\frac{x\sin x}{1-\cos x}<2$$

What is the value of $\lim_{x\to 0}\frac{x\sin x}{1-\cos x}$?

(a) 1
(b) 0
(c) 1/2
(d) 2

15. [MCQ] [GATE-CE-2019:1M]

Which of the following is correct?

(a) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=1$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(b) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=\infty$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(c) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=\infty$

(d) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

16. [MCQ] [GATE-ME-2019: 2M]

The derivative of f(x) = cos (x) can be estimated using the approximation $f'(x)=\frac{f(x+h)-f(x-h)}{2h}$. The percentage error is calculated as $\left(\frac{\text{Exact value} - \text{Approximate value}}{\text{Exact value}}\right)\times 100$. The percentage error in the derivative of $f(x)$ at $x=\frac{\pi}{6}$ radian, choosing h = 0.1 radian, is

(a) < 0.1%
(b) > 0.1% and < 1%
(c) >1% and < 5%
(d) >5%

17. [MCQ] [GATE-EE-2018 : 1M]

Let f be a real-valued function of a real variable defined as $f(x)=x^2$ for $x\ge 0$, and $f(x)=-x^2$ for $x<0$. Which one of the following statements is true?

(a) f(x) is discontinuous at x = 0
(b) f(x) is continuous but not differentiable x = 0.
(c) f(x)is differentiable but its first derivative is not continuous at x = 0.
(d) f(x)is differentiable but its first derivative is not differentiable at x = 0.

18. [MCQ] [GATE-EE-2017 : 2M]

Let $g(x)=\begin{cases}-x & x\le 1\\ x+1 & x\ge 1\end{cases}$ and

$$f(x) = \begin{cases} 1-x & x \le 0 \\ x^2, & x > 0 \end{cases}$$

Consider the composition of f and g. i.e., (f∘g) (x) = f (g(x)). The number of discontinuities in (f∘g) (x) present in the interval (-∞, 0) is:

(a) 0 (b) 1
(c) 2 (d) 4

19. [MCQ] [GATE-EE-2017 : 2M]

A function f(x) is defined as

$$f(x) = \begin{Bmatrix} e^x & ; & x < 1 \\ \ln x + ax^2 + bx & ; & x \ge 1 \end{Bmatrix},$$

where x ϵ R. Which one of the following statement is TRUE?

(a) f(x) is NOT differentiable at x = 1 for any values of a and b.
(b) f(x) is differentiable at x = 1 for the unique value of a and b.
(c) f(x) is differentiable at x = 1 for all values of a and b such that a + b = e.
(d) f(x) is differentiable at x = 1 for all values of a and b

20. [MCQ] [GATE-CS-2017:2M]

The vaue of $\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

(a) Is 0 (b) Is-1
(c) is 1 (d) Does not exist

21. [NAT] [GATE-CE-2017:1M]

$\lim_{x \to 0}\left(\frac{\tan x}{x^2 - x}\right)$ is equal to ____

22. [MCQ] [GATE-CE-2017:1M]

Let W = f (x,y) , where x and y are function of t. Then, according to the chain rule, $\frac{dw}{dt}$ is equal

(a) $\frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dt}{dt}$ (b) $\frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$

(c) $\frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$ (d) $\frac{dw}{dx}\frac{\partial x}{\partial t} + \frac{dw}{dy}\frac{\partial t}{\partial t}$

23. [MCQ] [GATE-ME-2017: 1M]

The value of $\lim_{x \to 0} \frac{x^3 - \sin(x)}{x}$ is

(a) 0 (b) 3
(c) 1 (d) –1

24. [NAT] [GATE-EC-2017: 1M]

The smaller angle (in degrees) between the planes x + y + z = 1 and 2x – y + 2z = 0 is ____.

25. [MCQ] [GATE-EC-2016:1M]

Given the following statements about a function f: R → R, select the right option.

P. If f(x) is continuous at $x = x_0$, then it is also differentiable at $x = x_0$.

Q. If f(x) is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R. If f(x) is differentiable at $x = x_0$, then it is also continuous at $x = x_0$

(a) P is true, Q is false, R is false
(b) P is false, Q is true, R is true
(c) P is false, Q is true, R is false
(d) P is true, Q is false, R is true

26. [NAT] [GATE-CS-2016: 1M]

$\lim_{x \to 4} \frac{\sin(x-4)}{x-4} =$ ______

27. [MCQ] [GATE-ME-2016: 1M]

$\underset{x \to 0}{\text{Lt}} \frac{\log_e(1+4x)}{e^{3x} - 1}$ is equal to

(a) 0 (b) $\frac{1}{12}$

(c) $\frac{4}{3}$ (d) 1

28. [MCQ] [GATE-ME-2016: 2M]

$\lim_{x\to\infty} \sqrt{x^2+x-1}-x$ is

(a) 0 (b) ∞
(c) $1/2$ (d) $-\infty$

29. [MCQ] [GATE-ME-2016: 1M]

The values of x for which the function

$$f(x)=\frac{x^2-3x-4}{x^2+3x-4}$$

is NOT continuous are

(a) 4 and −1 (b) 4 and 1
(c) − 4 and 1 (d) − 4 and −1

30. [NAT] [GATE-EE- 2016: 2M]

Let $S=\sum_{n=0}^{\infty} n\alpha^n$ where $|\alpha|<1$. The value of α in the range $0<\alpha<1$, such that $S=2\alpha$ is ___________.

31. [MCQ] [GATE-EE-2015 : 1M]

If a continuous function f (x) does not have a root in the interval [a, b], then which one of the following statements is TRUE?

(a) f (a). f (b) = 0 (b) f (a). f (b) < 0
(c) f (a). f (b) > 0 (d) f (a)/ f (b) ≤ 0

32. [MCQ] [GATE-CE-2015:1M]

$\lim_{x\to} (1+1/x)^{2x}$ is equal to

(a) e^{-2} (b) 1
(c) e (d) e^2

33. [MCQ] [GATE-ME-2015: 1M]

The value of $\lim_{x\to 0}\frac{1-\cos(x^2)}{2x^4}$ is

(a) 0 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) Undefined

34. [NAT] [GATE-ME-2015: 1M]

The value of $\lim_{x\to 0}\left(\frac{-\sin x}{2\sin x + x\cos x}\right)$ is _____.

35. [NAT] [GATE-ME-2015: 2M]

Consider an ant crawling along the curve $(x-2)^2+y^2=4$, where x and y are in meters. The ant starts at the point (4, 0) and moves counter-clockwise with a speed of 1.57 meters per second. The time taken by the ant to reach the point (2, 2) is (in seconds) _____

36. [MCQ] [GATE-CS-2015: 1M]

$\lim_{x\to\infty} x^{1/x}$

(a) ∞ (b) 0
(c) 1 (d) Not defined

37. [MCQ] [GATE- CS-2015:1M]

The value of $\lim_{x\to\infty}(1+x^2)e^{-x}$ is

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) ∞

38. [MCQ] [GATE-EC-2014: 1M]

The value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ is

(a) ln 2 (b) 1.0
(c) e (d) ∞

39. [NAT] [GATE-CS-2014:2M]

The function f(x) = x sin x satisfies the following equation: f''(x) + f(x) + t cos x = 0. The value of t is _______.

40. [MCQ] [GATE-CE-2014:1M]

$\text{Lim}_{x\to\infty}\left(\frac{x+\sin x}{x}\right)$ equal to

(a) $-\infty$ (b) 1
(c) 0 (d) ∞

41. [MCQ] [GATE-CE-2014:2M]

The expression $\lim_{\alpha \to 0} \frac{x^{\alpha}-1}{\alpha}$ is equal to

(a) log x (b) x log x
(c) 0 (d) ∞

42. [MCQ] [GATE-CE-2014:2M]

What should be the value of λ such that the function defined below is continuous at x = π/2?

$$f(x)=\begin{cases}\dfrac{\lambda \cos x}{\frac{\pi}{2}-x} & \text{if } x \neq \frac{\pi}{2} \\ 1 & \text{if } x=\frac{\pi}{2}\end{cases}$$

(a) 0 (b) 2/π
(c) 1 (d) π/2

43. [MCQ] [GATE-ME-2014: 1M]

$\mathop{\mathrm{Lt}}_{x \to 0} \frac{x-\sin x}{1-\cos x}$ is

(a) 0 (b) 1
(c) 3 (d) Not defined.

44. [MCQ] [GATE-ME-2014: 1M]

$\mathop{\mathrm{Lt}}_{x \to 0}\left(\frac{e^{2x}-1}{\sin(4x)}\right)$ is equal to

(a) 0 (b) 0.5
(c) 1 (d) 2

45. [MCQ] [GATE-ME-2014: 1M]

If a function is continuous at a point,

(a) The limit of the function may not exist at the point.
(b) The function must be derivable at the point.
(c) The limit of the function at the point tends to infinity.
(d) The limit must exist at the point and the value of limit should be same as the value of the function at that point.

46. [MCQ] [GATE-CS-2013:1M]

Which one of the following functions is continuous at x = 3?

(a) $f(x)=\begin{cases}2, & \text{if } x=3 \\ x-1, & \text{if } x>3 \\ \frac{x+3}{3}, & \text{if } x<3\end{cases}$

(b) $f(x)=\begin{cases}4, & \text{if } x=3 \\ 8-x & \text{if } x \neq 3\end{cases}$

(c) $f(x)=\begin{cases}x+3, & \text{if } x \leq 3 \\ x-4 & \text{if } x>3\end{cases}$

(d) $f(x)=\frac{1}{x^3-27}$, if $x \neq 3$

47. [MCQ] [GATE-ME-2013: 1M]

Choose the CORRECT set of functions, which are linearly dependent.

(a) sin x, $\sin^2 x$ and $\cos^2 x$
(b) cos x, sin x and tan x
(c) cos 2x, $\sin^2 x$ and $\cos^2 x$
(d) cos 2x, sin x and cos x

48. [MCQ] [GATE-ME-2012: 1M]

Consider the function f(x) = |x| in the interval $-1 \leq x \leq 1$. At the point x = 0, f(x) is

(a) Continuous and differentiable.
(b) Non-continuous and differentiable.
(c) Continuous and non-differentiable.
(d) Neither continuous nor differentiable.

49. [MCQ] [GATE-ME-2012: 1M]

$\lim_{x \to 0}\left(\frac{1-\cos x}{x^2}\right)$ is

(a) 1 / 4 (b) 1 / 2
(c) 1 (d) 2

50. [MCQ] [GATE-ME-2011: 1M]

What is $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$ equal to?

(a) θ (b) sin θ
(c) 0 (d) 1

51. [MCQ] [GATE- ME -2010: 1M]

The function $y = |2 - 3x|$

(a) is continuous $\forall$ x $\in$ R and differentiable $\forall$ x $\in$ R
(b) is continuous $\forall$ x $\in$ R and differentiable $\forall$ x $\in$ R except at x = 3/2
(c) is continuous $\forall$ x $\in$ R and differentiable $\forall$ x $\in$ R except at x = 2/3
(d) is continuous $\forall$ x $\in$ R except at x = 3 and differentiable $\forall$ x $\in$ R.

52. [MCQ] [GATE-CS-2010:1M]

What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$

(a) 0
(b) e^{-2}
(c) $e^{-1/2}$
(d) 1

53. [MCQ] [GATE-CE-2010:1M]

The $\lim_{x\to 0}\frac{\sin\left[\frac{2}{3}x\right]}{x}$ is

(a) 2/3
(b) 1
(c) 3/2
(d) ∞

54. [MCQ] [GATE-CS-2008:2M]

$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x}$ equals

(a) 1
(b) –1
(c) ∞
(d) $-\infty$

55. [MCQ] [GATE-ME-2008: 1M]

The value of $\lim_{x\to 8}\frac{x^{1/3}-2}{(x-8)}$ is

(a) $\frac{1}{16}$
(b) $\frac{1}{12}$
(c) $\frac{1}{8}$
(d) $\frac{1}{4}$

Applications of Differentiation

56. [NAT] [GATE 2023-EE :2M]

In the following differential equation, the numerically obtained value of y(t), at t = 1, is _______(Round off to 2 decimal places)

$$\frac{dy}{dt}=\frac{e^{-\alpha t}}{2+\alpha t}, \alpha = 0.01 \text{ and } y(0) = 0$$

57. [MCQ] [GATE-2023-EC : 2M]

Consider the following series:

$$\sum_{n=1}^{\infty}\frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

(a) c = 1, d = –1
(b) c = 2, d = 1
(c) c = 0.5, d = –10
(d) c = 1, d = –2

58. [MCQ] [GATE-2023-EC : 2M]

The function $f(x) = 8\log_e x - x^2 + 3$ attains its minimum over the interval [1, e] at x = _________.
(Here $\log_e x$ is the natural logarithm of x.)

(a) 2
(b) 1
(c) e
(d) $\frac{1+e}{2}$

59. [MCQ] [GATE-CS-2023: 1M]

Let f(x) = $x^3 + 15x^2 - 33x - 36$ be a real-valued function.
Which of the following statements is/are TRUE?

(a) f(x) does not have a local maximum.
(b) f(x) has a local maximum.
(c) f(x) does not have a local minimum.
(d) f(x) has a local minimum.

60. [MCQ] [GATE-CE-2023:2M]

For the function f(x) = $e^x|\sin x|$; x∈R, which of the following statements is/are TRUE?

(a) The function is continuous at all x
(b) The function is differentiable at all x
(c) The function is periodic
(d) The function is bounded

61. [NAT] [GATE-ME-2023: 2M]

The smallest perimeter that a rectangle with area of 4 square units can have is _____ units. (Answer in integer)

62. [MCQ] [GATE-CE-2022:1M]

Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S, given by $1 \leq x \leq 3$. The first and second derivatives are f'(x) and f'(x). Consider the following statements:

I) The given polynomial is zero at the boundary points x = 1and x =3

II) There exists on local maxima of f(x) within the domain S.

III) The second derivative $f''(x) > 0$ throughout the domains S.

IV) There exists one local minima f(x) within the domain S.

(a) Only statements II and IV are correct

(b) Only statements I and IV are correct.

(c) Only statements I, II and III are correct.

(d) Only statements I, II and IV are correct

63. [MSQ] [GATE-CE-2022:2M]

Let max (a, b) denote the maximum of two real numbers a and b. Which of the following statement(s) is/are TRUE about the function f(x) = max{3 – x, x – 1}?

(a) It is continuous on its domain

(b) It has a local minimum at x =2

(c) It has a local maximum at x = 2

(d) It is differentiable on its domain

64. [MCQ] [GATE-ME-2021: 2M]

Let the superscript T represent the transpose operation. Consider the function $f(x) = \frac{1}{2}x^TQx - r^Tx$, where x and r are n × 1 vectors and Q is a symmetric n × n matrix. The stationary point of f(x) is

(a) Q^Tr (b) $Q^{-1}r$

(c) $\frac{r}{r^Tr}$ (d) r

65. [NAT] [GATE-CS-2021:1M]

Suppose that f: R → R is a continuous function on the interval [–3, 3] and a differentiable function in the interval (-3, 3) such that for every x in the interval, $f'(x) \leq 2$. If f(-3) = 7, then f(3) is at most ________________.

66. [MCQ] [GATE-EE-2021 : 1M]

Let f(x) be a real-valued function such that $f'(x_0) = 0$ for some $x_0 \in (0,1)$, and $f''(x) > 0$ for all $x \in (0,1)$. Then f(x) has

(a) no local minimum in (0,1)

(b) one local maximum in (0,1)

(c) exactly one local minimum in (0,1)

(d) two distinct local minima in (0,1)

67. [NAT] [GATE-EE-2021: 1M]

Suppose the circles $x^2 + y^2 = 1$ and $(x - 1)^2 + (y - 1)^2 = r^2$ intersect each other orthogonally at the point (u, v). Then u + v =_______.

68. [MCQ] [GATE-EE-2021: 2M]

In the open interval (0,1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

(a) two real roots (b) one real root

(c) three real roots (d) no real roots

69. [MSQ] [GATE-EE-2020: 1M]

$ax^3 + bx^2 + cx + d$ is a polynomial on real x over real coefficients a, b, c, d wherein a≠0. Which of the following statements is true?

(a) d can be chosen to ensure that x=0 is a root for any given set a, b, c. identical.

(b) No choice of coeffecients can make all roots identical.

(c) a, b, c, d can be chosen to ensure that all roots are complex

(d) c alone cannot ensure that all roots are real

70. [MCQ] [GATE-EE-2020: 2M]

For real numbers, x and y, with $y = 3x^2 + 3x + 1$, the maximum and minimum value of y for $x \in [-2, 0]$ are respectively, _______.

(a) 7 and 1/4 (b) 7 and 1

(c) –2 and –1/2 (d) 1 and 1/4

71. [MCQ] [GATE-CS-2020:1M]

Consider the functions

I: e^{-x}

II: $x^2 - \sin x$

III: $\sqrt{x^3+1}$

Which of the above functions is/are increasing everywhere in [0, 1]?

(a) III only (b) II only

(c) II and III only (d) I and III only

72. [NAT] [GATE-CE-2020:2M]

The diameter and height of a right circular cylinder are 3 cm and 4 cm respectively. The absolute error in each of these two measurements is 0.02 cm. The absolute error in the compute volume (in cm^3, round off three decimal places), is

73. [MCQ] [GATE-EC-2019: 1M]

The families of curves represented by the solution of the equation

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

for n = –1 and n = +1, respectively, are

(a) Hyperbolas and Parabolas

(b) Parabolas and Circles

(c) Hyperbolas and Circles

(d) Circles and Hyperbolas

74. [MCQ] [GATE-CE-2019:2M]

Which one of the following is NOT a correct statement?

(a) The function $\sqrt[x]{x}, (x>0)$ has the global minima at x = e

(b) The function $\sqrt[x]{x}, (x>0)$ has the global maxima at x = e

(c) The function x^3 has neither global minima nor global maxima

(d) The function |x| has the global minima at x = 0

75. [MCQ] [GATE-CE-2019:1M]

For a small value of h, the Taylor series expansion for f(x + h) is

(a) $f(x)+hf'(x)+\frac{h^2}{2}f''(x)+\frac{h^3}{3}f''(x)+\ldots\infty$

(b) $f(x)-hf'(x)+\frac{h^2}{2!}f''(x)-\frac{h^3}{3!}f''(x)+\ldots\infty$

(c) $f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f''(x)+\ldots\infty$

(d) $f(x)-hf'(x)+\frac{h^2}{2}f''(x)-\frac{h^3}{3}f''(x)+\ldots\infty$

76. [MCQ] [GATE-CE-2018:1M]

At the point x = 0, the function f(x) = x^3 has

(a) Local maximum

(b) Local minimum

(c) Both local maximum and minimum

(d) Neither local maximum nor local minimum

77. [NAT] [GATE-EE-2018 : 2M]

Let f (x) = $3x^3 - 7x^2 + 5x + 6$. The maximum value of f(x) over the interval [0,2] is ____ (up to 1 decimal place).

78. [NAT] [GATE-EE-2017: 1M]

Let x and y be integers satisfying the following equations

$2x^2 + y^2 = 34$

$x + 2y = 11$

The value of $(x+y)$ is _______________.

79. [MCQ] [GATE-CE-2017:2M]

The tangent to the curve represented by y = x ln x is required to have 45° inclination with the x-axis. The coordinates of the tangent point would be

(a) (1, 0) (b) (1, 1)

(c) (0, 1) (d) $(\sqrt{2}, \sqrt{2})$

80. [MCQ] [GATE-EC-2017:2M]

Let $f(x)=e^{x+x^2}$ for real x. From among the following choose the Taylor series approximation of $f(x)$ around $x = 0$, which includes all powers of x less than equal to 3.

(a) $1 + x + x^2 + x^3$ (b) $1 + x + \frac{3}{2}x^2 + x^3$

(c) $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$ (d) $1 + x + 3x^2 + 7x^3$

81. [NAT] [GATE-EC-2017: 2M]

The minimum value of the function $f(x) = \frac{1}{3}x\left(x^2 - 3\right)$ in the interval $-100 \le x \le 100$ occurs at x = _______

82. [NAT] [GATE-2016-EC : 2M]

How many distinct values of *x* satisfy the equation $\sin(x) = \frac{x}{2}$, where *x* is in radians?

(a) 1 (b) 2

(c) 3 (d) 4 or more

83. [NAT] [GATE-EE-2016 : 1M]

The maximum value attained by the function $-f(x) = -x(x-1)(x-2)$ in the interval [1, 2] is _______.

84. [MCQ] [GATE-EC-2016: 1M]

As x varies from –1 to +3, which one of the following describes the behavior of the function $f(x) = x^3 - 3x^2 + 1$?

(a) f(x) increases monotonically

(b) f(x) increases, then decreases and increases again

(c) f(x) decreases, then increases and decreases again

(d) f(x) increases and then decreases.

85. [MCQ] [GATE-CE-2016:2M]

The quadratic approximation of $f(x) = x^3 - 3x^2 - 5$ at the point x = 0 is

(a) $3x^2 - 6x - 5$ (b) $-3x^2 - 5$

(c) $-3x^2 - 6x - 5$ (d) $3x^2 - 5$

86. [MCQ] [GATE-CE-2016:2M]

The angle of intersection of the curves $x^2 = 4y$ and $y^2 = 4x$ at point (0, 0) is

(a) 0° (b) 30°

(c) 45° (d) 90°

87. [MCQ] [GATE-CE-2016:1M]

The optimum value of the function $f(x) = x^2 - 4x + 2$ is

(a) 2 (maximum) (b) –2 (maximum)

(c) 2 (minimum) (d) –2 (minimum)

88. [NAT] [GATE-ME-2016: 2M]

Consider the function $f(x) = 2x^3 - 3x^2$ in the domain [–1, 2]. The global minimum of f(x) is ____________

89. [NAT] [GATE-EC-2015: 1M]

The value of $\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$ is _______

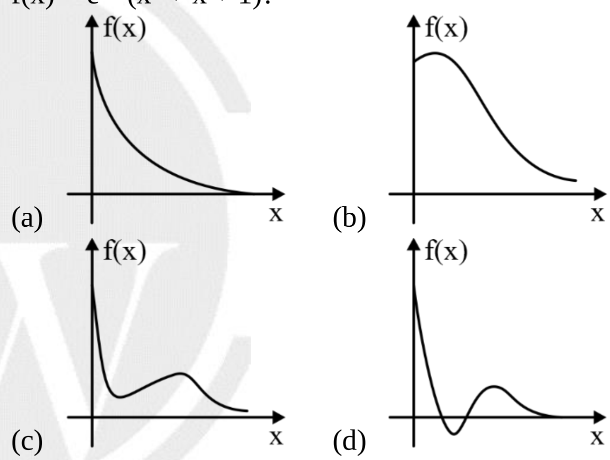
90. [NAT] [GATE-EC-2015: 2M]

The maximum area (in square units) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is _____.

91. [MCQ] [GATE-EC-2015: 2M]

Which one of the following graphs describes the function

$f(x) = e^{-x}(x^2 + x + 1)$?



92. [MCQ] [GATE-EC-2015: 1M]

A function $f(x) = 1 - x^2 + x^3$ is defined in the closed interval [–1, 1]. The value of x, in the open interval (–1, 1) for which the mean value theorem is satisfied, is

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

93. [MCQ] [GATE-CS-2015:2M]

Let $f(x) = x^{-\frac{1}{3}}$ and A denote the area of the region bounded by f(x) and the X-axis, when x varies from –1 to 1. Which of the following statements is/are TRUE?

(I) f is continuous in [–1, 1]
(II) f is not bounded in [–1, 1]
(III) A is nonzero and finite

(a) II only (b) III only
(c) II and III only (d) I, II and III

94. [MCQ] [GATE-CE-2015:1M]

While minimizing the function f(x), necessary and sufficient conditions for a point x_0 to be a minima are

(a) $f'(x_0) > 0$ and $f''(x_0) = 0$
(b) $f'(x_0) < 0$ and $f''(x_0) = 0$
(c) $f'(x_0) = 0$ and $f''(x_0) < 0$
(d) $f'(x_0) = 0$ and $f''(x_0) > 0$

95. [MCQ] [GATE-ME-2015: 1M]

At x = 0, the function f (x) = |x| has

(a) a minimum
(b) a maximum
(c) a point of inflexion
(d) neither a maximum nor minimum

96. [MCQ] [GATE-EC-2014:1M]

For $0 \le t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at

(a) $t = \log_e 4$ (b) $t = \log_e 2$
(c) $t = 0$ (d) $t = \log_e 8$

97. [NAT] [GATE-EC-2014:2M]

The maximum value of $f(x)= 2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is _____

98. [MCQ] [GATE-EC-2014:1M]

The series $\Sigma_{n=0}^{\infty} \frac{1}{n!}$ Converges to

(a) 2 ln 2 (b) $\sqrt{2}$
(c) 2 (d) e

99. [MCQ] [GATE-EC-2014: 1M]

The Taylor series expansion of 3 sin x + 2 cos x is

(a) $2 + 3x - x^2 - \frac{x^3}{2} + ...$

(b) $2 - 3x - x^2 - \frac{x^3}{2} + ...$

(c) $2 + 3x + x^2 + \frac{x^3}{2} + ...$

(d) $2 - 3x - x^2 + \frac{x^3}{2} + ...$

100.[MCQ] [GATE-EC-2014: 2M]

For a right-angled triangle, if the sum of the lengths of the hypotenuse and a side is kept constant, in order to have maximum area of the triangle, the angle between the hypotenuse and the side is

(a) 12° (b) 36°
(c) 60° (d) 45°

101.[NAT] [GATE-ME-2014: 1M]

The best approximation of the minimum value attained by $e^{-x}\sin(100x)$ for $x \ge 0$ is _______.

102.[NAT] [GATE-EE-2014: 1M]

A particle, starting from origin at t = 0 s, is traveling along x-axis with velocity

$$v = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) \text{m / s}$$

At t = 3s, the difference between the distance covered by the particle and the magnitude of displacement from the origin is ______.

103.[MCQ] [GATE-EE-2014 : 2M]

The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval [–3, 3] is

(a) 20 (b) 28
(c) 16 (d) 32

104.[MCQ] [GATE-EE-2014: 1M]

Minimum of the real valued function $f(x) = (x - 1)^{2/3}$ occurs at x equal to

(a) $-\infty$ (b) 0
(c) 1 (d) ∞

105.[MCQ] **[GATE-EE-2014: 1M]**

Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is

$(0, \infty)$ is

(a) e^{-1} (b) e

(c) $1 - e^{-1}$ (d) $1 + e^{-1}$

106.[MCQ] **[GATE-EE-2013: 2M]**

A function $y = 5x^2 + 10x$ is defined over an open interval $x \in (1, 2)$. At least at one point in this interval, $\frac{dy}{dx}$ is exactly

(a) 20 (b) 25

(c) 30 (d) 35

107.[MCQ] **[GATE-EC-2013: 1M]**

The maximum value of θ until which the approximation $\sin\theta \approx \theta$ holds to within 10% error is

(a) 10° (b) 18°

(c) 50° (d) 90°

108.[MCQ] **[GATE-EC-2012:2M]**

The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(a) 21 (b) 25

(c) 41 (d) 46

109.[MCQ] **[GATE-CS-2012:1M]**

Consider the function $f(x) = \sin(x)$ in the interval $x = [\pi/4, 7\pi/4]$. The number and location(s) of the local minima of this function are

(a) One, at $\pi/2$ (b) One, at $3\pi/2$

(c) Two, at $\pi/2$ and $3\pi/2$ (d) Two, at $\pi/4$ and $3\pi/2$

110.[MCQ] **[GATE-ME-2012: 1M]**

At x = 0, the function $f(x) = x^3 + 1$ has

(a) A maximum value (b) A minimum value

(c) A singularity (d) A point of inflection

111.[MCQ] **[GATE-EE-2012: 2M]**

The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(a) 21 (b) 25

(c) 41 (d) 46

112.[MCQ] **[GATE-EE-2011: 2M]**

The function $f(x) = 2x - x^2 + 3$ has

(a) A maximum at x = 1 and a minima at x = 5

(b) A maxima at x =1 and a minima at x = −5

(c) Only a maxima at x = 1

(d) Only a minima at x = 1

113.[MCQ] **[GATE-CE-2011:2M]**

The infinite series

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$ correspond to

(a) sec x (b) cos x

(c) e^x (d) $1 + \sin^2 x$

114.[MCQ] **[GATE-ME-2011: 1M]**

A series expansion for the function sin θ is

(a) $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$ (b) $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$

(c) $1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \cdots$ (d) $\theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$

115.[MCQ] **[GATE-EE-2010 : 1M]**

At t = 0, the function $f(t) = \frac{\sin t}{t}$ has

(a) a minimum (b) a discontinuity

(c) a point of inflection (d) a maximum

116.[MCQ] **[GATE-CE-2010:2M]**

Given a function $f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$

Total optimal value of f(x, y)

(a) Is a minimum equal to 10/3

(b) Is a maximum equal to 10/3

(c) Is a minimum equal to 8/3

(d) Is a maximum equal to 8/3

117.[MCQ] [GATE-EC-2010:2M]

If $e^{y} = x^{1/x}$, then y has a

(a) maximum at x = e

(b) minimum at x = e

(c) maximum at $x = e^{-1}$

(d) minimum at $x = e^{-1}$

118.[MCQ] [GATE-EC-2009: 2M]

The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by

(a) $1+\frac{(x-\pi)^2}{3!}+\cdots$ (b) $-1-\frac{(x-\pi)^2}{3!}+\cdots$

(c) $1-\frac{(x-\pi)^2}{3!}+\cdots$ (d) $-1+\frac{(x-\pi)^2}{3!}+\cdots$

119.[MCQ] [GATE-ME-2009: 2M]

The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is

(a) 1 (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) 2

120.[MCQ] [GATE-EE-2009 : 1M]

The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by

(a) $1+\frac{(x-\pi)^2}{3!}+\cdots$ (b) $-1-\frac{(x-\pi)^2}{3!}\ldots$

(c) $1-\frac{(x-\pi)^2}{3!}+\cdots$ (d) $-1+\frac{(x-\pi)^2}{3!}+\ldots.$

121.[MCQ] [GATE-EE -2008 : 2M]

Consider function $f(x) = (x^2 - 4)^2$ where x is real number. Then the function has

(a) only one minima (b) only two minima

(c) three minima (d) three maxima

122.[MCQ] [GATE-EC-2008: 2M]

In the Taylor series expansion of exp(x) + sin(x) about the point $x = \pi$, the coefficient of $(x - \pi)^2$ is

(a) $\exp(\pi)$ (b) $0.5 \exp(\pi)$

(c) $\exp(\pi) + 1$ (d) $\exp(\pi) - 1$

123.[MCQ] [GATE-EC-2008:1M]

For real values of x, the minimum value of the function $f(x) = \exp(x) + \exp(-x)$ is

(a) 2 (b) 1

(c) 0.5 (d) 0

124.[MCQ] [GATE-EC-2008: 1M]

Which of the following functions would have only odd powers of x in its Taylor series expansion about the point x = 0?

(a) $\sin(x^3)$ (b) $\sin(x^2)$

(c) $\cos(x^3)$ (d) $\cos(x^2)$

125.[MCQ] [GATE-CS-2008:2M]

A point on a curve is said to be an extremum if it is a local minimum or a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is

(a) 0 (b) 1

(c) 2 (d) 3

126.[MCQ] [GATE-ME-2008: 1M]

In the Taylor series expansion of e^x about x = 2, the coefficient of $(x - 2)^4$ is

(a) $\frac{1}{4!}$ (b) $\frac{2^4}{4!}$

(c) $\frac{e^2}{4!}$ (d) $\frac{e^4}{4!}$

Integral Calculus

127.[MCQ] [GATE-CE-2023:1M]

For the integral

$$I = \int_{-1}^{1} \frac{1}{x^2} dx$$

which of the following statements is TRUE?

(a) I = 0
(b) I = 2
(c) I = –2
(d) The integral does not coverage

128.[MCQ] **[GATE-ME-2022: 1M]**

Given $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$. If a and b are positive integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$, is

(a) $\sqrt{\pi a}$
(b) $\sqrt{\frac{\pi}{a}}$
(c) $b\sqrt{\pi a}$
(d) $b\sqrt{\frac{\pi}{a}}$

129.[MCQ] **[GATE-EE-2022 :2M]**

Let $f(x) = \int_0^x e^t (t-1)(t-2)dt$. Then f(x) decreases in the interval

(a) $x \in (1,2)$
(b) $x \in (2,3)$
(c) $x \in (0,1)$
(d) $x \in (0.5,1)$

130.[NAT] **[GATE-CE-2021:1M]**

The value (round off to one decimal place) if $\int_{-1}^{1} xe^{|x|}dx$ is_________.

131.[MCQ] **[GATE-ME-2021: 2M]**

Let $f(x) = x^2 - 2x + 2$ be a contineous function defined on $x \in [1, 3]$. The point x at which the tangent of $f(x)$ becomes parallel to the straight line joining $f(1)$ and $f(3)$ is

(a) 0
(b) 1
(c) 2
(d) 3

132.[MSQ] **[GATE-EE-2020 : 1M]**

Which of the following is true for all possible non-zero choices of integers m, n; m≠n, or all possible non-zero choices of real numbers p, q;p≠q, as applicable?

(a) $\frac{1}{\pi}\int_0^{\pi} \sin m\theta \sin n\theta d\theta = 0$
(b) $\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \sin p\theta \sin q\theta d\theta = 0$
(c) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin p\theta \cos q\theta \, d\theta$
(d) $\lim_{\alpha\to\infty} \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \sin p\theta \sin q\theta d\theta = 0$

133.[MCQ] **[GATE-EC-2019: 2M]**

Consider a differentiable function $f(x)$ on the set of real numbers such that $f(-1) = 0$ and $|f'(x)| \le 2$. Given these conditions, which one of the following inequalities is necessarily true for all $x \in [-2, 2]$?

(a) $f(x) \le 2|x|$
(b) $f(x) \le 2|x + 1|$
(c) $f(x) \le \frac{1}{2}|x|$
(d) $f(x) \le \frac{1}{2}|x+1|$

134.[NAT] **[GATE-ME-2019: 2M]**

The value of the following definite integral is____(round off to three decimal places) $\int_1^e (x \ln x)dx$

135.[NAT] **[GATE-EE-2018 :1M]**

Let f be a real-valued function of a real variable defined as f(x) = x–[x] where [x] denotes the largest less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)dx$ is _________(up to 2 decimal places) .

136.[MCQ] **[GATE-ME-2018: 1M]**

According to the Mean Value Theorem, for a continuous function f(x) in the interval [a, b] there exists a value ξ in this interval such that $\int_a^b f(x)dx =$

(a) $f(\xi)(b - a)$
(b) $f(b)(\xi - a)$
(c) $f(a)(b - \xi)$
(d) 0

137.[NAT] **[GATE-CS-2018: 1M]**

The values of $\int_0^{\frac{\pi}{2}} x\cos(x^2)dx$ Correct to three decimal places (Assuming that π = 3.14) is _____.

138.[MCQ] **[GATE-CS-2017:1M]**

If f(x) = $R\sin\left(\frac{\pi x}{2}\right)+S, f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)dx=\frac{2R}{\pi}$ then the constants R and S are, respectively

(a) $\frac{2}{\pi}$ and $\frac{16}{\pi}$
(b) $\frac{2}{\pi}$ and 0
(c) $\frac{4}{\pi}$ and 0
(d) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

139.[MCQ] **[GATE-CE-2017:2M]**

Consider the following definite integral:

$$I=\int_0^1\frac{\left(\sin^{-1}x\right)^2}{\sqrt{1-x^2}}dx$$

The value of the integral is

(a) $\frac{\pi^3}{24}$
(b) $\frac{\pi^3}{12}$
(c) $\frac{\pi^3}{48}$
(d) $\frac{\pi^3}{64}$

140.[MCQ] **[GATE-CE-2017:1M]**

Let x be a continuous variable defined over the interval $(-\infty,\infty)$ and f(x)= $e^{-x-e^{-x}}$. The integral g(x)=∫f(x)dx is equal to

(a) $e^{e^{-x}}$
(b) $e^{-e^{-x}}$
(c) $e^{-e^{x}}$
(d) e^{-x}

141.[MCQ] **[GATE-ME-2017: 2M]**

A parametric curve defined by x = $\cos\left(\frac{\pi u}{2}\right)$, y = $\sin\left(\frac{\pi u}{2}\right)$ in the range $0\le u\le 1$ is rotated about the X-axis by 360 degrees. Area of the surface generated is

(a) $\frac{\pi}{2}$
(b) π
(c) 2π
(d) 4π

142.[NAT] **[GATE-CS-2015: 2M]**

$$\int_{\pi}^{2/\pi}\frac{\cos\left(\frac{1}{x}\right)}{x^2}dx = \_\_\_\_\_\_$$

143.[MCQ] **[GATE-CS-2015:2M]**

If for non-zero x, $a\cdot f(x)+b\cdot f\left(\frac{1}{x}\right)=\frac{1}{x}-25$ where $a\neq b$ then $\int_1^2 f(x)dx$

(a) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)+\frac{47b}{2}\right]$
(b) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)-\frac{47b}{2}\right]$
(c) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)+\frac{47b}{2}\right]$
(d) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)-\frac{47b}{2}\right]$

144.[NAT] **[GATE-CS-2014:1M]**

If $\int_0^{2x}|x\cdot\sin x|dx=k\pi$ then the value of k is equal to ______.

145.[MCQ] **[GATE-CS-2014:2M]**

The value of the integral given below is $\int_0^{\pi}x^2\cos x\,dx$

(a) -2π
(b) π
(c) $-\pi$
(d) 2π

146.[NAT] **[GATE-CE-2014:2M]**

The value determined from

$\iiint 8xyz\,dV$ for $V=[2,3]\times[1,2]\times[0,1]$ will be (in integer)

147.[MCQ] **[GATE-ME-2014: 1M]**

The value of the integral $\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2+\cos(x-1)}dx$ is

(a) 3 (b) 0
(c) –1 (d) –2

148.[MCQ] **[GATE-ME-2013: 2M]**

The value of the definite integral $\int_1^e \sqrt{x}\ln(x)dx$

(a) $\frac{4}{9}\sqrt{e^3}+\frac{2}{9}$ (b) $\frac{2}{9}\sqrt{e^3}-\frac{4}{9}$
(c) $\frac{2}{9}\sqrt{e^3}+\frac{4}{9}$ (d) $\frac{4}{9}\sqrt{e^3}-\frac{2}{9}$

149.[MCQ] **[GATE-CS-2011:2M]**

Given $i=\sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}dx$

(a) 0 (b) 2
(c) –i (d) i

150.[MCQ] **[GATE-CE-2011:2M]**

The value of $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta d\theta$

(a) 0 (b) 1
(c) $\frac{1}{15}$ (d) $\frac{8}{3}$

151.[MCQ] **[GATE-CE-2011:2M]**

What is the value of the definite integral

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

(a) 0 (b) a
(c) a/2 (d) 2a

152.[MCQ] **[GATE-CE-2011:2M]**

$\int\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+....\right)dx$ is equal to

(a) $-\frac{1}{1-x^2}+\text{constant}$ (b) $-\frac{1}{1-x}+\text{constant}$
(c) $\frac{1}{1+x^2}+\text{constant}$ (d) $\frac{1}{1+x}+\text{constant}$

153.[MCQ] **[GATE-ME-2011: 1M]**

If f(x) is an even function and a is a positive real number, then $\int_{-a}^{a} f(x)dx$ equals

(a) 0 (b) a
(c) 2a (d) $2\int_0^a f(x)dx$

154.[MCQ] **[GATE-EE-2010: 1M]**

The value of the quantity P, where $P=\int_0^1 xe^x dx$, is equal to

(a) 0 (b) 1
(c) e (d) 1/e

155.[MCQ] **[GATE-ME-2010: 1M]**

The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is

(a) $-\pi$ (b) $-\pi/2$
(c) $\pi/2$ (d) π

156.[MCQ] **[GATE-CS-2009:2M]**

$\int_0^{\pi/4} (1-\tan x)/(1+\tan x)dx$ evaluates to

(a) 0 (b) 1
(c) ln 2 (d) 1/2 ln 2

157.[MCQ] **[GATE-CE-2008:2M]**

The value of $\int_0^3\int_0^x (6-x-y)dx\,dy$ is

(a) 13.5 (b) 27.90
(c) 40.5 (d) 54.0

158.[MCQ] **[GATE-ME-2008: 2M]**

Which of the following integrals is unbounded?

(a) $\int_0^{\pi/4} \tan x\, dx$ (b) $\int_0^{\infty} \frac{1}{x^2+1} dx$

(c) $\int_0^{\infty} x e^{-x} dx$ (d) $\int_0^{1} \frac{1}{1-x} dx$

Applications of Integration

159.[MSQ] **[GATE-EE-2023: 2M]**

Consider the following equation in a 2-D real-space

$|x_1|^p + |x_2|^p = 1$ for $p > 0$

Which of the following statement(s) is/are true.

(a) When p = 2, the area enclosed by the curve is π

(b) When p tends to ∞, the area enclosed by the curve tends to 4.

(c) When p tends to 0, the area enclosed by the curve is 1.

(d) When p = 1, the area enclosed by the curve is 2.

160.[NAT] **[GATE-ME-2023: 1M]**

A linear transformation maps a point (x, y) in the plane to the point $(\hat{x}, \hat{y})$ according to the rule

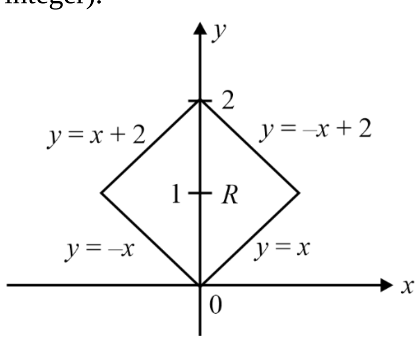
$$\hat{x} = 3y, \hat{y} = 2x .$$

Then, the disc $x^2 + y^2 < 1$ gets transformed to a region with an area equal to _____. (Rounded off to two decimals)

Use $\pi = 3.14$

161.[NAT] **[GATE-EC-2023 : 2M]**

The value of the integral $\iint_R xy\, dx\, dy$ over the region R, given in the figure, is _______ (rounded off to the nearest integer).

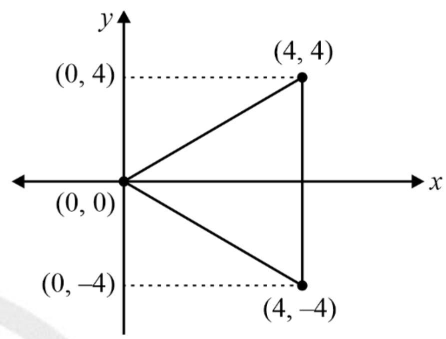


162.[NAT] **[GATE-EC-2022 : 2M]**

The value of the integral,

$$\iint_D 3(x^2 + y^2)\, dx\, dy,$$

where D is the shaded triangular region shown in the diagram, is _____ (rounded off to the nearest integer).



163.[MCQ] **[GATE-CE-2020:1M]**

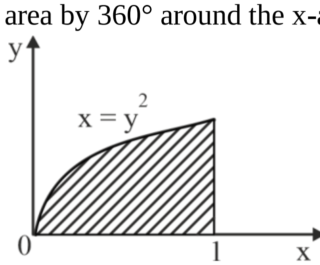
The area of an ellipse represented by an equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $\frac{4\pi ab}{3}$ (b) πab

(c) $\frac{\pi ab}{2}$ (d) $\frac{\pi ab}{4}$

164.[MCQ] **[GATE-ME-2019: 1M]**

A parabola $x = y^2$ with $0 \le x \le 1$ is shown in the figure. The volume of the solid of rotation obtained by rotating the shaded area by 360° around the x-axis is



(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) 2π

165.[MCQ] [GATE-CE-2018:1M]

The value of the integral $\int_0^{\pi} x\cos^2 x dx$

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{2}$ (d) π^2

166.[NAT] [GATE-CE-2016:2M]

The area between the parabola $x^2 = 8y$ and the straight-line $y = 8$ is________.

167.[MCQ] [GATE-CE-2016:2M]

The area of the region bounded by the parabola $y = x^2 + 1$ and the straight-line $x + y = 3$ is

(a) $\frac{59}{6}$ (b) $\frac{9}{2}$

(c) $\frac{10}{3}$ (d) $\frac{7}{6}$

168.[NAT] [GATE-ME-2015: 2M]

Consider a spatial curve in three-dimensional space given in parametric form by

$$x(t) = \cos t,\ y(t) = \sin t,\ z(t) = \frac{2}{\pi}t; \quad 0 \le t \le \frac{\pi}{2}.$$

The length of the curve is_______.

169.[MCQ] [GATE-ME-2012: 1M]

The area enclosed between the starigth-line $y = x$ and the parabola $y = x^2$ in the $x - y$ plane is

(a) 1/6 (b) 1/4

(c) 1/3 (d) 1/2

170.[MCQ] [GATE-2010-CE:2M]

The parabolic cable is held between two supports at the same level. The horizontal span between the supports is L. The sag at the mid-span is h. The equation of the parabola is $y = 4h\frac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is

(a) $\int_0^L \sqrt{1 + 64\frac{h^2x^2}{L^4}}dx$

(b) $\int_0^L \sqrt{1 + 64\frac{h^2x^2}{L^4}}dx$

(c) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^3x^2}{L^4}}dx$

(d) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}dx$

171.[MCQ] [GATE-ME-2010: 1M]

The parabolic arc $y = \sqrt{x}, 1 \le x \le 2$ is revolved around the x-axis. The volume of the solid revolution is

(a) $\pi/4$ (b) $\pi/2$

(c) $3\pi/4$ (d) $3\pi/2$

172.[MCQ] [GATE-ME-2009: 2M]

The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

(a) $\frac{16}{3}$ (b) 8

(c) $\frac{32}{3}$ (d) 16

173.[MCQ] [GATE-ME-2008: 2M]

The length of curve $y = \frac{2}{3}x^{3/2}$ between $x = 0$ and $x = 1$ is

(a) 0.27 (b) 0.67

(c) 1 (d) 1.22

Multi Variable Calculus

174.[NAT] [GATE-CS-2023:1M]

The value of the definite integral $\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}\left(4x^2y - z^3\right)dzdydx$ is__________ . (Rounded off to the nearest integer)

175.[MCQ] **[GATE-ME-2021: 2M]**

The value of $\int_0^{\pi/2}\int_0^{\cos\theta} r\sin\theta\, dr\, d\theta$ is

(a) 0 (b) $\frac{1}{6}$

(c) $\frac{4}{3}$ (d) π

176.[MCQ] **[GATE-EC-2020: 1M]**

The partial derivative of the function

$$f(x, y, z) = e^{1-x\cos y} + x z e^{-1/(1+y^2)}$$

with respect to x at the point (1, 0, e) is

(a) –1 (b) 0

(c) 1 (d) $\frac{1}{e}$

177.[MCQ] **[GATE-ME-2020: 1M]**

Let $I = \int_{x=0}^{1}\int_{y=0}^{x^2} xy^2\, dy\, dx$. Then, I may also be expressed as

(a) $\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} xy^2\, dx\, dy$ (b) $\int_{y=0}^{1}\int_{x=\sqrt{y}}^{0} yx^2 dx\, dy$

(c) $\int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} xy^2 dx\, dy$ (d) $\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} yx^2 dx\, dy$

178.[MCQ] **[GATE-EC-2018: 1M]**

Let, $f(x, y) = \frac{ax^2 + by^2}{xy}$, where a and b ae constants.

If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ at x = 1 and y = 2, then the relation between a and b is

(a) $a = \frac{b}{4}$ (b) $a = \frac{b}{2}$

(c) a = 2b (d) a = 4b

179.[NAT] **[GATE-EC-2018:2M]**

Let $r = x^2 + y - z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At (x,y,z) = (2, −1, 1), , the value (correct to two decimal places) of $\frac{\partial r}{\partial x}$ is

180.[NAT] **[GATE-EE-2017: 1M]**

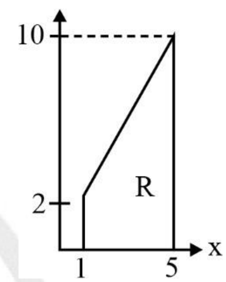
Consider a function f (x, y, z) given by

$f(x, y, z) = (x^2 + y^2 - 2z^2)(y^2 + z^2)$

The partial derivative of this function with respect x at the point x = 2, y = 1 z = 3 is ________.

181.[NAT] **[GATE-EE-2017: 1M]**

Let $I = c\int\int_R xy^2.dx.dy$ where R is the region shown in the figure and $c = 6 \times 10^{-4}$. The value of I equals ____. (Give the answer up to two decimal places).



182.[NAT] **[GATE-EC-2017:2M]**

A three-dimensional region R of finite volume is described by $x^2 + y^2 \le z^3; 0 \le z \le 1$. Where x, y, z are real. The volume of R (up to two decimal place) is ______

183.[NAT] **[GATE-EC-2016: 2M]**

The region specified by $\{(\rho, \phi, z): 3 \le \rho \le 5, \frac{\pi}{8} \le \phi \le \frac{\pi}{4}, 3 \le z \le 4.5\}$ in cylindrical coordinates has volume of ____.

184.[NAT] **[GATE-EC-2016: 2M]**

A triangle in the xy-plane is bounded by the straight lines 2x = 3y, y = 0 and x = 3. The volume above the triangle and under the plane x + y + z = 6 is ______________ .

185.[NAT] **[GATE-EE-2015: 2M]**

The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by the lines x = y; x = 0; y = 1 in the xy plane is _______.

186.[MCQ] [GATE-EC-2015: 1M]

The contour on the x-y plane, where the partial derivative of $x^2 + y^2$ with respect to y is equal to the partial derivative of 6y + 4x with respect to x, is

(a) y = 2 (b) x = 2

(c) x + y = 4 (d) x – y = 0

187.[MCQ] [GATE-EE-2014: 2M]

To evaluate the double integral

$\int_0^8\left(\int_{y/2}^{(y/2)+1}\left(\frac{2x-y}{2}\right)dx\right)dy$, we make the substitution

$u=\left(\frac{2x-y}{2}\right)$ and $v=\frac{y}{2}$. The integral will reduce to

(a) j (b) $\int_0^4\left(\int_0^1 2u\,du\right)dv$

(c) $\int_0^4\left(\int_0^1 u\,du\right)dv$ (d) $\int_0^4\left(\int_0^2 u\,du\right)dv$

188.[NAT] [GATE-EC-2014: 2M]

The volume under the surface z(x,y) = x + y and above the triangle in the x – y plane defined by {0 ≤ y ≤ x and 0 ≤ x ≤ 12} is ______

189.[MCQ] [GATE-ME-2014: 2M]

The value of the integral $\int_0^2\int_0^x e^{x+y}\cdot dy\cdot dx$ is

(a) $\frac{1}{2}(e-1)$ (b) $\frac{1}{2}(e^2-1)^2$

(c) $\frac{1}{2}(e^2-e)$ (d) $\frac{1}{2}\left(e-\frac{1}{e}\right)^2$

190.[MCQ] [GATE-EE-2009: 2M]

F(x, y) is a continuous function defined over (x, y) ϵ [0, 1] × [0, 1]. Given the two constraints, x > y^2 and y > x^2, the volume under f(x, y) is

(a) $\int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} f(x,y)\,dx\,dy$

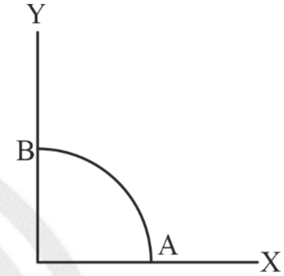
(b) $\int_{y=x^2}^{y=1}\int_{x=y^2}^{x=1} f(x,y)\,dx\,dy$

(c) $\int_{y=0}^{y=1}\int_{x=0}^{x=1} f(x,y)\,dx\,dy$

(d) $\int_{y=0}^{y=\sqrt{x}}\int_{x=0}^{x=\sqrt{y}} f(x,y)\,dx\,dy$

191.[MCQ] [GATE-ME-2009: 2M]

A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x + y)^2$ on path of AB traversed in a counter-clockwise sense

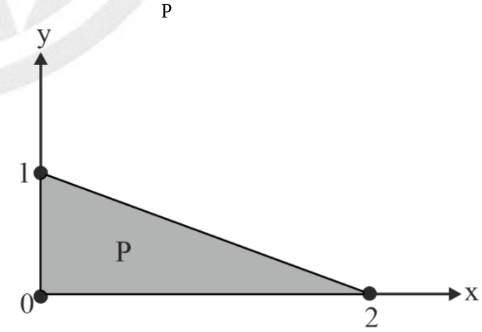


(a) $\frac{\pi}{2}-1$ (b) $\frac{\pi}{2}+1$

(c) $\frac{\pi}{2}$ (d) 1

192.[MCQ] [GATE-ME-2008: 2M]

Consider the shaded triangular region P shown in the figure. Where is $\iint_P xy\,dx\,dy$?



(a) $\frac{1}{6}$ (b) $\frac{2}{9}$

(c) $\frac{7}{16}$ (d) 1

❑❑❑

ANSWER KEY

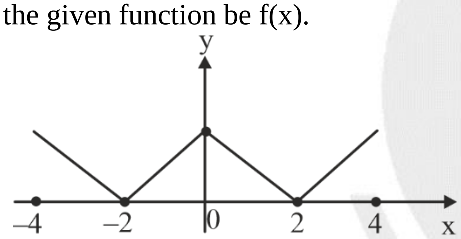
1.	(b)	**2.**	(a)	**3.**	(–0.5)	**4.**	(0.25)
5.	(c)	**6.**	(d)	**7.**	(0.5)	**8.**	(b)
9.	(d)	**10.**	(c)	**11.**	(c)	**12.**	(a)
13.	(c)	**14.**	(d)	**15.**	(d)	**16.**	(b)
17.	(d)	**18.**	(a)	**19.**	(b)	**20.**	(c)
21.	(–1)	**22.**	(c)	**23.**	(d)	**24.**	(54 to 55)
25.	(b)	**26.**	(1)	**27.**	(c)	**28.**	(c)
29.	(c)	**30.**	(0.28 to 0.31)	**31.**	(c)	**32.**	(d)
33.	(c)	**34.**	(–0.35 to –0.30)	**35.**	(1.9 to 2.1)	**36.**	(c)
37.	(a)	**38.**	(c)	**39.**	(–2)	**40.**	(b)
41.	(a)	**42.**	(c)	**43.**	(a)	**44.**	(b)
45.	(d)	**46.**	(a)	**47.**	(c)	**48.**	(c)
49.	(b)	**50.**	(d)	**51.**	(c)	**52.**	(b)
53.	(a)	**54.**	(a)	**55.**	(b)	**56.**	(0.50)
57.	(b)	**58.**	(b)	**59.**	(d)	**60.**	(a)
61.	(7.999 to 8.001)	**62.**	(d)	**63.**	(a, b)	**64.**	(b)
65.	(19)	**66.**	(c)	**67.**	(1)	**68.**	(b)
69.	(a, d)	**70.**	(a)	**71.**	(a)	**72.**	(5.18 to 5.2)
73.	(c)	**74.**	(a)	**75.**	(c)	**76.**	(d)
77.	(11.5 to 12.5)	**78.**	(7)	**79.**	(a)	**80.**	(c)
81.	(–100)	**82.**	(c)	**83.**	(0 to 0)	**84.**	(b)
85.	(b)	**86.**	(d)	**87.**	(d)	**88.**	(–5.1 to –4.9)
89.	(2)	**90.**	(1)	**91.**	(b)	**92.**	(b)
93.	(c)	**94.**	(d)	**95.**	(a)	**96.**	(a)
97.	(6)	**98.**	(d)	**99.**	(a)	**100.**	(c)
101.	(–1.00 to –0.94)	**102.**	(2)	**103.**	(b)	**104.**	(c)
105.	(a)	**106.**	(b)	**107.**	(c)	**108.**	(c)
109.	(b)	**110.**	(d)	**111.**	(c)	**112.**	(c)
113.	(c)	**114.**	(b)	**115**	(d)	**116.**	(a)
117.	(a)	**118.**	(c)	**119.**	(a)	**120.**	(d)
121.	(b)	**122.**	(b)	**123.**	(a)	**124.**	(a)
125.	(b)	**126.**	(c)	**127.**	(d)	**128.**	(b)
129.	(a)	**130.**	(0)	**131.**	(c)	**132.**	(a, c, d)
133.	(b)	**134.**	(2.090 to 2.104)	**135.**	(0.50)	**136.**	(a)
137.	(0.27 to 0.3)	**138.**	(c)	**139.**	(a)	**140.**	(b)

141.	(c)	**142.**	(–1)	**143.**	(a)	**144.**	(4)
145.	(a)	**145.**	(15)	**147.**	(b)	**148.**	(c)
149.	(d)	**150.**	(c)	**151.**	(c)	**152.**	(*)
153.	(b)	**154.**	(b)	**155.**	(d)	**156.**	(d)
157.	(a)	**158.**	(d)	**159.**	(a, b, d)	**160.**	(18.80 to 18.90)
161.	(0)	**162.**	(171)	**163.**	(b)	**164.**	(b)
165.	(b)	**166.**	(85 to 85.5)	**167.**	(b)	**168.**	(1.85 to 1.87)
169.	(a)	**170.**	(d)	**171.**	(d)	**172.**	(a)
173.	(d)	**174.**	(0)	**175.**	(b)	**176.**	(b)
177.	(c)	**178.**	(d)	**179.**	(4.4 to 2.6)	**180.**	(40)
181.	(1.00)	**182.**	(0.7 to 0.85)	**183.**	(4 to 5)	**184.**	(10)
185.	(0.7 to 0.76)	**186.**	(a)	**187.**	(b)	**188.**	(864)
189.	(b)	**190.**	(a)	**191.**	(b)	**192.**	(a)

SOLUTIONS

1. (b)

Let the given function be f(x).



(i) f(x) is symmetric about y-axis.

$\Rightarrow \quad f(-x) = -f(x)$

(ii) $f(x) > 0 \ \forall \ x \in [-4, 4]$

(iii) At $x = 2$, $f(x) = 0$

(iv) $0 \le f(x) \le 2 \ \forall \ x \in [-4, 4]$

The only function that satisfies the above conditions is option B.

$\Rightarrow \quad f(x) = |2 - |x||$

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2. (a)

Given limit is

$$P = \lim_{x\to\pi}\left(\frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x}\right)$$

$$P = \lim_{x\to\pi}\frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x} = \frac{3\pi^2 + \alpha\pi}{0}$$

But for ‘P’ has to be finite,

$$3\pi^2 + \alpha\pi = 0 \ \boxed{\Rightarrow \alpha = -3\pi}$$

If $\alpha = -3\pi$,

$$P = \lim_{x\to\pi}\frac{x^2 - 3\pi x + 2\pi^2}{x - \pi + 2\sin x} \qquad \left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\dfrac{0}{0}(\text{or})\dfrac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to\pi}\frac{\frac{d}{dx}\left(x^2 - 3\pi x + 2\pi^2\right)}{\frac{d}{dx}\left(x - \pi + 2\sin x\right)}$$

$$P = \lim_{x\to\pi}\frac{2x - 3\pi}{1 + 2\cos x} = \frac{-\pi}{-1} = \pi$$

$\therefore \quad \boxed{\alpha = -3\pi \text{ and } P = \pi}$

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3. **(–0.5)**

Given limit is

$$P = \lim_{x\to 0^+} \frac{\sqrt{x}}{1-e^{2\sqrt{x}}} = \frac{\sqrt{0}}{1-e^{2\left(\sqrt{0}\right)}} = \frac{0}{1-1} = \left(\frac{0}{0}\right)$$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)} \end{array}\right]$$

$$P = \lim_{x\to 0^+} \frac{\frac{d}{dx}\left(\sqrt{x}\right)}{\frac{d}{dx}\left(1-e^{2\sqrt{x}}\right)} = \lim_{x\to 0^+} \frac{\frac{1}{\cancel{2\sqrt{x}}}}{\left(0-e^{2\sqrt{x}}\,2\,\frac{1}{\cancel{2\sqrt{x}}}\right)}$$

$$P = \frac{1}{-2} = -0.5$$

$$\therefore \boxed{P = \lim_{x\to 0^+} \frac{\sqrt{x}}{1-e^{2\sqrt{x}}} = -0.5}$$

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4. **(0.25)**

Given limit is $P = \lim\limits_{x\to -3} \frac{\sqrt{2x+22-4}}{x+3} = \left(\frac{0}{0}\right)$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)} \end{array}\right]$$

$$P = \lim_{x\to -3} \frac{\frac{d}{dx}\sqrt{2x+22-4}}{\frac{d}{dx}(x+3)} = \lim_{x\to -3} \frac{\frac{1}{\cancel{2}\sqrt{2x+22}}\cdot\cancel{2}-0}{1}$$

$$P = \lim_{x\to -3} \frac{1}{\sqrt{2x+22}}$$

$$P = \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$$

$$\therefore \boxed{\lim_{x\to -3} \frac{\sqrt{2x+22}-4}{x+3} = 0.25}$$

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5. **(c)**

Given $P = \lim\limits_{x\to 0} \frac{1-\cos x}{x^2}$

$$P = \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1-1}{0} = \left(\frac{0}{0}\right)$$

$$\left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)} \end{array}\right]$$

$$P = \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}\left(x^2\right)} = \lim_{x\to 0} \frac{\sin x}{2x}$$

$$P = \frac{1}{2} \times \lim_{x\to 0} \frac{\sin x}{x}$$

$$\left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$P = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$\therefore \boxed{\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}}$$

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6. **(d)**

$$P = \lim_{x\to\infty} \frac{x\ln x}{1+x^2} \qquad \left(\frac{\infty}{\infty}\text{form}\right)$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to\infty}\frac{\frac{d}{dx}(x\ln x)}{\frac{d}{dx}\left(1+x^2\right)} = \lim_{x\to\infty}\frac{x.\frac{1}{x}+\ln x.1}{2x}$$

$$P = \lim_{x\to\infty}\frac{1+\ln x}{2x}$$

$$\left(\frac{\infty}{\infty}\text{form, apply L Hospital's Rule}\right)$$

$$P = \lim_{x\to\infty}\frac{\frac{d}{dx}(1+\ln x)}{\frac{d}{dx}(2x)} = \lim_{x\to\infty}\frac{\frac{1}{x}}{2} = \frac{0}{2} = 0$$

$$\therefore \quad \boxed{\lim_{x\to\infty}\frac{x\ln x}{1+x^2} = 0}$$

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7. (0.5)

$$P = \lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$$

$$P = \lim_{x\to 1}\frac{(x-1)-\ln x}{(x-1)\ln x} \qquad \text{(0/0 form)}$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to 0}\frac{\frac{d}{dx}((x-1)-\ln x)}{\frac{d}{dx}((x-1)\ln x)}$$

$$P = \lim_{x\to 1}\frac{1-\frac{1}{x}}{(x-1)\frac{1}{x}+\ln x} = \left(\frac{0}{0}\right)\text{form}$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to 1}\frac{\frac{d}{dx}\left(1-\frac{1}{x}\right)}{\frac{d}{dx}\left\{(x-1)\frac{1}{x}+\ln x\right\}} = \lim_{x\to 1}\frac{\frac{d}{dx}\left(1-\frac{1}{x}\right)}{\frac{d}{dx}\left\{1-\frac{1}{x}+\ln x\right\}}$$

$$P = \lim_{x\to 1}\frac{-\left(-\frac{1}{x^2}\right)}{-\left(-\frac{1}{x^2}\right)+\frac{1}{x}} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

$$\therefore \quad \boxed{\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = 0.5}$$

Scan for Video solution

8. (b)

$$P = \lim_{x\to\infty}\frac{\sqrt{9x^2+2020}}{x+7} = \lim_{x\to\infty}\frac{3x\sqrt{1+\frac{2020}{9x^2}}}{x\left(1+\frac{7}{x}\right)}$$

$$P = \lim_{x\to\infty}\frac{3\sqrt{1+\frac{2020}{9x^2}}}{\left(1+\frac{7}{x}\right)} = \lim_{x\to\infty}\frac{3\sqrt{1+0}}{(1+0)} = 3$$

$$\therefore \quad \boxed{\lim_{x\to\infty}\frac{\sqrt{9x^2+2020}}{x+7} = 3}$$

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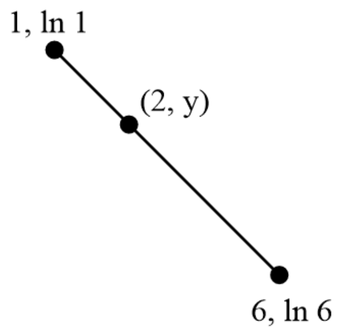
9. (d)

True value of ln 2 = 0.69

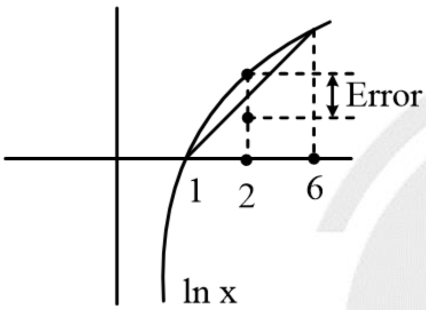
$x_1 = 1 \qquad y_1 = \ln 1$

$x_2 = 6$ $\qquad$ $y_2 = \ln 6$



$$\frac{y-\ln 1}{2-1} = \frac{\ln 6.0 - \ln 1}{6-1}$$

y = 0.358 (Approximate value of ln 2)



Hence % error =

$$\frac{\text{True value- Approximate value}}{\text{Truevalue}} \times 100\%$$

$$= \frac{0.69-0.358}{0.69} \times 100 = 48.11\%$$

= 48%

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10. (c)

Method I:

$$P = \lim_{x\to\infty} \frac{x^2-5x+4}{4x^2+2x} = \left(\frac{\infty}{\infty}\text{form}\right)$$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)} \end{array}\right]$$

$$P = \lim_{x\to\infty} \frac{\frac{d}{dx}\left(x^2-5x+4\right)}{\frac{d}{dx}\left(4x^2+2x\right)} = \lim_{x\to\infty}\frac{2x-5}{8x+2} = \left(\frac{\infty}{\infty}\text{form}\right)$$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)} \end{array}\right]p$$

$$P = \lim_{x\to\infty} \frac{\frac{d}{dx}(2x-5)}{\frac{d}{dx}(8x+2)} = \lim_{x\to\infty}\frac{2}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\therefore \boxed{\lim_{x\to\infty}\frac{x^2-5x+4}{4x^2+2x} = \frac{1}{4}}$$

Method II:

$$P = \lim_{x\to\infty} \frac{x^2-5x+4}{4x^2+2x}$$

$$P = \lim_{x\to\infty} \frac{x^2\left(1-\frac{5}{x}+\frac{4}{x^2}\right)}{x^2\left(4+\frac{2}{x}\right)} = \lim_{x\to\infty}\frac{\left(1-\frac{5}{x}+\frac{4}{x^2}\right)}{\left(4+\frac{2}{x}\right)}$$

$$P = \frac{(1-0+0)}{(4+0)} = \frac{1}{4}$$

$$\therefore \boxed{\lim_{x\to\infty}\frac{x^2-5x+4}{4x^2+2x} = \frac{1}{4}}$$

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11. (c)

Given $P = \lim\limits_{x\to 1}\left(\frac{1-e^{-c(1-x)}}{1-xe^{-c(1-x)}}\right)$

$$P = \lim_{x\to 1}\frac{1-e^{-c(1-x)}}{1-x\cdot e^{-c(1-x)}} = \frac{1-e^{-0}}{1-1\cdot e^{-0}} = \left(\frac{0}{0}\right)$$

$$\left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P=\lim_{x\to 1}\frac{\frac{d}{dx}\left(1-e^{-c(1-x)}\right)}{\frac{d}{dx}\left(1-x\cdot e^{-c(1-x)}\right)}$$

$$P=\lim_{x\to 1}\frac{0-e^{-c(1-x)}\cdot c}{0-\left\{x\cdot c\cdot e^{-c(1-x)}+e^{-c(1-x)}\cdot 1\right\}}$$

$$P=\lim_{x\to 1}\frac{-c\cdot e^{-c(1-x)}}{-e^{-c(1-x)}\{1+cx\}}$$

$$P=\lim_{x\to 1}\frac{c}{1+cx}$$

$$\boxed{\lim_{x\to 1}\frac{1-e^{-c(1-x)}}{1-x\cdot e^{-c(1-x)}}=\frac{c}{1+c}}$$

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12. (a)

Given; $x=\psi\ln\phi$ ...(i)

and $y=\phi\ln\psi$ ...(ii)

Partially differentially equation (i) w.r.t. x

$$1=\psi_x\ln\phi+\psi.\frac{\phi_x}{\phi}\quad\text{...(iii)}$$

Partially differentially equation (ii) w.r.t. x

$$0=\phi_x\ln\psi+\phi\frac{1}{\psi}.\psi_x$$

$$\frac{\psi\phi_x}{\phi}=-\frac{\psi_x}{\ln\psi}\quad\text{...(iv)}$$

Put this term in (iii)

$$1=\psi_x\ln\phi-\frac{\psi_x}{\ln\psi}$$

$$1=\psi_x\left[\ln\phi-\frac{1}{\ln\psi}\right]$$

$$1=\psi_x\left[\frac{\ln\phi\ln\psi-1}{\ln\psi}\right]$$

$$\psi_x=\frac{\ln\psi}{\ln\phi\ln\psi-1}$$

$$\boxed{\frac{\partial\psi}{\partial x}=\frac{\ln\psi}{\ln\phi\ln\psi-1}}$$

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13. (c)

Given limits is

$$P=\lim_{x\to -3}\frac{x^4-81}{2x^2-5x-3}=\frac{81-81}{18-15-3}=\left(\frac{0}{0}\right)$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P=\lim_{x\to -3}\frac{\frac{d}{dx}\left(x^4-81\right)}{\frac{d}{dx}\left(2x^2-5x-3\right)}$$

$$P=\lim_{x\to -3}\frac{4x^3}{4x-5}=\frac{4(3)^3}{4(3)-5}=\frac{4\times 27}{7}=\frac{108}{7}$$

$$\therefore\ \boxed{\lim_{x\to -3}\frac{x^4-81}{2x^2-5x-3}=\frac{108}{7}}$$

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14. (d)

Method I:

$$P=\lim_{x\to 0}\frac{x\sin x}{1-\cos x}$$

$$P=\lim_{x\to 0}\frac{\sin x}{\frac{1-\cos x}{x}}$$

$$P=\lim_{x\to 0}\frac{\frac{\sin x}{x}}{\frac{1-\cos x}{x^2}}$$

$$\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and} \because \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}\right]$$

$$P=\frac{1}{1/2}=2$$

Method II:

$$P=\lim_{x\to 0}\frac{x\sin x}{1-\cos x}=\left(\frac{0}{0}\right)\text{form}$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P=\lim_{x\to 0}\frac{\frac{d}{dx}(x\sin x)}{\frac{d}{dx}(1-\cos x)}=\lim_{x\to 0}\frac{x\cos x+\sin x}{\sin x}=\left(\frac{0}{0}\right)\text{form}$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P=\lim_{x\to 0}\frac{\frac{d}{dx}(x\cos x+\sin x)}{\frac{d}{dx}(\sin x)}$$

$$P=\lim_{x\to 0}\frac{-x\sin x+\cos x+\cos x}{\cos x}=\frac{0+1+1}{1}=2$$

$$\therefore \boxed{\lim_{x\to 0}\frac{x\sin x}{1-\cos x}=2}$$

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15. (d)

Let, $P_1=\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$

Method I:

$$P_1=\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=\left(\frac{0}{0}\right)\text{Form}$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P_1=\lim_{x\to 0}\frac{\frac{d}{dx}\sin 4x}{\frac{d}{dx}\sin 2x}=\frac{4\cos 4x}{2\cos 2x}$$

$$P_1=\frac{4}{2}=2$$

Method II:

$$P_1=\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$$

$$P_1=\lim_{x\to 0}\frac{4x.\left(\frac{\sin 4x}{4x}\right)}{2x.\left(\frac{\sin 2x}{2x}\right)} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$P_1=2.\frac{1}{1}=2$$

Let, $P_2=\lim_{x\to 0}\frac{\tan x}{x}$

$$P_2=\lim_{x\to 0}\frac{\tan x}{x}=\left(\frac{0}{0}\right)\text{ form}$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P_2=\lim_{x\to 0}\frac{\frac{d}{dx}\tan x}{\frac{d}{dx}x}=\lim_{x\to 0}\frac{\sec^2 x}{1}$$

$$P_2=\lim_{x\to 0}\frac{\sec^2 0}{1}=1$$

$\lim_{x\to 0}\frac{\tan x}{x}=1\leftarrow$ (This expression we can remember also)

$$\therefore \boxed{\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=2\ \&\ \lim_{x\to 0}\frac{\tan x}{x}=1}$$

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16. (b)

Given

f(x) = cos x …(i)

Approximate value of $f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

At $x = \dfrac{\pi}{6}$ and h = 0.1:

Approximate value of $f'(x) = \dfrac{\left[\begin{array}{l}\cos\left(\pi/6 + 0.1\right) \\ -\cos\left(\pi/6 - 0.1\right)\end{array}\right]}{2(0.1)}$

Approximate value of $f'(x) = \dfrac{0.81178 - 0.9116}{0.2}$

Approximate value of $f'(x) = -0.49916$

Exact value of $f'(x)$:

Differentiating equation (i) with respect to x,

Exact value of $f'(x) = -\sin x$

At $x = \dfrac{\pi}{6}$:

Exact value of $f'(x) = -\sin\dfrac{\pi}{6}$

Exact value of $f'(x) = -0.5$

$$\boxed{\therefore\ \%\ \text{error} = \left(\frac{-0.5 + 0.49916}{-0.5}\right) \times 100 = 0.166\%}$$

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17. (d)

$$f(x) = \begin{cases} x^2 & ; \quad x \geq 0 \\ -x^2 & ; \quad x < 0 \end{cases}$$

Now,

Left hand limit (LHL):

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(x^2\right) = 0$$

Right hand limit (RHL):

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(-x^2\right) = 0$$

$$f(0) = 0^2 = 0$$

Since $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = f(0)$, **therefore** $f(x)$ **is continuous at x = 0.**

Now,

$$f'(x) = \begin{cases} 2x & ; \quad x \geq 0 \\ -2x & ; \quad x < 0 \end{cases}$$

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} (-2x) = 0$$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (2x) = 0$$

$$f'(0) = 2 \times 0 = 0$$

Since $\lim\limits_{x \to 0^-} f'(x) = \lim\limits_{x \to 0^+} f'(x) = f'(0)$, **therefore** $f'(x)$ **is continuous at x = 0 i.e. f (x) is differentiable at x = 0.**

Now,

$$f''(x) = \begin{cases} 2 & ; \quad x \geq 0 \\ -2 & ; \quad x < 0 \end{cases}$$

$$\lim_{x \to 0^-} f''(x) = \lim_{x \to 0^-} (-2) = -2$$

$$\lim_{x \to 0^+} f''(x) = \lim_{x \to 0^+} (2) = 2$$

Since $\lim\limits_{x \to 0^-} f''(x) \neq \lim\limits_{x \to 0^+} f''(x)$, **therefore** $f''(x)$ **is not continuous at x = 0 i.e.** $f'(x)$ **is not differentiable at x = 0.**

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18. (a)

Given Data:

$$g(x) = \begin{cases} -x & ; x \leq 1 \\ x + 1 & : x \geq 1 \end{cases} \quad \text{…(i)}$$

$$f(x) = \begin{cases} 1 - x & : x \leq 0 \\ x^2 & : x > 0 \end{cases} \quad \text{…(ii)}$$

(f∘g) (x) = f (g(x))

From equation (ii),

$$(fog)(x) = \begin{cases} 1 - g(x) & ; \quad g(x) \leq 0 \\ [g(x)]^2 & ; \quad g(x) > 0 \end{cases}$$

From equation (i).

$$(fog)(x)=\begin{cases}1-(-x) & ; \ -x\le 0 \ \& \ x\le 1\\ 1-(x+1) & ; \ x+1\le 0 \ \& \ x\ge 1\\ (-x)^2 & ; \ -x>0 \ \& \ x\le 1\\ (x+1)^2 & ; \ x+1>0 \ \& \ x\ge 1\end{cases}$$

$$(fog)(x)=\begin{cases}1-(-x) & ; \ x\ge 0 \ \& \ x\le 1 \ \Rightarrow 0\le x\le 1\\ 1-(x+1) & ; \ x\le -1 \ \& \ x\ge 1 \Rightarrow \text{No interval}\\ (-x)^2 & ; \ x<0 \ \& \ x\le 1 \ \Rightarrow x<0\\ (x+1)^2 & ; \ x>-1 \ \& \ x\ge 1 \Rightarrow x\ge 1\end{cases}$$

$$(fog)(x)=\begin{cases}1+x & ; \ 0\le x\le 1\\ x^2 & ; \ x<0\\ (x+1)^2 & ; \ x\ge 1\end{cases}$$

In interval (-∞, 0) i.e. $x < 0$, the $(fog)(x)$ is x^2 which is a continuous function. Therefore, number of discontinuities in interval (-∞, 0) is 0.

Note:

If examinor is asking discontinous point in interval (–∞, ∞) for $(fog)(x)$, then we need to check continuity only at x = 0 and x =1.

Checking continuity at x = 0:

Left hand limit (LHL):

$$\lim_{x\to 0^-}(fog)(x)=\lim_{x\to 0^-}\left(x^2\right)=0$$

Right hand limit (RHL):

$$\lim_{x\to 0^+}(fog)(x)=\lim_{x\to 0^+}(1+x)=1$$

$\because$ LHL $\ne$ RHL, therefore $(fog)(x)$ is not continuous at x = 0.

Checking continuity at x = 1:

Left hand limit (LHL):

$$\lim_{x\to 1^-}(fog)(x)=\lim_{x\to 1^-}(1+x)=2$$

Right hand limit (RHL):

$$\lim_{x\to 1^+}(fog)(x)=\lim_{x\to 1^+}\left\{(x+1)^2\right\}=4$$

$\because$ LHL $\ne$ RHL, therefore $(fog)(x)$ is not continuous at x = 1.

Hence in the interval (–∞, 0) there is no discontinuity but if we consider overall interval (–∞, ∞) then f [g(x)] is discontinuous at x = 0 and x = 1.

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19. (b)

$$f(x)=\begin{cases}e^x & ; x<1\\ \ln x+ax^2+bx & ; x\ge 1\end{cases}$$

For f (x) to be differential, first f(x) should be continuous at x =1. And for that,

$$\lim_{x\to 1^-}f(x)=\lim_{x\to 1^+}f(x)=f(1)$$

$$\lim_{x\to 1^-}\left(e^x\right)=\lim_{x\to 1^+}\left(\ln x+ax^2+bx\right)$$

$\Rightarrow \quad e=(\ln 1+a+b)$

$\Rightarrow \quad e=a+b \qquad \ldots$(i)

Now,

$$f'(x)=\begin{cases}e^x & ; x<1\\ \dfrac{1}{x}+2ax+b & ; x\ge 1\end{cases}$$

For f (x) to be differentiable

$f'(1^-) = f'(1^+)$

$\Rightarrow \quad e = 1 + 2a(1) + b(1) \qquad \ldots$ (ii)

From (i) and (ii)

$a + b = 1 + 2a + b$

$\Rightarrow \quad a = –1$

Put value of a in equation (i),

$b = 1 + e$

Hence, f(x) is differentiable at x = 1 for the unique value of a and b.

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20. (c)

Given limit is

$$P = \lim_{x\to 1}\frac{x^7-2x^5+1}{x^3-3x^2+2} = \frac{1-2+1}{1-3+2} = \left(\frac{0}{0}\right)$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to 1}\frac{\frac{d}{dx}\left(x^7-2x^5+1\right)}{\frac{d}{dx}\left(x^3-3x^2+2\right)} = \lim_{x\to 1}\frac{7x^6-10x^4}{3x^2-6x}$$

$$P = \frac{7-10}{3-6} = \frac{-3}{-3} = 1$$

$$\therefore \boxed{\lim_{x\to 1}\frac{x^7-2x^5+1}{x^3-3x^2+2} = 1}$$

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21. (–1 to –1)

$$P = \lim_{x\to 0}\frac{\tan x}{x^2-x} = \left(\frac{0}{0}\right) \qquad \left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to 0}\frac{\frac{d}{dx}(\tan x)}{\frac{d}{dx}\left(x^2-x\right)} = \lim_{x\to 0}\frac{\sec^2 x}{2x-1}$$

$$P = \frac{\sec^2 0}{2(0)-1} = \frac{1}{-1} = -1$$

$$\therefore \boxed{\lim_{x\to 0}\frac{\tan x}{x^2-x} = -1}$$

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22. (c)

Given, w = f(x, y)

By Chain rule of differentiation,

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}.\frac{dx}{dt} + \frac{\partial w}{\partial y}.\frac{dy}{dt}$$

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23. (d)

$$\lim_{x\to 0}\frac{x^3-\sin x}{x} = \lim_{x\to 0} x^2 - \frac{\sin x}{x}$$

$$= \lim_{x\to 0} x^2 - \lim_{x\to 0}\frac{\sin x}{x}$$

$$= 0 - 1 = -1$$

$$\therefore \boxed{\lim_{x\to 0}\frac{x^3-\sin x}{x} = -1}$$

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24. (54 to 55)

Given planes are x + y + z –1 = 0

and 2x – y + 2z = 0

The angle between the planes $a_1x+ b_1y + c_1z + d_1 = 0$ and $a_2x+ b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\Rightarrow \cos\theta = \frac{1(2)+1(-1)+1(2)}{\sqrt{1^2+1^2+1^2}\cdot\sqrt{2^2+(-1)^2+2^2}}$$

$$\Rightarrow \cos\theta = \frac{3}{\sqrt{3}\times 3}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \boxed{\theta = 54.73°}$$

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25. (b)

(i) If f(x) is continuous at $x = x_o$, then it may not be differentiable at x_o, ex: |x| at x = 0

(ii) If f(x) is differentiable at $x = x_o$, then it is definitely continuous at $x = x_o$.

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26. (1)

$\lim\limits_{x\to 4} \dfrac{\sin(x-4)}{(x-4)}$ is the given limit

As $x \to 4 \Rightarrow (x-4) \to 0$

Let $x - 4 = t$

$\Rightarrow$ Given limits is $\lim\limits_{t\to 0} \dfrac{\sin t}{t} = 1$

$\therefore \boxed{\lim\limits_{t\to 4} \dfrac{\sin(x-4)}{(x-4)} = 1}$

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27. (c)

$$\lim_{x\to 0} \frac{\log_e(1+4x)}{e^{3x}-1} = \frac{0}{0}$$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For} \left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to a} \dfrac{f'(x)}{g'(x)} \end{array}\right]$$

$$\lim_{x\to 0} \frac{\frac{1}{1+4x}\times 4}{3\cdot e^{3x}} = \frac{4\times(1)}{3\cdot e^{0}} = \frac{4}{3}$$

$$\therefore \boxed{\lim_{x\to 0} \frac{\log_e(1+4x)}{e^{3x}-1} = \frac{4}{3}}$$

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28. (c)

$$\lim_{x\to\infty} \sqrt{x^2+x-1} - x = \lim_{x\to\infty} \left(\sqrt{x^2+x-1} - x\right) \frac{\left(\sqrt{x^2+x-1}+x\right)}{\left(\sqrt{x^2+x-1}-x\right)}$$

$$= \lim_{x\to\infty} \frac{x^2+x-1-x^2}{\sqrt{x^2+x-1}+x} = \lim_{x\to\infty} \frac{x\left(1-\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{1}{x}-\left(\frac{1}{x}\right)^2}+1\right)}$$

$$= \lim_{\frac{1}{x}\to 0} \frac{\left(1-\frac{1}{x}\right)}{\sqrt{1+\left(\frac{1}{x}\right)-\left(\frac{1}{x}\right)^2}+1}$$

As $x \to \infty; \dfrac{1}{x} \to 0$

$$= \frac{1}{1+1} = \frac{1}{2} = 0.5$$

$$\therefore \boxed{\lim_{x\to\infty} \left(\sqrt{x^2+x-1} - x\right) = \frac{1}{2}}$$

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29. (c)

The function $f(x) = \dfrac{\phi(x)}{\psi(x)}$ where $\phi(x)$ and $\psi(x)$ are polynomial is discontinuous at the points where $\psi(x) = 0$.

Hence, given function is not continuous at the points Where, $x^2 + 3x - 4 = 0$

$\Rightarrow \quad (x + 4)(x - 1) = 0$

$\Rightarrow \quad x = -4, 1$

$\boxed{\therefore\ \textbf{f(x) is NOT continuous at x = –4, 1}}$

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30. (0.28 to 0.31)

$$S = \sum_{n=0}^{\infty} n\alpha^n$$

$\Rightarrow \quad S = 0 + \alpha + 2\alpha^2 + 3\alpha^3 + \ldots\ldots \quad \ldots(1)$

$\Rightarrow \quad \alpha S = 0 + \alpha^2 + 2\alpha^3 + 3\alpha^4 + \ldots \quad \ldots(2)$

Equation (1) – (2)

$\Rightarrow \quad S(1-\alpha) = \alpha + \alpha^2 + \alpha^3 + \ldots\ldots \quad \ldots(3)$

Here,

$\alpha + \alpha^2 + \alpha^3 + \ldots\ldots$ is infinite G.P. series with common ratio $r = \alpha$

Now, sum of infinite G.P. series $= \dfrac{a}{1-r} \left[\text{If } r < 1\right]$

Therefore,

$$\alpha + \alpha^2 + \alpha^3 + \ldots\ldots = \frac{\alpha}{1-\alpha}$$

From equation (3),

$$S(1-\alpha) = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow \quad S = \frac{\alpha}{(1-\alpha)^2} \qquad \left[\because S = 2\alpha\right]$$

$$\Rightarrow \quad 2\alpha = \frac{\alpha}{(1-\alpha)^2}$$

$$\Rightarrow \quad (1-\alpha)^2 = \frac{1}{2}$$

$$\Rightarrow \quad 1-\alpha = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad \alpha = 0.2928$ and $\alpha = 1.7071$

Discarding $\alpha = 1.7071$ because $0 < \alpha < 1$

$\Rightarrow \quad \therefore \boxed{\alpha = 0.2928}$

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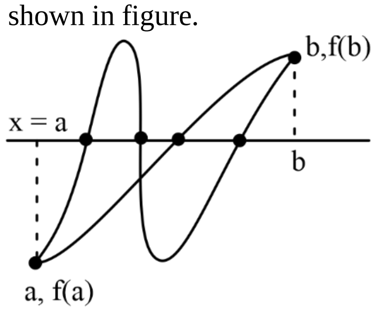
31. (c)

According to intermediate value theorem,

If f (x) is continuous in [a, b] and changing its sign between *a* and *b* (i.e f(a) and f(b) have opposite signs) then there exists at least one root of f (x) in [a, b] as shown in figure.



If f(x) does not have a root in the interval [a, b] then it implies that f (x) is not changing its sign in interval [a, b] i.e. both f (a) and f (b) are either positive or negative.

$\therefore$ f (a) . f (b) > 0

Scan for Video solution

32. (d)

$$P = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$$

$$P = \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^2$$

$$\left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e\right]$$

$$P = e^2$$

$$\therefore \quad \boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x} = e^2}$$

Scan for Video solution

33. (c)

Given:

$$P = \lim_{x\to 0}\frac{1-\cos(x^2)}{2x^4}$$

$$P = \frac{1-1}{4} = \left(\frac{0}{0}\right)$$

$$\left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\dfrac{0}{0}(\text{or})\dfrac{\infty}{\infty}\right)\text{form} \\ \displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to 0}\frac{\frac{d}{dx}\left(1-\cos(x^2)\right)}{\frac{d}{dx}\left(2x^4\right)} = \lim_{x\to 0}\frac{\sin(x^2)\times 2x}{8x^3}$$

$$P = \frac{1}{4}\times \lim_{x\to 0}\frac{\sin(x^2)}{x^2}$$

As $x \to 0$, $x^2 \to 0$ and let $x^2 = t$, therefore,

$$P = \frac{1}{4}\times \lim_{t\to 0}\frac{\sin(t)}{t} \qquad \left[\because \lim_{t\to 0}\frac{\sin t}{t} = 1\right]$$

$$P = \frac{1}{4}\times 1 = \frac{1}{4}$$

Hence, $\boxed{\lim_{x\to 0}\frac{1-\cos(x^2)}{2x^4} = \frac{1}{4}}$

Scan for Video solution

34. (–0.35 to –0.30)

Given:

$$P = \lim_{x\to 0}\left(\frac{-\sin x}{2\sin x + x\cos x}\right)$$

$$P = \lim_{x\to 0}\left(\frac{-\left(\frac{\sin x}{x}\right)}{2\left(\frac{\sin x}{x}\right)+\left(\frac{x\cdot\cos x}{x}\right)}\right)$$

$$P = \frac{-\lim_{x\to 0}\frac{\sin x}{x}}{2.\lim_{x\to 0}\cdot\frac{\sin x}{x} + \lim_{x\to 0}.\cos x}$$

$$\left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$P = \frac{-1}{2+1} = \frac{-1}{3} = -0.33$$

$$\therefore \quad \boxed{\lim_{x\to 0}\frac{-\sin x}{\sin x + x\cdot\cos x} = -0.33}$$

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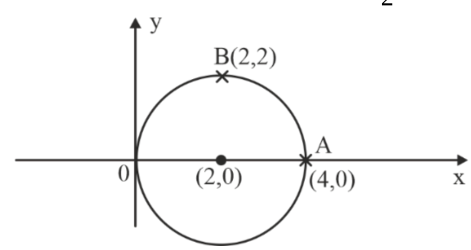
35. (1.9 to 2.1)

Given

curve $(x-2)^2 + y^2 = 4$

Starting point = (4, 0) and final point = (2, 2)

Speed of the ant = 1.57 m/s = $\frac{\pi}{2}$ m/s



$$\text{Time required for travel} = \frac{\text{length of the arc AB}}{\text{speed of ant}}$$

$$= \frac{\frac{1}{4}\times 2\pi r}{\left(\frac{\pi}{2}\right)} = 2 \text{ sec}$$

$\boxed{\therefore \text{ Time for travelling from A to B = 2 sec}}$

Scan for Video solution

36. (c)

Given limit is $\lim_{x\to\infty} x^{\frac{1}{x}} = \infty^{\circ}$ (undetermined form)

Let $y = x^{\frac{1}{x}}$

$$\Rightarrow \log_e y = \frac{1}{x}\log_e x$$

$$\Rightarrow \lim_{x\to\infty}\left(\log_e y\right) = \lim_{x\to\infty}\frac{\log_e x}{x} = \left(\frac{\infty}{\infty}\right)$$

Apply L-Hospital Rule

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$\Rightarrow \log_e\left(\lim_{x\to\infty} y\right) = \lim_{x\to\infty}\frac{\frac{d}{dx}\log_e x}{\frac{d}{dx}x} = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0$$

$$\Rightarrow \log_e\left(\lim_{x\to\infty} y\right) = 0 \qquad \Rightarrow \lim_{x\to\infty} y = e^0 = 1$$

$$\Rightarrow \boxed{\lim_{x\to\infty} x^{\frac{1}{x}} = 1}$$

Scan for Video solution

37. (a)

Given limit is

$$P = \lim_{x\to\infty}\left(1+x^2\right)\cdot e^{-x} = \lim_{x\to\infty}\frac{\left(1+x^2\right)}{e^x} = \left(\frac{\infty}{\infty}\right)$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$\Rightarrow \quad P = \lim_{x\to\infty}\frac{\frac{d}{dx}\left(1+x^2\right)}{\frac{d}{dx}e^x} = \lim_{x\to\infty}\frac{2x}{e^x} = \left(\frac{\infty}{\infty}\right)$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$\Rightarrow \quad P = \lim_{x\to\infty}\frac{\frac{d}{dx}(2x)}{\frac{d}{dx}e^x} = \lim_{x\to\infty}\frac{2}{e^x} = \frac{2}{e^0} = 0$$

$$\therefore \quad \boxed{P = \lim_{x\to\infty}\frac{1+x^2}{e^x} = 0}$$

Scan for Video solution

38. (c)

Let $P = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = 1^\infty$

For $\lim_{x\to a}\left(f(x)\right)^{g(x)} = 1^\infty$,

$$\lim_{x\to a}\left(f(x)\right)^{g(x)} = e^{\lim_{x\to a} g(x)\cdot\{f(x)-1\}}$$

Therefore,

$$P = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e^{\lim_{x\to\infty} x\left(1+\frac{1}{x}-1\right)}$$

$$P = e^{\lim_{x\to\infty} 1} = e^1 = e$$

$$\therefore \quad \boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right) = e}$$

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39. (–2)

Given f(x) = x·sinx

⇒ f’(x) = x·cosx + sinx

And,

⇒ f’’(x) = x·(–sinx) + cosx + cosx

⇒ f’’(x) = –x·sinx + 2cosx

Considering f’’(x) + f(x) + t·cosx = 0

⇒ –x·sinx + 2cosx + x·sinx + t·cosx = 0

⇒ (2 + t) cosx = 0

⇒ (2 + t) = 0

⇒ $\boxed{t = -2}$

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40. (b)

$$P = \lim_{x\to\infty}\left(\frac{x+\sin x}{x}\right)$$

$$P = \lim_{x\to\infty}\left(1+\frac{\sin x}{x}\right)$$

As $x \to \infty$; the value of sin x oscillated between +1 & –1 which is a finite value.

$$P = \lim_{x\to\infty}\left(1+\frac{\text{Finite value}}{\infty}\right)$$

$$P = 1 + 0 = 1$$

$$\therefore \quad \boxed{\lim_{x\to\infty}\left(\frac{x+\sin x}{x}\right)=1}$$

Scan for Video solution

41. (a)

$$P = \lim_{\alpha\to 0}\frac{x^{\alpha}-1}{a} = \left(\frac{0}{0}\right)$$

$$\left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{\alpha\to 0}\frac{\frac{d}{d\alpha}\left(x^{\alpha}-1\right)}{\frac{d}{d\alpha}(a)} = \lim_{\alpha\to 0}\frac{x^{\alpha}\log_e x - 0}{1}$$

$$P = \lim_{\alpha\to 0}\frac{x^{0}\log_e x - 0}{1}$$

$$P = \log_e x$$

$$\therefore \quad \boxed{\lim_{\alpha\to 0}\frac{x^{\alpha}-1}{a} = \log_e x}$$

Scan for Video solution

42. (c)

Given, $f(x) = \begin{cases}\dfrac{\lambda\cos x}{\frac{\pi}{2}-x} & \text{if } x \neq \dfrac{\pi}{2}\\ 1 & \text{if } x = \dfrac{\pi}{2}\end{cases}$

For function to be continuous at $x = \frac{\pi}{2}$,

LHL = RHL = f(π/2)

Now,

$$\lim_{x\to\frac{\pi}{2}}\frac{\lambda\cos x}{\frac{\pi}{2}-x} = \left(\frac{0}{0}\right)\text{form}$$

$$\left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\lambda\cos x}{\frac{\pi}{2}-x} = \lim_{x\to\frac{\pi}{2}}\frac{\frac{d}{dx}(\lambda\cos x)}{\frac{d}{dx}\left(\frac{\pi}{2}-x\right)}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\lambda\cos x}{\frac{\pi}{2}-x} = \lim_{x\to\frac{\pi}{2}}\frac{\lambda(-\sin x)}{-1} = \lambda$$

Therefore, LHL = RHL = λ …(i)

For function to be continuous at $x = \frac{\pi}{2}$,

LHL = RHL = f(π/2)

From equation 9i),

λ = f(π/2)

$\boxed{\lambda = 1}$

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43. (a)

Given: $P = \lim\limits_{x\to 0} \dfrac{x - \sin x}{1 - \cos x}$

$P = \lim\limits_{x\to 0} \dfrac{x - \sin x}{1 - \cos x} = \left(\dfrac{0}{0}\right)$

$\left[\left(\dfrac{0}{0}\right)\text{form}\right]$

$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\dfrac{0}{0}(\text{or})\dfrac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}\end{array}\right]$

$P = \lim\limits_{x\to 0} \dfrac{\frac{d}{dx}(x - \sin x)}{\frac{d}{dx}(1 - \cos x)}$

$P = \lim\limits_{x\to 0} \dfrac{1 - \cos x}{\sin x} = \left(\dfrac{0}{0}\right)$

Again apply L-hospital rule.

$P = \lim\limits_{x\to 0} \dfrac{\frac{d}{dx}(1 - \cos x)}{\frac{d}{dx}(\sin x)}$

$P = \lim\limits_{x\to 0} \dfrac{0 + \sin x}{\cos x} = \lim\limits_{x\to 0} \tan x = 0$

$\therefore \boxed{\lim\limits_{x\to 0} \dfrac{x - \sin x}{1 - \cos x} = 0}$

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44. (b)

Given: $P = \lim\limits_{x\to 0} \dfrac{e^{2x} - 1}{\sin 4x}$

$P = \lim\limits_{x\to 0} \dfrac{e^{2x} - 1}{\sin 4x} = \dfrac{1-1}{0} = \left(\dfrac{0}{0}\right)$

$\left[\left(\dfrac{0}{0}\right)\text{form}\right]$

$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\dfrac{0}{0}(\text{or})\dfrac{\infty}{\infty}\right)\text{form} \\ \lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}\end{array}\right]$

$P = \lim\limits_{x\to 0} \dfrac{\frac{d}{dx}(e^{2x} - 1)}{\frac{d}{dx}(\sin 4x)} = \lim\limits_{x\to 0} \dfrac{2\cdot e^{2x} - 0}{4\cdot \cos 4x}$

$P = \dfrac{2\cdot e^0}{4\cdot \cos 0} = \dfrac{2}{4} = \dfrac{1}{2} = 0.5$

$\therefore \boxed{\lim\limits_{x\to 0} \dfrac{e^{2x} - 1}{\sin 4x} = 0.5}$

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45. (d)

For f(x) to be continueous at a point x = a

$\lim\limits_{x\to a} f(x) = f(a)$

$\Rightarrow$ **Limit should exist and must be equal to functional value at that point.**

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46. (a)

For a funcitn f(x) to be continous at x = 3

$\lim\limits_{x\to 3} f(x) = f(3)$

(i) $f(x) = \begin{cases} 2; & \text{if } x = 3 \\ x - 1; & \text{if } x > 3 \\ \dfrac{x+3}{3}; & \text{if } x < 3 \end{cases}$

$\therefore \lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} \dfrac{x+3}{3} = 2$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} x - 1 = 2$

$\therefore \lim\limits_{x\to 3} f(x) = 2$

and f(3) = 2

Since $\lim_{x\to 3} f(x) = f(3)$

$\therefore$ f(x) is continous at x = 3

In all other options, $\underset{x\to 3}{\text{It}} f(x)$ doesn't exist. (Since left hand limits are not equal to right hand limits)

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47. (c)

The functions f(x), g(x), h(x) are said to be linearly dependent if $f(x) = C_1 g(x) + C_2 . h(x)$

Where C_1, C_2 are non-zero constants.

We have $\cos 2x = \cos^2 x - \sin^2 x$

$\Rightarrow \cos 2x = 1.\cos^2 x + (-1).\sin^2 x$

$\boxed{\therefore \cos 2x, \cos^2 x, \sin^2 x \text{ are linearly dependent.}}$

Scan for Video solution

48. (c)

Method I:

$$y = f(x) = |x|$$

$$y = f(x) = \begin{cases} x & ; x \ge 0 \\ -x & ; x < 0 \end{cases}$$

Checking for continuity:

Left hand limit (LHL):

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0$$

Right hand limit (RHL):

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x) = 0$$

$f(0)$:

$$f(0) = |0| = 0$$

$$\because \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$\boxed{\text{Therefore, function is continuous at}}\ x = 0.$

Checking for differentiability:

$$y' = f'(x) = \begin{cases} 1 & ; x > 0 \\ -1 & ; x < 0 \end{cases}$$

Left hand derivative (LHD):

$$\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} (-1) = -1$$

Right hand derivative (RHD):

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} 1 = 1$$

$\because$ LHD $\ne$ RHD , therefore, function is not differentiable at $x = 0$.

$\therefore$ **$|x|$ is continuous at x =0 and not differentiable at x =0.**

Method II:

Given: $f(x) = |x|; \quad -1 \le x \le 1$

$|x|$ is continuous $\forall x\in R$ and differentiable $\forall x\in R$ except at x = 0.

$\therefore$ At x = 0, $|x|$ is continuous but not differentiable.

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49. (b)

Given: $P = \lim_{x\to 0} \frac{1-\cos x}{x^2}$

Method I:

$$P = \lim_{x\to 0} \frac{1-\cos x}{x^2} = \left(\frac{0}{0}\right)$$

$$\left[\left(\frac{0}{0}\right) \text{form}\right]$$

$$\left[\begin{array}{l} \text{Apply L – Hospital rule, For} \left(\frac{0}{0} (\text{or}) \frac{\infty}{\infty}\right) \text{form} \\ \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} \end{array}\right]$$

$$P = \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{\sin x}{2x}$$

$$P == \frac{1}{2} \times \lim_{x\to 0} \frac{\sin x}{x}$$

$$P = \frac{1}{2}\times 1 = \frac{1}{2}$$

$$\therefore \quad \boxed{\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}}$$

Method II:

$$\left(\lim_{x\to 0}\frac{1-\cos ax}{x^2} = \frac{a^2}{2}\right)$$

$$P = \lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1^2}{2} = \frac{1}{2}$$

$$\therefore \quad \boxed{\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}}$$

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50. (d)

Given: $P = \lim_{\theta\to 0}\frac{\sin\theta}{\theta}$

$$P = \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \left(\frac{0}{0}\right)\quad\left[\left(\frac{0}{0}\right)\text{form}\right]$$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form}\\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{\theta\to 0}\frac{\frac{d}{d\theta}(\sin\theta)}{\frac{d}{d\theta}(\theta)} = \lim_{\theta\to 0}\frac{\cos\theta}{1} = \cos 0 = 1$$

$$\therefore \quad \boxed{\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1}$$

Note: Remember above expression directly.

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51. (c)

Method I:

$$y = f(x) = |2-3x|$$

$$\Rightarrow \quad y = f(x) = \begin{cases} 2-3x & ;2-3x \ge 0 \\ -(2-3x) & ;\ 2-3x < 0\end{cases}$$

$$\Rightarrow \quad y = f(x) = \begin{cases} 2-3x & ;x \le \frac{2}{3} \\ -(2-3x) & ;\ x > \frac{2}{3}\end{cases}$$

For f(x) we need to check its continuity and differentiability at $x = \frac{2}{3}$ only.

Checking for continuity:

Left hand limit (LHL):

$$\lim_{x\to \frac{2}{3}^-} f(x) = \lim_{x\to \frac{2}{3}^-}(2-3x) = \left(2-3\times\frac{2}{3}\right)$$

$$\Rightarrow \quad \lim_{x\to \frac{2}{3}^-} f(x) = 0$$

Right hand limit (RHL):

$$\lim_{x\to \frac{2}{3}^+} f(x) = \lim_{x\to \frac{2}{3}^+} -(2-3x) = -\left(2-3\times\frac{2}{3}\right)$$

$$\Rightarrow \quad \lim_{x\to \frac{2}{3}^+} f(x) = 0$$

$$f\left(\frac{2}{3}\right):$$

$$f\left(\frac{2}{3}\right) = \left|2-3\times\frac{2}{3}\right| = 0$$

$$\because \lim_{x\to \frac{2}{3}^-} f(x) = \lim_{x\to \frac{2}{3}^+} f(x) = f\left(\frac{2}{3}\right)$$

Therefore, function is continuous at $x = \frac{2}{3}$.

Checking for differentiability:

$$y' = f'(x) = \begin{cases} -3 & ;x < \frac{2}{3} \\ 3 & ;x > \frac{2}{3}\end{cases}$$

Left hand derivative (LHD):

$$\lim_{x\to \frac{2}{3}^-} f'(x) = \lim_{x\to \frac{2}{3}^-}(-3) = -3$$

Right hand derivative (RHD):

$$\lim_{x\to\frac{2}{3}^+} f'(x) = \lim_{x\to\frac{2}{3}^+} 3 = 3$$

$\because$ LHD $\neq$ RHD , therefore, function is not differentiable at $x = \frac{2}{3}$.

$\therefore$ $|2-3x|$ **is continuous $\forall$ x∈R and not differentiable at** $x = 2/3$.

Method II:

Given: function $y = f(x) = |2-3x|$

- In $|f(x)|$ (Where $f(x)$ is polynomial) is continuous $\forall$ x∈R and not differentiable at the points where $f(x) = 0$.

  $|2-3x|$ is continuous $\forall$ x ∈ R and differentiable $\forall$ x∈R except at the value where $2-3x = 0$ $x = \frac{2}{3}$.

$\therefore$ $|2-3x|$ **is continuous $\forall$ x∈R and not differentiable at** $x = 2/3$.

Scan for Video solution

52. (b)

Given limit is

$$P = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = 1^{\infty}$$

If $\lim_{n\to\infty}\left(f(x)\right)^{g(x)} = 1^{\infty}$, then

$$\lim_{n\to\infty}\left(f(x)\right)^{g(x)} = e^{\lim_{x\to a} g(x)\cdot\{f(x)-1\}}$$

Therefore,

$$\therefore\ P = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = e^{\lim_{n\to\infty} 2n\left(1-\frac{1}{n}-1\right)}$$

$$\Rightarrow\quad P = e^{\lim_{n\to\infty} -2} = e^{-2}$$

$$\therefore\ \boxed{\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = e^{-2}}$$

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53. (a)

$$P = \lim_{x\to 0}\frac{\sin\frac{2}{3}x}{x}$$

$$P = \lim_{x\to 0}\frac{\frac{2}{3}\sin\frac{2}{3}x}{\frac{2}{3}x}$$

If $x \to 0$, then $\frac{2}{3}x \to 0$

$$P = \frac{2}{3}\times\lim_{x\to 0}\frac{\sin\frac{2}{3}x}{\frac{2}{3}x} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$P = \frac{2}{3}\times 1 = \frac{2}{3}$$

$$\therefore\ \boxed{\lim_{x\to 0}\frac{\sin\frac{2}{3}x}{x} = \frac{2}{3}}$$

Scan for Video solution

54. (a)

Given limit is

$$P = \lim_{x\to\infty}\frac{x-\sin x}{x+\cos x} = \lim_{x\to\infty}\frac{x\left(1-\frac{\sin x}{x}\right)}{x\left(1+\frac{\cos x}{x}\right)}$$

For any x ∈ R, sin x ∈ [–1, 1] and cos x ∈ [–1, 1]

$$P = \frac{1-\frac{\text{A finite value in }[-1,1]}{\infty}}{1+\frac{\text{A finite value in }[-1,1]}{\infty}}$$

$$P = \frac{1-0}{1+0} = 1$$

$$\therefore\ \boxed{\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x} = 1}$$

Scan for Video solution

55. (b)

Given: $P = \lim_{x\to 8} \frac{x^{1/3}-2}{(x-8)}$

$$P = \lim_{x\to 8} \frac{x^{1/3}-2}{(x-8)} = \frac{8^{1/3}-2}{(8-8)} = \left(\frac{0}{0}\right)$$

$\left[\left(\frac{0}{0}\right)\text{form}\right]$

$$\left[\begin{array}{l}\text{Apply L – Hospital rule, For}\left(\frac{0}{0}(\text{or})\frac{\infty}{\infty}\right)\text{form} \\ \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}\right]$$

$$P = \lim_{x\to 8}\frac{\frac{d}{dx}\left(x^{1/3}-2\right)}{\frac{d}{dx}(x-8)} = \lim_{x\to 8}\frac{\frac{1}{3}\cdot x^{-2/3}-0}{1-0} = \frac{1}{3}\cdot(8)^{-2/3}$$

$$P = \frac{1}{3}\cdot(8)^{-2/3} = \frac{1}{3}\cdot\left(2^3\right)^{-2/3} = \frac{1}{3}.2^{-2} = \frac{1}{12}$$

$\therefore \quad \boxed{\lim_{x\to 8}\frac{x^{1/3}-2}{(x-8)} = \frac{1}{12}}$

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56. (0.50)

Given $\frac{dy}{dt} = \frac{e^{-\alpha t}}{2+\alpha t}, \alpha = 0.01$ and $y(0) = 0$

$$\frac{dy}{dt} = \frac{e^{-\alpha t}}{2+\alpha t} \qquad \ldots(i)$$

At t = 0;

$$\Rightarrow \quad \frac{dy}{dt} = y'(0) = \frac{1}{2}$$

Differentiating equation (i) with respect to t,

$$\frac{d^2y}{dt^2} = \frac{-\alpha(2+\alpha t)e^{-\alpha t} - e^{-\alpha t}(\alpha)}{(2+\alpha t)^2}$$

At t = 0;

$$\Rightarrow \quad \frac{d^2y}{dt^2} = y''(0) = \frac{-\alpha(2)\times 1 - 1\times(\alpha)}{(2)^2} = \frac{-3\alpha}{4}$$

By Taylor series,

$$y(t) = y(0) + ty'(0) + \frac{t^2}{2!}y''(0)\ldots.$$

$$\left[\because y(0) = 0,\ y'(0) = \frac{1}{2}, y''(0) = -\frac{3\alpha}{4}\right]$$

$$\Rightarrow \quad y(t) = 0 + t\left(\frac{1}{2}\right) + \frac{t^2}{2}\left(-\frac{3\alpha}{4}\right) + \ldots\ldots..$$

$\therefore$ At t = 1

$$\Rightarrow \quad y(1) = 0 + \frac{1}{2} + \frac{1}{2}\times\left(-\frac{3\alpha}{4}\right) + \ldots\ldots..$$

$$[\because \alpha = 0.01]$$

$$\Rightarrow \quad y(1) = \frac{1}{2} + \frac{1}{2}\left(-\frac{3\times 0.01}{4}\right) + \ldots$$

$$\Rightarrow \quad y(1) = 0.5 - \frac{3}{8}\times 0.01 = 0.496$$

$$\Rightarrow \quad y(1) = 0.50$$

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57. (b)

Given, series is $\sum_{n=1}^{\infty}\frac{n^d}{c^n} = \sum u_n$

By ratio test, $\sum u_n$ converges if $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1$

$\therefore$ Series converges if

$$\lim_{n\to\infty}\left|\frac{\left(\frac{(n+1)^d}{c^{n+1}}\right)}{\left(\frac{n^d}{c^n}\right)}\right| < 1$$

$$\Rightarrow \quad \lim_{n\to\infty}\left|\frac{(n+1)^d}{c^{n+1}}\times\frac{c^n}{n^d}\right| < 1$$

$$\Rightarrow \quad \frac{1}{|c|}\times\lim_{n\to\infty}\left|\frac{n+1}{n}\right|^d < 1$$

$$\Rightarrow \quad \frac{1}{|c|}\times \lim_{n\to\infty}\left|\frac{\left(1+\frac{1}{n}\right)}{1}\right|^{d} < 1$$

$$\Rightarrow \quad \frac{1}{|c|}\times 1 < 1 \qquad \text{(For any finite number '}d\text{')}$$

$\Rightarrow$ $|c| > 1$ and 'd' is a finite number.

∴ out of the given options, c = 2 and d = 1 is possible.

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58. (b)

Given:

$$f(x) = 8\log_e x - x^2 + 3 \text{ in } [1, e] \qquad \ldots(i)$$

Calculating $f'(x)$

$$f'(x) = \frac{8}{x} - 2x$$

Stationary point: [At stationary point f '(x) =0]

$$f'(x) = \frac{8}{x} - 2x = 0$$

$\Rightarrow$ x = ±2

But, x = 2 is the only value that lies in [1, e].

Finding value of f(x) at initial and final point (i.e at x = 1 and e) and in stationary point (i.e. at x =2) :

$$f(x) = 8\log_e x - x^2 + 3$$

At x =1,

$$f(1) = 8\log_e 1 - 1^2 + 3 = 8(0) - 1 + 3 = 2$$

At x =e,

$$f(e) = 8\log_e e - e^2 + 3 = 8 - e^2 + 3 = 3.611$$

At x =2,

$$f(2) = 8\log_e 2 - 2^2 + 3 = 8\log_e 2 - 2^2 + 3 = 4.545$$

∴ Minimum of f(x) is at x = 1.

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59. (d)

Given $f(x) = x^3 + 15x^2 - 33x - 36$

$\Rightarrow$ $f'(x) = 3x^2 + 30x - 33$

Equating $f'(x) = 0$

$\Rightarrow$ $3x^2 + 30x - 33 = 0$

$\Rightarrow$ $x^2 + 10x - 11 = 0$

$\Rightarrow$ $x = -11, x = 1$

Considering $f''(x) \Rightarrow f''(x) = 6x + 30$

At $x = -11$. $f''(-11) = -66 + 30 < 0$

∴ f(x) has local maxima at $x = -11$

At $x = 1$; $f''(1) = 6 + 30 = 36 > 0$

∴ f(x) has local minima at x = 1

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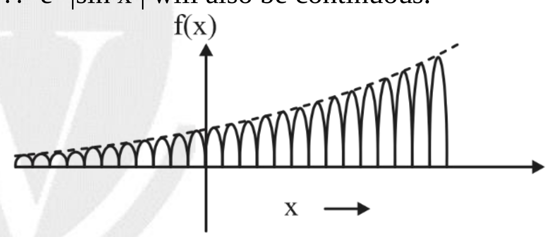
60. (a)

→ Since e^x and $|\sin x|$ both are continuous.

∴ $e^x|\sin x|$ will also be continuous.



→ f(x) is not differentiable for all $x \in R$ because there are sharp edges.

→ f(x) is not periodic

→ f(x) is unbounded

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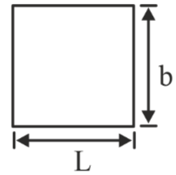
61. (7.999 to 8.001)

Let 'L' be length of Rectangle and 'b' be breadth of Rectangle.



Given : area of rectangle; Lb = 4

Find: Minimum value of perimeter (P)

P= 2(L+ b)

$$\left[\because Lb = 4 \Rightarrow b = \frac{4}{L}\right]$$

$$P = 2\left(L + \frac{4}{L}\right)$$

$\therefore$ For minimum of perimeter,

$$\frac{dP}{dL} = 0 \Rightarrow 2\left(L - \frac{4}{L^2}\right) = 0$$

L= 2

$\therefore$ For minimum perimeter, L = 2

and $b = \frac{4}{L} = \frac{4}{2} = 2$

Hence minimum perimeter,

P = 2(L + b) = 2(4) = 8 units

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62. (d)

$f(x) = x^3 - 6x^2 + 11x - 6;\ x \in [1, 3]$

for x = 1

$f(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 0$

for x = 3

$f(3) = 3^3 - 6(3^2) + 11(3) - 6 = 0$

$\boxed{\text{Therefore statement I is correct}}$

Differentiating equation (i) with respect to x,

$f'(x) = 3x^2 - 12x + 11 \quad \ldots(ii)$

Differentiating equation (ii) with respect to x,

$f''(x) = 6x - 12 \quad \ldots(iii)$

Stationary point:

At stationary point $f'(x) = 0$

$f'(x) = 0$

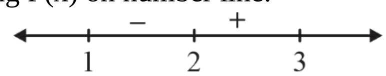
$\Rightarrow\ 3x^2 - 12x + 11 = 0$

$$\Rightarrow\ x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 3 \times 11}}{2 \times 3}$$

$\Rightarrow\ x = 2.577,\ 1.423 \in [1, 3]$

Now $f''(x) = 6(x - 2)$

Drawing $f'(x)$ on number line:



$$f''(x) = \begin{cases} < 0 \text{ for } & x < 2 \\ > 0 \text{ for } & x > 0 \end{cases}$$

At stationary point x = 1.423 (x < 2) $f''(x)$ is <0, therefore x = 1.423 is local maxima and at stationary point x = 2.577 (ie. x > 2) $f''(x)$ is > 0, therefore x = 2.577 is local minima.

Hence is domain S. There exit a local maxima and a local minima.

$\therefore$ Both statement II and IV are correct.

Since $f''(x)$ is not > 0 for throughout domain s, therefore statement III is incorrect.

$\therefore$ Correct statements are I, II and III therefore option D is correct

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63. (a, b)

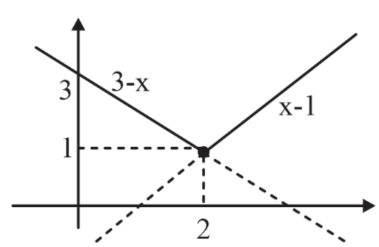
For intersection of 3 – x and x – 1

3 – x = x – 1

$\therefore$ At x = 2 these lines will intersect.

$$f(x) = \max\{3 - x, x - 1\}$$

$$f(x) = \begin{cases} 3 - x & x < 2 \\ x - 1 & x > 2 \\ 1 & x = 2 \end{cases}$$



Check for continuity:-

$$LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3 - x) = 1$$

$$LHL = \lim_{x \to 2^=} f(x) = \lim_{x \to 2^+} (x - 1) = 1$$

f(2) =1

Since at x= 2 $\text{LHL} = \text{RHL} = f(2)$ ∴ It is continuous in its domain.

Check for differentiability: –

From graph we can infer that f(x) has sharp edge at x =2, therefore, function is not differential at x =2.

From graph of this function it is evident that function has local minimum at x = 2

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64. (b)

Given

$f(x) = \frac{1}{2}x^TQx - r^Tx$; and

Q is symmetric matrix ⇒ $Q = Q^T$

$$f'(x) = \frac{1}{2} \times \frac{d}{dx}\left(x^TQx\right) - \frac{d}{dx}\left(r^Tx\right)$$

$$\left(\because \frac{d}{dx}\left(x^TQx\right) = x^T\left(Q + Q^T\right)\right)$$

$$f'(x) = \frac{1}{2} \cdot \left\{x^T\left(Q + Q^T\right)\right\} - r^T$$

$$\left(\because Q^T = Q\right)$$

$$f'(x) = \frac{1}{2} \cdot \left\{x^T(2Q)\right\} - r^T$$

$$f'(x) = x^TQ - r^T$$

At stationary point, $f'(x) = 0$:

$f'(x) = 0$

$x^TQ - r^T = 0$

$(x^TQ - r^T)^T = 0^T$

$(x^TQ - r^T)^T = 0$

$Q^T.\, x - r = 0$

$x = Q^{-1}.\, r$

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65. (19)

Given f:R → R is continuous in [–3, 3] and differentiable in (–3, 3)

By mean value theorem, $f'(c) = \frac{f(b) - f(a)}{b - a}$

Where $c \in (a, b)$

$\Rightarrow f'(x) = \frac{f(3) - f(-3)}{3 - (-3)}$

$\Rightarrow f(3) = 6f'(x) + f(-3)$

Since $f'(x) \le 2$

$\Rightarrow f(3) \le 6(2) + f(-3)$

$\Rightarrow f(3) \le 12 + 7 \le 19$

$\therefore f(3) \le 19$

⇒ **f(3) is equal to 19 at most.**

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66. (c)

Given, $f'(x_0) = 0$ at $x_0 \in (0,1)$.

∴ x_0 is a stationary or turning point. One stationary point is coming in interval (0,1). Now if $f''(x_0) > 0$ at stationary point, then it will local minimum. Conversely, if $f''(x_0) < 0$, then it will be local maximum.

Since $f''(x_0) > 0$ in all x in interval (0, 1), it follows that at stationary point x_0 also $f''(x_0) > 0$. Therefore, the stationary point x_0 is local minimum. **Hence, there exists only one local minimum in interval (0,1).**

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67. (1)

Condition for orthogonality: -

$m_1 \cdot m_2 = -1$ …(i)

$m_1 \rightarrow$ slope of curve –1 at (u,v)

$m_2 \rightarrow$ slope of curve –2 at (u,v)

Curve 1:

$x^2 + y^2 = 1$

Differentiate above equation with respect to x,

$$\Rightarrow 2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

Slope of curve –1 at (u,v):

$$m_1 = \left.\frac{dy}{dx}\right|_{(u,v)} = -\frac{u}{v} \qquad \ldots(ii)$$

Curve-2:-

$(x-1)^2 + (y-1)^2 = r^2$

Differentiate above equation with respect to x,

$$\Rightarrow 2(x-1) + 2(y-1)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{x-1}{1-y}\right)$$

Slope of curve –2 at (u,v):

$$m_2 = \left.\frac{dy}{dx}\right|_{(u,v)} = -\left(\frac{u-1}{1-v}\right) \qquad \ldots(iii)$$

Since both curves are intersecting at (u, v), therefore

$m_1 \cdot m_2 = -1$

$$\Rightarrow \left(-\frac{u}{v}\right)\cdot\left(\frac{u-1}{1-v}\right) = -1$$

$\Rightarrow u^2 - u = v - v^2$

$\Rightarrow u + v = u^2 + v^2 \qquad \ldots(v)$

Since (u, v) is intersection point of curve 1and 2, therefore it will satisfy the equation of both curve 1 and 2. In equation of curve (i) put x =u and y =v,

$u^2 + v^2 = 1$

From equation (v),

$\Rightarrow \boxed{\mathbf{u + v = 1}}$

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68. **(b)**

Given:

$P(x) = x^4 - 4x^3 + 2 \ldots\ldots(i)$

To determine the number of real roof in open interval (0, 1), we can anlyaze the behaviour of polynomial and its derivative differentiating equation (i) with respect to x.

$P'(x) = 4x^3 - 12x^2$

$\Rightarrow P'(x) = 4x^2(x-3)$

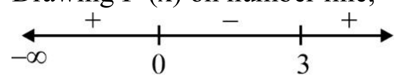
Finding stationary point:

$P'(x) = 0$

$\Rightarrow 4x^2(x-3) = 0$

$\Rightarrow x = 0$ and 3

Drawing P'(x) on number line;



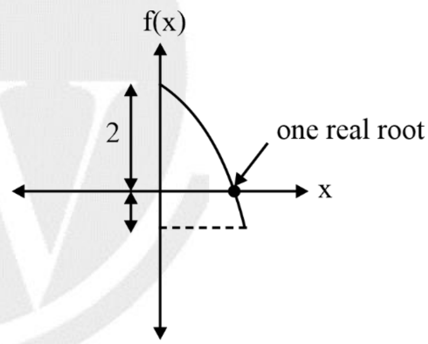
Since in interval (0, 1) P'(x) is negative, therefore function in interval (0, 1) is decreasing function.

Now we will find value of function P(x) at x = 0 and at x = 1.

$P(0) = 0^4 - 4 \times (0)^3 + 2 = 2$ (+ve)

$P(1) = 1^4 - 4 \times (1)^3 + 2 = 1$ (–ve)

Since sign of P(0) and P(1) is opposite and P(x) is decreasing function in inteval [0, 1], therefore graph of p(x) in interval [0, 1] will be as given below.



(0, 1) in interval therefore f(x) has only one real root.

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69. **(a, d)**

Given polynomial:- $ax^3 + bx^2 + cx + d \qquad a \neq 0$

Option a) If d = 0; $ax^3 + bx^2 + cx = 0$

$x(ax^2 + bx + c) = 0$

$\therefore$ x = 0 is a solution. Hence option (a) is correct.

Option b)

If all roots are identical then the given polynomial can be expressed as

$ax^3 + bx^2 + cx + d = (x - r)^3$

$\Rightarrow \quad ax^3 + bx^2 + cx + d = x^3 - 3x^2r + 3xr^2 - r^3$

On comparing, $a = 1$, $b = -3r$, $c = 3r^2$, $d = -r^3$

$\therefore$ The above choice of coefficients can make all roots same. Hence option (b) is incorrect.

Option c)

All 3 roots cannot be complex because complex roots occur in conjugate pairs. Either 0 or 2 complex roots are possible. Hence option (c) is incorrect.

Option d) All coefficient along with c will decide the nature of roots. Hence option (d) is correct.

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70. (a)

$y = f(x) = 3x^2 + 3x + 1$ …(i)

f (x) is continuous and differential function in interval $[-2, 0]$.

Differentiate equation (i) with respe’t to ‘,

$y' = f'(x) = 6x + 3$

Stationary point: [At station‘ry point f ’(x) =‘]

$y' = f'(x) = 6x + 3 = 0$

$\Rightarrow x = -1/2 \in [-2, 0]$

Finding value of f(x) at initial and final point (i.e at $x = -2$ and 0) in stationary point (i.e. at $x = -\frac{1}{2}$) :

$y = f(x) = 3x^2 + 3x + 1$

At $x = -2$:

$f(-2) = 3(-2)^2 + 3(-2) + 1 = 7$

At $x = 0$:

$f(0) = 3(0)^2 + 3(0) + 1 = 1$

At $x = \frac{1}{2}$:

$f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1 = \frac{1}{4}$

$\therefore$ **Maximum and minimum values of *y* in [–2, 0] are 7 and $\frac{1}{4}$ respectively.**

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71. (a)

Given function are

(i) e^{-x}

(ii) $x^2 - \sin x$

(iii) $\sqrt{x^3 + 1}$

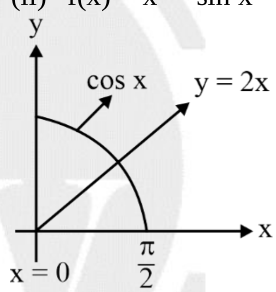
For a function f(x) to be increasing in [0, 1], $f'(x) > 0$ in [0, 1]

(i) let, f(x) e^{-x} :

Then, $f'(x) = - e^{-x}$

Since $f'(x) = - e^{-x}$ is always less than xero therefore f(x) e^{-x} is decreasing function $\forall x \epsilon R$

(ii) $f(x) = x^2 - \sin x \Rightarrow f'(x) = 2x - \cos x$



Since $\cos x > 2x$ for some $x \epsilon [0, 1]$

$\Rightarrow 2x - \cos x < 0$ for some $x \epsilon [0, 1]$

$\therefore x^2 - \sin x$ is not completely increasing in [0, 1]

(iii) $f(x) = \sqrt{x^3 + 1}$

$\Rightarrow f'(x) = \dfrac{3x^2}{2\sqrt{x^3 + 1}} > 0 \ \forall x \epsilon [0, 1]$

$\therefore$ $f(x) = \sqrt{x^3 + 1}$ is the only function that is increasing $\forall x \epsilon [0, 1]$

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72. (5.18 to 5.2)

Given: Diameter (D) = 3cm ; Height (H) = 4 cm

Error (dD or dH) = $\pm$ 0.02 cm

Volume $V = \frac{\pi}{4} D^2 H$

Absolute error in volume;

$$dV = \frac{\partial V}{\partial D}.dD + \frac{\partial V}{\partial H}.dH$$

$$dV = \left(\frac{\pi DH}{2}\right)dD + \left(\frac{\pi}{4}D^2\right)dH$$

$$dV = \left(\frac{\pi}{2}\times 3\times 4\right)\times 0.02 + \left(\frac{\pi}{4}\times 3^2\right)\times 0.02$$

$dV = 1.65\ \pi\ cm^3 = 5.1836\ cm^3$

Absolute error in volume= 5.184 cm^3

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73. (c)

Given: Family of curves $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$

(i) for n = –1, $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-1} = -\frac{y}{x}$

$\Rightarrow \frac{1}{y}dy = \frac{-1}{x}dx$

$\Rightarrow \int \frac{1}{y}dy = \int \frac{-1}{x}dx$

$\Rightarrow \ln y = -\ln x + c$

$\Rightarrow xy = c$

$\therefore$ **$xy = c \rightarrow$ hyperbolas**

(ii) For n = 1; $\frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow ydx = -xdx$

$\Rightarrow \int ydx = \int -xdx$

$\Rightarrow \frac{y^2}{2} = \frac{-x^2}{2} + C \qquad \Rightarrow x^2 + y^2 = C'$

$\therefore \quad x^2 + y^2 = C' \rightarrow$ circles

Hence, the family of curves for n = –1 and n = +1 are hyperbolas and circles respectively.

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74. (a)

Checking for function $\sqrt[x]{x}$:

Let $y = \sqrt[x]{x} = x^{1/x}$

$$\log y = \frac{\log x}{x}$$

$$y = e^{\frac{\log x}{x}}$$

Maxima and minima of y will depends on $\frac{\log x}{x}$

Let $f(x) = \frac{\log x}{x}$

$$f'(x) = \frac{x.\frac{1}{x} - \log x.1}{x^2} = \frac{1-\log x}{x^2}$$

At stationary point:

$f'(x) = 0$

$\frac{1-\log x}{x^2} = 0$

$\log x = 1$

$x = e$

Now,

$$f''(x) = \frac{x^2.\left(-\frac{1}{x}\right) - (1-\log x).2x}{x^4} = \frac{-3x + 2x\log x}{x^4}$$

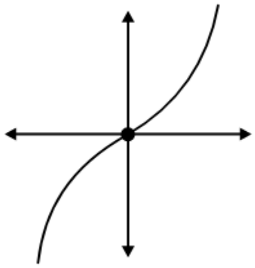
At x = e;

$$f''(e) = \frac{-3e + 2e\log e}{e^4} = -\frac{e}{e^4} < 0$$

Since at stationary point x = e, $f''(x)$ is less than 0, hence, the function $\sqrt[x]{x}, (x > 0)$ has the global maxima at x = e.

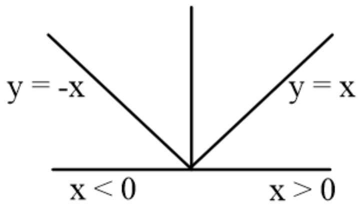
Hence statement in options (a) is incorrect.

Checking for function x^3:



From graph of x^3, we infer it has neither global maxima nor global minima.

Checking for function $|x|$:



From graph of $|x|$, we conclude it has global minimum at x = 0 .

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75. (c)

Taylor's series expansion about x for f(x + h) is

f(x + h) = f(x) + (x + h – x) f'(x) +

$$f(x+h) = f(x) + (x+h-x)f'(x) + \frac{(x+h-x)^2}{2!}f''(x) + \ldots$$

$$\therefore \boxed{f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \ldots}$$

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76. (d)

$f(x) = x^3$

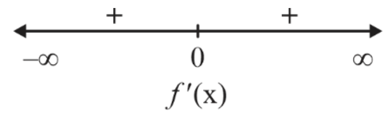
$f'(x) = 3x^2$

For stationary points,

$f'(x) = 3x^2 = 0$

$\Rightarrow$ x = 0 is a stationary point.

f′(x) on number line:

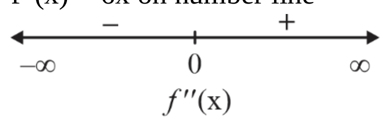


$\therefore$ f′(x) is not changing its sign at x = 0, $\therefore$ f(x) at x = 0 is neither local maxima nor minima now,

$$f''(x) = \frac{d}{dx}(3x^2) = 6x$$

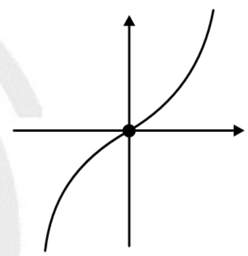
At x = 0, f″(x) = 0

f″(x) = 6x on number line



$\because$ f″(x) is changing its sign at

x = 0, hence $f(x) = x^3$ has point of inflection at x = 0



From the graph of x^3, we can also conclude that at x = 0; point of inflection occurs because nature of curve is changing at x = 0.

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77. (11.5 to 12.5)

$f(x) = 3x^3 - 7x^2 + 5x + 6$ …(i)

f (x) is continuous and differential function in interval [0, 2].

Differentiate equation (i) with respect'to x,

$f'(x) = 9x^2 - 14x + 5$

Stationary point: [At station'ry point f ′(') =0]

$f'(x) = 9x^2 - 14x + 5 = 0$

$\Rightarrow$ (9x – 5) (x – 1) = 0

$\Rightarrow \quad x = 1$ and $\frac{5}{9} \in [0, 2]$

Finding value of f(x) at initial and final point (i.e at x = 0 and 2) in stationary points (i.e. at x = 1 and $\frac{5}{9}$) :

$f(x) = 3x^3 - 7x^2 + 5x + 6$

At x = 0:

$f(0) = 3(0)^3 - 7(0)^2 + 5(0) + 6 = 6$

At x = 2:

$f(2) = 3(2)^3 - 7(2)^2 + 5(2) + 6 = 12 \leftarrow$ maximum

At x = 1:

$f(1) = 3(1)^3 - 7(1)^2 + 5(1) + 6 = 7$

At $x = \frac{5}{9}$:

$$f\left(\frac{5}{9}\right) = 3\left(\frac{5}{9}\right)^3 - 7\left(\frac{5}{9}\right)^2 + 5\left(\frac{5}{9}\right) + 6 = 7 \cdot 13$$

Hence, Maximum value of f (x) in interval [0, 2] is 12.

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78. (7)

$2x^2 + y^2 = 34$ …..(1)

$x + 2y = 11$

$y = \frac{11 - x}{2}$ … (2)

From value of y from equation (2) to equation (1),

$$2x^2 + \left(\frac{11 - x}{2}\right)^2 = 34$$

$\Rightarrow \quad 8x^2 + 121 + x^2 - 22x = 136$

$\Rightarrow \quad 9x^2 - 22x - 15 = 0$

$\Rightarrow \quad x = \frac{-10}{18}, 3$

Discarding x = –10/18 because it is not on integer.

Hence $x = 3$.

Put value of x in equation (2),

$$y = \frac{11 - x}{2} = \frac{11 - 3}{2} = 4$$

Hence,

$x + y = 3 + 4$

$\Rightarrow \quad \boxed{x + y = 7}$

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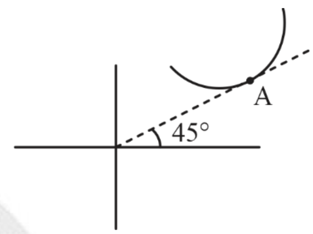
79. (a)

Function is y = x ln x …(i)

Say point is A at which inclination of tangent with the x-axis is 45°.



Slope at point A = tan 45°

$\frac{dy}{dx} = \tan 45°$

$\frac{d}{dx}(x \ln x) = 1$

$x.\frac{1}{x} + \ln x.1 = 1$

$1 + \ln x = 1$

ln x = 0

x = 1

At x = 1 from equation (i),

y = x ln x = 1 ln 1 = 1 × 0 = 0

∴ **Point is (1, 0)**

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80. (c)

Given $f(x) = e^{x + x^2}$

The taylor series expansion is written about x= 0

$f(x) = e^{x + x^2}$

$f'(x) = e^{x + x^2}(1 + 2x)$

$\Rightarrow \quad f'(x)|_{x=0} = f'(0) = 1$

$$f''(x) = e^{x+x^2}(2) + (1+2x)^2 e^{x+x^2}$$

$\Rightarrow \quad f''(x) = e^{x+x^2}\{4x^2 + 4x + 3\}$

$\Rightarrow \quad f''(x)|_{x=0} = f''(0) = 3$

$$f'''(x) = \begin{bmatrix} e^{x+x^2}\{8x+4\} + \\ (4x^2+4x+3)(1+2x)e^{x+x^2} \end{bmatrix}$$

$\Rightarrow \quad f'''(x)|_{x=0} = f'''(0) = 4 + 3 = 7$

$\therefore$ coefficeint of $x^2 = \frac{f''(0)}{2!} = \frac{3}{2!} = \frac{3}{2}$

Coefficient of $x^3 = \frac{f'''(0)}{3!} = \frac{7}{6}$

$\therefore$ $f(x) = 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$

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81. (–100)

Given, $f(x) = \frac{1}{3}x(x^2-3)$; $-100 \le x \le 100$

$$f(x) = \frac{1}{3}(x^3 - 3x)$$

Calculating f'(x):

$$f'(x) = \frac{1}{3}(3x^2 - 3) = x^2 - 1$$

At stationary point:

$$f'(x) = 0$$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x = 1, -1 \in [-100, 100]$

Finding value of f(x) at initial and final point (i.e at x = −100 and 100) in stationary point (i.e. at x =1 and –1) :

$$f(x) = \frac{1}{3}x(x^2-3)$$

At x = −100,

$$f(-100) = \frac{1}{3}(-100)((-100)^2 - 3) = -333233.33$$

At x = 100,

$$f(100) = \frac{1}{3}(100)((100)^2 - 3) = 333233.33$$

At x = 1,

$$f(1) = \frac{1}{3}(1)((1)^2 - 3) = -0.667$$

At x = –1,

$$f(-1) = \frac{1}{3}(-1)((-1)^2 - 3) = 0.667$$

$\therefore$ Minimum value of f(x) in [–100, 100]

= min{f(–100), f(1), f(–1), f(100)}

= min {–333233.33, 0.667, –0.667, 333233.33}

$\therefore$ Min of f(x) in [–100, 100] is –333233.33 **at x = –100**

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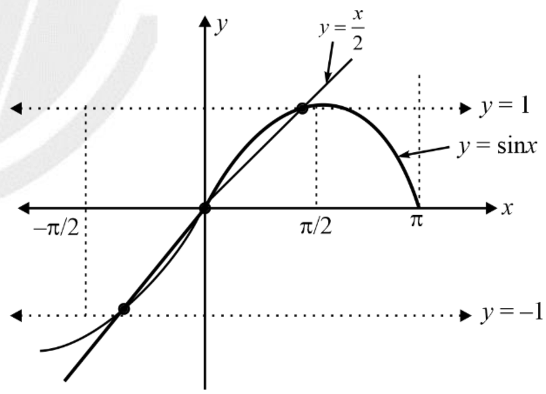
82. (c)

Given, $\sin x = \frac{x}{2}$



The curves $y = \sin x$ and $y = \frac{x}{2}$ intersect at 3 locations.

$\therefore$ Number of solutions of $\sin x = \frac{x}{2}$ is 3.

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83. (0 to 0)

f (x) = x (x – 1)(x – 2)

$\Rightarrow$ f(x) = $x^3 - 3x^2 + 2x$ …(i)

f (x) is continuous and differential function in interval [1, 2].

Differentiate equation (i) with respec‘ to x,

f '(x) = $3x^2 - 6x + 2$ …(ii)

Stationary point: [At station‘ry point f '‘x) =0]

f '(x) = $3x^2 - 6x + 2 = 0$

$$\Rightarrow x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$\Rightarrow x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$\Rightarrow x = 1 + \frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 1 + \frac{1}{\sqrt{3}} \in [1,2] \;\&\; x = 1 - \frac{1}{\sqrt{3}} \notin [1,2]$$

Hence stationary point in interval [–3, 3] is

$$\Rightarrow x = 1 + \frac{1}{\sqrt{3}}.$$

Finding value of f(x) at initial and final point (i.e at x = 1 and 2) in stationary points

(i.e. at $x = 1 + \frac{1}{\sqrt{3}}$) :

f (x) = x (x – 1) (x – 2)

At x = 1:

f (1) = 1 (1 – 1) (1 – 2) =0

At x = 2:

f (2) = 2 (2 – 1) (2 – 2) = 0

At $x = 1 + \frac{1}{\sqrt{3}}$:

$$f\left(1 + \frac{1}{\sqrt{3}}\right) = \left(1 + \frac{1}{\sqrt{3}}\right)\left(1 + \frac{1}{\sqrt{3}} - 1\right)\left(1 + \frac{1}{\sqrt{3}} - 2\right)$$

$$\Rightarrow f\left(1 + \frac{1}{\sqrt{3}}\right) = -0.3849$$

Hence maximum value of f (x) is 0.

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84. (b)

Given f(x) = $x^3 - 3x^2 + 1$ in [–1, 3]

$\Rightarrow$ f'(x) = $3x^2 - 6x = 3x(x - 2)$

For f(x) to be increasing,

f'(x) > 0

$\Rightarrow$ 3x(x – 2) > 0

$\Rightarrow$ x > 0 and x >2 or x < 0 and x < 2

$\Rightarrow$ x > 2 or x < 0 [In interval -1 to 3]

$\Rightarrow (2,3) \text{ or } (-1,0)$

For f(x) to be decreasing,

f'(x) < 0

$\Rightarrow$ 3x(x – 2) < 0

$\Rightarrow$ x < 0 and x >2 or x > 0 and x < 2

$\Rightarrow$ 0 < x <2 [In interval -1 to 3]

$\Rightarrow (0,2)$

Therefore, in interval -1 to 3, f(x) is initially increasing in $(-1,0)$ then decreasing in $(0,2)$ and again increasing in $(2,3)$.

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85. (b)

By Taylor's expansion of f(x) about x = a

$$f(x) = f(a) + (x - a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots.$$

About x = 0 $\Rightarrow a = 0$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots.. \quad \text{….(i)}$$

Now,

f(x) = $x^3 - 3x^2 - 5 = 0^3 - 3(0)^2 - 5 = -5$

f '(0) = $3x^2 - 6x = 3(0)^2 - 6(0) = 0$

f '(0) = 6x – 6 = 6(0) – 6 = –6

Put the values in (i)

$$f(x) = -5 + (x \times 0) + \frac{x^2}{2}(-6) = -3x^2 - 5$$

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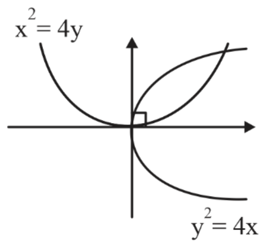
86. (d)



Slope of $x^2 = 4y$:

$x^2 = 4y \Rightarrow y = \dfrac{x^2}{4}$

Slope, $\dfrac{dy}{dx} = \dfrac{2x}{4} = \dfrac{x}{2}$

Slope at (0, 0) = $\dfrac{x}{2} = \dfrac{0}{2} = 0$ (hence tangent at origin will be horizontal)

Slope of $y^2 = 4x$:

$y^2 = 4x \Rightarrow y = 2\sqrt{x}$

Slope, $\dfrac{dy}{dx} = 2.\dfrac{1}{2\sqrt{x}} = \dfrac{1}{\sqrt{x}}$

Slope at (0, 0) = $\dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{0}} = \infty$ (hence tangent at origin will be vertical)

Hence angle between the curves at (0, 0) is 90°

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87. (d)

$f(x) = x^2 - 4x + 2$

$f'(x) = 2x - 4$ and

$f''(x) = 2$

Find stationary points using $f'(x) = 0$

$f'(x) = 2x - 4 = 0$

$x = 2$

At $x = 2$, $f''(x) = 2 > 0 \Rightarrow$ hence $x = 2$ is minima.

Therefore,

$f_{min} = x^2 - 4x + 2$ at $x = 2$

$f_{min} = 2^2 - 4(2) + 2$

$f_{min} = -2$

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88. (–5.1 to –4.9)

Given

$f(x) = 2x^3 - 3x^2$

$\Rightarrow f'(x) = 6x^2 - 6x$

For stationary point,

$f'(x) = 0$

$\Rightarrow 6x^2 - 6x = 0$

$\Rightarrow x = 0, 1$

Considering f (0), f (1), f (–1), f (2)

$f(0) = 2(0)^3 - 3(0)^2 = 0$

$f(1) = 2(1)^3 - 3(1)^2 = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 = -5$

$f(2) = 2(2)^3 - 3(2)^2 = 4$

Global minimum of f(x)

= min {f (0), f (1), f (–1), f (2)}

= min {0, –1, –5, 4}

= –5

$\therefore$ **Global minimum in [–1, 2] is –5.**

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89. (2)

$$\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = 0 + 1\left(\frac{1}{2}\right)^1 + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \ldots$$

$$\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)\left\{1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots\right\} \quad \ldots\text{(i)}$$

For $|x| < 1$, $1 + 2x + 3x^2 + 4x^3 + \ldots = (1 - x)^{-2}$

$$\Rightarrow \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)\left\{\left(1 - \frac{1}{2}\right)^{-2}\right\}$$

$$\Rightarrow \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right) \times 2^2 = 2$$

$$\boxed{\therefore \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = 2}$$

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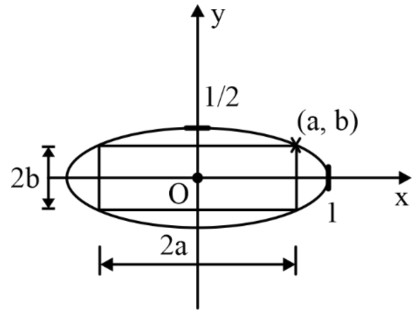
90. (1)

Given ellipse is $x^2 + 4y^2 = 1$



$$\Rightarrow \frac{x^2}{1} + \frac{y^2}{\left(\frac{1}{4}\right)} = 1$$

$$\Rightarrow \frac{x^2}{1^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

Let 2a, 2b are lengths are breadths of the rectangle.

$\therefore$ Area of the rectangle = A = 4ab

And also $a^2 + 4b^2 = 1$

$$\Rightarrow A = 4\sqrt{1-4b^2}\cdot b$$

For 'A' to be maximum

$$\frac{dA}{db} = 0 \Rightarrow \frac{d}{db}\left(b\cdot\sqrt{1-4b^2}\right) = 0$$

$$\Rightarrow b\frac{1}{2\sqrt{1-4b^2}} \times (-8b) + \sqrt{1-4b^2}\cdot(1) = 0$$

$$\Rightarrow -\frac{4b^2}{\sqrt{1-4b^2}} + \sqrt{1-4b^2} = 0$$

$$\Rightarrow \frac{-8b^2+1}{\sqrt{1-4b^2}} = 0$$

$$\Rightarrow 1 - 8b^2 = 0$$

$$\Rightarrow b = \frac{1}{2\sqrt{2}}$$

$$\therefore 2b = \frac{1}{\sqrt{2}}$$

Since $a^2 + 4b^2 = 1$

$$\Rightarrow a^2 + \frac{1}{2} = 1 \Rightarrow a = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 2a = \sqrt{2}$$

$$\boxed{\therefore A_{max} = 4ab = 4 \times \frac{1}{\sqrt{2}}\cdot\frac{1}{2\sqrt{2}} = 1 \text{ sq units}}$$

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91. (b)

Given $f(x) = e^{-x}(x^2 + x + 1)$

$$= e^{-x}\left\{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right\}$$

For $x > 0$, $f(x) > 0$

Consider $f'(x) = e^{-x}\{2x+1\} + (x^2+x+1)(-e^{-x})$

$$= e^{-x}\{2x+1-x^2-x-1\}$$

$$= e^{-x}\{x - x^2\} = e^{-x}x(1-x)$$

For $x > 0$; $e^{-x} > 0$;

And for $0 < x < 1$; $x(1-x) > 0$

and for $x > 1$; $x(1-x) < 0$

Therefore,

For $0 < x < 1$; $f'(x) > 0$

For $x > 1$, $f'(x) < 0$

f(x) is increasing in $0 < x < 1$ and decreasing in $x > 1$.

$\therefore$ Option 'B' is correct

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92. (b)

Given $f(x) = 1 - x^2 + x^3$; [–1, 1]

By mean value theorem of f(x) in the interval [a, b]

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

for $f(x) = 1 - x^2 + x^3$

$\Rightarrow f'(x) = 3x^2 - 2x$

$\Rightarrow$ By mean value theorem

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)}$$

$\Rightarrow 3c^2 - 2c = \frac{1-(-1)}{1-(-1)}$

$\Rightarrow 3c^2 - 2c - 1 = 0$

$\Rightarrow 3c^2 - 3c + c - 1 = 0$

$\Rightarrow 3c(c-1) + 1(c-1) = 0 \Rightarrow c = \frac{-1}{3}$ and $c = 1$

Since $c \in (-1, 1)$, the mean value 'c' is equal to $\frac{-1}{3}$

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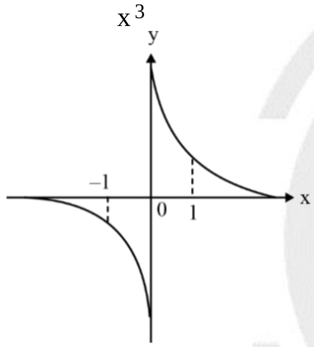
93. (c)

Given $f(x) = x^{-\frac{1}{3}} = \frac{1}{x^{\frac{1}{3}}}$



(a) At x= 0; f(x) is not bounded
As $x \to 0$; $f(x) \to \infty$

(b) f(x) is not continuous at x = 0

(c) $A = \int_{-1}^{1} x^{-1} dx = 3 \leftarrow$ finite

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94. (d)

If $f'(x_0) = 0$

If $f''(x_0) > 0 \Rightarrow$ local minimum

If $f''(x_0) < 0 \Rightarrow$ local maximum

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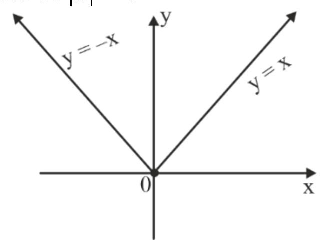
95. (a)

At x = 0, the function |x| has a minimum and minimum of |x| = 0



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96. (a)

Given,

$f(t) = e^{-t} - 2e^{-2t}$

Calculating $f'(t)$,

$f'(t) = -e^{-t} + 4e^{-2t}$

At stationary point:

$\Rightarrow f'(t) = -e^{-t} + 4e^{-2t} = 0$

$\Rightarrow e^{-t}(-1 + 4e^{-t}) = 0$

$\Rightarrow e^{-t} = 0 \Rightarrow t \to \infty \notin [0, \infty)$

or $-1 + 4e^{-t} = 0$

$\Rightarrow e^{-t} = \frac{1}{4} \Rightarrow t = \log_e 4$

Calculating $f''(t)$

$f''(t) = e^{-t} - 8e^{-2t} = e^{-t}\left(1 - 8e^{-t}\right)$

At $t = \log_e 4$,

$f''(t) = e^{-\log_e 4}\left(1 - 8e^{-\log_e 4}\right) = \frac{1}{4}\left(1 - \frac{8}{4}\right)$

$\Rightarrow f''(t) = -\frac{1}{4} < 0$

Since at stationary point $t = \log_e 4$, $f''(t)$ is less than 0,

$\boxed{\therefore f(t) \text{ has maximum at } t = \log_e^4}$

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97. (b)

Given:

$f(x) = 2x^3 - 9x^2 + 12x - 3$; $0 \le x \le 3$ i.e. $x \in [0,3]$

Calculating $f'(x)$,

$$f'(x) = 6x^2 - 18x + 12$$

Stationary point: [At stationary point f '(x) =0]

$$f'(x) = 0$$

$\Rightarrow 6x^2 - 18x + 12 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1, 2 \in [0,3]$

Finding value of f(x) at initial and final point (i.e at *x* = 0 and 3) and in stationary points (i.e. at x = 1 and 2) :

$f(x) = 2x^3 - 9x^2 + 12x - 3$

At x =0,

$f(0) = 2(0)^3 - 9(0)^2 + 12(0) - 3 = -3$

At x =3,

$f(3) = 2(3)^3 - 9(3)^2 + 12(3) - 3 = 6$

At x =1,

$f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 3 = 2$

At x =2,

$f(2) = 2(2)^3 - 9(2)^2 + 12(2) - 3$

$\therefore$ Maximum of f(x) in [0, 3]

= maximum {f(0), f(1), f(2), f(3)}

= maximum {–3, 2, 1, 6} = 6

$$\boxed{\text{Maximum of } f(x) \text{ in}[0, 3] = 6}$$

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98. (d)

$$\sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \ldots\infty$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\infty \quad \ldots\text{(i)}$$

Now, expansion of e^x,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\infty$$

For x =1,

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\infty \quad \ldots\text{(ii)}$$

Comparing equation (i) and (ii),

$$\boxed{\sum_{n=0}^{\infty} \frac{1}{n!} = e}$$

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99. (a)

Expansion of sin x is:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots..$$

And expansion of cos x is:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots..$$

$\therefore$ expansion of 3 sin x + 2 cos x is:

$$= \begin{bmatrix} 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots..\right) \\ +2\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots..\right) \end{bmatrix}$$

$$= 2 + 3x - x^2 - \frac{x^3}{2} + \ldots\ldots.$$

$$\therefore \quad \boxed{3\sin x + 2\cos x = 2 + 3x - x^2 - \frac{x^3}{2} + \ldots\ldots.}$$

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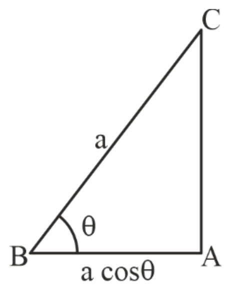
100. (c)

Given sum of length of a side and hypotenuse is constant.



$\Rightarrow$ $a + a\cos\theta = \text{constant} = c$

$\Rightarrow$ $a(1 + \cos\theta) = c$

$\Rightarrow$ $a = \dfrac{c}{(1+\cos\theta)}$ …(i)

$\therefore$ Area of triangle (A)

$$A = \frac{1}{2} \times a\cos\theta \times a\sin\theta$$

$$A = \frac{a^2}{2} \times \sin\theta\cos\theta$$

$$A = \frac{a^2}{4} \times \sin 2\theta$$

From equation (i),

$$A = \frac{c^2}{4(1+\cos\theta)^2} \cdot \sin 2\theta$$

For area to be maximum,

$$\frac{d}{d\theta}(A) = 0$$

$$\frac{d}{d\theta}\left(\frac{c^2 \cdot \sin 2\theta}{4(1+\cos\theta)^2}\right) = 0$$

$\Rightarrow$ $\dfrac{d}{d\theta}\left(\dfrac{\sin 2\theta}{(1+\cos\theta)^2}\right) = 0$

$\Rightarrow$ $(1 + \cos\theta)^2 . 2. \cos 2\theta - \sin 2\theta . 2(1 + \cos\theta) .(-\sin\theta) = 0$

$\Rightarrow$ $2(1 + \cos\theta)\{\cos 2\theta (1 + \cos\theta) + \sin\theta . \sin 2\theta\} = 0$

$\Rightarrow$ $2(1 + \cos\theta)\{\cos 2\theta + \cos 2\theta \cos\theta + \sin 2\theta. \text{Sin } \theta\} = 0$

$\Rightarrow$ $2(1 + \cos\theta)\{\cos 2\theta + \cos\theta\} = 0$

$\Rightarrow$ $\cos 2\theta + \cos\theta = 0$

$\Rightarrow$ $2\cos^2\theta - 1 + \cos\theta = 0$

$\Rightarrow$ $\cos\theta = 0.5, -1$

$\Rightarrow$ $\theta = 60°, 180°$

$\therefore$ For area to be maximum, angle between sides $= 60°$.

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101. (–1.00 to –0.94)

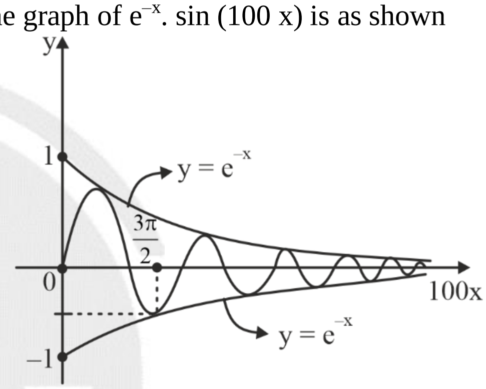
Let $f(x) = e^{-x}. \sin 100x$

We have for $x > 0$,

$-1 \le \sin(100x) \le 1$

$\Rightarrow$ $-e^{-x} \le e^{-x}. \sin(100x) \le e^{-x}$

The graph of $e^{-x}. \sin(100x)$ is as shown



$\therefore$ Minimum value of $e^{-x}. \sin(100x)$ is at

$$100x = \frac{3\pi}{2} \Rightarrow x = \frac{3\pi}{200}$$

$\Rightarrow$ Min value $= e^{-\left(\frac{3\pi}{200}\right)} \cdot \sin\left(100 \times \dfrac{3\pi}{200}\right)$

$$= e^{-\frac{3\pi}{200}} \cdot \sin\left(\frac{3\pi}{2}\right) = -0.9539$$

$\therefore$ $\boxed{\text{Best approximate minimum value} = -0.954}$

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102. (2)

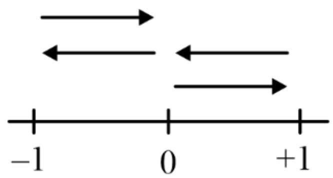
We know, $v = \dfrac{dx}{dt}$

$\therefore$ The displacement of particle $\int_0^x dx = \int_0^t v\,dt$

$$x = \int_0^t \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)dt$$

$$\Rightarrow \quad x = \left[\sin\frac{\pi}{2}t\right]_0^t$$

$$\Rightarrow \quad x = \sin\frac{\pi}{2}t$$



t = 0

$\therefore$ At t = 0, x = 0

t = 1, x = 1

t = 2, x = 0

t = 3, x = – 1

In 3 sec, distance covered = 3 units

In 3 sec, displacement is = –1 units

$\therefore$ Required difference = 3 – |–1| = 2 units

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103. (b)

$f(x) = x^3 - 3x^2 - 24x + 100$

f (x) is continuous and differential function in interval [–3, 3].

Stationary p'ints:

$f'(x) = 0$

$\Rightarrow 3x^2 - 6x - 24 = 0$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x + 2)(x - 4) = 0$

$\Rightarrow x = -2, 4$

$\Rightarrow x = -2 \in [-3,3]$ & $x = 4 \notin [-3,3]$

Hence stationary point in interval [–3, 3] is x = –2.

Finding value of f(x) at initial and final point (i.e at x = –3and 3) in stationary points (i.e. at x = –2) :

$f(x) = x^3 - 3x^2 - 24x + 100$

$\Rightarrow f(-3) = (-3)^3 - 3(-3)^2 - 24(-3) + 100 = 118,$

$\Rightarrow f(3) = 3^3 - 3(3)^2 - 24(3) + 100 = 28, \leftarrow$ minimum

$\Rightarrow f(-2) = (-2)^3 - 3(-2)^2 - 24(-2) + 100 = 128,$

Hence minimum value of f (x) in interval [–3, 3] = 28

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104. (c)

$f(x) = (x-1)^{2/3}$

f (x) is continuous function.

Differentiating above equation with respect to x,

$$f'(x) = \frac{2}{3(x-1)^{1/3}} \qquad \ldots(i)$$

f (x) is not differential at point where x – 1 = 0 i.e. x = 1.

$f'(x)$ is never zero.

Here x = 1is critical point.

Now, when $x < 1$, $f'(x) < 0$ and when $x > 1$, $f'(x) > 0$. Therefore, in interval $(-\infty,1)$ $f(x)$ is decreasing function and in interval $(1,\infty)$ $f(x)$ is increasing function.

$\therefore$ at x = 1 f (x) has minimum.

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105. (a)

$f(x) = xe^{-x}$

On Differentiating,

$f'(x) = -x.e^{-x} + e^{-x}.1$

$\Rightarrow f'(x) = e^{-x}(1-x) \qquad \ldots(i)$

Stationary point:

$f'(x) = 0$

$\Rightarrow e^{-x}(1-x) = 0$

$\Rightarrow x = 1$

Differentiating equation (i),

$$f''(x) = \frac{d}{dx}\left\{e^{-x}(1-x)\right\}$$

$\Rightarrow \quad f''(x) = -e^{-x}(1-x) + e^{-x}(-1)$

$\Rightarrow \quad f''(x) = -e^{-x}(1-x+1)$

$\Rightarrow \quad f''(x) = -e^{-x}(2-x)$

At $x = 1$

$$f''(1) = -e^{-1}(2-1) = -\frac{1}{e} < 0$$

Hence x =1 is maxima.

Hence maximum value of f(x)

= f (1) = e^{-1}

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106. (b)

Given: y = f (x) = $5x^2 + 10x$ …(i)

Since y is continuous in interval [1, 2] & is differentiable in interval (1, 2).

Thus, both the con'ition of Lagrange's Mean Value Theorem is satisfying, therefore there exist at least one value c ∈ (1, 2) such that

$$f'(c) = \frac{f(b) - f(a)}{b-a} \quad \text{…(ii)}$$

Here, a =1 and b = 2, from equation (ii)

$$\Rightarrow \quad f'(c) = \frac{f(2) - f(1)}{2-1} \quad \text{…(iii)}$$

Now, from equation (i);

f (1) = $5(1)^2 + 10(1) = 15$

f (2) = $5(2)^2 + 10(2) = 40$

Put value of f (1) and f (2) in equation (iii),

$$\therefore f'(c) = \frac{40-15}{1} = 25$$

Hence in interval (1, 2) atleast point point will exist,

where $\boxed{\frac{dy}{dx} = f'(c) = 25}$.

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107. (c)

Given error in sinθ ≈ θ holds is < 10%.

Let 'θ' be the value with in which error is less than 10%.

For small values of θ, $\sin\theta \approx \theta - \frac{\theta^3}{3!}$

$$\therefore \quad \frac{\sin\theta - \left(\theta - \frac{\theta^3}{3!}\right)}{\sin\theta} \times 100 < 10$$

$$\Rightarrow \quad \sin\theta - \theta + \frac{\theta^3}{3!} < 0.1 \sin\theta$$

$$\Rightarrow \quad \frac{\sin\theta}{\theta} - 1 + \frac{\theta^2}{3!} < 0.1\frac{\sin\theta}{\theta}$$

As θ is small, $\theta \to 0 \Rightarrow \frac{\sin\theta}{\theta} \to 1$

$$\Rightarrow \quad 1 - 1 + \frac{\theta^2}{6} < 0.1(1)$$

$$\Rightarrow \quad \theta^2 < 0.6 \Rightarrow \theta < \sqrt{0.6}$$

∴ 'θ' value should be less than $\sqrt{0.6} \cong 0.7746$

∴ $\theta \cong 44.38°$ (Close to 50°).

∴ For error to be less than 10%, θ ≤ 50°.

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108. (c)

Given:

$f(x) = x^3 - 9x^2 + 24x + 5$ …(i)

Differentiate equation (i) with respect to x,

$f'(x) = 3x^2 - 18x + 24$

Stationary point: [At stationary point f '(x) =0]

$f'(x) = 0$

$\Rightarrow \quad 3x^2 - 18x + 24 = 0$

$\Rightarrow \quad x^2 - 6x + 8 = 0$

$\Rightarrow \quad (x-2)(x-4) = 0$

$\Rightarrow \quad x = 2$ (or) $x = 4 \in [1, 6]$

Finding value of f(x) at initial and final point (i.e at *x* = 1 and 6) and in stationary points (i.e. at x = 2 and 4) :

$f(x) = x^3 - 9x^2 + 24x + 5$

At x =1,

$f(1) = (1)^3 - 9(1)^2 + 24(1) + 5 = 21$

At x =6,

$f(6) = (6)^3 - 9(6)^2 + 24(6) + 5 = 41$

At x =2,

$f(2) = (2)^3 - 9(2)^2 + 24(2) + 5 = 25$

At x =4,

$f(4) = (4)^3 - 9(4)^2 + 24(4) + 5 = 25$

Maximum of f(x) in [1, 6] = max{f (2), f (4), f (1), f (6)}

= max{21, 25, 21, 41}= 41

$\boxed{\therefore \text{ Maximum of } x^3 - 9x^2 + 24x + 5 \text{ in } [1, 6] \text{ is } 41}$

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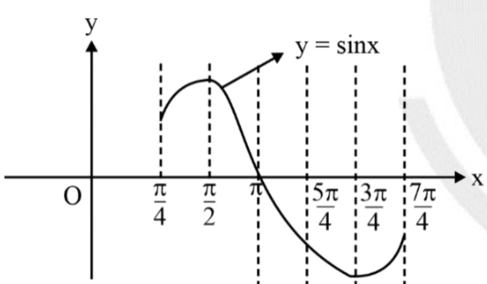
109. (b)

Given f(x) = sinx; $x \in \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$



$\therefore$ f(x) has one minima at $x = \frac{3\pi}{2}$

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110.(d)

Given $f(x) = x^3 + 1$

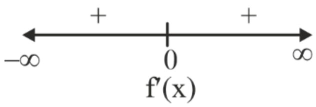
$\Rightarrow \quad f'(x) = 3x^2$

For stationary points,

$f'(x) = 3x^2 = 0$

$\Rightarrow \quad$ x = 0 is a stationary point.

f′ (x) on number line:



$\because$ f′(x) is not changing it's sign at x = 0,

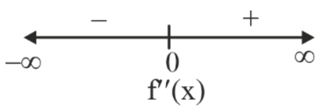
$\therefore$ f(x) at x = 0 is neither local maximum nor minimum.

Now,

$$f''(x) = \frac{d}{dx}\left(3x^2\right) = 6x$$

at x = 0, $f''(x) = 0$

$f''(x) = 6x$ on number line.



$\because$ f″ (x) is changing its sign at x = 0, hence $f(x) = x^3 + 1$ has point of inflection at x = 0.

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111. (c)

$f(x) = x^3 - 9x^2 + 24x + 5 \qquad \ldots$(i)

f (x) is continuous and differential function in interval [1, 6].

Differentiate equation (i) with respect to x,

$f'(x) = 3x^2 - 18x + 24$

Stationary point: [At s'ationary point f '(x) =0]

$f'(x) = 3x^2 - 18x + 24 = 0$

$\Rightarrow \quad x^2 - 6x + 8 = 0$

$\Rightarrow \quad (x - 2)(x - 4) = 0$

$\Rightarrow \quad x = 2, 4 \in [1, 6]$

Finding value of f(x) at initial and final point (i.e at *x* = 1 and 6) in stationary points (i.e. at x = 2 and 4):

$f(x) = x^3 - 9x^2 + 24x + 5$

At x = 1:

$f(1) = 1^3 - 9(1)^2 + 24(1) + 5 = 21$

At x = 6:

$f(6) = 6^3 - 9(6)^2 + 24(6) + 5 = 41 \leftarrow$ maximum

At x = 2:

$f(2) = 2^3 - 9(2)^2 + 24(2) + 5 = 25$

At x = 4:

$f(4) = 4^3 - 9(4)^2 + 24(4) + 5 = 21$

Values of f (x) at boundaries

$\therefore$ **Maximum value of f (x) occurs at x = 6 and is equal to 41.**

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112. (c)

f (x) = 2x –x^2 + 3 …(i)

Differentiate equation (i) ‘ith respect to x,

f '(x) = 2 – 2x …(ii)

Stationary point:‘[At stationar‘ point f '(x) =0]

f '(x) = 2 – 2x = 0

$\Rightarrow$ x = 1

Differentiate equation (ii) “ith respect to x,

f " (x“ = –2

At x = 1:

f " (x) = –2 < 0

Since at st“tionary point x = 1, f " (x) < 0

$\therefore$ **At x = 1, f (x) is a maximum**

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113. (c)

By McLaurin’s series, expansion of e^x about x = 0 is

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$$

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114.(b)

Given: f (θ) = sin θ

The general Taylor series expansion of sin θ about θ = 0 is given by

$$\boxed{\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\infty}$$

Useful series

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\infty$$

$$e^\theta = 1 + \frac{\theta}{1!} + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \cdots\infty$$

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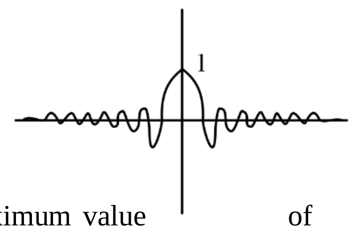
115. (d)

$$f(t) = \frac{\sin t}{t}$$



From the graph of $\frac{\sin t}{t}$ we can identify that maximum value of function lies at t tends to zero.

Note: The maximum of the function can’t be determined from the standard maxima and minima concept since the point x=0 is a critical point.

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116. (a)

Given f(x, y) = $4x^2 + 6y^2 - 8x - 4y + 8$

$$p = \frac{\partial f}{\partial x} = 8x - 8$$

$$q = \frac{\partial f}{\partial y} = 12y - 4$$

$$r = \frac{\partial^2 f}{\partial x^2} = 8$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(12y - 4) = 0$$

$$t = \frac{\partial^2 f}{\partial y^2} = 12$$

For stationary point: $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

$$\frac{\partial f}{\partial x} = 0 \Rightarrow 8x - 8 = 0 \Rightarrow x = 1$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow 12y - 4 = 0 \Rightarrow y = 1/3$$

Hence stationary point is (1, 1/3)

Now,

$rt - s^2 = 8 \times 12 - 0^2 = 96 > 0$

and $r = 8 > 0$

Since $rt - s^2 > 0$ and $r > 0$, therefore stationary point (1, 1/3) is minima. And at stationary point (1, 1/3) function takes minimum value:

$f_{min} = 4x^2 + 6y^2 - 8x - 4y + 8$

$$f_{min} = 4(1)^2 + 6\left(\frac{1}{3}\right)^2 - 8(1) - 4\left(\frac{1}{3}\right) + 8$$

$$f_{min} = 4 + \frac{6}{9} - 8 - \frac{4}{3} + 8 = 4 - \frac{2}{3}$$

$$\boxed{f_{min} = \frac{10}{3}}$$

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117. (a)

Given: $e^y = x^{1/x} \Rightarrow y = \ln x^{1/x} \Rightarrow y = \frac{1}{x}\ln x$

Calculating y',

$$y' = \frac{1}{x}\left(\frac{1}{x}\right) + (\ln x)\left(\frac{-1}{x^2}\right) \quad \Rightarrow \quad y' = \frac{1 - \ln x}{x^2}$$

At stationary point:

$$y' = 0 \quad \Rightarrow \frac{1 - \ln x}{x^2} = 0 \Rightarrow 1 - \ln x = 0$$

$\Rightarrow \quad x = e$

Calculating y''

$$y'' = \frac{x^2\left(\frac{-1}{x}\right) - (1 - \ln x)2x}{x^4} = \frac{-x - 2x + 2x\ln x}{x^4}$$

$$\Rightarrow \quad y'' = \frac{-3x + 2x\ln x}{x^4}$$

At $x = e$,

$$\Rightarrow \quad y''\big|_{x=e} = \frac{-3e + 2e\ \ln e}{e^4} = \frac{-e}{e^4} < 0$$

Since at stationary point $x = e$, y'' is less than 0,

$\therefore$ $y = \ln x^{1/x}$ **has maximum at x = e.**

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118. (c)

The Taylor series expansion of f(x) = sinx at x = a is

$$\sin x = (x-a) - \frac{(x-a)^3}{3!} + \frac{(x-a)^5}{5!} - \frac{(x-a)^7}{7!} + \cdots$$

Hence expansion of f(x) = sinx at x = π is:

$$\sin x = (x-\pi) - \frac{(x-\pi)^3}{3!} + \frac{(x-\pi)^5}{5!} - \frac{(x-\pi)^7}{7!} + \cdots$$

Divide both side by $x - \pi$,

$$\boxed{\frac{\sin x}{x - \pi} = 1 - \frac{(x-\pi)^2}{3!} + \frac{(x-\pi)^4}{5!} - \frac{(x-\pi)^6}{7!} + \cdots}$$

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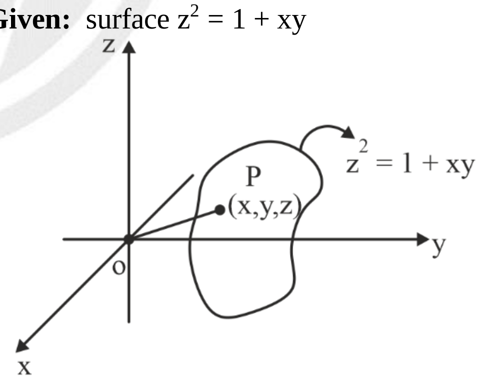
119.(a)

Given: surface $z^2 = 1 + xy$



Let 'P' be the nearest point on $z^2 = 1 + xy$ to origin O.

For OP to be minimum, OP^2 should be minimum.

$OP^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2$

$\Rightarrow OP^2 = x^2 + y^2 + z^2 = x^2 + y^2 + 1 + xy$

For OP^2 to be minimum,

$x^2 + y^2 + 1 + xy$ should be minimum

Let $f = x^2 + y^2 + 1 + xy$

Stationary point;

$$p = \frac{\partial f}{\partial x} = 2x + y = 0 \qquad \ldots(i)$$

$$q = \frac{\partial f}{\partial y} = 2y + x = 0 \qquad \ldots(ii)$$

Solving equation (i) and (ii),

x =0, y=0

$\Rightarrow$ (0, 0) is stationary point

$$\text{Now } r = \frac{\partial^2 f}{\partial x^2} = 2;\ s = \frac{\partial^2 f}{\partial x \partial y} = 1;\ t = \frac{\partial^2 f}{\partial y^2} = 2$$

Since $r > 0$ and $rt - s^2 = 2(2) - 1^2 > 0$

$\therefore$ f has minimum at (0, 0)

$\therefore$ Minimum of $OP^2 = 0^2 + 0^2 + 1 + (0.0) = 1$

$\therefore$ Minimum of OP = $\sqrt{1} = 1$

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120. (d)

$$f(x) = \frac{\sin x}{x - \pi}$$

$$\Rightarrow f(x) = \frac{\sin(\pi + x - \pi)}{x - \pi} \qquad (\because \sin(\pi + \theta) = -\sin\theta)$$

$$\Rightarrow f(x) = -\frac{\sin(x - \pi)}{(x - \pi)}$$

$$\left(\because \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots \infty\right)$$

Therefore,

$$\Rightarrow f(x) = -\frac{1}{(x - \pi)}\left[(x - \pi) - \frac{(x - \pi)^3}{3!} + \ldots\right]$$

$$\Rightarrow \boxed{f(x) = -1 + \frac{(x - \pi)^2}{3!} - \ldots}$$

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121. (b)

$$f(x) = (x^2 - 4)^2$$

Differenti‘ting with respect to x,

$$f'(x) = 4x(x^2 - 4) \qquad \ldots(i)$$

Again different“ating with respect to x,

$$f''(x) = 4x(2x) + (x^2 - 4)4$$

“$\Rightarrow$ $f''(x) = 12x^2 - 16 \qquad \ldots(ii)$

‘To find stationary points, $f'(x) = 0$

$$f'(x) = 4x(x^2 - 4) = 0$$

$\Rightarrow$ $4x(x + 2)(x - 2) = 0$

$\Rightarrow$ x = 0, 2, –2

Now,

At x = 0 ; $f''(x) = 12x^2 - 16 = -16 < 0$

$\therefore$ f(x) has maxima at x = 0

At x = 2 ; $f''(x) = 12(2)^2 - 16 = 32 > 0$

$\therefore$ f(x) has minima at x = 2

At x = –2 ; $f''(x) = 12(2)^2 - 16 = 32 > 0$

$\therefore$ f(x) has minima at x = –2

$\therefore$ f(x) has two minima

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122. (b)

Given: Function $f(x) = e^x + \sin x$

The coefficient of $(x - \pi)^2$ in Taylor series expansion of f(x) is $\frac{f''(\pi)}{2!}$.

Now, $f(x) = e^x + \sin x$

$$\therefore \quad f'(x) = e^x + \cos x$$

And $f''(x) = e^x - \sin x$

At x = π,

$$f''(x) = e^\pi - \sin\pi = e^\pi$$

Coefficient of $(x - \pi)^2$ in Taylor series expansion

$$= \frac{e^\pi}{2!} = 0.5e^\pi$$

$\therefore$ **coefficient of $(x - \pi)^2 = 0.5(e^\pi) = 0.5\exp(\pi)$**

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123. (a)

Given: $f(x) = e^x + e^{-x}$ …(i)

Method I:

For minimum value of f(x), $f'(x) = 0$ and $f''(x) > 0$

Differentiating equation (i) with respect to x,

$$f'(x) = e^x - e^{-x} \quad \text{…(ii)}$$

At stationary points:

$$f'(x) = 0$$

$\Rightarrow$ $e^x - e^{-x} = 0$

$\Rightarrow$ $(e^x)^2 - 1 = 0$

$\Rightarrow$ $e^x = 1$

$\Rightarrow$ $x = 0$

Now, Differentiating equation (ii) with respect to x,

$$f''(x) = e^x + e^{-x} \quad \text{…(iii)}$$

At stationary point x = 0,

$$f''(0) = e^0 + e^{-0} = 2 > 0$$

Since at stationary point x = 0, $f''(x)$ is greater than 0, $\therefore$ f(x) has minimum at x = 0

The minimum value of f(x) is $e^0 + e^{-0} = 2$

Method II:

For any two positive numbers a, b,

A.M $\geq$ GM

$\Rightarrow$ $\dfrac{e^x + e^{-x}}{2} \geq \sqrt{e^x e^{-x}}$

$\Rightarrow$ $e^x + e^{-x} \geq 2$

$\Rightarrow$ **Minimum value of $f(x)$= 2**

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124. (a)

The Taylor series expansion of $f(x)$ about x = a is

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \text{.....}\infty$$

For the expansion to have only odd powers of x coefficients of even powers of 'x' at x = 0 should be zero

$\Rightarrow$ $f(a) = f''(a) = f''''(a) = \text{…….} = 0$

Option A:

$f(x) = \sin(x^3)$

$\Rightarrow$ $f'(x) = 3x^2\cos(x^3)$

$\Rightarrow$ $f''(x) = 3\{x^2(-\sin(x^3)\, 3x^2) + \cos(x^3)\, 2x\}$

$= -9x^4 \sin(x^3) + 6x\cos(x^3)$

At x =0,

f (0) = sin (0) = 0,

$\Rightarrow$ $f''(0) = 0$

$$\therefore \quad \sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \text{………}\infty$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} + \text{……}\infty$$

$$\cos(x^3) = 1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} + \text{….}\infty$$

$$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} + \text{.....}\infty$$

$\therefore$ Option A where $f(x) = \sin(x^3)$, is having odd powersof 'x'.

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125. (b)

Given $f(x) = 3x^4 - 16x^3 + 24x^2 + 37$

$\Rightarrow f'(x) = 12x^3 - 48x^2 + 48x$

$\Rightarrow f'(x) = 12(x^3 - 4x^2 + 4x)$

$\Rightarrow f'(x) = 12x(x^2 - 4x + 4)$

$\Rightarrow f'(x) = 12x(x - 2)^2$

For existance of maximum (or) minima,

$f'(x) = 0$

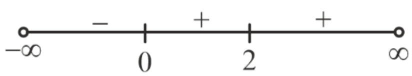
$\Rightarrow 12\,x\,(x - 2)^2 = 0$

$\Rightarrow x = 0, x = 2, 2$

$\therefore$ The function has distinct stationary points x = 0,. x= 2.

Now find the sign of f''(x) around stationary point.

$f'(x)$ on number line:



Since around x = 0 $f'(x)$ is changong is sign from negative to positive. Therefore, x = 0 I total minima.

Since around x = 2, $f'(x)$ is not changing its sign. Therefore, x= 2 is neither maxima nor minima.

$\therefore$The function has extreme only at x = 0

$\therefore$ Number of points of extrema = 1

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126. (c)

Given: $f(x) = e^x$

The coefficient of $(x - a)^n$ in Taylor series expansion of f(x) about x = a is $\frac{f^n(a)}{n!}$

$\therefore$ The coefficient of $(x-2)^4 = \frac{f^4(2)}{4!}$

$\Rightarrow \quad f^1(x) = f^2(x) = f^3(x) = f^4(x) = e^x \quad (\because f(x) = e^x)$

$\therefore \quad \boxed{\text{Coefficient of } (x-2)^4 = \frac{e^x\big|_{x=2}}{4!} = \frac{e^2}{4!}}$

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127. (d)

$f(x) = \frac{1}{x^2}$ is not defined at x = 0.

$$f(x) = \int_{-1}^{+1} \frac{1}{x^2}dx \qquad [\because f(x) = f(-x)]$$

$$= 2\int_0^1 \frac{1}{x^2}dx$$

$$= 2\left[-\frac{1}{x}\right]_0^1 = 2(-1+\infty) = \infty$$

Divergent

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128. (b)

Given $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$

$$I = \int_{-\infty}^{\infty} e^{-a(x+b)^2}dx$$

Let $x + b = t \quad \Rightarrow \quad dx = dt$

L.L: $x \to -\infty$, $x + b = t \to -\infty$,

U.L: $x \to \infty$, $x + b = t \to \infty$

$$\therefore \quad I = \int_{-\infty}^{\infty} e^{-at^2}\cdot dt = \int_{-\infty}^{\infty} e^{-(\sqrt{a}t)^2}\cdot dt$$

Let $\sqrt{a}t = v \quad \Rightarrow \sqrt{a}\cdot dt = dv$

L.L: $t \to -\infty$; $\sqrt{a}t \to -\infty$

U.L: $t \to \infty$; $\sqrt{a}t \to \infty$

$$\therefore \quad I = \frac{1}{\sqrt{a}}\cdot\int_{-\infty}^{\infty} e^{-v^2}\cdot dv$$

$$\left[\because \int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi} \Rightarrow \int_{-\infty}^{\infty} e^{-v^2}\cdot dv = \sqrt{\pi}\right]$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{a}}\times\sqrt{\pi} = \sqrt{\frac{\pi}{a}}$$

$$\therefore \quad \boxed{I = \int_{-\infty}^{\infty} e^{-a(x+b)^2}dx = \sqrt{\frac{\pi}{a}}}$$

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129. (a)

$$f(x) = \int_0^x e^t (t-1)(t-2)dt$$

To identify the nature of the function, f'(x) has to be calculated.

Differentiating the above function on both sides with respect to x, by Newton – Leibnitz rule,

$$f'(x) = \left[e^x (x-1)(x-2)\frac{dx}{dx}\right] - \left[e^0 (0-1)(0-2)\frac{d0}{dx}\right]$$

$$f'(x) = e^x (x-1)(x-2)$$

For f(x) to be decreasing, $f'(x) < 0$

$$\Rightarrow f'(x) = e^x (x-1)(x-2) < 0$$

Since e^x is always positive, therefore

$$\Rightarrow (x-1)(x-2) < 0 \Rightarrow x \in (1,2)$$

Therefore, f(x) is decreasing in $x \in (1,2)$.

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130. (0)

$$\int_{-1}^{+1} x\, e^{|x|} dx$$

Let, $f(x) = x\, e^{|x|}$

Then, $f(-x) = -x\, e^{|-x|} \Rightarrow f(-x) = -x\, e^{|x|} = -f(x)$

Since $f(x) = f(-x)$, hence $f(x)$ is an odd function. For odd function:

$$\int_{-a}^{+a} f(x)dx = 0$$

$$\boxed{\int_{-1}^{+1} x\, e^{|x|} dx = 0}$$

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131. (c)

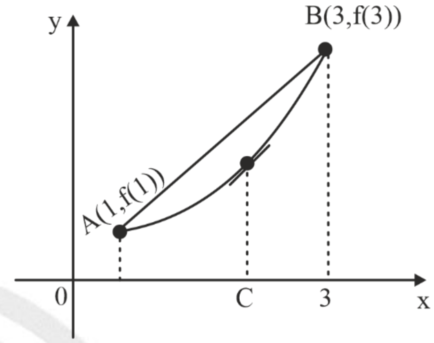
Given

$f(x) = x^2 - 2x + 2$ in $x \in [1, 3]$

$f'(x) = 2x - 2$

$f(3) = 9 - 6 + 2 = 5$

$f(1) = 1 - 2 + 2 = 1$



By Lagrange's mean value theorem,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 2c - 2 = \frac{5-1}{3-1} = 2$$

$$\Rightarrow \quad c = 2$$

$\therefore$ **x = 2 is the point where tangent is parallel to the straight line joining f(1) and f(3).**

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132. (a, c, d)

option (a)

$$I = \frac{1}{\pi}\int_0^{\pi} \sin m\theta \sin n\theta\, d\theta$$

$$\left[\begin{array}{l} \because \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \\ \therefore \cos(A-B) - \cos(A+B) = \frac{1}{2}\sin A \sin B \end{array}\right]$$

$$I = \frac{1}{2\pi}\int_0^{\pi} \cos(m-n)\theta - \cos(m+n)\theta \cdot d\theta$$

$$I = \frac{1}{2\pi}\left[\frac{\sin(m-n)\theta}{m-n} - \frac{\sin(m+n)\theta}{m+n}\right]_0^{\pi} = 0$$

option (b)

$$I = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \sin p\theta \sin q\theta \, d\theta$$

$$\begin{bmatrix} \because \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \\ \therefore \cos(A - B) - \cos(A + B) = \frac{1}{2}\sin A \sin B \end{bmatrix}$$

$$I = \frac{1}{4\pi}\int_{-\pi/2}^{\pi/2} \cos(p-q)\theta - \cos(p+q)\theta \, d\theta$$

$$I = \frac{2}{4\pi}\left[\frac{\sin(p-q)\theta}{p-q} - \frac{\sin(p+q)\theta}{p+q}\right]_0^{\pi/2}$$

$\rightarrow$ will depend on p and q it will not be necessarily be 0.

option (c)

$$I = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin p\theta \cos q\theta \, d\theta$$

$$\begin{bmatrix} \because \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \\ \therefore \sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\} \end{bmatrix}$$

$$I = \frac{1}{4\pi}\int_{-\pi}^{\pi} \{\sin(p+q)\theta + \sin(p-q)\theta\} \, d\theta$$

Let, $f(\theta) = \sin(p+q)\theta + \sin(p-q)\theta$ then,

$$f(-\theta) = \sin\{(p+q)(-\theta)\} + \sin\{(p-q)(-\theta)\}$$

$$f(-\theta) = -\sin(p+q)(\theta) - \sin(p-q)(\theta)$$

$$f(-\theta) = -\{\sin(p+q)\theta + \sin(p-q)\theta\} = -f(\theta)$$

Hence $f(\theta) = \sin(p+q)\theta + \sin(p-q)\theta$ is odd function, therefore

$I = 0$

option (d)

$$L = \lim_{\alpha\to\infty} \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \sin p\theta \sin q\theta \, d\theta$$

$$L = \lim_{\alpha\to\infty} \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \cos(p-q)\theta - \cos(p+q)\theta d\theta$$

$$L = \lim_{\alpha\to\infty} \frac{2}{2\alpha}\left[\frac{\sin(p-q)\theta}{p-q} - \frac{\sin(p+q)\theta}{p+q}\right]_0^{\alpha}$$

$$L = \lim_{\alpha\to\infty}\left[\frac{\sin(p-q)\alpha}{(p-q)\alpha} - \frac{\sin(p+q)\alpha}{(p+q)\alpha}\right]$$

$$\left[\because \lim_{x\to\infty}\frac{\sin x}{x} = 0\right] \quad L = 0$$

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133. (b)

Given: $f(-1) = 0$ and $|f'(x)| \le 2$

$\Rightarrow$ $-2 \le f'(x) \le 2$

$\Rightarrow$ $-2\,dx \le f'(x)\,dx \le 2dx$

On integrating

$$\int -2\,dx \le \int f'(x)\,dx \le \int 2\,dx$$

$\Rightarrow$ $-2x + c_1 \le f(x) \le 2x + c_2$ ...(i)

$\because$ $f(-1) = 0$; hence from equation (i),

$\Rightarrow$ $0 \le -2 + c_1$ $\Rightarrow c_1 \ge 2$

Or, $2 + c_2 \le 0$ $\Rightarrow c_2 \le -2$

$\therefore$ $-2x - 2 \le f(x) \le 2x + 2$ ...(i)

Or, $|f(x)| \le 2x + 2$

$\Rightarrow$ $f(x) \le |2x + 2|$ $\Rightarrow$ $f(x) \le 2|x + 1|$

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134. (2.090 to 2.104)

$$I = \int_1^e (x \ln x)dx = ?$$

$$I = \int_1^e (x \ln x)dx = \int_1^e (\ln x)\cdot x \cdot dx$$

$$I = (\ln x)\cdot\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx\Big|_1^e$$

$$I = \frac{x^2}{2}\ln x - \int\frac{x}{2}dx\Big|_1^e$$

$$I = \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_1^e$$

$$I = \left(\frac{e^2}{2}\times\ln e - \frac{e^2}{4}\right) - \left(0 - \frac{1}{4}\right)$$

$$I = \frac{e^2}{2}(1) - \frac{e^2}{4} + \frac{1}{4} = \frac{e^2+1}{4} = 2.097$$

$$\boxed{\therefore \int_1^e (x\cdot\ln x)dx = 2.097}$$

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135. (0.50)

$$I = \int_{0.25}^{1.25} f(x)dx = ?$$

f(x) = x – [x]

Siince $[x] = \begin{cases} 0 & ; 0.25 \le x < 1 \\ 1 & ; 1 \le x < 2 \end{cases}$

Hence,

$$\Rightarrow \quad f(x) = \begin{cases} x - 0 = x & 0.25 \le x < 1 \\ x - 1 & 1 \le x < 2 \end{cases}$$

$$I = \int_{0.25}^{1.25} f(x)dx$$

$$\Rightarrow \quad I = \int_{0.25}^{1} x dx + \int_{1}^{1.25} (x-1)dx$$

$$\Rightarrow \quad I = \left[\frac{x^2}{2}\right]_{0.25}^{1} + \left[\frac{x^2}{2} - x\right]_{1}^{1.25}$$

$$\Rightarrow I = \left[\frac{1^2}{2} - \frac{(0.25)^2}{2}\right] + \left[\left(\frac{1.25^2}{2} - 1.25\right) - \left(\frac{1^2}{2} - 1\right)\right]$$

$$\Rightarrow I = 0.5$$

$$\therefore \quad \boxed{\int_{0.25}^{1.25} f(x)dx = 0.5}$$

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136. (a)

Mean value theorem of integration states that if a function f(x) is continuous in [a, b] and differentiable in (a, b) then in the interval [a, b] at least one value 'ξ' exists, such that,

$$f(\xi) = \frac{\int_a^b f(x)dx}{(b-a)}$$

$$\int_a^b f(x)dx = (b-a) \cdot f(\xi)$$

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137. (0.27 to 0.3)

Given Integral is

$$I = \int_0^{\frac{\pi}{4}} x \cdot \cos\left(x^2\right)dx$$

Let $x^2 = t \Rightarrow 2x \cdot dx = dt = x \cdot dx = \frac{1}{2} \cdot dt$

L.L $x^2 = t \Rightarrow 0^2 = t \Rightarrow t = 0$

U.L $x^2 = t \Rightarrow \left(\frac{\pi}{4}\right)^2 = t \Rightarrow t = \frac{\pi}{16}^2$

$$\therefore \ I = \int_0^{\frac{\pi^2}{16}} \cos t \frac{dt}{2} = \frac{1}{2} \times \sin\Big|_0^{\frac{\pi^2}{16}}$$

$$= \frac{1}{2} \times \left\{\sin\left(\frac{\pi}{16}\right)^2 - \sin 0\right\}$$

$$= \frac{1}{2} \times 0.5784 = 0.2892$$

$$\therefore \ \boxed{\int_0^{\frac{\pi}{4}} x \cdot \cos\left(x^2\right)dx = 0.2892}$$

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138. (c)

Given

$$f(x) = R\sin\left(\frac{\pi x}{2}\right) + S, \ f'\left(\frac{1}{2}\right) = \sqrt{2}; \int_0^1 f(x)dx = \frac{2R}{\pi}$$

Now,

$$f(x) = R\sin\left(\frac{\pi x}{2}\right) + S$$

$$\Rightarrow f'(x) = R\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi x}{2}\right)$$

Given. $f'\left(\frac{1}{2}\right) = \sqrt{2}$; therefore,

$$\Rightarrow f'\left(\frac{1}{2}\right) = R\frac{\pi}{2}\cos\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$\Rightarrow \frac{R\pi}{2\sqrt{2}} = \sqrt{2}$$

$$\Rightarrow \boxed{R = \frac{4}{\pi}}$$

Hence, $f(x) = R\sin\left(\frac{\pi x}{2}\right) + S = \frac{4}{\pi}\sin\left(\frac{\pi x}{2}\right) + S$

Now,

$$\Rightarrow \int_0^1 f(x)\,dx = \frac{2R}{\pi}\ dx$$

$$\Rightarrow \int_0^1 \left(\left(\frac{4}{\pi}\sin\frac{\pi x}{2}\right) + S\right)dx = \frac{2}{\pi}\left(\frac{4}{\pi}\right)$$

$$\Rightarrow \frac{4}{\pi}\left.\frac{-\cos\left(\frac{\pi x}{2}\right)}{\left(\frac{\pi}{2}\right)}\right|_0^1 + S\cdot x\Big|_0^1 = \frac{8}{\pi^2}$$

$$\Rightarrow \frac{8}{\pi^2}\{-0-(-1)\} + S = \frac{8}{\pi^2}$$

$$\Rightarrow \frac{8}{\pi^2} + S = \frac{8}{\pi^2}$$

$$\Rightarrow \boxed{S = 0}$$

$$\therefore \boxed{R = \frac{4}{\pi} \text{ and } S = 0}$$

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139. (a)

$$I = \int_0^1 \frac{\left(\sin^{-1}x\right)^2}{\sqrt{1-x^2}}\,dx$$

Let $\sin^{-1} x = t \quad \Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

At $x = 0$; $t = \sin^{-1} 0 = 0$

At $x = 1$; $t = \sin^{-1} 1 = \frac{\pi}{2}$

Hence,

$$I = \int_0^{\pi/2} t^2 dt = \left[\frac{t^3}{3}\right]_0^{\pi/2}$$

$$I = \frac{(\pi/2)^3}{3} = \frac{\pi^3}{24}$$

$$\therefore \boxed{\int_0^1 \frac{\left(\sin^{-1}x\right)^2}{\sqrt{1-x^2}}\,dx = \frac{\pi^3}{24}}$$

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140. (b)

$$g(x) = \int f(x)\,dx$$

$$g(x) = \int e^{-x-e^{-x}}\,dx$$

$$g(x) = \int e^{-x}.\,e^{-e^{-x}}.\,dx \qquad \ldots(i)$$

Let $e^{-x} = t \quad \Rightarrow \quad -e^{-x}\,dx = dt$

Then, equation (i) will become,

$$g(x) = -\int e^{-t}dt$$

$$g(x) = -\left[\frac{e^{-t}}{-1}\right] = e^{-t} \qquad \left[\because e^{-x} = t\right]$$

$$\boxed{g(x) = e^{-e^{-x}}}$$

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141. (c)

Given,

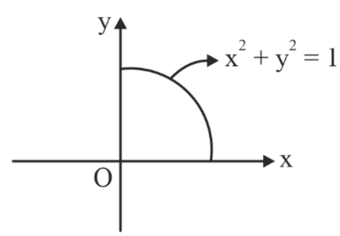
$$x = \cos\left(\frac{\pi u}{2}\right) \text{ and } y = \sin\left(\frac{\pi u}{2}\right)$$

$$\text{and } 0 \le u \le 1 \qquad \Rightarrow \quad 0 \le \frac{\pi u}{2} \le \frac{\pi}{2}$$

$$\Rightarrow \quad x^2 + y^2 = \cos^2\left(\frac{\pi u}{2}\right) + \sin^2\left(\frac{\pi u}{2}\right) = 1$$

The curve is $\quad x^2 + y^2 = 1$

$$\Rightarrow \quad y = \sqrt{1-x^2} \text{ between } \theta = 0 \text{ to } \theta = \frac{\pi}{2}$$



Revolving of $x^2 + y^2 = 1$ about x-axis by 360° forms a hemisphere of radius r=1.

$\therefore$ Surface area $= 2\pi r^2 = 2\pi(1)^2 = 2\pi$

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142. (–1)

$$\text{Given Integral is } I = \int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\cos\left(\frac{1}{x}\right)}{x^2}dx$$

$$\text{Let } \frac{1}{x} = t \Rightarrow \frac{-1}{x^2}dx = dt \Rightarrow \frac{1}{x^2}dx = -dt$$

$$\text{Lower limit : } \frac{1}{x} = t \Rightarrow \frac{1}{\left(\frac{1}{\pi}\right)} = t \Rightarrow t = \pi$$

$$\text{Upper limit : } \frac{1}{x} = t \Rightarrow \frac{1}{\left(\frac{2}{\pi}\right)} = t \Rightarrow t = \frac{\pi}{2}$$

$$\therefore\ I = \int_{\pi}^{\frac{\pi}{2}} \cos t(-dt) = \int_{\frac{\pi}{2}}^{\pi} \cos t(dt)$$

$$\Rightarrow I = \sin t\Big|_{\frac{\pi}{2}}^{\pi} = -1$$

$$\therefore\ \boxed{I = \int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\cos\left(\frac{1}{x}\right)}{x^2}dx = -1}$$

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143. (a)

Given,

$$a \cdot f(x) + b \cdot f\left(\frac{1}{x}\right) = \frac{1}{x} - 25 \quad \text{.......(1)}$$

(Replacing x with $\frac{1}{x}$)

$$\Rightarrow a \cdot f\left(\frac{1}{x}\right) + b \cdot f(x) = x - 25 \quad \text{......(2)}$$

In equation 1 multiply a both side,

$$a^2 \cdot f(x) + ab \cdot f\left(\frac{1}{x}\right) = \frac{a}{x} - 25a \quad \text{.......(3)}$$

In equation 2 multiply b both side,

$$ab\left(\frac{1}{x}\right) + b^2 \cdot f(x) = bx - 25b \quad \text{......(4)}$$

Equation 3 – equation 4,

$$\Rightarrow (a^2 - b^2)f(x) = \frac{a}{x} - bx + 25(b - a)$$

$$\Rightarrow f(x) = \frac{1}{(a^2 - b^2)}\left\{\frac{a}{x} - bx + 25(b - a)\right\}$$

On integrating,

$$\Rightarrow \int_1^2 f(x)dx = \frac{1}{(a^2 - b^2)}\left\{a \cdot \ln x - \frac{bx^2}{2} + 25(b - a)x\Big|_1^2\right\}$$

$$\Rightarrow \int_1^2 f(x)dx = \frac{1}{(a^2 - b^2)}\left[a \cdot \ln 2 - \frac{b}{2}(3) + 25(b - a)\right]$$

$$\Rightarrow \boxed{\int_1^2 f(x)dx = \frac{1}{(a^2 - b^2)}\left\{a(\ln 2 - 25) + \frac{47}{2}b\right\}}$$

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144. (4)

$$\text{Given } \int_0^{2x} |x \cdot \sin x|\,dx = k\pi \cdot$$

|x. sinx| is not differentiable at the points where,

x. sinx = 0

$\Rightarrow$ $x = 0, \pm\pi, \pm2\pi, \dots$

$\therefore$ π is a value in $[0, 2\pi]$ where |x. sinx| is not differentiable.

$$\therefore \int_0^{2\pi} |x \cdot \sin x|\,dx = \int_0^{\pi} |x \cdot \sin x|\,dx + \int_{\pi}^{2\pi} |x \cdot \sin x|\,dx$$

When $0 < x < \pi$, x.sinx > 0 $\Rightarrow$ |x. sinx| = x.sinx

$\pi < x < 2\pi$; x.sinx < 0 $\Rightarrow$ |x. sin x| = –x.sinx

$$\therefore \int_0^{2\pi} |x \cdot \sin x|\,dx = \int_0^{\pi} x \cdot \sin x\,dx + \int_{\pi}^{2\pi} x \cdot \sin x\,dx$$

$$= x\cdot(-\cos x)+1\cdot\sin x\,\Big|_0^{\pi}-(-x\cos x+\sin x)\Big|_{\pi}^{2\pi}$$

$= \pi(1) - (-2\pi(1) + 0 - (+\pi)) = 4\pi$

$$\therefore \int_0^{2\pi}|x\cdot\sin x|\,dx = 4\pi \;-K\pi$$

$\Rightarrow K = 4$

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145. (a)

Given integral is

$$I=\int_0^{\pi}x^2\cdot\cos x\;dx$$

$$\left[\begin{array}{l}\because \int u.v\,dx = u\int v dx-\int\left(u'\int v dx\right)dx\\ \text{here, consider } u = x^2\ \&\ v=\cos x\end{array}\right]$$

$$I = x^2\cdot\sin x\Big|_0^{\pi}-\int_0^{\pi}2x\sin x dx$$

$$I=(0-0)-2\int_0^{\pi}x\sin x dx$$

$$\left[\begin{array}{l}\because \int u.v\,dx = u\int v dx-\int\left(u'\int v dx\right)dx\\ \text{In } \int_0^{\pi}2x\sin x dx,\ \text{consider } u = x\ \&\ v=\sin x\end{array}\right]$$

$$I=-2\left[-x\cos x\Big|_0^{\pi}-\int_0^{\pi}(-\cos x)dx\right]$$

$$I=-2\left[(\pi-0)+\sin x\Big|_0^{\pi}\right]$$

$$I=-2\left[\pi+(0-0)\right]=-2\pi$$

$$\therefore\ \boxed{\int_0^{\pi}x^2\cdot\cos x\;dx=-2\pi}$$

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146. (15)

$2\le x\le 3;\ 1\le y\le 2;\ 0\le z\le 1$

Therefore,

$$I=\int_{z=0}^{z=1}\int_{y=1}^{y=2}\int_{x=2}^{x=3}8xyz\,dx\,dy\,dz$$

$$I=8\left[\frac{x^2}{2}\right]_2^3\left[\frac{y^2}{2}\right]_1^2\left[\frac{z^2}{2}\right]_0^1$$

$$I=8\left(\frac{9-4}{2}\right)\left(\frac{4-1}{2}\right)\left(\frac{1-0}{2}\right)$$

$$I=8\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}$$

$$\boxed{I=15}$$

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147. (b)

Find: $I=\displaystyle\int_0^2\frac{(x-1)^2\cdot\sin(x-1)}{(x-1)^2+\cos(x-1)}dx$

$$I=\int_0^2\frac{(x-1)^2\cdot\sin(x-1)}{(x-1)^2+\cos(x-1)}dx \qquad \ldots(i)$$

$$\left\{\begin{array}{l}\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\\ \text{Hence replacing x by } 0+2-x \text{ in equation (i)}\end{array}\right\}$$

$$I=\int_0^2\frac{(0+2-x-1)^2\cdot\sin(0+2-x-1)}{(0+2-x-1)^2+\cos(0+2-x-1)}dx$$

$$I=\int_0^2\frac{(1-x)^2\cdot\sin(1-x)}{(1-x)^2+\cos(1-x)}dx$$

$$I=\int_0^2\frac{(x-1)^2\cdot\sin\{-(x-1)\}}{(x-1)^2+\cos\{-(x-1)\}}dx$$

$$I=\int_0^2\frac{(x-1)^2\cdot\{-\sin(x-1)\}}{(x-1)^2+\cos(x-1)}dx$$

$$I = -\int_0^2 \frac{(x-1)^2 \cdot \sin(x-1)}{(x-1)^2 + \cos(x-1)}dx \quad \ldots(ii)$$

Adding equation (i) and (ii),

$$2I = \int_0^2 \frac{(x-1)^2 \cdot \sin(x-1)}{(x-1)^2 + \cos(x-1)}dx - \int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}dx$$

$$2I = 0$$

$$I = 0$$

$$\therefore \boxed{I = \int_0^2 \frac{(x-1)^2 \cdot \sin(x-1)}{(x-1)^2 + \cos(x-1)}dx = 0}$$

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148. (c)

Find: $I = \int_1^e \sqrt{x}\ln(x)dx$

$$I = \int_1^e (\sqrt{x} \cdot \ln x)dx = \int_1^e (\ln x) \cdot \sqrt{x}\, dx$$

$$I = \left\{ \ln x \cdot \frac{2}{3} \cdot x^{3/2} - \int \frac{1}{x} \cdot \frac{2}{3} \cdot x^{3/2} dx \Big|_1^e \right\}$$

$$I = \left\{ \ln x \cdot \frac{2}{3} \cdot x^{3/2} - \frac{2}{3} \int x^{1/2} dx \Big|_1^e \right\}$$

$$I = \frac{2}{3} \cdot \ln x \cdot x^{3/2} - \frac{2}{3} \times \frac{2}{3} \times x^{3/2} \Big|_1^e$$

$$I = \left[\left(\frac{2}{3} \cdot \ln e \cdot e^{3/2} - \frac{2}{3} \times \frac{2}{3} \times e^{3/2} \right) - \left(\frac{2}{3} \cdot \ln 1 \cdot 1^{3/2} - \frac{2}{3} \times \frac{2}{3} \times 1^{3/2} \right) \right]$$

$$I = \left[\left(\frac{2}{3} e^{3/2} - \frac{4}{9} e^{3/2} \right) - \left(0 - \frac{4}{9} \right) \right]$$

$$I = \frac{2}{3} e^{3/2} + \frac{4}{9} = \frac{2}{9}\sqrt{e^3} + \frac{4}{9}$$

$$\therefore \boxed{\int_1^e \sqrt{x}\ln(x)dx = \frac{2}{9}\sqrt{e^3} + \frac{4}{9}}$$

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149. (d)

Let $I = \int_0^{\pi/2} \frac{\cos x + i \cdot \sin x}{\cos x - i \sin x} dx = \int_0^{\pi/2} \frac{e^{ix}}{e^{-ix}} dx$

$$= \int_0^{\pi/2} e^{i(2x)} dx$$

$$= \frac{e^{i2x}}{2i}\Big|_{x=0}^{x=\pi/2} = \frac{1}{2i}\left\{ e^{i2(\pi/2)} - e^{i2(0)} \right\}$$

$$= \frac{1}{2i}\{-1-1\} = \frac{-2}{2i} = \frac{-1}{i} = i$$

$$\therefore \boxed{\int_0^{\pi/2} \frac{\cos x + i \cdot \sin x}{\cos x - i \sin x} dx = i}$$

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150. (c)

$$I = \int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta\, d\theta$$

Let $3\theta = t$, then $3\,d\theta = dt \Rightarrow d\theta = \frac{dt}{3}$

If $\theta = 0$; $t = 3\theta = 0$

If $\theta = \pi/6$; $t = 3\theta = \frac{\pi}{2}$

$$I = \int_0^{\pi/2} \cos^4 t \sin^3 2t \frac{dt}{3}$$

$$I = \int_0^{\pi/2} \cos^4 t (2 \sin t \cos t)^3 \frac{dt}{3}$$

$$I = \frac{8}{3}\int_0^{\pi/2} \left(\sin^3 t \times \cos^7 t \right) dt$$

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$$\left[\begin{array}{l}\because \int_0^{\pi/2} \sin^m t \cos^n t \, dt = \dfrac{((m-1)(m-3)\ldots 2 \text{ (or) } 1)((n-1)(n-3)\ldots 2 \text{ (or) } 1)}{((m+n)(m+n-2)\ldots\ldots 2 \text{ (or)} 1)} \times k \\ k = \begin{cases} \pi/2, \text{when both m and n are even} \\ 1, \text{ otherwise} \end{cases}\end{array}\right]$$

$$I = \frac{8}{3}\left[\frac{2\times 6\times 4\times 2}{10\times 8\times 6\times 4\times 2}\times 1\right] = \frac{1}{15}$$

$$\therefore \quad \boxed{\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta = \frac{1}{15}}$$

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151. (c)

Let $I = \int_0^a \dfrac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx \qquad \ldots \text{(i)}$

$$\left\{\begin{array}{l}\because \int_0^a f(x)dx = \int_0^a f(a-x)dx \\ \text{Hence replacing x by } a-x \text{ in equation (i)}\end{array}\right\}$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{(a-x)}+\sqrt{a-(a-x)}}$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx \qquad \ldots \text{(ii)}$$

On adding (i) and (ii),

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$$

$$2I = \int_0^a 1 \, dx = a$$

$$I = \frac{a}{2}$$

$$\therefore \quad \boxed{I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx = \frac{a}{2}}$$

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152. (MTA)

$$\int \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right) dx$$

$$= \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^5}{20} + \ldots$$

Checking all the options follows

$$\frac{1}{1+x} = (1+x)^{-1} = \left(1 - x + x^2 - x^3 + \ldots.\infty\right)$$

$$-\frac{1}{1-x} = (1-x)^{-1} = -\left(1 + x + x^2 + x^3 + \ldots.\infty\right)$$

$$\frac{1}{1+x^2} = \left(1+x^2\right)^{-1} = \left(1 - x^2 + x^4 - x^6 + \ldots\infty\right)$$

$$-\frac{1}{1-x^2} = \left(1-x^2\right)^{-1} = \left(1 + x^2 + x^4 + x^6 + \ldots\infty\right)$$

All options a, b, c and d are incorrect. (Hence marks to all)

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153. (b)

Given: f(x) is even function: $a > 0$

$\Rightarrow \quad f(-x) = f(x)$

For any $a > 0$ if $\qquad f(-x) = f(x)$

Then,

$$\boxed{\int_{-a}^{a} f(x)dx = 2\times\int_0^a f(x)dx}$$

Important point: if $f(-x) = -f(x)$, then

$$\int_{-a}^{a} f(x)dx = 0$$

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154. (b)

$$P = \int_0^1 x e^x dx$$

$$\begin{bmatrix} \because \int u.v\,dx = u\int v dx - \int\left(u'\int v dx\right)dx \\ \text{here, consider } u = x \ \& \ v = e^x \end{bmatrix}$$

$$\Rightarrow \quad P = \left[x\, e^x\right]_0^1 - \int_0^1 1.e^x dx \ \lim_{x\to\infty}$$

$$\Rightarrow P = \left(\left(1\times e^1\right) - 0\right) - \left[e^x\right]_0^1$$

$$\Rightarrow \quad P = e - \left(\left(e^1 - e^0\right)\right)$$

$$\Rightarrow P = e - (e-1)$$

$$\Rightarrow \boxed{P = 1}$$

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155. (d)

$$I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = ?$$

$$I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

$$\left\{\because \int_{-a}^{a} f(x)dx = 2\times\int_0^a f(x)\,dx \quad \text{if } f(-x) = f(x)\right\}$$

Let, $f(x) = \frac{1}{1+x^2}$, then

$$f(-x) = \frac{1}{1+(-x)^2} = \frac{1}{1+x^2} = f(x)$$

Since, $f(-x) = f(x)$, hence

$$I = \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = 2\times\int_0^{\infty}\frac{dx}{1+x^2}$$

$$I = 2\times \tan^{-1} x\Big|_0^{\infty} = 2\times(\tan^{-1}\infty - \tan^{-1} 0)$$

$$I = 2\times\left(\frac{\pi}{2} - 0\right) = \pi$$

$$\therefore \quad \boxed{\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \pi}$$

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156. (d)

Given integral is $I = \int_0^{\pi/4}\frac{1-\tan x}{1+\tan x}dx$

$$\Rightarrow \quad I = \int_0^{\pi/4}\frac{1-\tan x}{1+\tan x}dx = \int_0^{\pi/4}\frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\times\tan x}dx$$

$$\left(\because \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\cdot\tan B}\right)$$

$$\therefore \quad I = \int_0^{\pi/4}\tan\left(\frac{\pi}{4} - x\right)dx$$

$$\left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

$$\Rightarrow \quad I = \int_0^{\pi/4}\tan\left(\frac{\pi}{4} - \left(0+\frac{\pi}{4} - x\right)\right)dx$$

$$\Rightarrow I = \int_0^{\pi/4}\tan x dx$$

$$\Rightarrow \quad I = \log_e|\sec x|_0^{\pi/4} = \log_e\frac{\sqrt{2}}{1} = \frac{1}{2}\cdot\log_e^2$$

$$\therefore \quad \boxed{\int_0^{\pi/4}\frac{1-\tan x}{1+\tan x}dx = \frac{1}{2}\ln 2}$$

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157. (a)

$$\int_0^3\int_{y=0}^{y=x}(6-x-y)dydx$$

$$= \int_0^3 \left[6y - xy - \frac{y^2}{2} \right]_{y=0}^{y=x} dx$$

$$= \int_0^3 \left(6x - x^2 - \frac{x^2}{2} \right) dx$$

$$= \int_0^3 6x - \frac{3x^2}{2} dx = \left[\frac{6x^2}{2} - \frac{3}{2} \times \frac{x^3}{3} \right]_0^3$$

$$= \left[3x^2 - \frac{x^3}{2} \right]_0^3$$

$$= \left(3 \times 3^2 - \frac{3^3}{2} \right) = 27 - \frac{27}{2} = \frac{27}{2} = 13.5$$

$$\therefore \boxed{\int_0^3 \int_{y=0}^{y=x} (6 - x - y)\, dy dx = 13.5}$$

Scan for Video solution

158.(d)

(i) $I = \int_0^{\pi/4} \tan x\, dx = \ln|\sec x|_0^{\pi/4}$

$I = \ln\sqrt{2} - \ln(1) = \ln\sqrt{2}$ $\rightarrow$ **finite**

(ii) $I = \int_0^{\infty} \frac{1}{x^2+1} dx = \tan^{-1} x|_0^{\infty} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\rightarrow$ **finite**

(iii) $I = \int_0^{\infty} x \cdot e^{-x} dx = \int_0^{\infty} e^{-x} \cdot x\, dx = \int_0^{\infty} e^{-x} \cdot x^{2-1} dx$

$$\left(\because \int_0^{\infty} e^{-x} \cdot x^{n-1} dx = \overline{|n} \right)$$

$I = \overline{|2} = 1! = 1 \rightarrow$ **finite**

(iv) $I = \int_0^1 \frac{1}{1-x} dx = \frac{\ln(1-x)}{-1}\Big|_0^1 = -1(\ln 0 - \ln 1)$

$I = -\ln 0 \rightarrow \infty \rightarrow$ **infinite hence unbounded**

$$\boxed{\therefore \int_0^1 \frac{1}{1-x} dx \text{ is unbounded}}$$

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159. **(a, b, d)**

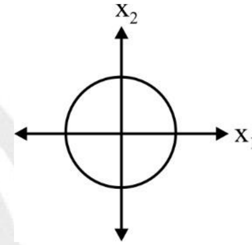
$|x_1|^p + |x_2|^p = 1$ for $p > 0$

From option a):

$p = 2$

$\therefore |x_1|^2 + |x_2|^2 = 1$

Above equation is equation of circle of radius 1 unit.



Area of circle = $\pi r^2 = \pi \times 1^2 = \pi$

Hence option (a) is correct.

From option b):

$p \to \infty$ $|x_1|^p + |x_2|^p = 1$

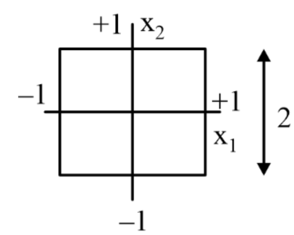
Case I: If $|x_1| < 1$ then $\lim_{p\to\infty} |x_1|^p \to 0$

$\therefore |x_2| = 1 \Rightarrow x_2 = 1 \text{ or } x_2 = -1$

Case II: If $|x_2| < 1$ then $\lim_{p\to\infty} |x_2|^p \to 0$

$\therefore |x_1| = 1$

$\Rightarrow x_1 = 1 \text{ or } x_1 = -1$



When $p \to \infty$, the given function represents a square bounded by the lines $|x_1| = 1$ and $|x_2| = 1$.

Side of square = 2, therefore,

Area enclosed in this case

= 2 × 2 = 4

From option c):

When p tends to 0, the function does not enclose any area.

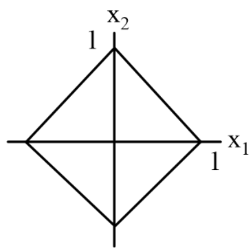
From option (d):

$p = 1$

$|x_1| + |x_2| = 1$

The above equation represents a square bounded by the lines $x_1 + x_2 = 1$, $x_1 - x_2 = 1$, $-x_1 + x_2 = 1$ and $-x_1 - x_2 = 1$.

Side of square = $\sqrt{1^2 + 1^2} = \sqrt{2}$, therefore,



Area = $(\sqrt{2})^2 = 2$

Hence option a, b and d are correct

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160. (18.80 to 18.90)

Given

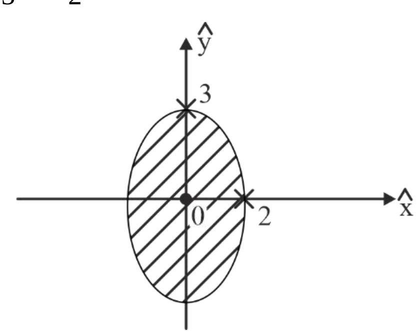
$\hat{x} = 3y, \hat{y} = 2x \quad \Rightarrow \quad y = \frac{\hat{x}}{3}; x = \frac{\hat{y}}{2}$

The disc $x^2 + y^2 < 1$ is given.

$\Rightarrow \quad x^2 + y^2 < 1$

$\Rightarrow \quad \left(\frac{\hat{y}}{2}\right)^2 + \left(\frac{\hat{x}}{3}\right)^2 < 1$

$\Rightarrow \quad \frac{\hat{x}^2}{3^2} + \frac{\hat{y}^2}{2^2} < 1 \rightarrow$ Elliptic Region



The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab

Area of required region = π (3) (2) = (3.14)6 = **18.84**

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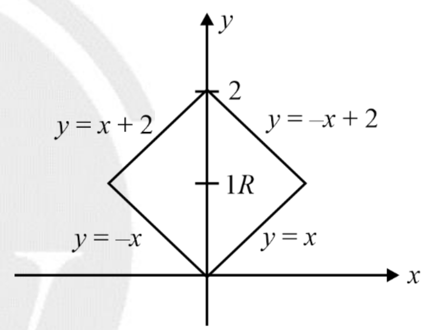
161. (0)

Given:

Region is a shown and the integral is (Say),

$$I = \iint_R xy\, dx\, dy$$



For any region 'R' that is symmetric about x-axis (or) *y*-axis,

$$\iint_R xy\, dx\, dy = 0$$

Since given region 'R' is symmetric about *y*-axis,

$$\iint_R xy\, dx\, dy = 0$$

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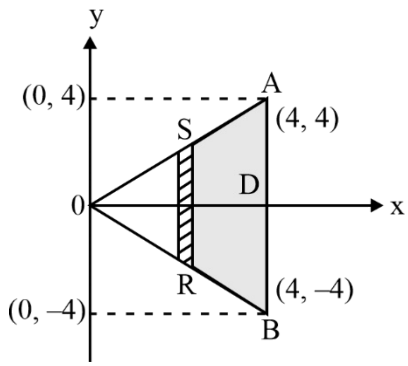
162. (171)

Given integral is,

$$I = \iint_D 3(x^2 + y^2)\, dx\, dy$$



From the given diagram,

Equation of $OA \Rightarrow y = x$

Equation of $OB \Rightarrow y = -x$

Taking vertical strip RS in required region D.

Now for entire region the limit of y at any x will be,

Lower limit $\rightarrow$ $y = -x$ [At R]

Upper limit $\rightarrow$ $y = x$ [At S]

Now by sliding strip RS in horizontal direction from extreme left to right in required region D we get the limit of x as given below,

Lower limit $\rightarrow x = 0$

Upper limit $\rightarrow x = 4$

Therefore,

$$I = \int_{x=0}^{x=4}\int_{y=-x}^{y=x} 3(x^2 + y^2)\,dy\,dx$$

$$I = \int_{x=0}^{x=4} 3\left(x^2 y + \frac{y^3}{3}\Big|_{-x}^{x} \right) dx$$

$$I = \int_{x=0}^{x=4} 3\left(x^2 (2x) + \frac{2x^3}{3} \right) dx$$

$$I = \int_{x=0}^{x=4} 8x^3\,dx = 2x^4\Big|_0^4$$

$$I = 2(4^4 - 0^4) = 512$$

$\therefore$ $\boxed{I = \iint_D 3(x^2 + y^2)\,dx\,dy = 512}$

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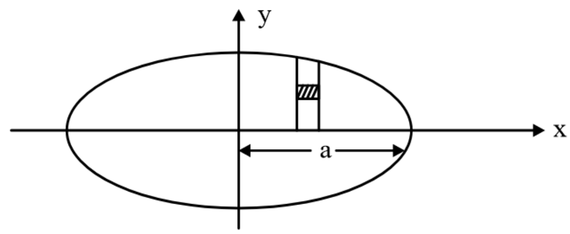
163. (b)



Given, curve is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\Rightarrow \frac{y^2}{b^2} = \frac{a^2 - x^2}{a^2}$$

$$\Rightarrow y^2 = b^2\left(\frac{a^2 - x^2}{a^2}\right)$$

$$\Rightarrow y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$$

Area of ellipse (A)

$$A = 4\int_{x=0}^{x=a}\int_{y=0}^{y=\frac{b}{a}\sqrt{a^2-x^2}} dy\ dx$$

$$A = 4\int_{x=0}^{x=a}\frac{b}{a}\sqrt{a^2 - x^2}\,dx = 4\frac{b}{a}\int_{x=0}^{x=a}\sqrt{a^2 - x^2}\,dx$$

Since, $\int\sqrt{(a^2 - x^2)}\,dx = \frac{x\sqrt{(a^2 - x^2)}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$,

hence

$$A = \frac{4b}{a}\left[\frac{x\sqrt{(a^2 - x^2)}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\Big|_0^a\right]$$

$$A = \frac{4b}{a}\left[\left(0 + \frac{a^2}{2}\sin^{-1}1\right) - \left(0 + \frac{a^2}{2}\sin^{-1}0\right)\right]$$

$$A = \frac{4b}{a}\left[\left(\frac{a^2}{2}\times\frac{\pi}{2}\right) - (0+0)\right] = 4\left(\frac{\pi ab}{4}\right) = \pi ab$$

$\therefore$ $\boxed{\text{Area of ellipse} = \pi ab}$

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164. (b)

Given

$y^2 = x$; and $0 \le x \le 1$

Volume of solid obtained by revolving a curve

$y = f(x)$; $a \le x \le b$ by 360° about x-axis is $\int_a^b \pi y^2 dx$.

$$\therefore \quad \text{Volume} = \int_0^1 \pi y^2 dx = \int_0^1 \pi(x)dx = \pi \left.\frac{x^2}{2}\right|_0^1 = \frac{\pi}{2}$$

$$\therefore \quad \boxed{\text{Volume of solid generated} = \frac{\pi}{2}}$$

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165. (b)

Let $I = \int_0^{\pi} x\cos^2 x \; dx$ ...(i)

$$\left\{ \begin{array}{l} \because \int_0^a f(x)dx = \int_0^a f(a-x)dx \\ \text{Hence replacing x by } \pi - x \text{ in equation (i)} \end{array} \right\}$$

$$I = \int_0^{\pi} (\pi - x)\cos^2(\pi - x)dx$$

$$I = \int_0^{\pi} (\pi - x)\cos^2 x$$

$$I = \int_0^{\pi} \pi\cos^2 x \; dx - \int_0^{\pi} x\cos^2 x \; dx$$

$$I = \int_0^{\pi} \pi\cos^2 x \; dx - I$$

$$2I = \pi\int_0^{\pi} \cos^2 x \; dx$$

$$\left[\begin{array}{l} \text{Here } \cos^2(\pi - x) = \cos^2 x \\ \text{\& When } f(x) = f(2a - x) \\ \Rightarrow \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx \end{array} \right]$$

$$2I = 2\pi\int_0^{\pi/2} \cos^2 x \; dx$$

$$I = \pi\int_0^{\pi/2} \cos^2 x \, dx = \pi \times \frac{1}{2} \times \frac{\pi}{2}$$

$$\boxed{I = \frac{\pi^2}{4}}$$

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166. (85 to 85.5)

Given: curves are $y = 8$ ...(i)

and $x^2 = 8y$ ...(ii)

Intersection points of curve (i) and (ii):

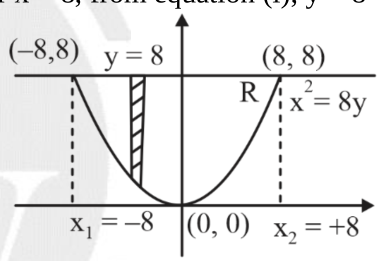
From (i) putting $y = 8$ in equation (ii),

$x^2 = 8 \times 8$

$x = \pm 8$

For $x = -8$, from equation (i); $y = 8$

And for $x = 8$, from equation (i); $y = 8$



Area bounded by curves

$$= \int_{x_1}^{x_2} y_1 - y_2 \; dx$$

$$= \int_{-8}^{8} 8 - \frac{x^2}{8} dx = \left[8x - \frac{x^3}{24}\right]_{-8}^{+8}$$

$$= 2\left(64 - \frac{64}{3}\right) \boxed{= 85.33}$$

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167. (b)

curves are $x + y = 3$ ...(i)

and $y = x^2 + 1$ ...(ii)

Intersection point of curve (i) and (ii):

From (i) putting y = 3 − x in equation (ii),

$3 - x = x^2 + 1$

$x^2 + x - 2 = 0$

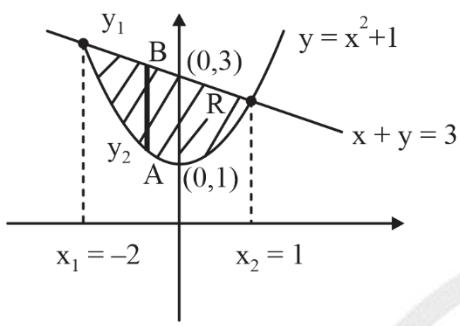
$(x + 2)(x - 1) = 0$

$x = -2, 1$

For x = –2, from equation (i); y = 0

And for x = 1, from equation (i); y = 1

$y = x^2 + 1$ and $x + y = 3$



Taking vertical strip AB in required region R. Now for entire region the limit of y at any x will be,

Lower limit $\rightarrow$ $y = y_2 = x^2 + 1$ [At A, from the equation of curve (ii)]

Upper limit $\rightarrow$ $y = y_1 = 3 - x$ [At B, from the equation of curve (i)]

Now by sliding strip AB in horizontal direction from extreme left to right in required region R we get the limit of x as given below,

Lower limit $\rightarrow x = -2$

Upper limit $\rightarrow x = 1$

Hence required area (A),

$$A = \int_{x=-2}^{x=1} \int_{y=x^2+1}^{y=3-x} dydx$$

$$A = \int_{x=-2}^{x=1} \left(\int_{y=x^2+1}^{y=3-x} dy \right) dx$$

$$A = \int_{x=-2}^{x=1} (3 - x) - (x^2 + 1) dx$$

$$A = \int_{x=-2}^{x=1} (-x^2 - x + 2) dx = \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right]_{-2}^{1}$$

$$A = \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(\frac{8}{3} - 2 - 4 \right)$$

$$\boxed{A = \frac{9}{2}}$$

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168. (1.85 to 1.87)

Given

$x(t) = \cos t$; $y(t) = \sin t$; $z(t) = \frac{2}{\pi} t$ and $0 \le t \le \frac{\pi}{2}$

$$\Rightarrow \frac{dx}{dt} = -\sin t; \frac{dy}{dt} = \cos t; \frac{dz}{dt} = \frac{2}{\pi}$$

The length of the parametric curve (L) is

$$L = \int_{t=0}^{t=\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, . \, dt$$

$$L = \int_{t=0}^{t=\frac{\pi}{2}} \sqrt{(-\sin t)^2 + (\cos t)^2 + \left(\frac{2}{\pi}\right)^2} \, .dt$$

$$L = \int_{t=0}^{t=\frac{\pi}{2}} \sqrt{1 + \frac{4}{\pi^2}} \, . \, dt = \sqrt{1 + \frac{4}{\pi^2}} \times \frac{\pi}{2} = 1.862 \text{ units}$$

$\Rightarrow$ $\boxed{\text{Length of the curve} = 1.862 \text{ units}}$

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169. (a)

Given: curves are y = x …(i)

and $y = x^2$ …(ii)

Intersection point of curve (i) and (ii):

From (i) putting y = x in equation (ii),

$x = x^2$

$x(x - 1) = 0$

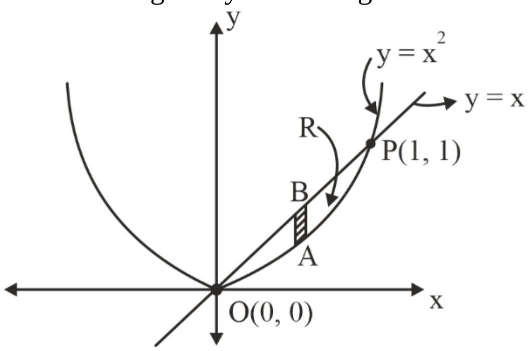
x = 0 and x = 1

For $x = 0$, from equation (i); $y = 0$

And for $x = 1$, from equation (i); $y = 1$

Hence intersection points of curve (i) and (ii) are O (0, 0) and P (1, 1)

The region R bounded by the curve $y = x^2$ and $y = x$ as shown in figure by shaded region R.



Taking vertical strip AB in required region R. Now for entire region the limit of y at any x will be,

Lower limit $\rightarrow$ $y = x^2$ [At A, from the equation of parabola $y = x^2$]

Upper limit $\rightarrow$ $y = x$ [At B, from the equation of line $y = x$]

Now by sliding strip AB in horizontal direction from extreme left to right in required region R we get the limit of x as given below,

Lower limit $\rightarrow x = 0$

Upper limit $\rightarrow x = 1$

Hence required area (A),

$$A = \int_{x=0}^{x=1} \int_{y=x^2}^{y=x} dy dx$$

$$A = \int_{x=0}^{x=1} \left(\int_{y=x^2}^{y=x} dy \right) dx$$

$$A = \int_{x=0}^{x=1} (x - x^2) dx$$

$$A = \left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1$$

$$A = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\boxed{\therefore \text{ Area bounded} = \frac{1}{6} \text{units}}$$

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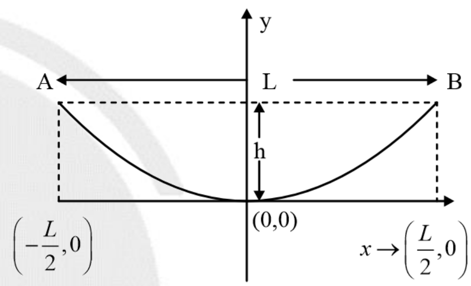
170. (d)

The length of the curve f(x) from x = a to x = b is given by

$$L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx \qquad \ldots\text{(i)}$$

$$\text{Now, } y = 4h\frac{x^2}{L^2} \quad \Rightarrow \frac{dy}{dx} = \frac{8hx}{L^2}$$



Here, A & B are supports. Put y = h in the equation of parabola to get co-ordinates of the supports

$$h = 4h\frac{x^2}{L^2} \Rightarrow x^2 = \frac{L^2}{4} \Rightarrow x = \pm\frac{L}{2}$$

$$\therefore A \rightarrow \left(-\frac{L}{2}, 0\right) \text{ and } B \rightarrow \left(\frac{L}{2}, 0\right)$$

Therefore, from equation (i),

$$\text{Length of cable} = \int_{-L/2}^{L/2} \sqrt{1 + \left(\frac{8hx}{L^2}\right)^2} .dx$$

Since $\sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}$ is even function, therefore

$$\boxed{\text{Length of cable} = 2\int_0^{L/2} \sqrt{1 + \frac{64h^2x^2}{L^4}} dx}$$

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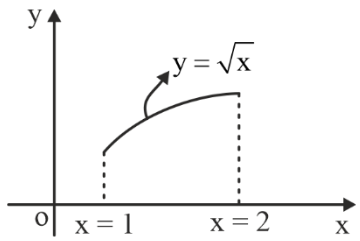
171. (d)

Given: $y = \sqrt{x}$; $1 \le x \le 2$

The volume of the solid obtained by revolving $y = f(x)$ about x-axis between x = a & x = b is given by

$$V = \int_{x=a}^{x=b} \pi y^2 dx$$

$$V = \int_{x=1}^{x=2} \pi y^2 dx$$



$$V = \int_{x=1}^{x=2} \pi\left(\sqrt{x}\right)^2 dx = \pi \cdot \left.\frac{x^2}{2}\right|_1^2$$

$$V = \frac{\pi}{2}(4-1) = \frac{3\pi}{2}$$

$\therefore$ $\boxed{\text{Volume of solid generated} = \frac{3\pi}{2}}$

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172. (a)

Given: curves are

$y^2 = 4x$ …(i)

and $x^2 = 4y$ …(ii)

Intersection point of curve (i) and (ii):

From (i) putting $x = \frac{y^2}{4}$ in equation (ii),

$$\left(\frac{y^2}{4}\right)^2 = 4y$$

$$y^4 - 64y = 0$$

$$y\left(y^3 - 64\right) = 0$$

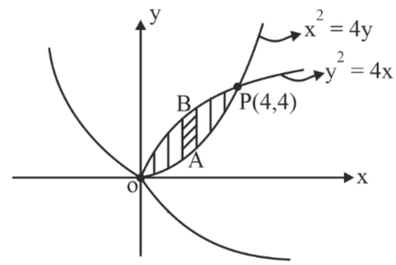
$$y = 0, y = 4$$

For y= 0, from equation (i); x = 0

And for y = 4, from equation (i); x = 4

Hence intersection points of curve (i) and (ii) are O (0, 0) and P (4, 4)

The region R bounded by the curve $y = x^2$ and $y = x$ is shown in figure.



Taking vertical strip AB in required region R. Now for entire region the limit of y at any x will be,

Lower limit $\rightarrow$ $y = 2\sqrt{x}$ [At A, from the equation of curve $y^2 = 4x$]

Upper limit $\rightarrow$ $y = \frac{x^2}{4}$ [At B, from the equation of $x^2 = 4y$]

Now by sliding strip AB in horizontal direction from extreme left to right in required region R we get the limit of x as given below,

Lower limit $\rightarrow x = 0$

Upper limit $\rightarrow x = 4$

Hence required area (A),

$$A = \int_{x=0}^{x=4} \int_{y=\frac{x^2}{4}}^{y=2\sqrt{x}} dydx$$

$$A = \int_{x=0}^{x=1} \left(\int_{y=\frac{x^2}{4}}^{y=2\sqrt{x}} dy \right) dx$$

Area between the curves $A = \int_{x=0}^{x=4} \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$

$$A = \left.\left(2 \cdot \frac{2}{3} \cdot x^{3/2} - \frac{x^3}{12}\right)\right|_0^4$$

$$A = \frac{4}{3}\left(4^{3/2}\right) - \frac{4^3}{12} = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

$\boxed{\therefore \text{ Area bounded} = \frac{16}{3} \text{units}}$

Scan for Video solution

173. (d)

Given:

$$y=\frac{2}{3}x^{3/2} \Rightarrow \frac{dy}{dx}=\frac{2}{3}\cdot\frac{3}{2}\cdot x^{1/2}=\sqrt{x}$$

The length of the curve

y = f(x) between x = a and x = b is given by

$$L=\int_{x=a}^{x=b}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$$

$$L=\int_{x=0}^{x=1}\sqrt{1+(\sqrt{x})^2}\,dx$$

$$L=\int_{x=0}^{x=1}(\sqrt{1+x})dx$$

$$L=\frac{2}{3}(1+x)^{3/2}\Big|_{x=0}^{x=1}=\frac{2}{3}(2^{3/2}-1^{3/2})$$

$$L=\frac{2}{3}\left(2\sqrt{2}-1\right)=1.2189$$

$\therefore$ $\boxed{\text{Length of the curve = 1.22}}$

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174. (0)

$$\text{Let } I=\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}\left(4x^2y-z^3\right)dzdydx$$

$$=\int_{-3}^{3}\int_{-2}^{2}\left(4x^2yz-\frac{z^4}{4}\Big|_{-1}^{1}\right)dydx$$

$$=\int_{-3}^{3}\int_{-2}^{2}\left\{\left(4x^2y-\frac{1}{4}\right)-\left(-4x^2y-\frac{1}{4}\right)\right\}dydx$$

$$=\int_{-3}^{3}\int_{-2}^{2}8x^2ydydx$$

$$=\int_{-3}^{3}\frac{8x^2y^2}{2}\Big|_{-2}^{2}dx=\int_{-3}^{3}\left\{\left(16x^2\right)-\left(16x^2\right)\right\}dx=0$$

$$\therefore \boxed{\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}\left(4x^2y-z^3\right)dzdydx=0}$$

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175. (b)

$$I=\int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=\cos\theta}r\sin\theta\,dr\,d\theta$$

$$I=\int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=\cos\theta}r\sin\theta\,dr\,d\theta$$

$$I=\int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=\cos\theta}\left(r\sin\theta dr\right)d\theta$$

$$I=\int_{\theta=0}^{\theta=\frac{\pi}{2}}\left(\sin\theta\cdot\frac{r^2}{2}\Big|_0^{\cos\theta}\right)d\theta$$

$$I=\frac{1}{2}\times\int_{\theta=0}^{\theta=\frac{\pi}{2}}\sin\theta\cdot\cos^2\theta\cdot d\theta$$

Let $\cos\theta = t \Rightarrow -\sin\theta\, d\theta = dt$

At $\theta = 0$, $t = \cos 0 = 1$ and $\theta = \pi/2$, $t = \cos\pi/2 = 0$

$$I=\frac{1}{2}\times\int_1^0 t^2\left(-dt\right)=-\frac{1}{2}\times\frac{t^3}{3}\Big|_1^0$$

$$I=-\frac{1}{2}\times\left(0-\frac{1}{3}\right)=\frac{1}{6}$$

$$\therefore \boxed{\int_{\theta=0}^{\theta=\frac{\pi}{2}}\int_{r=0}^{r=\cos\theta}r\cdot\sin\theta\cdot dr\,d\theta=\frac{1}{6}}$$

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176. (b)

Given:

$$f(x, y, z)= e^{1-x\cos y}+xze^{-1/(1+y^2)}$$

$$\Rightarrow f(x, y, z)=e\cdot e^{(-\cos y)x}+xze^{-1/(1+y^2)}$$

Differentiating above equation with respect to x,

$$\Rightarrow \frac{\partial f}{\partial x}=e\cdot e^{(-\cos y)x}\cdot(-\cos y)+z\cdot e^{-1/(1+y^2)}$$

At the point (1, 0, e)

$$\frac{\partial f}{\partial x}\Big|_{(1,0,e)}=e\cdot e^{(-\cos 0)1}\cdot\left(-\cos 0\right)+e\cdot e^{-1/(1+0^2)}$$

$$\Rightarrow \frac{\partial f}{\partial x}\Big|_{(1,0,e)}=e.e^{-1}(-1)+e.e^{-1}=0$$

∴ Partial deviation of f(x, y, z) at (1, 0, e) is 0.

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177. (c)

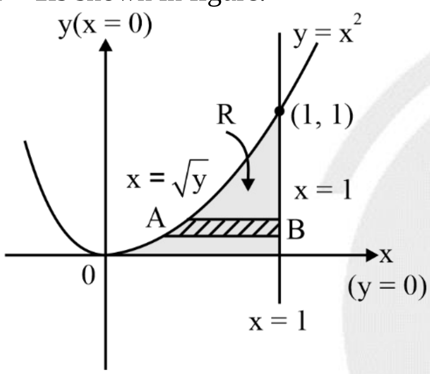
Given

$$I = \int_{x=0}^{1}\int_{y=0}^{x^2} xy^2 \, dy \, dx$$

The curves involved are $x = 0, x = 1$

$y = 0, y = x^2$

The region R bounded by curve $y = 0$, $y = x^2$, $x = 0$ and $x = 1$is shown in figure.



Now to change the order of integration takin a horizontal strip AB in region R.

Now for region R the limit of x at any y will be,

Lower limit → $x = \sqrt{y}$ [At A]

Upper limit → $x = 1$ [At B]

Now by sliding strip AB in vertical direction from bottom to top in required region R we get the limit of y as given below,

Lower limit → $y = 0$

Upper limit → $y = 4$

On changing the order of integration, integral I will become,

$$\boxed{I = \int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} xy^2 \, dx \, dy}$$

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178. (d)

Given: $f(x, y) = \dfrac{ax^2 + by^2}{xy}$ & $\left.\dfrac{\partial f}{\partial x}\right|_{\substack{x=1\\y=2}} = \left.\dfrac{\partial f}{\partial y}\right|_{\substack{x=1\\y=2}}$

$$f(x, y) = \frac{ax^2 + by^2}{xy}$$

$$\Rightarrow \quad f(x, y) = a.\frac{x}{y} + b.\frac{y}{x} \qquad \ldots(i)$$

Differentiating equation (i) partially with respect to x,

$$\Rightarrow \quad \frac{\partial f}{\partial x} = \frac{a}{y} - \frac{b.y}{x^2}$$

At x = 1 and y = 2:

$$\Rightarrow \quad \left.\frac{\partial f}{\partial x}\right|_{\substack{x=1\\y=2}} = \frac{a}{2} - 2b$$

Differentiating equation (i) partially with respect to y,

$$\Rightarrow \quad \frac{\partial f}{\partial y} = \frac{-ax}{y^2} + \frac{b}{x}$$

At x = 1 and y = 2:

$$\Rightarrow \quad \left.\frac{\partial f}{\partial x}\right|_{\substack{x=1\\y=2}} = \frac{-a}{4} + b$$

$$\because \quad \left.\frac{\partial f}{\partial x}\right|_{\substack{x=1\\y=2}} = \left.\frac{\partial f}{\partial y}\right|_{\substack{x=1\\y=2}}$$

$$\Rightarrow \quad \frac{a}{2} - 2b = \frac{-a}{4} + b \qquad \Rightarrow \quad \frac{3a}{4} = 3b$$

$$\Rightarrow \quad \boxed{a = 4b}$$

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179. (4.4 to 4.6)

Given $r = x^2 + y - z$ and $z^3 - xy + y^2 + y^3 = 1$

$r = x^2 + y - z$

Differentiating both sides partially with respect to 'x'

$$\Rightarrow \frac{\partial r}{\partial x} = 2x - \frac{\partial z}{\partial x} \ldots\ldots.(1)$$

$z^3 - xy + yz + y^3 = 1$

Differentiating both sides partially with respect to 'x'

$$\Rightarrow 3z^2.\frac{\partial z}{\partial x} - y + y\frac{\partial z}{\partial x} + 0 = 0$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{y}{(3z^2 + y)} \quad \ldots\ldots.(2)$$

Substituting $\frac{\partial z}{\partial x}$ in (1)

$$\Rightarrow \frac{\partial r}{\partial x} = 2x - \frac{y}{3z^2 + y}$$

At the point (2, –1, 1),

$$\left.\frac{\partial r}{\partial x}\right|_{(2,-1,1)} = 2\times 2 - \frac{(-1)}{3\times 1^2 + (-1)}$$

$$\boxed{\left.\frac{\partial r}{\partial x}\right|_{(2,-1,1)} = 4.5}$$

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180. **(40)**

f(x, y, z) = $(x^2 + y^2 - 2z^2)(y^2 + z^2)$

f(x, y, z) = $x^2y^2 + x^2z^2 + y^4 + y^2z^2 - 2y^2z^2 - 2z^4$

Differentiating partially with respect to x,

$$\Rightarrow \frac{\partial f}{\partial x} = 2xy^2 + 2xz^2 + 0 + 0 - 0 - 0$$

$$\Rightarrow \frac{\partial f}{\partial x} = 2xy^2 + 2xz^2$$

At (2, 1, 3)

$$\Rightarrow \frac{\partial f}{\partial x} = 2(2)(1)^2 + 2(2)(3)^2 = 4 + 36$$

$$\Rightarrow \boxed{\frac{\partial f}{\partial x} = 40}$$

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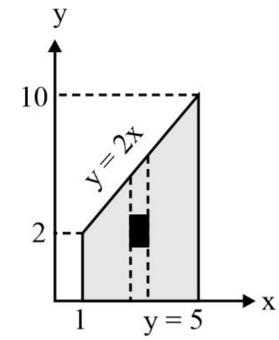
181. **(1.00)**

Equation of the line joining points (1, 2) and (5, 10) is:

$$\frac{x-1}{5-1} = \frac{y-2}{10-2}$$

$$\Rightarrow 2\times(x-1) = y-2$$

$$\Rightarrow y = 2x$$



Taking vertical strip AB in required region R.

Now for entire region the limit of y at any x will be,

Lower limit $\rightarrow$ y = 0 [At A]

Upper limit $\rightarrow$ y = 2x [At B]

Now by sliding strip AB in horizontal direction from extreme left to right in required region R we get the limit of x as given below,

Lower limit $\rightarrow x = 1$

Upper limit $\rightarrow x = 5$

Therefore,

$$I = c\int\int_R xy^2 dxdy$$

$$\Rightarrow I = c\int_{x=1}^{x=5}\int_{y=0}^{y=2x} xy^2 dxdy$$

$$\Rightarrow I = c\int_{x=1}^{5} x\left[\frac{y^3}{3}\right]_0^{2x} dx = c\int_{x=1}^{5} x\left(\frac{8x^3}{3}\right)dx$$

$$\Rightarrow I = c\times\frac{8}{3}\int_{x=1}^{5} x^4 dx$$

$$\Rightarrow I = 6\times 10^{-4}\times\frac{8}{3}\times\left[\frac{x^5}{5}\right]_1^5$$

$$\Rightarrow I = 6\times 10^{-4}\times\frac{8}{3}\times\frac{5^5 - 1}{5} = 0.99968$$

$$\Rightarrow \boxed{I = 1.00}$$

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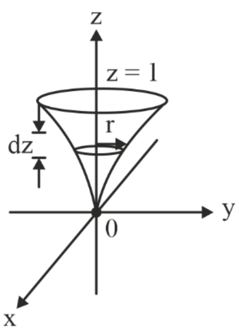
182. (0.7 to 0.85)



Given $x^2 + y^2 \le z^3$; $0 \le z \le 1$.
Let $z^3 = r^2$, $dv = \pi r^2 \cdot dz$

$$\therefore \text{ Volume} = \int dv = \int_0^1 \pi z^3 dz = \left.\frac{\pi z^4}{4}\right|_0^1$$

$$= \frac{\pi}{4} = 0.7854$$

$\therefore$ Volume of region R= 0.7854

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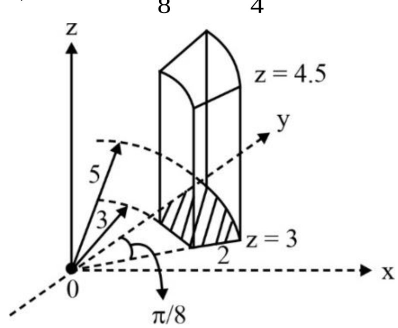
183. (4 to 5)

The region is given by

$$\left\{(\rho, \phi, z): 3 \le \rho \le 5, \frac{\pi}{8} \le \phi \le \frac{\pi}{4}, 3 \le z \le 4.5\right\}$$



Volume of the region
= Volume of the portion of cylinder
= base area × height

$$= \frac{\left(\frac{\pi}{4} - \frac{\pi}{8}\right)}{2\pi} \times \pi \times \left\{5^2 - 3^2\right\} \times (4.5 - 3)$$

$$= \frac{1}{16} \times \pi \times 16 \times 1.5$$

$= 1.5\ \pi = 4.712$

$\therefore$ $\boxed{\textbf{Volume of Region = 4.712 units}}$

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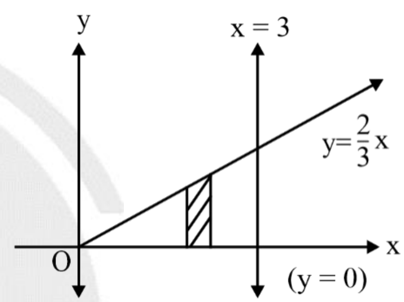
184. (10)

Given region in xy – plane is bounded by 2x = 3y

$\Rightarrow \frac{2}{3}x$, y = 0 and x = 3



$$\text{Volume of the region} = \int_{x=0}^{x=3} \int_{y=0}^{y=\frac{2}{3}x} \int_{z=0}^{z=6-x-y} dz\ dy\ dx$$

$$= \int_{x=0}^{x=3} \int_{y=0}^{y=\frac{2}{3}x} (6 - x - y) dy\ dx$$

$$= \int_{x=0}^{x=3} \left.(6 - x) y - \frac{y^2}{2}\right|_0^{\frac{2}{3}x} dx$$

$$= \int_{x=0}^{x=3} \left\{(6 - x) \cdot \frac{2x}{3} - \frac{1}{2}\left(\frac{4}{9}x^2\right)\right\} dx$$

$$= \int_{x=0}^{x=3} \left\{4x - \frac{2}{3}x^2 - \frac{2}{9}x^2\right\} dx$$

$$= \int_{x=0}^{x=3} \left(4x - \frac{8x^2}{9}\right) dx = \left. 2x^2 - \frac{8x^3}{27}\right|_0^3$$

= 18 – 8 = 10

$\boxed{\therefore \textbf{ Volume of the region = 10 units}}$

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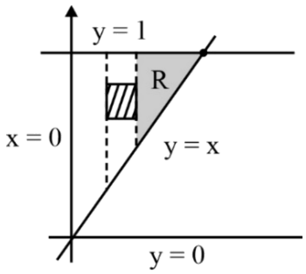
185. (0.7 to 0.76)

Triangle bounded by the lines x = y; x = 0; y = 1 is shown in figure:



Required volume

$$= \iint_R f(x,y)\,dx\,dy$$

$$= \int_{x=0}^{x=1}\int_{y=x}^{y=1} e^x\,dy\,dx$$

$$= \int_0^1 e^x [y]_x^1\,dx = \int_0^1 e^x[1-x]dx$$

$$= \left[e^x(1-x)\right]_0^1 - \int_0^1 (-1)\times e^x dx$$

$$= \left[(0)-(1)\right] + \int_0^1 e^x dx$$

$$= -1 + \left[e^x\right]_0^1$$

$$= -1 + (e-1)$$

$$\boxed{= 0.718}$$

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186. (a)

Given $\frac{\partial}{\partial y}\left(x^2+y^2\right) = \frac{\partial}{\partial x}(6y+4x)$

$\Rightarrow\quad 2y = 4$

$\Rightarrow\quad y = 2$

$\therefore$ **the required contour is y = 2**

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187. (b)

$$u = \frac{2x-y}{2} \qquad \ldots(i)$$

And $v = \frac{y}{2} \Rightarrow y = 2v \qquad \ldots(ii)$

Put $y = 2v$ in equation (i), $u = \frac{2x-2v}{2}$

$\Rightarrow\ u = x - v$

$\Rightarrow\ x = u + v \qquad \ldots(iii)$

Let, $I = \int_0^8\left(\int_{y/2}^{(y/2)+1}\left(\frac{2x-y}{2}\right)dx\right)dy$

By changing the variable above integration will become,

$$I = \int_{v_1}^{v_2}\left(\int_{u_1}^{u_1} u|J|du\right)dv \qquad \ldots(iv)$$

$$\left[\because \frac{2x-y}{2} = u\right]$$

Where J = Jacobian of transformation.

$x \to \frac{y}{2}$ to $\frac{y}{2}+1$ and $y \to 0$ to 8

Lower limit of u (u_1): Put $x = \frac{y}{2}$ & $y = 0$ in equation (i),

$$u_1 = \frac{2\left(\frac{y}{2}\right)-y}{2} = \frac{2\left(\frac{0}{2}\right)-0}{2} = 0$$

Upper limit of u (u_2): Put $x = \frac{y}{2}+1$ & $y = 8$ in equation (i),

$$u_2 = \frac{2\left(\frac{y}{2}+1\right)-y}{2} = \frac{2\left(\frac{8}{2}+1\right)-8}{2} = 1$$

Lower limit of v (v_1): Put $x = \frac{y}{2}$ & $y = 0$ in equation (ii),

$$v_1 = \frac{0}{2} = 0$$

Upper limit of v (v_2) : Put $x = \frac{y}{2} + 1$ & $y = 8$ in equation (ii), $v_2 = \frac{8}{2} = 4$

Jacobian matrix

$$J = J\left(\frac{x, y}{u, v}\right) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\Rightarrow \quad J = \begin{vmatrix} \frac{\partial (u+v)}{\partial u} & \frac{\partial (u+v)}{\partial v} \\ \frac{\partial (2v)}{\partial u} & \frac{\partial (2v)}{\partial v} \end{vmatrix}$$

$$\Rightarrow \quad J = \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2$$

Hence from equation (iv),

$$I = \int_0^4 \left(\int_0^1 2u\,du \right) dv$$

Therefore,

$$\boxed{I = \int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x - y}{2} \right) dx \right) dy = \int_0^4 \left(\int_0^1 2u\,du \right) dv}$$

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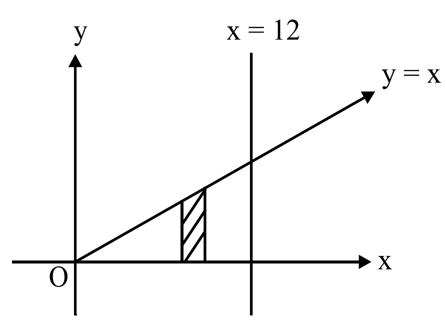
188. (864)



Given region is bounded by

xy – plane (z = 0) and z = x + y

$\{0 \le y \le x \text{ and } 0 \le x \le 12\}$

$\therefore$ Volume

$$= \int_{x=0}^{x=12} \int_{y=0}^{y=x} \int_{z=0}^{z=x+y} dz\, dy\, dx$$

$$= \int_{x=0}^{x=12} \int_{y=0}^{y=x} (x + y) dy\, dx = \int_{x=0}^{x=12} \left(xy + \frac{y^2}{2} \Big|_0^x \right) dx$$

$$= \int_{x=0}^{x=12} \left(x^2 + \frac{x^2}{2} \right) dx$$

$$= \int_{x=0}^{x=12} \frac{3}{2} x^2 dx = \frac{x^3}{2} \Big|_0^{12}$$

$$= \frac{(12)^3}{2} = 864$$

$\therefore$ **Volume bounded by the given curves is 864 units.**

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189. (b)

$$I = \int_0^2 \int_0^x e^{x+y} \cdot dy \cdot dx$$

$$I = \int_0^2 \int_0^x e^x \cdot e^y \cdot dy \cdot dx$$

$$I = \int_0^2 e^x \cdot \left(e^y \Big|_0^x \right) dx$$

$$I = \int_0^2 e^x \cdot (e^x - 1) dx$$

$$I = \int_0^2 (e^{2x} - e^x) dx = \frac{e^{2x}}{2} - e^x \Big|_0^2$$

$$I = \left(\frac{e^4}{2} - e^2 \right) - \left(\frac{1}{2} - 1 \right)$$

$$I = \frac{e^4}{2} - e^2 + \frac{1}{2} = \frac{1}{2}(e^4 - 2e^2 + 1)$$

$$\therefore \quad \boxed{\int_0^2 \int_0^x e^{x+2y} dy\, dx = \frac{1}{2}(e^2 - 1)^2}$$

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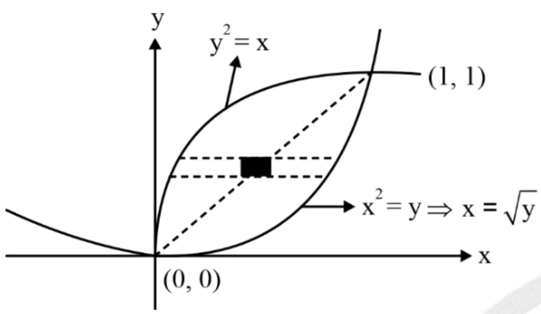
190. (a)

$x > y^2 \quad x \in [0, 1]$

$y > x^2 \quad y \in [0,1]$



In case of horizontal strip limits will be $x = y^2$ and $x = \sqrt{y}$, then this strip will move from y = 0 to y = 1 to give the required volume of f (x, y). Hence volume under f(x, y) is

$$\int_{y=0}^{y=1} \int_{x=y^2}^{x=\sqrt{y}} f(x, y)\, dx\, dy$$

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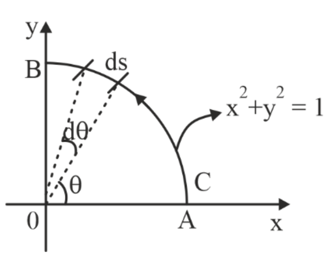
191. (b)

Given: function is $(x + y)^2$

Radius of quarter of a circle, r =1

Let $I = \int_C (x+y)^2\, ds$ ...(i)



$ds = r \cdot d\theta = d\theta$ $\quad [\because r = 1]$

$x = \cos\theta$; $y = \sin\theta$

From equation (i),

$$I = \int_{\theta=0}^{\theta=\frac{\pi}{2}} (\cos\theta + \sin\theta)^2 \cdot d\theta$$

$$I = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \left(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta\right) \cdot d\theta$$

$$I = \int_{\theta=0}^{\theta=\frac{\pi}{2}} (1 + \sin 2\theta) \cdot d\theta$$

$$I = \theta - \frac{\cos 2\theta}{2}\Bigg|_0^{\frac{\pi}{2}}$$

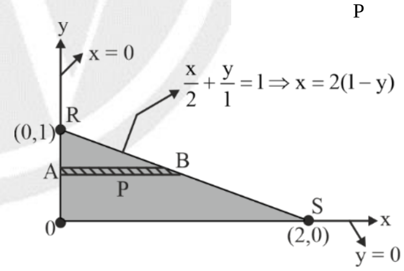
$$I = \left\{\frac{\pi}{2} - \left(-\frac{1}{2}\right)\right\} - \left\{0 - \frac{1}{2}\right\} = \frac{\pi}{2} + 1$$

$$\therefore \boxed{I = \int_{\theta=0} (x+y)^2\, ds = \frac{\pi}{2} + 1}$$

Scan for Video solution

192. (a)

Given: Integral $I = \iint_P xy\, dx\, dy$



Equation of line RS:

$$\frac{x}{2} + \frac{y}{1} = 1 \Rightarrow x = 2(1-y)$$

Taking horizontal strip AB in the region P.

Now for region P the limit of x at any y will be,

Lower limit $\rightarrow$ $x = 0$ [At A]

Upper limit $\rightarrow$ $x = 2(1-y)$ [At B]

Now by sliding strip AB in vertical direction from bottom to top in required region P we get the limit of y as given below,

Lower limit $\rightarrow y = 0$

Upper limit $\rightarrow y = 1$

Therefore,

$$I = \iint_P xy\,dx\,dy == \int_{y=0}^{y=1}\int_{x=0}^{x=2(1-y)} xy \cdot dx \cdot dy$$

$$I = \int_{y=0}^{y=1} y\left(\left.\frac{x^2}{2}\right|_0^{2(1-y)}\right)dy$$

$$I = \int_{y=0}^{y=1} \frac{y}{2}\cdot 4(1-y)^2\,dy$$

$$I = \int_{y=0}^{y=1} y\left(1-2y+y^2\right)dy = \int_{y=0}^{y=1}\left(y-2y^2+y^3\right)dy$$

$$I = 2\times\left(\left.\frac{y^2}{2}-\frac{2}{3}y^3+\frac{y^4}{4}\right|_0^1\right)$$

$$I = 2\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right) = \frac{1}{6}$$

$$\therefore \boxed{\iint_p xy\,dx\,dy = \frac{1}{6}}$$

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CHAPTER

3

PROBABILITY AND STATISTICS

Basic Probability

1. [NAT] [GATE-CE-2023:1M]

The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (***Rounded off to one decimal place***)?

2. [NAT] [GATE-EE-2023:2M]

The expected number of trials for first occurrence of a "head" in a biased coin is known to be 4. The probability of first occurrence of a "head" in the second trial is____.
(***Round off to 3 decimal places***)

3. [NAT] [GATE-CE-2022:2M]

A pair of six-faced dice is rolled twice, the probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is_________ (***round off to three decimal places***).

4. [MCQ] [GATE-EE-2021:2M]

Suppose the probability that a coin toss shows "head" is p, where $0 < p < 1$. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is

(a) $p/(1 - p)$ (b) $(1 - p)/p$
(c) $1/p$ (d) $1/p^2$

5. [NAT] [GATE-2020-CE:1M]

A fair (unbiased) coin is tossed 15 times. The probability of getting exactly 8 heads (***round off to three decimal places***), is____________.

6. [NAT] [GATE-ME-2020:1M]

A company is hiring to fill four managerial vacancies. The candidates are five men aand three women. If every candidate is eqaully likely to be chosen then the probability that at least one women will be selected is _____
(***round off to 2 decimal places***).

7. [NAT] [GATE-ME-2020:2M]

A fair coin is tossed 20 times. The probability that 'head' will appear exactly 4 times in the first ten tosses, and 'tail' will appear exactly 4 times in the next ten tosses is ______.
(***Round off to 3 decimal places***)

8. [NAT] [GATE-ME-2019:2M]

The probability that a part manufactured by a company will be defective is 0.05. If 15 such parts are selected randomly and inspected, then the probability that at least two parts will be defective is _____.(***Round off to two decimal places***)

9. [NAT] [GATE-EE-2019:2M]

The probability of a resistor being defective is 0.02. There are 50 such resistors in a circuit. The probability of two or more defective resistors in the circuit (***round off to two decimal places***) is ______.

10. [NAT] [GATE-CE-2018:1M]

Probability (up to one decimal place) of consecutively picking 3 red balls without replacement from a box containing 5 red balls and 1 white ball is ______.

11. [MCQ] [GATE-ME-2018:1M]

Four red balls, four green balls and four blue balls are put in a box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is

(a) $\frac{1}{72}$ (b) $\frac{1}{55}$

(c) $\frac{1}{36}$ (d) $\frac{1}{27}$

12. [MCQ] [GATE-ME-2018:1M]

A six-faced fair dice is rolled five times. The probability (in %) of obtaining "ONE" at least four times is

(a) 33.3 (b) 3.33

(c) 0.33 (d) 0.0033

13. [NAT] [GATE-CS-2018:1M]

Two people, P and Q, decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is _____.

14. [MCQ] [GATE-CS-2017:2M]

P and Q are considering to apply for a job. The probability that P applies for the job is 1/4, the probability that P applies for the job given that Q applies for the job is 1/2, and the probability that Q applies for the job given that P applies for the job is 1/3. Then the probability that P does not apply for the job given that Q does not apply for the job is

(a) $\frac{4}{5}$ (b) $\frac{5}{6}$

(c) $\frac{7}{8}$ (d) $\frac{11}{12}$

15. [NAT] [GATE-ME-2017:1M]

A six-face fair dice is rolled a large number of times. The mean value of othe outcomes is ______.

16. [NAT] [GATE-ME-2017:1M]

Two coins are tossed simultaneously. The probability (upto two decimal points accuracy) of getting at least one head is ______.

17. [NAT] [GATE-CE-2017:1M]

A two-faced coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes; H, H, H. If the coin is tossed one more time, the probability (up to one decimal place) of obtaining H again, given the previous realization of H, H and H, would be________.

18. [MCQ] [GATE-EE-2017:1M]

An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its color. The probability to get a red ball in the second draw is

(a) $\frac{1}{2}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{6}{9}$

19. [NAT] [GATE-EC-2017:1M]

Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place) _________.

20. [NAT] [GATE-EC-2017:2M]

Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40 % chance of getting reservation in any attempt by a passenger then the average number of attempts that passengers need to make to get a seat reserved is _______

21. [NAT] [GATE-EC-2017:1M]

Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place) _________.

22. [NAT] [GATE-EE-2016:2M]

Candidates were asked to come to an interview with 3 pens each. Black, Blue, green and red were the permitted pen colors that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is__________.

23. [NAT] [GATE-ME-2016:2M]

The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is _________.

24. [MCQ] [GATE-ME-2016:2M]

Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(a) $\frac{16}{5525}$ (b) $\frac{64}{2197}$

(c) $\frac{3}{13}$ (d) $\frac{8}{16575}$

25. [NAT] [GATE-EC-2016:1M]

The probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a "head" is obtained. If the tosses are independent, then the probability of getting "head" for the first time in the fifth toss is _________.

26. [NAT] [GATE-CS-2016:2M]

Consider the following experiment.

Step 1. Flip a fair coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step1.

The probability that the output of the experiment is Y is (up to two decimal places)_________.

27. [MCQ] [GATE-ME-2015:2M]

The probability of obtaining at least two "SIX" in throwing a fair dice 4 times is

(a) 425/432 (b) 19/144

(c) 13/144 (d) 125/432

28. [NAT] [GATE-ME-2015:1M]

Three vendors were asked to supply a very high precision component. The respective probabilities of their meeting the strict design specifications are 0.8, 0.7 and 0.5. Each vendor supplies one component. The probability that out of total three components supplied by the vendors, at least one will meet the design specifications is_______.

29. [MCQ] [GATE-ME-2015:2M]

The chance of a student passing an exam is 20%. The chance of a student passing the exam and getting above 90% marks in it is 5%. GIVEN that a student passes the examination, the probability that the student gets above 90% marks is

(a) $\frac{1}{18}$ (b) $\frac{1}{4}$

(c) $\frac{2}{9}$ (d) $\frac{5}{18}$

30. [MCQ] [GATE-ME-2015:1M]

If P(X)=1/4,P(Y)=1/3, and P(X ∩ Y)=1/12, the value of P(Y/X) is

(a) 1/4 (b) 4/25

(c) 1/3 (d) 29/50

31. [NAT] [GATE-EC-2015:2M]

Let the random variable X represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is_______.

32. [MCQ] [GATE-EC-2015:1M]

Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\bar{A}$ and $\bar{B}$ be their complements. Which one of the following statements is FALSE?

(a) $P(A \cap B) = P(A)P(B)$

(b) $P(A|B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$

(d) $P(\bar{A} \cap \bar{B}) = P(\bar{A})\, P((\bar{B})$

33. [NAT] [GATE-EC-2015:2M]

A fair die with faces {1,2,3,4,5,6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown. The expected value of X is ________.

34. [MCQ] [GATE-EE-2015:2M]

Two players, A and B, alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is

(a) $\frac{5}{11}$ (b) $\frac{1}{2}$

(c) $\frac{7}{13}$ (d) $\frac{6}{11}$

35. [MCQ] [GATE-EE-2015: 2M]

Two coins R and S are tossed. The 4 joint events H_RH_s, T_RT_s, H_RT_S, T_RH_s have probabilities 0.28, 0.18, 0.30, 0.24 respectively, where H represents head and T represents tail. Which one of the following is TRUE

(a) The coin tosses are independent

(b) R is fair, S is not.

(c) S is fair, R is not

(d) The coin tosses are dependent

36. [MCQ] [GATE-ME-2014:1M]

A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is

(a) $\frac{7}{20}$ (b) $\frac{42}{125}$

(c) $\frac{25}{29}$ (d) $\frac{5}{9}$

37. [NAT] [GATE-ME-2014:1M]

A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is _______.

38. [NAT] [GATE-ME-2014:2M]

Consider an unbiased cubic dice with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the dice. If the dice is thrown thrice, the probability of obtaining red colour on top face of the dice at least twice is _______.

39. [MCQ] [GATE-2014-CE:1M]

A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes: (i) Head, (ii) Head, (iii) Head, (iv) Head. The probability of obtaining a 'Tail' when the coin is tossed again is

(a) 0 (b) 1/2

(c) 4/5 (d) 1/5

40. [NAT] [GATE-EC-2014:1M]

In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _______.

41. [NAT] [GATE-EC-2014:1M]

A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is _________.

42. [MCQ] [GATE-EC-2014:1M]

An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(a) 0.067 (b) 0.073

(c) 0.082 (d) 0.091

43. [NAT] [GATE-EE-2014:1M]

Consider a dice with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is __________.

44. [NAT] [GATE-CS-2014:2M]

Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A)P(B)$ is ________.

45. [MCQ] [GATE-ME-2013:2M]

The probability that a student knows the correct answer to a multiple choice question is $2/3$. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is $1/4$. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{5}{6}$ (d) $\frac{8}{9}$

46. [MCQ] [GATE-ME-2012:2M]

A box contains 4 red balls nad 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black ball is

(a) 1/20 (b) 1/12

(c) 3/10 (d) 1/2

47. [MCQ] [GATE-CE-2012:2M]

In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

(a) 1/32 (b) 2/32

(c) 3/32 (d) 6/32

48. [MCQ] [GATE-EE-2012:2M]

A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

49. [MCQ] [GATE-EC-2012:2M]

A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(a) 1/3 (b) 1/2

(c) 2/3 (d) 3/4

50. [MCQ] [GATE-EC-2012:2M]

A fair coin is tossed independently four times. The probability of the event "the number of times heads show up is more than the number of times tails show up" is

(a) 1/16 (b) 1/8

(c) 1/4 (d) 5/16

51. [MCQ] [GATE-CE-2011:1M]

There are two containers, with one containing 4 Red and 3 Green balls and the other containing 3 Blue and

4 Green balls. One ball is drawn at random from each container. The probability that one of the balls is Red and the other is Blue will be

(a) 1/7 (b) 9/49

(c) 12/49 (d) 3/7

52. [MCQ] [GATE-ME-2011:2M]

A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

(a) 2/315 (b) 1/630

(c) 1/1260 (d) 1/2520

53. [MCQ] [GATE-ME-2011:2M]

An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. The probability of getting at least one head is

(a) 1/32 (b) 13/32

(c) 16/32 (d) 31/32

54. [MCQ] [GATE-EC-2011:2M]

A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is

(a) 2/36 (b) 2/6

(c) 5/12 (d) 1/2

55. [MCQ] [GATE-CS-2011:1M]

If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

(a) 1/3 (b) 1/4
(c) 1/2 (d) 2/3

56. [MCQ] [GATE-CS-2010:2M]

What is the probability that a divisor of 10^{99} is a multiple of 10^{96}?

(a) 1/625 (b) 4/625
(c) 12/625 (d) 16/625

57. [MCQ] [GATE-CE-2010:1M]

Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is

(a) 1/8 (b) 1/6
(c) 1/4 (d) 1/2

58. [MCQ] [GATE-EE-2010:2M]

A box contains 4 while balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3 (b) 3/7
(c) 1/2 (d) 4/7

59. [MCQ] [GATE-EE-2009:1M]

Assume for simplicity that N people, all born in April (a month of 30 days), are collected in a room. Consider the event of at least two people in the room being born on the same date of the month, even if in different year, e.g. 1980 and 1985. What is the smallest N so that the probability of this event exceeds 0.5?

(a) 20 (b) 7
(c) 15 (d) 16

60. [MCQ] [GATE-EC-2009:1M]

A fair coin is tossed 10 times. What is the probability that ONLY the first two tosses will yield heads?

(a) $\left(\frac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\frac{1}{2}\right)^2$

(c) $\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

61. [MCQ] [GATE-ME-2009:1M]

If three coins are tossed simultaneously, the probability of getting at least one head is

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$

(c) $\frac{1}{2}$ (d) $\frac{7}{8}$

62. [MCQ] [GATE-ME-2008:1M]

A coin is tossed 4 times. What is the probability of getting heads exactly 3 times?

(a) $\frac{1}{4}$ (b) $\frac{3}{8}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

63. [MCQ] [GATE-CE-2008:2M]

A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using the public transport, further choices available are bus and metro, out of which the probability commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively would be

(a) 0.45, 0.30 and 0.25
(b) 0.45, 0.25 and 0.30
(c) 0.45, 0.55 and 0.00
(d) 0.45, 0.35 and 0.20

64. [MCQ] [GATE-CS-2008:2M]

Aishwarya studies either computer science or mathematics everyday. If she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday?

(a) 0.24 (b) 0.36

(c) 0.4 (d) 0.6

Random Variables

65. [NAT] [GATE-EE-2023:2M]

Let the probability density function of a random variable x be given as $f(x) = ae^{-2|x|}$

The value of ' a ' is _____.

66. [MCQ] [GATE-EE-2023:1M]

One million random numbers are generated from a statistically stationary process with a Gaussian distribution with mean zero and standard deviation σ_o. The σ_o is estimated by randomly drawing out 10,000 numbers of samples (x_n). The estimates $\hat{\sigma}_1, \hat{\sigma}_2$ are computed in the following two ways.

$$\hat{\sigma}_1^2 = \frac{1}{10000}\sum_{n=1}^{10000} x_n^2 \quad \hat{\sigma}_2^2 = \frac{1}{9999}\sum_{n=1}^{10000} x_n^2$$

Which of the following statements is true?

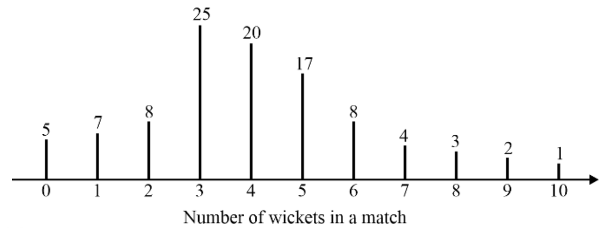
(a) $E\left(\hat{\sigma}_2^2\right) = \sigma_0^2$ (b) $E\left(\hat{\sigma}_2\right) = \sigma_0$

(c) $E\left(\hat{\sigma}_1^2\right) = \sigma_0^2$ (d) $E\left(\hat{\sigma}_1\right) = E\left(\hat{\sigma}_2\right)$

67. [NAT] [GATE-EC-2022:2M]

The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is ________.

(*Rounded off to one decimal place*)



68. [MCQ] [GATE-ME-2023:1M]

A machine produces a defective component with a probability of 0.015. The number of defective components in a packed box containg 200 components produced by the machine follows a Poisson distribution. The mean and the variance of the distribution are

(a) 3 and 3, respectively

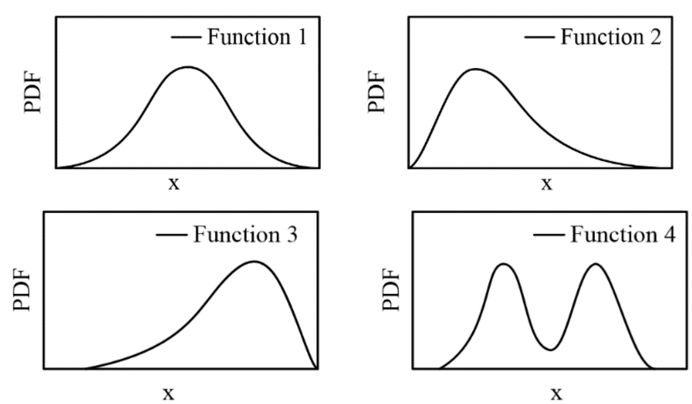
(b) $\sqrt{3}$ and $\sqrt{3}$, respectively

(c) 0.015 and 0.015, respectively

(d) 3 and 9, respectively

69. [MCQ] [GATE-CE-2023:1M]

Which of the following probability distribution functions (PDFs) has the mean greater than the median?



(a) Function 1 (b) Function 2

(c) Function 3 (d) Function 4

70. [NAT] [GATE-ME-2022:2M]

Let a random variable X follow Poisson distribution such that Prob(X = 1) = Prob(X = 2).
The value of Prob(X = 3) is ______.
(***Round off to 2 decimal places***)

71. [MCQ] [GATE-ME-2021:1M]

Consider a binomial random variable X. If X_1, X_2, ..., Xn are independent and identically distributed samples from the distribution of X with sum $Y = \sum_{i=1}^{n} X_i$, then the distribution of Y as $n \to \infty$ can be approximated as

(a) Exponential (b) Bernoulli
(c) Binomial (d) Normal

72. [MCQ] [GATE-ME-2021:1M]

The mean and variance, respectively, of a binomial distribution for *n* independent trials with the probability of success as *p*, are

(a) $\sqrt{np},\ np(1-2p)$ (b) $\sqrt{np},\ \sqrt{np(1-p)}$
(c) np, np (d) np, np(1 – p)

73. [NAT] [GATE-CS-2021:1M]

The lifetime of a component of a certain type is a random variable whose probability density function is exponentially distributed with parameter 2. For a randomly picked component of this type, the probability that its lifetime exceeds the expected lifetime (rounded to 2 decimal places) is _______.

74. [NAT] [GATE-CS-2021:1M]

For a given biased coin, the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1,000 times. Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of X (rounded to 2 decimal places) is ________.

75. [MCQ] [GATE-2021-CE:1M]

The shape of the cumulative distribution function of Gaussian distribution is

(a) Bell-shaped
(b) S-shaped
(c) Horizontal line
(d) Straight line at 45-degree angle

76. [NAT] [GATE-CS-2020:2M]

For n > 2, let $a \in \{0, 1\}^n$ be a non-zero vector. Suppose that x is chosen uniformly at random from $\{0, 1\}^n$. Then, the probability that $\sum_{i=1}^{n} a_i x_i$ is an odd number is ____.

77. [NAT] [GATE-ME-2020:2M]

Consider two exponentially distributed random variables X and Y, both having a mean of 0.50. Let Z = X + Y and *r* be the correlation coefficient between X and Y. If the variance of Z equals 0, then the value of *r* is _______.
(***Round off to 2 decimal places***)

78. [MCQ] [GATE-ME-2020:1M]

The sum of two normally distributed random variables X and Y is

(a) always normally distributed.
(b) normally distributed, only if X and Y are independent.
(c) normally distributed, only if X and Y have the same standard deviation.
(d) normally distributed, only if X and Y have the same mean.

79. [NAT] [GATE-EC-2020:2M]

x is a random variable with uniform probability density function in the interval [–2,10]. For y = 2x – 6, the conditional probability $P(y \le 7 | x \ge 5)$ (rounded off to three decimal places) is__________.

80. [NAT] [GATE-EC-2019:1M]

If X and Y are random variables such that E[2X+Y]=0 And E[X + 2Y] = 33, then E[X] + E[Y] = ______.

81. [NAT] [GATE-EC-2019:1M]

Let z be an exponential random variable with mean 1. That is, the cumulative distribution function of z is given by

$$F_Z(x) = \begin{cases} 1 - e^{-x} & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then Pr (z > 2|z > 1), rounded off to two decimal places, is equal to ______.

82. [MCQ] [GATE-ME-2019:1M]

The lengths of a large stock of titanium rods follow a normal distribution with a mean (μ) of 440 mm and a standard deviation (σ) of 1 mm. What is the percentage of rods whose lengths lie between 438 mm and 441 mm?

(a) 81.85% (b) 68.4%
(c) 99.75% (d) 86.64%

83. [MCQ] [GATE-ME-2019:2M]

The variable x takes a value between 0 and 10 with uniform probability distribution. The variable y takes a value between 0 and 20 with uniform probability distribution. The probability of the sum of variables (x + y) being greater than 20 is

(a) 0 (b) 0.25
(c) 0.33 (d) 0.50

84. [NAT] [GATE-ME-2019:1M]

If x is the mean of data 3, x, 2 and 4, then the mode is ______.

85. [MCQ] [GATE-CE-2019:2M]

The probability density function of a continuous random variable distributed uniformly between x and y (for y > x) is

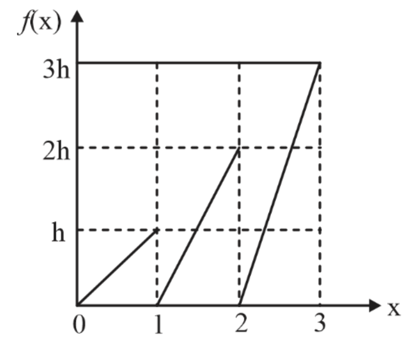
(a) y – x (b) 1/ y – x
(c) x – y (d) 1/ x – y

86. [NAT] [GATE-CS-2019:2M]

Suppose Y is distributed uniformly in the open interval (1,6).The probability that the polynomial $3x^2 + 6xY + 3Y + 6$ has only real roots is (rounded off to 1 decimal place) ________.

87. [MCQ] [GATE-CE-2018:1M]

The graph of a function f(x) is shown in the figure.

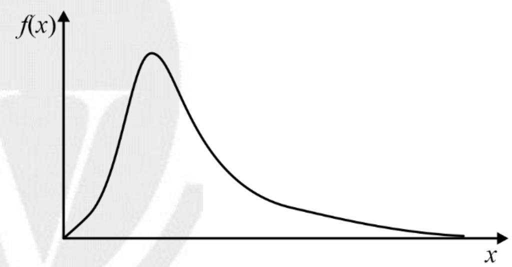


For f(x) to be valid probability density function, the value of h is

(a) 1/3 (b) 2/3
(c) 1 (d) 3

88. [MCQ] [GATE-CE-2018:1M]

A probability distribution with right skew is shown in the figure.



The correct statement for the probability distribution is

(a) Mean is equal to mode
(b) Mean is greater than median but less than mode.
(c) Mean is greater than median and mode
(d) Mode is greater than median

89. [NAT] [GATE-EC-2018:1M]

Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is ________.

90. [MCQ] [GATE-ME-2018:2M]

Let X_1, X_2 be two independent normal random variables with means μ_1, μ_2 and standard deviations σ_1, σ_2, respectively. Consider $Y = X_1 - X_2$; $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1$, $\sigma_2 = 2$. Then,

(a) Y is normally disdtributed with mean 0 and varience 1

(b) Y is normally distributed with mean 0 and variance 5

(c) Y has mean 0 and variance 5, but is NOT normally distributed

(d) Y has mean 0 and variance 1, but is NOT normally distributed

91. [MCQ] [GATE-ME-2018:2M]

Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then $Y = \min(X_1, X_2)$ is

(a) exponentially distributed with mean $1/6$

(b) exponentially distributed witn mean 2

(c) normally distributed with mean $3/4$

(d) normally distributed with mean $1/6$

92. [MCQ] [GATE-ME-2017:1M]

A sample of 15 data is as follows:17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is

(a) 4 (b) 13

(c) 17 (d) 20

93. [NAT] [GATE-EE-2017:1M]

Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of vehicles at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) for the vehicle at the junction is _________.

94. [MCQ] [GATE-EE-2017:2M]

A person decides to toss a fair coin repeatedly until he gets a head. He will make at most 3 tosses. Let the random variable Y denote the number of heads. The value of var(Y) where var(.) denotes the variance, equals:

(a) $\frac{7}{8}$ (b) $\frac{49}{64}$

(c) $\frac{7}{64}$ (d) $\frac{105}{64}$

95. [NAT] [GATE-CS-2017:1M]

Let x be a Gaussian random variable with mean 0 and variance σ^2. Let $y = \max(x, 0)$ where max(a, b) is the maximum of a and b. The median of y is _____.

96. [MCQ] [GATE-CS-2017:2M]

For any discrete random variable X, with probability mass function $P(x = j) = p_j$, $p_j \geq 0$, $j \in \{0, \ldots. N\}$, and $\sum_{j=0}^{N} p_j = 1$, define the polynomial function $g_x(z) = \sum_{j=0}^{N} P_j z^j$. For a certain discrete random variable Y, there exists a scalar $\beta \in [0, 1]$ such that $g_y(z) = (1 - \beta + \beta z)^N$. The expectation of y is

(a) $N\beta(1 - \beta)$

(b) $N\beta$

(c) $N(1 - \beta)$

(d) Not expressible in terms of N and β alone

97. [MCQ] [GATE-CE-2017:1M]

The number of parameters in the univariate exponential and Gaussian distributions, respectively, are

(a) 2 and 2 (b) 1 and 2

(c) 2 and 1 (d) 1 and 1

98. [MCQ] [GATE-CE-2017:2M]

For the function $f(x) = a + bx$, $0 \leq x \leq 1$, to be a valid probability density function, which one of the following statements is correct?

(a) a = 1, b = 4 (b) a = 0.5, b = 1

(c) a = 0, b = 1 (d) a = 1, b = –1

99. [NAT] [GATE-CS-2017:2M]

If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X+2)^2]$ equals _____.

100. [NAT] [GATE-CS-2016:1M]

Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is ___________ .

101. [MCQ] [GATE-CE-2016:1M]

Type II error in hypothesis testing is

(a) Acceptance of the null hypothesis when it is false and should be rejected

(b) Rejection of the null hypothesis when it is true and should be accepted

(c) Rejection of the null hypothesis when it is false and should be rejected

(d) Acceptance of the null hypothesis when it is true and should be accepted

102. [MCQ] [GATE-CE-2016:2M]

Probability density function of a random variable X is given below

$$f(x) = \begin{bmatrix} 0.25 & \text{if } 1 \le x \le 5 \\ 0 & \text{otherwise} \end{bmatrix}$$

$P(X \le 4)$ is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

103. [NAT] [GATE-CE-2016:1M]

The spot speeds (expressed in km/hr) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53 and 49. The median speed (expressed in km/hr) is___________.

(Note: Answer with one decimal accuracy)

104. [MCQ] [GATE-CE-2016:1M]

X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which one of the following is the value of $P(X \cup Y)$?

(a) 0.7 (b) 0.5

(c) 0.4 (d) 0.3

105. [MCQ] [GATE-CE-2016:2M]

If $f(x)$ and $g(x)$ are two probability density functions,

$$f(x) = \begin{cases} \frac{x}{a} + 1 & : -a \le x < 0 \\ -\frac{x}{a} + 1 & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} -\frac{x}{a} & : -a \le x < 0 \\ \frac{x}{a} & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

Which one of the following statements is true?

(a) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are same

(b) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are different

(c) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are same

(d) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are different

106. [MCQ] [GATE-ME-2016:1M]

Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

(a) $\sqrt{\mu}$ (b) μ^2

(c) μ (d) $1/\mu$

107. [NAT] [GATE-ME-2016:1M]

The area (in percentage) under standard normal distribution curve of random variable Z within limits from –3 to +3 is __________

108. [MSQ] [GATE-EE-2016:2M]

Let the probability density function of a random variable, X, be given as:

$$f_x(x)=\frac{3}{2}e^{-3x}u(x)+ae^{4x}u(-x)$$

where $u(x)$ is the unit step function.

Then the value of 'a' and Probability $\{X \le 0\}$, respectively, are

(a) $2, \frac{1}{2}$ (b) $4, \frac{1}{2}$

(c) $2, \frac{1}{4}$ (d) $4, \frac{1}{4}$

109. [NAT] [GATE-EC-2016:2M]

Two random variables x and y are distributed according to

$$f_{xy}(x,y)=\begin{cases}(x+y), & 0\le x\le 1, 0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$$

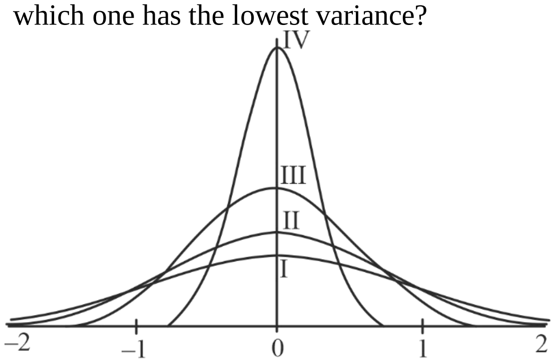
The probability P $(x + y \le 1)$ is ________.

110. [NAT] [GATE-CS-2016:1M]

A probability density function on the interval [a, 1] is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is ________.

111. [MCQ] [GATE-ME-2015:1M]

Among the four normal distributions with probability density functions as shown below, which one has the lowest variance?



(a) I (b) II

(c) III (d) IV

112. [NAT] [GATE-CE-2015:1M]

Consider the following probability mass function (p.m.f) of a random variable X.

$$p(X,q)=\begin{cases}q & \text{If } X=0\\ 1-q & \text{If } X=1\\ 0 & \text{otherwise}\end{cases}$$

If q = 0.4, the variance of X is_____________.

113. [NAT] [GATE-CE-2015:2M]

The probability density function of a random variable, x is

$$f(x)=\begin{cases}\frac{x}{4}\left(4-x^2\right) & 0\le x\le 2\\ 0 & \text{otherwise}\end{cases}$$

The mean, μ_x of the random variable is___________.

114. [NAT] [GATE-CE-2014:1M]

The probability density function of evaporation E on any day during a year in a watershed is given by

$$f(E)=\begin{cases}\frac{1}{5} & 0<E<5 \text{ mm/day}\\ 0 & \text{otherwise}\end{cases}$$

The probability that E lies in between 2 and 4 mm/day in a day in the watershed is (in decimal) _______.

115. [MCQ] [GATE-CE-2014:1M]

If {x} is a continuous, real valued random variable defined over the interval (–∞, +∞) and its occurrence is defined by the density function given as:

$$f(x)=\frac{1}{\sqrt{2\pi}\,b}e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$$

where 'a' and 'b' are the statistical attributes of the random variable {x}. The value of the integral

$$\int_{-\infty}^{a}\frac{1}{\sqrt{2\pi}\,b}e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}dx \quad \text{is}$$

(a) 1 (b) 0.5

(c) π (d) $\pi/2$

116. [NAT] [GATE-CE-2014:2M]

A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is_______.

117. [NAT] [GATE-CE-2014:2M]

In any given year, the probability of an earthquake greater than magnitude 6 occurring in the Garhwal Himalayas is 0.04. The average time between successive occurrence of such earthquake is ______ (in year).

118. [NAT] [GATE-EE-2014:1M]

Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years respectively, then the value of k is ________.

119. [NAT] [GATE-ME-2014:1M]

A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of Rs. 500 and a standard deviation of Rs. 50. The percentage of savings account holders, who maintain an average daily balance more than Rs. 500 is _______

120. [NAT] [GATE-ME-2014:2M]

The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

(a) 0.029 (b) 0.034
(c) 0.039 (d) 0.044

121. [MCQ] [GATE-ME-2014:2M]

In the following table, *x* is a discrete random variable and *p(x)* is the probability density. The standard deviation of *x* is

x	1	2	3
p(x)	0.3	0.6	0.1

(a) 0.18 (b) 0.36
(c) 0.54 (d) 0.6

122. [MCQ] [GATE-ME-2014:2M]

A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6, respectively. The mean value and the variance of the number of defective pieces produced by the machine in a day, respectively, are

(a) 1 and 1/3 (b) 1/3 and 1
(c) 1 and 4/3 (d) 1/3 and 4/3

123. [NAT] [GATE-EC-2014:1M]

Let X_1, X_2 and X_3 be independent and identically distributed random variables with the uniform distribution on [0,1]. The probability p{X_1 is the largest} is ________.

124. [NAT] [GATE-EC-2014:1M]

Let X be a zero mean unit variance Gaussian random variable. E[|X|] is equal to _________.

125. [MCQ] [GATE-EC-2014:2M]

Let X be a real-valued random variable with E[X] and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds true is

(a) $(E[x])^2 > E[x^2]$ (b) $E[x^2] \ge (E[x])^2$
(c) $E[x^2] = (E[x])^2$ (d) $E[x^2] > (E[x])^2$

126. [NAT] [GATE-EE-2014:1M]

Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

The probability P (0.5 < X < 5) is

127. [NAT] [GATE-CS-2014:1M]

Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is ________.

128. [MCQ] [GATE-EE-2013:1M]

A Continuous Random Variable X has a probability density function $f(x) = e^{-x}$, $0 < x < \infty$. Then P{X>1} is

(a) 0.368 (b) 0.5
(c) 0.632 (d) 1.0

129. [MCQ] [GATE-ME-2013:1M]

Let X be a normal random variable with mean 1 and variance 4. The probability P{X < 0} is

(a) 0.5
(b) Greater than zero and less than 0.5
(c) Greater than 0.5 and less than 1.0
(d) 1.0

130. [MCQ] [GATE-EC-2013:2M]

Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of x

(a) $F(x) - G(x) \le 0$
(b) $F(x) - G(x) \ge 0$
(c) $(F(x) - G(x))\, x \le 0$
(d) $(F(x) - G(x)).x \ge 0$

131. [MCQ] [GATE-EC-2013:2M]

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \ge 2U)$ is

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{5}{9}$

132. [NAT] [GATE-CE-2013:2M]

Find the value of λ such that function $f(x)$ is valid probability density function__________

$$f(x) = \begin{cases} \lambda(x-1)(2-x) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

133. [MCQ] [GATE-CS-2013:1M]

Suppose P is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and P has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(a) $\frac{8}{(3e^3)}$ (b) $\frac{9}{(2e^3)}$
(c) $\frac{17}{(2e^3)}$ (d) $\frac{26}{(2e^3)}$

134. [MCQ] [GATE-CE-2012:1M]

The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is ____

(a) < 50% (b) 50%
(c) 75% (d) 100%

135. [MCQ] [GATE-EE-2012:1M]

Two independent random variables X and Y are uniformly distributed in the interval [-1,1]. The probability that max[X,Y] is less than ½ is

(a) 3/4 (b) 9/16
(c) 1/4 (d) 2/3

136. [MCQ] [GATE-EC-2012:1M]

Two independent random variables *x* and *y* are uniformly distributed in the interval [–1, 1]. The probability that max [x, y] is less than 1/2 is

(a) 3/4 (b) 9/16
(c) 1/4 (d) 2/3

137. [MCQ] [GATE-CS-2012:1M]

Consider a random variable X that takes values +1 and –1 with probability 0.5 each. The values of the cumulative distribution function F(x) at x = – 1 and +1 are

(a) 0 and 0.5 (b) 0 and 1
(c) 0.5 and 1 (d) 0.25 and 0.75

138. [MCQ] [GATE-CS-2012:2M]

Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2, or 3, the die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6?

(a) $\frac{10}{21}$ (b) $\frac{5}{12}$
(c) $\frac{2}{3}$ (d) $\frac{1}{6}$

139. [MCQ] [GATE-CS-2011:2M]

Consider a finite sequence of random values $X = [x_1, x_2, \ldots x_n]$. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a * x_i + b$, where a and b are positive constants. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements is **INCORRECT**?

(a) Index position of mode of X in X is the same as the index position of mode of Y in Y
(b) Index position of median of X in X is the same as the index position of median of Y in Y.
(c) $\mu_y = a\mu_x + b$
(d) $\sigma_y = a\sigma_x + b$

140. [MCQ] [GATE-CS-2011:1M]

If the difference between the expectation of the square of a random variable ($E[X^2]$) and the square of the expectation of the random variable $(E[X])^2$ is denoted by R, then?

(a) $R = 0$ (b) $R < 0$
(c) $R \geq 0$ (d) $R > 0$

141. [MCQ] [GATE-CS-2011:2M]

A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(a) 1/5 (b) 4/25
(c) 1/4 (d) 2/5

142. [MCQ] [GATE-CS-2010:2M]

Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

(a) pq + (1–p) (1–q) (b) (1–q)p
(c) (1–p)q (d) pq

143. [MCQ] [GATE-EC-2009:2M]

Consider two independent random variables x and y with identical distributions. The variables x and y take values 0, 1 and 2 with probability 1/2, 1/4 and 1/4 respectively. What is the conditional probability $P(x + y = 2 \mid x - y = 0)$?

(a) 0 (b) 1/16
(c) 1/6 (d) 1

144. [MCQ] [GATE-EC-2009:2M]

A discrete random variable *x* takes values from 1 to 5 with probabilities as shown in the table. A student calculates the mean of *x* as 3.5 and her teacher calculates the variance to *x* as 1.5. Which of the following statement is true?

k	1	2	3	4	5
P(x=k)	0.1	0.2	0.4	0.2	0.1

(a) Both the student and the teacher are right.
(b) Both the student and the teacher are wrong.
(c) The student is wrong but the teacher is right.
(d) The student is right but the teacher is wrong.

145. [MCQ] [GATE-ME-2009:2M]

The standard deviation of a uniformly distributed random variable between 0 and 1 is

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

146. [MCQ] [GATE- CE-2009:2M]

The standard normal probability function can be approximate as

$$F(X_N) = \frac{1}{1 + \exp\left(-1.7255 X_N |X_N|^{0.12}\right)}$$

where X_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

(a) 66.7% (b) 33.3%
(c) 50.0% (d) 16.7%

147. [MCQ] [GATE-CS-2009:2M]

An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90 % of the probability that the face value is even. The probability of getting any even numbered face is the same.

If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closest to the probability that the face value exceeds 3?

(a) 0.453 (b) 0.468
(c) 0.485 (d) 0.492

148. [MCQ] [GATE- CE-2008:2M]

If probability density function of a random variable X is $f(x) = x^2$ for $-1 \le x \le 1$ and
= 0 for any other values of x

then, the percentage probability P $(-1/3 \le x \le 1/3)$ is

(a) 0.247 (b) 2.47
(c) 24.7 (d) 247

149. [MCQ] [GATE-CS-2008:2M]

Let $P = \sum_{\substack{1\le i\le 2k \\ i \text{ odd}}} i$ and $Q = \sum_{\substack{1\le i\le 2k \\ i \text{ even}}} i$ where k is a positive integer. Then

(a) P = Q – k (b) P = Q + k
(c) P = Q (d) P = Q + 2k

150. [MCQ] [GATE-CS-2008:2M]

Let X be a random variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean –1 and variance unknown, If $P(X \le -1) = P(Y \ge 2)$, the standard deviation of Y is

(a) 3 (b) 2
(c) $\sqrt{2}$ (d) 1

151. [MCQ] [GATE-EE-2008:1M]

X is a uniformly distributed random variable that takes values between 0 and 1. The value of $E(X^3)$ will be

(a) 0 (b) 1/8
(c) 1/4 (d) 1/2

152. [MCQ] [GATE-EC-2008:2M]

$P_x(x) = M\exp(-2|x|) + N\exp(-3|x|)$ is the probability density function for the real random variable X, over the entire x-axis. M and N are both positive real numbers. The equation relating M and N is

(a) $M + \frac{2}{3}N = 1$ (b) $2M + \frac{1}{3}N = 1$
(c) M + N = 1 (d) M + N = 3

Correlation & Regression

153. [NAT] [GATE-CE-2022:2M]

A set of observations of independent variable (x) and the corresponding dependent variable (y) is given below.

x	5	2	4	3
y	16	10	13	12

Based on the data, the coefficient a of the linear regression model y = a + bx is estimated as 6.1. The coefficient b is _____.

(Round off to one decimal place)

154. [MCQ] [GATE-CS-2021:2M]

Consider the two statements.

S_1: There exist random variables X and Y such that $(\mathbb{E}[(X - \mathbb{E}(X))(Y - \mathbb{E}(Y))])^2 > \text{Var}[X]\,\text{Var}[Y]$

S_2: For all random variables X and Y, $\text{Cov}[X, Y] = \mathbb{E}[|X - \mathbb{E}[X]|\,|Y - \mathbb{E}[Y]|]$

Which one of the following choices is correct?

(a) Both S_1 and S_2 are true
(b) S_1 is true, but S_2 is false
(c) S_1 is false, but S_2 is true
(d) Both S_1 and S_2 are false.

155. [MCQ] [GATE-2008-CE:2M]

Three values of x and y are to be fitted in a straight line in the form y = a + bx by the method of least squares. Given: $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$ and $\Sigma xy = 46$, the value of a and b are respectively.

(a) 2 and 3 (b) 1 and 2
(c) 2 and 1 (d) 3 and 2

❑❑❑

ANSWER KEY

1.	(0.9 to 0.9)	**2.**	(0.187 to 0.188)	**3.**	(0.018 to 0.020)	**4.**	(c)
5.	(0.190 to 0.200)	**6.**	(0.90 to 0.95)	**7.**	(0.041 to 0.043)	**8.**	(0.16 to 0.18)
9.	(0.25 to 0.27)	**10.**	(0.5 to 0.5)	**11.**	(b)	**12.**	(c)
13.	(0.021 to 0.024)	**14.**	(a)	**15.**	(3.5 to 3.5)	**16.**	(0.75 to 0.75)
17.	(0.5 to 0.5)	**18.**	(a)	**19.**	(0.027 to 0.028)	**20.**	(2.4 to 2.6)
21.	(0.027 to 0.028)	**22.**	(0.2 to 0.2)	**23.**	(0.39 to 0.43)	**24.**	(a)
25.	(0.07 to 0.08)	**26.**	(0.33 to 0.34)	**27.**	(b)	**28.**	(0.96 to 0.98)
29.	(b)	**30.**	(c)	**31.**	(1.5)	**32.**	(c)
33.	(6)	**34.**	(d)	**35.**	(d)	**36.**	(a)
37.	(0.64 to 0.66)	**38.**	(0.25 to 0.27)	**39.**	(b)	**40.**	(0.65 to 0.68)
41.	(2.9 to 3.1)	**42.**	(c)	**43.**	(0.13 to 0.15)	**44.**	(0.25 to 0.25)
45.	(d)	**46.**	(d)	**47.**	(d)	**48.**	(c)
49.	(c)	**50.**	(d)	**51.**	(c)	**52.**	(c)
53.	(d)	**54.**	(c)	**55.**	(a)	**56.**	(a)
57.	(c)	**58.**	(c)	**59.**	(b)	**60.**	(c)
61.	(d)	**62.**	(a)	**63.**	(a)	**64.**	(c)
65.	(0.99 to 1.01)	**66.**	(a)	**67.**	(4)	**68.**	(a)
69.	(b)	**70.**	(0.17 to 0.19)	**71.**	(d)	**72.**	(d)
73.	(0.35 to 0.39)	**74.**	15.00 to 16.00	**75.**	(b)	**76.**	(0.5 to 0.5)
77.	(–1.00 to –0.98)	**78.**	(Marks to all)	**79.**	(0.299 to 0.301)	**80.**	(11 to 11)
81.	(0.36 to 0.38)	**82.**	(a)	**83.**	(b)	**84.**	(3 to 3)
85.	(b)	**86.**	(0.8 to 0.8)	**87.**	(a)	**88.**	(c)
89.	(0.25)	**90.**	(b)	**91.**	(a)	**92.**	(c)
93.	(0.9)	**94.**	(c)	**95.**	(0.0 to 0.0)	**96.**	(b)
97.	(b)	**98.**	(b)	**99.**	(54 to 54)	**100.**	(0.55 to 0.55)
101.	(a)	**102.**	(a)	**103.**	(54.49 to 54.51)	**104.**	(a)
105.	(b)	**106.**	(a)	**107.**	(99.6 to 99.8)	**108.**	(a)
109.	(0.33)	**110.**	(0.5 to 0.5)	**111.**	(d)	**112.**	(0.23 to 0.25)
113.	(1.06 to 1.07)	**114.**	(0.4 to 0.4)	**115.**	(b)	**116.**	(0.26 to 0.27)
117.	(25)	**118.**	(0.4 to 0.5)	**119.**	(49 to 51)	**120.**	(b)
121.	(d)	**122.**	(a)	**123.**	(0.32 to 0.34)	**124.**	(0.79 to 0.81)
125.	(b)	**126.**	(0.35 to 0.45)	**127.**	(0.24 to 0.27)	**128.**	(a)
129.	(b)	**130.**	(d)	**131.**	(b)	**132.**	(6 to 6)
133.	(c)	**134.**	(a)	**135.**	(b)	**136.**	(b)
137.	(c)	**138.**	(b)	**139.**	(d)	**140.**	(c)
141.	(a)	**142.**	(a)	**143.**	(c)	**144.**	(b)
145.	(a)	**146.**	(d)	**147.**	(b)	**148.**	(b)
149.	(a)	**150.**	(a)	**151.**	(c)	**152.**	(a)
153.	(1.9 to 1.9)	**154.**	(d)	**155.**	(d)		

SOLUTIONS

1. **(0.9 to 0.9)**

Given: P(A) = 0.5 and P(B) = 0.8

Probability of occurrence of at least A or B

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A).\, P(B)$

[$\because$ A and B are independent events]

$= 0.5 + 0.8 - (0.5 \times 0.8) = 0.9$

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2. **(0.187 to 0.188)**

Let x$\rightarrow$ Number of trials for first occurrence of a 'Head'

x	1(H)	2(TH)	3(TTH)	....
P(x)	P	qp	qqp	....

$p \rightarrow$ Head

$q \rightarrow$ Tail

$p + q = 1$

Given E(x) = 4

$E(x) = \sum_{i=1}^{n} x\,P(x) = 1p + 2qp + 3q^2p + \ldots$

$= p(1 + 2q + 3q^2 + \ldots.)$

$= p(1 - q)^{-2}$

$= p(p)^{-2} = \frac{1}{p} = 4$

$\therefore\ p = \frac{1}{4}$ and $q = 1 - p = \frac{3}{4}$

P (Getting Head in second trial)

$= qp = \frac{3}{4}\cdot\frac{1}{4} = \frac{3}{16} = 0.1875 = 0.188$

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3. **(0.018 to 0.020)**

Favorable outcomes for (sum = 4)

S = (1, 3) (2, 2) (3, 1)

Let x $\rightarrow$ Event of getting (sum = 4)

$P(x) = \frac{n(x)}{n(S)} = \frac{3}{36} = \frac{1}{12}$ (sum = 4)

$\therefore\ P(\bar{x}) = 1 - \frac{1}{12} = \frac{11}{12}$ (Sum $\neq$ 4)

Required probability is

$P(x\,x\,\bar{x}) + P(x\,\bar{x}\,x) + P(\bar{x}\,x\,x)$

$= \frac{1}{12}.\frac{1}{12}.\frac{11}{12}. + \frac{1}{12}.\frac{11}{12}.\frac{1}{12}. + \frac{11}{12}.\frac{1}{12}.\frac{1}{12}.$

$= \frac{3 \times 11}{12 \times 12 \times 12} = 0.019$

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4. **(c)**

Let the probability of head is 'p'

The probability of tail is 'q'

Let head comes in r^{th} toss, then the favourable cases will be

H, TH, TTH, TTTH....(r = 1, 2, 3, 4 ...)

Probability distribution

x	1	2	3	4	...r
p(x)	p	qp	q^2p	q^3p	$q^{r-1}p$

$E(x) = \sum_{i=1}^{r} x\,p(x)$

$= p + 2qp + 3q^2p + 4q^3p + \ldots rq^{r-1}p$

$= p(1 + 2q + 3q^2 + 4q^3 + \ldots\, r\,q^{r-1})$

$= p(1 - q)^{-2} = \frac{p}{p^2} = \frac{1}{p}$ [By binomial expansion]

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5. (0.190 to 0.200)

Given: n = 15

$p = P(H) = \frac{1}{2}, \quad q = P(T) = \frac{1}{2}$

P (Probability of getting 8 Heads in 15 times)

$$P(x=8) = {}^{15}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{15-8} = 0.196$$

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6. (0.90 to 0.95)

Given

$5 \rightarrow$ Men

$3 \rightarrow$ Women

P (at least one women)

= 1 – P (no woman)

$$= 1 - \frac{5c_4}{8c_4} = 1 - \frac{5}{\left(\frac{8\times7\times6\times5}{4\times3\times2\times1}\right)}$$

$$= 1 - \frac{5}{70} = \frac{65}{70} = 0.9285 \simeq 0.93$$

$\therefore$ P (at least one woman) = 0.93

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7. (0.041 to 0.043)

Given

A fair coin is tossed 20 times.

$\Rightarrow$ Number of trails n = 20

Let $p \rightarrow$ Probability of getting a head in a toss $= \frac{1}{2}$

$q \rightarrow$ Probability of getting a tail in a toss $= \frac{1}{2}$

P (4 heads in 1st 10 tosses $\cap$ 4 tails in next 10 tosses)
= P (4 heads in 1st 10 tosses) × P (4 tails in next 10 tosses)

$$= \left(10_{C_4}.p^4.q^6\right)\cdot\left(10_{C_4}.q^4.p^6\right)$$

$$= \left(10_{C_4}\right)^2\cdot(pq)^{10} = \left(\frac{10\times9\times8\times7}{4\times3\times2\times1}\right)^2\times\left(\frac{1}{4}\right)^{10}$$

$$= (210)^2\times\frac{1}{4^{10}} = 0.042$$

$\boxed{\therefore \text{ Required Probability} = 0.042}$

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8. (0.16 to 0.18)

Given

Probability of defective = 0.05 = p(say)

Let 'x' be the number of defective parts and 'q' be the probability of non-defective.

$\Rightarrow \quad q = 1 - p = 0.95$

$P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$= 1 - {}^{15}C_0.\ P^0.q^{15} - {}^{15}C_1.p^1.\ q^{14}$

$= 1 - q^{14}\ ({}^{15}C_0.q + {}^{15}C_1.p)$

$= 1 - (0.95)^{14}\ (0.95 + 15(0.05))$

$= 1 - 0.829 = 0.171$

$\therefore$ P(at least 2 defective) = 0.17

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9. (0.25 to 0.27)

- probability of resistor being defective is p = 0.02
- Number of resistors (n) = 50

Approximate poisson distribution mean

$(\mu) = np = 50 \times 0.02 = 1$

$$P(x=r) = \frac{e^{-\mu}\mu^r}{r!}$$

p (2 or more defective) = 1 – p(no resistor defective) – p(1 resistor defective)

= 1 – p(r = 0) – p(r = 1)

$$= 1 - \frac{e^{-\mu}\cdot\mu^o}{0!} - \frac{e^{-\mu}\mu^1}{1!}$$

$= 1 - e^{-1} - e^{-1}$

$= 1 - 2e^{-1}$

= 0.26

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10. (0.5 to 0.5)

Given: Picking 3 red balls consecutively without replacement

$$\boxed{\begin{matrix}5R\\1W\end{matrix}} \rightarrow \boxed{\begin{matrix}4R\\1W\end{matrix}} \rightarrow \boxed{\begin{matrix}3R\\1W\end{matrix}}$$

$$P(R_1 \cap R_2 \cap R_3)$$

$$= P(R_1).P(R_2/R_1).P(R_3/R_1 \cap R_2)$$

$$= \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4} = 0.5$$

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11. (b)

Given,

$4 \rightarrow$ Red

$4 \rightarrow$ Green

$4 \rightarrow$ Blue

Three balls are pulled one after the other, without replacing

P (Red ⋂ Red ⋂ Red) = P (Red in 1st draw) · P(Red in 2nd draw) · P(Red in 3rd draw)

$$= \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{1}{55}$$

$\therefore$ P (Red ⋂ Red ⋂ Red) = $\frac{1}{55}$

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12. (c)

Given that a six-faced fair dice is rolled

P (getting one) = $\frac{1}{6}$

Let P be the probability of getting one

q be the probability of not getting one

$\Rightarrow$ $P = \frac{1}{6}$ and $q = \frac{5}{6}$

No. of Rolls = 5

Let 'x' be the number of times that ONE appears

$\therefore$ $P(x \geq 4) = P(x = 4) + P(x = 5)$

$$= 5_{C_4} \cdot P^4 \cdot q + 5_{C_5} \cdot P^5 \cdot q^0$$

$$= 5 \cdot \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right) + 1 \cdot \left(\frac{1}{6}\right)^5 = \left(\frac{1}{6}\right)^5 \{25 + 1\} = \frac{26}{6^5}$$

$\therefore$ $P(x \geq 4) = \frac{26}{6^5} = 0.00334 = 0.334\%$

$\therefore$ $P(x \geq 4) = 0.33\%$

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13. (0.021 to 0.024)

Given: A pair of dice is rolled.

Probability of getting same number (1,1), (2,2) …. (6,6)

$P(\text{Tie}) = \frac{6}{36} = \frac{1}{6}$ $\Rightarrow$ $P(\text{Not a tie}) = \frac{5}{6}$

P (Win in 3rd trials) = P (Tie ∩Tie ∩ Not a Tie)

$= (P(\text{Tie}))^2 \cdot P(\text{Not a Tie})$

$$= \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right) = \frac{5}{216} = 0.023$$

$\therefore$ P (win in 3rd Trial) = **0.023**

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14. (a)

Given: P (P applies) = $\frac{1}{4}$, $P\left(\frac{\text{P applies}}{\text{Q applies}}\right) = \frac{1}{2}$

And $P\left(\frac{\text{Q applies}}{\text{P applies}}\right) = \frac{1}{3}$

By multiplication theorem

$$P(P)\ P \cdot \left(\frac{Q}{P}\right) = P(Q) \cdot P\left(\frac{P}{Q}\right)$$

$$\Rightarrow \frac{1}{4} \times \frac{1}{3} = P(Q) \cdot \frac{1}{2}$$

$\Rightarrow$ $P(Q) = \frac{1}{6}$ $\Rightarrow$ $P(P \cap Q) = \frac{1}{12}$

$$P\left(\frac{\text{P doesn't apply}}{\text{Q doesn't apply}}\right)=\frac{P(P'\cap Q')}{P(Q')}=\frac{1-P(P\cup Q)}{1-P(Q)}$$

$$=\frac{1-\{P(Q)+P(P)-P(P\cap Q)\}}{1-P(Q)}$$

$$=\frac{1-\{1/6+1/4-1/12\}}{1-1/6}=\frac{1-\{1/3\}}{5/6}=\frac{2/3}{5/6}=\frac{4}{5}$$

$$\therefore\ \boxed{P\left(\frac{P'}{Q'}\right)=\frac{4}{5}}$$

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15. (3.5 to 3.5)

Let x be the number appeared when a dice is rolled

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

Mean of the outcome $=\sum_{i=1}^{6} x_i P(x=x_i)$

$$=\frac{1}{6}(1+2+3+4+5+6)=\frac{7}{2}=3.5$$

$\therefore$ Mean value of outcomes = 3.5

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16. (0.75 to 0.75)

Given

Two coins are tossed simultaneously.

P (at least one head) = 1 – P (No head)

= 1 – P (1^{st} coin tail $\cap$ 2^{nd} coin tail)

$= 1-(P(\text{tail}))^2 = 1-\frac{1}{4} = 0.75$

$\therefore$ P (at least one head) = 0.75

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17. (0.5 to 0.5)

Probability of getting 3 heads (H, H, H)

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$$

Probability of getting 4 Heads (H, H, H, H)

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times=\frac{1}{16}$$

Getting 4^{th} Head when 3 heads are already realized. Conditional probability of getting next read after 3 Head is

$$P(4H/3H)=\frac{P(4H\cap 3H)}{P(3H)}=\frac{1/16}{1/8}=0.5$$

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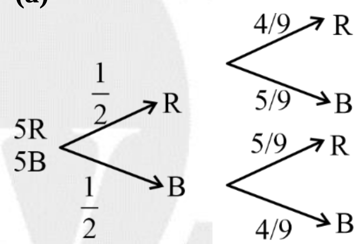
18. (a)



P(getting a red ball in 2^{nd} draw)

= P(R $\cap$ R) + P(B $\cap$ R)

$$=\frac{1}{2}\times\frac{4}{9}+\frac{1}{2}\times\frac{5}{9}=\frac{1}{2}$$

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19. (0.027 to 0.028)

Given: 3 fair dice are thrown simultaneously.

Total number of ways of outcomes = 6 × 6 × 6 = 216

For all the three dice to show the same number of dots i.e. the favorable outcomes are {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}

$\therefore$ Required probability $=\frac{6}{216}=\frac{1}{36}$

$= 0.0277 \approx 0.028$

Probability that all three dice show same number of dots is **0.028.**

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20. (2.4 to 2.6)

Given:

Probability of getting a reservation = 40%

= 0.4 = p(say)

Probability of not getting a reservation

= 1 – 0.4 = 0.6 = q (say)

Let 'x' be the number of attempts made by the passenger

The probability distribution table is as shown below

x	1	2	3	4	…….
P(x)	p	qp	q^2p	q^3p	……

∴ Average number of attempts

$E(x) = \Sigma x.\ P(x) = 1 \cdot p + 2 \cdot qp + 3 \cdot q^2p + 4 \cdot q^3p + ...$

$= p\,(1 + 2q + 3q^2 + 4q^3 + \ldots\ldots)$

$= p \cdot (1-q)^{-2} = \frac{p}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p} = \frac{1}{0.4} = 2.5$

∴ Average number of attempts to get a seat is **2.5.**

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21. (0.027 to 0.028)

Given:3 fair dice are thrown simultaneously.

Total number of ways of outcomes = 6 × 6 × 6 = 216

For all the three dice to show the same number of dots i.e. the favourable outcomes are {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}

∴ Required probability = = 0.0277 ≈ 0.028

Probability that all three dice show same number of dots is 0.028

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22. (0.2 to 0.2)

The number of ways in which all 3 pens are of same colour

$$= \,^4C_1 = 4$$

Out of 4 colours

Choose 1 at a time

The number of ways in which all 3 pens are of different colour = $^4C_3 = 4$

The number of ways in which 2 pens are of same colour and 3rd pen is of differnt colour.

$= \,^4C_2 \cdot \,^2C_1 = 6 \times 2 = 12$

Total number of ways 3 pens can be chosen

= 4 + 4 + 12 = 20 ways

P(All 3 pens are of same colour)

$$= \frac{\text{Favourable no. of ways}}{\text{Total no. of ways}} = \frac{4}{20} = \frac{1}{5} = 0.2$$

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23. (0.39 to 0.43)

Probability of a screw being defective = p(say) = 0.1

Number of screws in a packet = 5

Let 'q' be probability of non-defective, q = 1 – p = 0.9

If 'x' is the number of defective screws in a packet, then packet gets replaced if x ≥1

$P(\text{replacement}) = P\,(x \geq 1)$

$= 1 - P\,(x = 0)$

$= 1 - (^5C_0.\ P^0.\ q^5)$

$= 1 - (1 \times 1 \times (0.9)^5)$

$= 1 - (0.9)^5 = 0.40951$

∴ $\boxed{P(\text{packet being replaced}) = 0.41}$

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24. (a)

Given

That '3' cards are drawn at random.

In a pack of cards, we have 4- queens, 4- kings, 4- jacks

P(picking a king, a queen, a jack)

$$= \frac{\left({}^4C_1 \times {}^4C_1 \times {}^4C_1\right)}{{}^{52}C_3}$$

$$= \frac{4\times4\times4}{\left(\frac{52\times51\times50}{3\times2\times1}\right)} = \frac{16}{13\times17\times25} = \frac{16}{5525}$$

$\therefore$ $\boxed{\text{Required probability} = \frac{16}{5525}}$

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25. (0.07 to 0.08)

Given P(Head) = 0.3

$\Rightarrow$ P(tail) = 0.7

P(1^{st} head in 5^{th} toss) = P(4 tails in 1^{st} 4 tosses $\cap$ head in 5^{th} toss) = $(P(\text{Tail}))^4$ P(Head)

= $(0.7)^4 \cdot (0.3)$

= 0.072

$\therefore$ P(1^{st} head in 5^{th} toss) = 0.072

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26. (0.33 to 0.34)

P(Y) = P (Tail $\cap$ Head) + P (Tail $\cap$ Tail $\cap$ Tail $\cap$Head) + P (Tail $\cap$ Tail $\cap$ Tail $\cap$ Tail $\cap$ Tail $\cap$ Head) + ….........

Since all the tosses are independent

$$P(Y) = P(T)\cdot P(H) + (P(T))^3 \cdot P(H) + (P(T))^5\, (P(H)) + \ldots\ldots$$

$$= P(T)\cdot P(H)\,\{1 + P(T)^2 + P(T)^4 + \ldots\ldots\}$$

$$= P(T)\cdot P(H)\left\{\frac{1}{1-\left(P(T)\right)^2}\right\} = \frac{1}{2}\cdot\frac{1}{2}\left\{\frac{1}{1-1/4}\right\}$$

$$= \frac{1}{4}\times\frac{4}{3} = 0.33$$

$\Rightarrow$ $\boxed{P\ (\text{output Y}) = 0.33}$

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27. (b)

Number of trails = 4

Let ‘p’ be the probability of getting a ‘6’ $\Rightarrow p = \frac{1}{6}$

and ‘q’ be the probability of not getting a ‘6’ $\Rightarrow$ $q = \frac{5}{6}$

Let ‘x’ be the number of appearances of ‘6’ in 4 trails.

$\therefore$ P (x ≥ 2) = P (x = 2) + P (x = 3) + P (x = 4)

$$= {}^4C_2 \cdot p^2 \cdot q^2 + 4_{C_3} \cdot p^3 \cdot q^1 + 4_{C_4} \cdot p^4 \cdot q^0$$

$$\left\{\because P(x = r) = n_{C_r} \cdot p^r \cdot q^{n-r}\right\}$$

$$= 6\cdot\frac{25}{6^4} + 4\cdot\frac{5}{6^4} + 1\cdot\frac{1}{6^4}$$

$$= \frac{150}{6^4} + \frac{20}{6^4} + \frac{1}{6^4} = \frac{171}{6^4} = \frac{171}{6^3\times6}$$

$$= \frac{57}{6\times6\times6\times2} = \frac{19}{144}$$

$\therefore$ $\boxed{P(x \geq 2) = \frac{19}{144}}$

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28. (0.96 to 0.98)

Let A, B, C are the three vendors.

Let P(A) be the probability that the vendor ‘A’ meet design specification and similarly P(B) and P(C) for vendors ‘B’ and ‘C’.

$\Rightarrow$ P(A) = 0.8; P(B) = 0.7; P(C) = 0.5

P(at least one meeting design specification)

= 1 – P(None meeting design specification)

= 1 – $(A^C \cap B^C \cap C^C)$

= 1 – {$P(A^C)$. $P(B^C)$. $P(C^C)$)

= 1 – {(1– P(A)). (1– P(B)). (1– P(C))}

= 1 – {(0.2) (0.3) (0.5)} = 0.97

$\therefore$ $\boxed{\text{Required Probability} = 0.97}$

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29. (b)

Given

P(pass) = 20% = 0.2

P (pass ∩ (> 90%)) = 5% = 0.05

$$P((>90\%)\mid \text{pass}) = \frac{P((>90\%)\cap \text{pass})}{P(\text{pass})}$$

$$= \frac{0.05}{0.2} = 0.25$$

$$\boxed{\therefore\ P\ (\text{getting more than } 90\%|\text{pass}) = 0.25 = \frac{1}{4}}$$

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30. (c)

Given

$$P(X) = \frac{1}{4},\quad P(Y) = \frac{1}{3} \text{ and } P(X\cap Y) = \frac{1}{12}$$

$$\therefore\quad P(Y/X) = \frac{P(Y\cap X)}{P(X)} = \frac{P(X\cap Y)}{P(X)} = \frac{1/12}{1/4} = \frac{1}{3}$$

$$\Rightarrow\quad \boxed{P(Y/X) = \frac{1}{3}}$$

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31. (1.5)

Given

X → Number of times a coin is tossed until two consecutive heads

X	2	3	4	5	……
P(x)	p.p	qp.p	q^2.p.p	q^3p.p	……

∴ Expectation of X = Σx · P(x)

= $2p^2 + 3qp^2 + 4q^2p^2 + 5q^3p^2 + \ldots\ldots$

= $p^2(2 + 3q + 4q^2 + 5q^3 + \ldots.)$

Since

$$\therefore\quad \frac{p^2}{q}\left(2q+3q^2+4q^3\ldots..\right) = \frac{p^2}{q}\left(\frac{1}{(1-q)^2}-1\right)$$

Since p = q = 0.5

∴ Expectation of x = 0.5 × 3 = 1.5

∴ Average number of trails for getting two constative head = 1.5

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32. (c)

Given A, B are independent events.

1. P(A ∩B) = P(A).P(B)
2. $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\cdot P(B)}{P(B)} = P(A)$
3. P(A∪B) = P(A) + P(B) – P(A∩B)
   = P(A) + P(B) – P(A) ·P(B)
4. $P(\bar{A}\cap\bar{B}) = P(\bar{A})\cdot P(\bar{B})$

∴ option (c) is correct.

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33. (6)

Given X → Number of times the dice is rolled

$P(3) = \frac{1}{6}$ and $P(3') = \frac{5}{6}$

∴ X = 1, 2, 3

$P(3 \text{ on } 1^{st} \text{ roll}) = \frac{1}{6}$

$P(3 \text{ on } 2^{nd} \text{ roll}) = P(3'\cap 3) = \frac{5}{6}\times\frac{1}{6}$

$P(3 \text{ on } 3^{rd} \text{ roll}) = P(3'\cap 3'\cap 3) = \left(\frac{5}{6}\right)^2\times\frac{1}{6}$ and so on

∴ The probability distribution table

X	1	2	3	4	……
P(3)	$\frac{1}{6}$	$\frac{5}{6}\times\frac{1}{6}$	$\left(\frac{5}{6}\right)^2\times\frac{1}{6}$	$\left(\frac{5}{6}\right)^3\times\frac{1}{6}$	……

∴ Expectation of X = E[X]

$$E[X]=\left(\frac{1}{6}\times 1\right)+\left(\frac{5}{6}\times\frac{1}{6}\times 2\right)+\left(\left(\frac{5}{6}\right)^2\times\frac{1}{6}\times 3\right)+$$

$$\left(\left(\frac{5}{6}\right)^3\times\frac{1}{6}\times 4\right)+\ldots$$

$$E[X]=\frac{1}{6}\left[1+\left(2\times\frac{5}{6}\right)+\left(3\times\left(\frac{5}{6}\right)^2\right)+\left(4\times\left(\frac{5}{6}\right)^3\right)+\ldots..\right]$$

$$E[X]=\frac{1}{6}\left(1-\frac{5}{6}\right)^{-2}$$

$$\because (1-q)^{-2}=1+2q+3q^2+4q^3\ldots$$

$$E[X]=\frac{1}{6}\times\left(\frac{1}{6}\right)^{-2}=6$$

Scan for Video solution

34. (d)

$A \to$ A wins if he gets a six

P(A) = 1/6

$P(\bar{A})=1-\frac{1}{6}=\frac{5}{6}$ (Probability of A not getting a six)

$P(\bar{B})=\frac{5}{6}$ (Probability of B not getting a six)

P(Probability of A winning)

$= P(A)+P(\bar{A}\bar{B}A)+P(\bar{A}\bar{B}\bar{A}\bar{B}A)+\ldots$

$$=\frac{1}{6}+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$$

$$=\frac{1}{6}\left[1+\frac{25}{36}+\left(\frac{25}{36}\right)^2+\ldots\right]$$

$$=\frac{1}{6}\cdot\frac{1}{1-\frac{25}{36}}=\frac{6}{11}$$

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35. (d)

$P(H_R\, H_S) = 0.28$

$P(T_R\, T_S) = 0.18$

$P(H_R\, T_S) = 0.30$

$P(T_R\, H_S) = 0.24$

$\therefore P(H_R) = P(H_R\, H_S) + P(H_R\, T_S)$

$= 0.28 + 0.30 = 0.58$

$\therefore \quad P(T_R) = P(T_R\, T_S) + P(T_R\, H_S)$

$= 0.18 + 0.24 = 0.42$

$\therefore \quad P(H_S) = P(H_R\, H_S) + P(T_R\, H_S)$

$= 0.28 + 0.24 = 0.52$

$\therefore \quad P(T_R) = P(T_R\, T_S) + P(H_R\, T_S)$

$= 0.18 + 0.30 = 0.18$

Since $P(H_R) \neq P(T_R)$ & $P(H_S) \neq P(T_S)$

$\therefore$ R and S are not fair coin.

Let us suppose coins tosses are independent

$P(H_R H_S) = P(H_R)\cdot P(H_S)$

$\Rightarrow \quad 0.58 \times 0.52 \neq 0.28$

$\therefore$ Coin tosses are dependent.

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36. (a)

Total number of parts = 25

Number of defective = 10

$\Rightarrow$ Number of non-defective parts = 15

$\therefore$ P (both good) $=\frac{^{15}C_2}{^{25}C_2}$

$$=\frac{15\times 14}{25\times 24}=\frac{7}{20}$$

$\therefore$ P (both good) $=\frac{7}{20}$

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37. (0.64 to 0.66)

Given

A group consists of equal number of men and women

$\Rightarrow \quad P(\text{men}) = \frac{1}{2}$ and $P(\text{women}) = \frac{1}{2}$

Also, P (men $\cap$ unemployed) = 20% = $\frac{2}{10}$

And P (women $\cap$ unemployed) = 50% = $\frac{5}{10}$

P (Selecting Employed)

= P (men $\cap$ Employed) + P (Women $\cap$ Employed)

= P(men). P(Employed) + P(women). P(employed)

$= \frac{1}{2}\cdot\left(1-\frac{2}{10}\right)+\frac{1}{2}\cdot\left(1-\frac{5}{10}\right)$

$= \frac{1}{2}\left(\frac{8}{10}\right)+\frac{1}{2}\left(\frac{5}{10}\right) = \frac{13}{20} = 0.65$

$\therefore \boxed{P\ (\text{Selecting Employed}) = 0.65}$

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38. (0.25 to 0.27)

Opposite faces of a dice are coloured same

P(red) = $\frac{1}{3}$; P(green) = $\frac{1}{3}$; P(blue) = $\frac{1}{3}$

P(getting red at least twice) = P(exactly twice)+P (thrice)

P(exactly twice) = 3 × P(red $\cap$ red $\cap$ (blue $\cup$ green))

= 3 × P(red) × P(red) × P (blue $\cup$ green)

$= 3\times\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3} = \frac{2}{9}$

P(thrice) = P (red $\cap$ red $\cap$ red) = $(P(\text{red}))^3 = \frac{1}{27}$

$\therefore$ P (red at least twice) = $\frac{2}{9}+\frac{1}{27} = \frac{7}{27} = 0.2592$

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39. (b)

Given: A fair coin was tossed four times.

P (obtaining 'Tail' when coin is tossed) $= \frac{n(E)}{n(S)}$

n (E) = {T} = 1, n(S) = {H, T} = 2

$\Rightarrow \boxed{P(E) = \frac{1}{2}}$

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40. (0.65 to 0.68)

Let the total families in society be 'x'.

$\Rightarrow$ No. of families with single child = $\frac{'x'}{2}$

No of families with two children = $\frac{'x'}{2}$

$\therefore$ Total no of children in society =

$= \left(\frac{x}{2}\times 1+\frac{x}{2}\times 2\right) = \frac{3x}{2}$

$\therefore$ p (child having sibling) = $\frac{\left(\frac{x}{2}\times 2\right)_{C_1}}{\left(\frac{3x}{2}\right)_{C_1}} = \frac{x}{\left(\frac{3x}{2}\right)} = \frac{2}{3}$

= 0.67

$\therefore$ p (child selected at random having a sibling) = 0.67

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41. (2.9 to 3.1)

Given a coin is tossed repeatedly till both head and tail appear at least once.

Let x = Number of time,

T $\rightarrow$ Event of tail,

H $\rightarrow$Event of tail

$\therefore p(x=2) = p(T\cap H)+P(H\cap T)$

$= P(T).P(H)+P(H).P(T) = 2.\left(\frac{1}{2}\right)^2$

$P(x=3) = P(T\cap T\cap H)+P(H\cap H\cap T)$

$= P(T)^2.P(H) + (P(H))^2.P(T) = 2\left(\frac{1}{2}\right)^3$

$P(x = 4) = P(T \cap T \cap T \cap H) + P(H \cap H \cap H \cap T)$

$= (P(T))^3.P(H) + (P(H))^3.P(T) = 2.\left(\frac{1}{2}\right)^4$

And so on

$\therefore$ Probability distribution table is

x	2	3	4	5	.....
P(x)	$2.\left(\frac{1}{2}\right)^2$	$2.\left(\frac{1}{2}\right)^3$	$2.\left(\frac{1}{2}\right)^4$	$2.\left(\frac{1}{2}\right)^5$	.....

$\therefore E(x) = \Sigma x.P(x)$

$$= 2\times 2\left(\frac{1}{2}\right)^2 + 3.\times 2\left(\frac{1}{2}\right)^3 + 4\times 2\left(\frac{1}{2}\right)^4 + ...$$

$$= 2\times\left(\frac{1}{2}\right) + 3.\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{1}{2}\right)^3 + ... + 1 - 1$$

$$= (1 + 2\times\left(\frac{1}{2}\right) + 3.\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{1}{2}\right)^3 + ...) - 1$$

$$\left(\because 1 + 2x + 3x^2 + ... = (1-x)^{-2} \because |x| < 1\right)$$

$$= \left(1 - \frac{1}{2}\right)^{-2} - 1 = 4 - 1 = 3$$

$\therefore$ Average number of tosses required = 3

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42. (c)

Given coin is unbiased

$\Rightarrow P(H) = P(T) = \frac{1}{2}$

P(4th head in 10th toss) = P(3 Heads in first 9 tosses $\cap$ Head in 10th toss)

= P(3 Heads in first 9 tosses) × P(Head in 10th toss)

$$= {}^9C_3\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)$$

$$= \frac{9\times 8\times 7}{3\times 2\times 1} \times \frac{1}{2^{10}} = \frac{84}{2^{10}} = 0.082$$

$\boxed{\therefore\ P(4^{th}\ \text{Head in } 10^{th}\ \text{toss}) = 0.082}$

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43. (0.13 to 0.15)

Sample space = {1, 2, 3, 4, 5, 6 dots}

P(n dots) $\propto$ n

P(n dots) = kn

Sum of all probabilities is 1.

k + 2k + 3k + 4k + 5k + 6k = 1

$k = \frac{1}{21}$

$\therefore$ Probability of dice showing 3 dots = 3k

$= 3\times\frac{1}{21} = \frac{1}{7} = 0.142$

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44. (0.25 to 0.25)

Given: A and B are mutually exclusive

$\Rightarrow \quad A \cap B = \phi$ and $A \cup B = S$

By addition theorem,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(S) = P(A) + P(B) - 0$

$\Rightarrow P(A) + P(B) = 1$

Since $0 < P(A) < 1$ and $0 < P(B) < 1$,

A.M $\geq$ G.M

$$\Rightarrow \frac{P(A) + P(B)}{2} \geq \sqrt{P(A)\cdot P(B)}$$

$$\Rightarrow P(A)\cdot P(B) \leq \frac{1}{4}\cdot(P(A) + P(B))^2$$

$$\Rightarrow P(A)\cdot P(B) \leq \frac{1}{4}\cdot(1)^2$$

$\therefore$ Maximum value of P(A) · P(B) = **0.25**

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45. (d)

Given: P(know) = $\frac{2}{3} \Rightarrow$ P(Guess) = $\frac{1}{3}$

And P(correct/guess) = $\frac{1}{4}$

P(know/correct)

$$= \frac{P(\text{know} \cap \text{correct})}{P(\text{know} \cap \text{correct}) + P(\text{guess} \cap \text{correct})}$$

$$= \frac{P(\text{know}) \cdot P(\text{correct / know})}{P(\text{know}) \cdot P(\text{correct / know}) + P(\text{guess}) \cdot P(\text{correct / guess})}$$

$$= \frac{\frac{2}{3} \cdot (1)}{\frac{2}{3}(1) + \frac{1}{3} \cdot \frac{1}{4}} = \frac{2}{2 + \frac{1}{4}} = \frac{8}{9}$$

$\therefore \boxed{P(\text{know / correct}) = \frac{8}{9}}$

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46. (d)

Given: A box contains 4 red balls and 6 black balls.

4 $\rightarrow$ red$\rightarrow$ R

6 $\rightarrow$ black$\rightarrow$ B

P (1 R $\cap$ 2 B) = P (R $\cap$ B $\cap$ B) + P (B $\cap$ R $\cap$ B) + P (B $\cap$ B $\cap$ R)

= P(R). P(B). P(B)+ P(B). P(R). P(B)+ P(B). P(B).P(R)

$$= \left(\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8}\right) + \left(\frac{6}{10} \times \frac{4}{9} \times \frac{5}{8}\right) + \left(\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}\right) = \frac{1}{2}$$

$\therefore \boxed{P(\text{red} \cap 2\text{ black}) = \frac{1}{2}}$

Scan for Video solution

47. (d)

Given:

Since negative and positive value are equally likely, the distribution of number of negative values is binomial with n =5 and p = 1/2.

Let X represent number of negative values is 5trials

X $\rightarrow$ 0 to 5, P(–ve) = p = 1/2, P(+ve) = q =1/2

P (at most 1 negative value) = P (X = 0) + P (X = 1)

$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4$

$$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 + {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4$$

$$= \frac{1}{32} + \frac{5}{32} = \boxed{\frac{6}{32}}$$

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48. (c)

Let X be the random variable denoting number of tosses.

X	1 (H)	3 (TTH)	5 (TTTTH)	7 (TTTTTTH)
P(X)	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^5$	$\left(\frac{1}{2}\right)^7$

$\therefore$ The required probability = P(H) + P(TTH) + P(TTTTH)+...

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$$

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49. (c)

Given:

A fair coin is tossed till head appers for the first time

Possibilities to get head in odd tosses are

H(1^{st}), TTH(3^{rd}), TTTTH(5^{th}),.....................

P(1^{st} head in odd toss)

= P(H) + P(T$\cap$T$\cap$H) +P (T$\cap$T $\cap$T$\cap$T $\cap$H) +..........

$$=\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^5 \text{........}\infty$$

$$=\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}=\frac{\frac{1}{2}}{(3/4)}=\frac{2}{3}$$

$$\boxed{P\left(\text{getting } 1^{st} \text{head in odd toss}\right) = 2/3}$$

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50. (d)

Given: Number of tosses = 4

Number of heads more than number of tails

= (Number of heads = 3) + (Number of heads = 4)

If 'x' is number of heads in 4-tosses, then

P (x = 3) + P (x = 4)

$$= {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)+{}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$$

$$= 4\left(\frac{1}{16}\right)+1\left(\frac{1}{16}\right)=\frac{5}{16}$$

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51. (c)

Given:

Bag1	Bag 2
4R	3B
3G	4G

P (Drawing a red a ball from bag 1 and drawing a blue ball from Bag 2) = P (Red ball from Bag 1) × P (Blue ball from Bag 2)

P (R $\cap$ B) = P(R). P (B) ($\because$ independent events)

$$=\frac{4}{7}\times\frac{3}{7}=\mathbf{\frac{12}{49}}$$

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52. (c)

Given a box contains 2 washers, 3 nuts and 4 bolts

2 – Washers

3 – Nuts

4 – Bolts

P (2 washers $\cap$ 3 nuts $\cap$ 4 bolts)

= P(washer) × P(washer) × P(nuts) × P(nuts) × P(nuts) × P(bolt) × P(bolt) × P(bolt) × P(bolt)

$$=\frac{2}{9}\times\frac{1}{8}\times\frac{3}{7}\times\frac{2}{6}\times\frac{1}{6}\times\frac{4}{4}\times\frac{3}{3}\times\frac{2}{2}\times\frac{1}{1}$$

$$=\frac{1}{36\times35}=\frac{1}{1260}$$

$\therefore$ Required probability = $\frac{1}{1260}$

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53. (d)

Given: An unbiased coin is tossed 5 times

Let H $\rightarrow$ event of getting a head

T $\rightarrow$ event of getting a tail

P (at least one head) = 1 – P (No head)

= 1 – P(T$\cap$T$\cap$T$\cap$T$\cap$T)

= 1– {P(T). P(T). P(T). P(T). P(T)}

$$= 1 - \{P(\text{Tail})\}^5 = 1-\left(\frac{1}{2}\right)^5=\frac{31}{32}$$

$$\boxed{\therefore P\left(\text{at least one head}\right)=\frac{31}{32}}$$

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54. (c)

Given: A fair dice is rolled twice.

Let x$\rightarrow$ outcome in 1^{st} roll and y$\rightarrow$ outcome in 2^{nd} roll

P(x < y) = ?

considering followable cases

For $x = 1 \rightarrow \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)\}$

$x = 2 \rightarrow \{(2, 3), (2, 4), (2,5), (2,6)\}$

$x = 3 \rightarrow \{(3,4), (3,5), (3,6)\}$

$x = 4 \rightarrow \{(4, 5), (4, 6)\}$

$x = 5 \rightarrow \{(5, 6)\}$

No of favowable cases = 15

Requied probability = $\frac{15}{36} = \frac{5}{12}$

$\boxed{P\left(2^{nd}\text{roll value} > 1^{st}\text{roll value}\right) = 5/12}$

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55. (a)

P (Both heads | at least 1 head)

$$= \frac{P(\text{Both heads})}{P(\text{atleast one head})}$$

$$= \frac{P(H \cap H)}{P(T \cap H) + P(H \cap T) + P(H \cap H)}$$

$$= \frac{(P(H))^2}{2 \cdot P(T) \cdot P(H) + (P(H))^2} = \frac{\frac{1}{4}}{2\left(\frac{1}{4}\right) + \frac{1}{4}} = \frac{1}{3}$$

$\boxed{P\ (\text{Both Heads/at least one head}) = 1/3}$

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56. (a)

In general,

If $N = p \times q \times r \times s \times \ldots \times u$ (k terms) where p, q, r, s, …u are prime factors.

The number of divisors of $N^n = (n + 1)^k$

Number of divisors of $10^{99} = (2 \times 5)^{99}$ is

$(99 + 1)^2 = 100^2 = 10^4$

Number of multiples of 10^{96} that are factors of 10^{99}

$= \text{number of divisors of } \left(\frac{10^{99}}{10^{96}} = 10^3 = (2 \times 5)^3\right)$

$= (3+1)^2 = 16$

$\therefore$ Required probability = $\frac{16}{10^4} = \frac{16}{100 \times 100} = \frac{1}{625}$

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57. (c)

Given: Two coins are tossed

Possible Outcomes = {HH, HT, TH, TT}

P (2 Heads) = $\frac{\text{No of favourable outcomes}\{HH\}}{\text{No of total outcomes}}$

$\therefore$ The required probability = $\mathbf{\frac{1}{4}}$

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58. (c)

It is given that white ball is removed infirst draw, therefore sample space reduces

3W
3R

P(Probability of removing red ball in second draw)

$= \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

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59. (b)

P(at least two of them have same birthday)

= 1 – P(None of them have same birthday)

Assuming N = 2,

$P = 1 - \frac{30}{30} \times \frac{29}{30} \ngtr \frac{1}{2}$

Assuming N = 3,

$$P = 1 - \frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \not> \frac{1}{2}$$

Assuming N = 7,

$$P = 1 - \frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30} \times \frac{25}{30} \times \frac{24}{30}$$

$$= 0.5308 > \frac{1}{2} \qquad \therefore N = 7$$

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60. (c)

Given: A fair coin is tossed 10 times.

P (1st two heads $\cap$ Next 8 tails)

= P (H $\cap$ H $\cap$ T $\cap$ T $\cap$ ……. T)

= P(H). P(H). P(T). P(T)…. (P(T) occurs 8 times)

$$= \frac{1}{2} \times \frac{1}{2} \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \ldots \times \frac{1}{2}\right)_{8\,\text{times}} = \left(\frac{1}{2}\right)^{10}$$

$$\therefore P\,(1^{st}\text{ two as heads} \cap 8 \text{ tails}) = \left(\frac{1}{2}\right)^{10}$$

$$\boxed{P\left(\text{Number of heads} > \text{Number of tails}\right) = 5/16}$$

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61. (d)

Given: Three coins are simultaneously tossed

P (at least one head) = 1 – P (No head)

= 1 – P (tail $\cap$ tail $\cap$ tail)

= 1 – {P(tail). P(tail). P(tail)}

$$= 1 - \left(P\left(\text{tail}\right)\right)^3 = 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8}$$

$$\therefore \quad \boxed{P\left(\text{at least one head}\right) = 7/8}$$

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62. (a)

Given: A coin is tossed four times

Let H $\rightarrow$ denotes Head and T $\rightarrow$ denotes Tail

Assuming all the tosses are independent

P (3 Heads) = P (H$\cap$H$\cap$H$\cap$T) + P (H$\cap$H$\cap$T$\cap$H) + P (H$\cap$T$\cap$H$\cap$H) + P (T$\cap$H$\cap$H$\cap$H)

$$= 4 \times \left(\frac{1}{2}\right)^4 = \frac{4}{16} = \frac{1}{4}$$

$$\therefore \quad \boxed{\text{Probability of getting exactly 3 heads} = \frac{1}{4}}$$

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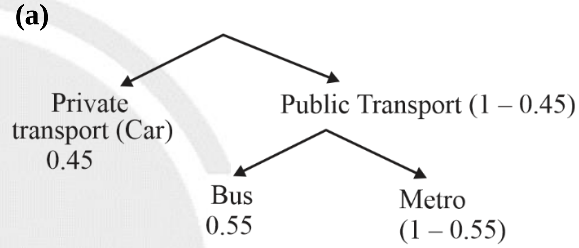
63. (a)



Obviously, the probability of using a car = 0.45

$$\left(\begin{matrix}\text{The probability}\\ \text{of using a bus}\end{matrix}\right) = \left(\begin{matrix}\text{Probability of using}\\ \text{public transport}\end{matrix}\right) \times \left(\begin{matrix}\text{Probability of}\\ \text{choosing bus}\end{matrix}\right)$$

= (1 – 0.45) × 0.55

= 0.3025 = **0.30**

$$\left(\begin{matrix}\text{The probability}\\ \text{of using metro}\end{matrix}\right) = \left(\begin{matrix}\text{Probability of using}\\ \text{public transport}\end{matrix}\right) \times \left(\begin{matrix}\text{Probability of}\\ \text{choosing a metro}\end{matrix}\right)$$

= (1 – 0.45) (1 – 0.55)

= 0.2475 = **0.25**

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64. (c)

Given: The students studies either computer science (S) or mathematics (M) every day.

P (S and then M) = 0.6

P (M and then C) = 0.4

P (C on Wednesday| C on Monday)

= P (C $\cap$ M $\cap$ C) + P (C $\cap$ C $\cap$ C)

= P (C and then M) · P (M and then C) + P (C and then C) · P (C and then C)

= (0.6 × 0.4) + (0.4 + 0.4) = 0.24 + 0.16 = 0.40

∴ P (computers on Wednesday/computers on Monday) is 0.4

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65. (0.99 to 1.01)

$f(x) = ae^{-2|x|}$

$\therefore \int_{-\infty}^{+\infty} f(x) = 1$ [for this function to be a valid probability density function.

$$\int_{-\infty}^{+\infty} ae^{-2|x|} = \int_{-\infty}^{0} ae^{2x} + \int_{0}^{\infty} ae^{-2x} = 1$$

$$\left[\therefore |x| = \begin{cases} x & ; \quad x^{3}0 \\ -x & ; \quad x<0 \end{cases}\right]$$

$$= \frac{a}{2}\left[e^{2x}\right]_{-\infty}^{0} + \frac{a}{-2}\left[e^{-2x}\right]_{0}^{\infty} = 1$$

$$= \frac{a}{2}(1-0) - \frac{a}{2}[0-1] = 1$$

$\Rightarrow a = 1$

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66. (a)

We know,

$E[(x-\bar{x})^2] = Var$

Given $\bar{x} = 0, E(x^2) = Var$

$E(x_n^2) = \sigma_0^2$

We have,

Standard deviation,

$$\sigma^2 = \frac{1}{N\text{-}1}\sum_{n=1}^{n} x_n^2 \Rightarrow \text{for sample}$$

$$= \frac{1}{N}\sum_{n=1}^{n} x_n^2 \Rightarrow \text{for population}$$

Here we taken 10000 samples out 10^6 numbers.

$\therefore E\left(\hat{\sigma}_2^2\right) = \sigma_0^2$ will be correct.

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67. (4)

From the given graph:

No. of wickets taken	0	1	2	3	4	5	6	7	8	9	10
No. of matches	5	7	8	25	20	17	8	4	3	2	1

Total number of matches

= 5 + 7 + 8 + 25 + 20 + 17 + 8 + 4 + 3 + 2 + 1 = 100

∴ Median is the average number of wickets taken in 50^{th} and 51^{st} match.

In the table above, data is arranged in ascending order.

∴ Number of wickets taken in 50^{th} and 51^{st} match are 4.

$\therefore \quad \text{Median} = \frac{4+4}{2} = 4$

∴ Median number of wickets taken in a match = 4.

Scan for Video solution

68. (a)

Probability of a component being defective = 0.015

Number of components in a box = 200

⇒ Average number of (Mean of) defective components in a box = 200 × 0.015 = 3

⇒ Mean = μ = 3

For Poisson distribution,

Mean = Variance = $\mu = \sigma^2 = 3$

∴ Mean = Variance = 3

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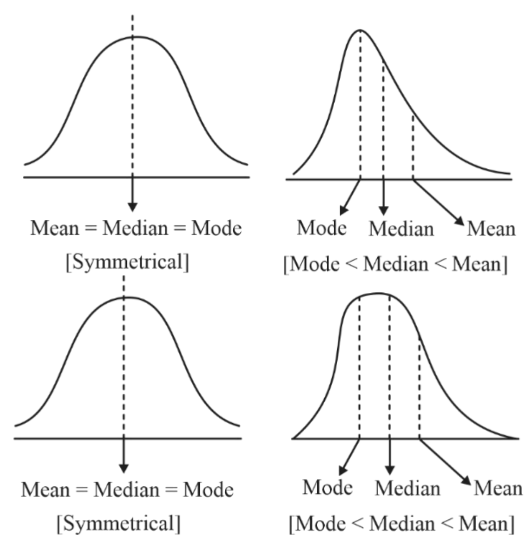
69. (b)



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70. (0.17 to 0.19)

Given 'X' follows Poisson distribution and

P (X = 1) = P (X = 2)

$$\Rightarrow \quad \frac{e^{-\lambda}\cdot\lambda^1}{1!} = \frac{e^{-\lambda}\cdot\lambda^2}{2!} \quad \Rightarrow \quad \lambda = 2$$

$$\therefore \quad P(X=3) = \frac{e^{-\lambda}\cdot\lambda^3}{3!} = \frac{e^{-2}\cdot(2)^3}{3!} = \frac{8}{6e^2}$$

$$\boxed{\therefore P(x=3) = 0.180}$$

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71. (d)

Binomial distribution is for finite samples.

When the number samples gets increased, the distribution can be approximated as continuous distribution.

So we can write the distribution as

$Y = \sum_{i=1}^{\infty} X_i = \int X_i dx$ and Y is normally distributed.

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72. (d)

For a Binomial distribution with 'n' trails, if 'P' is the probability of success and q = 1 – P is the probability of failure, then

Mean = np, Variance = npq = np(1 – p)

$\Rightarrow$ Standard deviation = $\sqrt{npq} = \sqrt{np(1-p)}$

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73. (0.35 to 0.39)

Given: P(x) is exponentially distribution with parameter '2'

$$\Rightarrow \quad P(x) = \begin{cases} 2e^{-2x}; & x \geq 0 \\ 0 \quad ; & x < 0 \end{cases}$$

Let 'x' be the life time of component

Expected life time = Mean life time = $\frac{1}{\lambda} = \frac{1}{2}$

$$P\left(x > \frac{1}{2}\right) = \int_{1/2}^{\infty} P(x)dx = \int_{1/2}^{\infty} 2e^{-2x}dx$$

$$= 2\left.\frac{e^{-2x}}{-2}\right|_{1/2}^{\infty} = -1\left(e^{-\infty} - e^{-2(1/2)}\right)$$

$$= e^{-1} = \frac{1}{e} = 0.3678 = 0.37 \because e^{-\infty} = 0$$

$$\boxed{P\ (\text{life time exceeds expected life time}) = 0.37}$$

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74. (15.00 to 16.00)

Given:

P(Head) = p = 0.4; P(Tail) = q = 1 – 0.4 = 0.6

Let X→ Number of heads in 1000 tosses.

Given random variable follows Binomial distribution Since number of possible outcomes = 2

For a random variable X

Where n → Total number of trails

p → Probability of success

q → Probability of failure.

Standard deviation = $\sigma = \sqrt{npq}$

$= \sqrt{1000(0.4)(0.6)} = \sqrt{240} = 15.49$

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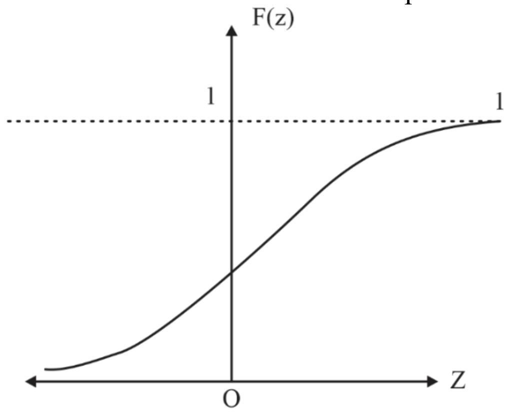
75. (b)

The cumulative distribution function for the Gaussian distribution will be "S–shaped".



Cumulative distribution curve

Scan for Video solution

76. (0.5 to 0.5)

Given:

For $n > 2$, $a \in \{0, 1\}^n$

$\Rightarrow$ 'a' is a $n \times 1$ vector with every element '0' or '1'

$\Rightarrow$ $a = [a_{ij}]\ \forall\ 1 \le i \le n,\ j = 1$ where $a_{ij} = 0$ (or) 1

Also, 'x' is a $n \times 1$ vector with element '0' or '1'.

$\Rightarrow$ $x = [x_{ij}]\ \forall\ 1 \le i \le n;\ j = 1$ where $x_{ij} = 0$ (or) 1.

$\because$ $a_{ij}.\ x_{ij}$ lies between 0 to '1'

$\therefore$ $\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_{ij}$ lies between 0 to 'n'

So, the probability distribution of $\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_{ij}$ is

$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_{ij}$	0	1	2...	n
$P\left(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_{ij}\right)$	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_n$

$\therefore$ Probability that $a_{ij}.x_{ij}$ is odd number is

$$= \frac{{}^nC_1 + {}^nC_3 + \cdots + \cdots}{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots} = \frac{2^{n-1}}{2^n} = \frac{1}{2} = 0.5$$

Scan for Video solution

77. (–1.00 to –0.98)

Given

X and Y are exponentially distributed with mean of 0.5.

$\Rightarrow$ $\mu_x = \mu_y = 0.5$

Also, $Z = X + Y$

$\therefore$ $\text{Var}(Z) = \text{Var}(X + Y)$

$\Rightarrow$ $0 = \text{Var}(X) + \text{Var}(Y) + 2.\ \text{Cov}(X, Y)$

$\Rightarrow$ $\text{Cov}(X, Y) = -\frac{1}{2}\{\text{Var}(X) + \text{Var}(Y)\}$

Correlation coeffecient is given by

$$r = \frac{\text{Cov}(X, Y)}{\sigma_X \cdot \sigma_Y} = \frac{-\frac{1}{2}\{\text{Var}(X) + \text{Var}(Y)\}}{\sigma_X \cdot \sigma_Y}$$

For exponential distribution; $\mu_x = \sigma_x$ and $\mu_y = \sigma_y$

$$\Rightarrow r = \frac{-\frac{1}{2}\{\mu_x^2 + \mu_y^2\}}{\mu_x \cdot \mu_y} = -\frac{1}{2}\frac{(2 \times 0.5^2)}{(0.5)^2} = -1$$

$\Rightarrow$ $\boxed{r = -1}$

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78. (Marks to all (MTA))

If $x \sim N(\mu_x, \sigma_x^2)$

$y \sim N(\mu_x, \sigma_y^2)$

Then

$z = x + y \sim N(\mu_x + \mu_y,\ a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\ \text{cov}(x, y))$

If x and y are independent, then cov (x, y) = 0

$\Rightarrow$ $z \sim N(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$

$\therefore$ Z is normally distributed if x and y are independent.

Scan for Video solution

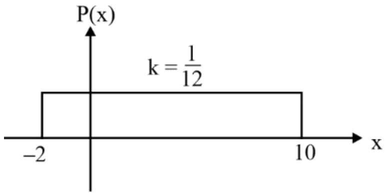
79. (0.299 to 0.301)

Given:

'x' is random variable with uniform probability density function in [–2, 10]

Let $P(x) = k$

$$\Rightarrow \int_{-2}^{10} k\,dx = 1 \Rightarrow k = \frac{1}{12}$$

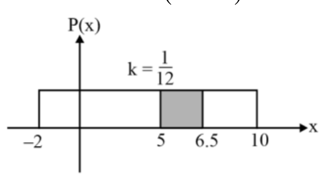


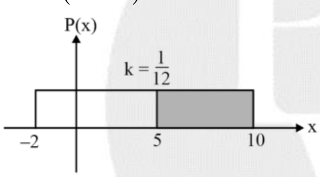
Now $y \le 7$ $\Rightarrow 2x - 6 \le 7$

$\Rightarrow x \le \frac{13}{2}$ $\Rightarrow x \le 6.5$

$$\therefore P(y \le 7 | x \ge 5) = \frac{P(y \le 7 \cap x \ge 5)}{P(x \ge 5)}$$

$$= \frac{P(x \le 6.5 \cap x \ge 5)}{P(x \ge 5)} = \frac{P(5 \le x \le 6.5)}{P(x \ge 5)}$$





$$\Rightarrow \frac{P(5 \le x \le 6.5)}{P(x \ge 5)} = \frac{1.5k}{5(k)} = \frac{1.5}{5} = 0.3$$

$\therefore P(y \le 7 | x \ge 5) = 0.3$

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80. (11 to 11)

Given: X and Y are random variable and

$E(2X + Y) = 0 \Rightarrow 2E(X) + E(Y) = 0$ … (1)

$E(X + 2Y) = 33 \Rightarrow E(X) + 2E(Y) = 33$ … (2)

Adding equation (1) and (2)

$\Rightarrow 3E(X) + 3E(Y) = 33 \Rightarrow 3(E(X) + E(Y)) = 33$

$\Rightarrow E(X) + E(Y) = 11$

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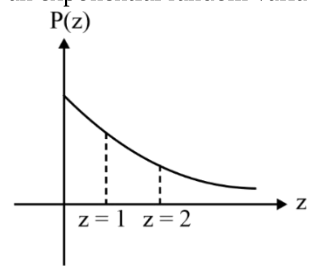
81. (0.36 to 0.38)

Given: z is an exponential random variable with mean = 1



$\Rightarrow P(z) = 1 \cdot e^{-1 \cdot z} = e^{-z}$

$$\Pr\left(z > 2 / z > 1\right) = \frac{\Pr(z > 2 \cap z > 1)}{\Pr(z > 1)} = \frac{P(z > 2)}{P(z > 1)}$$

$$= \frac{\int_2^\infty e^{-z} dz}{\int_1^\infty e^{-z} dz} = \frac{\left.\frac{e^{-z}}{-1}\right|_2^\infty}{\left.\frac{e^{-z}}{-1}\right|_1^\infty} = \frac{e^{-\infty} - e^{-2}}{e^{-\infty} - e^{-1}}$$

$$= \frac{e^{-2}}{e^{-1}} = \frac{1}{e} = 0.3678 \approx 0.37$$

$$\therefore \boxed{\Pr\left(z > 2 / z > 1\right) = 0.37}$$

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82. (a)

Given

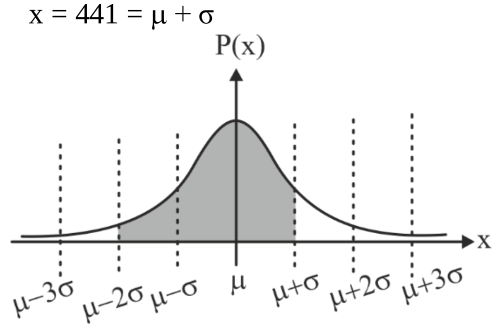
Length of rods follow normal distribution**.**

Let 'x' be the length of a randomly selected rod.

$\Rightarrow x \sim N(440, 1^2)$

$x = 438 = \mu - 2\sigma$

$x = 441 = \mu + \sigma$



$P(438 < x < 441) = P(\mu - 2\sigma < x < \mu + \sigma)$

$= P(\mu - 2\sigma < x < 0) + P(0 < x < \mu + \sigma)$

$= \frac{1}{2} \cdot P(\mu - 2\sigma < x < \mu + 2\sigma) + \frac{1}{2}.(P(\mu-\sigma<x<\mu+\sigma))$

$= \frac{1}{2} \times 0.9577 + \frac{1}{2}(0.6828) = 0.82025 = 82.025\%$

$\therefore$ P $(438 < x < 441) = 82.025\%$

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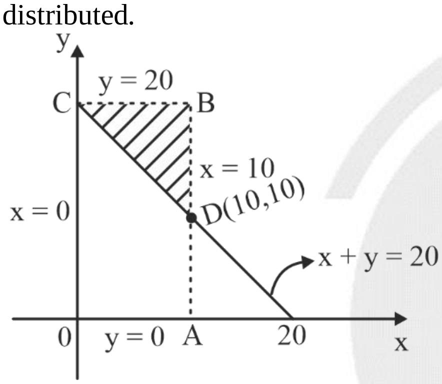
83. (b)

Given

$0 < x < 10$ and $0 < y < 20$ and x and y are uniformly distributed.



$$P(x + y > 20) = \frac{\text{area}(\Delta OBC)}{\text{area}(OABC)} = \frac{\frac{1}{2} \times BC \times BD}{BC \times AB}$$

$$\Rightarrow \quad P(x + y > 20) = \frac{\frac{1}{2} \times 10 \times 10}{10 \times 20} = \frac{1}{4} = 0.25$$

$\Rightarrow$ P $(x + y > 20) = 0.25$

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84. (3 to 3)

Given

x is the mean of 3, x, 2, 4

$$\Rightarrow \quad x = \frac{3 + x + 2 + 4}{4} \Rightarrow 3x = 9 \Rightarrow x = 3$$

$\therefore$ Given data is 3, 3, 2 and 4.

$\therefore$ $\boxed{\text{Most repeating data point is } x = 3}$

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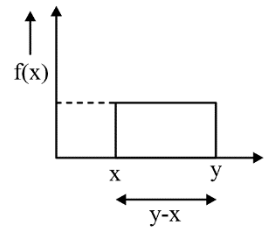
85. (b)

Uniform distribution is given by,



$$f(x) = \begin{cases} \frac{1}{y - x} & x \in (x, y) \\ 0 & \text{otherwise} \end{cases}$$

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86. (0.8 to 0.8)

Given: Y is uniformly distributed in (1, 6) and the polynomial is $3x^2 + 6xY + (3Y + 6)$

For the polynomial,

$Ax^2 + Bx + C$ to have real roots $B^2 - 4AC \geq 0$

$\Rightarrow \quad (6Y)^2 - 4(3)(3Y + 6) \geq 0$

$\Rightarrow \quad 36Y^2 - 12(3Y + 6) \geq 0$

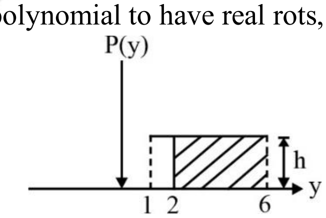
$\Rightarrow \quad 3Y^2 - 3Y - 6 \geq 0 \qquad \Rightarrow y^2 - y - 2 \geq 0$

$\Rightarrow \quad y^2 - 2y + y - 2 \geq 0 \qquad \Rightarrow (y + 1)(y - 2) \geq 0$

$\Rightarrow \quad y \leq -1$ and $y \geq 2$

Since 'y' is distribution only in (1, 6)

For the polynomial to have real rots, $y \geq 2$



$$P(Y \geq 2) = \frac{4.h}{5.h} = 0.8$$

$\boxed{\text{Probability of polynomial having real roots = 0.8.}}$

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87. (a)

For function to be a valid probability density function, the following condition must be satisfied,

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 f(x)\,dx + \int_1^2 f(x) + \int_2^3 f(x)\,dx = 1$$

$$\Rightarrow \left(\frac{1}{2}\times 1\times h\right) + \left(\frac{1}{2}\times 1\times 2h\right) + \left(\frac{1}{2}\times 1\times 3h\right) = 1$$

$$\Rightarrow \frac{h}{2} + h + \frac{3h}{2} = 1 \quad \Rightarrow \quad h = \frac{1}{3}$$

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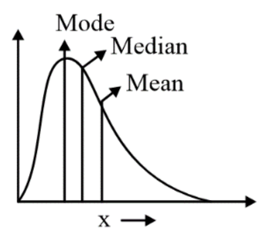
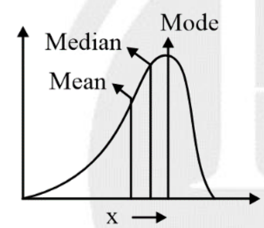
88. (c)



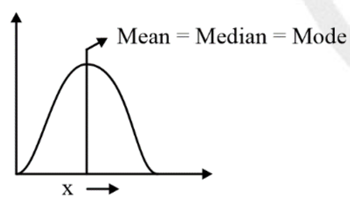
Right skewed curve

[Mode < Median < Mean]



Left skewed curve

[Mean < Median < Mode]



No skewness

[Symmetrical]

[Mean = Median = Mode]

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89. (0.25)

Given X_1, X_2, X_3 and X_4 be independent normal random variables with same mean (= 0) and variance ($\sigma^2 = 1$)

The probability that X_4 is smallest = ¼ = 0.25.

(Probability that a particular number from 4 randomly picked numbers to be smallest).

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90. (b)

Any normal random variable with mean 'μ' and variance 'σ^2' is denoted as $x \sim N(\mu, \sigma^2)$

Given, $X_1 \sim N(1, 1)$ and $X_2 \sim N(1, 4)$

For $\quad Y = aX_1 - bX_2$,

$Y \sim N(a\mu_1 - b\mu_2,\ a^2\sigma_1^2 + b^2\sigma_2^2 - 2ab\,\mathrm{Cov}(X_1, X_2))$

Since X_1, X_2 are independent variables

$\Rightarrow \quad \mathrm{Cov}(X_1, X_2) = 0$

For $a = 1$ and $b = 1$

$\Rightarrow \quad Y = X_1 - X_2$

$\Rightarrow \quad Y \sim N(\mu_1 - \mu_2,\ \sigma_1^2 + \sigma_2^2)$

$\Rightarrow \quad \boxed{Y \sim N(0, 5)}$

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91. (a)

Given,

$X_1 \rightarrow$ Exponentially distributed with Mean = 0.5

$$\Rightarrow \quad X_1 = \left(\frac{1}{0.5}\right)\cdot e^{-x\left(\frac{1}{0.5}\right)} = 2\cdot e^{-2x} \quad ; \quad x > 0$$

$X_2 \rightarrow$ Exponentially distributed with Mean = 0.25

$$\Rightarrow \quad X_2 = \left(\frac{1}{0.25}\right)\cdot e^{-x/0.25} = 4\cdot e^{-4x} \quad ; \quad x > 0$$

$$\text{Since } Y = \min(X_1, X_2) = \begin{cases} 2\cdot e^{-2x} & ; 0 < x < \alpha \\ 4\cdot e^{-4x} & ; \alpha < x < \infty \end{cases}$$

Where 'α' is value of 'x' where $2e^{-2x}$ and $4e^{-4x}$ intersect

$\therefore \quad$ 'Y' is exponentially distributed

$\Rightarrow \quad$ Mean of Y < Min {Mean of X_1, Mean of X_2}

$\Rightarrow \quad$ Mean of Y < Min {0.5, 0.25}

$\therefore \quad \boxed{\text{Mean of Y} < 0.25 = 1/6}$

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92. (c)

Mode is the most frequently occurring data point in a given data.

In the given data, 17 is the most repeating data point.

$\therefore$ Mode = 17.

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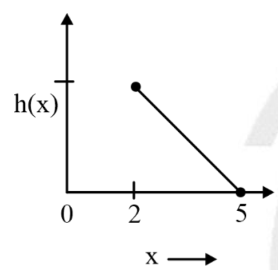
93. (0.9)



Let the time of arrival be a random variable x, then

its p.d.f. is $f(x) = \begin{cases} \dfrac{1}{5} & ; \quad 0 < x < 5 \\ 0 & ; \quad \text{otherwise} \end{cases}$

Assume waiting time be h(x), a function of arrival time

$$h(x) = \begin{cases} 0 & ; \quad 0 \le x \le 2 \text{ (GreenLight)} \\ 5 - x & ; \quad 2 < x \le 5 \text{ (Red Light)} \end{cases}$$

Average waiting time = E[h(x)]

$$= \int_0^5 h(x) \cdot f(x) dx$$

$$= \int_0^2 0 \cdot \frac{1}{5} dx + \int_2^5 (5 - x) \cdot \frac{1}{5} dx$$

$$= \int_2^5 \left(1 - \frac{x}{5}\right) dx$$

$$= \left[x - \frac{x^2}{10}\right]_2^5$$

= (5 – 2.5) – (2 – 0.4)

Average waiting time = 2.5 – 1.6 = 0.9

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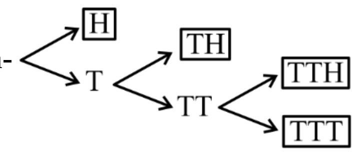
94. (c)

The maximum number of tosses = 3

Tree diagram-



Hence possible outcomes = {H, TH, TTH, TTT}

Here random variable Y denotes number of heads in the outcomes, hence possible values of Y will be 0 and 1.

$$p[y = 0] = P(TTT) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

p[y = 1] = p(H) + p(TH) + p(TTH)

$$= \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{7}{8}$$

$\therefore$ Probability distribution

y	0	1
p(y)	1/8	7/8

$$E(y) = \sum y(p(y)) = 0 \times \frac{1}{8} + 1 \times \frac{7}{8} = \frac{7}{8}$$

$$E(y^2) = \sum y^2 p(y) = 0^2 \times \frac{1}{8} + 1^2 \times \frac{7}{8} = \frac{7}{8}$$

$\therefore$ Var(y) = E(y^2) – [E(y)]^2

$$= \frac{7}{8} - \left(\frac{7}{8}\right)^2 = \frac{7}{64}$$

$$\text{Var}(Y) = \frac{7}{64}$$

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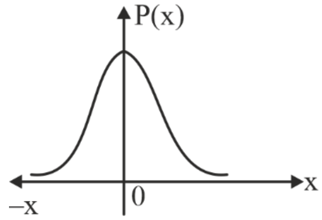
95. (0.0 to 0.0)

Given:

x: Gaussian random variable; $\mu_x = 0$; & variance = σ^2

$\because \quad y = \max(x, 0)$

$$\Rightarrow \quad y = \begin{cases} 0; & -\infty < x < 0 \\ x; & 0 < x < \infty \end{cases}$$



$\Rightarrow$ y = 0 is the line that is in middle of the complete data of y.

$\therefore$ Median of y = 0

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96. (b)

Given: 'X' is a discrete random variable and

$P(X = j) = p_j$, $p_j \ge 0$, $j \in \{0, \ldots. N\}$,

$$\Rightarrow \sum_{j=0}^{N} P_j = 1 \text{ (Sum of probabilities = 1)}$$

$$\because \quad g_x(z) = \sum_{j=0}^{N} P_j z^j = P_0 + P_1 z + P_2 z^2 + \ldots. P_N z^N$$

$\therefore$ $g_x(z)$ is polynomial function.

$$\because \quad g_y(z) = (1 - \beta + \beta z)^N = \sum_{j=0}^{N} N_{c_j} (1-\beta)^{N-j} \cdot (\beta z)^j$$

$\therefore$ Mean of Y = coefficient of 'z' in above expansion

$$= N_{C_1} (1-\beta)^{N-1} \beta = N\beta(1-\beta)^{N-1}$$

For $\beta \in [0, 1]$, $(1 - \beta)^{N-1} = 1 - (N - 1)\beta$

$\therefore$ Expectation of Y = $N\beta (1 - (N - 1)\beta)$

$= N\beta - N(N - 1)\beta^2$

Since 'β' is small, neglecting higher terms

$\Rightarrow$ Expectation of Y = $N\beta$

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97. (b)

In exponential distribution,

$f(x) = \lambda e^{-\lambda x}; \quad x \ge 0$

Parameter is λ. (One parameter)

In Gaussian distribution,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Parameters are μ and σ. (Two parameters)

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98. (b)

For function $f(x)$ to be a valid probability density function, following condition must be satisfied;

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 (a + bx)\,dx = \left[ax + \frac{bx^2}{2}\right]_0^1$$

$$\Rightarrow a + \frac{b}{2} = 1$$

Hence option (b) a = 0.5, b =1 will satisfy the relation.

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99. (54 to 54)

Given: 'x' is a Poisson distributed variable

$\therefore$ Mean E(x) = Variance = 5

$E(x + 2)^2 = E(x^2 + 4x + 4)$

$= E(x^2) + 4E(x) + E(4)$

$= \text{Variance} + (E(x))^2 + 4E(x) + 4$

$(\because \text{variance} = E(x^2) - E(x)^2)$

$\therefore E[(x+2)^2] = \text{Variance} + (E(x))^2 + 4E(x) + 4$

$= 5 + (5)^2 + 4(5) + 4 = 54$

$$\therefore \quad \boxed{E\left[(x+2)^2\right] = 54}$$

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100. (0.55 to 0.55)

Given:

Number of bulbs of type 1 = Number of bulbs of type 2

$\Rightarrow$ P (picking type1) =P (picking type 2)

= 0.5 P(type1> 100 hours)

= 0.7; P (type 2 > 100 hours)

= 0.4 P (bulb lasting > 100 hours)

= P (Type1 $\cap$ > 100 hrs + P (type 2) $\cap$ >100 hrs)

= P(Type–1) × P (type 1 > hrs) + P (type–2) × P (type 2 > 100 hrs)

= (0.5 × 0.7) + (0.5 × 0.4) = 0.35 + 0.2 = 0.55

$\therefore \boxed{P\left(\text{bulb lasting} > 100 \text{ hours}\right) = 0.55}$

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101. (a)

The decisions to accept or reject a null hypothesis after inspecting only from it may be expressed exhaustively in the following table:-

	Reject Null	Accept Null
Null True	Wrong (Type I error)	Correct
Null False	Correct	Wrong (Type II Error)

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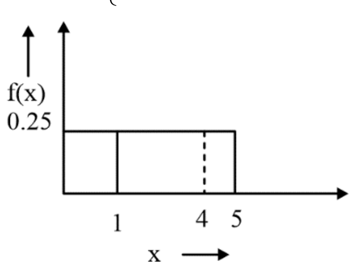
102. (a)

Given:

$$f(x) = \begin{cases} 0.25 & \text{if } 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$



$$P(x \le 4) = \int_{-\infty}^{4} f(x)dx$$

$$= \int_{-\infty}^{1} f(x)dx + \int_{1}^{4} f(x)dx$$

$$= \int_{-\infty}^{1} 0\ dx + \int_{1}^{4} 0.25\,dx = 0 + 0.25[x]_1^4 = \frac{1}{4}(4-1)$$

$$\boxed{p(x \le 4) = \frac{3}{4}}$$

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103. (54.49 to 54.51)

Median speed is the speed at the middle of series of spot speeds that are arranged in ascending order. 50% of speed values will be greater than the median and 50% of speed values will be less than median.

Ascending order of spot speed studies are

32, 45, 49, 51, 53, 56, 60, 62, 66, 79

Median speed $= \frac{53+56}{2} = 54.5$ km/hr

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104. (a)

Given: P(X) = 0.4, P (X $\cup$ Y^c) = 0.7

P (X $\cup$ Y^c) = P(X) + P(Y^c) – P (X $\cap$ Y^c)

= P(X)+ [1– P(Y)] – P(X). P(Y^c)

= P(X)+ [1 – P(Y) – P(X) {1 – P(Y)}]

$\Rightarrow$ 0.7 = 0.4 + 1– P(Y) – 0.4 {1 – P(Y)}

$\Rightarrow$ P(Y) – 0.4 P(Y) = 0.4 + 1 –0.7 – 0.4

$\Rightarrow$ 0.6 P(Y) = 0.3 $\Rightarrow$P(Y) = 0.3/0.6=0.5

$\therefore$ P (X $\cup$ Y) = P(X) + P(Y) – P (X $\cap$ Y)

= P(X) + P(Y) – P(X). P(Y)

[Since X & Y are independent events]

= 0.4 + 0.5 – 0.4 × 0.5

$\Rightarrow \boxed{P(X \cup Y) = 0.7}$

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105. (b)

Given:

$$f(x)=\begin{cases}\frac{x}{a}+1 & : -a\le x\le 0\\ -\frac{x}{a}+1 & : 0\le x\le a\\ 0 & : \text{otherwise}\end{cases},$$

$$g(x)=\begin{cases}-\frac{x}{a} & : -a\le x\le 0\\ \frac{x}{a} & : 0\le x\le a\\ 0 & : \text{otherwise}\end{cases}$$

Mean of f(x)

$$E[f(x)]=\int_{-a}^{0}x\left(\frac{x}{a}+1\right)dx+\int_{0}^{a}x\left(-\frac{x}{a}+1\right)dx$$

$$=\left[\frac{x^3}{3a}+\frac{x^2}{2}\right]_{-a}^{0}+\left[-\frac{x^3}{3a}+\frac{x^2}{2}\right]_{0}^{a}$$

$$=-\left(-\frac{a^2}{3}+\frac{a^2}{2}\right)+\left(-\frac{a^2}{3}+\frac{a^2}{2}\right)=\mathbf{0}$$

Variance of $f(x)=E[f(x)^2]-[E[f(x)]]^2$

$$E\left[f(x)^2\right]=\int_{-a}^{0}x^2\left(\frac{x}{a}+1\right)dx+\int_{0}^{a}x^2\left(-\frac{x}{a}+1\right)dx$$

$$=\left[\frac{x^4}{4a}+\frac{x^3}{3}\right]_{-a}^{0}+\left[-\frac{x^4}{4a}+\frac{x^3}{3}\right]_{0}^{a}=\mathbf{\frac{a^3}{6}}$$

Mean of g(x)

$$E[g(x)]=\int_{-a}^{0}x\left(-\frac{x}{a}\right)dx+\int_{0}^{a}x\left(\frac{x}{a}\right)dx$$

$$=\left[-\frac{x^3}{3a}\right]_{-a}^{0}+\left[\frac{x^3}{3a}\right]_{0}^{a}=\mathbf{0}$$

Variance of $g(x)=E[g(x)^2]-[E[g(x)]]^2$

$$E\left[g(x)^2\right]=\int_{-a}^{0}x^2\left(-\frac{x}{a}\right)dx+\int_{0}^{a}x^2\left(\frac{x}{a}\right)dx$$

$$=\left[-\frac{x^4}{4a}\right]_{-a}^{0}+\left[\frac{x^4}{4a}\right]_{0}^{a}=\mathbf{\frac{a^3}{2}}$$

$\therefore$ Mean of f(x) and g(x) are same (i.e., equal to 0) but variance of f(x) and g(x) are different.

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106. (a)

For a Poisson distribution over a discrete random variable 'x', Mean = Variance

$\Rightarrow \quad \mu=\sigma^2 \Rightarrow \sigma=\sqrt{\mu}$

$\therefore$ Standard deviation = $\sqrt{\mu}$

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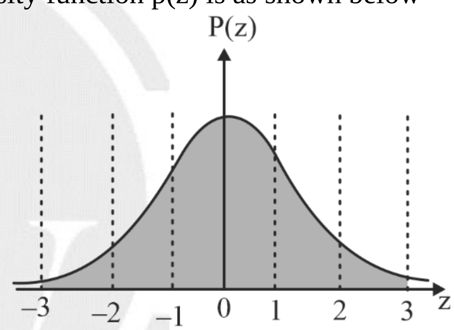
107. (99.6 to 99.8)

If 'z' is standard normal variable, then the probability density function p(z) is as shown below



And

P (–1 ≤ z ≤ 1) = 0.6828 = 68.28%

P (–2 ≤ z ≤ 2) = 0.9577 = 95.77%

P (–3 ≤ z ≤ 3) = 0.9973 = 99.73%

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108. (a)

$$f(x)=\frac{3}{2}e^{-3x}u(x)+ae^{4x}u(-x)$$

and, $u(x)=\begin{cases}1 & ; \quad x\ge 0\\ 0 & ; \quad x<0\end{cases}$

$$u(-x)=\begin{cases}0 & ; \quad x>0\\ 1 & ; \quad x\le 0\end{cases}$$

Since $= \int_{-\infty}^{+\infty} f(x)dx = 1$ for f(x) to a valid probability density function

$$\int_{-\infty}^{+\infty}\left[\frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)\right]dx$$

$$= \int_{-\infty}^{0} ae^{4x} + dx + \int_{0}^{\infty}\frac{3}{2}e^{-3x}dx$$

$$= a\left[\frac{e^{4x}}{4}\right]_{-\infty}^{0} + \frac{3}{2}\left[\frac{e^{-3x}}{-3}\right]_{0}^{\infty} = 1$$

$$\frac{a}{4} + \frac{1}{2} = 1$$

a = 2

$$\text{Prob } \{x \le 0\} = \int_{-\infty}^{0} ae^{4x}dx = 2\left[\frac{e^{4x}}{4}\right]_{-\infty}^{0}$$

$$2\left[\frac{1}{4} - 0\right]_{-\infty}^{0} = \frac{1}{2}$$

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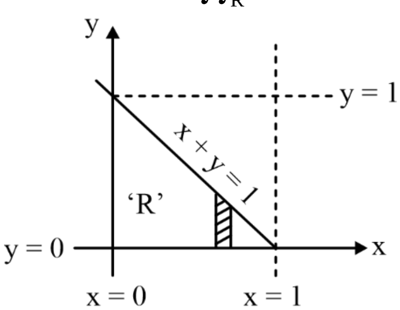
109. (0.33)

$$\text{Given } f(x, y) = \begin{cases}(x + y), & 0 \le x \le 1, 0 \le y \le 1\\ 0\ ; & \text{otherwise}\end{cases}$$

Considering the region 'R' as the region, where $x + y \le 1$

$$\therefore\ P(x + y \le 1) = \iint_R f(x, y)dy\,dx$$



$$\therefore\ P(x + y \le 1) = \iint_R f(x, y)dy\,dx$$

$$= \int_{x=0}^{x=1}\int_{y=0}^{y=1-x}(x + y)dy\,dx$$

$$= \int_{x=0}^{x=1} xy + \frac{y^2}{2}\Big|_0^{(1-x)} dx$$

$$= \int_{x=0}^{x=1}\left(x(1-x) + \frac{(1-x)^2}{2}\right)dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{(1-x)^3}{-6}\Big|_0^1 = \frac{1}{2} - \frac{1}{3} + 0 - \left(\frac{-1}{6}\right)$$

$$= \frac{1}{3} = 0.33$$

$$\boxed{\therefore\ P(x + y \le 1) = 0.33}$$

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110. (0.5 to 0.5)

Given: Probability density function,

$$P(x) = \begin{cases} 1/x^2; & a \le x \le 1\\ 0; & \text{otherwise}\end{cases}$$

For P(x) to be a valid p.d. function, following condition must be satisfied

$$\int_0^1 P(x)dx = 1 \Rightarrow \int_a^1 \frac{1}{x^2}dx = 1$$

$$\Rightarrow \frac{-1}{x}\Big|_a^1 = 1 \Rightarrow \left\{-1 + \frac{1}{a}\right\} = 1 \Rightarrow a = 0.5$$

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111. (d) For a given normal distribution,

$$\text{Mode} = \frac{1}{\sigma\cdot\sqrt{2\pi}} = \text{Max of } P(x)$$

For lowest variance, σ should be minimum.

If 'σ' is minimum, then $\frac{1}{\sigma\cdot\sqrt{2\pi}}$ is maximum.

∴ The graph with highest peak is having least variance.

∴ Graph IV has least variance.

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112. (0.23 to 0.25)

Given: $q = 0.4$

x	0	1
P(x)	0.4	0.6

$$E(x) = \sum_i x_i P(x_i) = 0 \times 0.4 + 1 \times 0.6 = 0.6$$

$$E(x^2) = \sum x_i^2 P(x_i) = 0^2 \times 0.4 + 1^2 \times 0.6 = 0.6$$

$\therefore$ $\text{Var}(x) = E(x^2) - [E(x)]^2$

$= 0.6 - (0.6)^2 = 0.6 - 0.36$

$\text{Var}(x) = 0.24$

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113. (1.06 to 1.07)

Given:

$$f(x) = \begin{cases} \dfrac{x}{4}(4 - x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Since it is continuous random variable,

$$\therefore \ \mu_x = \int_0^2 x f(x)\,dx = \int_0^2 x \cdot \frac{x}{4}(4 - x^2)\,dx$$

$$= \int_0^2 \left(x^2 - \frac{x^4}{4}\right)dx = \left[\frac{x^3}{3} - \frac{x^5}{20}\right]_0^2$$

$$= \frac{8}{3} - \frac{32}{20} = \frac{16}{15}$$

$\Rightarrow \mu_x = 1.066$

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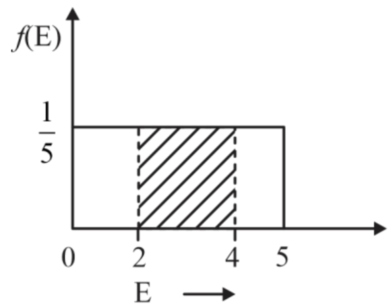
114. (0.4 to 0.4)

Given:

$$f(E) = \begin{cases} 1/5 & 0 < E < 5 \text{ mm/day} \\ 0 & \text{otherwise} \end{cases}$$

Required probability

$$P(2 < E < 4) = \int_2^4 f(E)\,dE = \int_2^4 \frac{1}{5}\,dE$$



$$= \frac{1}{5}[E]_2^4 = \frac{1}{5}(4 - 2) = \frac{2}{5} = 0.4$$

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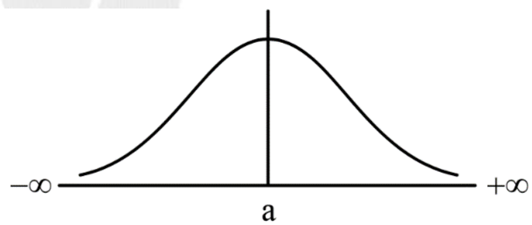
115. (b)

Given: The function $f(x)$ is normal distributed curve with mean as 'a' and standard deviation as 'b'



$$\therefore \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}\, b} e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx = 0.5$$

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116. (0.26 to 0.27)

Given: Mean $(\lambda) = 5$

For Poisson distribution

$$P(x = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

Required probability

$P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$$= \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!}$$

$$= e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6}\right] = e^{-5}\left(\frac{118}{3}\right)$$

$$= 0.265$$

$\therefore$ P (x < 4) = 0.265

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117. (25)

Given: Probability of earth quake in an year is

P(E) = 0.04

Average time interval $T = \frac{1}{P} = \frac{1}{0.04}$

$\Rightarrow$ $\boxed{T = 25 \text{ years}}$

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118.(0.4 to 0.5)

X$\rightarrow$Random variable denoting lifetime of a bulb.

$f(x) = kx^2$; $1 < x < 2$

For function to be a valid probability density function,

$$\therefore \int_1^2 f(x)dx = 1$$

$$\Rightarrow \int_1^2 kx^2 dx = 1$$

$$\Rightarrow k\left[\frac{x^3}{3}\right]_1^2 = 1$$

$$\Rightarrow k\left(\frac{8-1}{3}\right) = 1$$

$$\Rightarrow k = \frac{3}{7}$$

$$\Rightarrow k = 0.4285$$

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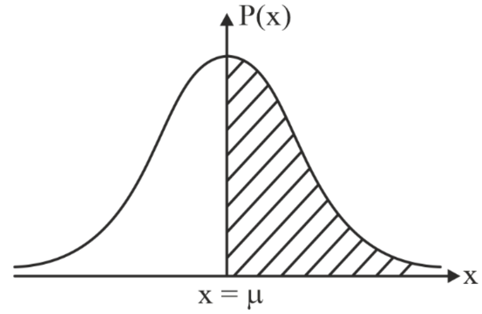
119. (49 to 51)

Given that average daily balance follows normal distribution

Let 'x' be the normal random variable.

μ = Rs. 500; σ = Rs. 50

$P(x > 500) = P(x > \mu) = 0.5 = 50\%$



$$\boxed{\therefore P\begin{Bmatrix}\text{Account having an} \\ \text{average daily balance} > 500\end{Bmatrix} = 50\%}$$

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120. (b)

Mean of Poisson distribution = 5.2

Let mean = $\lambda \Rightarrow \lambda = 5.2$

P(x < 2) = P(x = 0) + P(x = 1) (x $\rightarrow$ number of accidents)

From Poisson distribution

$$P(x = r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

$$P(x < 2) = P(x = 0) + P(x = 1) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!}$$

$$= \frac{(1+\lambda)}{e^{\lambda}} = \frac{1+5.2}{e^{5.2}} = 0.0342$$

$\therefore$ P (x < 2) = 0.034

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121. (d)

Given

x	1	2	3
p(x)	0.3	0.6	0.1

Standard deviation = $\sigma^2 = E(x^2) - (E(x))^2$

$$\Rightarrow \sigma^2 = \left(\sum_{i=1}^{n} x_i^2 p(x = x_i)\right) - \left(\sum_{i=1}^{n} x_i \cdot p(x = x_i)\right)^2$$

$$\Rightarrow \sigma^2 = (1^2(0.3) + 2^2(0.6) + 3^2(0.1)) - (1(0.3) + 2(0.6) + 3(0.1))^2$$

$$\Rightarrow = 3.6 - (1.8)^2 = 0.36$$

$$\Rightarrow \boxed{\sigma = \text{Standard deviation} = 0.6}$$

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122. (a)

Let 'x' be the Number of defective pieces in a day

x	0	1	2
P(x)	1/6	2/3	1/6

$$\text{Mean} = \sum_{i=1}^{n} x_i \cdot P(x = x_i)$$

$$= 0\left(\frac{1}{6}\right) + 1\left(\frac{2}{3}\right) + 2\left(\frac{1}{6}\right) = \frac{2}{3} + \frac{2}{6} = 1$$

$\therefore$ Mean = μ = 1

$$\text{Variance} = \sum_{i=1}^{n} x_i^2 \cdot P(x = x_i) - \mu^2$$

$$\Rightarrow \sigma^2 = \left(0^2\left(\frac{1}{6}\right) + 1^2 \cdot \left(\frac{2}{3}\right) + 2^2 \cdot \left(\frac{1}{6}\right)\right) - 1^2$$

$$\Rightarrow = \left(\frac{2}{3} + \frac{4}{6}\right) - 1 = \frac{1}{3}$$

$$\Rightarrow \sigma^2 = \text{Variance} = \frac{1}{3}$$

$$\therefore \boxed{\text{Mean} = \mu = 1 \text{ and Variance} = \sigma^2 = \frac{1}{3}}$$

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123. (0.32 to 0.34)

Given X_1, X_2 and X_3 be independent and identically distributed random valued variables

$P\{X_1 \text{ is largest}\} = \frac{1}{3}$ (X_1 being largest of X_1, X_2, X_3) = 0.33

$\therefore P\{X_1$ is largest$\}$ = 0.33

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124. (0.79 to 0.81)

Given x be a zero mean unit variance gaussian random variable

$\Rightarrow$ x ~ N(0, 1)

$\therefore$ Probability density function of x is

$$P(x) = \frac{1}{\sqrt{2\pi}} . e^{-\frac{-x^2}{2}}$$

$\therefore$ Expectation of |x| is

$$E(|x|) = \int_{-\infty}^{\infty} |x| \cdot P(X) dx = \int_{-\infty}^{\infty} |x| . \frac{1}{\sqrt{2\pi}} . e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \times \left\{ 2 \times \int_{0}^{\infty} x \cdot e^{-\frac{x^2}{2}} dx \right\}$$

Let $\frac{x^2}{2} = t \Rightarrow 2x dx = dt(2) \Rightarrow x.dx = dt$

Lower limit: $0^2 = t \Rightarrow t = 0$

Upper limit: $t \to \infty$

$$\therefore E|x| = \frac{2}{\sqrt{2\pi}} \times \int_{0}^{\infty} e^{-t} . dt = \sqrt{\frac{2}{\pi}} . \left(\frac{e^{-t}}{-1}\right)\Bigg|_{0}^{\infty}$$

$$= \sqrt{\frac{2}{\pi}} . \{0 + 1\} = 0.79788 \quad \boxed{\therefore E|x| \approx 0.8}$$

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125. (b)

For a real valued random variable x,

Variance = $\sigma^2 = E(x^2) - (E(x))^2 \geq 0$

$\boxed{E(x^2) \geq (E(x))^2}$

Note: If $\sigma = 0 \Rightarrow$ All the data points in a sample should be identical.

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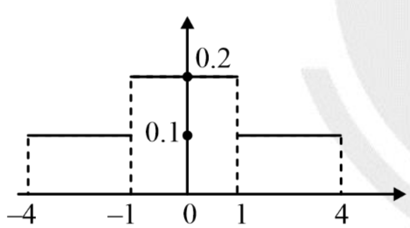
126. (0.35 to 0.45)

$$f(x) = \begin{cases} 0.2 & ; \quad -1 \leq x \leq 1 \\ 0.1 & ; \quad (-4 \leq x \leq -1) \cup (1 < x \leq 4) \\ 0 & ; \quad (x < -4) \cup (x > 4) \end{cases}$$



$$P(0.5 < x < 5) = \int_{0.5}^{5} f(x)$$

$$= \int_{0.5}^{1} (0.2)dx + \int_{1}^{4} (0.1)dx + \int_{4}^{5} (0)dx$$

$= 0.2(1 - 0.5) + 0.1\,(4 - 1) + 0$

$= 0.2 \times 0.5 + 0.1 \times 3 = 0.1 + 0.3 = 0.4$

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127. (0.24 to 0.27)

Given:

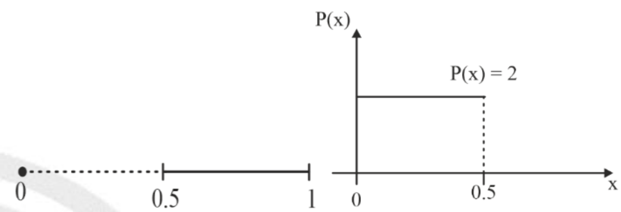
Length of stick = 1 unit

For the broken stick to be shorter,

0 < length of broken part < 0.5.

Let 'x' be the length of shorter part,

$\Rightarrow 0 < x < 0.5$



$$E(x) = \text{average of } (0, 0.5) = \frac{0 + 0.5}{2} = 0.25$$

$\therefore$ Expected length of shorter part is 0.25.

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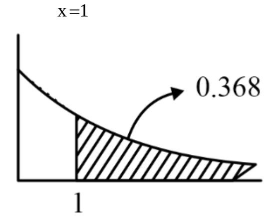
128.(a)

$$f(x) = \begin{cases} e^{-x} & ; \quad x > 0 \\ 0 & ; \quad x < 0 \end{cases}$$

$$\therefore\ P(x > 1) = \int_{x=1}^{x=\infty} f(x) \cdot dx$$



$$= \int_{1}^{\infty} e^{-x} dx \;\; - \left[e^{-x}\right]_1^{\infty} = -\left[e^{-\infty} - e^{-1}\right]$$

$$= -\left[0 - \frac{1}{e}\right]$$

$$= \frac{1}{e} = 0.368$$

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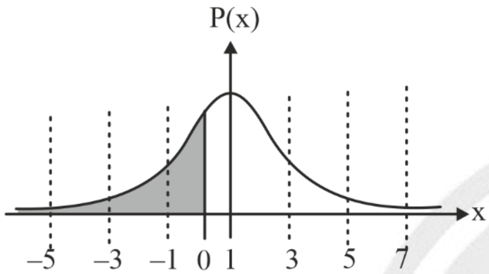
129.(b)

Given:

'X' is a normal random variable with mean 1 and variance 4 $\Rightarrow$ X is N(1, 4)

$\Rightarrow$ X = 0 lies the left of the mean X=1,

$\therefore$ P(X) for the given variable is as shown below



$\Rightarrow$ P (X < 0) = 0.5 – P (0 < X < 1)

$\Rightarrow$ 0 < P (X < 0) < 0.5

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130. (d)

Given U and V are identically distributed.

Let 2V = W

$\Rightarrow$ W = 2U.

Cumulative function of 'U' is $p(U \le x)$

Cumulative function of'W' is $p(W \le x)$

$$\Rightarrow (2U \le x)$$

$$= p(U \le \frac{x}{2})$$

$\therefore$ $F(x) = p(U \le x)$; $G(x) = p\left(U \le \dfrac{x}{2}\right)$

So, when x > 0; $F(x) \ge G(x) \Rightarrow (F(x) - G(x)) \ge 0$

When x < 0; $F(x) < G(x) \Rightarrow (F(x) - G(x)) < 0$

$\therefore$ In both cases for all x,

$$\boxed{(F(x) - G(x))x \ge 0}$$

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131. (b)

Given

μ_u = Mean of u = 0

μ_v = mean of v = 0

Both the variables are independent

$\sigma_u^2 = \dfrac{1}{4}$ and $\sigma_v^2 = \dfrac{1}{9}$

$p(3V \ge 2U) = p(3V - 2U) > 0$

Since $U \sim N\left(0, \dfrac{1}{4}\right)$ and $V \sim N\left(0, \dfrac{1}{9}\right)$

$3V - 2U \sim N\left(3.\mu_U - 2.\mu_V, 9\sigma_U^2 + 4\sigma_V^2\right)$

$$\sim N\left(0, 9\left(\frac{1}{4}\right) + 4\left(\frac{1}{9}\right)\right)$$

$$\sim N\left(0, \frac{9}{4} + \frac{4}{9}\right) \sim N\left(0, \frac{97}{36}\right).$$

$\therefore$ Mean of 3V – 2U is also '0' with normal distribution

$\therefore p(3V - 2U > 0) = \dfrac{p(-\infty < 3V - 2U < \infty)}{2} \quad \dfrac{1}{2} = 0.5$

Since mean of 3V-2U = 0; (3V – 2U) = 0 also

Acts as median $\boxed{\therefore p(3V - 2U > 0) = 0.5 = \dfrac{1}{2}}$

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132. (6 to 6)

Given:

$$f(x) = \begin{cases} \lambda(x-1)(2-x) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Condition for being valid p.d. function is,

$$\boxed{\int_{-\infty}^{+\infty} f(x)\,dx = 1}$$

$$\Rightarrow f(x) = \begin{cases} \lambda\left(-x^2+3x-2\right) & \text{for } 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \int_1^2 f(x)\,dx = 1$$

$$\Rightarrow \int_1^2 \lambda\left(-x^2+3x-2\right) = 1$$

$$\Rightarrow \lambda\left[-\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_1^2 = 1$$

$$\Rightarrow \lambda\left[-\frac{\left(2^3-1^3\right)}{3}+3\frac{\left(2^2-1^2\right)}{2}-2(2-1)\right] = 1$$

$$\Rightarrow \lambda\left[-\frac{7}{3}+\frac{9}{2}-2\right] = 1 \quad \Rightarrow \boxed{\lambda = 6}$$

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133. (c)

Given:

Average number of cars per minute passing is λ

'P' is following Poisson distribution with mean = λ = 3

P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)

$$= \frac{e^{-\lambda}\cdot\lambda^0}{0!}+\frac{e^{-\lambda}\cdot\lambda^1}{1!}+\frac{e^{-\lambda}\cdot\lambda^2}{2!}$$

$$\because P(x=r) = \frac{e^{-\lambda}\cdot\lambda^r}{r!}$$

$$= e^{-3}\left\{\frac{1}{1}+\frac{3}{1}+\frac{9}{2}\right\} = \frac{17}{2\cdot e^3}$$

$\therefore$ P (less than 3 cars) = $\mathbf{\frac{17}{2.e^3}}$

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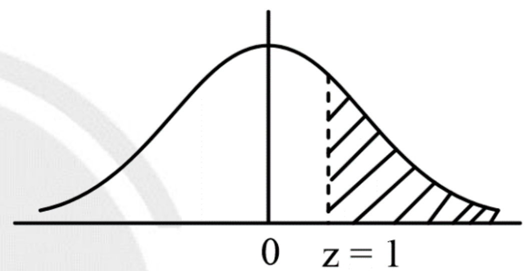
134. (a)

Given: The annual precipitation is normally distributed with μ = 1000 mm, σ = 200 mm

$$P(x > 1200\,\text{mm}) = P\left(z > \frac{x-\mu}{\sigma}\right)$$

$$= P\left(z > \frac{1200-1000}{200}\right) = P(z > 1)$$

Where z is the standard normal variate.



$\because$ P (–1 < z < 1) = 0.68

$$\Rightarrow P(0 < z < 1) = \frac{0.68}{2} = 0.34$$

So, P (z > 1) = 0.5 – 0.34 = 0.16

≈ 16% < 50%

Hence option (a) is correct

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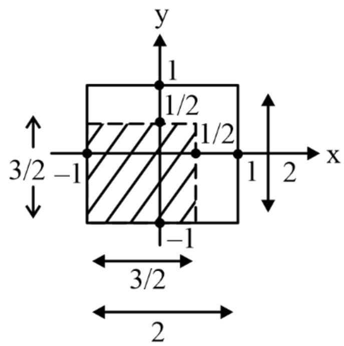
135. (b)

$1 \le x, y \le +1$

Probability of max(x, y) < $\frac{1}{2}$



$$= P\left(-1 < x, y < \frac{1}{2}\right)$$

$$= \frac{\text{Area of shaded region}}{\text{Total area}} = \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times 2} = \frac{9}{16}$$

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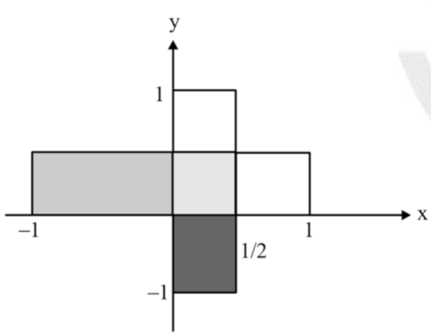
136. (b)

Given: x, y are independent random variables that are uniformly distributed in [–1, 1]

The probability distributions of both x and y is as shown



$$P(\text{Max }(x, y) < 1/2) = P\left(x < \frac{1}{2} \cap y < \frac{1}{2}\right)$$

$$= P\left(x < \frac{1}{2}\right).P\left(y < \frac{1}{2}\right)$$

$$= \frac{3}{4}.\frac{3}{4} = \frac{9}{16}$$

$$\boxed{\therefore P\left(\max(x, y) < \frac{1}{2}\right) = 9/16}$$

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137. (c)

Given: 'X' is a Random variable that can take values only +1 and -1 with equal probabilities. So the probability distribution of x is as below.

x	-1	+1
P(x)	0.5	0.5

Cumulative distribution function is defined as

$F(x) = P\ (X \le x)$

$\therefore$ $F\ (–1) = P\ (x \le –1) = P\ (x = –1) = 0.5$

$\therefore$ $F\ (+1) = P\ (x \le 1) = P\ (x = –1) + P\ (x = 1)$

$= 0.5 + 0.5 = 1$

$\therefore$ $F\ (–1) = 0.5$ and $F\ (+1) = 1$

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138. (b)

Given: If the value on the die is 1, 2, or 3, the die is rolled a second time i.e., if (4 or 5 or 6) appear than dice is not rolled out second time.

$P\ (\text{sum} \ge 6) = P\ (\text{sum} \ge 6)_{1\text{st roll}} + P\ (\text{sum} \ge 6)_{2^{nd}\text{ roll}}$

$\therefore$ $P\ (\text{sum} \ge 6)_{1\text{st roll}} = P\ (6) = \dfrac{1}{6}$

$P\ (\text{sum} \ge 6)_{2^{nd}}$

$= P\ (1 \cap \ge 5) + P\ (2 \cap \ge 4) + P\ (3 \cap \ge 3)$

$= P\ (1).\ P\ (\ge 5) + P\ (2).\ P\ (\ge 4) + P\ (3).\ P\ (\ge 3)$

$$= \frac{1}{6}\cdot\left(\frac{2}{6}\right) + \left(\frac{1}{6}\right)\cdot\left(\frac{3}{6}\right) + \left(\frac{1}{6}\right)\cdot\left(\frac{4}{6}\right)$$

$$= \frac{1}{36}(2 + 3 + 4) = \frac{1}{4}$$

$\therefore \quad P(\text{sum} \geq 6) = \frac{1}{6} + \frac{1}{4} = \mathbf{\frac{5}{12}}$

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139. (d)

- If $x_1, x_2, x_3, \ldots, x_n$ are 'n' data points and $y_i = a x_i + b$,

The index position of median and mode will be same as initial positions since all the points are modified uniformly.

$E(y) = E(a x_i + b) = a E(x_i) + b$

$\Rightarrow E(y) = a E(x) + b \qquad \Rightarrow \mu_y = a \mu_x + b$

But $\text{Var}(ax + b) = a^2 \text{Var}(x) \qquad \Rightarrow \sigma_y = a \cdot \sigma_x$

$\therefore$ The statements $\sigma_y = a \cdot \sigma_x + b$ is incorrect.

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140. (c)

For a random variable 'x'

Variance $\sigma^2 = E(x^2) - (E(x))^2 = R \geq 0$

$\therefore \quad R \geq 0$

Note:R = 0 if the data is ideal (i.e. all the data points are same).

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141. (a)

Given:

A deck contains 5 cards.

For 1st number to be more than the 2nd number by one, the possibilities are (5, 4), (4, 3), (3, 2), (2, 1)

Required probability

$= P(5). P(4) + P(4). P(3) + P(3).P(2)+P(2). P(1)$

$= \left(\frac{1}{5} \times \frac{1}{4}\right) \times 4 = \frac{4}{20} = \frac{1}{5}$

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142. (a)

Given:

Probability of a computer being faulty = P(E) = p

Probability that test gives correct result = P(C) = q

A computer is declared faculty in two cases

(1) Faulty and test gave correct result

(2) Not faulty but test gave incorrect result.

$P(\text{Declaring faulty}) = P(F \cap C) + P(F^c \cap C^c)$

$= P(E) \cdot P(C) + P(F^c) \cdot P(C^c)$

$= p \cdot q + (1 - p)(1 - q)$

$\therefore P(\text{Declaring faulty}) = pq + (1 - p)(1 - q)$

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143. (c)

Given: x, y are independent variables

x or y	0	1	2
P	1/2	1/4	1/4

$$P(x + y = 2 \mid x - y = 0) = \frac{P(x + y = 2 \cap x - y = 0)}{P(x - y = 0)}$$

$$P(x + y = 2 \cap x - y = 0) = P(x = 1 \cap y = 1)$$

$$= P(x = 1). P(y = 1) = \frac{1}{4} . \frac{1}{4} = \mathbf{\frac{1}{16}}$$

$P(x - y = 0) = P(x = y) = P(x = y = 0) + P(x = y = 1) + P(x = y = 2)$

$$= P(x = 0)\,P(y = 0) + P(x = 1)\,P(y = 1) + P(x = 2)\,P(y = 2)$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \mathbf{\frac{3}{8}}$$

$$\therefore\ P(x + y = 2 \mid x - y = 0) = \frac{1/16}{3/8} = \frac{1}{6}$$

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144. (b)

Given: 'x' is a random variable

k	1	2	3	4	5
P(x=k)	0.1	0.2	0.4	0.2	0.1

Mean of x = $E(x) = \sum_{k=1}^{5} k.P(x = k)$

$= 1(0.1) + 2(0.2) + 3(0.4) + 4(0.2) + 5(0.1)$

$= 0.1 + 0.4 + 1.2 + 0.8 + 0.5 = \mathbf{3}$

Variance of x = $E(x^2) - (E(x))^2$

$= \{1^2(0.1) + 2^2(0.2) + 3^2(0.4) + 4^2(0.2) + 5^2(0.1)\} - 3^2$

$= (0.1 + 0.8 + 3.6 + 3.2 + 2.5) - 9 = \mathbf{1.2}$

So, both teacher and student are wrong.

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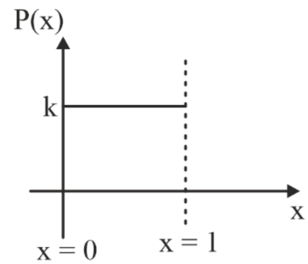
145.(a)

Given: A random variable is uniformly distributed.

Let probability distribution function P(x) = k.



Since total probability = 1

$$\Rightarrow \int_0^1 P(x)\,dx = \int_0^1 k\,dx = 1 \qquad \Rightarrow \quad P(x) = k = 1$$

$$\text{Variance } \sigma^2 = \left(\int_0^1 x^2 \cdot P(x)dx\right) - \left(\int_0^1 x \cdot P(x)dx\right)^2$$

$$= \int_0^1 x^2 dx - \left(\int_0^1 x dx\right)^2 = \left.\frac{x^3}{3}\right|_0^1 - \left(\left.\frac{x^2}{2}\right|_0^1\right)^2$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\therefore \boxed{\text{Standard deviation } = \sigma = \frac{1}{\sqrt{12}}}$$

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146. (d)

Given: $F(X_N) = \dfrac{1}{1 + \exp\left(-1.7255 X_N |X_N|^{0.12}\right)}$

μ (Mean) = 102 cm, Standard deviation (σ) = 27 cm

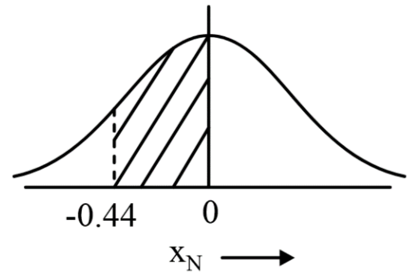
P (90 cm $\le$ X $\le$ 102 cm) =?

P (90 cm $\le$ X $\le$ 102 cm)

$$= P\left(\frac{90 - \mu}{\sigma} \le X_N \le \frac{102 - \mu}{\sigma}\right)$$

$$= P\left(\frac{90 - 102}{27} \le X_N \le \frac{102 - 102}{27}\right)$$

$$= P(-0.44 \le X_N \le 0) = F(0) - F(-0.44)$$



=

$$\frac{1}{1+\exp(0)}-\frac{1}{1+\exp\left(-1.7255\times(-0.44)\left|-0.44\right|^{0.12}\right)}$$

$$=\frac{1}{2}-\frac{1}{1+\exp(0.688)}=\frac{1}{2}-\frac{1}{2.9897}=0.1655$$

= **16.55%** (Approximate value)

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147. (b)

Given:

P(odd) = 0.9 P (even)

$\because$ P(odd) + P(even) = 1

0.9 P (even) + P(even) = 1

$\Rightarrow$ P (even) = $\frac{1}{1.9}$

Since probability of getting any even number is same.

$$P(2)=\frac{1}{3}\times\frac{1}{1.9};\ P(4)=\frac{1}{3}\times\frac{1}{1.9};\ P(6)=\frac{1}{3}\times\frac{1}{1.9}$$

P (Even | >3) = 0.75 $\Rightarrow \frac{P(\text{even}\cap>3)}{P(>3)}=0.75$

$$\Rightarrow \frac{P(4)+P(6)}{P(>3)}=0.75 \Rightarrow P(>3)=\frac{P(4)+P(6)}{P(>3)}$$

$$=\frac{2\left(\frac{1}{3\times1.9}\right)}{0.75}=0.46783\approx0.468$$

$\therefore$ Required probability = 0.468

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148. (b)

Given: $f(x)=\begin{bmatrix} x^2 & -1\le x\le 1 \\ 0 & \text{other wise}\end{bmatrix}$

$$\because\ P\left(-\frac{1}{3}\le x\le\frac{1}{3}\right)=\int_{-1/3}^{+1/3} f(x)\,dx=\int_{-1/3}^{+1/3} x^2\,dx$$

$$=2\int_0^{1/3}x^2dx \qquad [\because f(x)=f(-x)]$$

$$=2\left[\frac{x^3}{3}\right]_0^{1/3}=2\left(\frac{1}{81}\right)=\frac{2}{81}$$

$$\therefore \boxed{P\left(-\frac{1}{3}\le x\le\frac{1}{3}\right)=\frac{2}{81}\times100=2.47\ \%}$$

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149. (a)

Given:

$$P=\sum_{\substack{1\le i\le 2k \\ i\text{ odd}}} i = 1+3+5+\ldots\ldots+(2k-1)$$

$$Q=\sum_{\substack{1\le i\le 2k \\ i\text{ even}}} i = 2+4+6+\ldots\ldots+2k$$

$\therefore$ P = sum of 1st ‘k’ odd number = k^2

Q = sum of 1st ‘k’ even numbers

= 2 + 4 + 6 + ….+2k = 2(1 + 2 + 3 . ……+k)

$$=2\cdot\left(\frac{k(k+1)}{2}\right)=k^2+k$$

$\therefore P = k^2$; and $Q = k^2 + k$

$\Rightarrow \quad Q = P + k \Rightarrow P = Q - k$

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150. (a)

Given:

X is normal variable with Mean = +1 and variance = 4

Considering 'z' the standard normal variable as

$z = \dfrac{X - \mu_X}{\sigma_X} \quad \Rightarrow z = \dfrac{X-1}{2}$ i.e At X = –1; z = –1

Y is normal variable with Mean = –1 & variance = σ_Y

$\Rightarrow \; z = \dfrac{Y - \mu_Y}{\sigma_Y} = \dfrac{Y+1}{\sigma_Y}$ i.e. At Y = 2; $z = 3/\sigma_Y$)

$\because \; P(X \le -1) = P(Y \ge 2) \;\Rightarrow\; P(z < -1) = P\left(z \ge \dfrac{3}{\sigma_Y}\right)$

$\Rightarrow \; \left|\dfrac{3}{\sigma_Y}\right| = 1 \qquad \Rightarrow \sigma_Y = 3$

$\therefore$ standard deviation of Y = 3

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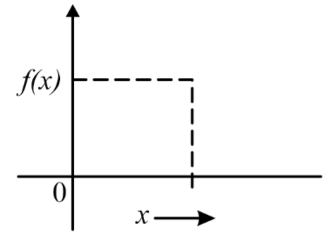
151. (c)

Since X is uniformly distributed between [0, 1]



$$f(X) = \begin{cases} \dfrac{1}{1-0} & ; \quad 0 < X < 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$\therefore$ f(X) = 1 ; 0 < X < 1 Now $E(X^3) = \int_0^1 X^3 f(x)dx$

$= \left[\dfrac{X^4}{4}\right]_0^1 \quad \therefore E(X^3) = \dfrac{1}{4}$

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152. (a)

Given: Probability density function,

$$P_x(x) = Me^{-2|x|} + N.e^{-3|x|}$$

Since Total probability = 1

$\Rightarrow \quad \int_{-\infty}^{\infty} P_X(x)dx = 1$

$\Rightarrow \quad \int_{-\infty}^{\infty} \left(Me^{-2|x|} + Ne^{-3|x|}\right)dx = 1$

$\Rightarrow \quad 2 \times \int_0^{\infty} \left(Me^{-2|x|} + Ne^{-3|x|}\right)dx = 1$ (Since |x| is even function)

$\Rightarrow \quad 2 \times \int_0^{\infty} \left(Me^{-2x} + Ne^{-3x}\right)dx = 1$

$\Rightarrow \quad 2 \times \left(\dfrac{Me^{-2x}}{-2} + \dfrac{N.e^{-3x}}{-3}\Bigg|_0^{\infty}\right) = 1$

$\Rightarrow \quad 2\left(-\dfrac{M}{-2} - \dfrac{N}{-3}\right) = 1 \quad \Rightarrow \quad \boxed{M + \dfrac{2N}{3} = 1}$

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153. (1.9 to 1.9)

Given:

x	5	2	4	3
y	16	10	13	12

x	y	x^2	XY
5	**16**	25	80
2	**10**	4	20
4	**13**	16	52
3	**12**	9	36

The normal equation for fitting of straight lines is given by,

$\Sigma y = an + b\Sigma x$ and $\Sigma xy = a\Sigma x + b\,\Sigma x^2$

Have, $n = 4$, $\Sigma x = 14$, $\Sigma y = 51$, $\Sigma x^2 = 54$, $\Sigma xy = 188$

$\Rightarrow \quad 51 = 4a + 14b \quad$ …..(i)

$\Rightarrow \quad 188 = 14a + 54b \quad$ …..(ii)

After solving (i) and (ii)

We get $a = 6.1$, $b =$ **1.9**

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154. (d)

Given: X, Y are random variables.

$$\Rightarrow \frac{\left[E\left(X-E(X)\right)\cdot\left(Y-E(Y)\right)\right]^2}{\sigma_X^2\cdot\sigma_Y^2}\le 1$$

$$\Rightarrow \left[E\left(X-E(X)\right)\cdot\left(Y-E(Y)\right)\right]^2\le\sigma_X^2\cdot\sigma_Y^2$$

$$\therefore \left[E\left(X-E(X)\right)\cdot\left(Y-E(Y)\right)\right]^2\le \mathrm{Var}(X)\cdot\mathrm{Var}(Y)$$

$\therefore$ Statement S_1 is false.

$\mathrm{Cov}[X,Y] = E\left(X-E(X)\right)\cdot\left(Y-E(Y)\right)$

Only if $X > E(X)$ and $Y > E(Y)$

Or $X < E(X)$ and $Y < E(Y)$

So it is not valid for any other relation between X and E(X).

$\therefore$ Statement-2 (for all X and Y) is also False.

$\Rightarrow$ Both S_1 and S_2 are False.

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155. (d)

Given: $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$, $\Sigma xy = 46$, $n = 3$

$y = a + bx$

$\Rightarrow \quad \Sigma y = a\,n + b\,\Sigma x$

$\Rightarrow \quad 21 = 3a + 6b \quad$ ……. (i)

Now, $\quad xy = a\,x + b\,x^2$

$\Rightarrow \quad \Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

$\Rightarrow \quad 46 = 6a + 14b$

$23 = 3a + 7b \quad$ ……. (ii)

Subtracting (i) from (ii) we get,

$\boxed{b = 2 \text{ and } a = 3}$

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❑❑❑

CHAPTER

4

DIFFERENTIAL EQUATIONS

Order and Degree of Differential Equations

1. **[MCQ]** **[GATE-CE-2022: 1M]**

For the equation $\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2y = 0$ the correct descripting is

(a) An ordinary differential equation of order 3 and degree 2.
(b) An ordinary differential equation of order 1 and degree 3.
(c) An ordinary differential equation of order 2 and degree 3.
(d) An ordinary differential equation of order 1 and degree 3/2

2. **[MCQ]** **[GATE-CE-2020:1M]**

The ordinary differential equation

$\frac{d^2u}{dx^2} - 2x^2u + \sin x = 0$ is

(a) Linear and homogeneous
(b) Nonlinear and homogeneous
(c) Nonlinear and nonhomogeneous
(d) Linear and nonhomogeneous

3. **[MCQ]** **[GATE-EC-2014:1M]**

Which ONE of the following is a linear non-homogeneous differential equation, where x and y are the independent and dependent variables respectively?

(a) $\frac{dy}{dx} + xy = e^{-x}$
(b) $\frac{dy}{dx} + xy = 0$
(c) $\frac{dy}{dx} + xy = e^{-y}$
(d) $\frac{dy}{dx} + e^{-y} = 0$

4. **[MCQ]** **[GATE-ME-2013: 1M]**

The partial differential equation $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$ is a

(a) Linear equation of order 2
(b) Non-linear equation of order 1
(c) Linear equation of order 1
(d) Non-linear equation of order 2

5. **[MCQ]** **[GATE-CE-2010:1M]**

The order and degree of the differential equation $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ are respectively

(a) 3 and 2
(b) 2 and 3
(c) 3 and 3
(d) 3 and 1

6. **[MCQ]** **[GATE-ME-2010: 1M]**

The Blasius equation, $\frac{d^3f}{d\eta^3} + \frac{f}{2}\frac{d^2f}{d\eta^2} = 0$, is a

(a) Second order nonlinear ordinary differential equation.
(b) Third order nonlinear ordinary differential equation.
(c) Third order linear ordinary differential equation.
(d) Mixed order nonlinear ordinary differential equation.

7. **[MCQ]** **[GATE-EC-2009:1M]**

The oder of the differential equation

$$\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^3 + y^4 == e^{-1} \text{ is}$$

(a) 1
(b) 2
(c) 3
(d) 4

First Order Differential Equations

8. [NAT] [GATE-CE-2023: 1M]

In the differential equation $\frac{dy}{dx} + \alpha x y = 0$, α is a positive constant. If y = 1.0 at x = 0.0, and y = 0.8 at x = 1.0, the value of α is _______________(rounded off to three decimal places).

10. [MCQ] [GATE-ME-2022: 2M]

For the exact differential equation

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2 u},$$

which one of the following is the solution?

(a) $u^2 + 2x^2$ = constant

(b) $xu^2 + u$ = constant

(c) $\frac{1}{2}x^2u^2 + 2u$ = constant

(d) $\frac{1}{2}ux^2 + 2x$ = constant

11. [MCQ] [GATE-ME-2021: 1M]

If $y(x)$ satisfies the differential equation

$$(\sin x)\frac{dy}{dx} + y\cos x = 1,$$

subject to be condition $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $y\left(\frac{\pi}{6}\right)$ is

(a) 0
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$

12. [MCQ] [GATE-ME-2021: 2M]

Consider the following differential equation

$$(1 + y)\frac{dy}{dx} = y$$

The solution of the equation that satisfies the condition y (1) = 1 is

(a) $2ye^y = e^x + e$
(b) $y^2e^y = e^x$
(c) $ye^y = e^x$
(d) $(1 + y)\, e^y = 2e^x$

13. [MCQ] [GATE-CE-2021:2M]

If k is a constant, the general solution of $\frac{dy}{dx} - \frac{y}{x} = 1$ will be in the form of

(a) y = xln(x)
(b) y = xkln (k)
(c) y = k ln (kx)
(d) y = x ln (kx)

14. [MCQ] [GATE-EC-2021:1M]

Consider the differential equation given below.

$$\frac{dy}{dx} + \frac{x}{1 - x^2}y = x\sqrt{y}$$

The integrating factor of the differential equation is

(a) $\left(1 - x^2\right)^{-3/4}$
(b) $\left(1 - x^2\right)^{-1/4}$
(c) $\left(1 - x^2\right)^{-3/2}$
(d) $\left(1 - x^2\right)^{-1/2}$

15. [NAT] [GATE-EE-2020 : 1M]

Consider the initial value problem below.

$$\frac{dy}{dx} = 2x - y,\ y(0) = 1$$

The value of y at x = ln 2, (rounded off to 3 decimal places) is_____________.

16. [MCQ] [GATE-ME-2019: 1M]

For the equation $\frac{dy}{dx} + 7x^2y = 0$, if $y(0) = \frac{3}{7}$, then the value of y (1) is

(a) $\frac{7}{3}e^{-7/3}$
(b) $\frac{7}{3}e^{-3/7}$
(c) $\frac{3}{7}e^{-7/3}$
(d) $\frac{3}{7}e^{-3/7}$

17. [MCQ] [GATE-ME-2019: 1M]

The differential equation $\frac{dy}{dx} + 4y = 5$ is valid in the domain $0 \le x \le 1$ with y (0) = 2.25. The solution of the differential equation is

(a) $y = e^{-4x} + 5$
(b) $y = e^{-4x} + 1.25$
(c) $y = e^{4x} + 5$
(d) $y = e^{4x} + 1.25$

18. [MCQ] [GATE-CE-2019:2M]

An ordinary differential equation is given below.

$$\left(\frac{dy}{dx}\right)(x\ln x) = y$$

The solution for the above equation is

Note: K denotes a constant in the options

(a) $y = Kx\,e^x$
(b) $y = Kx\,e^{-x}$
(c) y = K ln x
(d) Y = Kx ln x

19. [MCQ] [GATE-CE-2018:1M]

The solution of the equation $x\dfrac{dy}{dx} + y = 0$ passing through the point (1, 1) is

(a) x (b) x^2
(c) x^{-1} (d) x^{-2}

20. [MCQ] [GATE-ME-2018: 1M]

If *y* is the solution of the differential equation

$y^3\dfrac{dy}{dx} + x^3 = 0$, $y(0) = 1$, the value of $y(-1)$ is

(a) –2 (b) –1
(c) 0 (d) 1

21. [MCQ] [GATE-EC-2018:2M]

A curve passes through the point (x =1, y = 0) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}.$$

The equation that described the curve is

(a) $\ln\left(1 + \dfrac{y^2}{x^2}\right) = x - 1$

(b) $\dfrac{1}{2}\ln\left(1 + \dfrac{y^2}{x^2}\right) = x - 1$

(c) $\ln\left(1 + \dfrac{y}{x}\right) = x - 1$

(d) $\dfrac{1}{2}\ln\left(1 + \dfrac{y}{x}\right) = x - 1$

22. [MCQ] [GATE-EC-2017: 2M]

Which one of the following is the general solution of the first order differential equation

$\dfrac{dy}{dx} = (x + y - 1)^2$, where x, y are real?

(a) $y = 1 + x + \tan^{-1}(x + c)$, where c is a constant
(b) $y = 1 + x + \tan(x + c)$, where c is a constant
(c) $y = 1 - x + \tan^{-1}(x + c)$, where c is a constant
(d) $y = 1 - x + \tan(x + c)$, where c is a constant

23. [MCQ] [GATE-CE-2017:2M]

The solution of the equation $\dfrac{dQ}{dt} + Q = 1$ with Q = 0 at t = 0 is

(a) $Q(t) = e^{-t} - 1$ (b) $Q(t) = 1 + e^{-t}$
(c) $Q(t) = 1 - e^{t}$ (d) $Q(t) = 1 - e^{-t}$

24. [MCQ] [GATE-EE-2017 : 2M]

Consider the differential equation

$(t^2 - 81)\dfrac{dy}{dt} + 5ty = \sin(t)$ with $y(1) = 2\pi$. There exists a unique solution for this differential equation when t belongs to the interval

(a) (–2, 2) (b) (–10, 10)
(c) (–10, 2) (d) (0, 10)

25. [MCQ] [GATE-2015-EC: 1M]

The general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1 + \cos 2y}{1 - \cos 2x}$ is

(a) tan y – cot x = c (c is a constant)
(b) tan x – cot y = c (c is a constant)
(c) tan y + cot x = c (c is a constant)
(d) tan x + cot y = c (c is a constant)

26. [MCQ] [GATE-ME-2015: 2M]

Consider the following differential equation:

$\dfrac{dy}{dt} = -5y$; initial condition: y = 2 at t = 0.

The value of *y* at t = 3 is

(a) $-5e^{-10}$ (b) $2e^{-10}$
(c) $2e^{-15}$ (c) $-15e^{2}$

27. [MCQ] [GATE-CE-2015:2M]

Consider the following differential equation $x(ydx + xdy)\cos\dfrac{y}{x} = y(xdy - ydx)\sin\dfrac{y}{x}$ .which of the following is the solution of the above equation (c is an arbitrary constant)?

(a) $\dfrac{x}{y}\cos\dfrac{y}{x} = c$ (b) $\dfrac{x}{y}\sin\dfrac{y}{x} = c$

(c) $xy\cos\dfrac{y}{x} = c$ (d) $xy\sin\dfrac{y}{x} = c$

28. [MCQ] [GATE-EC-2015: 2M]

Consider the differential equation $\frac{dx}{dt} = 10 - 0.2x$ with initial condition x(0) = 1. The response x(t) for t > 0 is

(a) $2 - e^{-0.2t}$ (b) $2 - e^{0.2t}$

(c) $50 - 49e^{-0.2t}$ (d) $50 - 49e^{0.2t}$

29. [NAT] [GATE-EE-2015 : 2M]

A differential equation $\frac{di}{dt} - 0.2i = 0$ is applicable over –10 < t < 10. If i(4) = 10, then i(–5) is ______.

30. [MCQ] [GATE-CE-2014:2M]

The integrating factor for differential equation $\frac{dP}{dt} + k_2 P = k_1 L_0 e^{-k_1 t}$ is

(a) $e^{-k_1 t}$ (b) $e^{-k_2 t}$

(c) $e^{k_1 t}$ (d) $e^{k_2 t}$

31. [MCQ] [GATE-ME-2014: 1M]

The solution of the initial value problem $\frac{dy}{dx} = -2xy$; $y(0) = 2$ is

(a) $1 + e^{-x^2}$ (b) $2e^{-x^2}$

(c) $1 + e^{x^2}$ (d) $2e^{x^2}$

32. [MCQ] [GATE-ME-2014: 2M]

The general solution of the differential equation $\frac{dy}{dx} = \cos(x + y)$, with *c* as a constant, is

(a) y + sin (x + y) = x + c

(b) $\tan\left(\frac{x+y}{2}\right) = y + c$

(c) $\cos\left(\frac{x+y}{2}\right) = x + c$

(d) $\tan\left(\frac{x+y}{2}\right) = x + c$

33. [MCQ] [GATE-EE-2012 : 1M]

With initial condition $x(1) = 0.5$, the solution of the differential equation, $t\frac{dx}{dt} + x = t$ is

(a) $x = t - \frac{1}{2}$ (b) $x = t^2 - \frac{1}{2}$

(c) $x = \frac{t^2}{2}$ (d) $x = \frac{t}{2}$

34. [MCQ] [GATE-EE-2011: 1M]

With K as a constant the possible solution for the first order differential equation $\frac{dy}{dx} = e^{-3x}$ is

(a) $-\frac{1}{3}e^{-3x} + K$ (b) $-\frac{1}{3}e^{3x} + K$

(c) $-3e^{-3x} + K$ (d) $-3e^{-x} + K$

35. [MCQ] [GATE-ME-2011: 2M]

Consider the differential equation $\frac{dy}{dx} = (1 + y^2)x$. The general solution with constant c is

(a) $y = \tan\frac{x^2}{2} + \tan c$ (b) $y = \tan^2\left(\frac{x}{2} + c\right)$

(c) $y = \tan^2\left(\frac{x}{2}\right) + c$ (d) $y = \tan\left(\frac{x^2}{2} + c\right)$

36. [MCQ] [GATE-CE-2011:2M]

The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$, with the condition that y = 1 at x =1, is

(a) $y = \frac{2}{3x^2} + \frac{x}{3}$ (b) $y = \frac{x}{2} + \frac{1}{2x}$

(c) $y = \frac{2}{3} + \frac{x}{3}$ (d) $y = \frac{2}{3x} + \frac{x^2}{3}$

37. [MCQ] [GATE-CE-2011:2M]

The solution of the ordinary differential equation $\frac{dy}{dx} + 2y = 0$ for the boundary condition, y = 5 at x = 1 is

(a) $y = e^{-2x}$ (b) $y = 2e^{-2x}$

(c) $y = 10.95\, e^{-2x}$ (d) $y = 36.95\, e^{-2x}$

38. [MCQ] [GATE-EC-2011:1M]

The solution of the differential equation $\frac{dy}{dx} = ky$, $y(0) = c$ is

(a) $x = c\,e^{-ky}$ (b) $x = k\,e^{cy}$
(c) $y = c\,e^{kx}$ (d) $y = c\,e^{-kx}$

39. [MCQ] [GATE-ME-2009: 2M]

The solution of $x\frac{dy}{dx} + y = x^4$ with the condition $y(1) = \frac{6}{5}$ is

(a) $y = \frac{x^4}{5} + \frac{1}{x}$ (b) $y = \frac{4x^4}{5} + \frac{4}{5x}$

(c) $y = \frac{x^4}{5} + 1$ (d) $y = \frac{x^5}{5} + 1$

40. [MCQ] [GATE-CE-2009:2M]

Solution of the differential equation $3y\frac{dy}{dx} + 2x = 0$ represents a family of

(a) ellipses (b) circles
(c) parabolas (d) hyperbolas

41. [MCQ] [GATE-EC- 2009:2M]

Match each differential equation in Group I to its family of solution curves from Group II

	Group I		Group II
P.	$\frac{dy}{dx} = \frac{y}{x}$	1.	Circles
Q.	$\frac{dy}{dx} = \frac{-y}{x}$	2.	Straight lines
R.	$\frac{dy}{dx} = \frac{x}{y}$	3.	Hyperbolas
S.	$\frac{dy}{dx} = \frac{-x}{y}$		

Codes:

	P	Q	R	S
(a)	2	3	3	1
(b)	1	3	2	1
(c)	2	1	3	3
(d)	3	2	1	2

42. [MCQ] [GATE-CE-2008:2M]

Solution of $\frac{dy}{dx} = -\frac{x}{y}$ at $x = 1$ $y = \sqrt{3}$ is

(a) $x - y^2 = -2$
(b) $x + y^2 = 4$
(c) $x^2 - y^2 = -2$
(d) $x^2 + y^2 = 4$

43. [MCQ] [GATE-EC-2008:1M]

Which of the following is a solution to the differential equation $\frac{dx(t)}{dt} + 3x(t) = 0$?

(a) $x(t) = 3e^{-t}$
(b) $x(t) = 2e^{-3t}$
(c) $x(t) = -\frac{3}{2}t^2$
(d) $x(t) = 3t^2$

Higher Order Equations

44. [MCQ] [GATE-CE-2023:2M]

The solution of the differential equation

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

is expressed as $y = C_1e^{2.5x} + C_2e^{\alpha x} + C_3e^{\beta x}$, where C_1, C_2, C_3, α, and β are constants, with α and β being distinct and not equal to 2.5. Which of the following options is correct for the values of α and β?

(a) 1 and 2 (b) −1 and −2
(c) 2 and 3 (d) −2 and −3

45. [NAT] [GATE- EC-2023: 2M]

The general solution of the differential equation,

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$$

In terms of arbitrary constants K_1 and K_2 is

(a) $K_1e^{(-1+\sqrt{6})x} + K_2e^{(-1-\sqrt{6})x}$

(b) $K_1e^{(-1+\sqrt{8})x} + K_2e^{(-1-\sqrt{8})x}$

(c) $K_1e^{(-2+\sqrt{6})x} + K_2e^{(-2-\sqrt{6})x}$

(d) $K_1e^{(-2+\sqrt{8})x} + K_2e^{(-2-\sqrt{8})x}$

46. [MCQ] [GATE-CE-2021:2M]

The solution of the second-order differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ with boundary condition y (0) = 1 and y (1) = 3 is

(a) $e^{-x} + (3e - 1)\, xe^{-x}$

(b) $e^{-x} + \left[3e\sin\left(\frac{\pi x}{2}\right) - 1\right]xe^{-x}$

(c) $e^{-x} - (3e - 1)\, xe^{-x}$

(d) $e^{-x} - \left[3e\sin\left(\frac{\pi x}{2}\right) - 1\right]xe^{-x}$

47. [MCQ] [GATE-CE-2020:2M]

An ordinary differential equation is given below

$$6\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

The general solution of the above equation (with constants C_1 and C_2). is

(a) $y(x) = C_1e^{-\frac{x}{3}} + C_2xe^{\frac{x}{2}}$

(b) $y(x) = C_1e^{-\frac{x}{3}} + C_2e^{\frac{x}{2}}$

(c) $y(x) = C_1xe^{-\frac{x}{3}} + C_2e^{\frac{x}{2}}$

(d) $y\ \ x = C_1e^{\frac{x}{3}} + C_2e^{-\frac{x}{2}}$

48. [MCQ] [GATE-CE-2020:2M]

For the Ordinary Differential Equation $\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 10$. with initial condition x(0)=0 and $\frac{dx}{dt}(0) = 10$

(a) $-10e^{2t} + 10e^{3t}$ (b) $5e^{2t} + 6e^{3t}$

(c) $10e^{2t} + 10e^{3t}$ (d) $-5e^{2t} + 6e^{3t}$

49. [NAT] [GATE-ME-2018: 2M]

Given the ordinary differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

with y (0) = 0 and $\frac{dy}{dx}(0)$ = 1, the value of y (1) is ______ (Correct to two decimal places).

50. [NAT] [GATE-CE-2018: 2M]

The solution (up to three decimal places) at x =1 of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ subject to boundary conditions y(0) =1 and $\frac{dy}{dx}(x = 0) = -1$ is________.

51. [NAT] [GATE-EC-2018: 2M]

The position of a particle y(t) is described by the differential equation:

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4}.$$

The initial conditions are y (0) = 1 and $\left.\frac{dy}{dt}\right|_{t=0} = 0$.

The position (accurate to two decimal places) of the particle at t = π is ______

52. [MCQ] [GATE-CE-2017:2M]

Consider the following second-order differential equation: $y'' - 4y' + 3y = 2t - 3t^2$

The particular solution of the differential equation is

(a) $-2 - 2t - t^2$ (b) –2t –t2

(c) 2t –t2 (d) $-2 - 2t - 3t^2$

53. [MCQ] [GATE-ME-2017: 1M]

The differential equation $\frac{d^2y}{dx^2} + 16y = 0$ for y(x) with the two boundary conditions $\left.\frac{dy}{dx}\right|_{x=0} = 1$ and $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1$ has

(a) No solution

(b) Exactly two solutions

(c) Exactly one solution

(d) Infinitely many solutions

54. [NAT] [GATE-ME-2017: 2M]

Consider the differential equation $3y''(x) + 27\,y(x) = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 2000$. The value of y at x = 1 is ______.

55. [NAT] [GATE-ME-2016: 2M]

If $y = f(x)$ satisfies the boundary value problem $y'' + 9y = 0$, $y(0) = 0$, $y(\pi/2) = \sqrt{2}$ then $y(\pi/4)$ is ______.

56. [MCQ] [GATE-CE-2016:2M]

The respective expressions for complimentary function and particular integral part of the solution of the differential equation $\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2$ *are*

(a) $\left[C_1 + C_2x + C_3\sin\sqrt{3x} + C_4\cos\sqrt{3x}\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(b) $\left[C_2 + C_3\sin\sqrt{3x} + C_4\cos\sqrt{3x}\right]$ and $\left[5x^4 - 12x^2 + c\right]$

(c) $\left[C_1 + C_3\sin\sqrt{3}x + C_4\cos\sqrt{3}x\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(d) $\left[C_1 + C_2x + C_3\sin\sqrt{3x} + C_4\cos\sqrt{3x}\right]$ And $\left[5x^4 - 12x^2 + c\right]$

57. [MCQ] [GATE-EC-2016: 2M]

The particular solution of the initial value problem given below is

$$\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0$$

with $y(0) = 3$ $\left.\frac{dy}{dx}\right|_{x=0} = -36$

(a) $(3-18x)e^{-6x}$ (b) $(3+25x)e^{-6x}$

(c) $(3+20x)e^{-6x}$ (d) $(3-12x)e^{-6x}$

58. [MCQ] [GATE-EE-2016: 1M]

A function y(t), such that $y(0) = 1$ and $y(1) = 3e^{-1}$, is a solution of the differential equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$. Then y(2) is

(a) $5e^{-1}$ (b) $5e^{-2}$

(c) $7e^{-1}$ (d) $7e^{-2}$

59. [NAT] [GATE-EE-2016: 2M]

Let y(x) be the solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ with initial conditions $y(0) = 0$ and $\left.\frac{dy}{dx}\right|_{x=0} = 1$.

Then the value of y(1) is _______

60. [MCQ] [GATE 2016-EE- :1M]

The solution of the differential equation for $t > 0$, $y''(t) + 2y'(t) + y(t) = 0$ with initial condition $y(0) = 0$ and $y'(0) = 1$,is (u(t) denotes the unit step function),.

(a) $te^{-t}u(t)$

(b) $(e^{-t} - te^{-t})u(t)$

(c) $(-e^{-t} + te^{-t})u(t)$

(d) $e^{-t}u(t)$

61. [NAT] [GATE-EC-2015: 2M]

Consider the differential equation

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = 0$$

Given $x(0) = 20$ and $x(1) = 10/e$, where $e = 2.718$, the value of x(2) is ________ .

62.. [MCQ] [GATE-EC-2015: 2M]

The solution of the differential equation

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$ with $y(0) = y'(0) = 1$ is

(a) $(2 - t)e^{t}$ (b) $(1 + 2t)e^{-t}$

(c) $(2 + t)e^{-t}$ (d) $(1 - 2t)e^{t}$

63. [NAT] [GATE-CE-2015: 2M]

Consider the following second order linear differential equation $\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$.

The boundary conditions are: at x = 0, y = 5 and x = 2, y = 21

The value of y at x = 1 is __________

64. [MCQ] **[GATE-ME-2015: 2M]**

Find the solution of $\frac{d^2y}{dx^2} = y$ which passes through the origin and the point ($ln\ 2$, $\frac{3}{4}$).

(a) $y = \frac{1}{2}e^x - e^{-x}$ (b) $y = \frac{1}{2}(e^x + e^{-x})$

(c) $y = \frac{1}{2}(e^x - e^{-x})$ (d) $y = \frac{1}{2}e^x + e^{-x}$

65. [NAT] **[GATE-ME-2014: 2M]**

If y = f(x) is the solution of $\frac{d^2y}{dx^2} = 0$ with the boundary conditions y = 5 at x = 0 and $\frac{dy}{dx} = 2$ at x = 10, f (15) = _____.

66. [MCQ] **[GATE-ME-2014: 2M]**

Consider two solutions x(t) = x_1(t) and x(t) = x_2(t) of the differential equation

$\frac{d^2x(t)}{dt^2} + x(t) = 0$, t > 0, such that $x_1(0) = 1$,

$\left.\frac{dx_1(t)}{dt}\right|_{t=0} = 0,\ x_2(0) = 0,\ \left.\frac{dx_2(t)}{dt}\right|_{t=0} = 1.$

The Wronskian W(t) = $\begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_2(t)}{dt} \end{vmatrix}$ at $t = \frac{\pi}{2}$ is

(a) 1 (b) –1

(c) 0 (d) $\pi/2$

67. [MCQ] **[GATE-EC-2014: 1M]**

If the characteristic equation of the differential equation $\frac{d^2y}{dx^2} + 2\alpha\frac{dy}{dx} + y = 0$ has two equal roots, then the value of α are

(a) ± 1 (b) 0, 0

(c) ± j (d) $\pm\frac{1}{2}$

68. [MCQ] **[GATE-EC-2014: 1M]**

If a and b are constants, the most general solution of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ is

(a) ae^{-t} (b) $ae^{-t} + bte^{-t}$

(c) $ae^{t} + bte^{-t}$ (d) ae^{-2t}

69. [NAT] **[GATE-EC-2014: 2M]**

With initial values y(0) = y′ (0) = 1, the solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ at x = 1 is_______.

70. [MCQ] **[GATE-EE-2014: 1M]**

Consider the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

Which of the following is a solution to this differential equation for x > 0?

(a) e^x (b) X^2

(c) 1/x (d) ln x

71. [MCQ] **[GATE-EE-2014: 1M]**

The solution for the differential equation

$\frac{d^2x}{dt^2} = -9x$,

With initial conditions x(0) = 1 and $\left.\frac{dx}{dt}\right|_{t=0} = 1$, is

(a) $t^2 + t = 1$

(b) $\sin 3t + \frac{1}{3}\cos 3t + \frac{2}{3}$

(c) $\frac{1}{3}\sin 3t + \cos 3t$

(d) $\cos 3t + t$

72. [MCQ] **[GATE-ME-2013: 2M]**

The solution to the differential equation $\frac{d^2u}{dx^2} - k\frac{du}{dx} = 0$ where k is a constant, subjected to the boundary conditions u (0) = 0 and u(L) = U, is

(a) $u = U\frac{x}{L}$ (b) $u = U\left(\frac{1-e^{kx}}{1-e^{kL}}\right)$

(c) $u = U\left(\frac{1-e^{-kx}}{1-e^{-kL}}\right)$ (d) $u = U\left(\frac{1+e^{kx}}{1+e^{kL}}\right)$

73. [MCQ] **[GATE-CE-2010:2M]**

The solution to the ordinary differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ is

(a) $y = c_1e^{3x} + c_2e^{-2x}$ (b) $y = c_1e^{3x} + c_2e^{2x}$

(c) $y = c_1e^{-3x} + c_2e^{2x}$ (d) $y = c_1e^{-3x} + c_2e^{-2x}$

74. [MCQ] [GATE-EC-2010: 1M]

A function n(x) satisfies the differential equation $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ where L is a constant. The boundary conditions are n (0) = K and n (∞) = 0. The solution to this equation is

(a) $n(x) = K \exp(x/L)$
(b) $n(x) = K \exp(-x/\sqrt{L})$
(c) $n(x) = K^2 \exp(-x/L)$
(d) $n(x) = K \exp(-x/L)$

75 [MCQ] [GATE-EE-2010: 2M]

For the difference equation $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$ with initial condition x(0) = 1 and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.

(a) $x(t) = 2e^{-6t} - e^{-2t}$
(b) $x(t) = 2e^{-2t} - e^{-4t}$
(c) $x(t) = -e^{-6t} + e^{-4t}$
(d) $x(t) = e^{-2t} + 2e^{-4t}$

76. [MCQ] [GATE-ME-2008: 1M]

Given that $\ddot{x} + 3x = 0$, and x (0) = 1, $\dot{x}$ (0) = 0, what is x (1)?

(a) –0.99 (b) –0.16
(c) 0.16 (d) 0.99

77. [MCQ] [GATE-ME-2008: 2M]

It is given $y'' + 2y' + y = 0$, y (0) = 0, y (1) = 0. What is y (0.5)?

(a) 0 (b) 0.37
(c) 0.62 (d) 1.13

78. [MCQ] [GATE-CE-2008:1M]

The general solution of $\frac{d^2y}{dx^2} + y = 0$ is

(a) Y = P cos x + Q sin x
(b) y = p cos x
(c) y = P sin x
(d) y = P sin 2x

Euler -Cauchy Equations

79. [NAT] [GATE-ME-2023: 2M]

Consider the second-order linear ordinary differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$, $x \geq 1$

with the initial conditions

$$y(x=1) = 6, \quad \left.\frac{dy}{dx}\right|_{x=1} = 2$$

The value of y at x = 2 equals _____. (Answer in integer)

80. [MCQ] [GATE-ME-2019: 2M]

A differential equation is given as

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 4$$

The solution of the differential equation in terms of arbitrary constants C_1 and C_2 is

(a) $y = C_1x^2 + C_2x + 2$ (b) $y = \frac{C_1}{x^2} + C_2x + 2$
(c) $y = C_1x^2 + C_2x + 4$ (d) $y = \frac{C_1}{x^2} + C_2x + 4$

81. [NAT] [GATE-CE-2019:2M]

Consider the ordinary differential equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$. Given the values of y(1) = 0 and y(2) = 2, the value of y(3) (round off to 1 decimal place) is_________.

82. [NAT] [GATE-EC-2019: 2M]

Consider the homogeneous ordinary differential equation,

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0, \quad x > 0$$

with y(x) as a general solution.
Given that y(1) = 1 and y(2) = 14 the value of y(1.5). (rounded off to two decimal places), is________.

83. [MCQ] [GATE-ME-2012: 2M]

Consider the differemtial equation

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$ with the boundary conditions of y (0) = 0 and y (1) = 1. The complete solution of the differential equation is

(a) x^2 (b) $\sin\left(\frac{\pi x}{2}\right)$
(c) $e^x \sin\left(\frac{\pi x}{2}\right)$ (d) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

Partial Differential Equations

84. [NAT] [GATE-CE-2023-1M]

A 5 cm long metal rod AB was initially at a uniform temperature of T_0 °C. Thereafter, temperature at both the ends are maintained at 0 °C. Neglecting the heat transfer from the lateral surface of the rod, the heat transfer in the rod is governed by the one-dimensional diffusion equation $\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$, where D is the thermal diffusivity of the metal, given as 1.0 cm^2/s.
The temperature distribution in the rod is obtained as

$$T = (x,t) = \sum_{n=1,3,5\ldots}^{\infty} C_n \sin\frac{n\pi x}{5} e^{-\beta n^2 t},$$

where x is in cm measured from A to B with x=0 at A, t is in s, C_n are constants in °C, T is in °C, and β is in s^{-1}.
The value of β (in s^{-1}, rounded off to three decimal places) is ______________.

85. [NAT] [GATE-CE-2023-2M]

The steady-state temperature distribution in a square plate ABCD is governed by the 2-dimensional Laplace equation. The side AB is kept at a temperature of 100 °C and the other three sides are kept at a temperature of 0 °C. Ignoring the effect of discontinuities in the boundary conditions at the corners, the steady-state temperature at the center of the plate is obtained as To °C. Due to symmetry, the steady-state temperature at the center will be same (To °C), when any one side of the square is kept at a temperature of 100 °C and the remaining three sides are kept at a temperature of 0 °C. Using the principle of superposition, the value of T_0 is_______(rounded off to two decimal places).

86. [MCQ] [GATE-CE-2022-1M]

Consider the following expression:

Z = sin (y + it) + cos (y – it)

Where z, y and t are variables and $i = \sqrt{-1}$ is a complex number. The partial differential equation derived from the above expression is

(a) $\frac{\partial^2 Z}{\partial t^2} + \frac{\partial^2 Z}{\partial y^2} = 0$
(b) $\frac{\partial^2 Z}{\partial t^2} - \frac{\partial^2 Z}{\partial y^2} = 0$
(c) $\frac{\partial Z}{\partial t} - i\frac{\partial Z}{\partial y} = 0$
(d) $\frac{\partial Z}{\partial t} + i\frac{\partial Z}{\partial y} = 0$

87. [MCQ] [GATE- CE-2022:1M]

The function f(x, y) satisfies the Laplace equation $\Delta^2 f(x, y) = 0$ on a circular domain of radius r = 1 with its centre at point P with coordinates x = 0, y = 0. The value of this function on the circular boundary of this domain is equal to 3. The numerical value of f(0, 0) is

(a) 1
(b) 0
(c) 2
(d) 3

88. [MSQ] [GATE-EC-2022: 1M]

Consider the following partial differential equation (PDE),

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y),$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x, y) = e^{(\xi x + \eta y)}$ is a solution of the given PDE.

(a) $\xi = \frac{1}{\sqrt{2a}}, \eta = \frac{1}{\sqrt{2b}}$
(b) $\xi = \frac{1}{\sqrt{a}}, \eta = 0$
(c) $\xi = 0, \eta = 0$
(d) $\xi = \frac{1}{\sqrt{a}}, \eta = \frac{1}{\sqrt{b}}$

89. [MCQ] [GATE-CE-2020-1M]

In the following partial differential equation, θ is a function of t and z, and D and K are function of θ

$$D(\theta)\frac{\partial^2 \theta}{\partial z^2} + \frac{\partial K(\theta)}{\partial z} - \frac{\partial \theta}{\partial t} = 0$$

The above equation is

(a) A second order linear equation
(b) A second order non – linear equation
(c) A second degree non-linear equation
(d) A second degree linear equation

90. [MCQ] **[GATE-CE-2020-1M]**

The following partial differential equation is defined for

$$u : u(x, y)\frac{\partial u}{\partial y} = \frac{\partial^2 y}{\partial x^2} : y \geq 0, x_1 \leq x \leq x_2$$

The set of auxiliary conditions necessary to solve the equation uniquely, is

(a) One initial condition and two boundary conditions
(b) Three initial conditions
(c) Two initial conditions and one boundary condition
(d) Three boundary conditions

91. [MCQ] **[GATE-EE-2019 : 1M]**

The partial differential equation

$$\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0;$$

where $c \neq 0$ is known as

(a) heat equation (b) wave equation
(c) Poisson's equation (d) Laplace equation

92. [MCQ] **[GATE-ME-2018: 1M]**

Consider a function *u* which depends on position x and time t. The partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

is known as the

(a) Wave equation (b) Heat equation
(c) Laplace's equation (d) Elasticity equation

93. [MCQ] **[GATE-ME-2017: 1M]**

Consider the following partial differential equation for $u(x, y)$ with the constant $c > 1$:

$$\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0$$

Solution of this equation is

(a) $u(x, y) = f(x + cy)$ (b) $u(x, y) = f(x - cy)$
(c) $u(x, y) = f(cx + y)$ (d) $u(x, y) = f(cx - y)$

94. [NAT] **[GATE-CE-2017-1M]**

Consider the following partial differential equation:

$$3\frac{\partial^2 \phi}{\partial x^2} + B\frac{\partial^2 \phi}{\partial x \partial y} + 3\frac{\partial^2 \phi}{\partial y^2} + 4\phi = 0$$

For this equation to be classified as parabolic, the value of B^2 must be___________.

95. [MCQ] **[GATE-CE-2017-1M]**

The solution at x =1, t=1 of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial^2 u}{\partial t^2}$ subject to initial conditions of u(x,0) = 3x and $\frac{\partial u}{\partial t}(t = 0) = 3$ is_________.

(a) 1 (b) 2
(c) 3 (d) 6

96. [MCQ] **[GATE-CE-2016-1M]**

The type of partial differential equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + 3\frac{\partial^2 P}{\partial x \partial y} + 2\frac{\partial P}{\partial x} - \frac{\partial P}{\partial y} = 0 \text{ is}$$

(a) Elliptic (b) Parabolic
(c) Hyperbolic (d) None of these

97. [MCQ] **[GATE-CE-2016-1M]**

The solution of the partial differential equation $\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2}$ is of the form

(a) $C\cos(kt)\left|C_1 e^{\left(\sqrt{k/\alpha}\right)} + C_2 e^{-\left(\sqrt{k/\alpha}\right)x}\right|$

(b) $Ce^{kt}\left|C_1 e^{\left(\sqrt{k/\alpha}\right)x} + C_2 e^{-\left(\sqrt{k/\alpha}\right)x}\right|$

(c) $Ce^{kt}\left|C_1\cos\left(\sqrt{k/\alpha}\right)x + C_2\sin(-\left(\sqrt{k/\alpha}\right)x\right|$

(d) $C\sin(kt)\left|C_1\cos\left(\sqrt{k/\alpha}\right)x + C_2\sin(-\left(\sqrt{k/\alpha}\right)x\right|$

98. [MCQ] **[GATE-ME-2015: 2M]**

Solutions of Laplace's equation having continuous second-order partial derivatives are called

(a) Biharmonic functions
(b) Harmonic functions
(c) Conjugate harmonic functions
(d) Error functions

99. [MCQ] **[GATE-EC-2014: 1M]**

If z = xy ln (xy), then

(a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ (b) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(c) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$ (d) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

100. [MCQ] **[GATE-CE-2010-2M]**

The partial differential equation that can be formed from z = ax + by + ab has the form (with $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.)

(a) z = px + qy
(b) z = px + pq
(c) z = px + qy + pq
(d) z = qy + pq

101. [MCQ] **[GATE-ME-2008: 2M]**

Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at x = 2, y = 1?

(a) 0
(b) ln 2
(c) 1
(d) $\frac{1}{\ln 2}$

102. [MCQ] **[GATE-CE-2008-2M]**

The equation $k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$. can be transformed to $\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$. by substituting.

(a) $x_t = x \frac{k_z}{k_x}$
(b) $x_t = x \frac{k_x}{k_z}$
(c) $x_t = x \sqrt{\frac{k_x}{k_z}}$
(d) $x_t = x \sqrt{\frac{k_z}{k_x}}$

❑❑❑

ANSWER KEY

1.	(a)	2.	(d)	3.	(a)	4.	(d)
5.	(a)	6.	(b)	7.	(b)	8.	(0.446 to 0.446)
9.	(d)	10.	(c)	11.	(c)	12.	(c)
13.	(d)	14.	(b)	15.	(0.8774 to 0.8952)	16.	(c)
17.	(b)	18.	(c)	19.	(c)	20.	(c)
21.	(a)	22.	(d)	23.	(d)	24.	(a)
25.	(c)	26.	(c)	27.	(c)	28.	(c)
29.	(1.6 to 1.7)	30.	(d)	31.	(b)	32.	(d)
33.	(d)	34.	(a)	35.	(d)	36.	(d)
37.	(d)	38.	(c)	39.	(a)	40.	(a)
41.	(a)	42.	(d)	43.	(b)	44.	(a)
45.	(a)	46.	(a)	47.	(d)	48.	(a)
49.	(1.45 to 1.48)	50.	(0.36 to 0.37)	51.	(– 0.23 to – 0.19)	52.	(a)
53.	(a)	54.	(93 to 95)	55.	(–1.05 to –0.95)	56.	(a)
57.	(a)	58.	(b)	59.	(7.0 to 7.5)	60.	(a)
61.	(0.83 to 0.88)	62.	(b)	63.	(18)	64.	(c)
65.	(34 to 36)	66.	(a)	67.	(a)	68.	(b)
69.	(0.53 to 0.55)	70.	(c)	71.	(c)	72.	(b)
73.	(c)	74.	(d)	75.	(b)	76.	(b)
77.	(a)	78.	(a)	79.	(8.999 to 9.001)	80.	(a)
81.	(5.9 to 6.1)	82.	(5.24 to 5.26)	83.	(a)	84.	(0.395)
85.	(25)	86.	(a)	87.	(d)	88.	(a, b)
89.	(b)	90.	(a)	91.	(b)	92.	(b)
93.	(b)	94.	(36)	95.	(d)	96.	(c)
97.	(b)	98.	(b)	99.	(c)	100.	(c)
101.	(c)	102.	(d)				

SOLUTIONS

1. (a)

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^{3/2} + x^2y = 0$$

$$\Rightarrow \frac{d^3y}{dx^3} + x^2y = -x\left(\frac{dy}{dx}\right)^{3/2}$$

Square both side;

$$\Rightarrow \left[\frac{d^3y}{dx^3} + x^2y\right]^2 = x^2\left(\frac{dy}{dx}\right)^3$$

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 + x^4y^2 + 2x^2y\left(\frac{d^3y}{dx^3}\right) = x^2\left(\frac{dy}{dx}\right)^3$$

We have successfully eliminated radicals or square roots from this differential equation.

Order = Order of highest derivative = **3**

Degree = Exponent of highest derivative **= 2**

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2. (d)

Given:

$$\frac{d^2u}{dx^2} - 2x^2u + \sin x = 0$$

A differential equation is considered linear if it satisfies the following conditions:

(i) The dependent variable and differential coefficient appear in the differential equation only in a linear manner. This means that the power or exponent of the dependent variable and its derivatives is always 1.

(ii) The coefficients of the dependent variable and differential coefficient are constants or functions of the independent variables. There should not be product of dependent variable and differential coefficient or product of two differential coefficient.

Given that the provided differential equation satisfies the above conditions, it can be concluded that it is a linear differential equation.

U = f(x)

Here, u is dependent variable & x is independent variable. Since this equation can't be expressed as $x^n f\left(\frac{y}{x}\right)$ $\therefore$ **It is non-homogenous.**

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3. (a)

(i) $\frac{dy}{dx} + xy = e^{-x} \rightarrow$ Linear Non-Homogeneous DE

(ii) $\frac{dy}{dx} + xy = 0 \rightarrow$ Linear but Homogeneous DE

(iii) $\frac{dy}{dx} + xy = e^{-y} \Rightarrow \frac{dy}{dx} + xy = \left\{1 - \frac{y}{1!} + \frac{y^2}{2!} - \frac{y^3}{3!} + ...\infty\right\}$

$\rightarrow$ Non-linear DE

(iv) $\frac{dy}{dx} + e^{-y} = 0 \Rightarrow \frac{dy}{dx} = -e^{-y} \rightarrow$ Non-linear DE

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4. (d)

Given: $\frac{\partial u}{\partial t} + u\cdot\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$

$u\cdot\frac{\partial u}{\partial x} \rightarrow$ Non-linear term and $\frac{\partial^2 u}{\partial x^2} \rightarrow$ order '2'

$\boxed{\therefore \text{ Given DE is a non-linear DE of } 2^{nd} \text{order.}}$

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5. **(a)**

$$\frac{d^3y}{dx^3}+4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}=0$$

$$\Rightarrow \frac{d^3y}{dx^3}=-4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}$$

Making differential equation free from radicals, we get

$$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2=16\left[\left(\frac{dy}{dx}\right)^3+y^2\right]$$

Order = Order of highest differential coefficient **= 3**

Degree = Exponent of highest order derivative **= 2**

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6. **(b)**

Given: $\frac{d^3f}{d\eta^3}+\frac{f}{2}\cdot\frac{d^2f}{d\eta^2}=0$

Clearly given equation is non-linear (**Due to presence of** $f\cdot\frac{d^2f}{d\eta^2}$ **term**) and the highest order derivative that is involved is of 3rd order.

$\therefore$ Given DE is $\boxed{3^{rd}\text{order, non-linear ordinary DE}}$.

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7. **(b)**

Given:

DE is $\frac{d^2y}{dt^2}+\left(\frac{dy}{dt}\right)^3+y^4=e^{-t}$

For the given DE, highest order derivative that is involved is '2'

$\therefore$ **order of the given DE is 2**

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8. **(0.446 to 0.446)**

$y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$

$$\frac{dy}{dx}+\alpha xy=0$$

$\Rightarrow \frac{dy}{y}=-\alpha x\,dx$ [Variable separable form]

Integrating both sides,

$$\Rightarrow \ln y=-\alpha\frac{x^2}{2}+c \qquad \text{…(i)}$$

Given, y = 1 at x = 0; from equation (i),

$\ln 1 = 0 + c \Rightarrow c = 0$

Put value of c in equation (i),

$$\ln y=-\alpha\frac{x^2}{2} \qquad \text{…(ii)}$$

Given, y = 0.8 at x = 1; from equation (ii),

$$\ln 0.8=-\alpha\frac{1^2}{2}$$

$$\Rightarrow \boxed{\alpha=0.446}$$

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9. **(d)**

Given: $t.\frac{dx}{dt}+x=t;$ $\quad x(1) = 0.5$

$$t.\frac{dx}{dt}+x=t;$$

$$\Rightarrow \frac{dx}{dt}+\frac{1}{t}x=1$$

This is **Leibnitz linear equation** of form

$\frac{dx}{dt}+P'x=Q'$ where $P'=\frac{1}{t}$ and $Q'=1$.

Solution of above differentiable equation is given by,

$$x\cdot(\text{I.F})=\int Q'(\text{IF})dt+c$$

$$\Rightarrow x\cdot(\text{I.F})=\int(\text{IF})dt+c \qquad \text{…(i)}$$

Integrating factor

$$\text{I.F}=e^{\int\frac{1}{t}dt}=e^{\ln t}=t$$

From equation (i),

$$x \times t = \int t\,dt + c$$

$$\Rightarrow \quad x.t = \frac{t^2}{2} + c$$

$$\Rightarrow \quad x = \frac{t}{2} + \frac{c}{t} \qquad \ldots(ii)$$

At t = 1; x = 0.5, from equation (ii),

$$\Rightarrow 0.5 = \frac{1}{2} + c \ \Rightarrow \ c = 0$$

Put value of c in equation (ii),

$$\boxed{x = \frac{t}{2}}$$

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10. (c)

Method I:

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2u}$$

$\Rightarrow$ $2du + x^2.udu = -xu^2dx$

$\Rightarrow$ $2du + x^2udu + xu^2dx = 0$

$$\Rightarrow \quad 2du + d\left(\frac{x^2u^2}{2}\right) = 0$$

On integrating

$$\Rightarrow \quad \boxed{2u + \frac{x^2u^2}{2} = \text{constant}}$$

Method II:

$$\frac{du}{dx} = \frac{-xu^2}{2 + x^2u}$$

$$\Rightarrow (2 + x^2u)du + xu^2dx = 0 \qquad \ldots(1)$$

Comparing with Mdu + Ndx = 0

$M = 2 + x^2u$ and $N = xu^2$

The above differential equation

Will be exact if, $\frac{\delta M}{\delta x} = \frac{\delta N}{\delta u}$

Now, $\frac{\delta M}{\delta x} = 2xu$ and $\frac{\delta N}{\delta u} = 2xu$

$$\because \frac{\delta M}{\delta x} = \frac{\delta N}{\delta u}$$

$\therefore$ differential equation (i) is exact differential equation and solution of differential equation is:

$$\int_{x=c} M\,du + \int (\text{Terms of N not containing u})\,dx = c$$

$$\Rightarrow \quad \int_{x=c} (2 + x^2u)du + \int (0)\,dx = c$$

$$\Rightarrow \quad \boxed{2u + \frac{x^2u^2}{2} = c}$$

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11. (c)

Given $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ and $\sin x \frac{dy}{dx} + y\cos x = 1$

Find: $y\left(\frac{\pi}{6}\right) = ?$

$$\sin x \frac{dy}{dx} + y\cos x = 1$$

$$\Rightarrow \quad \frac{dy}{dx} + \cot x \cdot y = \operatorname{cosec} x$$

This is **Leibnitz linear equation** of form

$\frac{dy}{dx} + P'y = Q'$ where $P' = \cot x$ and $Q' = \operatorname{cosec} x$.

Solution of above differentiable equation is given by,

$$y \cdot (I.F) = \int Q'(IF)\,dx + c$$

$$\Rightarrow \quad y \cdot (I.F) = \int \operatorname{cosec} x\,(IF)\,dx + c \qquad \ldots(i)$$

Here, Integrating factor:

$$\text{I.F} = e^{\int \cot x dx} = e^{\int \frac{\cos x}{\sin x}dx} = e^{\ln|\sin x|} = \sin x$$

From equation (i),

$$y\,(\sin x) = \int \sin x \cdot \operatorname{cosec} x \ dx + c$$

$$\Rightarrow \quad y\,(\sin x) = \int dx + c$$

$$\Rightarrow \quad y\,(\sin x) = x + c \qquad \ldots(ii)$$

Given, At $x = \frac{\pi}{2}$, $y = \frac{\pi}{2}$; from equation (ii)

$\Rightarrow \frac{\pi}{2}(1) = \frac{\pi}{2} + c \qquad \Rightarrow \quad c = 0$

Put value of c in equation (ii),

y (sin x) = x

$\Rightarrow \quad y = \frac{x}{\sin x}$

At $x = \frac{\pi}{6}$;

$$y\left(\frac{\pi}{6}\right) = \frac{\frac{\pi}{6}}{\left(\frac{1}{2}\right)} = \frac{\pi}{3}$$

$$\Rightarrow \quad \boxed{y\left(\frac{\pi}{6}\right) = \frac{\pi}{3}}$$

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12. (c)

Given, $(1+y)\frac{dy}{dx} = y$

$\Rightarrow \quad \frac{1+y}{y} \cdot dy = dx \qquad$ [Variable separable form]

$\Rightarrow \quad \left(\frac{1}{y} + 1\right)dy = dx$

Since y (1) = 1, hence integrating above differential equation from x = 1 to x and y = 1 to y

$$\Rightarrow \int_{y=1}^{y} \left(\frac{1}{y} + 1\right)dy = \int_{x=1}^{x} dx$$

$\Rightarrow \quad \ln y + y|_1^y = x|_1^x$

$\Rightarrow$ (ln y + y) – (1) = x – 1

$\Rightarrow$ x = y + ln y

$\Rightarrow$ x – y = ln y

$\Rightarrow$ $e^{x-y} = y$

$\Rightarrow$ $e^x = y.e^y$

$\Rightarrow$ $\boxed{ye^y = e^x}$

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13. (d)

$$\frac{dy}{dx} - \frac{y}{x} = 1$$

This is **Leibnitz linear equation** of form

$\frac{dy}{dx} + P'y = Q'$ where $P' = -\frac{1}{x}$ and $Q' = 1$.

Solution of above differentiable equation is given by,

$y \cdot (\text{I.F}) = \int Q'(\text{IF})dx + c$

$\Rightarrow \quad y \cdot (\text{I.F}) = \int 1 \times (\text{IF})dx + c \qquad \ldots(i)$

Here, Integrating factor (I.F.);

$$\text{I.F.} = e^{\int P\, dx} = e^{\int -\frac{1}{x}.dx} = e^{-\ln x}$$

$\Rightarrow \quad \text{I.F.} = e^{\ln \frac{1}{x}} = \frac{1}{x}$

From equation (i),

$y\left(\frac{1}{x}\right) = \int \frac{1}{x}dx + c \qquad [c = \ln k]$

$\Rightarrow \quad \frac{y}{x} = \ln x + \ln k$

$\Rightarrow \quad \boxed{y = x\ln(kx)}$

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14. (b)

Given:

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = x \cdot \sqrt{y}$$

$\Rightarrow \quad \frac{1}{\sqrt{y}} \cdot \frac{dy}{dx} + \frac{x}{1-x^2} \cdot \sqrt{y} = x \qquad \ldots(i)$

Let $\sqrt{y} = z$

$\Rightarrow \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = \frac{dz}{dx} \Rightarrow \frac{1}{\sqrt{y}} \cdot \frac{dy}{dx} = 2 \cdot \frac{dz}{dx}$

Equation (i) will become,

$\Rightarrow \quad 2 \cdot \frac{dz}{dx} + \frac{x}{1-x^2}z = x$

$$\Rightarrow \frac{dz}{dx}+\frac{x}{2\left(1-x^2\right)}z=\frac{x}{2}$$

This is **Leibnitz linear equation** of form $\frac{dz}{dx}+P'z=Q'$ where $P'=\frac{x}{2\left(1-x^2\right)}$ and $Q'=\frac{x}{2}$.

$\therefore$ Integrating factor, I.F = $e^{\int P'dx}=e^{\int\frac{x}{2\left(1-x^2\right)}dx}$

$$=e^{\frac{-1}{4}\int\frac{-2x}{1-x^2}dx}=e^{\frac{-1}{4}\cdot\ell n\left(1-x^2\right)}$$

$$=e^{\ell n\left(1-x^2\right)^{-1/4}}=\left(1-x^2\right)^{-1/4}$$

$\therefore$ $\boxed{\text{Integrating factor} = \left(1-x^2\right)^{-1/4}}$

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15. (0.8774 to 0.8952)

Given: $\frac{dy}{dx}=2x-y,\qquad y(0)=1$

$$\frac{dy}{dx}=2x-y$$

$$\Rightarrow\quad \frac{dy}{dx}+y=2x$$

This is **Leibnitz linear equation** of form $\frac{dy}{dx}+P'y=Q'$ where $P'=1$ and $Q'=2x$.

Solution of above differentiable equation is given by,

$$y\cdot(\text{I.F})=\int Q'(\text{IF})dx+c$$

$$\Rightarrow\quad y\cdot(\text{I.F})=\int 2x(\text{IF})dx+c\ \ldots(i)$$

Here, Integrating factor:

$$\text{I.F.}=e^{\int Pdx}=e^{\int 1dx}=e^x$$

From equation (i),

$$y\cdot\left(e^x\right)=\int 2x\left(e^x\right)dx+c$$

$$\Rightarrow\quad ye^x=2\int xe^xdx+c$$

$$\left[\begin{array}{l}\left(\because \int uv=u\int vdx-\int u'\left(\int vdx\right)dx\right)\\ \text{In}\int xe^xdx,\text{consider } u=x\ \&\ v=e^x\end{array}\right]$$

$$\Rightarrow\quad y.e^x=2\left[xe^x-\int 1.e^x\right]+c$$

$$\Rightarrow\quad y.e^x=2xe^x-2e^x+c$$

$$\Rightarrow\quad y=2x-2+ce^{-x}\quad\ldots(ii)$$

Given y(0) = 1; from equation (ii)

$$1=2(0)-2+ce^0$$

$$\Rightarrow\quad c=3$$

Put value of c in equation (ii)

$$y=2x-2+3e^{-x}$$

At x = ln 2;

$$y=2\ln 2-2+3e^{-\ln 2}$$

$$\Rightarrow\quad \boxed{\mathbf{y=0.886}}$$

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16. (c)

Given:

$\frac{dy}{dx}+7x^2y=0;$ and $y(0)=\frac{3}{7}$

Find: y (1) = ?

$$\frac{dy}{dx}+7x^2y=0$$

$$\Rightarrow\quad \frac{dy}{dx}=-7x^2y$$

$$\Rightarrow\quad \frac{1}{y}dy=-7x^2dx\ \ \text{[Variable separable form]}$$

$y(0)=\frac{3}{7}$ i.e. at x = 0 $\Rightarrow y=\frac{3}{7}$ and let at x = 1; y = α

Integrating above differential equation from x = 0 to x = 1 and $y=\frac{3}{7}$ to y = α

$$\Rightarrow\quad \int_{y=3/7}^{y=\alpha}\frac{1}{y}dy=\int_{x=0}^{x=1}-7x^2dx$$

$$\Rightarrow \quad \ln y\Big|_{3/7}^{\alpha} = \left.\frac{-7x^3}{3}\right|_0^1$$

$$\Rightarrow \quad \ln\left(\frac{\alpha}{\left(\frac{3}{7}\right)}\right) = \frac{-7}{3}(1-0) \Rightarrow \alpha = \frac{3}{7}\cdot e^{-7/3}$$

$$\Rightarrow \quad \boxed{y(1) = \frac{3}{7}\cdot e^{-7/3}}$$

Scan for Video solution

17. (b)

Given $\frac{dy}{dx} + 4y = 5; \quad y(0) = 2.25$

$$\frac{dy}{dx} + 4y = 5$$

$$\Rightarrow \quad \frac{dy}{dx} = 5 - 4y$$

$$\Rightarrow \quad \frac{1}{5-4y}dy = dx \quad \text{[Variable separable form]}$$

On integrating,

$$\Rightarrow \quad \int \frac{1}{5-4y}dy = \int dx$$

$$\Rightarrow \quad \frac{\ln(5-4y)}{-4} = x + \ln C$$

$$\Rightarrow \quad \ln(5-4y) = -4x + \ln C$$

$$\Rightarrow \quad \ln\frac{(5-4y)}{C} = -4x$$

$$\Rightarrow \quad 5 - 4y = C.e^{-4x}$$

$$\Rightarrow \quad 4y = 5 - Ce^{-4x} \qquad \ldots(i)$$

At $x = 0$; $y = 2.25$; from equation (i),

$$4\times 2.25 = 5 - Ce^{-4\times 0} \Rightarrow 9 = 5 - C \quad \Rightarrow C = -4$$

Put value of C in equation (i),

$$4y = 5 - (-4)\times e^{-4x}$$

$$\Rightarrow \quad y = \frac{5}{4} + e^{-4x}$$

$$\Rightarrow \quad \boxed{y = e^{-4x} + 1.25}$$

Scan for Video solution

18. (c)

$$\frac{dy}{dx}(x\ln x) = y$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x\ln x} \qquad \text{[Variable separable form]}$$

$$\Rightarrow \ln y = \int \frac{dx}{x\ln x} + \ln K \qquad \text{[Integrating both sides]}$$

Let, $\ln x = t \Rightarrow \frac{1}{x}dx = dt$

$$\Rightarrow \ln y = \int \frac{dt}{t} + \ln K$$

$$\Rightarrow \ln y = \ln t + \ln K$$

$$\Rightarrow y = Kt \qquad \text{[Put } t = \ln x\text{]}$$

$$\Rightarrow \boxed{y = K\ln x}$$

Scan for Video solution

19. (c)

$$x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \frac{dy}{y} = -\frac{dx}{x} \qquad \textbf{[Variable separable form]}$$

Integrating both sides, we get

$\ln y = -\ln x + c$

At $x = 1$, $y = 1$

$\ln 1 = -\ln 1 + c \Rightarrow \quad c = 0$

$\therefore$ The solution is,

$\ln y = -\ln x \Rightarrow \ln(yx) = 0$

$$\Rightarrow yx = 1 \Rightarrow \boxed{y = x^{-1}}$$

Scan for Video solution

20. (c)

Given, $y^3\cdot\frac{dy}{dx} + x^3 = 0; \quad y(0) = 1$

Find: $y(-1) = ?$

$$y^3\cdot\frac{dy}{dx} + x^3 = 0$$

$\Rightarrow \quad y^3 \cdot dy = -x^3 \cdot dx$ [Variable separable form]

$y(0)=1$ i.e. at $x = 0; y=1$ and let at $x = -1$; $y = \alpha$, then integrating above differential equation from $x = 0$ to $x = -1$ and $y=1$ to $y = \alpha$,

$$\Rightarrow \int_{y=1}^{y=\alpha} y^3 dy = \int_{x=0}^{x=-1} -x^3 dx$$

$$\Rightarrow \left.\frac{y^4}{4}\right|_1^{\alpha} = \left.\frac{-x^4}{4}\right|_0^{-1}$$

$\Rightarrow \quad \alpha^4 - 1 = -(1-0) \quad \Rightarrow \alpha^4 = 0$

$\Rightarrow \quad \alpha = 0$

$\therefore \quad \boxed{y(-1) = 0}$

Scan for Video solution

21. (a)

Given: $\dfrac{dy}{dx} = \dfrac{x^2+y^2}{2y} + \dfrac{y}{x}$

And the curve passes through (1, 0).

$$\frac{dy}{dx} = \frac{x^2+y^2}{2y} + \frac{y}{x} \qquad \ldots(i)$$

Let $y = vx \Rightarrow \dfrac{dy}{dx} = v + x.\dfrac{dv}{dx}$

From equation (i),

$$\Rightarrow \quad v + x.\frac{dv}{dx} = \frac{x^2+v^2x^2}{2vx} + v$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\Rightarrow \quad \int \frac{2v}{1+v^2} dv = \int dx$$

Let, $1+v^2 = t$, then $2vdv = dt$

$$\Rightarrow \quad \int \frac{dt}{t} = \int dx$$

$\Rightarrow \quad \ln t = x + c \qquad \left[\because 1+v^2 = t\right]$

$\Rightarrow \quad \ln(1+v^2) = x + c \qquad \left[\because y = vx \Rightarrow v = \dfrac{y}{x}\right]$

$$\Rightarrow \ln\left(1+\frac{y^2}{x^2}\right) = x + c \qquad \ldots(ii)$$

At $x = 1$, $y = 0$; from equation (ii),

$$\Rightarrow \ln\left(1+\frac{0}{1}\right) = 1 + c \Rightarrow c = -1$$

Put value of c in equation (ii),

$$\Rightarrow \boxed{\ln\left(1+\frac{y^2}{x^2}\right) = x-1}$$

Scan for Video solution

22. (d)

Given: DE is $\dfrac{dy}{dx} = (x+y-1)^2 \qquad \ldots\ldots (1)$

Let $x + y - 1 = t \quad \Rightarrow 1 + \dfrac{dy}{dx} = \dfrac{dt}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{dt}{dx} - 1$

Put in equation (1),

$$\Rightarrow \frac{dt}{dx} - 1 = t^2$$

$$\Rightarrow \frac{dt}{dx} = 1 + t^2$$

$$\Rightarrow \frac{1}{1+t^2} dt = dx$$

On integrating both side

$$\int \frac{1}{1+t^2} dt = \int dx$$

$\Rightarrow \quad \tan^{-1} t = x + c$

$\Rightarrow \quad t = \tan(x + c) \qquad$ [Since $x + y - 1 = t$]

$\Rightarrow \quad x + y - 1 = \tan(x + c)$

$\Rightarrow \quad y = 1 - x + \tan(x + c)$

$\therefore$ y = 1 – x + tan (x + c), where c is a constant.

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23. (d)

$\frac{dQ}{dt}+Q=1$ and Q = 0 at t = 0

This is **Leibnitz linear equation** of form

$\frac{dy}{dx}+P'y=Q'$ or $\frac{dQ}{dt}+P'Q=Q'$ where $P'=1$

and Q' = 1

Solution of above differentiable equation is given by,

$$Q\,(\text{I.F.})=\int Q'\,(\text{I.F.})\;dt+c$$

$$\Rightarrow Q\cdot(\text{I.F})=\int 1\times(\text{IF})dt+c \qquad \ldots(i)$$

Here, Integrating factor;

$$\text{I.F} = e^{\int P'd}=e^{\int dt}=e^{t}$$

From equation (i),

$$Q\,e^{t}=\int 1.e^{t}\;dt+c \;\Rightarrow\; Qe^{t}=e^{t}+c$$

$$\Rightarrow \quad Q = 1 + ce^{-t} \qquad \ldots(ii)$$

when t = 0, Q = 0

$$0 = 1 + c\,e^{-0} \Rightarrow c=-1$$

Therefore, from equation (ii),

$$\boxed{Q(t)=1-e^{-t}}$$

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24. (a)

Given: $(t^2-81)\frac{dy}{dt}+5ty=\sin t$; $y(1)=2\pi$

$$(t^2-81)\frac{dy}{dt}+5ty=\sin t$$

$$\Rightarrow \quad \frac{dy}{dt}+\frac{5t}{t^2-81}y=\frac{\sin t}{t^2-81}$$

This is **Leibnitz linear equation** of form $\frac{dy}{dt}+P'y=Q'$ where $P'=\frac{5t}{t^2-81}$ and $Q'=\frac{\sin t}{t^2-81}$.

Solution of above differentiable equation is given by,

$$y\cdot(\text{I.F})=\int Q'(\text{IF})dt+c$$

$$\Rightarrow \quad y\cdot(\text{I.F})=\int\frac{\sin t}{t^2-81}(\text{IF})dt+c \qquad \ldots(i)$$

Here, Integrating factor:

$$\text{I.F.}=e^{\int P'dt}=e^{\int\frac{5t}{t^2-81}dt}$$

$$\Rightarrow \quad I.F.=e^{\frac{5}{2}\int\frac{2t}{t^2-81}dt}$$

Let, $t^2-81=0\Rightarrow 2tdt = du$,then

$$\Rightarrow \quad \text{I.F.}=e^{\frac{5}{2}\int\frac{du}{u}}=e^{\frac{5}{2}\ln u}=u^{\frac{5}{2}}$$

$$[\because u=t^2-81]$$

$$\Rightarrow \quad \text{I.F.}=(t^2-81)^{5/2}$$

From equation (i),

$$y(t^2-81)^{5/2}=\int\frac{\sin t}{t^2-81}\cdot(t^2-81)^{5/2}dt+c$$

$$\Rightarrow \quad y=\frac{\int \sin t\cdot(t^2-81)^{3/2}dt}{(t^2-81)^{5/2}}+\frac{c}{(t^2-81)^{5/2}}$$

For $t^2-81=0\Rightarrow t=\pm 9$ the solution will not be unique

∴ option b, c and d are incorrect as in their interval t becomes 9 or −9. Hence option b i.e. (–10, 10), c i.e. (–10, 2) and d i.e. (0, 10) are eliminated and correct option is option a i.e. (–2, 2).

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25. (c)

Given $\frac{dy}{dx}=\frac{1+\cos 2y}{1-\cos 2x}=\frac{1+2\cos^2 y-1}{1-\left(1-2\sin^2 x\right)}$

$$\Rightarrow \frac{dy}{dx}=\frac{2\cos^2 y}{2\sin^2 x}\Rightarrow \sec^2 y dy=\operatorname{cosec}^2 x dx$$

On Integrating,

$$\Rightarrow \int \sec^2 y\;dy=\int \operatorname{cosec}^2 x\;dx$$

$$\Rightarrow \tan\;y=-\cot x+c$$

$$\boxed{\Rightarrow \tan\;y+\cot x=c}$$

Scan for Video solution

26. (c)

Given: $\frac{dy}{dt} = -5y$; At $t = 0 \Rightarrow y = 2$

And at $t = 3 \Rightarrow y = ?$

$$\frac{dy}{dt} = -5y$$

$\Rightarrow \frac{1}{y}dy = -5dt$ [Variable separable form]

at $t = 0 \Rightarrow y = 2$ and let at $t = 3 \Rightarrow y = \alpha$, then integrating above differential equation from t = 0 to t = 3 and $y = 2$ to $y = \alpha$,

$$\Rightarrow \int_{y=2}^{y=\alpha} \frac{1}{y}dy = \int_{t=0}^{t=3} -5\,dt$$

$$\Rightarrow \ln y\Big|_2^{\alpha} = -5 \times t\Big|_0^3$$

$$\Rightarrow \ln\left(\frac{\alpha}{2}\right) = -5 \times (3-0) \Rightarrow \ln\left(\frac{\alpha}{2}\right) = -15$$

$$\Rightarrow \alpha = 2.\, e^{-15}$$

$$\Rightarrow \boxed{y(3) = 2.e^{-15}}$$

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27. (c)

$$x(y\,dx + x\,dy)\cos\frac{y}{x} = y(x\,dy - y\,dx)\sin\frac{y}{x}$$

$$\Rightarrow \frac{y\,dx + x\,dy}{x\,dy - y\,dx} = \frac{y}{x}\tan\frac{y}{x} \;.........(i)$$

Let $y = vx$

then, $dy = v\,dx + x\,dv$,

$\therefore$ from equation (i),

$$\frac{v\,x\,dx + x(v\,dx + x\,dv)}{x(v\,dx + x\,dv) - vx\,dx} = v\tan v$$

$$\Rightarrow \frac{2vx\,dx + x^2\,dv}{x^2\,dv} = v\tan v$$

$$\Rightarrow \frac{2v}{x}\frac{dx}{dv} + 1 = v\tan v \qquad \text{[divide both side by v]}$$

$$\Rightarrow 2\frac{dx}{x\,dv} + \frac{1}{v} = \tan v$$

$$2\frac{dx}{x} = \left(\tan v - \frac{1}{v}\right)dv$$

Integrating both sides,

$$\Rightarrow 2\log x = \log|\sec v| - \log v + \log c$$

$$\Rightarrow \log x^2 = \log\frac{c\sec v}{v} \Rightarrow x^2 = \frac{c\sec v}{v}$$

$$\Rightarrow x^2 = \frac{c\sec\frac{y}{x}}{y/x} \Rightarrow xy\frac{1}{\sec\frac{y}{x}} = c$$

$$\Rightarrow \boxed{xy\cos\frac{y}{x} = c}$$

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28. (c)

Given $\frac{dx}{dt} = 10 - 0.2x;\; x(0) = 1$

$$\frac{dx}{dt} = 10 - 0.2x \Rightarrow \frac{1}{10-0.2x}dx = dt$$

Integrating above equation from t=0 to t and x =1 to x,

$$\Rightarrow \int_{x=1}^{x=x} \frac{1}{10-0.2x}dx = \int_{t=0}^{t=t} dt$$

$$\Rightarrow \frac{\ln(10-0.2x)}{-0.2}\Bigg|_1^x = t$$

$$\Rightarrow (\ln(10-0.2x))\Big|_1^x = -0.2t$$

$$\Rightarrow \ln\left(\frac{10-0.2x}{9.8}\right) = -0.2t$$

$$\Rightarrow 10 - 0.2x = 9.8(e^{-0.2t})$$

$$\Rightarrow x = \frac{10 - 9.8(e^{-0.2t})}{0.2}$$

$$\Rightarrow x = 50 - 49e^{-0.2t}$$

$$\boxed{\therefore\; x(t) = 50 - 49e^{-0.2t}}$$

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29. (1.6 to 1.7)

Given: $\frac{di}{dt} - 0.2i = 0$ and i(4) = 10

$$\frac{di}{dt} - 0.2i = 0$$

$\Rightarrow \frac{di}{i} = 0.2dt$ (Variable separable form)

Integrating both side,

$$\ell n\, i = 0.2t + \ell n\, C$$

$\Rightarrow$ $i = Ce^{0.2t}$ …(i)

Given i(4) = 10; from equation (i)

$$10 = Ce^{0.2 \times 4}$$

$\Rightarrow$ $C = 10e^{-0.8}$

Put value of c in equation (i),

$$i = 10e^{-0.8}.e^{0.2t}$$

At t = –5;

$$i(-5) = 10e^{-0.8} \cdot e^{0.2(-5)} = 10e^{-1.8}$$

$\Rightarrow$ $\boxed{\mathbf{i(-5) = 1.652}}$

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30. (d)

$$\frac{dP}{dt} + k_2 P = k_2 L_0 e^{-k_1 t}$$

This is **Leibnitz linear equation** of form

$\frac{dP}{dt} + P'y = Q'$ where $P' = k_2$ and Q' = $k_2 L_0 e^{-k_1 t}$

Integrating factor (I.F.),

I.F. = $e^{\int P'dx}$ $\Rightarrow$ $\boxed{I.F. = e^{\int k_2 dt}}$

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31. (b)

Given $\frac{dy}{dx} = -2xy$ and y (0) = 2 i.e. At x = 0 $\Rightarrow$ y = 2

$$\frac{dy}{dx} = -2xy$$

$\Rightarrow$ $\frac{1}{y}dy = -2x\,dx$ [Variable separable form]

Since y (0) = 2, hence integrating above differential equation from x = 0 to x and y = 2 to y,

$$\Rightarrow \int_{y=2}^{y=y} \frac{1}{y}dy = \int_{x=0}^{x=x} -2x\,dx$$

$$\Rightarrow \ln y \Big|_2^y = -2\frac{x^2}{2}\Big|_0^x \quad \Rightarrow \quad \ln\left(\frac{y}{2}\right) = -x^2$$

$$\Rightarrow \boxed{y = 2 \cdot e^{-x^2}}$$

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32. (d)

Given: $\frac{dy}{dx} = \cos(x + y)$ …(i)

Let x + y = t

Then, $1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\therefore$ Given equation (i) will become,

$$\frac{dt}{dx} - 1 = \cos t$$

$$\Rightarrow \frac{dt}{dx} = 1 + \cos t$$

$$\Rightarrow \frac{dt}{dx} = 2\cos^2 t/2$$

$$\Rightarrow \frac{1}{\cos^2 t/2}dt = 2 \cdot dx$$

$$\Rightarrow \sec^2\frac{t}{2}dt = 2 \cdot dx$$

On integrating,

$$\Rightarrow \int \sec^2\frac{t}{2}dt = \int 2 \cdot dx$$

Let $\frac{t}{2} = V \Rightarrow \frac{dt}{2} = dV$

$$\Rightarrow \int \sec^2 V\cdot(2)\,dV = 2\cdot\int dx$$

$$\Rightarrow \tan V = x + c \qquad \left[\because \frac{t}{2} = V\right]$$

$$\Rightarrow \tan\frac{t}{2} = x + c \qquad [\because t = x + y]$$

$$\Rightarrow \boxed{\tan\left(\frac{x+y}{2}\right) = x + c}$$

Scan for Video solution

33. (d)

Given: $t\frac{dx}{dt} + x = t$; $x(1) = 0.5$,

$$t\frac{dx}{dt} + x = t$$

$$\Rightarrow \frac{dx}{dt} + \frac{x}{t} = 1$$

This is **Leibnitz linear equation** of form $\frac{dx}{dt} + P'x = Q'$ where $P' = \frac{1}{t}$ and $Q' = 1$.

Solution of above differentiable equation is given by,

$$x\cdot(\text{I.F}) = \int Q'(\text{IF})\,dt + c$$

$$\Rightarrow \quad x\cdot(\text{I.F}) = \int 1\times(\text{IF})\,dt + c \qquad \ldots\text{(i)}$$

Here, Integrating factor:

$$\text{IF} = e^{\int P'dt} = e^{\int \frac{1}{t}dt} = e^{\log t} = t$$

From equation (i)

$$xt = \int t\,dt + c$$

$$\Rightarrow \quad xt = \frac{t^2}{2} + c \qquad \ldots\text{(ii)}$$

Given, at t = 1 x = 0.5; from equation (ii),

$$(0.5)\,(1) = \frac{1^2}{2} + c \Rightarrow c = 0$$

Put value of c in equation (ii),

$$\boxed{x = \frac{t}{2}}$$

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34. (a)

$$\frac{dy}{dx} = e^{-3x}$$

$$\Rightarrow \quad dy = e^{-3x}\,dx$$

(Variable separable form)

Integrating both sides.

$$y = -\frac{1}{3}e^{-3x} + K$$

$$\Rightarrow \boxed{y = -\frac{1}{3}e^{-3x} + K}$$

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35. (d)

Given: $\frac{dy}{dx} = (1 + y^2)x$

$$\Rightarrow \frac{1}{1+y^2}dy = x\cdot dx \quad \text{[Variable Separation form]}$$

On integrating both sides

$$\Rightarrow \int \frac{1}{1+y^2}dy = \int x\,dx + c$$

$$\Rightarrow \tan^{-1} y = \frac{x^2}{2} + c$$

$$\Rightarrow \boxed{y = \tan\left(\frac{x^2}{2} + c\right)}$$

Scan for Video solution

36. (d)

Given, $\frac{dy}{dx} + \frac{y}{x} = x$ and y = 1 at x = 1

This is **Leibnitz linear equation** of form

$\frac{dy}{dx} + P'y = Q'$ where $P' = \frac{1}{x}$ and $Q' = x$.

Solution of above differentiable equation is given by,

$$y \cdot (I.F) = \int Q'(IF)dx + c$$

$$y \cdot (I.F) = \int x(IF)dx + c \quad \ldots(i)$$

Here, Integrating factor (I.F.):

$$I.F. = e^{\int P'dx} = e^{\int \frac{1}{x}.dx} = e^{\ln x}$$

$\Rightarrow$ I.F. = x

From equation (i),

$$yx = \int x \cdot x \, dx$$

$$\Rightarrow yx = \frac{x^3}{3} + c \quad \ldots(ii)$$

At x = 1, y = 1;

$\therefore$ from equation (ii)

$1 = \frac{1}{3} + c \Rightarrow c = \frac{2}{3}$, then

$$yx = \frac{x^3}{3} + \frac{2}{3} \Rightarrow \boxed{y = \frac{x^2}{3} + \frac{2}{3}}$$

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37. (d)

$$\frac{dy}{dx} + 2y = 0$$

$\Rightarrow \frac{dy}{y} = -2\,dx$ [Variable separable form]

On integrating, we get

$\ln y = -2x + c$

$\Rightarrow y = e^{-2x + c}$

$\Rightarrow y = e^{-2x}. e^{c}$ [replace e^c by c_1]

$\Rightarrow y = c_1 e^{-2x}$ …(i)

At x = 1, y = 5 so

$5 = c_1 e^{-2}$

$\Rightarrow c_1 = 5e^2 = 36.95$

Put value of c_1 in equation (i)

$\boxed{\mathbf{y = 36.95\, e^{-2x}}}$

Scan for Video solution

38. (c)

Given:

$\frac{dy}{dx} = ky$ and y (0) = c

$$\frac{dy}{dx} = ky$$

$$\Rightarrow \frac{dy}{y} = kdx$$

On integrating,

$$\Rightarrow \int \frac{1}{y} dy = \int k.dx$$

$\Rightarrow \ln y = k\,x + C_1$ …(i)

At x = 0; y = c; from equation (i)

$\Rightarrow \ln c = C_1$

Put value of C_1 in equation (i),

$\ln y = k\,x + \ln c$

$\Rightarrow \ln y - \ln c = k\,x$

$\Rightarrow \ln y/c = k\,x$

$\Rightarrow \boxed{y = c.e^{kx}}$

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39. (a)

Given: $x \cdot \frac{dy}{dx} + y = x^4; \quad y(1) = \frac{6}{5}$

$$x \cdot \frac{dy}{dx} + y = x^4$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x} \cdot y = x^3$$

This is **Leibnitz linear equation** of form

$\frac{dy}{dx} + P'y = Q'$ where $P' = \frac{1}{x}$ and $Q' = x^3$.

Solution of above differentiable equation is given by,

$$y \cdot (I.F) = \int Q'(IF)dx + c$$

$$y \cdot (\text{I.F}) = \int x^3 (\text{IF})\,dx + c \qquad \ldots(i)$$

Here, Integrating factor:

$$\text{IF} = e^{\int P'dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

From equation (i),

$$yx = \int x^3 \cdot x\,dx + c$$

$$\Rightarrow \qquad y\,x = \frac{x^5}{5} + c$$

$$\Rightarrow \qquad y = \frac{x^4}{5} + \frac{c}{x} \qquad \ldots(ii)$$

Given, $y(1) = \frac{6}{5}$; from equation (ii),

$$\frac{6}{5} = \frac{1}{5} + c \qquad \Rightarrow c = 1$$

Put value of c in equation (ii),

$$\boxed{y = \frac{x^4}{5} + \frac{1}{x}}$$

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40. (a)

$$3y\frac{dy}{dx} + 2x = 0$$

$\Rightarrow$ 3y dy + 2x dx = 0

On integrating, we get

$$\frac{3y^2}{2} + \frac{2x^2}{2} = c$$

$$\Rightarrow 2x^2 + 3y^2 = 2c \Rightarrow \boxed{2x^2 + 3y^2 = c'}$$

This equation represents family of ellipses.

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41. (a)

(P) $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \int \frac{1}{y}dy = \int \frac{1}{x}dx \Rightarrow \ln y = \ln x + c$

$\Rightarrow y = cx$ $\qquad \rightarrow$ **Straight line (2)**

(Q) $\frac{dy}{dx} = \frac{-y}{x}$

$\Rightarrow \int \frac{1}{y}dy = -\int \frac{1}{x}dx \quad \Rightarrow \ln y = -\ln x + c$

$\Rightarrow xy = c$ $\qquad \rightarrow$ **Rectangular Hyperbola (3)**

(R) $\frac{dy}{dx} = \frac{x}{y}$

$\Rightarrow \int ydy = \int xdx \Rightarrow y^2 - x^2 = c$ $\rightarrow$ **Hyperbolas (3)**

(S) $\frac{dy}{dx} = \frac{-x}{y}$

$\Rightarrow \int ydy = -\int xdx$

$\Rightarrow \quad y^2 - x^2 = c$ $\qquad \rightarrow$ **Circles (1)**

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42. (d)

$$\frac{dy}{dx} = -\frac{x}{y}$$

$\Rightarrow$ xdx + ydy = 0 $\qquad$ [Variable Separation form]

On integrating,

$$\frac{x^2}{2} + \frac{y^2}{2} = c \qquad \ldots(i)$$

At x = 1, y = $\sqrt{3}$

$$\frac{1^2}{2} + \frac{\sqrt{3}^2}{2} = c \Rightarrow c = 2$$

Put value of c in equation (i),

$$\frac{x^2}{2} + \frac{y^2}{2} = c \Rightarrow \boxed{x^2 + y^2 = 4}$$

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43. (b)

Given:

$$\frac{d}{dt}x(t) + 3 \times x(t) = 0$$

$\Rightarrow \frac{d}{dt}x(t) = -3x(t) \Rightarrow \frac{1}{x(t)}dx(t) = -3dt$

$\Rightarrow \int \frac{1}{x(t)} d\,x(t) = \int -3dt$

$\Rightarrow$ ln x(t) = – 3t + c

$\Rightarrow$ $x(t) = e^{-3t+c}$ $\left[\text{Let } e^c = C\right]$

$\Rightarrow$ $x(t) = C.e^{-3t}$

$\therefore$ The solution should be of the form

$x(t) = C.e^{-3t}$

For C = 2,

$x(t) = 2.e^{-3t}$

Scan for Video solution

44. (a)

Given,

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

Complete solution is,

$y = C_1e^{2.5x} + C_2e^{\alpha x} + C_3e^{\beta x}$

Replace, $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2 in differential equation,

$(D^3 - 5.5D^2 + 9.5D - 5)\, y = 0$

This is homogenous linear differential equation with constant coefficients. Therefore,

y = C.F. +P.I.

[Since homogenous equation $\therefore$ P.I. =0]

$\Rightarrow C_1e^{2.5x} + C_2e^{\alpha x} + C_3e^{\beta x} = \text{C.F.}$

$\Rightarrow \text{C.F.} = C_1e^{2.5x} + C_2e^{\alpha x} + C_3e^{\beta x}$

We will get above expression of C.F. when roots of the Auxiliary equation are 2.5, α and β.

Now, Auxiliary equation is:

$m^3 - 5.5\, m^2 + 9.5\, m - 5 = 0$

Now we can solve this problem by two methods.

Method I:

Auxiliary equation:

$m^3 - 5.5\, m^2 + 9.5\, m - 5 = 0$

Since one root of the A.E. should be 2.5, therefore we can express A.E. in the form of

$(m - 2.5)(am^2 + bm + c)$:

$m^3 - 2.5m^2 - 3m^2 + 7.5m + 2m - 5 = 0$

$\Rightarrow m^2(m - 2.5) - 3m(m - 2.5) + 2(m - 2.5) = 0$

$\Rightarrow (m - 2.5)(m^2 - 3m + 2) = 0$

$\Rightarrow (m - 2.5)(m^2 - m - 2m + 2) = 0$

$\Rightarrow (m - 2.5)[m(m - 1) - 2(m - 1)] = 0$

$\Rightarrow (m - 2.5)(m - 1)(m - 2) = 0$

$\Rightarrow m = 2.5,\ 1,\ 2$

Since roots of A.E. are 2.5, α and β therefore,

$\boxed{\alpha = 1 \text{ and } \beta = 2}$

Method II: (Check by options)

Auxiliary equation:

$m^3 - 5.5\, m^2 + 9.5\, m - 5 = 0$ …(i)

Since α and β are roots of A.E. (i), therefore correct option will satisfy equation (i)

Checking for Option A:

For m=1

L.H.S = $1^3 - 5.5(1)^2 + 9.5(1) - 5 = 0 \Rightarrow$ L.H.S = R.H.S

Hence m=1 is root of A.E.

For m=2

L.H.S = $2^3 - 5.5(2)^2 + 9.5(2) - 5 = 0 \Rightarrow$ L.H.S = R.H.S

Hence m=2 is also root of A.E.

Hence roots of A.E. other than 2.5 are 1 and 2, hence

$\boxed{\alpha = 1 \text{ and } \beta = 2}$

Scan for Video solution

45. (a)

Given: DE is, $\frac{d^2y}{dx^2} + 2\cdot\frac{dy}{dx} - 5y = 0$

This is homogenous linear differential equation with constant coefficients.

Replace $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$$(D^2 + 2D - 5)\,y = 0$$

Auxiliary equation is,

$$m^2 + 2m - 5 = 0$$

$\Rightarrow \quad (m+1)^2 - 6 = 0$

$\Rightarrow \quad m = -1 \pm \sqrt{6}$

$\therefore \quad m_1 = -1+\sqrt{6},\ m_2 = -1-\sqrt{6}$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = c_1 e^{(-1+\sqrt{6})x} + c_2 e^{(-1-\sqrt{6})x}$$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution:

$y = \text{C.F.} + \text{P.I.}$

$\Rightarrow y = c_1 e^{(-1+\sqrt{6})x} + c_2 e^{(-1-\sqrt{6})x}$

$\Rightarrow \quad y = K_1 e^{(-1+\sqrt{6})x} + K_2 e^{(-1-\sqrt{6})x}$

Scan for Video solution

46. (a)

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$; y (0) = 1 and y (1)

This is homogenous linear differential equation with constant coefficients. Replace,

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$(D^2 + 2D + 1)\,y = 0$

Auxiliary equation is:

$m^2 + 2m + 1 = 0$

$\Rightarrow (m + 1)^2 = 0$

$\Rightarrow m = -1, -1$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$\text{C.F.} = (C_1 + C_2 x)\,e^{-x}$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$\Rightarrow y = (C_1 + C_2x)e^{-x}$ …(i)

Given, y(0) = 1; from equation (i)

$(C_1 + C_2 \times 0)e^{-0} \;\Rightarrow\; C_1 = 1$

Given, y(1) = 3; from equation (i)

$3 = (C_1 + C_2 \times 1)e^{-1}$

$\Rightarrow C_1 + C_2 = 3e$ [Put $C_1 = 1$]

$\Rightarrow 1 + C_2 = 3e$

$\Rightarrow C_2 = 3e - 1$

Put value of C_1 and C_2 in equation (i),

$$y = \left[1 + (3e-1)\,x\right]e^{-x}$$

$\Rightarrow \boxed{y = e^{-x} + (3e-1)\,xe^{-x}}$

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47. (d)

$6\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

This is homogenous linear differential equation with constant coefficients. Replace,

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$(6D^2 + D - 1)\,y = 0$

Its auxiliary equation is:

$6m^2 + m - 1 = 0$

$\Rightarrow 6m^2 + 3m - 2m - 1 = 0$

$\Rightarrow 3m(2m + 1) - (2m + 1) = 0$

$\Rightarrow (3m - 1)(2m + 1) = 0$

$\Rightarrow m = \frac{1}{3}, -\frac{1}{2}$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$\text{C.F.} = C_1 e^{\frac{x}{3}} + C_2 e^{-\frac{x}{2}}$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$\Rightarrow \boxed{y = C_1 e^{\frac{x}{3}} + C_2 e^{-\frac{x}{2}}}$

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48. (a)

x(0)=0 and $\frac{dx}{dt}(0) = 10$ and

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 10$$

This is homogenous linear differential equation with constant coefficients. Replace,

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2

$(D^2 - 5D + 6)x = 0$

Its auxiliary equation is:

$m^2 - 5m + 6 = 0$

$\Rightarrow (m-3)(m-2) = 0$

$\Rightarrow m = 2, 3$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$C.F. = C_1 e^{2t} + C_2 e^{3t}$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $x = C.F. + P.I.$

$\Rightarrow x = C_1 e^{2t} + C_2 e^{3t}$ …(i)

Differentiating equation (i) with respect to t,

$\frac{dx}{dt} = 2C_1 e^{2t} + 3C_2 e^{3t}$ …(ii)

Given, x (0) = 0; from equation (i)

$0 = C_1 e^{2\times 0} + C_2 e^{3\times 0}$

$\Rightarrow C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$ …(iii)

Given, $\frac{dx}{dt}(0) = 10;$ from equation (ii)

$10 = 2C_1 e^{2\times 0} + 3C_2 e^{3\times 0}$

$\Rightarrow 2C_1 + 3C_2 = 10$ …(iv)

Put $C_1 = -C_2$;

$2(-C_2) + 3C_2 = 10 \Rightarrow C_2 = 10$

$\therefore C_1 = -C_2 \Rightarrow C_1 = 10$

Put value of C_1 and C_2 in equation (i),

$\boxed{x = -10e^{2t} + 10e^{3t}}$

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49. (1.45 to 1.48)

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$; y (0) = 0 and $\frac{dy}{dx}(0) = 1$

Find: y (1) =?

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$(D^2 + D - 6)\,y = 0$

The Auxiliary equation is

$m^2 + m - 6 = 0 \Rightarrow m^2 + 3m - 2m - 6 = 0$

$\Rightarrow m(m+3) - 2(m+3) = 0$

$\Rightarrow (m+3)(m-2) = 0$

$\Rightarrow m = -3, 2$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$C.F. = C_1 e^{-3x} + C_2 e^{2x}$

Since, this is homogenous equation, therefore

$P.I. = 0$

Now, Complete solution:

$y = C.F. + P.I.$

$\Rightarrow \boxed{y = C_1 e^{-3x} + C_2 e^{2x}}$ …(i)

Given, y (0) = 0 i.e. At x = 0, y = 0; from equation (i),

$0 = C_1 \cdot (1) + C_2$

$\Rightarrow C_2 = -C_1$ …(ii)

Put $C_2 = -C_1$ in equation (i)

$y = C_1(e^{-3x} - e^{2x})$ …(iii)

Differentiating above equation with respect to x,

$\frac{dy}{dx} = C_1(-3 \cdot e^{-3x} - 2 \cdot e^{2x})$ …(iv)

Given $\frac{dy}{dx}(0) = 1$ i.e. at $x = 0 \Rightarrow \frac{dy}{dx} = 1$; from equation (iv),

$$1 = C_1[-3(1) - 2(1)] \Rightarrow C_1 = -1/5$$

Put value of C_1 in equation (iii)

$$y = \frac{-1}{5}\left(e^{-3x} - e^{2x}\right)$$

At x = 1,

$$y(1) = y = \frac{-1}{5}\left(e^{-3\times1} - e^{2\times1}\right)$$

$$\Rightarrow \boxed{y(1) = 1.47}$$

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50. (0.36 to 0.37)

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ and $y(0) = 1$, $\frac{dy}{dx}(x = 0) = -1$

Find: y(1) =?

This is homogenous linear differential equation with constant coefficients.

Replace, $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$$\left(D^2 + 2D + 1\right)y = 0$$

Its auxiliary equation is:

$m^2 + 2m + 1 = 0 \Rightarrow (m + 1)^2 = 0 \Rightarrow m = -1, -1$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = \left(C_1 + C_2x\right)e^{-x}$$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$$\Rightarrow y = (C_1 + C_2x)e^{-x} \quad \ldots\text{(i)}$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \left(C_1 + C_2x\right)\left(-e^{-x}\right) + C_2e^{-x} \quad \ldots\text{(ii)}$$

Now, y(0) = 1 i.e. at x = 0, y = 1 then from equation (i),

$$1 = (C_1 + C_2 \times 0)e^{-0} \Rightarrow C_1 = 1$$

& at x = 0, $\frac{dy}{dx} = -1$ then from equation (ii),

$$-1 = \left(C_1 + C_2 \times 0\right)\left(-e^0\right) + C_2e^0$$

$$\Rightarrow -1 = -C_1 + C_2 \Rightarrow -1 = -1 + C_2$$

$$\Rightarrow C_2 = 0$$

Put value of C_1 and C_2 in equation (i),

$y = e^{-x}$

$\therefore$ At x = 1;

$$y(1) = e^{-1} \Rightarrow \boxed{y(1) = 0.368}$$

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51. (– 0.23 to – 0.19)

Given: DE is $\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5}{4}y$

And y (0) = 1 and $\left.\frac{dy}{dt}\right|_{t=0} = 0$.

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5}{4}y$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{dy}{dt} + \frac{5}{4}y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace $\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$$\Rightarrow \left(D^2 + D + \frac{5}{4}\right)y = 0$$

The auxiliary equation:

$$m^2 + m + \frac{5}{4} = 0$$

$$\Rightarrow 4m^2 + 4m + 5 = 0$$

$$\Rightarrow m = \frac{-4 \pm \sqrt{16 - 4(4)(5)}}{8}$$

$$\Rightarrow m = \frac{-1}{2} \pm i$$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$, where $\alpha = -\frac{1}{2}$ and $\beta = 1$, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = e^{\alpha t}\left(c_1 \cos\beta t + c_2 \sin\beta t\right)$$

$$\Rightarrow \quad \text{C.F.} = e^{-t/2}\{c_1 \cos t + c_2 \sin t\}$$

Since, this is homogenous equation, therefore

Particular integral $(\text{P.I.}) = 0$

Now, Complete solution:

$$y(t) = \text{C.F.} + \text{P.I.}$$

$$\therefore \quad y(t) = e^{-t/2}\{c_1 \cos t + c_2 \sin t\} \quad \ldots(i)$$

Now,

$$\frac{dy}{dt} = e^{-t/2}\left[\{-c_1 \sin t + c_2 \cos t\} - \frac{1}{2}\{c_1 \cos t + c_2 \sin t\}\right] \quad \ldots(ii)$$

At t = 0; y = 1; from equation (i),

$\Rightarrow$ 1 = 1{c_1 cos 0 + c_2 sin 0} $\Rightarrow$ $c_1 = 1$

At t = 0; $\frac{dy}{dt} = 0$; from equation (ii),

$$\Rightarrow 0 = 1\left[c_2 - \frac{1}{2}c_1\right]$$

$$\Rightarrow c_2 = \frac{c_1}{2} \quad [\text{Since } c_1 = 1]$$

$$\Rightarrow c_2 = \frac{1}{2}$$

Put value of c_1 and c_2 in equation (i),

$$y(t) = e^{-t/2}\left(\cos t + \frac{1}{2}\sin t\right)$$

At t = π

$$y(t) = e^{-\pi/2}\left(\cos\pi + \frac{1}{2}\sin\pi\right)$$

$$\Rightarrow \quad y(\pi) = e^{-\pi/2}(-1+0)$$

$$\Rightarrow \quad \boxed{y(\pi) = -0.2079}$$

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52. (a)

$y'' - 4y' + 3y = 2t - 3t^2$ and find: P.I. = ?

This is non homogenous linear differential equation with constant coefficients and can be written as:

$$\left(D^2 - 4D + 3\right)y = 2t - 3t^2$$

$$\text{P.I.} = \frac{1}{f(D)}\left(2t - 3t^2\right) = \frac{1}{D^2 - 4D + 3}\left(2t - 3t^2\right)$$

$$= \frac{1}{D^2 - 3D - D + 3}\left(2t - 3t^2\right)$$

$$= \frac{1}{-D(3-D) + 1(3-D)}\left(2t - 3t^2\right)$$

$$= \frac{1}{(1-D)(3-D)}\left(2t - 3t^2\right)$$

$$= \left[\frac{1/2}{1-D} - \frac{1/2}{3-D}\right]\left(2t - 3t^2\right)$$

$$= \frac{1}{2}(1-D)^{-1}\left(2t - 3t^2\right) - \frac{1}{6}\left(1 - \frac{D}{3}\right)^{-1}\left(2t + 3t^2\right)$$

$$\left[\because (1-D)^{-1} = 1 + D + D^2 + D^3 + \ldots\right]$$

$$= \left[\begin{array}{l} \frac{1}{2}\left(1 + D + D^2 + \ldots\right)\left(2t - 3t^2\right) \\ -\frac{1}{6}\left(1 + \frac{D}{3} + \frac{D^2}{9} + \ldots\right)\left(2t - 3t^2\right) \end{array}\right]$$

$$= \frac{1}{2}\left[\left(2t - 3t^2\right) + D\left(2t - 3t^2\right) + D^2\left(2t - 3t^2\right)\right]$$

$$-\frac{1}{6}\left[\left(2t - 3t^2\right) + \frac{D}{3}\left(2t - 3t^2\right) + \frac{D^2}{9}\left(2t - 3t^2\right)\right]$$

$$= \frac{1}{2}\left[2t - 3t^2 + 2 - 6t - 6\right] - \frac{1}{6}\left[2t - 3t^2 + \frac{2}{3} - 2t - \frac{2}{3}\right]$$

$$= \frac{1}{2}\left[-3t^2 - 4t - 4\right] - \frac{1}{6}\left[-3t^2\right] = -\frac{3t^2}{2} - 2t - 2 + \frac{t^2}{2}$$

$$\boxed{\text{P.I.} = -2 - 2t - t^2}$$

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53. (a)

Given $\frac{d^2y}{dx^2} + 16y = 0$; $\left.\frac{dy}{dx}\right|_{x=0} = 1$ & $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1$

$$\frac{d^2y}{dx^2} + 16y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$(D^2 + 16)\,y = 0$

The Auxiliary equation is

$m^2 + 16 = 0 \Rightarrow m = \pm 4i$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$,where $\alpha = 0$ and $\beta = 4$, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = e^{\alpha x}\left(c_1 \cos\beta x + c_2 \sin\beta x\right)$$

$$\Rightarrow \text{C.F.} = e^{0\times x}\left(c_1 \cos(4\times x) + c_2 \sin(4\times x)\right)$$

$$\Rightarrow \text{C.F.} = c_1 \cos 4x + c_2 \sin 4x$$

Since, this is homogenous equation, therefore

Particular integral $(\text{P.I.}) = 0$

Now, Complete solution:

$$y = \text{C.F.} + \text{P.I.}$$

$$\Rightarrow y = c_1 \cos 4x + c_2 \sin 4x \qquad \ldots(i)$$

Differentiating above equation with respect to x,

$$\frac{dy}{dx} = -4c_1 \sin 4x + 4c_2 \cos 4x \qquad \ldots(ii)$$

At $x = 0;\ \frac{dy}{dx} = 1$, hence from equation (ii),

$$1 = 0 + 4c_2 \Rightarrow c_2 = \frac{1}{4}$$

At $x = \frac{\pi}{2};\ \frac{dy}{dx} = -1$, , hence from equation (ii),

$$-1 + 0 + 4c_2 \Rightarrow c_2 = -\frac{1}{4}$$

Since c_2 is different for both the boundary conditions. ∴ The differential equation has no solution that satisfy the given boundary conditions.

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54. (93 to 95)

$3\cdot y''(x) + 27\cdot y(x) = 0$; $y(0) = 0$ and $y'(0) = 2000$

Find: $y(1) = ?$

$$3\cdot y''(x) + 27\cdot y(x) = 0$$

$$\Rightarrow y''(x) + 9\cdot y(x) = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$(D^2 + 9)\,y(x) = 0$

The Auxiliary equation is:

$m^2 + 9 = 0 \Rightarrow m = \pm 3i$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$,where $\alpha = 0$ and $\beta = 3$, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = e^{\alpha x}\left(c_1 \cos\beta x + c_2 \sin\beta x\right)$$

$$\Rightarrow \text{C.F.} = e^{0\times x}\left(c_1 \cos(3x) + c_2 \sin(3x)\right)$$

$$\Rightarrow \text{C.F.} = c_1 \cos 3x + c_2 \sin 3x$$

Since, this is homogenous equation, therefore

Particular integral $(\text{P.I.}) = 0$

Now, Complete solution:

$$y(x) = \text{C.F.} + \text{P.I.}$$

$$\Rightarrow y(x) = c_1 \cos 3x + c_2 \sin 3x \qquad \ldots(i)$$

Given $y(0) = 0$ i.e. at $x = 0$, $y = 0$; from equation (i),

$$0 = c_1 \cos 0 + c_2 \sin(0) \quad \Rightarrow \quad c_1 = 0$$

Put value of c_1 in equation (i),

$$y(x) = c_2 \sin 3x \qquad \ldots(ii)$$

Differentiating above equation with respect to x,

$$y'(x) = 3c_2 \cos 3x \qquad \ldots(iii)$$

At $x = 0$; $y'(x) = 2000$, from equation (iii),

$$2000 = 3c_2 \Rightarrow c_2 = \frac{2000}{3}$$

Put value of c_2 in equation (ii),

$$y(x) = \frac{2000}{3}\cdot \sin 3x$$

At $x = 1$,

$$y(1) = \frac{2000}{3}\cdot \sin(3\times 1)$$

$$\Rightarrow \boxed{\mathbf{y(1) = 94.08}}$$

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55. (–1.05 to –0.95)

$y'' + 9y = 0$, $y(0) = 0$, $y(\pi/2) = \sqrt{2}$ and $y(\pi/4) = ?$

$$y'' + 9y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2;

$$(D^2 + 9)\,y = 0$$

The Auxiliary equation is:

$$m^2 + 9 = 0 \Rightarrow m = \pm 3i$$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$,where $\alpha = 0$ and $\beta = 3$, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = e^{\alpha x}\left(c_1 \cos\beta x + c_2 \sin\beta x\right)$$

$$\Rightarrow \text{C.F.} = e^{0\times x}\left(c_1 \cos(3x) + c_2 \sin(3x)\right)$$

$$\Rightarrow \text{C.F.} = c_1 \cos 3x + c_2 \sin 3x$$

Since, this is homogenous equation, therefore

$$\text{Particular integral (P.I.)} = 0$$

Now, Complete solution:

$$y = \text{C.F.} + \text{P.I.}$$

$$\Rightarrow y = c_1 \cos 3x + c_2 \sin 3x \qquad \ldots(i)$$

At $x = 0$; $y = 0$; from equation (i),

$$0 = c_1(1) + c_2(0) \Rightarrow c_1 = 0$$

At $x = \frac{\pi}{2}, y = \sqrt{2}$; and $c_1 = 0$, $\therefore$ from equation (i),

$$\sqrt{2} = 0 \times \cos\left(\frac{3\pi}{2}\right) + c_2 \sin\left(\frac{3\pi}{2}\right)$$

$$\Rightarrow \sqrt{2} = 0 + c_2 \times (-1)$$

$$\Rightarrow c_2 = -\sqrt{2}$$

Put value of c_1 and c_2 in equation (i),

$$y(x) = -\sqrt{2}\cdot\sin 3x$$

At $x = \frac{\pi}{4}$;

$$y\left(\frac{\pi}{4}\right) = -\sqrt{2}\cdot\sin\left(\frac{3\pi}{4}\right) = -\sqrt{2}\cdot\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \boxed{y\left(\frac{\pi}{4}\right) = -1}$$

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56. (a)

Complimentary function (C.F.) = ?,

Particular Integral (P.I.) = ?

$$\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2$$

This is non homogenous linear differential equation with constant coefficients.

Replace $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$$\left(D^4 + 3D^2\right)y = 108x^2$$

Its auxiliary equation is:

$$m^4 + 3m^2 = 0 \Rightarrow m^2(m^2 + 3) = 0$$

$$\Rightarrow m = 0, 0, \pm\sqrt{3}i$$

Here two roots 0 and 0 are real and repeated and two roots are in the form of pair of complex number $\alpha \pm i\beta$,where $\alpha = 0$ and $\beta = \sqrt{3}$, C.F. is given by,

$$\text{C.F.} = (C_1 + C_2x)\, e^{0x} + e^{0x}\left(C_3 \sin\sqrt{3}x + C_4 \cos\sqrt{3}x\right)$$

$$\Rightarrow \boxed{\text{C.F.} = C_1 + C_2x + C_3 \sin\sqrt{3}x + C_4 \cos\sqrt{3}x}$$

Since, this is non-homogenous equation, therefore

$$\text{P.I.} = \frac{1}{f(D)}.108x^2 = 108\left(\frac{1}{D^4 + 3D^2}x^2\right)$$

$$\text{P.I.} = 108\left(\frac{1}{3D^2\left(1 + \frac{D^2}{3}\right)}\right)x^2 = \frac{36}{D^2}\left(1 + \frac{D^2}{3}\right)^{-1}\left(x^2\right)$$

$$\text{P.I.} = \frac{36}{D^2}\left[1 - \frac{D^2}{3} + \left(\frac{D^2}{3}\right)^2 \ldots\right]\left(x^2\right)$$

$$\text{P.I.} = \frac{36}{D^2}\left[x^2 - \frac{2}{3} + 0\right] = \frac{36}{D^2}\left[x^2 - \frac{2}{3}\right]$$

$$\text{P.I.} = 36\iint\left(x^2 - \frac{2}{3}\right)dx = 36\int\left(\frac{x^3}{3} - \frac{2}{3}x\right)dx$$

$$\text{P.I.} = 36\left[\frac{x^4}{12} - \frac{2}{3}\times\frac{x^2}{2}\right]$$

$$\boxed{\text{P.I.} = 3x^4 - 12x^2}$$

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57. (a)

$$\frac{d^2y}{dx^2} + 12\cdot\frac{dy}{dx} + 36y = 0 \text{ ; } y(0) = 3 \quad \left.\frac{dy}{dx}\right|_{x=0} = -36$$

$$\frac{d^2y}{dx^2} + 12\cdot\frac{dy}{dx} + 36y = 0$$

This is homogenous linear differential equation with constant coefficients.

Replace $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$$(D^2 + 12D + 36)\,y = 0$$

The Auxiliary equation is,

$$m^2 + 12m + 36 = 0$$

$\Rightarrow \quad (m+6)^2 = 0$

$\Rightarrow \quad m = -6, -6$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = (c_1x + c_2)\cdot e^{-6x}$$

Since, this is homogenous equation, therefore

$\text{P.I.} = 0$

Now, Complete solution:

$$y = \text{C.F.} + \text{P.I.}$$

$\Rightarrow \quad y = (c_1x + c_2)\cdot e^{-6x} \quad \ldots(i)$

Differentiating above equation with respect to x,

$$\frac{dy}{dx} = -6(c_1x + c_2)\cdot e^{-6x} + e^{-6x}c_1 \ldots(ii)$$

Given, $y(0) = 3$; $\therefore$ from equation (i),

$\Rightarrow \quad 3 = c_2\cdot e^{-6(0)} \Rightarrow c_2 = 3$

$\left.\frac{dy}{dx}\right|_{x=0} = -36$ and $c_2 = 3$, from equation (ii),

$\Rightarrow \quad -36 = -6(0+3)\cdot e^0 + e^0c_1$

$\Rightarrow \quad -36 = -18 + c_1$

$\Rightarrow \quad c_1 = -18$

Put value of c_1 and c_2 in equation (i),

$$y = (-18x + 3)\cdot e^{-6x}$$

$\therefore$ The solution is, $\boxed{y = (3 - 18x)e^{-6x}}$.

Scan for Video solution

58. (b)

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0 \text{ ; } y(0) = 1 \text{ and } y(1) = 3e^{-1}$$

Find: $y(2) = ?$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace $\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$$(D^2 + 2D + 1)\,y = 0$$

The Auxiliary equation is:

$$(m^2 + 2m + 1) = 0$$

$\Rightarrow \quad (m+1)^2 = 0$

$\Rightarrow \quad m = -1, -1$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = (C_1 + C_2t)\,e^{-t}$$

Since, this is homogenous equation, therefore

Particular integral $(\text{P.I.}) = 0$

Now, Complete solution:

$$y = \text{C.F.} + \text{P.I.}$$

$\Rightarrow y = (C_1 + C_2t)\,e^{-t} \quad \ldots(i)$

At $t = 0$; $y = 1$; hence from equation (i),

$$1 = (C_1 + 0)e^0 \Rightarrow C_1 = 1$$

At $t = 1$; $y = 3e^{-1}$ and $C_1 = 1$; hence from equation (i),

$3e^{-1} = (1 + C_2 \times 1)e^{-1}$

$\Rightarrow \quad 1 + C_2 = 3$

$\Rightarrow \quad C_2 = 2$

Put value of C_1 and C_2 in equation (i),

$\therefore \quad y = (1 + 2t)\, e^{-t}$

At $t = 2$;

$y(2) = (1 + 2 \times 2)e^{-2}$

$\boxed{y(2) = 5e^{-2}}$

Scan for Video solution

59. (7.0 to 7.5)

$\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$; $y(0) = 0$ and $\left.\dfrac{dy}{dx}\right|_{x=0} = 1$

Find: $y(1) = ?$

$\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$

This is homogenous linear differential equation with constant coefficients. Replace

$\dfrac{d}{dx}$ by D and $\dfrac{d^2}{dx^2}$ by D^2

$(D^2 - 4D - 4)\, y = 0$

The Auxiliary equation is

$m^2 - 4m + 4 = 0$

$\Rightarrow \quad (m - 2)^2 = 0$

$\Rightarrow \quad m = 2, 2$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

C.F. $= (C_1 + C_2x)\, e^{2x}$

Since, this is homogenous equation, therefore

P.I. = 0

Now, Complete solution:

$y =$ C.F. + P.I.

$\Rightarrow \quad y = (C_1 + C_2x)\, e^{2x} \quad \ldots$(i)

At $x = 0$; $y = 0$; hence from equation (i),

$0 = (C_1 + C_2 \times 0)e^0$

$\Rightarrow \quad C_1 = 0$

Put value of C_1 in equation (i),

$y = (C_2x)e^{2x} \quad \ldots$(ii)

Differentiating above equation with respect to x,

$\dfrac{dy}{dx} = 2C_2xe^{2x} + C_2.e^{2x} \quad \ldots$(iii)

At $x = 0$; $\dfrac{dy}{dx} = 1$; hence from equation (ii),

$1 = 2C_2(0)e^0 + C_2e^0$

$\Rightarrow \quad C_2 = 1$

Put value of C_2 in equation (ii),

$y = xe^{2x}$

At $x = 1$,

$y = 1 \times e^{2\times1} = e^2$

$\Rightarrow \quad \boxed{y = 7.389}$

Scan for Video solution

60. (a)

$y''(t) + 2y'(t) + y(t) = 0$

Auxiliary equation is $m^2 + 2m + 1 = 0$

$(m + 1)^2 = 0$

$m = -1, -1$

$\therefore$ $y(t)$ = complementary function $= (C_1 + C_2t)\, e^{-t}$

At $t = 0$; $y(t) = 0$

$0 = (C_1 + C_2 \times 0)\, e^0$

$C_1 = 0$

$y(t) = C_2te^{-t}$

$y'(t) = C_2e^{-t} - C_2te^{-t}$

At $t = 0$; $y'(0) = 1$

$1 = C_2e^0 - C_2 \times 0 \times e^0$

$C_2 = 1$

$\therefore$ $y(t) = te^{-t}$

$y(t) = te^{-t}u(t)$

$\{\because \; t > 0;\; u(t) = 1\}$

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61. (0.83 to 0.88)

Given:

$$\frac{d^2x(t)}{dt^2}+\frac{dx(t)}{dt}+2x(t)=0;$$

x(0) = 20 and x(1) = 10/e

$$\frac{d^2x(t)}{dt^2}+\frac{dx(t)}{dt}+2x(t)=0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$\Rightarrow (D^2 + 3D + 2)\, x(t) = 0$

Auxiliary equation is

$m^2 + 3m + 2 = 0$

$\Rightarrow$ (m + 1) (m + 2) = 0

$\Rightarrow$ m = –1, m = –2

Since roots are real and different, therefore complimentary function (C.F.) is given by,

C.F. = $C_1 \cdot e^{-t} + C_2 \cdot e^{-2t}$

Since, this is homogenous equation, therefore

P.I. = 0

Now, Complete solution:

$x(t) = \text{C.F.} + \text{P.I.}$

$x(t) = C_1 \cdot e^{-t} + C_2 \cdot e^{-2t}$ ……..(i)

Given x(0) = 20; ∴ from equation (i),

$\Rightarrow 20 = C_1 + C_2$ ……..(ii)

$x(1) = \frac{10}{e}$; ∴ from equation (i),

$$\Rightarrow \frac{10}{e} = \frac{C_1}{e} + \frac{C_2}{e^2} \Rightarrow 10 = C_1 + \frac{C_2}{e} \quad \text{….(iii)}$$

Solving (ii) and (iii)

$$\Rightarrow C_2 = \frac{10e}{e-1},\ C_1 = \frac{10(e-2)}{e-1}$$

Put value of C_1 and C_2 in equation (i),

$$x(t)=\frac{10(e-2)}{(e-1)}\cdot e^{-t}+\frac{10e}{(e-1)}e^{-2t}$$

At t =2,

$$x(2)=\frac{10(e-2)}{(e-1)}\cdot e^{-2}+\frac{10e}{(e-1)}e^{-4}$$

$$\Rightarrow \quad x(2)=\frac{10}{(e-1)}\left(e^{-1}-2e^{-2}+e^{-3}\right)$$

$\Rightarrow$ $\boxed{x(2) = 0.855}$

Scan for Video solution

62. (b)

$$\frac{d^2y}{dt^2} + 2\cdot\frac{dy}{dt} + y = 0; \qquad y(0) = y'(0) = 1$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$(D^2 + 2D + 1)\, y = 0$

∴ Auxiliary equation is:

$m^2 + 2m + 1 = 0$

$\Rightarrow (m + 1)^2 = 0$

$\Rightarrow$ m = –1, 1

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$\text{C.F.} = (C_1 t + C_2)e^{-t}$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: y = C.F. + P.I.

$y = (C_1 t + C_2) \cdot e^{-t}$ …(i)

Differentiating above equation with respect to t,

$y'(t) = -(C_1 t + C_2) \cdot e^{-t} + \cdot\, e^{-t} \cdot C_1$ …(ii)

y(0) = 0 ; from equation (i)

$\Rightarrow 1 = C_2 \cdot e^{-0} \Rightarrow C_2 = 1$

y′ (0) = 1 and C_2 = 1; ∴ from equation (i)

$\Rightarrow 1 = -1 + C_1 \Rightarrow C_1 = 2$

Put value of C_1 and C_2 in equation (i),

$\boxed{\mathbf{y(t) = (2t + 1) \cdot e^{-t}}}$

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63. (18)

Find: y (1) = ?

$$\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$$

Integrating both sides w.r.t. x

$$\Rightarrow \frac{dy}{dx} = -4x^3 + 12x^2 - 20x + c_1 \qquad \ldots(i)$$

Integrating both sides, w.r.t. x

$$\Rightarrow y = -x^4 + 4x^3 - 10x^2 + c_1x + c_2 \qquad \ldots(ii)$$

At x = 0, y = 5, hence from equation (ii)

$c_2 = 5$

At x = 2, y = 21, hence from equation (ii)

$$21 = -16 + 32 - 40 + 2c_1 + c_2$$

$$\Rightarrow 21 = -24 + 2c_1 + 5$$

$$\Rightarrow c_1 = 20$$

$\therefore$ From equation (ii),

$$y = -x^4 + 4x^3 - 10x^2 + 20x + 5$$

At x = 1

$$y(1) = -1 + 4 - 10 + 20 + 5$$

$$\boxed{y(1) = 18}$$

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64. (c)

Given $\frac{d^2y}{dx^2} = y$ and curve passes through (0, 0) i.e.

at x = 0; y = 1 and $\left(\ln 2, \frac{3}{4}\right)$ i.e. at x = ln 2; $y = \frac{3}{4}$.

$$\frac{d^2y}{dx^2} = y \Rightarrow \frac{d^2y}{dx^2} - y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2;

$$(D^2 - 1)y = 0$$

The Auxiliary equation is

$$m^2 - 1 = 0 \Rightarrow m = \pm 1$$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = c_1e^x + c_2e^{-x}$$

Since, this is homogenous equation, therefore

$$\text{P.I.} = 0$$

Now, Complete solution:

$$y = \text{C.F.} + \text{P.I.}$$

$$\Rightarrow y = c_1e^x + c_2e^{-x} \qquad \ldots(i)$$

Since at x = 0; y = 1; $\therefore$ from equation (i),

$$0 = c_1 + c_2 \Rightarrow c_2 = -c_1$$

Since at x = ln 2; $y = \frac{3}{4}$, $\therefore$ from equation (i),

$$\frac{3}{4} = c_1e^{\ln 2} + c_2e^{-\ln 2} \qquad [\text{Since } c_2 = -c_1]$$

$$\Rightarrow \frac{3}{4} = 2c_1 - \frac{c_1}{2} \Rightarrow \frac{3}{4} = \frac{3}{2}c_1$$

$$\Rightarrow c_1 = \frac{1}{2}$$

$$\text{and } c_2 = -c_1 \Rightarrow c_2 = \frac{-1}{2}$$

Put value of c_1 and c_2 in equation (i),

$$y = \frac{1}{2} \times e^x + \left(-\frac{1}{2}\right) \times e^{-x}$$

$$\Rightarrow \boxed{y = \frac{1}{2}\left(e^x - e^{-x}\right)}$$

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65. (34 to 36)

$\frac{d^2y}{dx^2} = 0$; at x = 0; y = 5 and at x = 10; $\frac{dy}{dx} = 2$

Find: $f(15) = ?$

$$\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = 0$$

On Integrating,

$$\frac{dy}{dx} = c_1 \qquad \ldots(i)$$

at x = 10; $\frac{dy}{dx} = 2$, hence from equation (i),

$$c_1 = 2$$

Put value of c_1 in equation (i),

$$\frac{dy}{dx} = 2$$

On Integrating,

$$y = 2x + c_2 \quad \ldots(ii)$$

At x = 0; y = 5; hence from equation (ii),

$$5 = 0 + c_2 \Rightarrow c_2 = 5$$

Put value of c_2 in equation (ii),

$$y = 2x + 5$$

$$\Rightarrow f(x) = 2x + 5$$

At x = 15

$$f(15) = 2(15) + 5 = 35$$

$$\Rightarrow \boxed{f(15) = 35}$$

Scan for Video solution

66. (a)

Given $\frac{d^2x(t)}{dt^2} + x(t) = 0$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$$(D^2 + 1)\,x(t) = 0$$

The Auxiliary equation is

$$m^2 + 1 = 0 \Rightarrow m = \pm i$$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$,where $\alpha = 0$ and $\beta = 1$, therefore complimentary function (C.F.) is given by,

$$\text{C.F.} = e^{\alpha t}\left(c_1 \cos\beta t + c_2 \sin\beta t\right)$$

$$\Rightarrow \text{C.F.} = e^{0\times x}\left(c_1 \cos(t) + c_2 \sin(t)\right)$$

$$\Rightarrow \text{C.F.} = c_1 \cos(t) + c_2 \sin(t)$$

Since, this is homogenous equation, therefore

$$\text{Particular integral } (\text{P.I.}) = 0$$

Now, Complete solution:

$$x(t) = \text{C.F.} + \text{P.I.}$$

$$\Rightarrow x(t) = c_1 \cos t + c_2 \sin t$$

At t =0, cos t is 1 and sin t is 0. Since $x_1(0) = 1$, therefore cos t is $x_1(t)$ and hence sin t will be $x_2(t)$.

$\Rightarrow x_1(t) = \cos t$ and $x_2(t) = \sin t$

$$W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_2(t)}{dt} \end{vmatrix}$$

$$\Rightarrow W(t) = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t$$

$$\Rightarrow W(t) = 1$$

At $t = \frac{\pi}{2}$;

$$\boxed{W(t) = 1}$$

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67. (a)

Given DE is $\frac{d^2y}{dx^2} + 2\alpha \cdot \frac{dy}{dx} + y = 0$ and having two equal roots.

$$\frac{d^2y}{dx^2} + 2\alpha \cdot \frac{dy}{dx} + y = 0$$

This is homogenous linear differential equation with constant coefficients. Replace,

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$\Rightarrow (D^2 + 2\alpha D + 1)y = 0$

$\therefore$ Auxiliary Equation is:

$m^2 + 2\alpha m + 1 = 0.$

Comparing with $ax^2 + bx + c = 0$: a =1, b = 2α, c = 1,

For the equation to have equal root,

$$b^2 - 4ac = 0$$

$\Rightarrow$ $(2\alpha)^2 - 4(1)(1) = 0$

$\Rightarrow$ $4\alpha^2 - 4 = 0$

$\Rightarrow$ $\alpha^2 - 1 = 0$

$\Rightarrow$ $\alpha = 1, -1$

$\therefore$ **For the DE to have equal roots, $\alpha = \pm 1$**

Scan for Video solution

68. (b)

Given DE is $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$\Rightarrow (D^2 + 2D + 1)\, x = 0$

$\therefore$ Auxiliary equation is:

$m^2 + 2m + 1 = 0$

$\Rightarrow$ $(m + 1)^2 = 0$

$\Rightarrow$ $m = –1, 1$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = C.F. + P.I.$

$$C.F. = (a + bt)e^{-t} = ae^{-t} + bte^{-t}$$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution:

$$x = C.F. + P.I.$$

$$\Rightarrow \boxed{x = ae^{-t} + bte^{-t}}$$

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69. (0.53 to 0.55)

$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$; $y(0) = y'(0) = 1$

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$\Rightarrow (D^2 + 4D + 4)y = 0$

$\therefore$ Auxiliary equation is:

$m^2 + 4m + 4 = 0$

$\Rightarrow$ $(m+ 2)^2 = 0$

$\Rightarrow$ $m = –2, 2$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

$$C.F. = (C_1 + C_2x)e^{-2x}$$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = C.F. + P.I.$

$y(x) = (C_1 + C_2x)e^{-2x}$ …(i)

Differentiating above equation with respect to x,

$$y'(x) = (C_1 + C_2x)(-2e^{-2x}) + C_2e^{-2x} \quad \text{…(ii)}$$

Since, $y(0) = 1$; from equation (i)

$\Rightarrow 1 = C_1 \cdot e^{-2(0)} \Rightarrow C_1 = 1$

Since, $y'(0) = 1$ and $C_1 = 1$; $\therefore$ from equation (ii)

$$1 = (1+0)(-2e^0) + C_2e^0$$

$\Rightarrow$ $-2+ C_2 = 1$

$\Rightarrow$ $C_2 = 3$

Put value of C_1 and C_2 in equation (i),

$$y(x) = (1+3x)e^{-2x}$$

$\therefore$ at x = 1,

$$y(1) = (1+3\times1)e^{-2\times1}$$

$$\Rightarrow y(1) = \frac{4}{e^2}$$

$$\Rightarrow y(1) = 0.541$$

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70. (c)

$$x^2\cdot\frac{d^2y}{dx^2}+x\cdot\frac{dy}{dx}-y=0 \quad \text{…(i)}$$

This differential equation is Euler Cauchy’s form of linear differential equation.

Let, $x = e^z$ and $\frac{d}{dz} = D$

$$\Rightarrow x\frac{d}{dx} = D \;\&\; x^2\frac{d^2}{dx^2} = D(D-1)$$

Then equation (i) can be written as:

$[D(D-1) + D - 1]\,y = 0$

$\Rightarrow (D^2 - 1)\,y = 0$ …(ii)

Equation (ii) is homogenous linear differential equation with constant coefficients.

Its auxiliary equation is:

$m^2 + 1 = 0 \Rightarrow m = \pm 1$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

C.F. $= C_1 e^z + C_2 e^{-z}$

Since, Equation (ii) is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$y = C_1 e^z + C_2 e^{-z}$ $\left[\because x = e^z\right]$

$\Rightarrow y = C_1 x + \frac{C_2}{x}$ …(iii)

In given option $y = \frac{1}{x}$ is one of the solution of given differential equation when C_1 is 0 and C_2 is 1.

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71. (c)

Given: $\frac{d^2x}{dt^2} + 9x = 0$; $x(0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 1$,

$$\frac{d^2x}{dt^2} + 9x = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$(D^2 + 9)\,x = 0$

The Auxiliary equation is

$m^2 + 9 = 0 \Rightarrow m = \pm 3i$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$, where $\alpha = 0$ and $\beta = 3$, therefore complimentary function (C.F.) is given by,

C.F. $= e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t)$

$\Rightarrow$ C.F. $= e^{0\times x}(c_1 \cos(3t) + c_2 \sin(3t))$

$\Rightarrow$ C.F. $= c_1 \cos 3t + c_2 \sin 3t$

Since, this is homogenous equation, therefore

Particular integral (P.I.) = 0

Now, Complete solution:

$x = \text{C.F.} + \text{P.I.}$

$x = (c_1 \cos 3t + c_2 \sin 3t)$ …(i)

Differentiating above equation with respect to t,

$\frac{dx}{dt} = -3c_1 \sin 3t + 3c_2 \cos 3t$ …(ii)

At t = 0; x = 1 ; hence from equation (i),

$1 = c_1 \times 1 + c_2 \times 0$

$\Rightarrow c_1 = 1$

At t = 0; $\frac{dx}{dt} = 1$, and $c_1 = 1$; hence from equation (ii),

$1 = -3 \times 1 \times 0 + 3c_2 \times 1$

$\Rightarrow c_2 = 1/3$

Put value of c_1 and c_2 in equation (ii),

$$\boxed{x = \cos 3t + \frac{1}{3}\sin 3t}$$

Scan for Video solution

72. (b)

Given: $\frac{d^2u}{dx^2} - k \cdot \frac{du}{dx} = 0$; $u(0) = 0$ & $u(L) = U$

$$\frac{d^2u}{dx^2} - k \cdot \frac{du}{dx} = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2;

$(D^2 - kD)\,u = 0$

The auxiliary equation is

$m^2 - km = 0 \quad \Rightarrow m(m - k) = 0$

$\Rightarrow \quad m = 0$ or $m = k$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$C.F. = c_1e^{0.x} + c_2.e^{kx}$

$\Rightarrow C.F. = c_1 + c_2.\ e^{kx}$

Since, this is homogenous equation, therefore

P.I. = 0

Now, Complete solution:

$u = C.F. + P.I.$

$\Rightarrow \quad u = c_1 + c_2.\ e^{kx} \qquad \ldots(i)$

Given u (0) = 0, ∴ from equation (i);

$0 = c_1 + c_2.\ e^0$

$\Rightarrow \quad c_1 + c_2 = 0$

$\Rightarrow \quad c_2 = -c_1$

Given u(L) = U , ∴ from equation (i);

$\Rightarrow \quad U = c_1 + c_2.\ e^{kL} \qquad [\text{Since } c_2 = -c_1]$

$\Rightarrow \quad U = c_1[1 - e^{kL}]$

$$\Rightarrow \quad c_1 = \frac{U}{1-e^{kL}}$$

And $c_2 = -c_1 \Rightarrow c_2 = \dfrac{-U}{1-e^{kL}}$

Put value of c_1 and c_2 in equation (i),

$$u = \frac{U}{1-e^{kL}} + \frac{-U}{1-e^{kL}} \cdot e^{kx}$$

$$\Rightarrow \boxed{u = U\left(\frac{1-e^{kx}}{1-e^{kL}}\right)}$$

Scan for Video solution

73. (c)

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

This is homogenous linear differential equation with constant coefficients.

Replace $\dfrac{d}{dx}$ by D and $\dfrac{d^2}{dx^2}$ by D^2

$$(D^2 + D - 6)y = 0$$

Its auxiliary equation is:

$m^2 + m - 6 = 0$

$\Rightarrow \quad m^2 + 3m - 2m - 6 = 0$

$\Rightarrow \quad (m + 3)(m - 2) = 0$

$\Rightarrow \quad m = -3, 2$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$C.F. = C_1e^{-3x} + C_2e^{2x}$

Since, this is homogenous equation, therefore

P.I. = 0

Now, Complete solution:

$y = C.F. + P.I.$

$$\Rightarrow \boxed{y = C_1e^{-3x} + C_2e^{2x}}$$

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74. (d)

$$\frac{d^2}{dx^2}n(x) - \frac{n(x)}{L^2} = 0; n(0) = K \text{ and } n(\infty) = 0$$

$$\frac{d^2}{dx^2}n(x) - \frac{n(x)}{L^2} = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\dfrac{d}{dx}$ by D and $\dfrac{d^2}{dx^2}$ by D^2;

$$\left(D^2 - \frac{1}{L^2}\right)n(x) = 0$$

The Auxiliary equation is:

$$m^2 - \frac{1}{L^2} = 0 \Rightarrow m = \pm\frac{1}{L}$$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$$C.F. = c_1 \cdot e^{\frac{x}{L}} + c_2 \cdot e^{-\frac{x}{L}}$$

Since, this is homogenous equation, therefore

P.I. = 0

Now, Complete solution:

$n(x) = C.F. + P.I.$

$$\Rightarrow \quad n(x) = c_1 \cdot e^{\frac{x}{L}} + c_2 \cdot e^{-\frac{x}{L}} \qquad \ldots(i)$$

n (0) = K; from equation (i),

$\Rightarrow\ K = c_1 + c_2$ …(ii)

$n(\infty) = 0$; from equation (i),

$\Rightarrow 0 = c_1 \cdot e^{\frac{\infty}{L}} + c_2(0)$

To avoid discontinuity, 'c_1' should be zero.

From equation (ii),

$\Rightarrow K = 0 + c_2 \Rightarrow c_2 = K$

Put value of c_1 and c_2 in equation (i),

$\therefore\ n(x) = K \cdot e^{-\frac{x}{L}}$

$\Rightarrow\ \boxed{n(x) = K \cdot \exp\left(-\frac{x}{L}\right)}$

Scan for Video solution

75. (b)

$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$; $x(0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 0$

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$$

This is homogenous linear differential equation with constant coefficients.

Replace $\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$$(D^2 + 6D + 8)\,x = 0$$

The Auxiliary equation is

$m^2 + 6m + 8 = 0$

$\Rightarrow\ m^2 + 4m + 2m + 8 = 0$

$\Rightarrow\ (m + 4)(m + 2) = 0$

$\Rightarrow\ m = -2, -4$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$CF = C_1e^{-2t} + C_2e^{-4t}$

Since, this is homogenous equation, therefore

P.I. = 0

Now, Complete solution:

$x = \text{C.F.} + \text{P.I.}$

$\Rightarrow\ x = C_1e^{-2t} + C_2e^{-4t}$ …(i)

Differentiating above equation with respect to t,

$\frac{dx}{dt} = -2\ C_1e^{-2t} - 4\ C_2e^{-4t}$ …(ii)

At t = 0; x = 1; hence from equation (i),

$1 = C_1 + C_2$ …..(iii)

At t = 0; $\frac{dx}{dt} = 0$; hence from equation (ii),

$0 = -2\,C_1 - 4\,C_2$ …(iv)

On solving (iii) and (iv)

$C_1 = 2$ and $C_2 = -1$

Put value of C_1 and C_2 in equation (ii),

$x = 2e^{-2t} - 1e^{-4t}$

$\Rightarrow\ \boxed{\mathbf{x = 2e^{-2t} - e^{-4t}}}$

Scan for Video solution

76. (b)

Given: $\ddot{x} + 3x = 0;\ x(0) = 1;\ \dot{x}(0) = 0$

Find: x (1) = ?

$$\ddot{x} + 3x = 0 \Rightarrow \frac{d^2x}{dt^2} + 3x = 0$$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dt}$ by D and $\frac{d^2}{dt^2}$ by D^2;

$(D^2 + 3)\,x = 0$

The auxiliary equation is given by

$m^2 + 3 = 0 \Rightarrow m = \pm\sqrt{3}\,i$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$, where $\alpha = 0$ and $\beta = \sqrt{3}$, therefore complimentary function (C.F.) is given by,

$\text{C.F.} = e^{\alpha x}(c_1 \cos\beta t + c_2 \sin\beta t)$

$\Rightarrow\ \text{C.F.} = e^{0\times t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right)$

$\Rightarrow\ \text{C.F.} = c_1 \cos\sqrt{3}t + c_2 \sin\sqrt{3}t$

Since, this is homogenous equation, therefore

Particular integral (P.I.) = 0

Now, Complete solution:

$x(t) = \text{C.F.} + \text{P.I.}$

$\Rightarrow x(t) = c_1 \cos\sqrt{3}t + c_2 \sin\sqrt{3}t$ …(i)

Differentiating with respect to t,

$\dot{x}(t) = -c_1\sqrt{3}\cdot\sin\sqrt{3}t + c_2\sqrt{3}\cdot\cos\sqrt{3}t$ …(ii)

$\because$ x (0) = 1, hence from equation (i),

$1 = c_1 + c_2(0) \Rightarrow c_1 = 1$

$\because \dot{x}(0) = 0$ and $c_1 = 1$, hence from equation (ii),

$0 = 0 + c_2\cdot\sqrt{3}\cdot(1) \Rightarrow c_2 = 0$

Put value of c_1 and c_2 in equation (i),

$x(t) = \cos\sqrt{3}t$

At t =1,

$x(1) = \cos\sqrt{3}(1) = -0.16$

$\Rightarrow \boxed{x(1) = -0.16}$

Scan for Video solution

77. (a)

Given: $y''+2y'+y=0$; y (0) = 0; y (1) = 0

Find: y (1) = ?

$y''+2y'+y=0$

This is homogenous linear differential equation with constant coefficients. Replace

$\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2;

The auxiliary equation is:

$m^2 + 2m + 1 = 0$

$\Rightarrow (m+1)^2 = 0$

$\Rightarrow m = -1, -1$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

$\text{C.F.} = (c_1 + c_2x)e^{-x}$

Since, this is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$y = (c_1 + c_2x)e^{-x}$ …(i)

$\because$ y (0) = 0 , hence from equation (i),

$0 = c_1.(1) \Rightarrow c_1 = 0$

$\because$ y (1) = 0 and $c_1 = 0$, hence from equation (i),

$0 = c_2.e^{-1} \Rightarrow c_2 = 0$

Put value of c_1 and c_2 in equation (i),

$y = 0$

at x = 0.5,

$\boxed{y(0.5) = 0}$

Scan for Video solution

78. (a)

$\frac{d^2y}{dx^2} + y = 0$

This is homogenous linear differential equation with constant coefficients.

Replace $\frac{d}{dx}$ by D and $\frac{d^2}{dx^2}$ by D^2

$(D^2+1)y = 0$

Its auxiliary equation is:

$m^2 + 1 = 0 \quad \Rightarrow m = \pm i$

Since roots are in the form of pair of complex number $\alpha \pm i\beta$, where $\alpha = 0$ and $\beta = 1$, therefore complimentary function (C.F.) is given by,

$\text{C.F.} = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

$\Rightarrow \text{C.F.} = e^{0\times x}(c_1\cos(1\times x) + c_2\sin(1\times x))$

$\Rightarrow \text{C.F.} = c_1\cos x + c_2\sin x$

Since, this is homogenous equation, therefore Particular integral (P.I.) = 0

Now, Complete solution:

$y = \text{C.F.} + \text{P.I.}$

$\Rightarrow y = c_1\cos x + c_2\sin x$

Replace arbitrary constants c_1 by P and c_2 by Q,

$\Rightarrow \boxed{y = P\cos x + Q\sin x}$

Scan for Video solution

79. (8.999 to 9.001)

$$x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - y = 0 \quad ;$$

$$y(x=1) = 6, \quad \left.\frac{dy}{dx}\right|_{x=1} = 2$$

Find: y (2) = ?

$$x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - y = 0 \quad \ldots(i)$$

This differential equation is Euler Cauchy's form of linear differential equation.

Let, $x = e^z$ and $\frac{d}{dz} = D$

$$\Rightarrow x\frac{d}{dx} = D \;\&\; x^2\frac{d^2}{dx^2} = D(D-1)$$

Then equation (i) can be written as:

$$[D(D-1) + D - 1]\,y = 0$$

$$\Rightarrow (D^2 - 1)\,y = 0 \quad \ldots(ii)$$

Equation (ii) is homogenous linear differential equation with constant coefficients.
Its auxiliary equation is:

$$m^2 + 1 = 0 \;\Rightarrow\; m = \pm 1$$

Since roots are real and same, therefore complimentary function (C.F.) is given by,
C.F. = $C_1 e^z + C_2 e^{-z}$
Since, Equation (ii) is homogenous equation, therefore P.I. = 0.
Now, Complete solution: y = C.F. + P.I.

$$\Rightarrow y = C_1 e^z + C_2 e^{-z} \quad [\because x = e^z]$$

$$\Rightarrow y = C_1 . x + \frac{C_2}{x} \quad \ldots(iii)$$

$$\Rightarrow \frac{dy}{dx} = C_1 - \frac{C_2}{x^2} \quad \ldots(iv)$$

At x = 1, y = 6, hence from equation (iii),

$$C_1 + C_2 = 6 \quad \ldots(v)$$

At x = 1, $\frac{dy}{dx} = 2$, hence from equation (iv),

$$C_1 - C_2 = 2 \quad \ldots(vi)$$

Solving equation (v) and (vi),

$$C_1 = 4 \text{ and } C_2 = 2$$

Put value of C_1 and C_2 in equation (iii),

$$y = 4x + \frac{2}{x}$$

At x = 2,

$$y(2) = 8 + 1 = 9$$

$$\Rightarrow \boxed{y(2) = 9}$$

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80. (a)

Given

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 4$$

$$\left(x^2\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 2\right)y = 4 \quad \ldots(i)$$

This differential equation is Euler Cauchy's form of linear differential equation.

Let, $x = e^z$ and $\frac{d}{dz} = D$

Then, $x \cdot \frac{d}{dx} = D$ and $x^2\frac{d^2}{dx^2} = D(D-1)$

Then equation (i) can be written as:

$$[D(D-1) - 2D + 2]\,y = 4$$

$$\Rightarrow (D^2 - 3D + 2)\,y = 4 \quad \ldots(ii)$$

Equation (ii) is non-homogenous linear differential equation with constant coefficients.

Its auxiliary equation is:

$$m^2 - 3m + 2 = 0 \Rightarrow (m-2)(m-1) = 0$$

$\Rightarrow$ m = 2, 1

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$$C.F = C_1 e^{2z} + C_2 e^z$$

Since, equation (ii) is non - homogenous equation, therefore Particular integral (P.I.)

$$P.I. = \frac{1}{(D^2 - 3D + 2)} \times 4$$

$$\Rightarrow \text{P.I.} = 4 \cdot \frac{1}{D^2 - 3D + 2} \cdot e^{0.z}$$

$$\Rightarrow \text{P.I.} = 4 \cdot \frac{1}{0^2 - 3\times 0 + 2} \cdot e^{0.z} = \frac{4}{2}$$

$$\Rightarrow \text{P.I.} = 2$$

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$\Rightarrow \quad y = C_1 e^{2z} + C_2 e^z + 2$ …(iii)

Since $x = e^z$ ∴ from equation (iii),

$$\boxed{y = C_1x^2 + C_2x + 2}$$

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81. (5.9 to 6.1)

$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$; y(1) = 0 and y(2) = 2

Given differential equation can be written as:

$\left(x^2 \frac{d^2}{dx^2} - 2x\frac{d}{dx} + 2\right)y = 0$ …(i)

This differential equation is Euler Cauchy's form of linear differential equation.

Let, $x = e^z$ and $\frac{d}{dz} = D$

$$\Rightarrow x\frac{d}{dx} = D \,\&\, x^2\frac{d^2}{dx^2} = D(D-1)$$

Then equation (i) can be written as:

$[D(D–1)–2D + 2]y = 0$

$[D^2\text{-}3D+2]y = 0$ …(ii)

Equation (ii) is homogenous linear differential equation with constant coefficients.

Its auxiliary equation is:

$m^2 – 3m + 2 = 0 \Rightarrow (m–1)(m–2) = 0$

$\Rightarrow m = 1, 2$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

$C.F = C_1 e^z + C_2 e^{2z}$

Since, Equation (ii) is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$y = C_1 e^z + C_2 e^{2z}$ …(iii)

Since $x = e^z$ ∴ from equation (iii),

$y = C_1 x + C_2 x^2$ …(iv)

Given, y(1) = 0; from equation (iv)

$0 = C_1 \times 1 + C_2\times 1^2$

$\Rightarrow C_1 + C_2 = 0$ ...(v)

Given, y(2) = 2; from equation (iv)

$2 = C_1 \times 2 + C_2 \times 2^2$

$\Rightarrow C_1 + 2C_2 = 1$ …(vi)

Equation (vi) − (v), we get

$C_2 = 1$ [Put in equation (v)]

$C_1 + 1 = 0 \Rightarrow C_1 = -1$

Put value of C_1 and C_2 in equation (iv),

$y = – x + x^2$

At x = 3;

$y(3) = –3 + 3^2$

$$\boxed{y(3) = 6}$$

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82. (5.24 to 5.26)

$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$; y(1) = 1 and y(2) = 14

Find: y (1.5) = ?

$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$ …(i)

This differential equation is Euler Cauchy's form of linear differential equation.

Let, $x = e^z$ and $\frac{d}{dz} = D$

$$\Rightarrow x\frac{d}{dx} = D \,\&\, x^2\frac{d^2}{dx^2} = D(D-1)$$

Then equation (i) can be written as:

$$(D(D-1) - 3D + 3)\,y = 0$$

$\Rightarrow \quad (D^2 - 4D + 3)\,y = 0$ …(ii)

Equation (ii) is homogenous linear differential equation with constant coefficients.

Its auxiliary equation is:

$$m^2 - 4m + 3 = 0$$

$$\Rightarrow \quad (m-1)(m-3)=0$$

$$\Rightarrow \quad m=1,3$$

Since roots are real and same, therefore complimentary function (C.F.) is given by,

C.F. = $c_1 e^z + c_2 e^{3z}$

Since, Equation (ii) is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$$\Rightarrow y = c_1 e^z + c_2 e^{3z} \qquad [\because x = e^z]$$

$$\Rightarrow y = c_1 x + c_2 x^3 \qquad \ldots(iii)$$

Given, y(1) = 1, hence from equation (iii),

$$\Rightarrow \quad 1 = c_1 + c_2 \qquad \ldots(iv)$$

Given, y(2) = 14, hence from equation (iii),

$$14 = 2c_1 + 8c_2 \qquad \ldots(v)$$

Solving equation (iv) and (v)

$$\Rightarrow \quad c_1 = -1 \text{ and } c_2 = 2$$

Put value of c_1 and c_2 in equation (iii),

$$y = -x + 2x^3$$

At, x = 1.5,

$$y(1.5) = -1.5 + 2(1.5)^3 = 5.25$$

$$\therefore \quad \boxed{y(1.5) = 5.25}$$

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83. (a)

$$x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - 4y = 0 \quad ; y(0) = 0 \;\&\; y(1) = 1$$

$$x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - 4y = 0 \qquad \ldots(i)$$

This differential equation is Euler Cauchy's form of linear differential equation.

Let, $x = e^z$ and $\frac{d}{dz} = D$

$$\Rightarrow x\frac{d}{dx} = D \;\&\; x^2\frac{d^2}{dx^2} = D(D-1)$$

Then equation (i) can be written as:

$$[D(D-1) + D - 4]\, y = 0$$

$$\Rightarrow \quad (D^2 - 4)\, y = 0 \qquad \ldots(ii)$$

Equation (ii) is homogenous linear differential equation with constant coefficients.

Its auxiliary equation is:

$$m^2 - 4 = 0 \Rightarrow (m-2)(m+2) = 0$$

$$\Rightarrow \quad m = 2, -2$$

Since roots are real and different, therefore complimentary function (C.F.) is given by,

C.F = $c_1 e^{2z} + c_2 e^{-2z}$

Since, Equation (ii) is homogenous equation, therefore P.I. = 0.

Now, Complete solution: $y = \text{C.F.} + \text{P.I.}$

$$y = c_1 e^{2z} + c_2 e^{-2z} \qquad [\because x = e^z]$$

$$\Rightarrow \quad y = c_1 x^2 + \frac{c_2}{x^2}$$

$$\Rightarrow \quad yx^2 = c_1 x^4 + c_2 \qquad \ldots(iii)$$

$\because$ y (0) = 0 , hence from equation (iii)

$$0 = 0 + c_2 \Rightarrow c_2 = 0$$

$\because$ y (1) = 1 and $c_2 = 0$, hence from equation (iii)

$$1 = c_1 \Rightarrow c_1 = 1$$

Put value of c_1 and c_2 in equation (iii),

$$yx^2 = x^4$$

$$\Rightarrow \quad \boxed{y = x^2}$$

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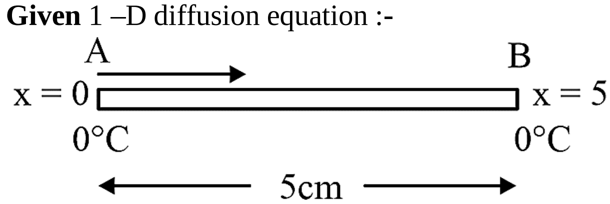
84. (0.395)

Given 1 –D diffusion equation :-



$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$$

Also D = 1,

T(x, 0) =T

T(0, t) = T(5, t) = 0

We know, solution of P.D.E is given by:-

$T(x, t) = (C_1 \cos \alpha x + C_2 \sin \alpha x) C_3 e^{-\alpha^2 t}$ _____(i))

For T (, t) = 0

$0 = (C_1 + 0) C_3\ e^{-\alpha t}$

$\Rightarrow \boxed{C_1 = 0}$

For T (5, t) = 0

$0 = C_2 \sin 5\alpha\, C_3\ e^{-\alpha^2 t}$

$\Rightarrow \sin 5\alpha = 0$

$\Rightarrow \alpha = \dfrac{n\pi}{5}$ [n → 1,2,3……]

Now from equation_____(i)

$$T(x,t) = C_2C_3 \sin\left(\frac{n\pi x}{5}\right) e^{-\frac{n^2\pi^2 t}{25}}$$

∴ The generalized solution will be

$$T(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{5}\right) e^{-\frac{n^2\pi^2}{25}t}$$

From Fourier series,

$b_n = 0$ for even value of n

$$\therefore \quad T(x,t) = \sum_{n=1,3,5}^{\infty} b_n \sin\left(\frac{n\pi x}{5}\right) e^{-\frac{n^2\pi^2}{25}t}$$

Given $T(x,t) = \sum_{n=1,3,5}^{\infty} C_n \sin\left(\frac{n\pi x}{5}\right) e^{-\beta n^2 t}$

On comparison:

$$\beta = \frac{\pi^2}{25} s^{-1} = 0.395 \sec^{-1}$$

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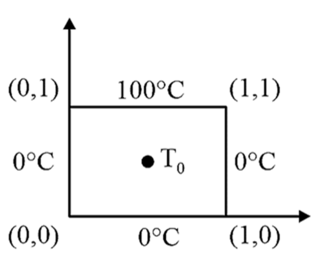
85. (25)

We know, Laplace equation is 2-dimension on a unit square with Dirichlet boundary condition:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad 0 < x, y < 1$$



This Laplace equation is also called harmonic function whose solution is given by

$$u(x,y) = (C_1 \cos \beta x + C_2 \sin \beta x)\left(C_3 e^{\beta y} + C_4 e^{-\beta y}\right)$$

Let square ABCD is unit square.

Now given u(x,0) = u(0, y) = u(1,y) = 0

u(x, 1) = 100

For u(x, 0) = 0

$\Rightarrow 0 = (C_1 \cos\beta x + C_2 \sin\beta x)(C_3 + C_4)$

$\boxed{C_3 = -C_4}$

For u(0, y) = 0

$\Rightarrow 0 = C_1\left(C_3 e^{\beta y} + C_4 e^{-\beta y}\right)$

$\boxed{C_1 = 0}$

Now $u(x,y) = C_2 \sin\beta x\left(C_3 e^{\beta y} - C_3 e^{-\beta y}\right)$

[$c_2c_3 = a_x$]

$u(x,y) = a_x \sin\beta x\left(e^{\beta y} - e^{-\beta y}\right)$_______(i)

For $u(1,y) = 0$

$0 = a_x \sin\beta\left(e^{\beta y} - e^{-\beta y}\right)$

$\sin\beta = 0$

$\boxed{\beta = n\pi}$

For u(x,1) =100,

$100 = a_x \sin n\pi x\left(e^{n\pi} - e^{-n\pi}\right)$

$$\Rightarrow \boxed{a_x = \frac{100}{\sin n\pi x\left(e^{n\pi} - e^{-n\pi}\right)}}$$

From equation (i)

$$u(x,y) = \sum_{n=1}^{\infty} \frac{100}{\sin n\pi x\left(e^{n\pi} - e^{-n\pi}\right)} \sin n\pi x\left(e^{n\pi y} - e^{-n\pi y}\right)$$

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} 100\left(\frac{e^{n\pi y} - e^{-n\pi y}}{e^{n\pi} - e^{-n\pi}}\right)}$$

Now, at mid-point (x, y) = $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\left(\frac{1}{2}, \frac{1}{2}\right)$ [Given]

$$\therefore \ T_0 = \sum_{n=1}^{\infty} 100 \frac{e^{n\pi/2} - e^{-n\pi/2}}{e^{n\pi} - e^{-n\pi}}$$

$$= 100\left[\frac{e^{\pi/2} - e^{-\pi/2}}{e^{\pi} - e^{\pi}} + \frac{e^{\pi} - e^{-\pi}}{e^{2\pi} - e^{-2\pi}} + \frac{e^{3\pi/2} - e^{-3\pi/2}}{e^{3\pi} - e^{-3\pi}} + \ldots\right]$$

$= 100\ [0.199 + 0.043 + 0.008 + \ldots]$

$\boxed{T_0 \approx 25°C}$

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86. **(a)**

$Z = \sin(y + it) + \cos(y - it)$ …(i)

Differentiating equation (i) partially with respect x,

$$\frac{\partial Z}{\partial y} = \cos(y + it) - \sin(y - it)$$

Again, differentiating equation (i) partially with respect x,

$$\frac{\partial^2 Z}{\partial y^2} = -\sin(y + it) - \cos(y - it) \quad \ldots(ii)$$

Differentiating equation (i) partially with respect t,

$$\frac{\partial Z}{\partial t} = i\cos(y + it) + i\sin(y - it)$$

Again, differentiating equation (i) partially with respect t,

$$\frac{\partial^2 Z}{\partial t^2} = -i^2\sin(y + it) - i^2\cos(y - it)$$

[Since, $i^2 = -1$]

$$\frac{\partial^2 Z}{\partial t^2} = \sin(y + it) + \cos(y - it) \quad \ldots(iii)$$

Adding equation (ii) and (i)

$$\frac{\partial^2 Z}{\partial t^2} + \frac{\partial^2 Z}{\partial y^2} = 0$$

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87. **(d)**

According to given condition function f(x,y) is nothing but constant function i.e. f(x, y) = 3 because this the only function whose value is 3 at any point on the boundary of unit circle and it also satisfies Laplace's equation,

$f(x, y) = 3$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

$\boxed{\therefore f(0,0) = 3}$

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88. **(a, b)**

Given, PDE is

$$a \cdot \frac{\partial^2 f}{\partial x^2} + b \cdot \frac{\partial^2 f}{\partial y^2} = f(x, y) \quad \ldots(1)$$

Given,

$$f(x, y) = e^{(\xi x + \eta y)} = e^{\xi x} \cdot e^{\eta y} \quad \ldots(2)$$

Differentiating partially with respect to x,

$$\Rightarrow \frac{\partial f}{\partial x} = \xi \cdot e^{\xi x} \cdot e^{\eta y}$$

Differentiating again partially with respect to x,

$$\Rightarrow \frac{\partial^2 f}{\partial x^2} = \xi^2 \cdot e^{\xi x} \cdot e^{\eta y} \quad \ldots(3)$$

Differentiating equation (2) partially with respect to y,

$$\frac{\partial f}{\partial y} = \eta \cdot e^{\xi x} \cdot e^{\eta y}$$

Differentiating again partially with respect to y,

$$\Rightarrow \frac{\partial^2 f}{\partial y^2} = \eta^2 \cdot e^{\xi x} \cdot e^{\eta y} \quad \ldots(4)$$

From equation (1), (2), (3) and (4),

$$\Rightarrow a(\xi^2 \cdot e^{\xi x} \cdot e^{\eta y}) + b(\eta^2 \cdot e^{\xi x} \cdot e^{\eta y}) = e^{\xi x} \cdot e^{\eta y}$$

$$\Rightarrow (a\xi^2 + b\eta^2) \times e^{\xi x} \cdot e^{\eta y} = e^{\xi x} \cdot e^{\eta y}$$

$\Rightarrow \quad a\xi^2 + b\eta^2 = 1$

Out of the given options, A and B satisfy the above equation. Hence option A and B are correct.

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89. (b)

1^{st} term of given equation contains product of dependent variable with its derivative, so it is non-linear and also we have 2^{nd} order derivative so its order is 2

$\therefore$ It is second order non - linear differential equation.

Scan for Video solution

90. (a)

The given P.D.E is parabolic which requires one initial condition and two boundary conditions to 777solve it uniquely.

Scan for Video solution

91. (b)

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

This is 2–D wave equation.

Scan for Video solution

92. (b)

The equation $\frac{\partial u}{\partial t} = C^2 \cdot \frac{\partial^2 u}{\partial x^2}$ is standard form of 1-dimensional heat equation.

$\therefore$ At C = 1, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

$\therefore$ Given equation is heat equation.

NOTE:

(1) $\frac{\partial u}{\partial t} = C^2 \cdot \frac{\partial^2 u}{\partial x^2} \rightarrow$ Heat equation

(2) $\frac{\partial^2 u}{\partial t^2} = C^2 \cdot \frac{\partial^2 u}{\partial x^2} \rightarrow$ Wave equation

(3) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow$ Laplace equation

Scan for Video solution

93. (b)

Given

$$\frac{\partial u}{\partial y} + c \cdot \frac{\partial u}{\partial x} = 0 \qquad \ldots(i)$$

Let $u = f(ax + by)$ is a solution, then

$\Rightarrow \quad \frac{\partial u}{\partial x} = a.f'(ax + by)$

And $\quad \frac{\partial u}{\partial y} = b.f'(ax + by)$

Hence from equation (i0,

$b \cdot f'(ax + by) + c.\ a\ f'(ax + by) = 0$

$\Rightarrow$ $(b + ac) f'(ax + by) = 0$

$\Rightarrow$ $ac + b = 0$

$\Rightarrow$ $b = -ac$

The solution is:

$u = f(ax + by)$ [Since $b = -ac$]

$\Rightarrow$ $u = f(ax - acy)$

$\Rightarrow$ $u = f\{a(x - cy)\}$

For $a = 1$,

$\Rightarrow \quad \boxed{u = f(x - cy)}$.

Scan for Video solution

94. (36)

Given,

$$3\frac{\partial^2\phi}{\partial x^2}+B\frac{\partial^2\phi}{\partial x\partial y}+3\frac{\partial^2\phi}{\partial y^2}+4\phi=0\ \ldots(i)$$

And equation is parabolic.

General from of second order partial differential equation,

$$A\frac{\partial^2\phi}{\partial x^2}+B\frac{\partial^2\phi}{\partial x\partial y}+C\frac{\partial^2\phi}{\partial y^2}+f\left(\phi_x,\phi_y,x,y,c\right)=0$$

If $B^2 - 4AC$	**Type of curve**
0	Parabolic
Less than 0	Elliptic
Greater than 0	Hyperbolic

Comparing General from of second order partial differential equation with given equation (i),

We have, A = 3, B = B , C = 3

For P.D.E to be parabolic,

$B^2 - 4AC = 0$

$B^2 - 4(3)(3) = 0$

$\boxed{B^2 = 36}$

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95. (d)

Standard form of wave equation:

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$$

Given, $\dfrac{\partial^2 u}{\partial x^2}=25\dfrac{\partial^2 u}{\partial t^2}$

$$\frac{\partial^2 u}{\partial t^2}=\frac{1}{25}\frac{\partial^2 u}{\partial x^2}$$

$\therefore$ From standard form value of $c=\frac{1}{5}$ and initial conditions f(x) = u(x,0) = 3x and $\frac{\partial u}{\partial t}(t=0)$ = g(y) = 3

So, by De-Alembert's' solution of wave equation:-

$$u(x,t)=\frac{1}{2}\left[f(x+ct)+f(x-ct)\right]+\frac{1}{2c}\int_{x-ct}^{x+ct}g(y)\,dy$$

$$=\frac{1}{2}\left[3(x+ct)+3(x-ct)\right]+\frac{1}{2c}\int_{x-ct}^{x+ct}3\,dy$$

$$=\frac{3}{2}[2x]+\frac{3}{2c}[x+ct-x+ct]$$

u(x,t) = 3x + 3t

At x = 1 and t = 1

u(1,1) = 3×1 + 3×1 = 6

Scan for Video solution

96. (c)

Given,

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+3\frac{\partial^2 P}{\partial x\partial y}+2\frac{\partial P}{\partial x}-\frac{\partial P}{\partial y}=0$$

…(i)

General from of second order partial differential equation,

$$A\frac{\partial^2 p}{\partial x^2}+B\frac{\partial^2 p}{\partial x\partial y}+C\frac{\partial^2 p}{\partial y^2}+f\left(p_x,p_y,x,y,c\right)=0$$

If $B^2 - 4AC$	**Type of curve**
0	Parabolic
Less than 0	Elliptic
Greater than 0	Hyperbolic

Comparing General form of second order partial differential equation with given equation (i),

We have, A = 1, B = 3 ,C = 1

Now,

$B^2 - 4AC = 3^2 - 4(1)(1) = 5 > 0$

Since $B^2 - 4AC > 0$ $\therefore$ partial differential equation is hyperbolic.

Scan for Video solution

97. (b)

The solution of $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ is

$$u(x,t) = (A\cos px + B\sin px)Ce^{-p^2\alpha t} \quad \_\_\_\_\_\_\_\_\_\_ (i)$$

Let $-p^2 \alpha = k$

$$\Rightarrow p = \sqrt{-\frac{k}{\alpha}} = \sqrt{\frac{k}{\alpha}}i$$

Putting value of p in equation (i)

$$u(x,t) = \left[A \quad \cos i\sqrt{\frac{k}{\alpha}}x + B\sin i\sqrt{\frac{k}{\alpha}}x\right]Ce^{kt}$$

$$u(x,t) = \left[A\cosh\sqrt{\frac{k}{\alpha}}x + iB\sin h\sqrt{\frac{k}{\alpha}}x\right]Ce^{kt}$$

$$u(x,t) = \left[A\left\{\frac{e^{\sqrt{\frac{k}{\alpha}}x} + e^{-\sqrt{\frac{k}{\alpha}}x}}{2}\right\} + iB\left\{\frac{e^{\sqrt{\frac{k}{\alpha}}x} - e^{-\sqrt{\frac{k}{\alpha}}x}}{2}\right\}\right]Ce^{kt}$$

$$u(x,t) = Ce^{kt}\left[e^{\sqrt{\frac{k}{\alpha}}x}\left\{\frac{A+iB}{2}\right\} + e^{-\sqrt{\frac{k}{\alpha}}x}\left\{\frac{A-iB}{2}\right\}\right]$$

$$\boxed{u(x,t) = Ce^{kt}\left(C_1 e^{\sqrt{\frac{k}{\alpha}}x} + C_2 e^{-\sqrt{\frac{k}{\alpha}}x}\right)}$$

Scan for Video solution

98. (b)

Laplace equation is $\nabla^2 f = 0$

Any solution of Laplace equation with 2nd order derivatives being continuous is called **Harmonic functions.**

Scan for Video solution

99. (c)

Given z = (xy) ln (xy) …(i)

Differentiating equation (i) partially with respect to x,

$$\frac{\partial z}{\partial x} = (xy)\frac{1}{xy}y + \ln(xy)y$$

$$\Rightarrow \frac{\partial z}{\partial x} = y(1 + \ln(xy))$$

Multiply above equation by x

$$\Rightarrow x\frac{\partial z}{\partial x} = xy(1 + \ln(xy)) \quad \ldots(ii)$$

Differentiating equation (i) partially with respect to y,

$$\frac{\partial z}{\partial y} = (xy)\frac{1}{xy}x + \ln(xy)x$$

$$\Rightarrow \frac{\partial z}{\partial y} = x(1 + \ln(xy))$$

Multiply above equation by y,

$$\Rightarrow y\frac{\partial z}{\partial y} = xy(1 + \ln(xy)) \quad \ldots(iii)$$

Equation (ii) ÷ (iii)

$$\frac{x\frac{\partial z}{\partial x}}{y\frac{\partial z}{\partial y}} = \frac{xy(1+\ln(xy))}{xy(1+\ln(xy))}$$

$$\Rightarrow \boxed{x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}}$$

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100. (c)

z = ax + by + ab …(i)

Differentiating partially equation (i) with respect to x,

$$p = \frac{\partial z}{\partial x} = a \Rightarrow a = p$$

Differentiating partially equation (i) with respect to y,

$$q = \frac{\partial z}{\partial y} = b \Rightarrow b = q$$

By substituting a and b in (i) in terms of p and q, we get

$$\boxed{z = px + qy + pq}$$

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101. (c)

Given:

$f = y^x$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(y^x \cdot \ln y\right)$$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = y^x \cdot \frac{1}{y} + \ln y \times x.y^{x-1}$$

$$\Rightarrow \frac{\partial^2 f}{\partial x\, \partial y} = y^x \left\{\frac{1}{y} + \frac{x}{y} \cdot \log_e y\right\}$$

$\therefore$ At x = 2 & y = 1

$$\Rightarrow \left.\frac{\partial^2 f}{\partial x\, \partial y}\right|_{(2,1)} = 1^2 \left\{\frac{1}{1} + \frac{2}{1} \cdot \log_e 1\right\} = 1$$

$$\Rightarrow \boxed{\left.\frac{\partial^2 f}{\partial x\, \partial y}\right|_{(2,1)} = 1}$$

Scan for Video solution

102. (d)

l Transform $k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$ to

$\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$.

Now,

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) = \frac{\partial}{\partial x_t} \cdot \frac{\partial x_t}{\partial x}\left(\frac{\partial h}{\partial x_t} \cdot \frac{\partial x_t}{\partial x}\right)$$

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial}{\partial x_t}\left(\frac{\partial h}{\partial x_t}\right) \cdot \left(\frac{\partial x_t}{\partial x}\right)^2$$

$$\frac{\partial^2 h}{\partial x^2} = \left(\frac{\partial^2 h}{\partial x_t^2}\right) \cdot \left(\frac{\partial x_t}{\partial x}\right)^2 \qquad \text{...(i)}$$

Now,

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

From equation (i),

$$k_x \left(\frac{\partial^2 h}{\partial x_t^2}\right) \cdot \left(\frac{\partial x_t}{\partial x}\right)^2 + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

$$\frac{k_x}{k_z}\left(\frac{\partial x_t}{\partial x}\right)^2 \left(\frac{\partial^2 h}{\partial x_t^2}\right) + \frac{\partial^2 h}{\partial z^2} = 0 \qquad \text{...(ii)}$$

We have to transform above equation as:

$$\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad \text{...(iii)}$$

Comparing equation (ii) and (iii),

$$\frac{k_x}{k_z}\left(\frac{\partial x_t}{\partial x}\right)^2 = 1$$

$$\frac{\partial x_t}{\partial x} = \sqrt{\frac{k_z}{k_x}}$$

$$\partial x_t = \sqrt{\frac{k_z}{k_x}}\partial x$$

$$x_t = \sqrt{\frac{k_z}{k_x}}x + \text{Constant}$$

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CHAPTER

5 VECTOR CALCULUS

Vector Algebra

1. [NAT] [GATE-ME-2022: 2M]

Consider two vectors:

$\vec{a} = 5\vec{i} + 7\vec{j} + 2\vec{k}$

$\vec{b} = 3\vec{i} - \vec{j} + 6\vec{k}$

Magnitude of the component of $\vec{a}$ orthogonal to $\vec{b}$ in the plane containing the vectors $\vec{a}$ and $\vec{b}$ is _____.(Round off to 2 decimal places)

2. [MCQ] [GATE-2021-CE:2M]

The unit normal vector to the surface $X^2 + Y^2 + Z^2 - 48 = 0$ at the point (4,4,4) is

(a) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{2}}, \frac{2}{\sqrt{2}}, \frac{2}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}$

3. [MCQ] [GATE-2021-CE:1M]

Let ϕ be a scalar field, and $\boldsymbol{u}$ be a vector field. Which of the following identities is true for $\text{div}(\phi\boldsymbol{u})$?

(a) $\text{div}(\phi\boldsymbol{u})=\phi\text{div}(\boldsymbol{u})+\boldsymbol{u}\cdot\text{grad}(\phi)$

(b) $\text{div}(\phi\boldsymbol{u})=\phi\text{div}(\boldsymbol{u})+\boldsymbol{u}\times\text{grad}(\phi)$

(c) $\text{div}(\phi\boldsymbol{u})=\phi\text{grad}(\boldsymbol{u})+\boldsymbol{u}\cdot\text{grad}(\phi)$

(d) $\text{div}(\phi\boldsymbol{u})=\phi\text{grad}(\boldsymbol{u})+\boldsymbol{u}\times\text{grad}(\phi)$

4. [NAT] [GATE-ME-2020: 1M]

For three vectors $\vec{A} = 2\hat{j} - 3\hat{k}$, $\vec{B} = -2\hat{i} + \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j}$, where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the axes of a right-handed rectangular /Cartesian coordinate system, the value of $\left(\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)+6\right)$ is ____.

5. [MCQ] [GATE-EE-2020: 2M]

The vector function expressed by $F = a_x(5y - k_1 z) + a_y(3z + k_2 x) + a_z(k_3 y - 4x)$ represents a conservative field, where a_x, a_y, a_z are unit vectors along x, y and z directions, respectively. The values of constants k_1, k_2, k_3 are given by:

(a) $k_1 = 3, k_2 = 3, k_3 = 7$

(b) $k_1 = 3, k_2 = 8, k_3 = 5$

(c) $k_1 = 4, k_2 = 5, k_3 = 3$

(d) $k_1 = 0, k_2 = 0, k_3 = 0$

6. [MCQ] [GATE-EC-2020: 1M]

For a vector field $\vec{A}$, which one of the following is FALSE?

(a) $\vec{A}$ is solenoidal if $\nabla\cdot\vec{A} = 0$.

(b) $\nabla\times\vec{A}$ is another vector field.

(c) $\vec{A}$ is irrotational if $\nabla^2\vec{A} = 0$.

(d) $\nabla\times\left(\nabla\times\vec{A}\right) = \nabla\left(\nabla\cdot\vec{A}\right) - \nabla^2\vec{A}$.

7. [NAT] [GATE-EE-2019 : 2M]

If $\boldsymbol{A} = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$ and $u = x^2 + y^2 + z^2$, then $\text{div}(u\boldsymbol{A})$ at(1, 1, 1) is ________

8. [MCQ] [GATE-2019-CE:1M]

Euclidean norm (length) of the vector $[4\ -2\ -6]^T$ is

(a) $\sqrt{48}$ (b) $\sqrt{56}$

(c) $\sqrt{24}$ (d) $\sqrt{12}$

9. [MCQ] [GATE-2019-CE:1M]

What is curl of the vector field $2x^2yi + 5z^2j - 4yzk$?

(a) $6zi + 4xj - 2x^2k$ (b) $6zi - 8xyj + 2x_2k$

(c) $-14zi + 6yj + 2x^2k$ (d) $-14zi - 2x^2k$

10. [MCQ] [GATE-CS-2017:2M]

Let u and v be two vectors in R^2 whose Euclidean norms satisfy $||u|| = 2||v||$. What is the value of a such that w = u + av bisects the angle between u and v?

(a) 2 (b) $\frac{1}{2}$

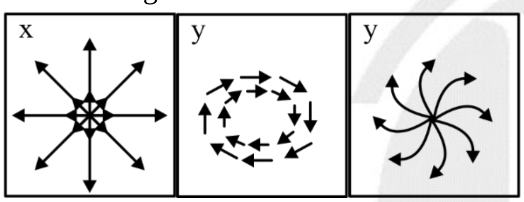
(c) 1 (d) $-\frac{1}{2}$

11. [NAT] [GATE-2017-CE:1M]

The divergence of the vector field $V = x^2 i + 2y^3 j + z^4 k$ at x =1, y = 2, z = 3 is___________.

12. [MCQ] [GATE-EE-2017: 1M]

The figures show diagrammatic representations of vector fields, $\vec{X}$, $\vec{Y}$ and $\vec{Z}$, respectively. Which one of the following choices is true?



(a) $\nabla \cdot \vec{X} = 0,\ \nabla \times \vec{Y} \neq 0,\ \nabla \times \vec{Z} = 0$

(b) $\nabla \cdot \vec{X} \neq 0,\ \nabla \times \vec{Y} = 0,\ \nabla \times \vec{Z} \neq 0$

(c) $\nabla \cdot \vec{X} \neq 0,\ \nabla \times \vec{Y} \neq 0,\ \nabla \times \vec{Z} \neq 0$

(d) $\nabla \cdot \vec{X} = 0,\ \nabla \times \vec{Y} = 0,\ \nabla \times \vec{Z} = 0$

13. [NAT] [GATE-EC-2016: 2M]

Consider the time-varying vector $I = \hat{x}15\cos(\omega t) + \hat{y}5\sin(\omega t)$ in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude $|I|$ is at its minimum value, the angle θ that I makes with the *x*-axis (in degree, such that $0 \le \theta \le 180$) is _____.

14. [MCQ] [GATE-ME-2014: 1M]

Which one of the following describes the relationship among the three vectors, $\hat{i} + \hat{j} + \hat{k},\ 2\hat{i} + 3\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 4\hat{k}$?

(a) The vectors are mutually perpendicular

(b) The vectors are linearly dependent

(c) The vectors are linearly independent

(d) The vectors are unit vectors

15. [NAT] [GATE-EC-2014: 1M]

The magnitude of the gradient for the function f(x,y,z) $= x^2 + 3y^2 + z^3$ at the point (1, 1, 1) is ____.

16. [MCQ] [GATE-EE-2013 :1M]

The curl of the gradient of the scalar field defined by $V = 2x^2 y + 3y^2 z + 4z^2 x$ is

(a) $4xya_x + 6yza_y + 8zxa_z$

(b) $4a_x + 6a_y + 8a_z$

(c) $(4xy + 4z^2)a_x + (2x^2 + 6yz)a_y + (3y^2 + 8zx)a_z$

(d) 0

17. [MCQ] [GATE-EC-2013: 1M]

Consider a vector field $\vec{A}(\vec{r})$. The closed loop line integral $\oint \bar{A} \cdot \overline{dl}$ can be expressed as

(a) $\iint (\nabla \times \bar{A}) \cdot \bar{d}s$ over the closed surface bounded by the loop

(b) $\iiint (\nabla \cdot \bar{A}) dv$ over the closed volume bounded by the loop

(c) $\iiint (\nabla \cdot \bar{A}) dv$ over the open volume bounded by the loop

(d) $\iint (\nabla \times \bar{A}) \cdot \bar{d}s$ over the open surface bounded by the loop

18. [MCQ] [GATE-EC-2013: 1M]

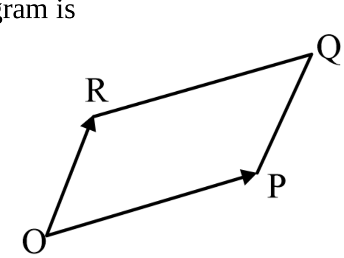
The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is

(a) 0 (b) 1/3

(c) 1 (d) 3

19. [MCQ] [GATE-2012-CE:2M]

For the parallelogram OPQR shown in the sketch $\overline{OP} = a\hat{\imath} + b\hat{\jmath}$ and $\overline{OR} = c\hat{\imath} + d\hat{\jmath}$. The area of the parallelogram is



(a) ad − bc (b) ac + bd

(c) ad + bc (d) ab − cd

20. [MCQ] [GATE-2011-CE:2M]

If $\vec{a}$ and $\vec{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $|\vec{a}\times\vec{b}|^2$ will be equal to

(a) $a^2b^2-(\overrightarrow{a.b})^2$ (b) $ab-\vec{a}.\vec{b}$

(c) $a^2b^2+(\vec{a}.\vec{b})^2$ (d) $ab+\vec{a}.\vec{b}$

21. [MCQ] [GATE-EE-2010 :1M]

Divergence of the three-dimensional radial vector field $\vec{r}$ is

(a) 3 (b) 1/r

(c) $\hat{i}+\hat{j}+\hat{k}$ (d) $3(\hat{i}+\hat{j}+\hat{k})$

22. [MCQ] [GATE-2009-CE:1M]

For a scalar function f (x, y, z) = $x^2 + 3y^2 + 2z^2$, the gradient at the point P (1, 2, –1) is

(a) $2\vec{i}+6\vec{j}+4\vec{k}$ (b) $2\vec{i}+12\vec{j}-4\vec{k}$

(c) $2\vec{i}+12\vec{j}+4\vec{k}$ (d) $\sqrt{56}$

23. [MCQ] [GATE-EC-2009: 1M]

If a vector field $\vec{V}$ is related to another field $\vec{A}$ through $\vec{V}=\nabla\times\vec{A}$, which of the following is true?

Note: C and S_C refer to any closed contour and any surface whose boundary is C.

(a) $\oint_C \overline{V}\cdot dl = \int_{S_c}\int \overline{A}\cdot\overline{ds}$

(b) $\oint_C \overline{A}\cdot dl = \int_{S_c}\int \overline{V}\cdot\overline{ds}$

(c) $\oint_C \nabla\times\overline{V}\cdot d\overline{l} = \int_{S_c}\int \nabla\times\overline{A}\cdot\overline{ds}$

(d) $\oint_C \nabla\times\overline{A}\cdot d\overline{l} = \int_{S_c}\int \overline{V}\cdot\overline{ds}$

24. [MCQ] [GATE-2008-CE:2M]

The inner (dot) product of two vectors $\vec{P}$ and $\vec{Q}$ is zero. The angle (degrees) between the two vectors is

(a) 0 (b) 30

(c) 90 (d) 120

Vector Differentiation

25. [MCQ] [GATE-EC-2023: 1M]

The rate of increase, of a scalar field f(x, y, z) = xyz, in the direction v = (2,1,2) at a point (0,2,1) is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$

(c) 2 (d) 4

26. [MCQ] [GATE-EE-2022 :2M]

Let, f(x, y, z) = $4x^2 + 7xy + 3xz^2$. The direction in which the function f(x, y, z) increases most rapidly at point P = (1, 0, 2) is

(a) $20\hat{i}+7\hat{j}$ (b) $20\hat{i}+7\hat{j}+12\hat{k}$

(c) $20\hat{i}+12\hat{k}$ (d) $20\hat{i}$

27. [NAT] [GATE-2021-CE:2M]

A function is defined in Cartesian coordinate system as $f(x,y) = xe^y$. The value of the directional derivative of the function (in integer) at the point (2, 0) along the direction of the straight-line segment from point (2,0) to point $\left(\frac{1}{2},2\right)$ is________.

28. [MCQ] [GATE-ME-2020: 2M]

The directional derivative of f(x, y, z) = xyz at point (–1, 1, 3) in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$ is

(a) $3\hat{i}-3\hat{j}-\hat{k}$ (b) $-\frac{7}{3}$

(c) $\frac{7}{3}$ (d) 7

29. [MCQ] [GATE-ME-2019: 1M]

The directional derivative of the function $f(x, y) = x^2 + y^2$ along a line directed from (0, 0) to (1, 1), evaluated at the point x = 1, y = 1 is

(a) $\sqrt{2}$ (b) 2

(c) $2\sqrt{2}$ (d) $4\sqrt{2}$

30. [MCQ] [GATE-ME-2018: 1M]

The divergence of the vector field $\vec{u}=e^x(\cos y\,\hat{i}+\sin y\,\hat{j})$ is

(a) 0 (b) $e^x \cos y + e^x \sin y$

(c) $2e^x \cos y$ (d) $2e^x \sin y$

31. [MCQ] [GATE-ME-2018: 2M]

For a position vector $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ the norm of the vector can be defined as $|\vec{r}|=\sqrt{x^2+y^2+z^2}$. Given a function $\phi=\ln|\vec{r}|$, its gradient $\nabla\phi$ is

(a) $\vec{r}$ (b) $\frac{\vec{r}}{|\vec{r}|}$

(c) $\frac{\vec{r}}{\vec{r}\cdot\vec{r}}$ (d) $\frac{\vec{r}}{|\vec{r}|^3}$

32. [MCQ] [GATE-EE-2018 : 1M]

The value of the directional derivative of the function $\phi(x, y, z) = xy^2 + yz^2 + zx^2$ at the point (2, –1, 1) in the direction of the vector $\mathbf{p} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ is

(a) 1 (b) 0.95

(c) 0.93 (d) 0.9

33. [NAT] [GATE-ME-2017: 2M]

For the vector $\vec{V} = 2yz\,\hat{i} + 3xz\,\hat{j} + 4xy\,\hat{k}$, the value of $\nabla\cdot(\nabla\times\vec{V})$ is ______.

34. [MCQ] [GATE-EC-2017:2M]

If the vector function

$\vec{F} = \hat{a}_x(3y - k_1z) + \hat{a}_y(k_2x - 2z) - \hat{a}_z(k_3y + z)$

is irrotational, then the values of the constants k_1, k_2 and k_3, respectively, are

(a) 0.3, –2.5, 0.5

(b) 0.0, 3.0, 2.0

(c) 0.3, 0.33, 0.5

(d) 4.0, 3.0, 2.0

35. [NAT] [GATE-ME-2017: 1M]

The divergence of the vector $-y\vec{i} + x\vec{j}$ is ______.

36. [MCQ] [GATE-ME-2015: 1M]

Curl of vector $\vec{V}(x, y, z) = 2x^2\vec{i} + 3z^2\vec{j} + y^3\vec{k}$ at x = y = z = 1 is

(a) $-3\vec{i}$ (b) $3\vec{i}$

(c) $3\vec{i} - 4\vec{j}$ (d) $3\vec{i} - 6\vec{k}$

37. [MCQ] [GATE-ME-2015: 1M]

Let ϕ be an arbitrary smooth real valued scalar function and $\vec{V}$ be an arbitrary smooth vector valued function in a three-dimensional space. Which one of the following is an identity?

(a) $\text{Curl}(\phi\vec{V}) = \nabla(\phi\,\text{Div}\,\vec{V})$

(b) $\text{Div}\,\vec{V} = 0$

(c) $\text{Div Curl}\,\vec{V} = 0$

(d) $\text{Div}(\phi\vec{V}) = \phi\ \text{Div}\vec{V}$

38. [NAT] [GATE-2015-CE:2M]

The directional derivative of the field $u(x, y, z) = x^2 - 3yz$ in the direction of the vector $(\bar{\imath} + \bar{\jmath} + 2\bar{k})$ at point (2, -1, 4) is_________.

39. [MCQ] [GATE-EC-2015: 2M]

A vector $\vec{P}$ is given by $\vec{P} = x^3y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\vec{a}_z$. Which one of the following statements is TRUE?

(a) $\vec{P}$ is solenoidal, but not irrotational

(b) $\vec{P}$ is irrotational, but not solenoidal

(c) $\vec{P}$ is neither solenoidal nor irrotational

(d) $\vec{P}$ is both solenoidal and irrotational

40. [NAT] [GATE-EC-2014:1M]

The directional derivative of $f(x,y) = \frac{xy}{\sqrt{2}}(x + y)$ at (1, 1) in the direction of the unit vector at an angle of $\frac{\pi}{4}$ with y-axis, is given by_______

41. [NAT] [GATE-EC-2014:2M]

If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and $|\vec{r}| = r$, then div $(r^2\nabla(\ln r))$ = _______.

42. [MCQ] [GATE-ME-2014: 1M]

Curl of vector $\vec{F} = x^2z^2\hat{i} - 2xy^2z\hat{j} + 2y^2z^3\hat{k}$ is

(a) $(4yz^3 + 2xy^2)\hat{i} + 2x^2z\hat{j} - 2y^2z\hat{k}$

(b) $(4yz^3 + 2xy^2)\hat{i} - 2x^2z\hat{j} - 2y^2z\hat{k}$

(c) $2xz^2\hat{i} - 4xyz\hat{j} + 6y^2z^2\hat{k}$

(d) $2xz^2\hat{i} + 4xyz\hat{j} + 6y^2z^2\hat{k}$

43. [MCQ] [GATE-ME-2014: 1M]

Divergence of the vector field $x^2z\hat{i} + xy\hat{j} - yz^2\hat{k}$ at (1, –1, 1) is

(a) 0 (b) 3

(c) 5 (d) 6

44. [MCQ] [GATE-EE-2014: 1M]

Let $\nabla\cdot(f\,\mathbf{V}) = x^2y + y^2z + z^2x$, where f and $\mathbf{V}$ are scalar and vector fields respectively. If $\vec{V} = y\hat{i} + z\hat{j} + x\hat{k}$, then $\vec{V}\cdot\nabla f$ is

(a) $x^2y + y^2z + z^2x$ (b) $2xy + 2yz + 2zx$

(c) $x + y + z$ (d) 0

45. [MCQ] [GATE-ME-2012: 1M]

For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by

(a) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$
(b) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$
(c) $\hat{k}$
(d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

46. [MCQ] [GATE-EE-2012 : 2M]

The direction of vector A is radially outward from the origin, with $|A| = Kr^n$ where $r^2 = x^2 + y^2 + z^2$ and K is constant. The value of n for which $\nabla \cdot \mathbf{A} = 0$ is

(a) –2
(b) 2
(c) 1
(d) 0

47. [MCQ] [GATE-2009-CE:2M]

For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, the directional derivative at the point P (1, 2, –1) in the direction of a vector $\vec{i} - \vec{j} + 2\vec{k}$ is

(a) –18
(b) $-3\sqrt{6}$
(c) $3\sqrt{6}$
(d) 18

48. [MCQ] [GATE-ME-2009: 1M]

The divergence of the vector field $3xz\,\hat{i} + 2xy\,\hat{j} - yz^2\,k$ at a point (1, 1, 1) is equal to

(a) 7
(b) 4
(c) 3
(d) 0

49. [MCQ] [GATE-ME-2008: 1M]

The divergence of the vector field

$(x-y)\hat{i} + (y-x)\hat{j} + (x+y+z)k$ is

(a) 0
(b) 1
(c) 2
(d) 3

50. [MCQ] [GATE-ME-2008: 2M]

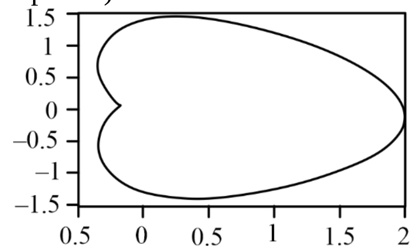
The directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point P = (1, 1, 2) in the direction of the vector $\vec{a} = 3\hat{i} - 4\hat{j}$ is

(a) –4
(b) –2
(c) –1
(d) 1

Vector Integration

51. [NAT] [GATE-EE-2023 : 2M]

The closed curve shown in the figure is described by $r = 1 + \cos\theta$, where $r = \sqrt{x^2 + y^2}$; $x = r\cos\theta$, $y = r\sin\theta$. The magnitude of the line integral of the vector field $F = -y\hat{i} + x\hat{j}$ around the closed is _____ (round off to 2 decimal places).



52. [NAT] [GATE-EE-2023 :2M]

Three points in the x-y plane are (–1, 0.8), (0, 2.2) and (1, 2.8). The value of the slope of the best fit straight in the least square sense is ______ (Round off to 2 decimal places)

53. [MCQ] [GATE-EC-2023: 2M]

The value of the line integral $\int_P^Q \left(z^2dx + 3y^2dy + 2xzdz\right)$ along the straight line joining the points P (1,1,2) and Q (2,3,1) is

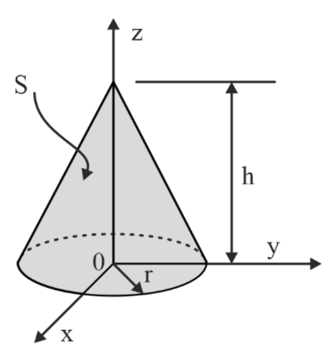
(a) 20
(b) 24
(c) 29
(d) –5

54. [NAT] [GATE-ME-2023: 1M]

A vector field

$\vec{B}(x, y, z) = x\hat{i} + y\hat{j} - 2z\hat{k}$

is defined over a conical region having h = 2, base radius r = 3 and axis along z, as shown in the figure. The base of the cone lies in the x-y plane and is centred at the origin. If $\hat{n}$ denotes the unit outward normal to the curved surface S of the cone, the value of the integral $\int_s \vec{B} \cdot \hat{n}\, ds$ equals _____. (Answer in integer)



55. [MCQ] **[GATE-ME-2022: 1M]**

Given a function $\varphi = \frac{1}{2}\left(x^2 + y^2 + z^2\right)$ in three-dimensional Cartesian space, the value of the surface integral $\oiint_s \hat{n} \cdot \nabla \varphi ds$, where s is the surface of a sphere of unit radius and $\hat{n}$ is the outward unit normal vector on s, is

(a) 4π (b) 3π

(c) $4\pi/3$ (d) 0

56. [MCQ] **[GATE-ME-2022: 1M]**

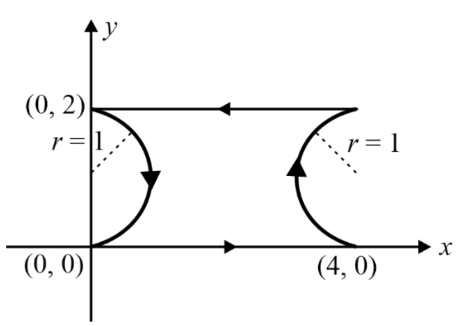
Consider a cube of unit edge length and sides parallel to co-ordinate axes, with its centroid at the point (1, 2, 3). The surface integral $\int_A \vec{F} \cdot d\vec{A}$ of a vector field $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$ over the entire surface A of the cube is

(a) 14 (b) 27

(c) 28 (d) 31

57. [MCQ] **[GATE-EC-2022: 1M]**

Consider the two-dimensional vector field $\vec{F}(x, y) = x\vec{i} + y\vec{j}$, where $\vec{i}$ and $\vec{j}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the x-y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semi-circular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral $\oint_C \vec{F}(x, y) \cdot \left(dx\,\vec{i} + dy\,\vec{j}\right)$ is _______ .



(a) 0 (b) 1

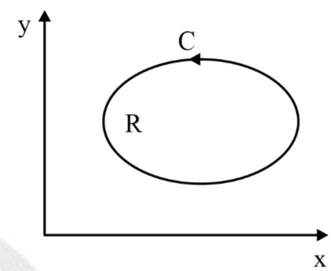
(c) $8 + 2\pi$ (d) -1

58. [MCQ] **[GATE-EE-2022: 2M]**

Let R be a region in the first quadrant of the xy plane enclosed by a closed curve C considered in counter-clockwise direction. Which of the following expressions does not represent the area of the region R?

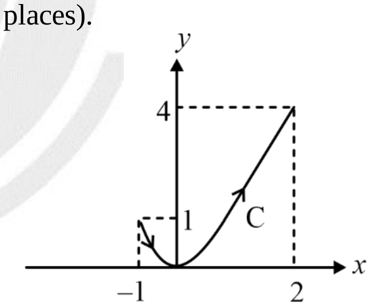
(a) $\iint_R dxdy$ (b) $\int_c xdy$

(c) $\int_c ydx$ (d) $\frac{1}{2}\int_c (xdy - ydx)$



59. [NAT] **[GATE-EE-2020 : 2M]**

Let $\mathbf{a_x}$ and $\mathbf{a_y}$ be unit vectors along x and y directions, respectively. A vector function is given by $\mathbf{F} = \mathbf{a_x}\, y - \mathbf{a_y} x$. The line integral of the above function $\int_C \mathbf{F} \cdot d\boldsymbol{l}$ along the curve C, which follows the parabola $y = x^2$ as shown below is _____ (rounded off to 2 decimal places).



60. [NAT] **[GATE-2020-CE:2M]**

If C represents a line segment between (0,0,0) and (1,1,1) in Cartesian coordinate system, the value (expressed a integer) of the line integral $\int_C [(y + z)dx + (x + z)dy + (x + y)dz]$ is_____________.

61. [MCQ] **[GATE-ME-2020: 2M]**

A vector field is defined as

$$\vec{f}(x, y, z) = \frac{x}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}}\hat{i} + \frac{y}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}}\hat{j} + \frac{z}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}}\hat{k}$$

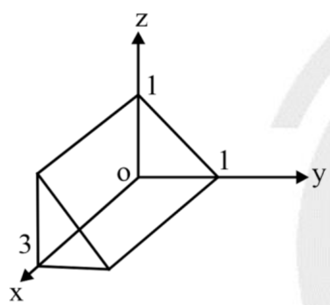
Where, $\hat{i}, \hat{j}, \hat{k}$ are unit vector along the axes of a right-handed rectangular/cartesian coordinate system. The surface integral $\iint \vec{f} \cdot d\vec{s}$ (where $d\vec{s}$ is an elemental surface area vector) evaluated over the inner and outer surfaces of a spherical shell formed by two concentric spheres with origin as the centre, and internal and external radii of 1 and 2, respectively is

(a) 0 (b) 2π

(c) 4π (d) 8π

62. [NAT] [GATE-EC-2020: 2M]

For a solid S shown below, the value of $\iiint_S x \cdot dx\, dy\, dz$ (rounded off to two decimal places) is________.



63. [NAT] [GATE-EE-2019 : 1M]

If $f = 2x^3 + 3y^2 + 4z$, the value of line integral $\int_C \text{grad}(f) \cdot \mathbf{dr}$ evaluated over contour C formed by the segments $(-3, -3, 2) \to (2, -3, 2) \to (2, 6, 2) \to (2, 6, -1)$ is ________.

64. [MCQ] [GATE-ME-2019: 2M]

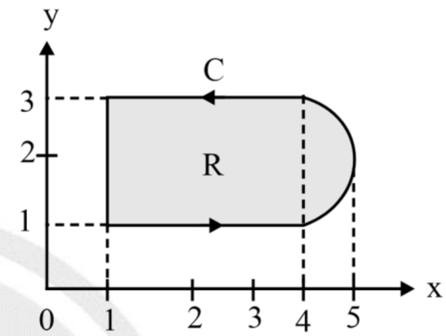
Given a vector $\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k}\right)$ and $\hat{n}$ is the unit normal vector to the surface of the hemisphere ($x^2 + y^2 + z^2 = 1$; $z \geq 0$), the value of integral $\int (\nabla \times \vec{u}) \cdot \hat{n}\, dS$ evaluated on the curved surface of the hemisphere S is

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

65. [MCQ] [GATE-EC-2019: 2M]

Consider the line integral $\int_C (x\,dy - y\,dx)$ the integral being taken in a counter clockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below. The region R is the area enclosed by the union of a 2×3 rectangle and a semi-circle of radius 1. The line integral evaluates to



(a) $6 + \frac{\pi}{2}$ (b) $16 + 2\pi$

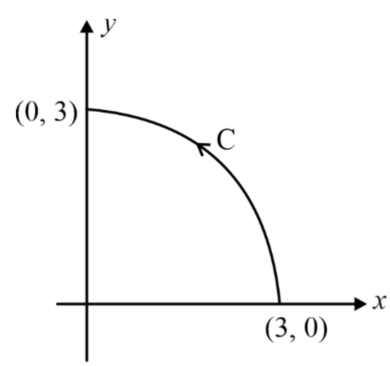
(c) $8 + \pi$ (d) $12 + \pi$

66. [NAT] [GATE-2018-CE:2M]

The value (up to two decimal places) of a line integral $\int_c \vec{F}(\vec{r}).\overrightarrow{dr}$ for $\vec{F}(\vec{r}) = x^2\hat{\imath} + y^2\hat{\jmath}$ along C which is a straight line joining (0, 0) to (1, 1) is _________.

67. [NAT] [GATE-EE-2018 : 2M]

As shown in the figure, C is the arc from the point (3,0) to the point (0,3) on the circle $x^2 + y^2 = 9$. The value of integral $\int_c \left(y^2 + 2yx\right)dx + \left(2xy + x^2\right)dy$ is _____ (up to 2 decimal places).



68. [MCQ] [GATE-ME-2018: 2M]

The value of the integral

$$\iint_s \vec{r} \cdot \vec{n}\, dS$$

over the closed surface S bounding a volume V, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector and $\vec{n}$ is the normal to the surface S, is

(a) V (b) 2V
(c) 3V (d) 4V

69. [NAT] [GATE-ME-2017: 2M]

The surface integral $\iint_s \vec{F} \cdot \hat{n}\, dS$ over the surface S of the sphere $x^2 + y^2 + z^2 = 9$, where $\vec{F} = (x + y)\vec{i} + (x + z)\vec{j} + (y + z)\vec{k}$ and $\hat{n}$ is the unit outward surface normal, yields ______.

70. [NAT] [GATE-EC-2017: 2M]

Let $I = \int_c (2zdx + 2ydy + 2xdz)$ where x, y, z are real, and let C be the straight-line segment from point A:(0, 2, 1) to point B:(4, 1, –1). The value of I is ___________.

71. [MCQ] [GATE-EE-2016 : 1M]

The value of the line integral $\int(2xy^2dx + 2x^2ydy+dz)$ along a path joining the origin (0, 0, 0) and the point (1, 1, 1) is

(a) 0 (b) 2
(c) 4 (d) 6

72. [NAT] [GATE-EE-2016 :2M]

The line integral of the vector field $F = 5xz\hat{i} + (3x^2 + 2y)\hat{j} + x^2z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is ________.

73. [NAT] [GATE-EC-2016: 2M]

Suppose C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anti-clockwise. The value of $\oint (xy^2dx + x^2y\,dy)$ over the curve C equals ____ .

74. [NAT] [GATE-ME-2016: 2M]

A scalar potential φ has the following gradient:
$\nabla\varphi = yz\hat{i} + xz\hat{j} + xy\hat{k}$. Consider the integral $\int_c \nabla\varphi \cdot d\vec{r}$ on the curve $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

The curve C is parameterized as follows:

$$\begin{cases} x = t \\ y = t^2 \quad \text{and } 1 \le t \le 3 \\ z = 3t^2 \end{cases}$$

The value of the integral is _____.

75. [NAT] [GATE-ME-2016: 2M]

The value of the line integral $\oint_c \bar{F} \cdot \bar{r}'ds$, where C is a circle of radius $\frac{4}{\sqrt{\pi}}$ is ____.

Here $\bar{F}(x, y) = y\hat{i} + 2x\hat{j}$ and $\bar{r}'$ is the **UNIT** tangent vector on the curve C at an arc length s from a reference point on the curve. $\hat{i}$ and $\hat{j}$ are the basis vectors in the x-y Cartesian reference. In evaluating the line integral, the curve has to be traversed in the counter-clockwise direction.

76. [NAT] [GATE-ME-2015: 2M]

The surface integral $\iint_s \frac{1}{\pi}(9x\vec{i} - 3y\vec{j}) \cdot n\, ds$ over the sphere given by $x^2 + y^2 + z^2 = 9$ is _____.

77. [NAT] [GATE-ME-2015: 2M]

The value of

$$\int_C \left[(3x - 8y^2)dx + (4y - 6xy)dy\right],$$

(Where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1) is ____.

78. [MCQ] [GATE-ME-2014: 2M]

The integral $\oint_c (ydx - xdy)$ is evaluated along the circle $x^2 + y^2 = \frac{1}{4}$ traversed in counter clockwise direction. The integral is equal to

(a) 0 (b) $-\frac{\pi}{4}$
(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

79. [MCQ] [GATE-EE-2013: 1M]

Given a vector field $\mathbf{F} = \mathbf{y}^2x\mathbf{a_x} - yz\mathbf{a_y} - x^2\mathbf{a_z}$, the line integral $\int \mathrm{F}.\,\mathbf{dl}$ evaluated along a segment on the x-axis from x = 1 to x = 2 is

(a) –2.33 (b) 0
(c) 2.33 (d) 7

80. [MCQ] [GATE-ME-2013: 2M]

The following surface integral is to be evaluated over a sphere for the given steady velocity vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ defined with respect to a Cartesian coordinate system having $\vec{i}, \vec{j}$ and $\vec{k}$ as unit base vectors.

$$\iint_S \frac{1}{4}(\vec{F}\cdot\hat{n})\,dA$$

where S is the sphere, $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ is the outward unit normal vector to the sphere. The value of the surface integral is

(a) π (b) 2π

(c) $3\pi/4$ (d) 4π

81. [MCQ] [GATE-EC-2011:1M]

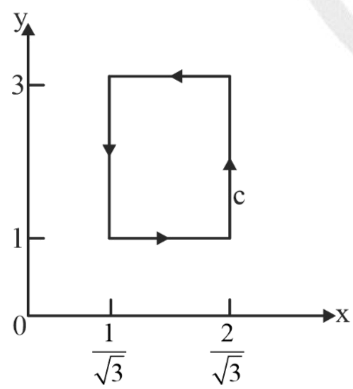
Consider a closed surface *s* surrounding a volume *v*. If $\vec{r}$ is the position vector of a point inside *s* with $\hat{n}$ the unit normal on *s*, the value of the integral $\oiint_s 5\vec{r}.\hat{n}\,ds$ is

(a) 3v (b) 5v

(c) 10 v (d) 15 v

82. [MCQ] [GATE-EC-2010: 2M]

If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$ then $\oint_C \vec{A}\cdot d\vec{l}$ over the path shown in the figure is



(a) 0 (b) $\frac{2}{\sqrt{3}}$

(c) 1 (d) $2\sqrt{3}$

83. [MCQ] [GATE-EE-2009: 2M]

$F(x,y) = (x^2 + xy)\hat{a}_x + (y^2 + xy)\hat{a}_y$. It's line integral over the straight line from (x, y) = (0, 2) to (2,0) evaluates to

(a) –8 (b) 4

(c) 8 (d) 0

84. [MCQ] [GATE-EC-2008: 2M]

Consider points P and Q in the x-y plane, with P = (1, 0) and Q = (0, 1). The line integral

$$2\int_P^Q (xdx + ydy)$$

along the semicircle with the line segment PQ as its diameter

(a) is –1

(b) is 0

(c) is 1

(d) depends on the direction (clockwise or anti clockwise) of the semicircle

85. [MCQ] [GATE-EC-2008: 2M]

The value of the integral of the function $g(x, y) = 4x^3 + 10y^4$ along the straight-line segment from the point (0,0) to the point (1, 2) in the x-y plane is

(a) 33 (b) 35

(c) 40 (d) 56

❑❑❑

ANSWER KEY

1.	**(7.90 to 8.70)**	**2.**	(a)	**3.**	(a)	**4.**	(6 to 6)
5.	(c)	**6.**	(c)	**7.**	(45 to 45)	**8.**	(b)
9.	(d)	**10.**	(a)	**11.**	(133.9 to 134.1)	**12.**	(c)
13.	(90 to 90)	**14.**	(b)	**15.**	(6.8 to 7.2)	**16.**	(d)
17.	(d)	**18.**	(d)	**19.**	(a)	**20.**	(a)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(b)	**27.**	(1 to 1)	**28.**	(c)
29.	(c)	**30.**	(c)	**31.**	(c)	**32.**	(a)
33.	(0 to 0)	**34.**	(b)	**35.**	(0 to 0)	**36.**	(a)
37.	(c)	**38.**	(–5.72 to –5.70)	**39.**	(a)	**40.**	(2.99 to 3.01)
41.	(2.9 to 3.1)	**42.**	(a)	**43.**	(c)	**44.**	(a)
45.	(a)	**46.**	(a)	**47.**	(b)	**48.**	(c)
49.	(d)	**50.**	(b)	**51.**	(9 to 10)	**52.**	(0.9 to 1.1)
53.	(b)	**54.**	(–0.001 to 0.001)	**55.**	(a)	**56.**	(a)
57.	(a)	**58.**	(a)	**59.**	(–3.05 to –2.95)	**60.**	(3 to 3)
61.	(a)	**62.**	(2.25 to 2.25)	**63.**	(139 to 139)	**64.**	(c)
65.	(d)	**66.**	(0.60 to 0.70)	**67.**	(0 to 0)	**68.**	(c)
69.	(225 to 227)	**70.**	(–11.1 to –10.9)	**71.**	(b)	**72.**	(4.40 to 4.45)
73.	(–0.03 to 0.03)	**74.**	(725.9 to 726.1)	**75.**	(15.9 to 16.1)	**76.**	(214 to 218)
77.	(1.60 to 1.70)	**78.**	(c)	**79.**	(b)	**80.**	(a)
81.	(d)	**82.**	(c)	**83.**	(d)	**84.**	(b)
85.	(a)						

SOLUTIONS

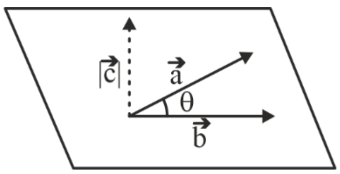
1. (7.90 to 8.70)

Given $\vec{a} = 5\vec{i} + 7\vec{j} + 2\vec{k}$

$\vec{b} = 3\vec{i} - \vec{j} + 6\vec{k}$

Component of $\vec{a}$ orthogonal to $\vec{b}$

$$|\vec{c}| = |\vec{a}| \cdot \sin\theta = |\vec{a}| \cdot \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\cdot|\vec{b}|} = \frac{|\vec{a}\times\vec{b}|}{|\vec{b}|}$$



$$\therefore \quad |\vec{a}\times\vec{b}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 7 & 2 \\ 3 & -1 & 6 \end{vmatrix} = \vec{i}(44) - \vec{j}(24) + \vec{k}(-26)$$

$$|\vec{b}| = \sqrt{3^2 + (-1)^2 + 6^2} = \sqrt{46}$$

$$\therefore \quad |\vec{a}\times\vec{b}| = \sqrt{44^2 + 24^2 + 26^2} = 56.46$$

$\therefore$ component of $\vec{a}$ orthogonal to $\vec{b}$ =

$$\frac{56.46}{\sqrt{46}} = 8.325$$

Scan for Video solution

2. (a)

Let the surface be $\phi = x^2 + y^2 + z^2 - 48$

$$\text{Grad } \phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\text{Grad } \phi = (2x)\hat{i} + (2y)\hat{j} + (2z)\hat{k}$$

At P (4, 4, 4) grad $\phi = 8\hat{i} + 8\hat{j} + 8\hat{k}$

$\therefore$ Normal vector $\vec{n} = 8\hat{i} + 8\hat{j} + 8\hat{k}$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{8\hat{i} + 8\hat{j} + 8\hat{k}}{\sqrt{8^2 + 8^2 + 8^2}}$$

$\therefore$ Unit normal vector; $\hat{n} = \dfrac{\hat{i}}{\sqrt{3}} + \dfrac{\hat{j}}{\sqrt{3}} + \dfrac{\hat{k}}{\sqrt{3}}$

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

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3. (a)

For any Scalar Ø and vector **u**,

div (ϕ. **u**) = ϕ div **u** + **u** grad ϕ

The above result is obtained by applying the product rule of differentiation.

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4. (6 to 6)

Given

$$\vec{A} = 2\hat{j} - 3\hat{k}, \vec{B} = -2\hat{i} - \hat{k}, \vec{C} = 3\hat{i} - \hat{j}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (2\hat{j} - 3\hat{k}) \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{vmatrix}$$

$$= (2\hat{j} - 3\hat{k}) \cdot \left(\vec{i}(1) - \vec{j}(-3) + \vec{k}(2)\right)$$

$$= 6 - 6 = 0$$

$$\therefore \quad \vec{A}\cdot(\vec{B}\times\vec{C}) = 0$$

$$\therefore \quad \boxed{\vec{A}\cdot(\vec{B}\times\vec{C}) + 6 = 0 + 6 = 6}$$

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5. (c)

$$\vec{F} = \hat{a}_x(5y - k_1 z) + \hat{a}_y(3z + k_2 x) + \hat{a}_z(k_3 y - 4x)$$

Since $\vec{F}$ is conservative field $\therefore$ curl $\vec{F} = 0$

$$\text{curl } \vec{F} = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 5y - k_1 z & 3z + k_2 x & k_3 y - 4x \end{vmatrix} = 0$$

$$\Rightarrow \hat{i}\left(\frac{\partial(k_3y-4x)}{\partial y}-\frac{\partial(3z+k_2x)}{\partial z}\right)$$

$$-\left(\frac{\partial(k_3y-4x)}{\partial x}-\frac{\partial(5y-k_1z)}{\partial z}\right)$$

$$+\hat{k}\left(\frac{\partial(3z+k_2x)}{\partial x}-\frac{\partial(5y-k_1z)}{\partial y}\right)$$

$$\Rightarrow \hat{i}(k_3-3)-\hat{j}(-4+k_1)+\hat{k}(k_2-5)=0$$

$\therefore k_3 - 3 = -4 + k_1 = k_2 - 5 = 0$

$\therefore k_1 = 4, k_2 = 5 =$ and $k_3 = 3$

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6. **(c)**

For any vector field $\vec{A}$,

- If $\nabla \cdot \vec{A} = 0$, then $\vec{A}$ is solenoidal
- If $\nabla \times \vec{A} = \vec{o}$, then $\vec{A}$ is irrotational
- $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \cdot \vec{A}$

For $\vec{A}$ to be irrotational, $\nabla \times \vec{A} = \vec{0}$

$\therefore$ options 'c' is false statement

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7. **(45 to 45)**

$$\text{grad } u = \frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}$$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\therefore \text{ grad } u \text{ at } (1,1,1) = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\text{and div } A = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

$$\because A = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$$

$\therefore$ div A = 2 + 3 + 4 = 9

Using vector identity,

div (uA) = u div **A** + **A** grad u

$$\text{div}(uA) = (x^2 + y^2 + z^2)9 + (2x\hat{i} + 3y\hat{j} + 4z\hat{k})$$
$$(2\hat{i} + 2\hat{j} + 2\hat{k})$$

At $x = 1, y = 1, z = 1$

$= (1^2 + 1^2 + 1^2)\ 9 + (4x + 6y + 8z)$

$= 27 + 4 \times 1 + 6 \times 1 + 8 \times 1$

$= 27 + 4 + 6 + 8$

div(u**A**) = 45

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8. (b)

$$\vec{x} = \begin{bmatrix} 4 & -2 & -6 \end{bmatrix}^T$$

$$\vec{x} = \begin{bmatrix} 4 \\ -2 \\ -6 \end{bmatrix}$$

Euclidean norm length = $|\vec{x}|$

$$= \sqrt{4^2 + (-2)^2 + (-6)^2} = \sqrt{56}$$

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9. **(d)**

$$\text{Curl } \vec{F} = \left(\vec{\nabla} \times \vec{F}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2y & 5z^2 & -4yz \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial(-4yz)}{\partial y} - \frac{\partial(5z^2)}{\partial z}\right] - \hat{j}\left[\frac{\partial(-4yz)}{\partial x} - \frac{\partial(2x^2y)}{\partial z}\right]$$

$$+\hat{k}\left[\frac{\partial}{\partial x}(5z^2) - \frac{\partial}{\partial y}(2x^2y)\right]$$

$$= \hat{i}(-4z - 10z) - \hat{i}(0-0) + \hat{k}(0 - 2x^2)$$

$$= -14z\hat{i} - 2x^2\hat{k}$$

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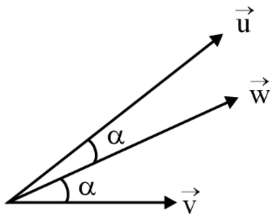
10. (a)

Given

$\|\vec{u}\| = 2\|\vec{v}\|$

$\vec{w} = \vec{u} + a\vec{v}$ bisects angle between $\vec{u}$ and $\vec{v}$



⇒ let 'α' be angle between $\vec{u}, \vec{w}$ and $\vec{v}, \vec{w}$

By dot product

$$\Rightarrow \frac{\vec{u}\cdot\vec{w}}{\|\vec{u}\|\cdot\|\vec{w}\|} = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\cdot\|\vec{w}\|}$$

$$\Rightarrow \vec{u}\cdot\vec{w} = 2\left(\vec{v}\cdot\vec{w}\right) \quad (\text{since } \|\vec{u}\| = 2\|\vec{v}\|)$$

$$\Rightarrow \vec{u}\left(\vec{u} + a\vec{v}\right) = 2\vec{v}\left(\vec{u} + 2a\vec{v}\right)$$

$$\Rightarrow \|\vec{u}\| + a\cdot\vec{v} = 2\vec{v}\vec{u} + 2a\|\vec{v}\|$$

$$\Rightarrow \|\vec{u}\| + a\cdot\vec{u}\vec{v} = 2\vec{v}\vec{u} + 2a\frac{\|\vec{u}\|}{4}$$

$$\Rightarrow \frac{2a}{4} = 1 \text{ and } a = 2$$

⇒ a = 2

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11. (133.9 to 134.1)

$$V = x^2\hat{i} + 2y^3\hat{j} + z^4\hat{k}$$

$$V = V_1\hat{i} + V_2\hat{j} + V_3\hat{k}$$

$$\text{div}V = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

$$\text{div}V = 2x + 6y^2 + 4z^3$$

At point (1,2,3,) div V = $2 \times 1 + 6 \times 2^2 + 4 \times 3^3$

= 2 + 24 + 108 = 134

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12. (c)

From fig.(i) $\vec{X}$ is a diverging field

$\therefore$ div $\vec{X} \neq 0$

$\nabla \cdot \vec{X} \neq 0$

from fig.(ii) $\vec{Y}$ is a circularly rotating field

$\therefore$ curl $\vec{Y} \neq 0$

$\nabla \times \vec{Y} \neq 0$

from fig.(ii) is a circularly rotating field

$\therefore$ curl $\vec{Z} \neq 0$

$\nabla \times \vec{Z} \neq 0$

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13. (90 to 90)

Given:

$$\vec{I} = \hat{x}\,15\cos(\omega t) + \hat{y}\,5\sin(\omega t)$$

$$\Rightarrow \quad \left|\vec{I}\right| = ((15\cos\omega t)^2 + (5\sin(\omega t)^2)^{1/2}$$

$$\Rightarrow \quad \left|\vec{I}\right| = (25 + 200\cos^2\omega t)^{1/2}$$

For $\left|\vec{I}\right|$ to be a minimum value, cosωt = 0

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$\therefore$ At minimum value of $\left|\vec{I}\right|$,

$$\vec{I} = \hat{x}\cdot 15(0) + \hat{y}\cdot 5(1) = \hat{y}\cdot(5)$$

$\therefore$ Angle made by $\vec{I}$ with *x*-axis is 90 degrees.

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14. (b)

Given Vectors are

$$\hat{i} + \hat{j} + \hat{k},\ \ 2\hat{i} + 3\hat{j} + \hat{k},\ \ 5\hat{i} + 6\hat{j} + 4\hat{k}$$

Let $\vec{a} = \vec{i} + \vec{j} + \vec{k}$

$\vec{b} = 2\vec{i} + 3\vec{j} + \vec{k}$

$\vec{c} = 5\vec{i} + 6\vec{j} + 4\vec{k}$

For $\vec{a}, \vec{b}, \vec{c}$ to linearly dependent,

Determinant of matrix $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{vmatrix} = 0$$

$\Rightarrow$ $1\times(6) - 1\times(3) + 1\times(-3) = 0$ (True)

$\therefore$ $\boxed{\text{Given vectors are linearly Dependent.}}$

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15. (6.8 to 7.2)

Given $f(x, y, z) = x^2 + 3y^2 + z^3$

$$\Rightarrow \nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$= 2x\vec{i} + 6y\vec{j} + 3z^2\vec{k}$$

$$\Rightarrow \nabla f\big|_{(1,1,1)} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\Rightarrow \left|\nabla f\big|_{(1,1,1)}\right| = \sqrt{2^2 + 6^2 + 3^2} = 7$$

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16. (d)

$V = 2x^2y + 3y^2z + 4z^2x$

$$\nabla V = (4xy + 4z^2)\hat{i} + (2x^2 + 6yz)\ \hat{j} + (3y^2 + 8zx)\hat{k}$$

$$\nabla \times \nabla V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial y & \partial z \\ 4xy + 4z^2 & 2x^2 + 6y2 & 3y^2 + 8zx \end{vmatrix}$$

$$= \hat{i}(6y - 6y) - \hat{j}(8z - 8z) + \hat{k}(4x - 4x)$$

$\therefore$ $\nabla \times \nabla V$ = **0**

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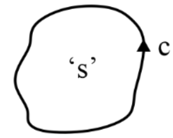
17. (d)

Given vector field is $\vec{A}(\vec{r})$

For any closed loop,

By stoke's theorem



$$\oint \vec{A}.d\vec{l} = \iint_s \text{curl}\vec{A}.d\vec{s}$$

$$= \iint_s (\nabla \times \vec{A}).d\vec{s}$$

$$\boxed{\therefore \oint_c \vec{A}.d\vec{l} = \iint_s (\nabla \times \vec{A}).d\vec{s}}$$

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18. (d)

Given $\overrightarrow{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

$$\Rightarrow \quad \nabla \cdot \overrightarrow{A} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$$

$= 1 + 1 + 1 = 3$

$\boxed{\therefore\ \nabla \cdot \overrightarrow{A} = 3}$

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19. (a)

Area of parallelogram = $\left|\overrightarrow{OP} \times \overrightarrow{OR}\right|$

Given $\overrightarrow{OP} = a\hat{i} + b\hat{j}$

$\overrightarrow{OR} = c\hat{i} + d\hat{j}$

$$\text{Area of parallelogram} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix}$$

$$= 0\hat{i} + 0\hat{j} + (ad - bc)\hat{k} = (ad - bc)\hat{k}$$

Area = ad – bc

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20. (a)

$\vec{a}.\vec{b} = a.b \ \cos\theta$

$\vec{a}\times\vec{b} = a.b \sin\theta \ \hat{n}$

$\therefore \ \left|\vec{a}\times\vec{b}\right|^2 = a^2b^2\sin^2\theta\left|\hat{n}\right|^2$

$= a^2b^2\left(1-\cos^2\theta\right) \qquad \left[\because \left|\hat{n}\right|^2 = 1\right]$

$= a^2b^2 - a^2b^2\cos^2\theta$

$= a^2b^2 - \left(\vec{a}.\vec{b}\right)^2$

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21. (a)

We know, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\nabla\vec{r} = \dfrac{\partial}{\partial x}(x) + \dfrac{\partial}{\partial y}(y) + \dfrac{\partial}{\partial z}(z)$

$= 1 + 1 + 1$

div $\vec{r} = 3$

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22. (b)

$f = x^2 + 3y^2 + 2z^2$

Grad $f = \dfrac{\partial f}{\partial x}\hat{i} + \dfrac{\partial f}{\partial y}\hat{j} + \dfrac{\partial f}{\partial z}\hat{k}$

Grad $f = (2x)\hat{i} + (6y)\hat{j} + (4z)\hat{k}$

At P (1, 2, –1)

Grad $f = 2\hat{i} + 12\hat{j} - 4\hat{k}$

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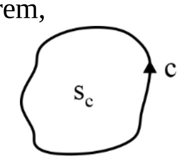
23. (b)

Given $\vec{V} = \nabla\times\vec{A} = \text{curl}\vec{A}$

S_c is any surface whose boundary is closed contour 'C,

By stokes theorem,



$\oint_c \vec{A}.d\vec{l} = \int\oint_{s_c} \text{curl}\vec{A}.d\vec{s}$

$= \int\oint_{s_c} \vec{V}.d\vec{s}$

$\boxed{\therefore \oint_c \vec{A}.d\vec{l} = \int_{s_c} \vec{V}.d\vec{s}} \rightarrow$ From stokes theorem

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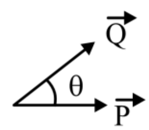
24. (c)

Given $\vec{P}.\vec{Q} = 0$



$\Rightarrow \ \left|\vec{P}\right|\left|\vec{Q}\right|\cos\theta = 0$

$\Rightarrow \ \cos\theta = 0$

$\Rightarrow \ \theta = 90°$

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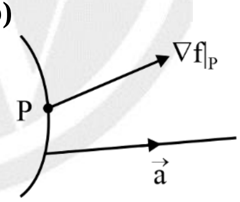
25. (b)



$f = xyz$

Given scalar field, f(x, 4, z) = xyz

Point is P(0, 2, 1) and in the direction of (2, 1, 2)

Rate of increase of f(x, y, z) at point 'P' in the direction of $\vec{a}$ is given by

$\nabla f|_P \cdot \hat{a}$

$= \nabla f|_P \cdot \dfrac{\vec{a}}{\left|\vec{a}\right|}$

$\therefore \ \nabla f = yz\vec{i} + x^2\vec{j} + xy\vec{k}$

$\nabla f|_P = 2\vec{i} + 0\vec{j} + 0\vec{k}$

Direction is $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

$\therefore$ Rate of increase = $\left(2\vec{i}\right)\cdot\dfrac{\left(2\vec{i}+\vec{j}+2\vec{k}\right)}{\sqrt{2^2+1^2+2^2}}$

$= \dfrac{4}{\sqrt{9}} = \dfrac{4}{3}$

$\therefore$ Rate of increase of f at (0, 2, 1) in the direction of (2, 1, 2) is $\dfrac{4}{3}$

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26. (b)

$f = 4x^2 + 7xy + 3xz^2$

$\nabla f = \dfrac{\partial f}{\partial x}\hat{i}+\dfrac{\partial f}{\partial y}\hat{j}+\dfrac{\partial f}{\partial z}\hat{k}$

$\nabla f = (8x+7y+3z^2)\hat{i}+(7x)\hat{j}+(6xz)\hat{k}$

At point (1,0,2)

$\nabla f = (8+0+12)\hat{i}+7\hat{j}+12\hat{k}$

$= 20\hat{i}+7\hat{j}+12\hat{k}$

In this direction function will increase most rapidly.

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27. (1 to 1)

$f(x,y) = xe^y$

P (2, 0) and $Q\left(\dfrac{1}{2},2\right)$

$\therefore$ Required directional Derivative = grad f. PQ at point (2,0)

Grad $f = \dfrac{\partial f}{\partial x}\hat{i}+\dfrac{\partial f}{\partial y}\hat{j}$

$= e^y\hat{i}+xe^y\hat{j}$

So, grad $f_{(2,0)} = e^0\hat{i}+2e^0\hat{j}$

$= \hat{i}+2\hat{j}$

Now $\overrightarrow{PQ} = \overrightarrow{OQ}-\overrightarrow{OP} = \left(\dfrac{1}{2}\hat{i}+2\hat{j}\right)-\left(2\hat{i}+0\hat{j}\right)$

$\overrightarrow{PQ} = -\dfrac{3}{2}\hat{i}+2\hat{j}$

$PQ = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \dfrac{-\dfrac{3}{2}\hat{i}+2\hat{j}}{\sqrt{\left(\dfrac{-3}{2}\right)^2+2^2}}$

$\therefore$ Directional derivative = grad f. $P\hat{Q}$

$\therefore$ Directional derivative at (2,0) =

$\left(\hat{i}+2\hat{j}\right).\dfrac{\left(-\dfrac{3}{2}\hat{i}+2\hat{j}\right)}{\sqrt{\dfrac{9}{4}+4}} = \dfrac{-\dfrac{3}{2}+4}{\sqrt{\dfrac{9+16}{4}}} = \dfrac{\dfrac{5}{2}}{\dfrac{5}{2}} = 1$

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28. (c)

Given

$f(x, y, z) = x\,y\,z$ and P(–1, 1, 3)

The directional derivative of $f(x, y, z)$ at 'P' in the directionof $\vec{a}$ is given by

$\nabla f\,|_P \cdot \hat{a} = \nabla(xyz)|_{(-1,1,3)}\cdot\dfrac{\vec{i}-2\vec{j}+2\vec{k}}{|\sqrt{1^2+(-2)^2+2^2}|}$

$= yz\vec{i}+xz\vec{j}+xy\vec{k}\,|_{(-1,1,3)}\cdot\dfrac{\left(\vec{i}-2\vec{j}+2\vec{k}\right)}{3}$

$= \left(3\vec{i}-3\vec{j}-\vec{k}\right)\cdot\dfrac{\left(\vec{i}-2\vec{j}+2\vec{k}\right)}{3} = \dfrac{7}{3}$

$\therefore$ Directional derivative = $\dfrac{7}{3}$

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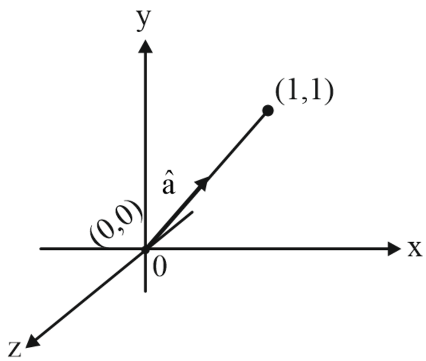
29. (c)

Given

$f = x^2 + y^2$; x = 1 & y = 1

Directional derivative of a function 'f' at point 'P' in direction of $\vec{a}$ is $\nabla f\,|_P \cdot \hat{a}$



Directional derivative = $\nabla f\,|_{(1,1)} \cdot \hat{a}$

$$= \nabla\left(x^2 + y^2\right)\Big|_{(1,1)} \cdot \left(\frac{\vec{i} + \vec{j}}{\sqrt{2}}\right)$$

$$= \left(2x\vec{i} + 2y\vec{j}\right)\Big|_{(1,1)} \cdot \left(\frac{\vec{i} + \vec{j}}{\sqrt{2}}\right)$$

$$= \left(2\vec{i} + 2\vec{j}\right) \cdot \frac{\left(\vec{i} + \vec{j}\right)}{\sqrt{2}} = 2\sqrt{2}$$

$\therefore$ Directional derivative = $2\sqrt{2}$

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30. (c)

Given,

$$\vec{u} = e^x \cos y\,\hat{i} + e^x \cdot \sin y\,\hat{j}$$

$$\Rightarrow \quad \nabla \cdot \vec{u} = \frac{\partial}{\partial x}(e^x \cos y) + \frac{\partial}{\partial y}(e^x \cdot \sin y)$$

$$= \cos y \cdot (e^x) + e^x \cdot (\cos y)$$

$$= 2e^x \cdot \cos y$$

$$\Rightarrow \quad \boxed{\nabla \cdot \vec{u} = 2e^x \cdot \cos y}$$

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31. (c)

Given,

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\Rightarrow \quad |\vec{r}| = \sqrt{x^2 + y^2 + z^2} = (x^2 + y^2 + z^2)^{1/2}$$

$$\phi = \ln|\vec{r}| = \frac{1}{2} \cdot \ln(x^2 + y^2 + z^2)$$

$$\Rightarrow \quad \text{grad } \phi = \nabla\phi = \frac{\partial \phi}{\partial x}\vec{i} + \frac{\partial \phi}{\partial y}\vec{j} + \frac{\partial \phi}{\partial z} \cdot \vec{k}$$

$$= \frac{1}{2} \cdot \frac{1}{(x^2 + y^2 + z^2)} 2x\vec{i} + \frac{1}{2} \cdot \frac{1}{(x^2 + y^2 + z^2)} \cdot 2y\vec{j} + \frac{1}{2} \cdot \frac{1}{(x^2 + y^2 + z^2)} \cdot 2z\vec{k}$$

$$= \frac{x\vec{i} + y\vec{j} + z\vec{k}}{(x^2 + y^2 + z^2)} = \frac{\vec{r}}{|\vec{r}|^2} = \frac{\vec{r}}{\vec{r} \cdot \vec{r}}$$

$$\therefore \quad \boxed{\text{grad } \phi = \nabla\phi = \frac{\vec{r}}{\vec{r} \cdot \vec{r}}}$$

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32. (a)

$\phi(x, y, z) = xy^2 + yz^2 + zx^2$

$$\text{grad } \phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$\text{grad } \phi = (y^2 + 2xz)\hat{i} + (2xy + z^2)\hat{j} + (2yz + x^2)\hat{k}$$

$\therefore$ At point (2, –1, 1)

$$\text{grad } \phi = ((-1)^2 + 2.2.1)\hat{i} + (2.2(-1) + 1^2)\hat{j} + (2(-1)(1) + 2^2)\hat{k}$$

$$\text{grad } \phi = 5\hat{i} - 3\hat{j} + 2\hat{k}$$

$\therefore$ The required directional derivative is given by,

$$\text{Directional Derivative} = \text{grad } \phi \cdot \frac{\vec{P}}{|\vec{P}|}$$

$$= (5\hat{i} - 3\hat{j} + 2\hat{k}) \cdot \frac{(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{5 - 6 + 4}{\sqrt{9}} = \frac{3}{3} = 1$$

Scan for Video solution

33. (0 to 0)

Let $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

$$\nabla\cdot(\nabla\times\vec{F}) = \nabla\cdot\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \nabla\cdot\left(\vec{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \vec{j}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \vec{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$= \frac{\partial^2 F_z}{\partial x\partial y} - \frac{\partial^2 F_y}{\partial x\partial z} - \frac{\partial^2 F_z}{\partial y\partial x} + \frac{\partial^2 F_x}{\partial y\partial z} + \frac{\partial^2 F_y}{\partial z\partial x} - \frac{\partial^2 F_x}{\partial z\cdot\partial y} = 0$$

$\therefore$ For any vector point function $\vec{F}$, $\nabla\cdot(\nabla\times\vec{F}) = 0$

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34. (b)

Given

$$\vec{F} = \hat{a}_x(3y - k_1 z) + \hat{a}_y(k_2 x - 2z) - \hat{a}_z(k_3 y + z)$$

For $\vec{F}$ to be irrotational, $\nabla\times\vec{F} = 0$

$$\Rightarrow \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y - k_1 z & k_2 x - 2z & -k_3 y - z \end{vmatrix} = 0$$

$\Rightarrow \hat{a}_x(-k_3 + 2) - \hat{a}_y(k_1) + \hat{a}_z(k_2 - 3) = 0$

$\Rightarrow -k_3 + 2 = 0,\ k_1 = 0;\ k_2 - 3 = 0$

$\therefore\ k_1 = 0;\ k_2 = 3;\ k_3 = 2$

Scan for Video solution

35. (0 to 0)

Given

Vector is $-y\vec{i} + x\vec{j} = \vec{F}(\text{say})$

$$\Rightarrow \text{div } \vec{F} = \nabla\cdot\vec{F} = \frac{\partial}{\partial x}(-y) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(0)$$

$$= 0 + 0 + 0 = 0$$

$\therefore\ \boxed{\nabla\cdot\left(-y\vec{i} + x\vec{j}\right) = 0}$

Scan for Video solution

36. (a)

Given

$$\vec{V}(x,y,z) = 2x^2\vec{i} + 3z^2\vec{j} + y^3\vec{k}$$

$$\Rightarrow \text{curl } \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix}$$

$$= \vec{i}\left(3y^2 - 6z\right) - \vec{j}(0 - 0) + \vec{k}(0)$$

$$= \left(3y^2 - 6z\right)\vec{i}$$

At $x = y = z = 1$

$$\text{curl}\vec{V} = \left(3(1)^2 - 6(1)\right)\vec{i}$$

$\Rightarrow\ \boxed{\text{curl}\vec{V}\,|_{(1,1,1)} = -3\vec{i}}$

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37. (c)

For any vector point function, $\vec{V}$

$$\nabla\cdot(\nabla\times\vec{V}) = 0$$

Ex: If $\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$

$$\Rightarrow \nabla\times\vec{V} = \vec{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \vec{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \vec{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\therefore \quad \nabla \cdot (\nabla \times \vec{V}) = \frac{\partial^2 V_z}{\partial x \cdot \partial y} - \frac{\partial^2 V_y}{\partial x \cdot \partial z} - \frac{\partial^2 V_z}{\partial x \cdot \partial y} + \frac{\partial^2 V_x}{\partial y \cdot \partial z} + \frac{\partial^2 V_y}{\partial z \cdot \partial x} - \frac{\partial^2 V_x}{\partial y \cdot \partial z}$$

$$= 0$$

$\boxed{\text{For any Vector } \vec{V}, \text{Div Curl } \vec{V} = 0}$

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38. (–5.72 to –5.70)

$u = x^2 - 3yz$

Directional derivative = grad $u.\hat{a}$

$$\Rightarrow \nabla u = \frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}$$

$$\nabla u_{(2,-1,4)} = (2x)\hat{i} + (-3z)\hat{j} + (-3y)\hat{k} = 4\hat{i} - 12\hat{j} + 3\hat{k}$$

$$\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$$

The required directional derivative is

$$= \text{grad } u.\frac{\vec{a}}{|\vec{a}|}$$

$$= (4\hat{i} - 12\hat{j} + 3\hat{k})\frac{(\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{1^2 + 1^2 + (-2)^2}}$$

$$= \frac{4 - 12 - 6}{\sqrt{6}} = -\frac{14}{\sqrt{6}} = -5.715$$

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39. (a)

$$\vec{P} = x^3y\,\vec{a}_x - x^2y^2\,\vec{a}_y - x^2yz\,\vec{a}_z$$

Considering

$$\text{div}\vec{P} = \nabla \cdot \vec{P}$$

$$= \frac{\partial}{\partial x}(x^3y) + \frac{\partial}{\partial y}(-x^2y^2) + \frac{\partial}{\partial z}(-x^2yz)$$

$$= 3x^2y - 2x^2y - x^2y = 0$$

$$\therefore \quad \nabla \cdot \vec{P} = 0$$

Considering curl $\vec{P} = \nabla \times \vec{P} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$

$$= \vec{a}_x(-x^2{}_z + 0) - \vec{a}_y(-2xyz - 0) + \vec{a}_z(-2xy^2 - x^3)$$

$$\neq \vec{0}$$

Since

$\nabla \cdot \vec{P} = 0$ & $\nabla \times \vec{P} \neq 0$

$\vec{P}$ is solenoidal but not irrotational.

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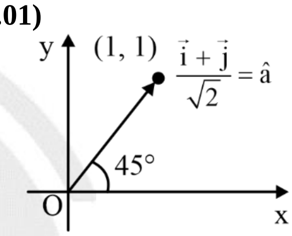
40. (2.99 to 3.01)



Given $f(x, y) = \frac{xy}{\sqrt{2}}(x + y)$

Directional derivative of f at (1, 1): In the direction of 45° to y-axis is

$$= \nabla f\big|_{(1,1)} \cdot \hat{a}$$

$$\nabla f\big|_{(1,1)} = \frac{1}{\sqrt{2}}\nabla(x^2y + xy^2)\Big|_{(1,1)}$$

$$= \frac{1}{\sqrt{2}}\left\{(2xy + y^2)\vec{i} + (x^2 + 2xy)\vec{j}\Big|_{(1,1)}\right\}$$

$$= \frac{1}{\sqrt{2}}\{3\vec{i} + 3\vec{j}\}$$

$$\hat{a} = \frac{\vec{i} + \hat{j}}{\sqrt{2}}$$

$\therefore$ Directional derivative

$$= \nabla f\big|_{(1,1)} \cdot \hat{a} = \frac{1}{\sqrt{2}} \cdot (3\vec{i} + 3\vec{j}) \cdot \frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$$

$$= \frac{1}{2}(3 + 3) = 3$$

$\boxed{\therefore \text{ Directional Derivative} = 3}$

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41. (2.9 to 3.1)

Given $\vec{r} = x \cdot \hat{a}_x + y \cdot \hat{a}_y + z \cdot \hat{a}_z$

$|\vec{r}| = r$

$$\nabla \cdot (r^2 \nabla \ln r) = \nabla \cdot \left(r^2 \nabla \left(\frac{1}{2} \cdot \ln(x^2 + y^2 + z^2)\right)\right)$$

$$= \nabla \cdot \left(r^2 \cdot \frac{1}{2}\left(\frac{1}{r^2}(2x\,\hat{a}_x + 2y\,\hat{a}_y + 2z\hat{a}_z)\right)\right)$$

$$= \nabla \cdot (x \cdot \hat{a}_x + y \cdot \hat{a}_y + z\hat{a}_z) = 1 + 1 + 1 = 3$$

$$\boxed{\therefore\ \nabla \cdot (r^2 \nabla \ln r) = 3}$$

Scan for Video solution

42. (a)

Given

$$\vec{F} = x^2z^2\hat{i} - 2xy^2z\hat{j} + 2y^2z^3\hat{k}$$

$$\Rightarrow \text{Curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2z^2 & -2xy^2z & 2y^2z^3 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(2y^2z^3) - \frac{\partial}{\partial z}(-2xy^2z)\right)$$

$$- \hat{j}\left(\frac{\partial}{\partial x}(2y^2z^3) - \frac{\partial}{\partial z}(x^2z^2)\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x}(-2xy^2z) - \frac{\partial}{\partial y}(x^2z^2)\right)$$

$$\Rightarrow \boxed{\text{Curl } \vec{F} = \hat{i}(4yz^3 + 2xy^2) + \hat{j}(2x^2z) - \hat{k}(2y^2z)}$$

Scan for Video solution

43. (c)

Given

Vector field $x^2z\hat{i} + xy\hat{j} - yz^2\hat{k} = \vec{F}$ (say)

$$\Rightarrow \nabla \cdot \vec{F} = \text{div} \cdot \vec{F} = \frac{\partial}{\partial x}(x^2z) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(-yz^2)$$

$$= 2\,x\,z + x - 2\,y\,z$$

$$\therefore \quad \nabla \cdot \vec{F}|_{(1,-1,1)} = 2(1)(1) + 1 - 2(-1)(1) = 5$$

$$\therefore \quad \boxed{\nabla \cdot \vec{F}|_{(1,-1,1)} = 5}$$

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44. (a)

$$\nabla \cdot (f\,\vec{V}) = x^2y + y^2z + z^2x$$

$$\vec{V} = y\hat{i} + z\hat{j} + x\hat{k}$$

$$\therefore \text{div}\,\vec{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

Using vector identity; $\nabla \cdot V = 0$

$$\text{div}(f\,\vec{V}) = f\,\text{div}\,\vec{V} + \vec{V}\,\text{grad}\,f$$

$$\nabla \cdot (fV) = f(\nabla \cdot V) + V \cdot (\nabla \cdot f)$$

$$x^2y + y^2z + z^2x = 0 + \vec{V} \cdot \nabla f$$

$$\vec{V} \cdot \nabla f = x^2y + y^2z + z^2x$$

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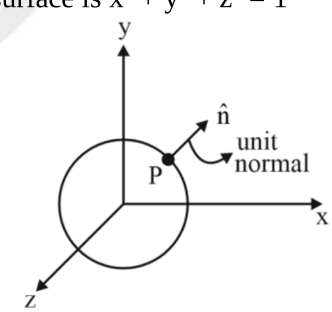
45. (a)

Given: surface is $x^2 + y^2 + z^2 = 1$



For any surface $\phi(x, y, z) = c$, the outward normal vector at point 'P' is given by $\dfrac{\Delta\phi|_P}{|\Delta\phi|_P|} = \hat{n}$

$$\Delta\phi|_P = \left(\frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}\right)\Bigg|_P$$

$$= 2x\vec{i} + 2y\vec{j} + 2z\vec{k}\,\Big|_{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)} = \sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j} + 0.\vec{k}$$

$$\therefore \quad \hat{n} = \frac{\nabla\phi|_P}{|\nabla\phi|_P|} = \frac{\sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j}}{2}$$

$$\Rightarrow \quad \boxed{\hat{n} = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}}$$

Scan for Video solution

46. (a)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

If **f** $\rightarrow$ f(r) then using standard result

$$\text{div}\, \boldsymbol{f} = \vec{\nabla}.\vec{f} = \frac{1}{r^2}\left(\frac{\partial}{\partial r}r^2 |\vec{f}|\right)$$

$\therefore$ div **A** = ∇. **A** = 0 (Given)

$$\frac{1}{r^2}\left(\frac{\partial}{\partial r}\cdot r^2 |A|\right) = 0$$

$$\frac{1}{r^2}\left(\frac{\partial}{\partial r}\cdot r^2 \cdot Kr^n\right) = 0$$

$$\frac{1}{r^2}\left(\frac{\partial}{\partial r}\cdot K \cdot r^{n+2}\right)$$

$$\frac{1}{r^2}(n+2)Kr^{n+1} = 0$$

$(n + 2)K\, r^{n-1} = 0$

$\therefore n + 2 = 0$

$n = -2$

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47. (b)

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$

Directional derivative is given by,

(Grad. f). $\hat{a} = \text{grad } f \cdot \frac{\vec{a}}{|\vec{a}|}$

$$= \left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right).\frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{1^2 + (-1)^2 + 2^2}}$$

$$= \left(2x\hat{i} + 6y\hat{j} + 4z\hat{k}\right).\frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{6}}$$

The required directional derivative at P (1,2,–1) is

$$= \left(2(1)\hat{i} + 6(2)\hat{j} + 4(-1)\hat{k}\right).\frac{\left(\hat{i} - \hat{j} + 2\hat{k}\right)}{\sqrt{6}}$$

$$= \frac{2 - 12 - 8}{\sqrt{6}} = -\frac{18}{\sqrt{6}} = -3\sqrt{6}$$

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48. (c)

Given:

vector function $\vec{F} = 3xz\,\hat{i} + 2xy\,\hat{j} - yz^2\,k$

Divergence of $\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,k$ is given by

$$\nabla\cdot\vec{F} = \frac{\partial}{\partial x}(F_x) + \frac{\partial}{\partial y}(F_y) + \frac{\partial}{\partial z}(F_z)$$

$$= \frac{\partial}{\partial x}(3xz) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(-yz^2)$$

$= 3z + 2x - 2yz$

$$\nabla\cdot\vec{F}\big|_{(1,1,1)} = 3z + 2x - 2yz\big|_{(1,1,1)}$$

$= 3(1) + 2(1) - 2(1)(1) = 3$

$$\boxed{\therefore \nabla\cdot\vec{F}\big|_{(1,1,1)} = 3}$$

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49. (d)

Given:

Vector $\vec{F} = (x - y)\hat{i} + (y - x)\hat{j} + (x + y + z)k$

If $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$, then divergence of $\vec{F}$ is

$$\nabla\cdot\vec{F} = \frac{\partial}{\partial x}(F_x) + \frac{\partial}{\partial y}(F_y) + \frac{\partial}{\partial z}(F_z)$$

$$\Rightarrow \quad \nabla\cdot\vec{F} = \frac{\partial}{\partial x}(x - y) + \frac{\partial}{\partial y}(y - x) + \frac{\partial}{\partial z}(x + y + z)$$

$= 1 + 1 + 1 = 3$

$$\therefore \quad \boxed{\nabla\cdot\vec{F} = 3}$$

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50. (b)

Given: f (x, y, z) = $x^2 + 2y^2 + z$;

P (1, 1, 2); and $\vec{a} = 3\vec{i} - 4\vec{j}$

The directional derivation of f (x, y, z) at P in direction of $\vec{a}$ is given by $\nabla f\big|_P \cdot \frac{\vec{a}}{|\vec{a}|}$

$$\nabla f\big|_P \cdot \frac{\vec{a}}{|\vec{a}|} = \left(\frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}\right)\Bigg|_P \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$= \left((2x)\vec{i} + (4y)\vec{j} + 1\cdot\vec{k}\right)\Big|_{(1,1,2)} \cdot \frac{(3\vec{i} - 4\vec{j})}{|3\vec{i} - 4\vec{j}|}$$

$$= (2\vec{i} + 4\vec{j} + \vec{k}) \cdot \frac{(3\vec{i} - 4\vec{j})}{5} = \frac{6 - 16}{5} = -2$$

$\therefore$ $\boxed{\text{Directional derivative} = -2}$

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51. (9 to 10)

C: r = 1 + cosθ

$$\int_c \vec{F}\cdot\vec{dl} = \oint_c (-y\hat{i} + x\hat{j})(dx\hat{i} + dy\hat{j})$$

$$= \int_c -ydx + xdy$$

According to Green theorem;

$$\int_c Mdx + Ndy = \iint\left(\frac{\partial N}{\partial X} - \frac{\partial M}{\partial y}\right)dxdy$$

$$\therefore \int_c -ydx + xdy = \iint 1 - (-1)dxdy$$

$$= 2\iint dxdy = 2\,(\text{Area of closed curve c})$$

$$= 2\int_0^{2\pi}\int_0^r r\,dr\,d\theta = 2\int_{\theta=0}^{\theta=2\pi}\frac{r^2}{2}d\theta = \int_0^{2\pi}(1+\cos\theta)^2 d\theta$$

$$= \int_0^{2\pi}[1 + \cos^2\theta + 2\cos\theta]d\theta$$

$$= [\theta]_0^{2\pi} + 4\int_0^{\pi/2}\cos^2\theta + 4\int_0^{\pi}\cos\theta$$

$$2\pi + 4\,\frac{1}{2}.\frac{\pi}{2} + 4\times 0$$

$$= 2\pi + \pi + 0$$

$$= 3\pi = 9.42$$

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52. (0.9 to 1.1)

Given that

x	–1	0	1
y	0.8	2.2	2.8

Let line of best fit is y = ax + b

By normal equations

$$\sum y = a\sum x + bn \quad .....(1)$$

$$\sum xy = a\sum x^2 + b\sum x \quad ......(2)$$

x	y	x^2	xy
–1	0.8	1	–0.8
0	2.2	0	0
1	2.8	1	2.8

$$\sum x = 0, \sum y = 5.8$$

$$\sum x^2 = 2, \sum xy = 2, n = 3$$

putting in (1) and (2)

5.8 = a(0) + b(3)

2 = a(2) + b(0)

∴ on solving a = 1, b = 1.93

∴ line of best fit y = ax + b

y = x + 1.93

∴ Slope of line of best fit = 1

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53. (b)

Given integral is $\int_P^Q \left(z^2dx + 3y^2dy + 2xz\ dz\right) = I(say)$

$$\therefore\ I = \int_P^Q \left(z^2\vec{i} + 3y^2\vec{j} + 2xz\ \vec{k}\right)\cdot\left(dx\vec{i} + dy\vec{j} + dz\vec{k}\right)$$

For $I = \int_A^B \vec{F}\cdot d\vec{r}$, if $\vec{F} = \nabla\phi$ for some scalar function ϕ,

Then $\int_A^B \vec{F}\cdot d\vec{r} = \phi|_B - \phi|_A$

$$\vec{F} = z^2\vec{i} + 3y^2 + 2xz\vec{k} = \nabla\left(xz^2 + y^3\right)$$

$$\therefore\ \int_P^Q \vec{F}\cdot d\vec{r} = xz^2 + y^3\Big|_P^Q = \left(xz^2 + y^3\right)\Big|_{(1,1,2)}^{(2,3,1)}$$

$= (2(1)^2 + 3^3) – (1\ (2)^2 + 1^3) = 29 – 5 = 24$

$$\therefore\ \int_P^Q z^2dx + 3y^2dy + 2xz\ dz = 24$$

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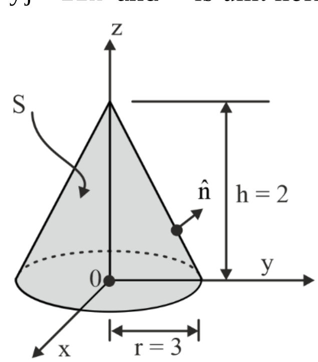
54. (–0.001 to 0.001)

Given

$\vec{B} = x\vec{i} + y\vec{j} – 2z\vec{k}$ and $\hat{n}$ is unit normal to 'S'



$$\int_S \vec{B}\cdot\hat{n}\,ds = \iiint_V (\nabla\cdot\vec{B})\,dv$$

(From Gauss-divergence theorem)

$$\nabla\cdot\vec{B} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(-2z) = 1 + 1 – 2 = 0$$

$$\Rightarrow\ \iiint_V (\nabla\cdot\vec{B})\,dv = 0 \text{ Since } \nabla\cdot\vec{B} = 0$$

$$\Rightarrow\ \boxed{\int_S \vec{B}\cdot\hat{n}\,ds = 0}$$

Scan for Video solution

55. (a)

Given $\phi = \frac{1}{2}\left(x^2 + y^2 + z^2\right)$

$$\Rightarrow\ \nabla\phi = x\vec{i} + y\vec{j} + z\vec{k}\quad \left\{\because \nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right\}$$

's' is surface of sphere of unit radius.

$$\oiint_s \hat{n}\cdot\nabla\phi\,ds = \oiint_s \nabla\phi\cdot\hat{n}\,ds$$

From Gauss-divergence theorem,

$$\oiint_s \nabla\phi\cdot\hat{n}\,ds = \iiint_v \nabla\cdot(\nabla\phi)\,dv$$

$$= \iiint_v \nabla\cdot\left(x\vec{i} + y\vec{j} + z\vec{k}\right)dv = 3\cdot\iiint_v dv$$

$= 3 \times$ Volume of sphere $= 3\times\frac{4}{3}\pi(1)^3 = 4\pi$

$$\therefore\ \boxed{\oiint_s \nabla\phi\cdot\hat{n}\,ds = 4\pi}$$

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56. (a)

Given $\vec{F} = 3x\hat{i} + 5y\hat{j} + 6z\hat{k}$

and A: Area of surface of a cube

From Gauss-divergence Theorem

$$\oiint_A \vec{F}\cdot d\vec{A} = \iiint_V (\nabla\cdot\vec{F})\,dV = \iiint_V 14\,dV$$

$= 14 \times$ Volume of the cube

$= 14 \times (a)^3 = 14 \times (1)^3 = 14$

$$\therefore\ \boxed{\oiint_A \vec{F}\cdot d\vec{A} = 14}$$

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57. (a)

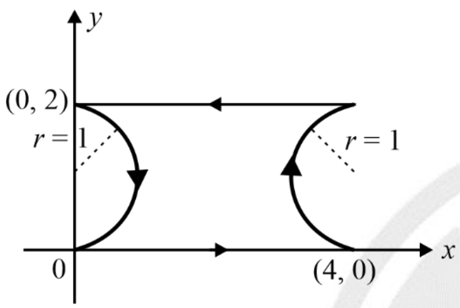
Given: $\vec{F}(x, y) = x\vec{i} + y\vec{j}$

Integral is (Say)

$$I = \oint_c \vec{F}(x, y)\cdot(dx\vec{i} + dy\vec{j})$$

Curl, $\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & 0 \end{vmatrix}$

$$\vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(0-0) = \vec{0}$$



Since, curl $\vec{F}$ is zero, $\vec{F}$ is irrotational (or) conservative field.

$\therefore$ For any closed path,

$$\oint_c \vec{F}\cdot d\vec{r} = \oint_c \vec{F}(x, y)\cdot(dx\vec{i} + dy\vec{j}) = 0$$

Scan for Video solution

58. (a)

Area $= \int\int dxdy$

$\therefore$ option (a) is correct

From Green's theorem;

$$\int M dx + N dy = \int\int\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$$

From option b

$$\int_c xdy = \int\int(1-0)dxdy = \text{Area}$$

$\therefore$ option b is correct

From option c)

$$\int_c ydx = \int\int(0-1)dxdy \neq \text{Area}$$

$\therefore$ Option c) is incorrect

From option d)

$$\frac{1}{2}\int_c xdy - ydx = \frac{1}{2}\int\int 1-(-1)dxdy =$$

$$\int\int dxdy = \text{Area}$$

$\therefore$ option d) is also correct.

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59. (–3.05 to –2.95)

$$\vec{F} = \boldsymbol{a}_x y - \boldsymbol{a}_y x$$

$$\int_c \vec{F}\cdot d\vec{\ell} = (\mathbf{a_x}y - \mathbf{a_y}x)(\mathbf{a_x}dx + \mathbf{a_y}dy)$$

$$\because d\vec{\ell} = dx\boldsymbol{a}_x + dy\boldsymbol{a}_y$$

$$\int_c \vec{F}\cdot d\vec{\ell} = \int ydx - xdy \qquad x \to -1 \text{ to } 2$$

$$\therefore y = x^2, dy = 2xdx$$

$$= -\int_{x=-1}^{x=2} x^2 dx - =x(2xdx)$$

$$= \int_{-1}^{2} x^2 dx - = \left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= -\frac{1}{3}[8+1]$$

$$\int_c F\cdot d\ell = -3.00$$

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60. (3 to 3)

$$\int_c (y+z)dx + (x+z)dy + (x+y)dz = I \text{ (say)}$$

$$\Rightarrow I = \int_c ydx + zdx + xdy + zdy + xdz + ydz$$

$$\Rightarrow I = \int_c (x\,dy + y\,dx) + (y\,dz + z\,dy) + (x\,dz + z\,dx)$$

$$\left[\because d(xy) = xdy + ydx\right]$$

$$= \int_c d(xy) + d(yz) + d(xz)$$

$$= [xy + yz + xz]_{(0,0,0)}^{(1,1,1)} = (1+1+1) - (0+0+0) = 3$$

Scan for Video solution

61. (a)

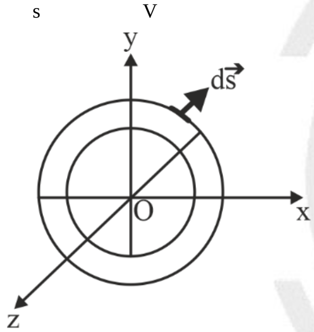
Given: The function can be written as

$$\vec{f} = \frac{\vec{r}}{|\vec{r}|^3}$$

where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

From Gauss-divergence theorem

$$\oint_s \vec{F}.d\vec{s} = \iiint_V (\text{div } \vec{f})\, dv$$



$$\text{div } \vec{f} = \nabla \cdot \vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= \sum \frac{\partial}{\partial x}\left(\frac{x}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$= \sum \frac{(x^2+y^2+z^2)^{\frac{3}{2}} \cdot (1) - x \cdot \frac{3}{2} \cdot (x^2+y^2+z^2)^{\frac{1}{2}} \cdot 2x}{(x^2+y^2+z^2)^3}$$

$$= \sum \frac{y^2+z^2-2x^2}{(x^2+y^2+z^2)^{5/2}} = 0 \quad (\because \text{ The term is cyclic})$$

$$\Rightarrow \boxed{\oiint_s \vec{F} \cdot d\vec{s} = 0}$$

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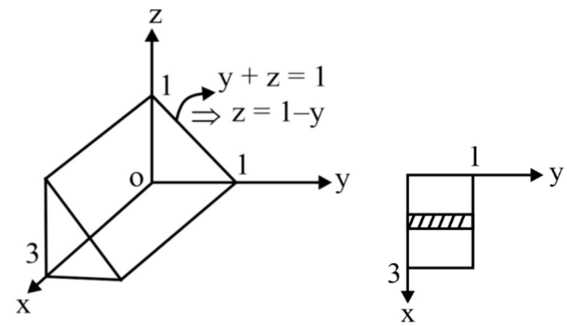
62. (2.25 to 2.25)

Given:

Solid 's' is as shown below



Let $I = \iiint_s x \cdot dx\, dy\, dz$

$$\Rightarrow I = \int_{x=0}^{x=3}\int_{y=0}^{y=1}\int_{z=0}^{z=1-y} x \cdot dx\,dy\,dz$$

$$= \int_{x=0}^{x=3}\int_{y=0}^{y=1}\left(x \cdot z\Big|_0^{1-y}\right)dy\ dx$$

$$\Rightarrow = \int_{x=0}^{x=3}\int_{y=0}^{y=1} x(1-y)dy\ dx = \int_{x=0}^{x=3}\left(xy - \frac{x \cdot y^2}{2}\right)_0^1 dx$$

$$= \int_{x=0}^{x=3}\left(x - \frac{x}{2}\right)dx = \int_{x=0}^{x=3}\frac{x}{2}dx = \frac{x^2}{4}\Bigg|_0^3 = \frac{9}{4} = 2.25$$

$$\therefore \boxed{\iiint_s x \cdot dx\ dy\ dz = 2.25}$$

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63. (139 to 139)

We know $f = 2x^3 + 3y^2 + 4z$

$$\text{grad } f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\text{grad } f = 6x^2\hat{i} + 6y\hat{j} + 4\hat{k}$$

also $\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\int_c \text{grad} f \cdot \boldsymbol{dr} = \int (6x^2\hat{i} + 6y\hat{j} + 4\hat{k})$$

$$(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int (6x^2dx + 6ydy + 4dz) = \int d(2x^3 + 3y^2 + 2z^2)$$

$$= 2x^3 + 3y^2 + 2z^2 \Big|_{(-3,-3,-2)}^{(2,6,-1)}$$

$$= 139$$

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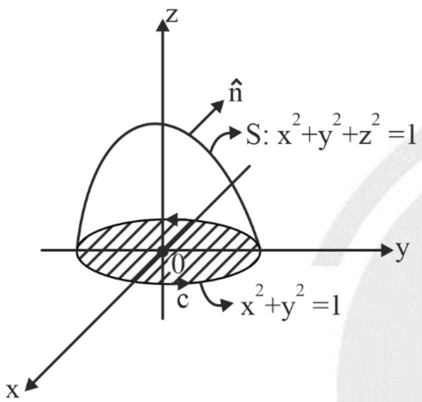
64. (c)

Given

$\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k}\right)$ and S: $x^2 + y^2 + z^2 = 1;\ z \geq 0$



Let 'c' be the path enclosing the surface 'S'.

From stoke's theorem,

$$\iint_S (\nabla \times \vec{u}) \cdot \hat{n}\, ds = \oint_c \vec{u} \cdot d\vec{r}$$

$$= \oint_c \frac{-y^3}{3}dx + \frac{x^3}{3}dy$$

From Green's theorem,

$$\oint_c \frac{-y^3}{3}dx + \frac{x^3}{3}dy = \iint_R \frac{\partial}{\partial x}\left(\frac{x^3}{3}\right) - \frac{\partial}{\partial y}\left(\frac{-y^3}{3}\right)dxdy$$

$$=$$

$$\iint_R \left(x^2 + y^2\right)dxdy = \frac{\pi}{2}(1)^4 = \frac{\pi}{2}$$

$$\therefore \quad \boxed{\int (\nabla \times \vec{u}) \cdot \hat{n}\, ds = \frac{\pi}{2}}$$

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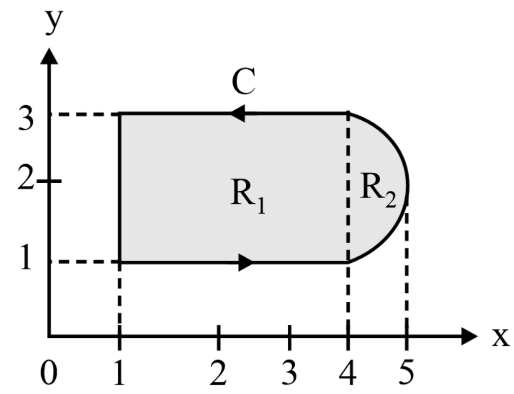
65. (d)

Given: Integral $I = \int_C (xdy - ydx)$ and C is as shown



Since 'C' is a closed path, from Green's theorem

$$\int_C -ydx + xdy = \iint_R \left(\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y)\right)dxdy$$

$$= 2 \cdot \iint_R dx \cdot dy = 2 \times \{\text{Area of region 'R'}\}$$

$$= 2 \times \{\text{Area of rectangle } R_1 + \text{Area of semicircle } R_2\}$$

$$= 2 \times \left\{(3 \times 2) + \left(\frac{\pi}{2} \times (1)^2\right)\right\} = 2 \times \left\{6 + \frac{\pi}{2}\right\} = 12 + \pi$$

$$\boxed{\oint_c (xdy - ydx) = 12 + \pi}$$

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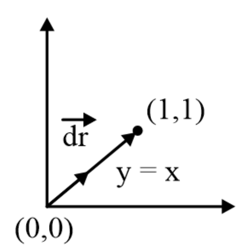
66. (0.60 to 0.70)

$\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$

Let r be the position vector of point (x, y) then,

$\vec{r} = x\hat{i} + y\hat{j} \quad d\vec{r} = dx\hat{i} + dy\hat{j}$

$\therefore \int_c \vec{F}(\vec{r}).d\vec{r} = \int_c \left(x^2\hat{i} + y^2\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$



Equation of line from (0, 0) to (1,1) is y = x

$$\therefore \int_c \vec{F}\cdot d\vec{r} = \int_c x^2dx + y^2dy$$

$$= \int_{x=0}^{x=1} x^2dx + x^2\,dx \quad \because y = x \Rightarrow dy = dx$$

$$= \int_0^1 2x^2dx = \left[2\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

$$\therefore \int_c \vec{F}(\vec{r})\cdot d\vec{r} = 0.67$$

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67. (0 to 0)

$$\int_c (y^2 + 2xy)dx + (x^2 + 2xy)dy$$

$\therefore \vec{F} = (y^2 + 2xy)\hat{i} + (2xy + x^2)\hat{j}$ is conservative field because curl $\vec{F} = 0$, therefore line integral is path independent and hence we will choose an easiest path i.e. a straight line. The equation of straight line will be,

$$\frac{x}{3} + \frac{y}{3} = 1 \Rightarrow x + y = 3 \Rightarrow dx = -dy$$

$$I = \int (y^2 + 2xy)dx + (2xy + x^2)dy$$

$$= \int_{y=0}^{y=3} y^2 + 2(3-y)y(-dy) + \int_{x=3}^{x=0} (2x(3-x) + x^2)(-dx)$$

$$= \int_0^3 -y^2 - 6y + 2y^2 + \int_3^0 -6x + 2x^2 - x^2$$

$$= \int_0^3 (y^2 - 6y)dy + \int_3^0 (x^2 - 6x)dx$$

$$= \left[\frac{y^3}{3} - \frac{6y^2}{2}\right]_0^3 + \left[\frac{x^3}{3} - \frac{6x^2}{2}\right]_3^0$$

= [9 – 27] + – (9 – 27) = 0

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68. (c)

Given: $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

and S: closed surface with a volume 'V'

From Gauss-divergence theorem

$$\oiint_s (\vec{r}\cdot\vec{n})ds = \iiint_V (\text{div}\,\vec{r})dV$$

$$= \iiint_V \left(\frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)\right)dV = \iiint_V 3\,dV$$

$$= 3\times\iiint_V dV = 3V$$

$$\therefore \boxed{\oiint_s (\vec{r}\cdot\vec{n})ds = 3V}$$

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69. (225 to 227)

Given,

$$\vec{F} = (x+y)\vec{i} + (x+z)\vec{j} + (y+z)\vec{k}$$

and S: $x^2 + y^2 + z^2 = 9 \quad \rightarrow$ Sphere

From Gauss-Divergence theorem

$$\oiint_s \vec{F}\cdot\hat{n}ds = \iiint_v (\text{div}\,\vec{F})dV$$

$$\text{div}\,\vec{F} = \nabla\cdot\vec{F} = \frac{\partial}{\partial x}(x+y) + \frac{\partial}{\partial y}(x+z) + \frac{\partial}{\partial z}(y+z)$$

$$= 1 + 0 + 1 = 2$$

$$\therefore \oiint_s \vec{F}\cdot\hat{n}\,ds = \iiint_v 2\cdot dV = 2\times\text{Volume of sphere}$$

$$= 2\times\frac{4}{3}\times\pi\times(3)^3 = 72\pi$$

= 226.19 Cubic units

$$\Rightarrow \boxed{\oiint_s \vec{F}\cdot\hat{n}ds = 226.19}$$

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70. (–11.1 to –10.9)

Given line integral is $I = \int_c (2zdx + 2ydy + 2xdz)$

and A: (0, 2, 1) to point B:(4, 1, –1)

$$I = \int_c (2zdx + 2ydy + 2xdz)$$

$$I = \int_c \left(2z\vec{i} + 2y\vec{j} + 2x\vec{k}\right)\cdot\left(dx\vec{i} + dy\vec{j} + dz\vec{k}\right)$$

$I = \int_c \vec{F}\cdot d\vec{r}$ where $\vec{F} = 2\left(z\hat{i} + y\hat{j} + x\hat{k}\right)$

Curl $\vec{F} = 0$

$\therefore \quad \vec{F} = \nabla\phi$ where $\phi = 2xz + y^2$

$\therefore \quad \int_A^B \vec{F}\cdot d\vec{r} = \phi\Big|_A^B = (2xz + y^2)\Big|_A^B$

$\Rightarrow \quad \int_C 2zdx + 2ydy + 2xdz = (2xz + y^2)\Big|_{(0,2,1)}^{(4,1,-1)}$

$= 2(-4) + 1 - (4) = -11$

$\therefore \quad I = \int_C (2zdx + 2ydy + 2xdz) = -11$

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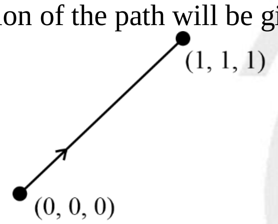
71. (b)

The equation of the path will be given as



$$\frac{x-0}{1-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0}$$

Let x = y = z = t

$\therefore \quad dx = dy = dz = dt$

When $x = y = z = 0 \Rightarrow t = 0$ & $x = y = z = 1 \Rightarrow t = 1$

$I = \int_c 2xy^2dx + 2x^2ydy + dz$

$= \int_{t=0}^{t=1} 2t(t)^2dt + 2(t)^2tdt + dt$

$= \int_0^1 (4t^3 + 1)dt = [t^4 + t]_0^1 = 2$

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72. (4.40 to 4.45)

We know, Line integral = $\int \vec{F}.d\vec{r}$

$$\int_{(0,0,0)}^{(1,1,1)} [5xz\hat{i} + (3x^2 + 2y)\hat{j} + x^2z\hat{k}]\ [dx\hat{i} + dy\hat{j} + dz\hat{k}]$$

$\int 5x^2\ dx + (3x^2 + 2y)\ dy + x^2z\ dz$

$x = t \quad dx = dt$

$y = t^2 \quad dy = 2t\ dt$

$z = t \quad dz = dt$

$\vec{F}.d\vec{r} = 5(t)(t)\cdot dt + (3t^2 + 2t^2)2tdt + t^3dt$

$\int_0^1 5t^2dt + 11t^3dt\ t = 0\ + (3t^2 + 2t^2)2tdt$

$$\left[5\frac{t^3}{3} + 11\frac{t^4}{4}\right]_0^1$$

$$\frac{5}{3} + \frac{11}{4} = \frac{53}{12} = 4.4167$$

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73. (–0.03 to 0.03)

Given, Integral is $I = \oint_c (xy^2dx + x^2y\,dy)$

$\Rightarrow \quad I = \oint_c d\left(\frac{x^2y^2}{2}\right) = 0$

Since, when the path is closed, initial and final points of integration are same.

$\therefore$ For any closed curve '*c*', $\oint_c (xy^2dx + x^2y\,dy) = 0$

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74. (725.9 to 726.1)

Given

$\nabla\varphi = yz\hat{i} + xz\hat{j} + xy\hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\int_C \nabla\phi\cdot d\vec{r} = \int_C yz\cdot dx + xz\cdot dy + xy\cdot dz$

The curve 'C' is

$x = t;\ y = t^2;\ z = 3t^2; \qquad 1 \le t \le 3$

$\Rightarrow \quad dx = dt;\ dy = 2\ t\ dt;\ dz = 6\ t\ dt$

$$\therefore \quad \int_c \nabla\phi \cdot d\vec{r} = \int_1^3 3t^4 \cdot dt + 3t^3 \cdot 2t \cdot dt + t^3 \cdot 6t \cdot dt$$

$$= \int_1^3 15t^4 \cdot dt = 3t^5\Big|_1^3 = 3\left(3^5 - 1\right) = 726$$

$\therefore$ Required value = 726

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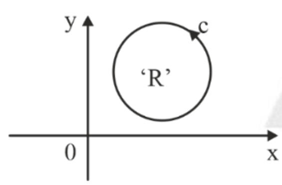
75. (15.9 to 16.1)

Given

$\vec{F} = y\hat{i} + 2x\hat{j}$

$\vec{r}'$ is unit Tangent vector on 'c'

$\Rightarrow \quad \vec{r}' \cdot ds = d\vec{s}$



$$\therefore \quad \oint_c \vec{F} \cdot \vec{r}' ds = \oint_c \vec{F} \cdot d\vec{s} = \oint_c y\,dx + 2x\,dy$$

c is closed path

From Green's Theorem,

$$\oint_c y\,dx + 2x\,dy = \iint_R \left(\frac{\partial}{\partial x}(2x) - \frac{\partial}{\partial y}(y) \right) dx\,dy$$

$$= \iint_R dx\,dy = \text{Area of circle}$$

$$\boxed{\therefore \oint_c \vec{F} \cdot \vec{r}\,ds = \text{Area of circle} = \pi\left(\frac{4}{\sqrt{\pi}}\right)^2 = 16}$$

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76. (214 to 218)

Given :

Integral $\iint_s \frac{1}{\pi}\left(9x\vec{i} - 3y\vec{j}\right) \cdot \hat{n}\,ds$ and $s : x^2 + y^2 + z^2 = 9$

S is surface of a sphere of radius '3'

By Gauss divergence theorem,

$$\iint_s \frac{1}{\pi}\left(9x\vec{i} - 3y\vec{j}\right) \cdot \hat{n}\,ds = \frac{1}{\pi} \cdot \iiint_V \nabla \cdot \left(9x\vec{i} - 3y\vec{j}\right) dV$$

$$= \frac{1}{\pi} \cdot \iiint_V 6 \cdot dV$$

$$= \frac{6}{\pi} \times \text{volume of the sphere}$$

$$= \frac{6}{\pi} \times \frac{4}{3} \times \pi \times (3)^3 = 216 \text{ units}$$

$$\therefore \quad \boxed{\iint_s \frac{1}{\pi}\left(9x\vec{i} - 3y\vec{j}\right) \cdot \hat{n}\,ds = 216}$$

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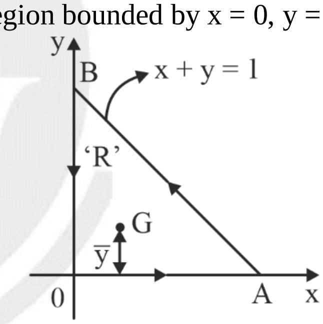
77. (1.60 to 1.70)

Given

Integral $\int_C \left(3x - 8y^2\right)dx + \left(4y - 6xy\right)dy$

C: is region bounded by x = 0, y = 0, x + y = 1



$\Rightarrow \quad \Delta OAB$ is closed path.

From Green's Theorem

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx \cdot dy$$

$$\Rightarrow \quad \oint_C \left(3x - 8y^2\right)dx + \left(4y - 6xy\right)dy$$

$$= \iint_R \left\{ \frac{\partial}{\partial x}\left(4y - 6xy\right) - \frac{\partial}{\partial y}\left(3x - 8y^2\right) \right\} dx \cdot dy$$

$$= \iint_R \left(-6y - (-16y)\right) dx \cdot dy$$

$$= \iint_R 10y\,dx \cdot dy$$

$$=10\times\iint_R y\,dx\cdot dy$$

= 10 × (Area of ΔOAB) × (y –coordinate of centroid)

$$\left(\because \iint_R y\,dA = A\times\overline{y}\right)$$

$$= 10\times\left(\frac{1}{2}\times1\times1\right)\times\frac{1}{3}=\frac{10}{6}$$

$$\therefore \quad \boxed{\int_C (3x-8y^2)dx+(4y-6xy)dy=\frac{10}{6}=1.667}$$

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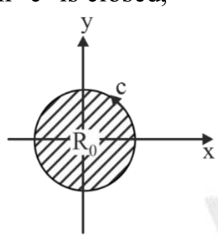
78. (c)

Integral, $\oint_c (y\,dx - x\,dy)$ and $c:\ x^2+y^2=\frac{1}{4}$

Since path 'c' is closed,



By Green's theorem,

$$\oint_c M\,dx+N\,dy=\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dx\,dy$$

$$\Rightarrow \oint_c (y\,dx-x\,dy)=\iint_R\left(\frac{\partial}{\partial x}(-x)-\frac{\partial}{\partial y}(y)\right)dx\,dy$$

$$= -2\cdot\iint_R dx\,dy$$

$$=-2\times\text{Area of Region 'R'}= -2\times\pi\left(\frac{1}{4}\right)=\frac{-\pi}{2}$$

$$\therefore \quad \boxed{\oint_c (y\,dx-x\,dy)=\frac{-\pi}{2}}$$

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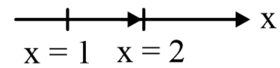
79. (b)



Along x-axis

$y = z = 0$

$dy = dz = 0$

$$I=\int \vec{F}.\vec{dl}$$

$$=\int (y^2xa_x-y^2a_y-x^2a_z)(dx\ a_x+d_ya_y+d_za_z)$$

$$=\int y^2xdx-yzdy-x^2dz$$

$$\int 0^2xdx-0.0.0-x^2.0$$

$\therefore$ I = 0

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80. (a)

Given:

$$\iint_S \frac{1}{4}(\vec{F}\cdot\hat{n})dA$$

Where S → surface of sphere $x^2 + y^2 + z^2 = 1$

By Gauss-divergence theorem,

$$\iint_S \frac{1}{4}(\vec{F}\cdot\hat{n})dA=\iiint_V\frac{1}{4}(\nabla.\vec{F})dV$$

$$=\frac{1}{4}\times\iiint_V\left(\frac{\partial(x)}{\partial x}+\frac{\partial(y)}{\partial y}+\frac{\partial(z)}{\partial z}\right)dV$$

$$=\frac{3}{4}\times\iiint_V dV=\frac{3}{4}\times\text{volume of sphere}$$

$$=\frac{3}{4}\times\frac{4}{3}\times\pi\times(1)^3=\pi$$

$$\boxed{\therefore \iint_s \frac{1}{4}(\vec{F}\cdot\hat{n})dA=\pi}$$

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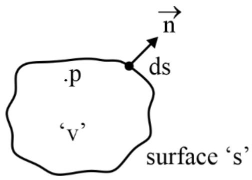
81. (d)

Given:

's' is a closed durrface enclosing volume 'v'

Position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$



From Gauss-divergence theorem

$$\oiint_s 5\,\vec{r}.\vec{n}\,ds = \iiint_v 5(\nabla.\vec{r})dv$$

$$= \iiint_v 5\left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right)dv = 15.\iiint_v dv = 15v$$

$$\boxed{\therefore \oiint_s 5\vec{r}.\vec{n}\,ds = 15v}$$

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82. (c)

Given $\vec{A} = xy\hat{a}_x + x^2 \cdot \hat{a}_y$

By Green's theorem,

$$\oint_c \vec{A}\cdot d\vec{l} = \oint_c xydx + x^2dy$$

$$= \iint_R \left(\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy)\right)dxdy$$

$$= \iint_R (2x - x)dxdy = \iint_R x\, dxdy$$

$\therefore \iint_R x \cdot dxdy =$ Area of 'R' $\times$ 'x' co-ordinate of centroid.

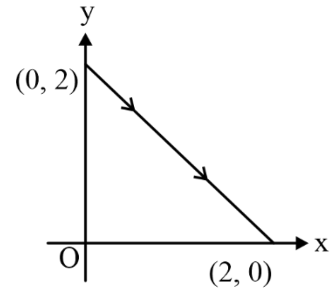
$$= \left(\frac{1}{\sqrt{3}} \times 2\right) \times \frac{1.5}{\sqrt{3}} = 1$$

$$\therefore \oint_c \vec{A}\cdot d\vec{l} = 1$$

Scan for Video solution

83. (d)

Equation of straight line;



$$\frac{x}{2} + \frac{y}{2} = 1$$

$x + y = 2$ $\qquad [\because \vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y]$

$$\int \vec{F}(x,y).\vec{dl} = \int\left\{(x^2 + xy)\hat{a}_x + (y^2 + xy)\hat{a}_y\right\}.\left\{dx\,\hat{a}_x + dy\,\hat{a}_y\right\}$$

$$= \int \left(x^2 + xy\right)dx + (y^2 + xy)dy$$

$$= \int_{x=0}^{x=2} x^2 + x(2 - x)dx + \int_{y=2}^{y=0} y^2 + (2 - 4)y\,dy$$

$$[x^2]_0^2 + [y^2]_2^0 = (4 - 0) + (0 - 4) = \boxed{0}$$

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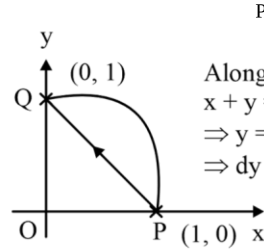
84. (b)

Given: Points are P (1, 0) and Q (0, 1)

$$\text{Line integral } I = 2 \times \int_P^Q (xdx + ydy)$$

$$= 2 \times \int_P^Q (x\vec{i} + y\vec{j}).(dx\vec{i} + dy\vec{j})$$



Along straight line
$x + y = 1$
$\Rightarrow y = 1 - x$
$\Rightarrow dy = -dx$

For the integral $\int_c \vec{F}.\vec{dr}$, if curl $\vec{F} = \vec{0}$, then $\int_c \vec{F}.\vec{dr}$ is independent of path.

Consider curl $\vec{F} = \nabla \times (x\vec{i} + y\vec{j}) = \vec{0}$

$$\therefore \quad I = 2 \times \int_{y=0}^{y=1} (1 - y)(-dy) + ydy$$

$$= 2 \times \int_0^1 -dy + 2y dy = 2 \times \{-y + y^2 \Big|_0^1\} = 2 \times 0 = 0$$

$$\boxed{\therefore 2 \times \int_p^Q x dx + y dy = 0}$$

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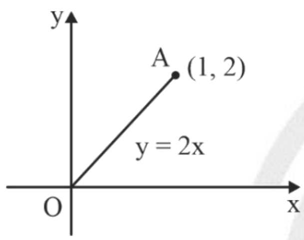
85. (a)

Given g (x, y) = $4x^3 + 10y^4$

The path is the line segment joining (0, 0) and (1, 2)

$\therefore$ g (x, y) = $4x^3 + 10y^4$



Along OA, y = 2x

$$\therefore \int_O^A g(x, y) = \int_{x=0}^{x=1} 4x^3 + 10(2x)^4 \cdot dx$$

$$= \int_{x=0}^{x=1} \left(4x^3 + 160x^4\right) \cdot dx$$

$$= x^4 + 32 \cdot x^5 \Big|_0^1 = 1 + 32 = 33$$

$\therefore$ The integration of g(x, y) along OA is 33.

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CHAPTER

6

COMPLEX CALCULUS

Complex Numbers and their Properties

1. [MCQ] [GATE-EC-2023: 1M]

Let $w^4 = 16j$. Which of the following cannot be a value of w?

(a) $2e^{\frac{j2\pi}{8}}$ (b) $2e^{\frac{j\pi}{8}}$

(c) $2e^{\frac{j5\pi}{8}}$ (d) $2e^{\frac{j9\pi}{8}}$

2. [MCQ] [GATE-ME-2021: 1M]

Value of $(1 + i)^8$, where $i = \sqrt{-1}$, is equal to

(a) 4 (b) 16

(c) 4i (d) 16i

3. [MCQ] [GATE-EE-2017 : 1M]

For a complex number z, $\lim_{Z \to i} \frac{z^2+1}{z^3+2z-i(z^2+2)}$ is

(a) –2i (b) –i

(c) i (d) 2i

4. [NAT] [GATE-EC-2015:1M]

Let $f(z) = \frac{az+b}{cz+d}$, If $f(z_1) = f(z_2)$ for all $z_1 \neq z_2$, a = 2, b = 4 and c = 5, then d should be equal to_________.

5. [MCQ] [GATE-EE-2015 : 1M]

Given $f(z) = g(z) + h(z)$ where f, g, h are complex valued functions of a complex variable z. Which one of the following statements is TRUE?

(a) If f(z) is differentiable at z_o, then g(z) and h(z) are also differentiable at z_o

(b) If g(z) and h(z) are differentiable at z_o, then f(z) is also differentiable at z_o.

(c) If f(z) is continuous at z_o, then it is differentiable at z_o.

(d) If f(z) is differentiable at z_o, then so are its real and imaginary parts.

6. [MCQ] [GATE-ME-2015: 1M]

Given two complex numbers $z_1 = 5 + (5\sqrt{3})i$ and $z_2 = \frac{2}{\sqrt{3}} + 2i$, the argument of $\frac{z_1}{z_2}$ in degrees is

(a) 0 (b) 30

(c) 60 (d) 90

7. [MCQ] [GATE-ME-2014: 1M]

The argument of the complex number $\frac{1+i}{1-i}$, where $i = \sqrt{-1}$, is

(a) $-\pi$ (b) $\frac{-\pi}{2}$

(c) $\frac{\pi}{2}$ (d) π

8. [MCQ] [GATE-ME-2014: 2M]

If z is a complex variable, the value of $\int_5^{3i} \frac{dz}{z}$ is

(a) –0.511 – 1.57i (b) –0.511 + 1.57i

(c) 0.511 – 1.57i (d) 0.511 + 1.57i

9. [MCQ] [GATE-EE-2014 : 1M]

All the values of the multi-valued complex function 1^i, where $i = \sqrt{-1}$, are

(a) Purely imaginary

(b) Real and non-negative

(c) On the unit circle
(d) Equal in real and imaginary parts

10. [MCQ] [GATE-EE-2013 : 1M]

Square roots of –i, where i = $\sqrt{-1}$, are

(a) i, –i
(b) $\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right),\cos\left(\frac{3\pi}{4}\right)+i\ \sin\left(\frac{3\pi}{4}\right)$
(c) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right),\cos\left(\frac{3\pi}{4}\right)+i\ \sin\left(\frac{\pi}{4}\right)$
(d) $\cos\left(\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right),\cos\left(-\frac{3\pi}{4}\right)+i\ \sin\left(\frac{3\pi}{4}\right)$

11. [MCQ] [GATE-EE-2012 : 1M]

If x = $\sqrt{-1}$, then the value of x^x is

(a) $e^{-\frac{\pi}{2}}$
(b) $e^{\frac{\pi}{2}}$
(c) x
(d) 1

12. [MCQ] [GATE-EC-2012:1M]

If x = $\sqrt{-1}$, then the value of x^x is

(a) $e^{-\pi/2}$
(b) $e^{\pi/2}$
(c) x
(d) 1

13. [MCQ] [GATE-EE-2011 : 1M]

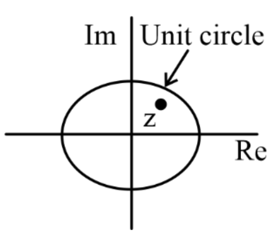
Roots of the algebraic equation $x^3 + x^2 + x + 1 = 0$ are

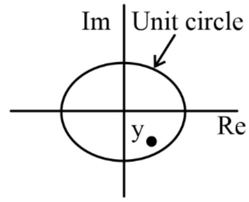
(a) (+1, +j, –j)
(b) (+1, –1, +1)
(c) (0, 0 , 0)
(d) (–1, +j, –j)

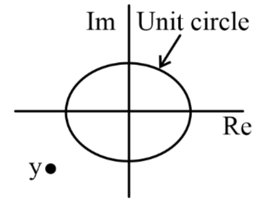
14. [MCQ] [GATE-EE-2011 :1M]

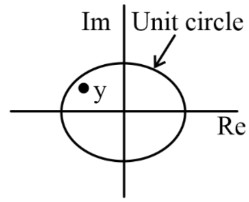
A point z has been plotted in the complex plane, as shown in figure below.

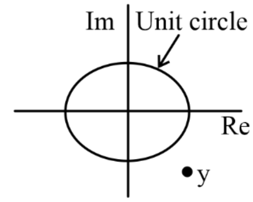
The plot of the complex number $y=\frac{1}{z}$ is



(a)


(b)


(c)


(d)


15. [MCQ] [GATE-ME-2011: 1M]

The product of two complex numbers 1 + i and 2 – 5i is

(a) 7 – 3i
(b) 3 – 4i
(c) –3 – 4i
(d) 7 + 3i

16. [MCQ] [GATE-ME-2010: 1M]

The modulus of the complex number $\left(\frac{3+4i}{1-2i}\right)$ is

(a) 5
(b) $\sqrt{5}$
(c) $1/\sqrt{5}$
(d) 1/5

Complex Functions

17. [NAT] [GATE-ME-2023: 1M]

The value of k that makes the complex-valued function

$f(z) = e^{-kx}(\cos 2y - i \sin 2y)$ analytic, where $z = x + iy$, is _____. (Answer in integer)

18. [MCQ] [GATE-EE-2021 : 1M]

Let $p(z)=z^3+(1+j)z^2+(2+j)z+3$, where z is a complex number. Which one of the following is true?

(a) Conjugate{p(z)} = p (conjugate{z}) for all z
(b) The sum of the roots of p(z)=0 is a real number
(c) The complex roots of the equation $p(z) = 0$ come in conjugate pairs
(d) All the roots cannot be real

19. [MCQ] [GATE-ME-2020: 1M]

Which of the following function f(z), of the complex variable z, is NOT analytic at all the points of the complex plane?

(a) $f(z) = z^2$ (b) $f(z) = e^z$
(c) $f(z) = \sin z$ (d) $f(z) = \log z$

20. [NAT] [GATE-ME-2020: 2M]

An analytic function of a complex variable z = x + iy $\left(i = \sqrt{-1}\right)$ is defined as

$$f(z) = x^2 - y^2 + i\,\psi(x, y),$$

where ψ (x, y), is a real function. The value of the imaginary part of f(z) at z = (1 + i) is _____. (Round off to 2 decimal places).

21. [MCQ] [GATE-ME-2020: 2M]

The function f(z) of complex variable z = x + iy, where $i = \sqrt{-1}$, is given as $f(z) = (x^3 - 3xy^2) + i\,v(x, y)$. For this function to be analytic, v (x, y) should be

(a) $(3xy^2 - y^3)$ + constant
(b) $(3x^2y^2 - y^3)$ + constant
(c) $(x^3 - 3x^2y)$ + constant
(d) $(3x^2y - y^3)$ + constant

22. [MCQ] [GATE-EC-2019:1M]

Which one of the following functions is analytic over the entire complex plane?

(a) ln (z) (b) $e^{1/z}$
(c) $\dfrac{1}{1-z}$ (d) cos(z)

23. [MCQ] [GATE-EE-2019 : 1M]

Which one of the following functions is analytic in the region $|z| \le 1$?

(a) $\dfrac{z^2-1}{z}$ (b) $\dfrac{z^2-1}{z+2}$
(c) $\dfrac{z^2-1}{z-0.5}$ (d) $\dfrac{z^2-1}{z+j0.5}$

24. [MCQ] [GATE-ME-2019: 2M]

A harmonic function is analytic if it satisfies the Laplace equation. If $u(x, y) = 2x^2 - 2y^2 + 4xy$ is a harmonic function, then its conjugate harmonic function v(x, y) is

(a) $4xy - 2x^2 + 2y^2$ + constant
(b) $4y^2 - 4xy$ + constant
(c) $2x^2 - 2y^2 + xy$ + constant
(d) $-4xy + 2y^2 - 2x^2$ + constant

25. [MCQ] [GATE-ME-2019: 2M]

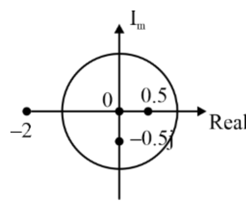
An analytic function f(z) of complex variable z = x + iy may be written as f(z) = u(x, y) + i v (x, y). Then u(x, y) and v(x, y) must satisfy

(a) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$

(b) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

(c) $\dfrac{\partial u}{\partial x} = -\dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$

(d) $\dfrac{\partial u}{\partial x} = -\dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$



26. [MCQ] [GATE-ME-2018: 1M]

F(z) is a function of the complex variable z = x + iy given by F(z) = i z + k Re(z) + i Im(z).

For what value of k will F(z) satisfy the Cauchy-Riemann equations?

(a) 0 (b) 1
(c) –1 (d) y

27. [MCQ] [GATE-ME-2017: 2M]

If $f(z) = (x^2 + ay^2) + i\,bxy$ is a complex analytic function of z = x + iy, where $i = \sqrt{-1}$, then

(a) a = –1, b = –1 (b) a = –1, b = 2
(c) a = 1, b = 2 (d) a = 2, b = 2

28. [MCQ] [GATE-ME-2016: 1M]

f(z) = u (x, y) + iv (x, y) is an analytic function of complex variable z = x + iy where $i = \sqrt{-1}$.

If u (x, y) = 2xy, then v (x, y) may be expressed as

(a) $-x^2 + y^2$ + constant (b) $x^2 - y^2$ + constant
(c) $x^2 + y^2$ + constant (d) $-(x^2 + y^2)$ + constant

29. [NAT] [GATE-ME-2016: 1M]

A function f of the complex variable z = x + i y, is given as f (x, y) = u (x, y) + i v (x, y), where

$u(x, y) = 2kxy$ and $v(x, y) = x^2 - y^2$. The value of k, for which the function is analytic, is ______

30. [NAT] [GATE-EC-2016:1M]

Consider the complex valued function $f(z) = 2z^3 + b|z|^3$ where z is a complex variable. The value of b for which the function f(z) is analytic is ______.

31. [MCQ] [GATE-EE-2016 : 1M]

Consider the function f (z) = z + z* where z is a complex variable and z* denotes its complex conjugate. Which one of the following is TRUE?

(a) f(z) is both continuous and analytic
(b) f(z) is a continuous but not analytic
(c) f(z) is not continuous but is analytic
(d) f(z) is neither continuous nor analytic

32. [NAT] [GATE-EC-2014:2M]

The real part of an analytic function f(z) where z = x + jy is given by $e^{-y}\cos(x)$. The imaginary part of f(z) is

(a) $e^{y}\cos(x)$ (b) $e^{-y}\sin(x)$
(c) $-e^{y}\sin(x)$ (d) $-e^{-y}\sin(x)$

33. [MCQ] [GATE-EE-2014 : 1M]

Let S be the set of points in the complex plane corresponding to the unit circle. (That is S = {z : |z| = 1}. Consider the function f(z) = zz*where z* denotes the complex conjugate of z. The f(z) maps S to which one of the following in the complex plane

(a) Unit circle
(b) Horizontal axis line segment from origin to (1, 0)
(c) The point (1, 0)
(d) The entire horizontal axis

34. [MCQ] [GATE-ME-2014: 2M]

An analytic function of a complex variable z = x + iy is expressed as f(z) = u(x, y) + iv(x, y), where $i = \sqrt{-1}$. If u (x, y) = 2xy, then v (x, y) must be

(a) $x^2 + y^2$ + constant (b) $x^2 - y^2$ + constant
(c) $-x^2 + y^2$ + constant (d) $-x^2 - y^2$ + constant

35. [MCQ] [GATE-ME-2014: 2M]

An analytic function of a complex variable z = x + iy is expressed as f(z) = u(x, y) + iv(x, y), where $i = \sqrt{-1}$. If $u(x, y) = x^2 - y^2$, then expression for v (x, y) in terms of x, y and a general constant c would be

(a) $xy + c$ (b) $\frac{x^2 + y^2}{2} + c$
(c) $2xy + c$ (d) $\frac{(x - y)^2}{2} + c$

36. [MCQ] [GATE-ME-2009: 2M]

An analytic function of a complex variable z = x + iy is expressed as f(z) = u (x, y) + i v(x, y) wheere $i = \sqrt{-1}$. If u = xy, then the expression for v should be

(a) $\frac{(x + y)^2}{2} + k$ (b) $\frac{x^2 - y^2}{2} + k$
(c) $\frac{y^2 - x^2}{2} + k$ (d) $\frac{(x - y)^2}{2} + k$

37. [MCQ] [GATE-EC-2008: 1M]

The equation sin(z) = 10 has

(a) no real (or) complex solution
(b) exactly two distinct complex solutions
(c) a unique solution
(d) an infinite number of complex solutions

Complex Integration

38. [MCQ] [GATE-EC-2023:1M]

The value of the contour integral, $\oint\left(\frac{z+2}{z^2+2z+2}\right)dz$, where the contour C is $\left\{z : \left|z + 1 - \frac{3}{2}j\right| = 1\right\}$ taken in the counter clockwise direction, is

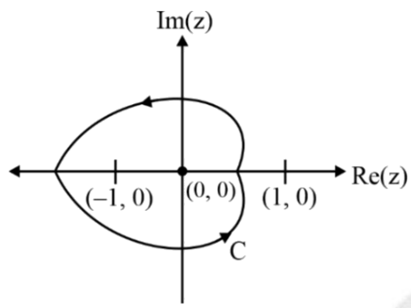
(a) $-\pi(1 + j)$ (b) $\pi(1 + j)$
(c) $\pi(1 - j)$ (d) $-\pi(1 - j)$

39. [NAT] [GATE-EC-2022:2M]

A simple closed path *C* in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{z^2-1}dz = -i\pi A,$$

where $i=\sqrt{-1}$, then the value of *A* is _______ (rounded off to two decimal places).



40. [MCQ] [GATE-ME-2022: 2M]

The value of the integral

$$\oint\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)dz$$

evaluated over a counter-clockwise circular contour in the complex plane enclosing only the pole $z = i$, where i is the imaginary unit, is

(a) $(-1+i)\pi$ (b) $(1+i)\pi$

(c) $2(1-i)\pi$ (d) $(2+i)\pi$

41. [NAT] [GATE-ME-2022: 2M]

Given $z = x + iy$, $i=\sqrt{-1}$. C is a circle of radius 2 with the center at the origin.

If the contour C is traversed anticlockwise, then the value of the integral $\frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$ is ____.

(Round off to one decimal place).

42. [MCQ] [GATE-ME-2021: 2M]

Let C represent the unit circle centered at origin in the complex plane and complex variable, $z = x + iy$. The value of the contour integral $\oint_C \frac{\cosh 3z}{2z}dz$ (where integration is taken counter clockwise) is

(a) 0 (b) 2

(c) πi (d) $2\pi i$

43. [MCQ] [GATE-EE-2021: 1 M]

Let $(-1-j), (3-j), (3+j)$ and $(-1+j)$ be the vertices of a rectangle C in the complex plane. Assuming that C is traversed in counter-clockwise direction, the value of the contour integral $\oint_c \frac{dz}{z^2(z-4)}$ is

(a) $j\pi/2$ (b) 0

(c) $-j\pi/8$ (d) $j\pi/16$

44. [MCQ] [GATE-EE-2020 : 1M]

The value of the following complex integral, with C representing the unit circle centered at origin in the counter clock wise sense, is:

$$\int_c \frac{z^2+1}{z^2-2z}dz$$

(a) $8\pi i$ (b) $-8\pi i$

(c) $-\pi i$ (d) πi

45. [MCQ] [GATE-EE-2019 : 2M]

The closed loop line integral

$$\oint_{|z|=5}\frac{z^3+z^2+8}{z+2}dz$$

evaluated counter-clockwise, is

(a) $+8\pi j$ (b) $-8\pi j$

(c) $-4j\pi$ (d) $+4j\pi$

46. [NAT] [GATE-EC-2019: 1M]

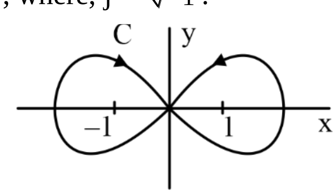
The value of the contour integral

$$\frac{1}{2\pi j}\oint_c\left(z+\frac{1}{z}\right)^2 dz$$

Evaluated over the unit circle $|z| = 1$ is ______ .

47. [NAT] [GATE-EC-2018:2M]

The contour C given below is on the complex plane $z = x + jy$, where, $j=\sqrt{-1}$.



The value of the integral $\frac{1}{\pi j}\oint_C \frac{dz}{z^2-1}$ is _______

48. [MCQ] **[GATE-EE-2018 : 1M]**

The value of the integral $\oint_c \frac{z+1}{z^2-4}dz$ in counter clockwise direction around a circle C of radius 1 with center at the point z = –2 is

(a) $\frac{\pi i}{2}$ (b) $2\pi i$

(c) $-\frac{\pi i}{2}$ (d) $-2\pi i$

49. [MCQ] **[GATE-EE-2018 : 2M]**

If C is a circle $|Z| = 4$ and $f(z) = \frac{z^2}{\left(z^2-3z+2\right)^2}$,

then $\oint_c f(z)dz$ is

(a) 1 (b) 0

(c) –1 (d) –2

50. [MCQ] **[GATE-ME-2018: 2M]**

Let z be a complex variable. For a counter-clockwise integration around a unit circle C, centred at origin, $\oint_C \frac{1}{5z-4}dz = A\pi i$ the value of A is

(a) 2/5 (b) 1/2

(c) 2 (d) 4/5

51. [MCQ] **[GATE-EC-2017:1M]**

The residues of a function f(z) $= \frac{1}{(z-4)(z+1)^3}$ are,

(a) $\frac{-1}{27}$ and $\frac{-1}{125}$ (b) $\frac{1}{125}$ and $\frac{-1}{125}$

(c) $\frac{-1}{27}$ and $\frac{1}{5}$ (d) $\frac{1}{125}$ and $\frac{-1}{5}$

52. [MCQ] **[GATE-EC-2017:2M]**

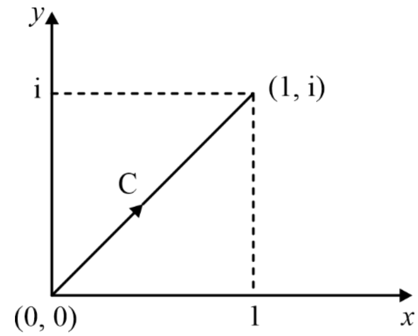
An integral I over a counter clockwise circle C is given by $I = \oint_C \frac{z^2-1}{z^2+1}e^z dz$. If C is defined as |z| = 3, then the value of I is

(a) $-\pi i \sin(1)$ (b) $-2\pi i \sin(1)$

(c) $-3\pi i \sin(1)$ (d) $-4\pi i \sin(1)$

53. [MCQ] **[GATE-EE-2017 : 2M]**

Consider the line integral $I = \int_C \left(x^2+iy^2\right)dz$, where $z = x + iy$. The line c is shown in the figure below.



The value of I is

(a) $\frac{1}{2}i$ (b) $\frac{2}{3}i$

(c) $\frac{3}{4}i$ (d) $\frac{4}{5}i$

54. [MCQ] **[GATE-EE-2017 : 2M]**

The value of the contour integral in the complex-plane $\oint \frac{z^3-2z+3}{z-2}dz$ along the contour |z| = 3, taken counter clockwise is:

(a) $-18\pi i$ (b) 0

(c) $14\pi i$ (d) $48\pi i$

55. [MCQ] **[GATE-EE-2016 : 1M]**

The value of the integral $\oint_c \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}.dz$ over the contour |z| = 1, taken in the anti-clockwise direction, would be

(a) $\frac{24\pi i}{13}$ (b) $\frac{48\pi i}{13}$

(c) $\frac{24}{13}$ (d) $\frac{12}{13}$

56. [MCQ] **[GATE-EC-2016:2M]**

The values of the integral $\frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz$ along a closed contour c in anti-clockwise direction for

(i) The point $z_0 = 2$ inside the contour c, and

(ii) The point $z_0 = 2$ outside the contour c, respectively, are

(a) (i) 2.72, (ii) 0 (b) (i) 7.39, (ii) 0
(c) (i) 0, (ii) 2.72 (d) (i) 0, (ii) 7.39

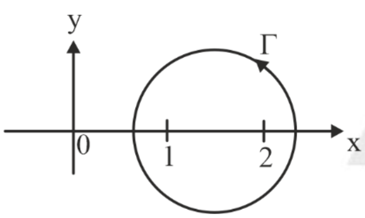
57. [NAT] [GATE-EC-2016:2M]

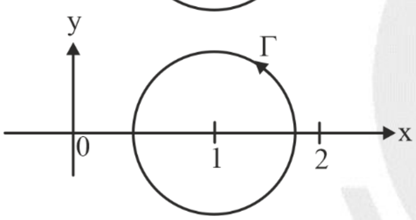
In the following integral, the contour C encloses the points $2\pi j$ and $-2\pi j$.

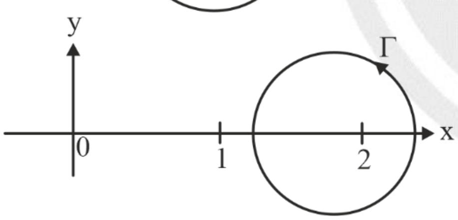
$-\frac{1}{2\pi}\oint_C \frac{\sin z}{(z-2\pi j)^3}dz$. The value of the integral is______.

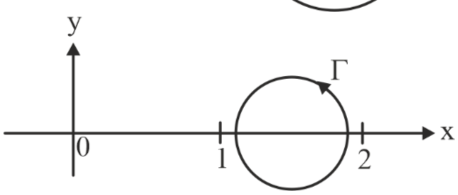
58. [MCQ] [GATE-ME-2016: 2M]

The value of $\oint_\Gamma \frac{3z-5}{(z-1)(z-2)}dz$ along a closed path Γ is equal to $(4\,\pi\,i)$, where $z = x + iy$ and $i = \sqrt{-1}$. The correct path Γ is

(a)


(b)


(c)


(d)


59. [MCQ] [GATE-ME- 2016: 2M]

The value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin x}{x^2+2x+2}dx$$

evaluated using contour integration and the residue theorem is

(a) $-\pi\sin(1)/e$ (b) $-\pi\cos(1)/e$
(c) $\sin(1)/e$ (d) $\cos(1)/e$

60. [MCQ] [GATE-EC-2015:1M]

Let $z = x + iy$ be a complex variable. Consider that contour integration performed along the unit circle in anticlockwise direction. Which one of the following statements is NOT TRUE?

(a) The residue of $\frac{z}{z^2-1}$ at $z = 1$ is $\frac{1}{2}$

(b) $\oint_c z^2 dz = 0$

(c) $\frac{1}{2\pi i}\oint_c \frac{1}{z}dz = 1$

(d) $\bar{z}$ (complex conjugate of z) is an analytical function

61. [MCQ] [GATE-EC-2015:1M]

If C is a circle of radius r with centre z_0, in the complex z-plane and if n is a non-zero integer, then $\oint_c \frac{dz}{(z-z_0)^{n+1}}$ equals

(a) $2\pi nj$ (b) 0
(c) $\frac{nj}{2\pi}$ (d) $2\pi n$

62. [NAT] [GATE-EC-2015:2M]

If C denotes the counter clockwise unit circle, the value of the contour integral $\frac{1}{2\pi j}\oint_C \mathrm{Re}\{z\}dz$ is ______.

63. [MCQ] [GATE-EC-2014:1M]

C is a closed path in the z-plane given by $|z| = 3$. The value of the integral $\oint_C \left(\frac{z^2-z+4j}{z+2j}\right)dz$

(a) $-4\pi(1 + j2)$ (b) $4\pi(3 - j2)$
(c) $-4\pi(3 + j2)$ (d) $4\pi(1 - j2)$

64. [MCQ] [GATE-EE-2014 : 2M]

Integration of the complex function $f(z) = \frac{z^2}{z^2-1}$, in the counter clockwise direction, around $|z - 1| = 1$, is

(a) $-\pi i$ (b) 0
(c) πi (d) $2\pi i$

65. [MCQ] [GATE-EE-2013 : 2M]

$\oint \frac{z^2-4}{z^2+4}dz$ evaluated anticlockwise around the circle $|z-i| = 2$, where $i = \sqrt{-1}$, is

(a) -4π (b) 0
(c) $2+\pi$ (d) $2+2i$

66. [MCQ] [GATE-EE-2012:1M]

Given $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$ If C is a counterclockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)dz$ is

(a) -2 (b) -1
(c) 1 (d) 2

67. [MCQ] [GATE-EC-2012:1M]

Given $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$. If C is counterclockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)dz$ is

(a) -2 (b) -1
(c) 1 (d) 2

68. [MCQ] [GATE-EC-2011:1M]

The value of the integral $\oint_c \frac{-3z+4}{(z^2+4z+5)}dz$ where c is the circle $|z| = 1$ is given by

(a) 0 (b) 1/10
(c) 4/5 (d) 1

69. [MCQ] [GATE-EC-2010:2M]

The residues of a complex function $X(z) = \frac{1-2z}{z(z-1)(z-2)}$ at its poles are

(a) $\frac{1}{2}, -\frac{1}{2}$ and 1 (b) $\frac{1}{2}, \frac{1}{2}$ and -1
(c) $\frac{1}{2}, 1$ and $-\frac{3}{2}$ (d) $\frac{1}{2}, -1$ and $-\frac{3}{2}$

70. [MCQ] [GATE-EC-2009:1M]

If $f(z) = c_0 + c_1 z^{-1}$, then $\oint_{\text{unit circle}} \frac{1+f(z)}{z}dz$ is given by

(a) $2\pi c_1$ (b) $2\pi(1+c_0)$
(c) $2\pi j c_1$ (d) $2\pi j(1+c_0)$

71. [MCQ] [GATE-EC-2008:2M]

The residue of the function $f(z) = \frac{1}{(z+2)^2(z-2)^2}$ at $z = 2$ is

(a) $-\frac{1}{32}$ (b) $-\frac{1}{16}$
(c) $\frac{1}{16}$ (d) $\frac{1}{32}$

72. [MCQ] [GATE-EE-2008 : 2M]

Given $X(z) = \frac{z}{(z-a)^2}$ with $|z| > a$, the residue of $X(z)\,z^{n-1}$ at $z = a$ for $n \geq 0$ will be

(a) a^{n-1}
(b) a^n
(c) n an
(d) $n\,a^{n-1}$

73. [MCQ] [GATE-ME-2008: 2M]

The integral $\oint f(z)dz$ evaluated around the unit circle on the complex plane for $f(z) = \frac{\cos z}{z}$ is

(a) $2\pi i$ (b) $4\pi i$
(c) $-2\pi i$ (d) 0

Series of Complex Functions

74. [NAT] [GATE-EC-2016:1M]

For $f(z) = \frac{\sin(z)}{z^2}$, the residue of the pole at $z = 0$ is ______.

❑❑❑

ANSWER KEY

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(10 to 10)
5.	(b)	**6.**	(a)	**7.**	(c)	**8.**	(b)
9.	(b)	**10.**	(b)	**11.**	(a)	**12.**	(a)
13.	(d)	**14.**	(d)	**15.**	(a)	**16.**	(b)
17.	(1.999 to 2.001)	**18.**	(d)	**19.**	(d)	**20.**	(1.99 to 2.01)
21.	(d)	**22.**	(d)	**23.**	(b)	**24.**	(a)
25.	(b)	**26.**	(b)	**27.**	(b)	**28.**	(a)
29.	(–1.1 to –0.9)	**30.**	(0 to 0)	**31.**	(b)	**32.**	(b)
33.	(c)	**34.**	(c)	**35.**	(c)	**36.**	(c)
37.	(d)	**38.**	(b)	**39.**	(0.50 to 0.50)	**40.**	(a)
41.	(0.2 to 0.2)	**42.**	(c)	**43.**	(c)	**44.**	(c)
45.	(b)	**46.**	(–0.0001 to 0.0001)	**47.**	(2 to 2)	**48.**	(a)
49.	(b)	**50.**	(a)	**51.**	(b)	**52.**	(d)
53.	(b)	**54.**	(c)	**55.**	(b)	**56.**	(b)
57.	(–136 to –132)	**58.**	(b)	**59.**	(a)	**60.**	(d)
61.	(b)	**62.**	(0.5 to 0.5)	**63.**	(c)	**64.**	(c)
65.	(a)	**66.**	(c)	**67.**	(c)	**68.**	(a)
69.	(c)	**70.**	(d)	**71.**	(a)	**72.**	(d)
73.	(a)	**74.**	(1 to 1)				

SOLUTIONS

1. (a)

Given $W^4 = 16\,j$

$\Rightarrow\ W = (16)^{1/4} \cdot (e^{j\pi/2})^{1/4}$

By De-movier's theorem

$$\Rightarrow W = 2\ c\ is\left(\frac{2x\pi + \frac{\pi}{2}}{4}\right) \text{ where } n = 0, 1, 2, 3$$

$$\Rightarrow W = 2\,\text{Cis}\left(\frac{\pi}{8}\right),\ 2\cdot\text{Cis}\left(\frac{5\pi}{8}\right),\ 2\cdot\text{Cis}\left(\frac{9\pi}{8}\right),$$

$$2\cdot\text{Cis}\left(\frac{13\pi}{8}\right)$$

$$\Rightarrow W = 2\cdot e^{j\pi/8},\ 2\cdot e^{\frac{j5\pi}{8}},\ 2\cdot e^{\frac{j9\pi}{8}},\ 2\cdot e^{\frac{j15\pi}{8}}$$

Out of the given options, $2\cdot e^{\frac{j2\pi}{8}}$ is not in the above values of z.

$\therefore\ 2\cdot e^{\frac{j2\pi}{8}}$ is not value of 'w'.

Scan for Video solution

2. (b)

$$(1+i)^8 = \left(\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)\right)^8$$

$$= \left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}\right)\right)^8 \quad (\because e^{i\theta} = \cos\theta + i\sin\theta)$$

$$= \left(\sqrt{2}\cdot e^{i\frac{\pi}{4}}\right)^8 = 2^4\cdot e^{i\left(\frac{8\pi}{4}\right)}$$

$= 16.\ (e^{i(2\pi)})$

$= 16\{\cos 2\pi + i.\ \sin 2\pi\}$

$= 16(1)$

$\therefore \quad (1+i)^8 = 16$

Scan for Video solution

3. (d)

$$\lim_{z\to i}\frac{z^2+1}{z^3+2z-i(z^2+2)}$$

Since it is $\frac{0}{0}$ form use L Hospital's rule and differentiate both numerator and denominator.

$$\lim_{z\to i}\frac{2z}{3z^2+2-2zi} = \frac{2i}{3i^2+2-2i^2} = \frac{2i}{-3+2+2} = 2i$$

Scan for Video solution

4. (10 to 10)

For all $z_1 \neq z_2$ it is given that

$F(z_1) = f(z_2)$

$\Rightarrow$ f(z) is a constant function

$$\Rightarrow f'(z) = 0 \Rightarrow \frac{d}{dz}\left(\frac{az+b}{cz+d}\right) = 0$$

$$\Rightarrow \frac{ad-bc}{(cz+d)^2} = 0$$

$\Rightarrow ad - bc = 0 \Rightarrow 2d - (20) = 0$

$\Rightarrow d = 10$

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5. (b)

(a) Option (a) is incorrect as g and h may individually be indifferentiable inspite of their sum f being differentiable at a point.

(b) Option (b) is correct because if both g and h are individually differentiable then their sum f is definitely differentiable.

(c) Option (c) is incorrect as continuity doesn't ensure differentiability.

(d) Option (d) is incorrect as it is not correct to say that if f is differentiable at z_0 then real and imaginary parts of f are differentiable at z_0

Scan for Video solution

6. (a)

Given

$$z_1 = 5+(5\sqrt{3})i \quad \text{and} \quad z_2 = \frac{2}{\sqrt{3}}+2i$$

$$\arg(x+iy) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1)-\arg(z_2)$$

$$= \tan^{-1}\left(\frac{5\sqrt{3}}{5}\right)-\tan^{-1}\left(\frac{2}{\left(\frac{2}{\sqrt{3}}\right)}\right)$$

$$= \tan^{-1}\sqrt{3}-\tan^{-1}\sqrt{3} = 0$$

$$\Rightarrow \quad \boxed{\arg\left(\frac{z_1}{z_2}\right) = 0}$$

Scan for Video solution

7. (c)

Given

Complex number $\frac{1+i}{1-i} = z$ (say)

Rationalizing the denominator

$$\Rightarrow \quad z = \frac{1+i}{1-i}\left(\frac{1+i}{1+i}\right) = \frac{(1+i)^2}{1-i^2} = \frac{2i}{2} = i$$

$\therefore \quad z = i = 0 + 1\ I$

$$\Rightarrow \quad \boxed{\arg(z) = \tan^{-1}(1/0) = \tan^{-1}\infty = \frac{\pi}{2}}$$

Scan for Video solution

8. (b)

Given: Integral $I = \int_5^{3i} \frac{dz}{z}$

$$\Rightarrow \quad I = \ln z\Big|_5^{3i} = \ln(3i) - \ln(5) = \ln\left(\frac{3}{5}\cdot i\right)$$

$$\Rightarrow \quad I = \ln\left(\frac{3}{5}\right) + \ln i = \ln\left(\frac{3}{5}\right) + \ln e^{i\pi/2} \quad (\text{since } e^{i\pi/2} = i)$$

$$\Rightarrow \quad I = \ln 0.6 + i\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad I = -0.5108 + i\,(1.5707)$$

$$\therefore \quad \boxed{\int_5^{3i}\frac{dz}{z} = -0.511 + i(1.571)}$$

Scan for Video solution

9. (b)

$f(z) = 1^i$

$= (e^{2\pi i})^i = e^{-2n\pi}$

Where n is any integer

Hence, f is real and non-negative.

Scan for Video solution

10. (b)

$$(-i)^{\frac{1}{2}} = \left[e^{-i\pi/2}\right]^{\frac{1}{2}}$$

$$= \left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right]^{\frac{1}{2}}$$

$$= \left[\cos\left(2n\pi - \frac{\pi}{2}\right) + i\sin\left(2n\pi - \frac{\pi}{2}\right)\right]^{\frac{1}{2}}$$

$$= \left[\cos(4n-1)\frac{\pi}{4} + i\sin(4n-1)\frac{\pi}{4}\right]$$

(By De-Moivre's Theorem)

Now put n = 0 and 1

$$\sqrt{-i} = \cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$$

Scan for Video solution

11. (a)

If $x = \sqrt{-1} = i$

$$\therefore\ x^x = (i)^i = \left(e^{\frac{i\pi}{2}}\right)^i = e^{\frac{-\pi}{2}}$$

$$\left[\therefore\ e^{\frac{i\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i\right]$$

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12. (a)

Given: $x = \sqrt{-1} = j$, $(\because e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 = j)$

$$\Rightarrow \quad x^x = j^j = \left(e^{j\pi/2}\right)^j = e^{j^2\pi/2} = e^{-\pi/2}$$

$\boxed{\text{For } x = j;\ x^x = e^{-\pi/2}}$

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13. (d)

$\Rightarrow \quad x^3 + x^2 + x + 1 = 0$

$\Rightarrow \quad x^2(x + 1) + (x + 1) = 0$

$\Rightarrow \quad (x + 1)(x^2 + 1) = 0$

$\Rightarrow \quad (x + 1)(x + j)(x - j) = 0$

$\Rightarrow \quad x = -1, +j, -j$

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14. (d)

We know, $z = x + iy$ is in I[st] quadrant & within unit circle.

$\therefore\ 0 < x < 1,\ 0 < y < 1\ \&\ 0 < \sqrt{x^2 + y^2} < 1$

Now $y = \frac{1}{z} = \frac{1}{x + iy}$

$= \frac{1}{x+iy} \times \frac{x-iy}{x-iy} = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$

Since real part is positive and imaginary part is negative

$\therefore\ y = \frac{1}{z}$ lies in IV[th] quadrant.

$$\left|\frac{1}{z}\right| = \left|\frac{1}{x+iy}\right| = \frac{1}{\sqrt{x^2+y^2}} > 1 \quad \left(\because\ 0 < \sqrt{x^2+y^2} < 1\right)$$

$\therefore\ y = \frac{1}{z}$ lies outside the unit circle.

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15. (a)

Given:

$(i + 1).(2 - 5i) = 2 - 5i + i(2 - 5i) = 2 - 5i + 2i + 5$

$= 7 - 3i$

$\therefore\ \boxed{(1+i)\ (2-5i) = 7 - 3i}$

Scan for Video solution

16. (b)

Given

complex number is $\frac{3+4i}{1-2i} = z$ (say)

$$\Rightarrow \quad |z| = \left|\frac{3+4i}{1-2i}\right| = \frac{|3+4i|}{|1-2i|} \qquad \left\{\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right\}$$

$$\Rightarrow \quad |z| = \frac{\sqrt{3^2 + (4)^2}}{\sqrt{1^2 + (-2)^2}} = \frac{\sqrt{25}}{\sqrt{5}} = \sqrt{5}$$

$$\Rightarrow \quad \boxed{\left|\frac{3+4i}{1-2i}\right| = \sqrt{5}}$$

Scan for Video solution

17. (1.999 to 2.001)

Given

$f(z) = e^{-kx} (\cos 2y - i \sin 2y)$

$= e^{-kx}.\cos 2y + i(-e^{-kx}.\sin 2y)$

For $f(z) = u(x, y) + i.v(x, y)$ to be analytic,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\Rightarrow \quad \frac{\partial}{\partial x}\left(e^{-kx}.\cos 2y\right) = \frac{\partial}{\partial y}\left(-e^{-kx}.\sin 2y\right)$$

$\Rightarrow \quad \cos 2y.(-k.e^{-kx}) = -e^{-kx}.2\cos 2y$

$\Rightarrow \quad k = 2$

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18. (d)

$ax^3 + bx^2 + cx + d = 0$

Let the roots be p, q and r

We know, sum of the roots $(p + q + r) = -b/a$

Product of the roots $(pqr) = -d/a$

$p(z) = z^3 + (1 + j)z^2 + (2 + j)z + 3$

$\therefore$ Sum of roots $= -(1 + j)$

Product of roots $= -3$

3 roots

1 Real root

2 complex roots (conjugate pairs)

$P(\overline{z}) = (\overline{z})^3 + (1+j)(\overline{z})^2 + (z+j)\overline{z} + 3$

$\therefore$ $p(z) \neq p(\overline{z})$. So only option – d is correct.

Scan for Video solution

19. (d)

Given

(i) $f(z) = z^2 \Rightarrow f'(z) = 2z \quad \exists \ \forall z \in C$

(ii) $f(z) = e^z \Rightarrow f'(z) = e^z \quad \exists \ \forall z \in C$

(iii) $f(z) = \sin z \Rightarrow f'(z) = \cos z \quad \exists \ \forall z \in C$

(iv) $f(z) = \log_e z \Rightarrow f'(z) = \frac{1}{z} \quad \nexists$ when $z = 0$

$\therefore$ $f(z) = \log_e z$ is not analytic $z = 0$

Scan for Video solution

20. (1.99 to 2.01)

Given

$f(z) = (x^2 - y^2) + i\,\psi(x, y)$

$= u(x, y) + i.\ \psi(x, y)$

$\Rightarrow \frac{\partial u}{\partial x} = 2x;$ and $\frac{\partial u}{\partial y} = -2y$

When $f(z) = u + iv$ is analytic,

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} + i\left(\frac{-\partial u}{\partial y}\right)$$

$\Rightarrow \quad = 2x - i(-2y) = 2(x + iy) = 2z$

Integrating both sides

$\Rightarrow \int f'(z)\cdot dz = \int 2z \cdot dz$

$\Rightarrow$ $f(z) = z^2 + c$

$\Rightarrow$ $f(z) = (x + iy)^2 + c = (x^2 - y^2) + i(2xy) + c$

By taking $c = 0$

$\psi(x, y) = 2xy$

At $z = 1 + i$; i.e., $x = 1$ & $y = 1$

$\psi(1, 1) = 2(1)(1) = 2$

$\therefore$ $\boxed{\psi(1, 1) = 2}$

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21. (d)

Given $f(z) = x^3 - 3xy^2 + i\,v(x, y)$ is analytic.

$\Rightarrow f'(z) = 3x^2 - 3y^2 + i\frac{\partial v}{\partial x}$

For analytic function $f(z) = u + iv$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \ \& \ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$\Rightarrow f'(z) = 3x^2 - 3y^2 + i\left(\frac{-\partial u}{\partial y}\right)$

$\Rightarrow$ $f'(z) = 3x^2 - 3y^2 - i(-6xy)$

Apply Milne- Thomson method

Replace ‘x’ with ‘z’ and 'y' with ‘0’.

$\Rightarrow \int f'(z)\,dz = \int 3z^2\,dz$

$\Rightarrow$ $f(z) = z^3 + c$

$\Rightarrow$ $f(z) = (x + iy)^3 + c$

$= x^3 - 3xy^2 + i(3x^2y - y^3) + c$

$\therefore$ $v(x, y) = 3x^2y - y^3 +$ constant

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22. (d)

$f(z) = \cos z \Rightarrow f'(z) = -\sin z$; $-\sin z$ exists for all ‘z’

$f(z) = \ln(z) \Rightarrow f'(z) = \frac{1}{z}$; $f'(z)$ doesn’t exist at $z = 0$

$f(z) = \frac{1}{1-z} \Rightarrow f'(z) = \frac{1}{(1-z)^2}$; $f'(z)$ doesn’t exist at $z = 1$

$f(z) = e^{1/z} \Rightarrow f'(z) = \frac{-e^{1/z}}{z^2}$; $f'(z)$ doesn’t exist at $z = 0$

$\therefore$ **f(z) = cos z** is the function that is analytic over the complete complex plane.

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23. (b)

For options a, c and d the singular points are lying within the given region $|z| < 1$

$\Rightarrow |z| \le 1$

$\Rightarrow |x + iy| \le 1$

$\Rightarrow x^2 + y^2 \le 1$

$|z| \le 1$ represents a Circle with unit radius

Option a) $z = 0$

Option b) $z = -2$

Option c) $z = 0.5$

Option d) $z = -0.5j$

Only option b) will be analytic because it's singular point $z = -2$ is not lying in the region $|z| \le 1$.

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24. (a)

If f(z) = u + iv is analytic function, then u and 'v' are Harmonic conjugate pair

Given $u = 2x^2 - 2y^2 + 4xy$

$\Rightarrow \quad f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

$\Rightarrow \quad f'(z) = 4x + 4y - i(-4y + 4x)$

Applying Milne Thomson method, Replace 'x' with 'z' & 'y' with '0'.

$\Rightarrow \quad \int f'(z)dz = \int (4z - i4z)dz$

$\Rightarrow \quad f(z) = (1 - i)2z^2 + c$

$= (2 - 2i)(x^2 - y^2 + i2xy) + c$

$= 2(x^2 - y^2) + 4xy + i(4xy - 2x^2 + 2y^2) + c$

$\Rightarrow \quad v(x, y) = 4xy - 2x^2 + 2y^2 + c$

Scan for Video solution

25. (b)

Given

$f(z) = u(x, y) + iv(x, y)$

For f(z) to be analytic function, u (x, y) and v (x, y) should satisfy Cauchy-Riemann equations.

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}}$$

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26. (b)

Given,

$F(z) = iz + k \cdot \text{Re}(z) + i\,\text{Im}(z)$

$= i(x + iy) + kx + iy$

$= (kx - y) + i(x + y)$

For satisfying Cauchy-Riemann equations

$\frac{\partial}{\partial x}(kx - y) = \frac{\partial}{\partial y}(x + y)$

$\Rightarrow \quad k = 1$

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27. (b)

Given,

$f(z) = (x^2 + ay^2) + ibxy$ is analytic function.

$\therefore \quad \frac{\partial}{\partial x}(x^2 + ay^2) = \frac{\partial}{\partial y}(bxy)$

$\Rightarrow \quad 2x = bx$

$\Rightarrow \quad \boxed{b = 2}$

and $\quad \frac{\partial}{\partial y}(x^2 + ay^2) = -\frac{\partial}{\partial x}(bxy)$

$\Rightarrow \quad 2ay = -by$

$\Rightarrow \quad a = \frac{-b}{2} = -1$

$\Rightarrow \quad \boxed{a = -1}$

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28. (a)

Given

f(z) = u (x, y) + i. v (x, y) and u (x, y) = 2xy

$\Rightarrow \quad f'(z) = \frac{\partial u}{\partial x} + i \cdot \frac{\partial v}{\partial x}$

When f(z) is analytic,

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$\Rightarrow \quad f'(z) = \frac{\partial u}{\partial x} - i \cdot \frac{\partial u}{\partial y}$

$\Rightarrow \quad f'(z) = 2y - i(2x)$

Applying Milne-Theorem method, Replace 'x' with z and 'y' with '0'.

$\Rightarrow \quad f'(z) = -2iz$

$\Rightarrow \quad \int f'(z)dz = -2i\int z \cdot dz$

$\Rightarrow \quad f(z) = -iz^2 + ic$

$= -i(x + iy)^2 + ic$

$= i(y^2 - x^2) + 2xy + ic$

$= 2xy + i(y^2 - x^2 + \text{constant})$

$\therefore \quad v(x, y) = y^2 - x^2 + \text{constant}$

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29. (–1.1 to –0.9)

Given

f(z) = u (x, y) + i v (x, y)

and u (x, y) = 2kxy and v (x, y) = $x^2 - y^2$

If f(z) is analytic, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$\Rightarrow \quad \frac{\partial}{\partial x}(2kxy) = \frac{\partial}{\partial y}(x^2 - y^2)$

$\Rightarrow \quad 2ky = -2y \Rightarrow \boxed{k = -1}$

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30. (0 to 0)

Given $f(z) = 2z^3 + b|z|^3$

$\Rightarrow f(z) = 2(x + iy)^3 + b(x^2 + y^2)^{3/2}$

$= 2\{x^3 - iy^2 + i3x^2y - 3xy^2\} + b(x^2 + y^2)^{3/2}$

$= 2x^3 - 6xy^2 + + b(x^2 + y^2)^{3/2} + i(6x^2y - 2y^3)$

$\therefore$ For a f(z) = u(x, y) + iv(x, y) to be analytic

$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$

$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left\{2x^3 - 6xy^2 + b\left(x^2 + y^2\right)^{3/2}\right\}$

$\Rightarrow \frac{\partial v}{\partial x} = \frac{\partial}{\partial y}\left\{6xy^2 - 2y^3\right\}$

$\Rightarrow 6x^2 - 6y^2 + \frac{3b}{2}\cdot(2x)\left(x^2 + y^2\right)^{1/2} = 6x^2 - 6y^2$

$\Rightarrow 3bx\left(x^2 + y^2\right)^{1/2} = 0$

$\Rightarrow b = 0$

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31. (b)

f(z) = z + z*

z = x + iy z* = x – iy

$\therefore$ f(z) = z + z* = x + iy + x – iy

f(z) = 2x

Since f(x) is polynomial and hence continuous.

f(z) = u + iv = 2x

$\therefore$ u = 2x and v = 0

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Since f(z) does not satisfy C-R equations therefore f(z) is not analytic.

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32. (b)

Given f(z) = u(x, y) + iv(x, y) is analytic

u(x, y) $\rightarrow$ Real part = $e^{-y}\cos x$

since f(z) is analytic,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\Rightarrow\ e^{-y}(-\sin x) = \frac{\partial v}{\partial y}$$

$$\Rightarrow\ \int \partial v = \int -e^{-y}\cdot \sin x \cdot \partial y$$

$$\Rightarrow\ V = -\sin x \cdot \frac{e^{-y}}{-1} + \phi(x)$$

$$\Rightarrow\ v(x, y) = e^{-y}\cdot \sin x + \phi(x)$$

Where $\phi(x)$ is arbitrary function

For $\phi(x) = 0$

$$\boxed{V(x, y) = e^{-y}\cdot \sin x}$$

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33. (c)

f (z) = zz*

$= |z|^2$ $\quad (\because zz^* = |z|^2)$

$= 1^2$ $\quad (\because |z| = 1)$

f (z) = 1 + 0i = (1, 0)

$\therefore$ f (z) maps S to the point (1, 0) in the complex plane.

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34. (c)

Given

f(z) = u + i v is analytic and u = 2x y

$$\Rightarrow\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

We have f(z) = u + i v

$$\Rightarrow\ f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$\Rightarrow\ = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$\Rightarrow\ f'(z) = 2y - i(2x)$$

Applying Milne Thomson method replace 'x' with z and 'y' with '0'

$$\Rightarrow\ f'(z) = 2(0) - i(2z) = -i2z$$

$$\Rightarrow\ \int f'(z)\,dz = -i(2)\cdot \int z\,dz$$

$$\Rightarrow\ f(z) = -iz^2 + c = -i(x + iy)^2 + c$$

$$= 2xy - i(x^2 - y^2) + ic$$

$$\therefore\ \boxed{v(x, y) = -x^2 + y^2 + c}$$

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35. (c)

Given

f(z) = u + i v is analytic, and u = $x^2 - y^2$

$$\Rightarrow\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \Rightarrow\ 2x = \frac{\partial v}{\partial y}$$

$$\Rightarrow\ \int \partial v = \int 2x\,\partial y \qquad \Rightarrow\ v = 2xy + h(x)$$

Where h(x) is arbitrary function

From options, h(x) = constant = c

$$\therefore\ \boxed{v(x, y) = 2xy + c}$$

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36. (c)

Given:

f(z) = u + iv is analytic function; u (x, y) = xy

$$\Rightarrow\ f'(z) = \frac{\partial u}{\partial x} + i\left(\frac{\partial v}{\partial x}\right)$$

$\because$ f(z) is analytic, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$$\Rightarrow\ f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$\Rightarrow\ f'(z) = \frac{\partial}{\partial x}(xy) - i\frac{\partial}{\partial y}(xy) = y - ix$$

Applying Milne-Thomson method (Replacing 'x' with z & 'y' with 0)

$\Rightarrow \quad f'(z) = 0 - iz \qquad \Rightarrow \quad f'(z) = -iz$

$\Rightarrow \quad f'(z)\cdot dz = -i\,z\,dz$

On integrating both sides

$$f'(z) = \frac{-i}{2}z^2 + c = \frac{-i}{2}\left\{(x+iy)^2\right\} + C$$

$$f'(z) = \frac{-i}{2}\left\{x^2 - y^2 + 2i\,xy\right\} + C$$

$$= xy + i\left(\frac{y^2 - x^2}{2} + C\right)$$

$$\boxed{\therefore\ u = xy,\ \text{and}\ v = \frac{y^2 - x^2}{2} + k}$$

where k = C= constant

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37. (d)

Given: sin z = 10

$\Rightarrow \quad \dfrac{e^{iz} - e^{-iz}}{2i} = 10 \qquad \Rightarrow$

$e^{iz} - e^{-iz} - 20i = 0$

$\Rightarrow \quad \left(e^{iz}\right)^2 - 20i\left(e^{iz}\right) - 1 = 0$

(Multiplied by e^{iz})

$$\therefore e^{iz} = \frac{20i \pm \sqrt{(-20i)^2 - 4(1)(-1)}}{2} = \frac{20i \pm \sqrt{-400+4}}{2}$$

$\Rightarrow \quad e^{iz} = 10i \pm i3\sqrt{11} = i(10 \pm 3\sqrt{11})$

Since e^{iz} is periodic function, for infinitely many values of 'z' that satisfy given equation.

$\therefore$ Number of complex solutions is infinite.

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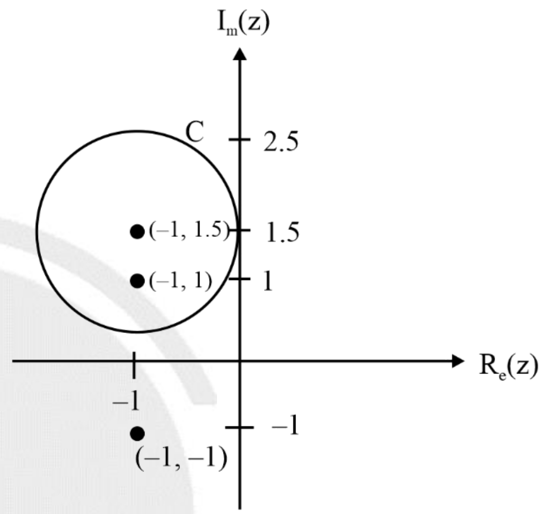
38. (b)

Given integral is (say) I = $\oint\left(\dfrac{z+2}{z^2+2z+2}\right)dz$

C: $\left|z + 1 - \dfrac{3}{2}j\right| = 1$

$\Rightarrow \left|z - \left(-1 + \dfrac{3}{2}j\right)\right| = 1$



$\therefore$ C is a circle with center at $\left(-1, \dfrac{3}{2}\right)$ and radius 1.

Singular points of given integral

Are at $z^2 + 2z + 2 = 0$

$\Rightarrow (z+1)^2 + 1 = 0$

$\Rightarrow z = -1 \pm j$

i.e. at z = –1 + j and z = –1 – j

out of the two singular points, only z = –1 + j lies inside 'c'

$\therefore$ By Cauchy's Residue theorem

$$\oint\left(\frac{z+2}{z^2+2z+2}\right)dz =$$

$$2\pi j.\text{Res}\left(\left.\frac{(z+2)}{(z-(-1+j))(z-(-1-j))}\right|_{z=-1+j}\right)$$

$$= 2\pi j\left\{\frac{-1+j+2}{(-1+j)-(-1-j)}\right\}$$

$$= 2\pi j\left\{\frac{j+1}{2j}\right\} = \pi(1+j)$$

$$\therefore \quad \oint\left(\frac{z+2}{z^2+2z+2}\right)dz = \pi(1+j)$$

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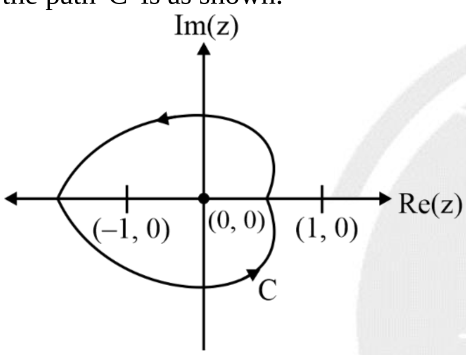
39. (0.50 to 0.50)

Given, integral is

$$\oint_c \frac{2^z}{z^2-1}dz = -i\pi A$$

and the path 'C' is as shown.



Singularities of the given integral are at

$$z^2 - 1 = 0 \Rightarrow z = 1, -1$$

In the given curve, only z = –1 lies with in 'C'.

$$\oint_c \frac{2^z}{(z-1)(z+1)}dz = 2\pi i\left\{\text{Res}\left(\frac{2^z}{(z-1)(z+1)}\right)\Bigg|_{z=-1}\right\}$$

$$= 2\pi i\left\{\frac{2^{-1}}{-2}\right\} = -\frac{\pi i}{2}$$

$$\therefore \quad A = \frac{1}{2} = 0.50$$

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40. (a)

Given: $I = \oint_C\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)dz$

where 'c' is enclosing the pole z = i

From Cauchy's residue theorem

$$I = 2\pi i \times \text{Res}\left(\frac{6z}{2z^4-3z^3+7z^2-3z+5}\right)_{z=i}$$

$$I = \left\{\lim_{z\to i}\frac{6z}{2z^4-3z^3+7z^2-3z+5}(z-i)\right\}$$

$$= 2\pi i \times\left(\frac{0}{2+3i-7-3i+5}\right) = 2\pi i\times\left(\frac{0}{0}\right)$$

Apply L-Hospital rule for the limit

$$\Rightarrow \quad I = 2\pi i\left\{\lim_{z\to i}\frac{12z-6i}{8z^3-9z^2+14z-3}\right\}$$

$$= 2\pi i\left\{\frac{6i}{-8i+9+14i-3}\right\} = 2\pi i\left\{\frac{6i}{6(1+i)}\right\}$$

$$\therefore \quad I = \frac{-2\pi}{(1+i)}\times\frac{(1-i)}{(1-i)} = \frac{-2\pi}{2}(1-i)$$

$$\Rightarrow \quad I = -\pi(1-i) = \pi(-1+i)$$

$$\boxed{\therefore \oint_c \frac{6z}{2z^4-3z^3+7z^2-3z+5}dz = \pi(-1+i)}$$

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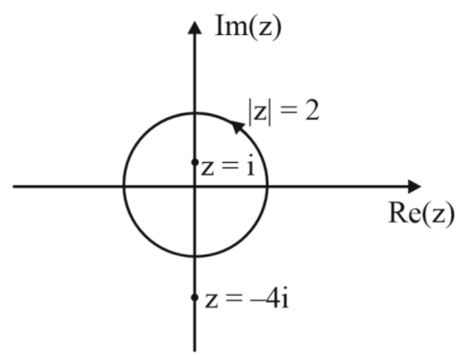
41. (0.2 to 0.2)

Given

Integral is $I = \frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz$

Poles are z = i, z = –4i

The pole which is inside 'C' is only z = i



$$I = \frac{1}{2\pi}\times 2\pi i\left\{\text{Res}\frac{1}{(z-i)(z+4i)}\Bigg|_{z=i}\right\}$$

$$= i\left\{\lim_{z\to i}\frac{1}{(z-i)(z+4i)}\times(z-i)\right\} = i\left(\frac{1}{5i}\right) = \frac{1}{5} = 0.2$$

$$\boxed{\therefore \frac{1}{2\pi}\int_C \frac{1}{(z-i)(z+4i)}dz = 0.2}$$

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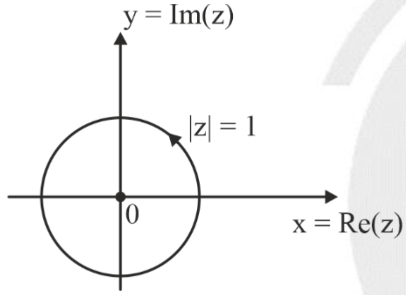
42. (c)

Given

$$\oint_C \frac{\cosh 3z}{2z}dz = I \text{ (say)}$$

$z = 0$ is a singular point of $\frac{\cosh 3z}{2z}$ within 'C'.



From Cauchy's residue theorem,

$$\oint_C \frac{\cosh 3z}{2z}dz = 2\pi i\{\text{Res } f(z)|_{z=0}\}$$

$$= 2\pi i\left\{\lim_{z\to 0}\frac{\cosh 3z}{2z}\times z\right\} = 2\pi i\left\{\left.\frac{e^{3z}+e^{-3z}}{2\times 2}\right|_{z=0}\right\}$$

$$= 2\pi i\left(\frac{1}{2}\right) = \pi i$$

$$\left\{\because \cosh z = \left(\frac{e^z+e^{-z}}{2}\right)\right\}$$

$$\therefore \boxed{\oint_C \frac{\cosh 3z}{2z}dz = \pi i}$$

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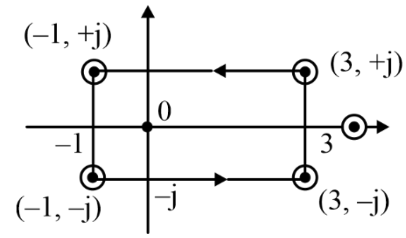
43. (c)

$$I = \int_c \frac{dz}{z^2(z-4)}$$



Only pole at z = 0 is there as it lies within the contour C.

Pole of order 2 at z = 0 exists.

$$\underset{z\to a}{Res}\left[f(a)\right] = \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{dz^{n-1}}f(z)\times(z-a)^n\right]$$

$$= \frac{1}{(2-1)!}\cdot\left[\frac{d}{dz}\cdot\frac{1}{z^2(z-4)}\cdot z^2\right]_{z=0} = -\left.\frac{1}{(z-4)^2}\right|_{z=0}$$

$$\text{Res}[f(0)] = -\frac{1}{16}$$

By Cauchy Residue Theorem

$$\int f(z)dz = 2\pi j\,\text{Res}[f(a)]$$

$$\int_c \frac{dz}{z^2(z-4)} = 2\pi j\left(-\frac{1}{16}\right) = \frac{-\pi j}{8}$$

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44. (c)

Using Cauchy's Integral theorem,

$$\int_c \frac{f(z)}{z-a}dz = 2\pi i(\text{Res}_{z=a})$$

$$I = \int_c \frac{z^2+1}{z^2-2z}dz = \int_c \frac{z^2+1}{z(z-2)}dz = \int_c \frac{\left(\frac{z^2+1}{z-2}\right)}{z}dz = \int_c \frac{f(z)}{z}dz$$

Poles are at z = 0 and z = 2 but only z = 0 is lying inside the unit circle centered at origin.

$$\text{Residue }(z=0) = \lim_{z\to 0} f(z) = \frac{z^2+1}{z-2} = -\frac{1}{2}$$

$$\therefore \int_c \frac{z^2+1}{z^2-2z} = 2\pi i(\text{Res}_{z=0}) = 2\pi i\left(-\frac{1}{2}\right) = -\pi i$$

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45. **(b)**

Using Cauchy's Integral formula:-

$$f(a) = \frac{1}{2\pi j}\oint \frac{f(z)}{z-a}$$

(if f(z) is analytic within and on the closed curve C)

$f(z) = z^3 + z^2 + 8$

$f(-2) = (-2)^3 + (-2)^2 + 8$

$f(-2) = -8 + 4 + 8$

$\therefore \quad f(-2) = 4$

$$I = \oint \frac{z^3+z^2+8}{z+2}dz \qquad \therefore a = -2$$

$= 2\pi j\ f(a)$

$= 2\pi j\ f(-2)$

$= 2\pi j\ (-4)$

$= -8\pi j$

$$\therefore \oint_{|z|=5} \frac{z^3+z^2+8}{z+2}dz = -8\pi j$$

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46. **(–0.0001 to 0.0001)**

Given: Integral $\frac{1}{2\pi j}\oint_c \left(z+\frac{1}{z}\right)^2 dz$ = I (say), c: |z| = 1

$$\Rightarrow \quad I = \frac{1}{2\pi j}\oint_c \left(z+\frac{1}{z}\right)^2 dz = \frac{1}{2\pi j}\oint_c \left(z^2+2+\frac{1}{z^2}\right)dz$$

$$= \frac{1}{2\pi j}\oint_c \left(z^2+2\right)dz + \frac{1}{2\pi j}\oint_c \left(\frac{1}{z^2}\right)dz$$

Since $z^2 + 2$ is analytic over complex plane,

$$\oint_c \left(z^2+2\right)dz = 0$$

$$\therefore \quad I = \frac{1}{2\pi j}\oint_c \left(\frac{1}{z^2}\right)dz$$

z = 0 is a pole of order 2 and z = 0 lies in |z| = 1

$$\therefore \ I = \frac{1}{2\pi j}\oint_c \frac{1}{z^2}dz = = \frac{1}{2\pi j}\times 2\pi j \times \left\{ \text{Res}\left(\frac{1}{z^2}\right)\Bigg|_{z=0}\right\}$$

$$= \lim_{z\to 0}\frac{d}{dz}\left(z^2\cdot\frac{1}{z^2}\right) = 0$$

$$\therefore \ \boxed{I = \frac{1}{2\pi j}\oint_c \left(z+\frac{1}{z^2}\right)dz = 0}$$

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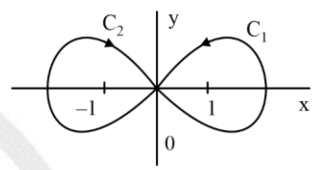
47. **(2 to 2)**



Given:

Integal $I = \frac{1}{\pi j}\oint_C \frac{dz}{z^2-1}$

$$\Rightarrow \quad I = \frac{1}{\pi j}\left\{\oint_{C_1}\frac{dz}{z^2-1} - \oint_{C_2}\frac{dz}{z^2-1}\right\}$$

C_2 is negative since it is in clockwise direction.

$$\Rightarrow \quad \frac{1}{\pi j}\left\{\text{Res}\frac{1}{z^2-1}\Bigg|_{z=1}\times 2\pi j - \text{Res}\frac{1}{z^2-1}\Bigg|_{z=-1}\times 2\pi j\right\}$$

$$\Rightarrow \quad \left\{\lim_{z\to 1}(z-1)\frac{1}{z^2-1}\times 2 - \lim_{z\to -1}(z+1)\frac{1}{(z^2-1)}\times 2\right\}$$

$$\left\{\lim_{z\to 1}(z-1)\frac{1}{(z+1)(z-1)}.2 - \lim_{z\to -1}(z+1)\frac{1}{(z+1)(z-1)}.2\right\}$$

$\Rightarrow \quad = 1 - (-1) = 2$

$$\boxed{\therefore \frac{1}{\pi j}\oint_C \frac{dz}{z^2-1} = 2}$$

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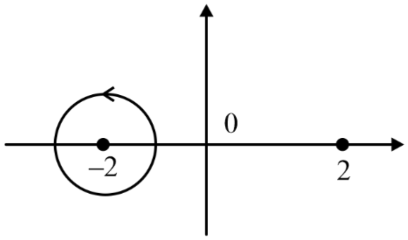
48. (a)

Equation of circle C: $|z + 2| = 1$



$$I = \oint_c \frac{z+1}{z^2-4}dz$$

Only pole $z = -2$ is lying withing the contour therefore

$$\text{Res } (z = -2) = \lim_{z\to -2}[f(z)(z+2)] = \lim_{z\to -2}\frac{z+1}{z-2} = \frac{1}{4}$$

$\because$ By Cauchy's Residue theorem,

$$\oint f(z)dz = 2\pi i[\text{Res} f(a)]$$

$$I = \oint \frac{z+1}{z^2+4}dz = 2\pi i\left(\frac{1}{4}\right) = \frac{\pi i}{2}$$

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49. (b)

$$f(z) = \frac{z^2}{(z^2-3z+2)^2}$$

$$= \frac{z^2}{(z-1)^2(z-2)^2}$$

$\because$ At $z = 1$ and $z = 2$ poles of order 2 exists and they are lying within the contour $|z| = 4$

$$\underset{z=a}{\text{Res}}[f(a)] = \frac{1}{(n-1)!}\lim_{z\to a}\left[\lim \frac{d^{n-1}}{dz^{n-1}}f(z)(z-a)^n\right]$$

$$\therefore \underset{z=1}{\text{Res}}[f(1)] = \frac{1}{(2-1)!}\lim_{z\to 1}\left[\frac{d}{dz}\frac{z^2}{(z-2)^2}\right]$$

$$= \frac{(z-2)^2 \cdot 2z - z^2.2(z-2)}{(z-2)^4} = 2 + 2 = 4$$

$$\therefore \underset{z=2}{\text{Res}}[f(2)] = \frac{1}{(2-1)!}\lim_{z\to 2}\left[\frac{d}{dz}\cdot\frac{z^2}{(z-1)^2}\right]$$

$$= \frac{(z-1)^2 2z - z^2 2(z-1)}{(z-1)^4} = 4 - 8 = -4$$

By Cauchy Residue Theorem,

$$\oint f(z)dz = 2\pi i[\text{Res} f(a_1) + \text{Res} f(a_2)]$$

$$I = \oint \frac{z^2}{(z^2-3z+2)}dz = 2\pi i[\text{Res} f(1) + \text{Res} f(z)]$$

$$= 2\pi i[4 - 4] = 0$$

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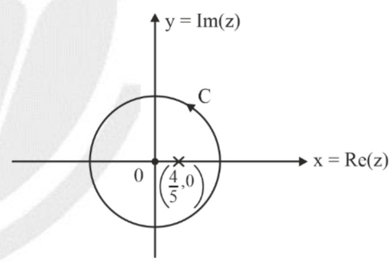
50. (a)

Given,

$\oint_c \frac{1}{5z-4}dz = A\pi i$; 'c' is unit circle centered at origin.

For $f(z) = \frac{1}{\left(z-\frac{4}{5}\right)}$, $z = \frac{4}{5}$ is a singular point and it lies within 'C'



$\therefore$ From Cauchy's Residue theorem,

$$\frac{1}{5}\cdot\oint_c \frac{dz}{\left(z-\frac{4}{5}\right)} = \frac{1}{5}\times 2\pi i\left(\text{Res } f(z)\Big|_{z=\frac{4}{5}}\right)$$

$$= \frac{1}{5}\times 2\pi i \times (1) = \frac{2}{5}\pi i$$

$$\Rightarrow \boxed{A = \frac{2}{5}}$$

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51. (b)

Given, $f(z) = \dfrac{1}{(z-4)(z+1)^3}$

$\Rightarrow$ z = 4 is a simple pole.
z = –1 is a pole of order 3.

(i) Residue at z = 4

$$\text{Res } f(z)\Big|_{z=4} = (z-4)\cdot f(z)\Big|_{z=4} = \frac{1}{(z+1)^3}\Bigg|_{z=4}$$

$$= \frac{1}{5^3} = \frac{1}{125}$$

(ii) Residue at z = –1

$$\text{Res } f(z)\Big|_{z=-1} = \frac{1}{(3-1)!}$$

$$\lim_{z\to -1}\frac{d^2}{dz^2}\left\{(z+1)^3\cdot\frac{1}{(z-4)(z+1)^3}\right\}$$

$$= \frac{1}{2!}\cdot\lim_{z\to -1}\frac{2}{(z-4)^3} = \frac{1}{2}\times\left(\frac{2}{-125}\right) = \frac{-1}{125}$$

$\therefore$ Residues of f(z) are $\dfrac{1}{125}$ and $\dfrac{-1}{125}$

Scan for Video solution

52. (d)

Given:

$$\oint_C \frac{z^2-1}{z^2+1}e^z dz = I \ ; \quad C: |z| = 3$$

Singular points of given function are

At $z^2 + 1 = 0 \Rightarrow z^2 = -1 \Rightarrow z = \pm i$

Both the singular points z = i; z = –i lies in 'C'

$$\oint_C \frac{z^2-1}{z^2+1}e^z dz = 2\pi i\left\{\text{res } f(z)\Big|_{z=i} + \text{res } f(z)\Big|_{z=-i}\right\}$$

$$I = 2\pi i\left\{\lim_{z\to i}\frac{z^2-1}{(z+i)}e^z + \lim_{z\to -i}\frac{z^2-1}{z-i}e^z\right\}$$

$$= 2\pi i\left\{\frac{-1-1}{2i}e^i + \frac{-1-1}{-2i}e^{-i}\right\} = \pi\{-2e^i + 2e^{-i}\}$$

$$= -2\pi\{e^i - e^{-i}\} = -2\pi\{2i\sin 1\} = -4\pi i\sin 1$$

$\therefore$ $I = \mathbf{-4\pi i \sin 1}$

Scan for Video solution

53. (b)

The line integral is being evaluated while moving from point (0, 0) to (1, i) therefore the equation of straight line will be y = x and z = x + iy.

$\therefore$ dy = dx $\therefore$ dz = dx + idy

$$I = \int_c (x^2 + iy^2)dz$$

$$= \int_{x=0}^{x=1}(x^2 + ix^2)(dx + idx)$$

$$= \int_0^1 x^2 dx(1+i)^2 = \int_0^1 x^2(1-1+2i)dx$$

$$= 2i\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}i$$

Scan for Video solution

54. (c)

$f(z) = \dfrac{z^3-2z+3}{z-2}$ is having pole at z = 2 which is lying within the contour |z| = 3, therefore we have to find residue at z = 2.

$$\text{Res}(z=2) = \lim_{z\to 2} f(z)(z-2) = \lim_{z\to 2}\frac{z^3-2z+3}{z-2}(z-2)$$

$= 2^3 - 2(2) + 3 = 7$

So, by Cauchy's Reisdue Theorem,

$$\oint f(z)dz = 2\pi i(\text{Res } f(a))$$

$= 2\pi i[\text{Res } f(2)] = 2\pi i\ (7) = 14\pi i$

Scan for Video solution

55. **(b)**

$$I = \oint_c \frac{2z+5}{\left(z-\frac{1}{2}\right)(z^2-4z+5)}dz$$

Poles of the given function are $z = \frac{1}{2}$, $z = 2 + i$, $z = 2 - i$.

Only pole $z = \frac{1}{2}$ is lying within the contour $|z| = 1$

$\therefore$ By Cauchy's Residue theorem,

$$\oint_c \frac{f(z)}{z-a}\cdot dz = 2\pi i.[\text{Res } f(a)]$$

$$= 2\pi i\left[\frac{2\times\frac{1}{2}+5}{\left(\frac{1}{2}\right)^2-4\left(\frac{1}{2}\right)+5}\right] \qquad \left\{\because a = \frac{1}{2}\right\}$$

$$= 2\pi i\left(\frac{24}{13}\right) = \frac{48}{13}\pi i$$

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56. **(b)**

Given integral is

$$I = \frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz$$

(i) When z = 2 is inside the contour

$$I = \frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz$$

$$= I = \frac{1}{2\pi j}\times 2\pi j\times \text{Res}\left(\frac{e^z}{z-2}\right)_{z=2}$$

$= e^2 = 7.389$

(ii) When z = 2 is outside the contour

Then $I = \frac{1}{2\pi j}\oint_C \frac{e^z}{z-2}dz = 0$ (From Cauchy's integral theorem).

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57. **(–136 to –132)**

Given that contour 'C' encloses $2\pi j$ and $-2\pi j$

Let $I = \frac{-1}{2\pi}\cdot\oint_C \frac{\sin z}{(z-2\pi j)^3}dz$

For the given integrand, $z = 2\pi j$ is a pole of order '3'

$$\therefore \quad \oint_C \frac{\sin z\, dz}{(z-2\pi j)^3} = 2\pi j\cdot \text{Res}\left\{\left.\frac{\sin z}{(z-2\pi j)^3}\right|_{z=2\pi j}\right\}$$

$$= 2\pi j\left\{\frac{1}{(3-1)!}\lim_{z\to 2\pi j}\frac{d^2}{dz^2}(\sin z)\right\}$$

$$= \frac{2\pi j}{2!}\cdot -\sin(2\pi j)$$

$$= -\pi j(\sin 2\pi j) = -\pi j\left(\frac{e^{j(2\pi j)}-e^{-j(2\pi j)}}{2j}\right)$$

$$= \frac{-\pi}{2}(e^{-2\pi}-e^{2\pi})$$

$$\therefore \quad I = \frac{-1}{2\pi}\cdot\oint_C \frac{\sin z}{(z-2\pi j)^3}dz = \frac{-1}{2\pi}\cdot\left(-\frac{\pi}{2}(e^{-2\pi}-e^{2\pi})\right)$$

$$= \frac{+e^{-2\pi}-e^{2\pi}}{4} = -133.87$$

$$\therefore \quad \frac{-1}{2\pi}\cdot\oint_C \frac{\sin z}{(z-2\pi j)^3}dz = -133.87$$

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58. **(b)**

Given

Integral $\oint_\Gamma \frac{3z-5}{(z-1)(z-2)}dz = \oint_\Gamma f(z)\,dz$

Consider $\text{Res } f(z)|_{z=1} = \frac{3(1)-5}{(1-2)} = 2$

$$\text{Res } f(z)|_{z=2} = \frac{3(2)-5}{(2-1)} = 1$$

For $\oint_{\Gamma} \frac{3z-5}{(z-1)(z-5)} = 4\pi i = 2\pi i(2)$,

$\therefore$ $\text{Res } f(z)|_{z=1} = 2$ is only to be considered.

$\Rightarrow$ z = 1 has to be inside Γ and z = 2 has to be outside Γ.

$\therefore$ option 'B' is correct.

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59. (a)

Given integral is $\int_{-\infty}^{\infty} \frac{\sin x}{x^2 + 2x + 2} dx$

As per the Contour Integration,

$-\infty < x < \infty \Rightarrow$ Implies the upper half of the circle with center at origin and radius $\to \infty$.

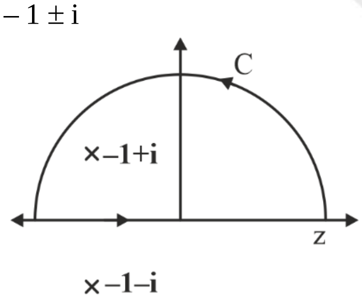
$$\therefore I = \int_{-\infty}^{\infty} \frac{\sin x}{x^2 + 2x + 2} dx = \oint_{c} \frac{\sin z}{z^2 + 2z + 2} dz$$

For the given functions, the poles are at

$z^2 + 2z + 2 = 0$

$\Rightarrow$ $(z + 1)^2 + 1 = 0$

$\Rightarrow$ $z = -1 \pm i$



$\therefore$ Poles are at z = – 1 + i and z = –1 – i.

Out of the above poles z = –1 + i lies in 'C'.

$$\therefore \oint_{C} \frac{\sin z}{z^2 + 2z + 2} dz = 2\pi i \cdot \text{Res}\left(\frac{\sin z}{z^2 + 2z + 2}\right)\Bigg|_{z=-1+i}$$

$$= 2\pi i \cdot \frac{\sin(-1+i)}{2i}$$

$$= \pi \cdot \sin(-1+i) = \pi \cdot \text{Im}(e^{i(-1+i)})$$

$$= \pi \cdot \text{Im}(e^{-i} \cdot e^{-1})$$

$$\Rightarrow \frac{\pi}{e} \cdot \text{Im}(\cos 1 - i \sin 1) = \frac{-\pi}{e} \cdot \sin 1$$

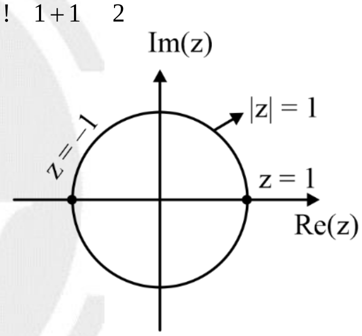
Scan for Video solution

60. (d)

(i) Residue of $\frac{z}{z^2 - 1}$ at z = 1

$$= \frac{1}{(1-1)!} \underset{z\to 1}{\text{Lt}} \frac{d^0}{dz^0}\left\{(z-1)\frac{z}{(z+1)(z-1)}\right\}$$

$$= \frac{1}{0!} \times \frac{1}{1+1} = \frac{1}{2}$$



(ii) $\oint z^2 dx = 0$, since z^2 is analytic at all points in complex plane.

(iii) $\frac{1}{2\pi i} \oint_{c} \frac{1}{z} dz = \frac{1}{2\pi i} \times 2\pi i(1) = 1$

(iv) $\bar{z}$ is not an analytic function since C-R equations are not satisfied.

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61. (b)

If the path 'C' is given by

$|z - z_0| = r$

Then $\oint_{c} (z-a)^n dz = \begin{cases} 0; \text{ if } n \neq -1 \\ 2\pi i; \text{ if } n = -1 \end{cases}$ $\rightarrow$ Cauchy's integral theorem

For $n = -(n+1) \neq -1$

$$\Rightarrow \boxed{\oint_c \frac{dz}{(z-z_0)^{n+1}} = 0}$$

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62. (0.5 to 0.5)

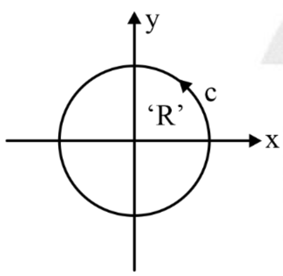
Given 'C' denotes a counter clockwise unit circle

Integral is

$$I = \frac{1}{2\pi j}\oint_C \text{Re}(z)\,dz$$

$$= \frac{1}{2\pi j}\oint_C x\cdot(dx + j\,dy)$$

$$= \frac{1}{2\pi j}\left\{\oint_C x\,dx + (jx)\,dy\right\}$$



Applying Green's theorem

$$\oint_C M\,dx + N\,dy = \iint_R\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dy\cdot dx$$

$$\therefore \quad \oint_C x\,dx + (jx)\,dy = \iint_R\left(\frac{\partial}{\partial x}(jx) - \frac{\partial}{\partial y}(x)\right)dy\,dx$$

$$= j\iint_R dx\,dy$$

$\therefore$ Integral $I = \frac{1}{2\pi j}\cdot j\cdot\iint_R dx\,dy$

$$= \frac{1}{2\pi}\text{ Area of 'R'} = \frac{1}{2\pi}\times\pi(1)^2 = \frac{1}{2} = 0.5$$

$$\therefore \quad \oint_C \frac{1}{2\pi j}\cdot\text{Re}(z)\,dz = 0.5$$

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63. (c)

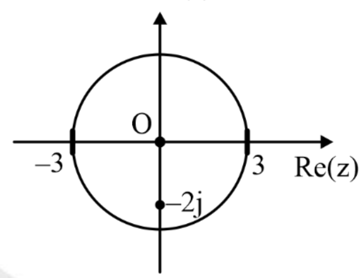
Given

C: $|z| = 3$

$\Rightarrow$ C is a circle with center at origin and radius =

Given Integral is $\oint_C \frac{z^2 - z + 4j}{z + 2j}dz$

Singular point is $z + 2j = 0 \Rightarrow z = -2j$



$$\Rightarrow \oint_C \frac{z^2 - z + 4j}{z + 2j}dz = 2\pi j\cdot\text{Res}\left\{\left.\frac{z^2 - z + 4j}{z + 2j}\right|_{z=-2j}\right\}$$

$$= 2\pi j\left\{(-2j)^2 - (-2j) + 4j\right\}$$

$$= 2\pi j\{-4 + 6j\}$$

$$= 4\pi j\{-2 + 3j\} = 4\pi\{-3 - 2j\}$$

$$\boxed{\therefore \oint_C \frac{z^2 - z + 4j}{z + 2j}dz = -4\pi(3 + 2j)}$$

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64. (c)

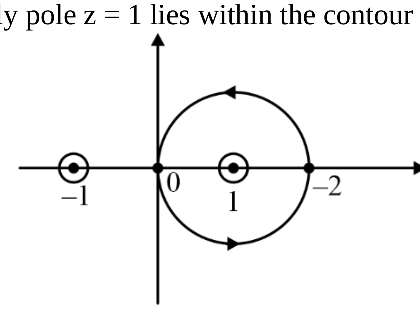
$f(z) = \frac{z^2}{z^2 - 1}$ C : $|z - 1| = 1$

Poles of f(z) are at $z^2 - 1 = 0$

$\Rightarrow$ $(z-1)(z+1) = 0$

$\Rightarrow$ $z = \pm 1$

Only pole $z = 1$ lies within the contour C.



$$\underset{z=a}{\text{Res}}[f(a)] = \lim_{z\to a} f(z)(z-a)$$

$$= \lim_{z\to 1}\frac{z^2}{z^2-1}\cdot(z-1) = \lim_{z\to 1}\left(\frac{z^2}{z+1}\right) = \frac{1^2}{1+1} = \frac{1}{2}$$

By Cauchy Residue Theorem,

$$\int_c \frac{z^2}{z^2-1}dz = 2\pi i\,[\text{Res } f(1)] = 2\pi i\left(\frac{1}{2}\right) = \pi i$$

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65. (a)

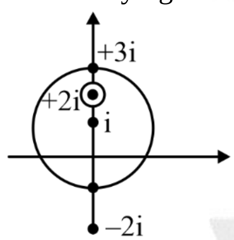
$I = \oint_c \frac{z^2-4}{z^2+4}$ C : |z – i| = z

Poles of the given function are at $z^2 + 4 = 0$

$\Rightarrow$ $(z + 2i)(z - 2i) = 0$

$\Rightarrow$ $z = +2i, -2i$

$\therefore$ Only pole z = 2i is lying within the contour C.



$$\underset{(z=a)}{\text{Res}}[f(a)] = \lim_{z\to a} f(z)(z-a)$$

$$\underset{(z=2i)}{\text{Res}}[f(2i)] = \lim_{z\to 2i}\frac{z^2-4}{z^2+4}\cdot(z-2i)$$

$$= \frac{4i^2-4}{2i+2i} = \frac{-8}{4i} = -\frac{-2}{i}$$

$\therefore$ By Cauchy Residue theorem,

$$I = \oint_c \frac{z^2-4}{z^2+4}dz = 2\pi i[\text{Res}\, f(2i)] = 2\pi i\left(\frac{-2}{i}\right) = -4\pi$$

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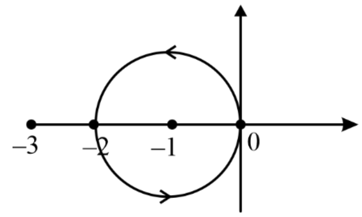
66. (c)

$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$ C: |z + 1|

Only z = –1 is lying within the contour C.



$$\underset{z=a}{\text{Res}}[f(a)] = \lim_{z\to a} f(z)(z-a)$$

$$= \lim_{z\to -1}\left[\frac{1}{z+1} - \frac{2}{z+3}\right](z+1)$$

$$\text{Res}[f(-1)] = \left[1 - \frac{2(z+1)}{(z+3)}\right]_{z=-1} = 1$$

By Cauchy Residue Theorem,

$$= \frac{1}{2\pi j}\oint_c\left[\frac{1}{z+1} - \frac{1}{z+3}\right]dz = \frac{1}{2\pi j}2\pi j\text{Res}[f(-1)] = 1$$

$$\therefore \frac{1}{2\pi j}\oint_c f(z)\,dz = 1$$

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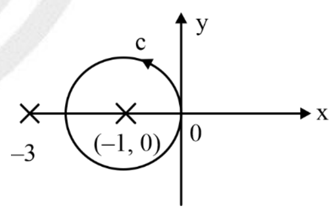
67. (c)

Given: $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$ and c: |z + 1| = 1

$\Rightarrow$ c is circle with centre (–1, 0) and radius '1'

The poles of f(z) are at z = –1, z = –3

$\because$ z = –1 is the only pole within 'c'



$$\frac{1}{2\pi j}\oint_c f(z)\,dz = \frac{1}{2\pi j}\oint_c \frac{-z+1}{(z+1)(z+3)}dz$$

$$= \frac{1}{2\pi j}\times 2\pi j\left\{\text{Res}\left(\left.\frac{1-z}{(z+1)(z+3)}\right|_{z=-1}\right)\right\} = \frac{2}{2} = 1$$

$$\boxed{\therefore \frac{1}{2\pi j}\oint_c\left(\frac{1}{z+1} - \frac{1}{z+3}\right)dz = 1}$$

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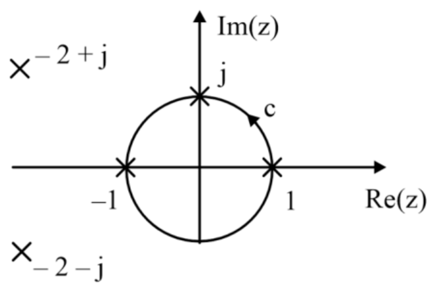
68. (a)

Given: Integral $\oint_c \frac{-3z+4}{(z^2+4z+5)}dz$; c: |z| = 1-unit circle



For $\frac{-3z+4}{z^2+4z+5}$, poles are at the points

Where $z^2 + 4z + 5 = 0$

$\Rightarrow \quad (z+2)^2 + 1 = 0 \qquad \Rightarrow z = -2 \pm i$

Since both the poles are outside the path, given function is analytic within 'c'.

$\therefore \boxed{\oint_{|z|=1} \frac{-3z+4}{z^2+4z+5}dz = 0}$

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69. (c)

Given function is $X(z) = \frac{1-2z}{z(z-1)(z-2)}$

Poles of X(z) are at the points where,

z (z – 1) (z – 2) = 0

$\Rightarrow$ z = 0, z = 1, z = 2 are simple poles.

$\therefore$ Residue of

$$(z)|_{z=0} = \lim_{z\to\infty} z \cdot X(z) = \lim_{z\to 0} \frac{1-2z}{(z-1)(z-2)} = \frac{1}{2}$$

Residue of

$$X(z)|_{z=1} = \lim_{z\to 1}(z-1)\cdot X(z) = \lim_{z\to 1}\frac{1-2z}{z(z-2)} = \frac{-1}{-1} = 1$$

Residue of

$$X(z)|_{z=2} = \lim_{z\to 2}(z-2)\cdot X(z) = \lim_{z\to 2}\frac{1-2z}{z(z-1)} = \frac{-3}{2}$$

$\therefore$ Residue at poles are $\frac{1}{2}$, 1 and $\frac{-3}{2}$.

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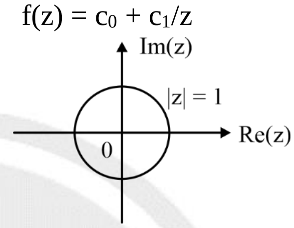
70. (d)

Given:

$f(z) = c_0 + c_1/z$



$$\oint_{\substack{\text{unit}\\ \text{circle}}} \frac{1+f(z)}{z}dz = \oint_{|z|=1} \frac{1+f(z)}{z}dz$$

$$= \oint_c \left[\left(\frac{1+c_0}{z}\right) + \left(\frac{c_1}{z^2}\right)\right]dz$$

$$= 2\pi j\left\{\text{Res}\left.\frac{(1+c_0)}{z}\right|_{z=0} + \text{Res}\left.\left(\frac{c_1}{z^2}\right)\right|_{z=0}\right\}$$

$$= 2\pi j\left\{(1+c_0) + \frac{1}{1!}\lim_{z\to 0}\frac{d}{dz}\left(z^2.\frac{c_1}{z^2}\right)\right\} = 2\pi j(1+c_0)$$

$$\boxed{\oint_{\substack{\text{unit}\\ \text{circle}}} \frac{1+f(z)}{z}dz = 2\pi j(1+c_0)}$$

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71. (a)

Given: $f(z) = \frac{1}{(z+2)^2(z-2)^2}$ and z = 2 is a pole of order '2'.

For any f(z), where z = a is a pole of order 'n'

$$\text{Res } f(z)\big|_{z=a} = \frac{1}{(n-1)!}\left\{\lim_{z\to a}\frac{d^{n-1}}{dz^{n-1}}(z-a)^n f(z)\right\}$$

$$\text{Res } f(z)\big|_{z=2} = \frac{1}{(2-1)!}\lim_{z\to 2}\frac{d}{dz}\left\{(z-2)^2\frac{1}{(z-2)^2(z+2)^2}\right\}$$

$$= \frac{1}{1!}\left\{\lim_{z\to 2} -2(z+2)^{-3}\right\} = \frac{-2}{64} = \frac{-1}{32}$$

$$\therefore \boxed{\text{Res } f(z)\big|_{z=2} = \frac{-1}{32}}$$

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72. **(d)**

$X(z) = \dfrac{z}{(z-a)^2}; |z| > a$

$f(z) = X(z)\cdot z^{n-1}$

$f(z) = \dfrac{z}{(z-a)^2}\cdot z^{n-1} = \dfrac{z^n}{(z-a)^2}$

$\therefore$ We have a pole at z = a of order 2.

$$\underset{z=a}{\text{Res}}[f(a)] = \frac{1}{(n-1)!}\lim_{z\to a}\left[\frac{d^{n-1}}{dz^{n-1}} f(z)\cdot(z-a)^n\right]$$

$$\underset{z=a}{\text{Res}}[f(a)] = \frac{1}{(2-1)!}\lim_{z\to a}\left[\frac{d}{dz}\frac{z^n}{(z-a)^n}\cdot(z-a)^n\right]$$

$$= \lim_{z\to a} nz^{n-1} = na^{n-1}$$

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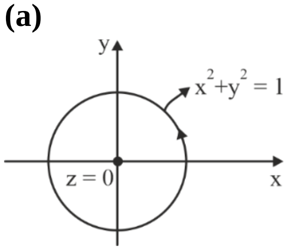
73. **(a)**



Given:

$$f(z) = \frac{\cos z}{z}$$

$\Rightarrow$ z = 0 is a singular point

From Cauchy Residue Theorem

$$\oint f(z)\,dz = 2\pi i \times \{\text{sum of residue of } f(z)\}$$

$$\Rightarrow \oint_{|z|=1}\frac{\cos z}{z}dz = 2\pi i\left\{\text{Res of } f(z)\big|_{z=0}\right\}$$

$$= 2\pi i\left\{\cos z\,|_{z=0}\right\} = 2\pi i(1) = 2\pi i$$

$$\therefore \boxed{\oint_{|z|=1}\frac{\cos z}{z}dz = 2\pi i}$$

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74. **(1 to 1)**

Given

$F(z) = \dfrac{\sin z}{z^2}$

$$\frac{1}{z^2}\left\{z - \frac{z^3}{3!} + \frac{z^5}{5!}.......\right\}$$

$$\frac{1}{z} - \frac{z}{3!} + \frac{z^3}{5!} + ...$$

$\text{Resf}(z)|_{z=0}$ = coefficient of $\dfrac{1}{z}$ in series expansion of f(z)

$\Rightarrow \text{resf}(z)|_{z=0} = 1$

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CHAPTER

7

FOURIER SERIES

General Fourier Expansions

1. [MSQ] [GATE-CE-2023-2M]

The following function is defined over the interval [–L, L]:

$f(x) = px^4 + qx^5$

If it is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left\{a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right)\right\},$$

Which options amongst the following are true?

(a) a_n, n = 1,2,⋯,∞ depend on p

(b) a_n, n = 1,2,⋯,∞ depend on q

(c) b_n, n = 1,2,⋯,∞ depend on p

(d) b_n, n = 1,2,⋯,∞ depend on q

2. [NAT] [GATE-CE-2022-1M]

The Fourier cosine series of a function is given by:

$$f(x) = \sum_{n=0}^{\infty} f_n \cos nx$$

For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is_______. (round off to three decimal places).

3. [MCQ] [GATE-ME-2022:1M]

The Fourier series expasion of x^3 in the interval $-1 \le x < 1$ with periodic continuation has

(a) Only sine terms

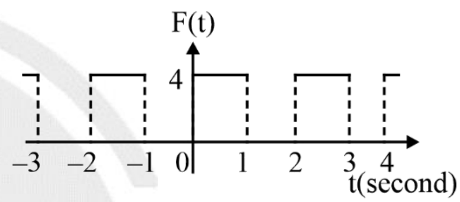
(b) Only cosine terms

(c) Both sine and cosin terms

(d) Only sine terms and a non-zero constant

4. [MCQ] [GATE-ME-2022:1M]

F(t) is a periodic square wave funciton as shown. It takes only two values, 4 and 0, and stay at each of these values for 1 second before changing. What is the constant term in the Fourier series expansion of F(t)?



(a) 1 (b) 2

(c) 3 (d) 4

5. [MCQ] [GATE-EE-2021:2M]

Let f(t) be an even function, i.e. f(–t) = f(t) for all t. Let the Fourier transform of f(t) be defined as $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$.

Suppose $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω, and F(0) = 1. Then

(a) f(0) < 1 (b) f(0) > 1

(c) f(0) = 1 (d) f(0) = 0

6. [NAT] [GATE-CE-2020-2M]

The Fourier series to represent $x - x^2$ for $-\pi \le x \le \pi$ is given by

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

The value of a_0 (round off to two decimal places), is ____________.

7. [MCQ] [GATE-EE-2019:2M]

A periodic function f(t), with a period of 2π, is represented as its Fourier series,

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

If $f(t) = \begin{cases} A\sin t, & 0 \le t \le \pi \\ 0, & \pi < t < 2\pi \end{cases}$, the Fourier series coefficients a_1 and b_1 of f(t) are

(a) $a_1 = \frac{A}{\pi}; b_1 = 0$
(b) $a_1 = \frac{A}{2}; b_1 = 0$
(c) $a_1 = 0; b_1 = \frac{A}{\pi}$
(d) $a_1 = 0; b_1 = \frac{A}{2}$

8. [MCQ] [GATE-ME-2018:1M]

The fourier cosine series for an even function f(x) is given by $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$. The value of the coefficient a_2 for the function $f(x) = \cos^2(x)$ in $[0, \pi]$ is

(a) –0.5
(b) 0.00
(c) 0.5
(d) 1.0

9. [MCQ] [GATE-EC-2017:1M]

For the function $f(x) = \begin{cases} -2 & -\pi < x < 0 \\ 2 & 0 < x < \pi \end{cases}$. The value of a_n in the Fourier Series expansion of f(x) is

(a) 2
(b) 4
(c) 0
(d) –2

10. [MCQ] [GATE-CE-2016-1M]

The Fourier series of the function

$$f(x) = \begin{cases} 0 & -\pi < x \le 0 \\ \pi - x & 0 < x < \pi \end{cases}$$

In the interval $[-\pi, \pi]$ is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \ldots\right] + \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots\right]$$

The convergence of the above Fourier series at x = 0 gives

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$
(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$
(c) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$
(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$

11. [NAT] [GATE-EE-2014:2M]

Let g: $[0, \infty) \to [0, \infty)$ be a function defined by g(x) = x – [x], where [x] represents the integer part of x. (That is, the largest integer which is less than or equal to x). The value of the constant term in the Fourier Series expansion of g(x) is _____.

12. [MCQ] [GATE-EE-2010:1M]

The period of the signal $x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$

(a) 0.4π s
(b) 0.8π s
(c) 1.25 s
(d) 2.5 s

❑❑❑

ANSWER KEY

1.	(b, c)	**2.**	(0.120 to 0.130)	**3.**	(a)	**4.**	(b)
5.	(a)	**6.**	(–6.28 to –6.68)	**7.**	(d)	**8.**	(c)
9.	(c)	**10.**	(c)	**11.**	(0.5 to 0.5)	**12.**	(d)

SOLUTIONS

1. (b, c)

$$f(x) = px^4 + qx^5$$

Using Fourier coefficient formula,

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}\left[px^4 + qx^5\right]\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} px^4 \sin\left(\frac{n\pi x}{L}\right)dx + \frac{1}{L}\int_{-L}^{L} qx^5 \sin\left(\frac{n\pi x}{L}\right)dx$$

$$a_n = 0 + \frac{1}{L}\int_{-L}^{L} qx^5 \sin\left(\frac{n\pi x}{L}\right)dx$$

$$\left[\because px^4 \sin\left(\frac{n\pi x}{L}\right) \text{is an odd function}\right]$$

So, a_n depends on q.

$$f(x) = px^4 + qx^5$$

Using Fourier coefficient formula,

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cdot\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}\left[px^4 + qx^5\right]\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} px^4 \cos\left(\frac{n\pi x}{L}\right)dx + \frac{1}{L}\int_{-L}^{L} qx^5 \cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} px^4 \cos\left(\frac{n\pi x}{L}\right)dx + 0$$

$$\left[\because qx^5 \cos\left(\frac{n\pi x}{L}\right) \text{is an odd function}\right]$$

So, b_n depends on p.

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2. (0.120 to 0.130)

$$f(x) = \cos^4 x = \left(\cos^2 x\right)^2 = \left[\frac{1+\cos 2x}{2}\right]^2$$

$$\Rightarrow \quad f(x) = \frac{1}{4}\left[1 + \cos^2 2x + 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1+\cos 4x}{2} + 2\cos 2x\right]$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{\cos 4x}{8} + \frac{\cos 2x}{2}$$

$$= \frac{3}{8} + 0\cos x + \frac{1}{2}\cos 2x + 0\cos 3x$$
$$+ \frac{1}{8}\cos 4x + 0\cos 5x + \ldots$$

∴ From cosine Fourier series

$$f_4 = \frac{1}{8} \text{ and } f_5 = 0$$

∴ $f_4 + f_5 = \frac{1}{8} = 0.125$

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3. (a)

Given data:

$f(x) = x^3$ $[-1, 1)$

The General Fourier Series Expansion is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n.\cos\left(\frac{n\pi x}{c}\right) + \sum_{n=1}^{\infty} b_n.\sin\left(\frac{n\pi x}{c}\right)$$

Comparing $(\alpha, \alpha + 2c)$ with $[-1, 1]$

$\Rightarrow$ $\alpha = -1; c = 1$

$$a_0 = \frac{1}{c}.\int_{-1}^{1} f(x)dx = \frac{1}{c}\int_{-1}^{1} x^3 dx = 0$$

{∵ f(x) is an odd function}

$$a_n = \frac{1}{c}.\int_{-1}^{1} f(x)\cos nx dx = \frac{1}{c}\int_{-1}^{1} x^3 \cos nx dx = 0$$

$$b_n = \frac{1}{c}.\int_{-1}^{1} f(x)\sin nx dx = \frac{1}{c}\int_{-1}^{1} x^3 \sin nx dx \neq 0$$

∴ The Fourier Series expansion of x^3 in $[-1, 1)$ has only sine terms in it.

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4. (b)

Given data: f(t) takes two values: 4 and 0.

Let us consider [–1, 1]

(∵ since the period of function is 2 seconds)

The general Fourier Series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n.\cos\left(\frac{n\pi x}{c}\right) + \sum_{n=1}^{\infty} b_n.\sin\left(\frac{n\pi x}{c}\right)$$

Where

$$a_0 = \frac{1}{c}.\int_{\alpha}^{\alpha+2c} f(x)dx = \frac{1}{1}.\int_{-1}^{1} f(x)dx$$

$$= \int_{-1}^{0} 0.dx + \int_{0}^{1} 4.dx = 4$$

Since $a_0 = 4$

$$\Rightarrow \quad \frac{a_0}{2} = 2$$

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5. (a)

Given function is even

$$\frac{d}{d\omega}F(\omega) = -\omega F(\omega) \qquad \ldots (1)$$

From differentiation property,

$$t\, f(t) = j\frac{d}{d\omega}F(\omega)$$

Applying inverse Fourier transfer to the above equation,

$$-j\, t\, f(t) = j\frac{d}{d\omega}f(t)$$

$$\frac{d}{dt}f(t) = -t\,f(t) \qquad \ldots (2)$$

From equation (1) and (2) it is clear that f(t) is Gaussian function, it can be written as,

$$f(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$$

$$\therefore \quad f(0) = \frac{1}{\sqrt{2\pi}} = 0.3989$$

$$\therefore \quad f(0) < 1$$

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6. (–6.28 to –6.68)

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\left(x - x^2\right)dx$$

$$= \frac{1}{\pi}\cdot 2\int_0^{\pi} -x^2 dx \qquad [\because x \text{ is an odd function}]$$

$$= -\frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = -\frac{2}{\pi}\cdot\frac{\pi^3}{3}$$

$$= -\frac{2\pi^2}{3} = -6.58$$

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7. (d)

Given data: $f(t) = \begin{cases} A.\sin t; & 0 \le t \le \pi \\ 0; & \pi < t < 2\pi \end{cases}$

$$a_1 = \frac{1}{\pi}.\int_0^{2\pi} f(t).\cos t.dt$$

$$\Rightarrow \quad a_1 = \frac{1}{\pi}.\left\{\int_0^{\pi} A.\sin t.\cos t.dt\right\}$$

$$\Rightarrow \quad a_1 = \frac{1}{\pi}\times\frac{A}{2}\times\int_0^{\pi}\sin 2t.dt$$

$$\Rightarrow \quad a_1 = \frac{A}{2\pi}\times\left\{\frac{-\cos 2t}{2}\Bigg|_0^{\pi}\right\} = \frac{-A}{4\pi}\{1-1\} = 0$$

$$b_1 = \frac{1}{\pi}.\int_0^{2\pi} f(t).\sin t.dt$$

$$\Rightarrow \quad b_1 = \frac{1}{\pi}.\left\{\int_0^{\pi} A.\sin t.\sin t.dt\right\}$$

$$\Rightarrow \quad b_1 = \frac{A}{\pi}\times\int_0^{\pi}\sin^2 t.dt$$

$$\Rightarrow \quad b_1 = \frac{2A}{\pi}\times\int_0^{\pi/2}\sin^2 t.dt = \frac{2A}{\pi}\times\frac{1}{2}\times\frac{\pi}{2}$$

$$\Rightarrow \quad \boxed{b_1 = \frac{A}{2} \;\&\; a_1 = 0}$$

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8. (c)

Given data: $f(x) = \cos^2 x$, $[0, \pi]$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n.\cos nx$$

We know that

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\Rightarrow \quad \cos^2 x = \frac{1}{2} + \frac{1}{2}.\cos 2x = a_0 + a_2.\cos 2x$$

On comparing $\Rightarrow$ $\boxed{a_2 = \frac{1}{2} = 0.5}$

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9. (c)

Given data:

$$f(x) = \begin{cases} -2; & -\pi < x < 0 \\ 2; & 0 < x < \pi \end{cases}$$

The general Fourier Series expansion of f(x) is given

by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

Where $a_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) . \cos nx . dx$

$$\Rightarrow a_n = \frac{1}{\pi} \times \left\{ \int_{-\pi}^{0} -2.\cos nx.dx + \int_{0}^{\pi} 2.\cos nx.dx \right\}$$

$$\Rightarrow \quad \frac{1}{\pi}.\left\{ -2.\frac{\sin nx}{n}\bigg|_{-\pi}^{0} + 2.\frac{\sin nx}{n}\bigg|_{0}^{\pi} \right\}$$

$$\Rightarrow \quad \frac{1}{\pi}.\left\{ \frac{-2}{n}(0) + 2.(0) \right\}$$

$$= 0$$

$\therefore \quad a_n = 0$

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10. (c)

The function $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 \le x < \pi \end{cases}$

And Fourier series is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \ldots \right]$$

$$+ \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots \right]$$

At x = 0, it is a point of discontinuity, the Fourier series converges to $\frac{1}{2}\left[f\left(0^-\right) + f\left(0^+\right) \right]$

Where $f\left(0^-\right) = \lim_{x \to 0} (\pi - x) = \pi$

$f\left(0^+\right) = 0$

Put x = 0 in fouries series,

$$f(0) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \right]$$

$$\frac{\pi}{2} = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \right]$$

$$\frac{\pi}{2}\left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \right]$$

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}}$$

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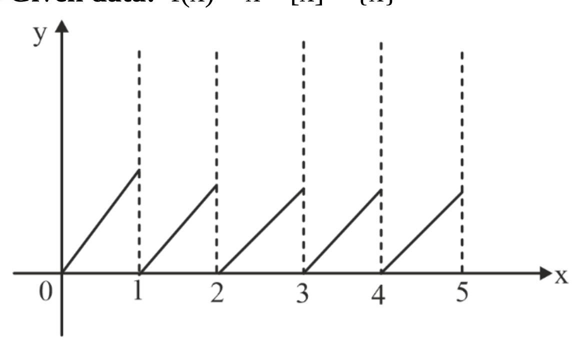
11. (0.5 to 0.5)

Given data: f(x) = x – [x] = {x}



From the graph, period of f(x) = 1

The general Fourier Series expansion of f(x) in the interval (α, α + 2c) is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n.\cos\left(\frac{n\pi x}{c}\right) + \sum_{n=1}^{\infty} b_n.\sin\left(\frac{n\pi x}{c}\right)$$

Where $a_0 = \frac{1}{c}\int_{\alpha}^{\alpha+2c} f(x)dx$

Considering the interval [0, 1]

$\Rightarrow$ $\alpha = 0$ and $c = \frac{1}{2}$

$$\frac{a_0}{2} = \frac{1}{2}\left\{\frac{1}{c}\int_{\alpha}^{\alpha+2c} f(x)dx\right\}$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{1}{2}\right)} \times \int_0^1 (x - [x])dx$$

$$\Rightarrow \frac{a_0}{2} = \int_0^1 (x-0)dx = \frac{1}{2}$$

$$\Rightarrow \frac{a_0}{2} = \frac{1}{2} = 0.5$$

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12. (d)

The given signal is

$$x(t) = 8.\sin\left(0.8\pi t + \frac{\pi}{4}\right)$$

Comparing with the standard form of the signal

$$x(t) = A.\sin(\omega t + \phi)$$

$\Rightarrow$ $\omega = 0.8\pi$

$\Rightarrow$ $\frac{2\pi}{T} = 0.8\pi$

$\Rightarrow$ $T = \frac{2}{0.8} = 2.5$ s

$\therefore$ $T = 2.5$ s

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CHAPTER

8

LAPLACE TRANSFORMS

Laplace Transforms of Standard Functions

1. [MCQ] [GATE-ME-2023:2M]

Which of the following options given is the inverse Laplace transform of $\frac{1}{s^3 - s}$?

u(t) denotes the unit-step function.

(a) $\left(-1+\frac{1}{2}e^{-t}+\frac{1}{2}e^{t}\right)u(t)$

(b) $\left(\frac{1}{3}e^{-t}-e^{t}\right)u(t)$

(c) $\left(-1+\frac{1}{2}e^{-(t-1)}+\frac{1}{2}e^{(t-1)}\right)u(t-1)$

(d) $\left(-1-\frac{1}{2}e^{-(t-1)}-\frac{1}{2}e^{(t-1)}\right)u(t-1)$

2. [MCQ] [GATE-ME-2022:1M]

Solution of $\nabla^2 T = 0$ in a square domain $(0 < x < 1$ and $0 < y < 1)$ with boundary conditions:
$T(x, 0) = x$; $T(0, y) = y$; $T(x, 1) = 1 + x$; $T(1, y) = 1 + y$ is

(a) $T(x, y) = x - xy + y$
(b) $T(x, y) = x + y$
(c) $T(x, y) = -x + y$
(d) $T(x, y) = x + xy + y$

3. [MCQ] [GATE-ME-2021:1M]

The Dirac-delta function $(\delta(t - t_0))$ for $t, t_0 \in \Re$, has the following property.

$$\int_a^b \varphi(t)\delta(t-t_0)\,dt = \begin{cases} \varphi(t_0) & a < t_0 < b \\ 0 & \text{otherwise} \end{cases}$$

The Laplace transform of the Dirac-delta function $\delta(t - a)$ for $a > 0$; $L(\delta(t - a)) = F(s)$ is

(a) 0
(b) ∞
(c) e^{sa}
(d) e^{-sa}

4. [MCQ] [GATE-ME-2021:1M]

If the Laplace transform of a function $f(t)$ is given by $\frac{s+3}{(s+1)(s+2)}$ then $f(0)$ is.

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) $\frac{3}{2}$

5. [MCQ] [GATE-ME-2020:1M]

The Laplace transform of a function $f(t)$ is

$L(f) = \frac{1}{(s^2+\omega^2)}$. Then, $f(t)$ is

(a) $f(t) = \frac{1}{\omega^2}(1-\cos\omega t)$

(b) $f(t) = \frac{1}{\omega}\cos\omega t$

(c) $f(t) = \frac{1}{\omega}\sin\omega t$

(d) $f(t) = \frac{1}{\omega^2}(1-\sin\omega t)$

6. [MCQ] [GATE-CE-2019:1M]

The Laplace transform of sin h (at) is

(a) $\frac{s}{s^2+a^2}$
(b) $\frac{s}{s^2-a^2}$
(c) $\frac{a}{s^2-a^2}$
(d) $\frac{a}{s^2+a^2}$

7. [MCQ] [GATE-CE-2018:2M]

The Laplace transform F(s) of the exponential function f(t) = e^{at} when t $\geq$ 0, where a is a constant and (s–a) > 0 , is

(a) $\frac{1}{s+a}$ (b) $\frac{1}{s-a}$

(c) $\frac{1}{a-s}$ (d) ∞

8. [NAT] [GATE-ME-2018:2M]

F(s) is the Laplace transform of the function

$$f(t) = 2t^2 e^{-t}$$

F(1) is ______ (Correct to two decimal places).

9. [MCQ] [GATE-ME-2017:1M]

The Laplace transform of $t e^t$ is

(a) $\frac{s}{(s+1)^2}$ (b) $\frac{1}{(s-1)^2}$

(c) $\frac{1}{(s+1)^2}$ (d) $\frac{s}{s-1}$

10. [MCQ] [GATE-ME-2016:1M]

If *f*(t) is a function defined for all t ≥ 0, its Laplace transform F(s) is defined as

(a) $\int_0^\infty e^{st} f(t)dt$ (b) $\int_0^\infty e^{-st} f(t)dt$

(c) $\int_0^\infty e^{ist} f(t)dt$ (d) $\int_0^\infty e^{-ist} f(t)dt$

11. [MCQ] [GATE-ME-2016:1M]

Laplace transform of $\cos(\omega t)$ is

(a) $\frac{s}{s^2+\omega^2}$ (b) $\frac{\omega}{s^2+\omega^2}$

(c) $\frac{s}{s^2-\omega^2}$ (d) $\frac{\omega}{s^2-\omega^2}$

12. [MCQ] [GATE-CE-2016:2M]

The value of $\int_0^\infty \frac{1}{1+x^2}dx + \int_0^\infty \frac{\sin x}{x}dx$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 1

13. [NAT] [GATE-2015-EC: 2M]

The value of the integral

$\int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{4\pi t}dt$ is ____

14. [MCQ] [GATE-ME-2015:1M]

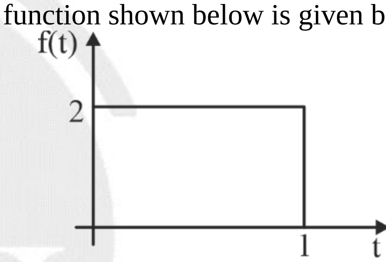
The Laplace transform of e^{i5t} where $i = \sqrt{-1}$, is

(a) $\frac{s-5i}{s^2-25}$ (b) $\frac{s+5i}{s^2+25}$

(c) $\frac{s+5i}{s^2-25}$ (d) $\frac{s-5i}{s^2+25}$

15. [MCQ] [GATE-ME-2015:2M]

Laplace transform of the function *f*(t) is given by $F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$. Laplace transform of the function shown below is given by



(a) $\frac{1-e^{-2s}}{s}$ (b) $\frac{1-e^{-s}}{2s}$

(c) $\frac{2-2e^{-s}}{s}$ (d) $\frac{1-2e^{-s}}{s}$

16. [MCQ] [GATE-ME-2014:1M]

Laplace transform of $\cos(\omega t)$ is $\frac{s}{s^2+\omega^2}$. The Laplace transformation of $e^{-2t}\cos(4t)$ is

(a) $\frac{s-2}{(s-2)^2+16}$ (b) $\frac{s+2}{(s-2)^2+16}$

(c) $\frac{s-2}{(s+2)^2+16}$ (d) $\frac{s+2}{(s+2)^2+16}$

17. [MCQ] [GATE-ME-2013: 2M]

The function $f(t)$ satisfies the differential equation $\frac{d^2 f}{dt^2} + f = 0$ and the auxiliary conditions, $f(0) = 0, \frac{df}{dt}(0) = 4$. The Laplace transform of $f(t)$ is given by

(a) $\frac{2}{s+1}$ (b) $\frac{4}{s+1}$

(c) $\frac{4}{s^2+1}$ (d) $\frac{2}{s^4+1}$

18. [MCQ] [GATE-ME-2012:2M]

The inverse Laplace transform of the function $F(s) = \frac{1}{s(s+1)}$ is given by

(a) $f(t) = \sin t$ (b) $f(t) = e^{-t} \sin t$

(c) $f(t) = e^{-t}$ (d) $f(t) = 1 - e^{-t}$

19. [MCQ] [GATE-ME-2010:2M]

The Laplace transform of a function $f(t)$ is $\frac{1}{s^2(s+1)}$.

The function $f(t)$ is

(a) $t - 1 + e^{-t}$ (b) $t + 1 + e^{-t}$

(c) $-1 + e^{-t}$ (d) $2t + e^{t}$

20. [MCQ] [GATE-ME-2009:1M]

The inverse Laplace transform of $\frac{1}{(s^2+s)}$ is

(a) $1 + e^{t}$ (b) $1 - e^{t}$

(c) $1 - e^{-t}$ (d) $1 + e^{-t}$

21. [MCQ] [GATE-CE-2009:2M]

Laplace transform for the function f(x) =cos h(ax) is

(a) $\frac{a}{s^2-a^2}$ (b) $\frac{s}{s^2-a^2}$

(c) $\frac{a}{s^2+a^2}$ (d) $\frac{s}{s^2+a^2}$

Roperties of Laplace Transforms

22. [MCQ] [GATE-ME-2020:1M]

The solution of

$$\frac{d^2y}{dt^2} - y = 1,$$

which additionally satisfies $y|_{t=0} = \left.\frac{dy}{dt}\right|_{t=0} = 0$ in the Laplace s-domain is

(a) $\frac{1}{s(s+1)(s-1)}$ (b) $\frac{1}{s(s+1)}$

(c) $\frac{1}{s(s-1)}$ (d) $\frac{1}{(s-1)}$

23. [MCQ] [GATE-EE 2016:2M]

The value of the integral $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt$ is equal to

(a) 0 (b) 0.5

(c) 1 (d) 2

24. [MCQ] [GATE-EE-2014:1M]

Let X(s) = $\frac{3s+5}{s^2+10s+21}$ be the Laplace Transform of a signal x(t). Then, $x(0^+)$ is

(a) 0 (b) 3

(c) 5 (c) 21

25. [MCQ] [GATE-2012 : 2M]

Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$$ with

$y(t)|_{t=0} = -2$ and $\left.\frac{dy}{dt}\right|_{t=0^+} = 0$ The numerical value of $\left.\frac{dy}{dt}\right|_{t=0}$ is

(a) –2 (b) –1

(c) 0 (d) 1

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ANSWER KEY

1.	**(a)**	**2.**	**(b)**	**3.**	**(d)**	**4.**	**(c)**
5.	**(c)**	**6.**	**(c)**	**7.**	**(b)**	**8.**	**(0.48 to 0.52)**
9.	**(b)**	**10.**	**(b)**	**11.**	**(a)**	**12.**	**(b)**
13.	**(3 to 3)**	**14.**	**(b)**	**15.**	**(c)**	**16.**	**(d)**
17.	**(c)**	**18.**	**(d)**	**19.**	**(a)**	**20.**	**(c)**
21.	**(b)**	**22.**	**(a)**	**23.**	**(d)**	**24.**	**(b)**
25.	**(d)**						

SOLUTIONS

1. (a)

Given

$$L\{f(t)\} = \frac{1}{s^3 - s}$$

u(t) is unit step function $\Rightarrow$ $u(t) = 1 \ \forall \ t > 0$

$$\Rightarrow \quad L\{f(t)\} = \frac{1}{s(s^2 - 1)}$$

$$\Rightarrow \quad f(t) = L^{-1}\left\{\frac{1}{s(s-1)(s+1)}\right\}$$

$$\Rightarrow \quad = L^{-1}\left\{\frac{-1}{s} + \frac{\left(\frac{1}{2}\right)}{s-1} + \frac{\left(\frac{1}{2}\right)}{s+1}\right\}$$

$$\Rightarrow \quad f(t) = L^{-1}\left\{\frac{-1}{s}\right\} + \frac{1}{2} \cdot L^{-1}\left\{\frac{1}{s-1}\right\} + \frac{1}{2} \cdot L^{-1}\left\{\frac{1}{s+1}\right\}$$

$$\Rightarrow \quad f(t) = -e^{0.t} + \frac{1}{2} \cdot e^{t} + \frac{1}{2} \cdot e^{-t}$$

$$\left(\because L\{e^{at}\} = \frac{1}{s-a}\right)$$

$$\Rightarrow \quad f(t) = -1 + \frac{1}{2}e^{t} + \frac{1}{2}e^{-t}$$

Since u(t) = 1

$$\Rightarrow \quad \boxed{f(t) = \left(-1 + \frac{1}{2} \cdot e^{t} + \frac{1}{2} \cdot e^{-t}\right)u(t)}$$

Scan for Video solution

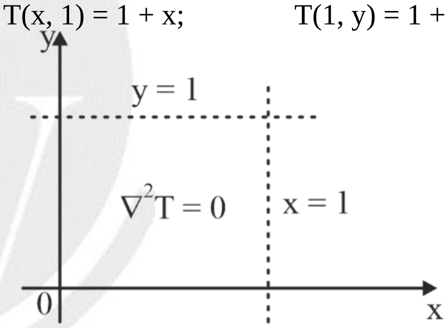
2. (b)

Given

$\nabla^2 T = 0$ in $(0 < x < 1$ and $0 < y < 1)$

$T(x, 0) = x;$ $T(0, y) = y,$

$T(x, 1) = 1 + x;$ $T(1, y) = 1 + y$



From given options, $T(x, y) = x + y$ is the only option that satisfies the boundary condition

$T(x, 1) = 1 + x$

$\therefore$ $T(x, y) = x + y$

Scan for Video solution

3. (d)

Given

For Dirac-delta function, $\delta(t - t_0)$

$$\int_a^b \phi(t) \cdot \delta(t - t_0)\,dt = \begin{cases} \phi(t_0); & a < t < b \\ 0 \quad ; & \text{otherwise} \end{cases}$$

For $0 < t < \infty$ $\quad \int_0^\infty \phi(t) \cdot \delta(t - t_0)\,dt = \phi(t_0)$

For $\phi(t) = e^{-st}$ and $t_0 = a$

$$\Rightarrow \int_0^\infty e^{-st} \cdot \delta(t-a)\,dt = \phi(a) = e^{-sa}$$

$$\Rightarrow L\{\delta(t-a)\} = e^{-sa} \left\{\because \int_0^\infty e^{-st} \cdot f(t)\,dt = L\{f(t)\}\right\}$$

Scan for Video solution

4. (c)

Given

$$L\{f(t)\} = \frac{s+3}{(s+1)(s+2)}$$

$$\Rightarrow L\{f(t)\} = \frac{2}{s+1} - \frac{1}{s+2}$$

Applying L^{-1} on both sides

$$\Rightarrow L^{-1}\{L\{f(t)\}\} = L^{-1}\left\{\frac{2}{s+1} - \frac{1}{s+2}\right\}$$

$$\Rightarrow f(t) = 2.\, e^{-t} - e^{-2t}$$

$$\left(\because L\{e^{at}\} = \frac{1}{s-a}\right)$$

$$\therefore f(0) = 2.e^{-0} - e^{-2(0)} = 2 - 1 = 1$$

$$\Rightarrow f(0) = 1$$

Scan for Video solution

5. (c)

Given

$$L\{f(t)\} = \frac{1}{s^2+\omega^2}$$

We have $L\{\sin \omega t\} = \dfrac{\omega}{s^2+\omega^2}$

$$\Rightarrow \frac{1}{s^2+\omega^2} = \frac{1}{\omega}.L\{\sin\omega t\} = L\left\{\frac{1}{\omega}\cdot\sin\omega t\right\}$$

$$\therefore \boxed{f(t) = \frac{1}{\omega}\cdot\sin\omega t}$$

Scan for Video solution

6. (c)

$$\sinh at = \frac{e^{at} - e^{-at}}{2}$$

Taking Laplace Transform, we get

$$= \frac{1}{2}\left[L\{e^{at}\} - L\{e^{-at}\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{2a}{s^2-a^2}\right]$$

$$\boxed{L[\sinh at] = \frac{a}{s^2-a^2}}$$

Scan for Video solution

7. (b)

$$L[f(t)] = \int_0^\infty e^{-st} \cdot f(t)\,dt$$

$$L[e^{at}] = \int_0^\infty e^{-st} \cdot e^{at}dt = \int_0^\infty e^{-(s-a)t}dt$$

$$\left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = -\frac{1}{s-a}(0-1)$$

$$\boxed{L(e^{at}) = \frac{1}{s-a}}$$

Scan for Video solution

8. (0.48 to 0.52)

Given,

$f(t) = 2 \cdot t^2 \cdot e^{-t}$

$$L\{e^{-t}\} = \frac{1}{s+1} \qquad \left\{\because \; L\{e^{at}\} = \frac{1}{s-a}\right\}$$

$$L\{t^n \cdot e^{at}\} = (-1)^n \cdot \frac{d^n}{ds^n}\left(\frac{1}{s+1}\right)$$

$$\Rightarrow \; L\{t^2 \cdot e^{-t}\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left(\frac{1}{s+1}\right)$$

$$= \frac{d}{ds}\left(\frac{-1}{(s+1)^2}\right) = \frac{2}{(s+1)^3}$$

$$\therefore \; L\{2 \cdot t^2 \cdot e^{-t}\} = \frac{4}{(s+1)^3} = F(s)$$

$$\therefore \; F(s) = \frac{4}{(s+1)^3}$$

$$\Rightarrow \; F(1) = \frac{4}{(1+1)^3} = \frac{4}{8} = 0.5$$

$$\Rightarrow \; \boxed{F(1) = 0.5}$$

Scan for Video solution

9. (b)

$L\ \{t \cdot e^t\} = ?$

$$L\{e^{at}\} = \frac{1}{s-a}$$

$$\Rightarrow \; L\{t^n \cdot e^{at}\} = (-1)^n \cdot \frac{d^n}{ds^n}\left\{\frac{1}{s-a}\right\}$$

$$\Rightarrow \; L\{t \cdot e^{at}\} = (-1) \cdot \frac{d}{ds}\left(\frac{1}{s-a}\right) = \frac{1}{(s-a)^2}$$

$$\Rightarrow \; \boxed{L\{t \cdot e^t\} = \frac{1}{(s-1)^2}} \qquad \text{For } a = 1$$

Scan for Video solution

10. (b)

For a function $f(t)$ defined $\forall\ t \geq 0$; the Laplace transform of $f(t)$ is given by

$$\boxed{L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\,dt}$$

Scan for Video solution

11. (a)

$$L\{\cos \omega t\} = \int_0^\infty e^{-st} \cdot \cos \omega t\,dt = \frac{s}{s^2 + \omega^2}$$

Scan for Video solution

12. (b)

$$I = \int_0^\infty \frac{1}{1+x^2}dx + \int_0^\infty \frac{\sin x}{x}dx = ?$$

$I = I_1 + I_2$

Where,

$$I_1 = \int_0^\infty \frac{1}{1+x^2} = \left[\tan^{-1} x\right]_0^\infty$$

$$I_2 = \int_0^\infty \frac{\sin x}{x}dx$$

Considering Laplace Transform of sin at

$$\Rightarrow \; L\{\sin at\} = \frac{a}{s^2 + a^2}$$

By the Property of division with 't'

$$\Rightarrow \; L\left\{\frac{\sin at}{t}\right\} = \int_s^\infty \frac{a}{s^2+a^2} \cdot ds$$

$$\Rightarrow \; \int_s^\infty e^{-st} \cdot \frac{\sin at}{t} \cdot dt = \frac{1}{a} \cdot \operatorname{Tan}^{-1}\left(\frac{s}{a}\right)\Bigg|_s^\infty$$

$$\Rightarrow \; \int_s^\infty e^{-st} \cdot \frac{\sin at}{t}dt = \frac{1}{a} \cdot \left\{\frac{\pi}{2} - \operatorname{Tan}^{-1}\left(\frac{s}{a}\right)\right\}$$

Substituting s = 0 and a = 1 in both sides of above equation

$$\Rightarrow \int_{s}^{\infty} \frac{\sin at}{t} dt = 1\left\{\frac{\pi}{2} - 0\right\} = \frac{\pi}{2}$$

$$\therefore \int_{s}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

Therefore,

$$I = \left[\tan^{-1} x\right]_0^{\infty} + \int_0^{\infty} \frac{\sin x}{x} dx$$

$$\left[\because \int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}\right]$$

$$I = \tan^{-1}\infty - \tan^{-1}0 + \frac{\pi}{2}$$

$$I = \frac{\pi}{2} - 0 + \frac{\pi}{2} = \pi$$

$$\therefore \boxed{\int_0^{\infty} \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{\sin x}{x} dx = \pi}$$

Scan for Video solution

13. (3 to 3)

Let the given integral,

$$I = \int_{-\infty}^{\infty} 12 \cdot \cos(2\pi t) \frac{\sin(4\pi t)}{4\pi t} dt$$

$$= 2 \times \int_0^{\infty} 12 \cdot \cos(2\pi t) \frac{\sin(4\pi t)}{4\pi t} dt$$

$$\left(\because \int_{-\infty}^{\infty} f(t) dt = 2 \times \int_0^{\infty} f(t) dt, \text{ If } f(-t) = f(t)\right)$$

$$\Rightarrow I = 3 \times \int_0^{\infty} \frac{2\sin(4\pi t) \cdot \cos(2\pi t)}{\pi t} dt$$

$$\Rightarrow I = \frac{3}{\pi} \times \int_0^{\infty} \frac{\sin(6\pi t) + \sin(2\pi t)}{t} dt$$

$$\left(\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)\right)$$

$$\Rightarrow I = 3 \times \left\{\int_0^{\infty} \frac{\sin(6\pi t)}{\pi t} dt + \int_0^{\infty} \frac{\sin(2\pi t)}{\pi t} dt\right\}$$

$$I = I_1 + I_2$$

$$I_1 = 3x \int_0^{\infty} \frac{\sin 6\pi t}{\pi t} dt$$

Let when

$$6\pi t = u \qquad t = 0, u = 0$$

$$6\pi dt = du \qquad t = \infty, u = \infty$$

$$dt = \frac{du}{6\pi}$$

$$\therefore I_1 = 3\int_0^{\infty} \frac{\sin u}{u/6} \cdot \frac{du}{6\pi} = \frac{3}{\pi} \int_0^{\infty} \frac{\sin u}{u} du$$

We know that $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$

$$\therefore I_1 = \frac{3}{\pi} \int_0^{\infty} \frac{\sin u}{u} dv = \frac{3}{\pi} \times \frac{\pi}{2} = \frac{3}{2}$$

Now, $I_2 = 3 \int_0^{\infty} \frac{\sin 2\pi t}{\pi t} dt$

Let when

$$6\pi t = v \qquad t = 0, v = 0$$

$$6\pi dt = dv \qquad t = \infty, v = \infty$$

$$dt = \frac{dv}{2\pi}$$

$$\therefore I_2 = 3\int_0^{\infty} \frac{\sin v}{v/2} \cdot \frac{dv}{2\pi} = \frac{3}{\pi} \int_0^{\infty} \frac{\sin v}{v} dv$$

Similarly,

$$I_2 = \frac{3}{\pi} \int_0^{\infty} \frac{\sin v}{v} dv = \frac{3}{\pi} \times \frac{\pi}{2} = \frac{3}{2}$$

$$\therefore I = I_1 + I_2 = \frac{3}{2} + \frac{3}{2} = 3$$

Scan for Video solution

14. (b)

$$L\{e^{i5t}\} = L\{\cos 5t + i \sin 5t\}$$
$$= L\{\cos 5t\} + i\, L\{\sin 5t\}$$
$$= \frac{s}{s^2+5^2} + i\frac{5}{s^2+5^2}$$
$$\left(\because L\{\cos at\} = \frac{s}{s^2+a^2}, L\{\sin at\} = \frac{a}{s^2+a^2}\right)$$
$$= \frac{s+i5}{s^2+25}$$
$$\boxed{\therefore L\{e^{i5t}\} = \frac{s+5i}{s^2+25}}$$

Scan for Video solution

15. (c)

From the given graph,

$$f(t) = \begin{cases} 2; & 0 < t < 1 \\ 0; & t > 1 \end{cases}$$
$$\therefore \quad L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\,at$$
$$= \int_0^1 e^{-st} \cdot 2 \cdot dt + \int_1^\infty e^{-st} \cdot (0) \cdot dt$$
$$\Rightarrow \quad L\{f(t)\} = 2 \cdot \left.\frac{e^{-st}}{-s}\right|_0^1 = \frac{-2}{s}\{e^{-3} - 1\} = \frac{2}{s}(1 - e^{-s})$$
$$\Rightarrow \quad L\{f(t)\} = \frac{2-2e^{-s}}{s}$$

Scan for Video solution

16. (d)

$$L\{\cos \omega t\} = \frac{s}{s^2+\omega^2}$$

By first shift property,

$$L\left(e^{at} \cdot \cos \omega t\right) = \frac{(s-a)}{(s-a)^2+\omega^2}$$

Substitute a = – 2 & ω = 4

$$\Rightarrow \quad L\{e^{-2t} \cdot \cos 4t\} = \frac{(s-(-2))}{(s-(-2))^2+4^2} = \frac{(s+2)}{(s+2)^2+16}$$
$$\Rightarrow \quad \boxed{L\{e^{-2t} \cdot \cos 4t\} = \frac{s+2}{(s+2)^2+16}}$$

Scan for Video solution

17. (c)

Given: $\frac{d^2 f}{dt^2} + f = 0; \quad f(0) = 0, \frac{df}{dt}(0) = 4$

The auxiliary equation is $m^2 + 1 = 0$

$\Rightarrow$ $m = \pm i$

The solution is $f(t) = c_1 \cos t + c_2 \sin t$

Given $f(0) = 0 \Rightarrow 0 = c_1 \cos 0 + c_2 \sin 0 \Rightarrow c_1 = 0$

$\therefore$ $f(t) = c_2 \sin t$

$$\Rightarrow \quad \frac{d\,f(t)}{dt} = c_2 \cos t$$

Given $\frac{df}{dt}(0) = 4 \Rightarrow 4 = c_2.(1) \Rightarrow c_2 = 4$

$\therefore$ $f(t) = 4. \text{Sin } t$

$$\Rightarrow \quad L\{f(t)\} = 4.\, L\{\sin t\} = 4 \cdot \left(\frac{1}{s^2+1^2}\right)$$
$$= \frac{4}{s^2+1} \qquad \left(\because L(\sin at) = \frac{a}{s^2+a^2}\right)$$
$$\therefore \quad \boxed{L\{f(t)\} = \frac{4}{s^2+1}}$$

Scan for Video solution

18. (d)

Given: $F(s) = \frac{1}{s(s+1)}$

$$\Rightarrow \quad F(s) = \frac{1}{s} - \frac{1}{s+1}$$

Applying inverse Laplace on both sides

$$\Rightarrow L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{1}{s+1}\right\} = 1 - e^{-t}$$
$$\left(\because L\{t^n\} = \frac{n!}{s^{n+1}}; L\{e^{at}\} = \frac{1}{s-a}\right)$$
$$\Rightarrow \quad \boxed{f(t) = 1 - e^{-t}}$$

Scan for Video solution

19. (a)

Given:

$$\bar{F}(s)=\frac{1}{s^2(s+1)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}$$

$$\Rightarrow \quad \bar{F}(s)=\frac{A\,s(s+1)+B(s+1)+C\,s^2}{s^2(s+1)}=\frac{1}{s^2(s+1)}$$

$\Rightarrow$ A + C = 0; A + B = 0; B = 1

$\Rightarrow$ A = –1; B = 1; C = 1

$$\Rightarrow \quad \frac{1}{s^2(s+1)}=\frac{-1}{s}+\frac{1}{s^2}+\frac{1}{(s+1)}$$

Applying inverse Laplace on both sides

$$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\}=L^{-1}\left\{\frac{-1}{s}\right\}+L^{-1}\left\{\frac{1}{s^2}\right\}+L^{-1}\left\{\frac{1}{(s+1)}\right\}$$

$$L^{-1}\left\{\frac{1}{s^2(s+1)}\right\}=-1+t+e^{-t}$$

$$\left\{\because L\{t^n\}=\frac{n!}{s^{n+1}}\right\}$$

$$\Rightarrow \quad \boxed{f(t)=-1+t+e^{-t}}$$

$$\left\{\because L\{e^{at}\}=\frac{1}{s-a}\right\}$$

Scan for Video solution

20. (c)

Given: $\bar{F}(S)=\frac{1}{(S^2+S)}=\frac{1}{S(S+1)}=\frac{1}{S}-\frac{1}{S+1}$

$$\Rightarrow \quad \bar{F}(S)=\frac{1}{S}-\frac{1}{S+1}$$

$$\therefore \quad L^{-1}\{\vec{F}(S)\}=L^{-1}\left\{\frac{1}{S}\right\}-L^{-1}\left\{\frac{1}{S+1}\right\}$$

$$=L^{-1}\left\{\frac{1}{S-0}\right\}-L^{-1}\left\{\frac{1}{S-(-1)}\right\}$$

$$=e^{0.t}-e^{-1.t}=1-e^{-t}$$

$$\left\{\because L\left(e^{at}\right)=\frac{1}{S-a}\right\}$$

$$\Rightarrow \quad \boxed{L^{-1}\left\{\frac{1}{S^2+S}\right\}=1-e^{-t}}$$

Scan for Video solution

21. (b)

$$\cosh ax=\frac{e^{ax}+e^{-ax}}{2}$$

Taking Laplace transform, we get

$$=\frac{1}{2}L\{e^{ax}\}+L\{e^{-ax}\}$$

$$=\frac{1}{2}\left[\frac{1}{s-a}+\frac{1}{s+a}\right]$$

$$\frac{1}{2}\left[\frac{2s}{s^2-a^2}\right]$$

$$L(\cosh ax)=\frac{s}{s^2-a^2}$$

Scan for Video solution

22. (a)

Given

$$\frac{d^2y}{dt^2}-y=1; \qquad y(0)=0 \ \ \& \ \ \left.\frac{dy}{dt}\right|_{t=0}=0$$

Applying Laplace transform on both sides of DE.

$$\Rightarrow \quad \left(s^2\cdot F(s)-s\cdot y(0)-y'(0)\right)-F(s)=\frac{1}{s}$$

$$\Rightarrow \quad (s^2-1)F(s)=\frac{1}{s}$$

$$\Rightarrow \quad F(s)=\frac{1}{s(s^2-1)}$$

$$\Rightarrow \quad F(s) = \frac{1}{s(s-1)(s+1)}$$

Scan for Video solution

23. (d)

Let $f(t) = \frac{\sin 2\pi t}{\pi t}$

$f(t) = f(-t) = \frac{\sin 2\pi t}{t}$

$\therefore$ f(t) is even

$$I = 2\int_{-\infty}^{\infty} \frac{\sin 2\pi t}{\pi t} = 4\int_0^{\infty} \frac{\sin 2\pi t}{\pi t} = \frac{4}{\pi}\boxed{\int_0^{\infty} \frac{\sin 2\pi t}{t}}$$

From Laplace transform $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} f(s)ds$

We know, $\int_0^{\infty} \frac{\sin at}{t} = \frac{\pi}{2}$

$\therefore \quad \int_0^{\infty} \frac{\sin 2\pi t}{t} = \frac{\pi}{2}$

Hence, $I = \frac{4}{\pi}.\frac{\pi}{2} = 2$

Scan for Video solution

24. (b)

Using initial value theorem,

$x(0^+) = \lim_{s\to\infty} s\,X(s)$

$$= \lim_{s\to\infty} s\left(\frac{3s+5}{s^2+10s+21}\right)$$

$$\lim_{s\to\infty} \frac{3+\frac{5}{s}}{1+\frac{10}{s}+\frac{21}{s^2}} = 3$$

Scan for Video solution

25. (d)

Given data, $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$

$y(t)\big|_{t=0} = -2 \Rightarrow y(0) = -2$

$\frac{dy}{dt}\Big|_{t=0} = 0 \Rightarrow y'(0) = 0$

Differential equation can be written as:-

$y'' + 2y' + y = \delta(t)$

Taking Laplace transform on both sides

$$\left[s^2Y(s) - sy(0) - y'(0)\right] + 2[sY(s) - y(0)] + Y(s) = 1$$

$$[s^2 + 2s + 1]Y(s) + 2s + 4 = 1$$

$$Y(s) = \frac{-3-2s}{(s+1)^2}$$

$$Y(s) = -\frac{3}{(s+1)^2} - 2\left[\frac{(s+1)-1}{(s+1)^2}\right]$$

$$Y(s) = -\frac{3}{(s+1)^2} - \frac{2}{(s+1)} + \frac{2}{(s+1)^2} = -\frac{1}{(s+1)^2} - \frac{2}{s+1}$$

$$y(t) = L^{-1}Y(s) = L^{-1}\left[-\frac{1}{(s+1)^2} - \frac{2}{(s+1)}\right]$$

$y = -te^{-t} - 2e^{-t}$

$\frac{dy}{dt} = te^{-t} - e^{-t} + 2e^{-t}$

$= e^{-t}(t - 1 + 2)$

$\frac{dy}{dt} = e^{-t}(t+1)$

At t = 0; $\boxed{\frac{dy}{dt} = 1}$

Scan for Video solution

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CHAPTER

9

NUMERICAL METHODS

Numerical Solutions of Linear Equations

1. **[NAT]** **[GATE-ME-2016:2M]**
Gauss-Seidel method is used to solve the following equations (as per the given order):

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 3$$

Assuming initial guess as $x_1 = x_2 = x_3 = 0$, the value of x_3 after the first iteration is ___________

2. **[MCQ]** **[GATE- CE-2009:2M]**
In the solution of the following set of linear equations by Gauss elimination using partial pivoting $5x + y + 2z = 34$: $4y - 3z = 12$: and $10x - 2y + z = -4$: the pivots for elimination of x and y are
(a) 10 and 4
(b) 10 and 2
(c) 5 and 4
(d) 5 and – 4

Numerical Solution of Algebraic & Transcendental Equations

3. **[NAT]** **[GATE- CE-2022:1M]**
Consider the following recursive iteration scheme for different values of variable P with the initial guess

$$x_1 = 1: \; x_{n+1} = \frac{1}{2}\left(x_n + \frac{P}{x_n}\right), n = 1, 2, 3, 4, 5$$

For P = 2, x_5 is obtained to be 1.414, rounded off to three decimal places. For P = 3, x_5 is obtained to be 1.732, rounded-off to three decimal places.
If P = 10, the numerically value of x_5 is________(*round off to three decimal place*)

4. **[NAT]** **[GATE-ME-2021:2M]**
Find the positive real root of $x^3 - x - 3 = 0$ using Newton-Raphson method. If the starting guess (x_0) is 2, the numerical value of the root after two iterations (x_2) is ______ (*round off to two decimal places*).

5. **[NAT]** **[GATE- CE-2018:1M]**
The quadratic equation $2x^2 - 3x + 3 = 0$ is to be solved numerically starting with an initial guess as $x_0 = 2$. The new estimate of x after the first iteration using Newton-Raphson method is ___________.

6. **[NAT]** **[GATE-EE-2017: 2M]**
Only one of the real roots f (x) = $x^6 - x - 1$ lies in the interval $1 \le x \le 2$ and bisection method is used to find its value. For achieving an accuracy of 0.001, the required minimum number of iterations is _____.

7. **[NAT]** **[GATE-EE-2017 : 1M]**
Let $y^2 - 2y + 1 = x$ and $\sqrt{x} + y = 5$. The value of $x + \sqrt{y}$ equals ____. (*Given the answer up to three decimal*)

8. **[NAT]** **[GATE-EC-2017:2M]**
Starting with x = 1. the solution of the equation $x^3 + x = 1$, after two iterations of Newton-Raphson's method (*up to two decimal places*) is _____.

9. **[NAT]** **[GATE- CE-2016-:1M]**
Newton-Raphson method is to be used to find root of equation $3x - e^x + \sin x = 0$. If the initial trial value for the roots is taken as 0.333, the next approximation for the root would be_________ (*answer up to three decimal places*).

10. [NAT] [GATE-ME-2016:1M]

Solve the equation x = 10 cos(x) using the Newton-Raphson method. The initial guess is $x = \pi/4$. The value of the predicted root after the first iteration, up to second decimal, is ________

11. [MCQ] [GATE-ME-2016:1M]

The root of the function $f(x) = x^3 + x - 1$ obtained after first iteration on application of Newton-Raphson scheme using an initial guess of $x_0 = 1$ is

(a) 0.682 (b) 0.686
(c) 0.750 (d) 1.000

12. [NAT] [GATE-ME-2015:2M]

Newton-Raphson method is used to find the roots of the equation, $x^3 + 2x^2 + 3x - 1 = 0$. If the initial guess is $x_0 = 1$, then the value of x after 2nd iteration is_____

13. [NAT] [GATE-EC-2015:2M]

The Newton-Raphson method is used to solve the equation $f(x) = x^3 - 5x^2 + 6x - 8 = 0$. Taking the initial guess as x = 5, the solution obtained at the end of the first iteration is________.

14. [NAT] [GATE-CE-2015:2M]

The quadratic equation $x^2 - 4x + 4 = 0$ is to be solved numerically, starting with the initial guess $x_0 = 3$. The Newton-Rapshon method is applied once to get a new estimated and then the Secant method is applied once using the initial guess and this new estimate. The estimated value of the root after the application of the Secant method is_________.

15. [NAT] [GATE-CE-2015:1M]

In Newton-Raphson iterative method, the initial guess value (x_{ini}) is considered as zero while finding the roots of the equation: $f(x) = -2 + 6x - 4x^2 + 0.5x^3$.

The correction, Δx, to be added to x_{ini} in the first iteration is _______.

16. [NAT] [GATE-ME-2014:2M]

The real root of the equation $5x - 2\cos x - 1 = 0$ (up to two decimal accuracy) is _______

17. [NAT] [GATE-EE-2014:2M]

The function $f(x) = e^x - 1$ is to be solved using Newton-Raphson method. If the initial value of x_0 is taken as 1.0, then the absolute error observed at 2nd iteration is ______.

18. [MCQ] [GATE-EE-2013:2M]

When the Newton-Raphson method is applied to solve the equation $f(x) = x^3 + 2x - 1 = 0$, the solution at the end of the first iteration with the initial guess value as $x_0 = 1.2$ is

(a) –0.82 (b) 0.49
(c) 0.705 (d) 1.69

19. [MCQ] [GATE-EE-2011:1M]

Solution of the variables x_1 and x_2 for the following equations is to be obtained by employing the Newton-Raphson iterative method.

Equation(i) $10x_2 \sin x_1 - 0.8 = 0$

Equation(ii) $10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0$

Assuming the initial values $x_1 = 0.0$ and $x_2 = 1.0$, the Jacobian matrix is

(a) $\begin{bmatrix} 10 & -0.8 \\ 0 & -0.6 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -0.8 \\ 10 & -0.6 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 0 \\ 10 & -10 \end{bmatrix}$

20. [MCQ] [GATE-EC-2011:2M]

A numerical solution of the equation $f(x) = x + \sqrt{x} - 3 = 0$ can be obtained using Newton Raphson method. If the starting value is x = 2 for the iteration, the value of x that is to be used in the next step is

(a) 0.306 (b) 0.739
(c) 1.694 (d) 2.306

21. [MCQ] [GATE- CE-2011:1M]

The square root of a number N is to be obtained by applying the Newton Raphson iterations to the equation $x^2 - N = 0$. If i denotes the iteration index, the correct iterative scheme will be

(a) $x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$ (b) $x_{i+1} = \frac{1}{2}\left(x_i^2 + \frac{N}{x_i^2}\right)$

(c) $x_{i+1} = \frac{1}{2}\left(x_i + \frac{N^2}{x_i}\right)$ (d) $x_{i+1} = \frac{1}{2}\left(x_i - \frac{N}{x_i}\right)$

22. [MCQ] [GATE-EE-2009:2M]

Let $x^2 - 117 = 0$. The iterative steps for the solution using Newton-Raphson's Method is given by

(a) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

(b) $x_{k+1} = x_k - \frac{117}{x_k}$

(c) $x_{k+1} = x_k - \frac{x_k}{117}$

(d) $x_{k+1} = x_k - \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

23. [MCQ] [GATE-EE-2008:2M]

Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then, after one step of Newton's method, estimate x_1 the solution will be given by

(a) 0.71828 (b) 0.36784
(c) 0.20587 (d) 0.00000

24. [MCQ] [GATE-EC-2008:2M]

The recursion relation to solve $x = e^{-x}$ using Newton-Raphson method is

(a) $x_{n+1} = e^{-x_n}$

(b) $x_{n+1} = x_n - e^{-x_n}$

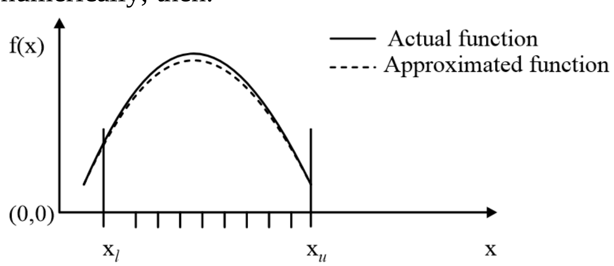
(c) $x_{n+1} = (1 + x_n)\frac{e^{-x_n}}{1 + e^{-x_n}}$

(d) $x_{n+1} = \frac{x_n^2 - e^{-x_n}(1 + x_n) - 1}{x_n - e^{-x_n}}$

Numerical Integration

25. [MCQ] [GATE- CE-2023:1M]

A function $f(x)$, that is smooth and convex-shaped between interval (x_l, x_u) is shown in the figure. This function is observed at odd number of regularly spaced points. If the area under the function is computed numerically, then.



(a) the numerical value of the area obtained using the trapezoidal rule will be less than the actual

(b) the numerical value of the area obtained using the trapezoidal rule will be more than the actual

(c) the numerical value of the area obtained using the trapezoidal rule will be exactly equal to the actual

(d) with the given details, the numerical value of area cannot be obtained using trapezoidal rule

26. [MCQ] [GATE-ME-2022:1M]

Consider the definite integral $\int_1^2 \left(4x^2 + 2x + 6\right)dx$.

Let I_e be the exact value of the integral. If the same integral is estimated using Simpson's rule with 10 equal subintervals, the value is I_s. The percentage error is defined as $e = 100 \times (I_e - I_s)/I_e$. The value of e is

(a) 2.5 (b) 3.5
(c) 1.2 (d) 0

27. [NAT] [GATE-CE-2021:2M]

The values of abscissa (x) and ordinate (y) of a curve are as follows:

X	Y
2.0	5.00
2.5	7.25
3.0	10.00
3.5	13.25
4.0	17.00

By Simpson's 1/3rd rule, the area under the curve (*round off to two decimal places*) is ___________.

28. [MCQ] [GATE-CE-2021:2M]

The value of $\int_0^1 e^x dx$ using the trapezoidal rule with four equal subintervals is

(a) 1.718 (b) 2.192
(c) 1.727 (d) 2.718

29. [NAT] [GATE- CE-2021:2M]

Numerically integrate, $f(x) = 10x - 20x^2$ from lower limit a = 0 to upper limit b = 0.5. Using Trapezoidal rule with five equal subdivisions. The value (*in units, round off to two decimal places*) obtained is_______.

30. [MCQ] [GATE-ME-2021:1M]

Value of $\int_4^{5.2} \ln x \, dx$ using Simpson's one-third rule with interval size 0.3 is

(a) 1.83 (b) 1.60
(c) 1.51 (d) 1.06

31. [MCQ] [GATE-ME-2020:2M]

The evaluation of the definite integral $\int_{-1}^{1.4} x|x| \, dx$ by using Simpson's 1/3rd (one-third) rule with step size $h = 0.6$ yields.

(a) 0.914 (b) 1.248
(c) 0.581 (d) 0.592

32. [MCQ] [GATE-CE-2020:1M]

The integral $\int_0^1 (5x^3 + 4x^2 + 3x + 2)dx$ is estimated numerically using three alternative methods namely the rectangular, trapezoidal and Simpson's rules with a common step size. In this context, which one of the following statements is TRUE?

(a) Simpson's rule as well as rectangular rule of estimation will give non-zero error.
(b) Only Simpson's rule of estimation will give zero error.
(c) Simpson's rule, rectangular rule as well as trapezoidal rule of estimation will give non-zero error.
(d) Only the rectangular rule of estimation will give zero error.

33. [NAT] [GATE-ME-2020:2M]

For the integral $\int_0^{\pi/2} (8 + 4\cos x) dx$,the absolute percentage error in numerical evaluation with the Trapezoidal rule, using only the end points, is__________ (*round off to one decimal place*).

34. [NAT] [GATE-ME-2019:1M]

Evaluation of $\int_2^4 x^3 dx$ using a 2-equal-segment trapezoidal rule gives a value of ____

35. [MCQ] [GATE-ME-2017:2M]

P(0, 3), Q(0.5, 4), and R(1, 5) are three points on the curve defined by $f(x)$. Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits x = 0 and x = 1 for the curve. The difference between the two results will be

(a) 0 (b) 0.25
(c) 0.5 (d) 1

36. [MCQ] [GATE-ME-2016:1M]

Numerical integration using trapezoidal rule gives the best result for a single variable function, which is

(a) Linear (b) Parabolic
(c) Logarithmic (d) Hyperbolic

37. [NAT] [GATE-ME-2016:2M]

The error in numerically computing the integral $\int_0^{\pi} (\sin x + \cos x) dx$ using the trapezoidal rule with three intervals of equal length between 0 and π is ____.

38. [NAT] [GATE-ME-2015:1M]

Simpson's $\frac{1}{3}$ rule is used to integrate the function $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$ between x = 0 and x = 1 using the least number of equal sub-intervals. The value of the integral is ____.

39. [MCQ] [GATE-CE-2015:1M]

The integral $\int_{x_1}^{x_2} x^2 dx$ with $x_2 > x_1 > 0$ is evaluated analytically as well as numerically using a single application of the trapezoidal rule. If I is the exact value of the integral obtained analytically and J is the approximate value obtained using the trapezoidal rule, which of the following statements is correct about their relationship?

(a) J > I
(b) J < I
(c) J = I
(d) Insufficient data to determine the relationship

40. [NAT] [GATE-CE-2015:2M]

For step-size, Δx = 0.4, the value of following integral using Simpson's 1/3 rule is __________.

$$\int_0^{0.8} \left(0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5\right) dx$$

41. [NAT] [GATE-ME-2015:2M]

The value of function $f(x)$ at 5 discrete points are given below:

x	0	0.1	0.2	0.3	0.4
$f(x)$	0	10	40	90	160

Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x) dx$ is ____

42. [NAT] [GATE-ME-2014:1M]

Using a unit step size, the value of integral $\int_1^2 x \ln x \, dx$ by trapezoidal rule is ________

43. [NAT] [GATE-ME-2014:1M]

The definite integral $\int_1^3 \frac{1}{x} dx$ is evaluated using Trapezoidal rule with a step size of 1. The correct answer is _______

44. [NAT] [GATE-ME-2014:2M]

The value of $\int_{2.5}^4 \ln(x) dx$ calculated using the Trapezoidal rule with five subintervals is _____

45. [NAT] [GATE-ME-2014:2M]

Using the trapezoidal rule, and dividing the interval of integration into three equal subintervals, the definite integral $\int_{-1}^{+1} |x| dx$ is _____

46. [NAT] [GATE-CE-2013:2M]

Find the magnitude of error (Correct to two decimal places) in the estimation of following integral using Simpson 1/3 rule. Take the step length as 1

$$\int_0^4 \left(x^4 + 10\right) dx$$

47. [MCQ] [GATE-ME-2013:1M]

Match the CORRECT pairs

Numerical Integration Scheme	Order of Fitting Polynomial
P. Simpson's $3/8$ rule	1. First
Q. Trapezoidal rule	2. Second
R. Simpson's $1/3$ rule	3. Third

(a) P-2, Q-1, R-3
(b) P-3, Q-2, R-1
(c) P-1, Q-2, R-3
(d) P-3, Q-1, R-2

48. [MCQ] [GATE-CE-2011:2M]

The estimated of $\int_{0.5}^{1.5} \frac{dx}{x}$ Obtained using Simpson's rule with three-point function evaluation exceeds the exact value by

(a) 0.235
(b) 0.068
(c) 0.024
(d) 0.012

49. [MCQ] [GATE-ME-2011:2M]

The integral $\int_1^3 \frac{1}{x} dx$, when evaluated by using Simpson's 1/3 rule on two equal subintervals each of length 1, equals

(a) 1.000
(b) 1.098
(c) 1.111
(d) 1.120

50. [MCQ] [GATE-ME-2010:2M]

Torque exerted on a flywheel over a cycle is listed in the table. Flywheel energy (in J per unit cycle) using Simpson's rule is

Angle (degree)	0	60	12	180	240	300	360
Torque (N m)	0	1066	–323	0	323	–355	0

(a) 542
(b) 993
(c) 1444
(d) 1986

51. [MCQ] [GATE-CE-2010:2M]

The table below gives values of a function F(x) obtained for values of x at intervals of 0.25.

The value of the integral of the function between the limits 0 to 1 using Simpson's rule is

x	0	0.25	0.5	0.75	1.0
F(x)	1	0.9412	0.8	0.64	0.50

(a) 0.7854
(b) 2.3562
(c) 3.1416
(d) 7.5000

Numerical Solution of Differential Equations

52. [MCQ] [GATE-CE-2023:2M]

The differential equation, $\frac{du}{dt} + 2tu^2 = 1$, is solved by employing a backward difference scheme within the finite difference framework. The value of u at the $(n-1)^{th}$ time-step, for some n, is 1.75. The corresponding time (t) is 3.14 s. Each time step is 0.01 s long. Then, the value of $(u_n - u_{n-1})$ is _______ (*round off to three decimal places*).

53. [NAT] [GATE-ME-2023:2M]

The initial value problem

$$\frac{dy}{dt} + 2y = 0, \quad y(0) = 1$$

is solved numerically using the forward Euler's method with a constant and positive time step of Δt. Let y_n represent the numerical solution obtained after n steps. The condition $|y_{n+1}| - |y_n|$ is satisfied if and only if Δt does not exceed _____. (*Answer in integer*)

54. [NAT] [GATE-CE-2022:2M]

Consider the differential equation $\frac{dy}{dx} = 4(x+2) - y$

For the initial condition y = 3 at x = 1, the value of y at x =1.4 obtained using Euler's method with a step-size of 0.2 is__________.(*round off to one decimal place*)

55. [MCQ] [GATE-ME-2022:1M]

A polynomial $\varphi(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$ of degree $n > 3$ with constant real coefficients a_n, $a_n - 1$, …a_0 has triple roots at $s = -\sigma$. Which one of the following conditions must be satisfied?

(a) $\phi(s) = 0$ at all the three values of s satisfying $s^3 + \sigma^3 = 0$

(b) $\phi(s) = 0, \frac{d\phi(s)}{ds} = 0$ and $\frac{d^2\phi(s)}{ds^2} = 0$ at $s = -\sigma$

(c) $\phi(s) = 0, \frac{d^2\phi(s)}{ds^2} = 0$, and $\frac{d^4\phi(s)}{ds^4} = 0$, at $s = -\sigma$

(d) $\phi(s) = 0$ and $\frac{d^3\phi(s)}{ds^3} = 0$, at $s = -\sigma$

56. [MCQ] [GATE-ME-2021:1M]

The ordinary differential equation $\frac{dy}{dt} = -\pi y$ subjected to an initial condition *y* (0) = 1 is solved numerically using the following scheme:

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

where h is the time step, $t_n = nh$, and $\pi = 0, 1, 2, \ldots$ This numerical scheme is stable for all values of *h* in the interval _____.

(a) $0 < h < \frac{2}{\pi}$ (b) $0 < h < 1$

(c) $0 < h < \frac{\pi}{2}$ (d) for all $h > 0$

57. [MCQ] [GATE- CE-2020:2M]

A continuous function f(x) is defined. If the thrid derivative at x_i is to be computed by using the fourth order central finite-divided-difference scheme (with step length = h), the correct formula is

(a) $f'''(x_i) = \frac{-f(x_{i+3}) + 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) + f(x_{i-3})}{8h^3}$

(b) $f'''(x_i) = \frac{f(x_{i+3}) - 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) + 8f(x_{i-2}) + f(x_{i-3})}{8h^3}$

(c) $f'''(x_i) = \frac{-f(x_{i+3}) - 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) + 8f(x_{i-2}) - f(x_{i-3})}{8h^3}$

(d) $f'''(x_i) = \frac{f(x_{i+3}) - 8f(x_{i+2}) + 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) - f(x_{i-3})}{8h^3}$

58. [NAT] [GATE-ME-2018:2M]

An explicit forward Euler method is used to numerically integrate the differential equation

$$\frac{dy}{dt} = y$$

using a time step of 0.1. With the initial condition *y* (0) = 1, the value of *y* (1) computed by this method is ______ (*Correct to two decimal places*).

59. [NAT] [GATE-CE-2017:2M]

Consider the equation $\frac{du}{dt} = 3t^2 + 1$ with u = 0 at t = 0. This is numerically solved by using the forward Euler method with a step size. $\Delta t = 2$. The absolute error in the solution in the end of the first-time step is

60. [NAT] [GATE-EC-2016:1M]

Consider the first order initial value problem $y' = y + 2x - x^2$, y(0) = 1, $(0, \leq x < \infty)$ with exact solutions $y(x) = x^2 + e^x$. For x = 0.1, the percentage difference between the exact solution and the solution obtained using a single iteration of the second-order Runge Kutta method with step-size h = 0.1 is _______.

61. [NAT] [GATE-EC-2016:2M]

The ordinary differential equation

$$\frac{dx}{dt} = -3x + 2; \text{ with } x(0) = 1$$

is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is __________.

62. [MCQ] [GATE-ME-2014:2M]

Consider an ordinary differential equation $\frac{dx}{dt} = 4t + 4$.

If $x = x_0$ at t = 0, the increment in *x* calculated using Runge-Kutta fourth order multi-step method with a step size of $\Delta t = 0.2$ is

(a) 0.22 (b) 0.44

(c) 0.66 (d) 0.88

63. MCQ [GATE-EC-2014:1M]

Match the application to appropriate numerical method.

Application

P1: Numerical integration

P2: Solution to a transcendental equation

P3: Solution to a system of linear equation

P4: Solution to a differential equation

Numerical Method

M1: Newton-Raphson Method

M2: Runge-Kutta Method

M3: Simpson's 1/3-rule

M4: Gauss Elimination Method

(a) P1-M3, P2-M2, P3-M4, P4-M1
(b) P1-M3, P2-M1, P3-M4, P4-M2
(c) P1-M4, P2-M1, P3-M3, P4-M2
(d) P1-M2, P2-M1, P3-M3, P4-M4

64. [MCQ] [GATE-CE-2012:2M]

The error in $\frac{d}{dx}f(x)\Big|_{x=x_0}$ for a continuous function estimated with h = 0.03 using the central difference formula $\frac{d}{dx}f(x)\Big|_{x=x_0} \approx \frac{f(x_0+h)-f(x_0-h)}{2h}$, is 2×10^{-3}. The value of x_0 and $f(x_0)$ are 19.78 and 500.01, respectively. The corresponding error in the central difference estimate for h = 0.02 is approximately

(a) 1.3×10^{-4} (b) 4.5×10^{-4}
(c) 3.0×10^{-4} (d) 9.0×10^{-4}

65. [MCQ] [GATE-EC-2010: 2M]

Consider a differential equation $\frac{dy(x)}{dx} - y(x) = x$ with the initial condition y (0) = 0. Using Euler's first order method with a step size of 0.1 the value of y (0.3) is

(a) 0.01 (b) 0.031
(c) 0.0631 (d) 0.1

66. [MCQ] [GATE-EE-2008:2M]

A differential equation $\frac{dx}{dt} = e^{-2t}u(t)$ has to be solved using trapezoidal rule of integration with a step size h = 0.01s. Function $u(t)$ indicates a unit step function. If $x(0^-) = 0$, then value of x at t = 0.01s will be given by

(a) 0.00099 (b) 0.00495
(c) 0.0099 (d) 0.0198

Interpolation

67. [MCQ] [GATE-CE-2019:1M]

The value of the function f(x) is given at n distinct value of x and its value is to be interpolated at the point x*, using all the-n points. The estimate is obtained first by the Lagrange polynomial denoted by I_L and then by the Newton polynomial, denoted by I_N. Which one of the following statements is correct?

(a) I_L is always greater than I_N
(b) No definite relation exists between I_L and I_N
(c) I_L and I_N are always equal
(d) I_L is always less than I_N

❑❑❑

ANSWER KEY

1.	(–6 to –6)	**2.**	(a)	**3.**	(3.100 to 3.200)	**4.**	(1.66 to 1.68)
5.	(1 to 1)	**6.**	(10 to 10)	**7.**	(5.7 to 5.8)	**8.**	(0.65 to 0.72)
9.	(0.355 to 0.365)	**10.**	(1.53 to 1.59)	**11.**	(c)	**12.**	(0.29 to 0.31)
13.	(4.25 to 4.35)	**14.**	(2.32 to 2.34)	**15.**	(0.3 to 0.4)	**16.**	(0.53 to 0.56)
17.	(0.05 to 0.07)	**18.**	(c)	**19.**	(b)	**20.**	(c)
21.	(a)	**22.**	(a)	**23.**	(a)	**24.**	(c)
25.	(a)	**26.**	(d)	**27.**	(20.00 to 21.00)	**28.**	(c)
29.	(0.38 to 0.42)	**30**	(a)	**31.**	(d)	**32.**	(b)
33.	(5.1 to 5.5)	**34.**	(63 to 63)	**35.**	(a)	**36.**	(a)
37.	(0.175 to 0.195)	**38.**	(2 to 2)	**39.**	(a)	**40.**	(1.36 to 1.37)
41.	(21.8 to 22.2)	**42.**	(0.68 to 0.70)	**43.**	(1.1 to 1.2)	**44.**	(1.74 to 1.76)
45.	(1.10 to 1.12)	**46.**	(0.50 to 0.53)	**47.**	(d)	**48.**	(d)
49.	(c)	**50.**	(b)	**51.**	(a)	**52.**	(–0.152 to –0.149)
53.	(0.999 to 1.001)	**54.**	(6.3 to 6.5)	**55.**	(b)	**56.**	(a)
57.	(a)	**58.**	(2.55 to 2.65)	**59.**	(7.95 to 8.05)	**60.**	(0.060 to 0.063)
61.	(0.6 to 0.7)	**62.**	(d)	**63.**	(b)	**64.**	(d)
65.	(b)	**66.**	(c)	**67.**	(c)		

SOLUTIONS

1. **(–6 to –6)**

Given System is $x_1 + 2x_2 + 3x_3 = 5$

$2x_1 + 3x_2 + x_3 = 1$

$3x_1 + 2x_2 + x_3 = 3$

Taking equation in same order

$$x_1^{(k+1)} = 5 - 2x_2^{(k)} - 3x_3^{(k)}$$

$$x_2^{(k+1)} = \frac{1}{3}\left\{1 - 2x_1^{(k+1)} - x_3^{(k)}\right\}$$

$$x_3^{(k+1)} = 3 - 3x_1^{(k+1)} - 2x_2^{(k+1)}$$

since $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$

$\Rightarrow$ $x_1^{(1)} = 5 - 2(0) - 3(0) = 5$

$$x_2^{(1)} = \frac{1}{3}\{1 - 2(5) - 0\} = -3$$

$$x_3^{(1)} = 3 - 3(5) - 2(-3)$$

$\Rightarrow$ $x_3^{(1)} = 3 - 15 + 6 = -6$

$\therefore$ x_3 after 1st iteration = –6

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2. **(a)**

The equation are, $5x + y + 2z = 34$

$4y - 3z = 12$ and $10x - 2y + z = -4$

The augmented matrix for gauss elimination is,

$$(A / B) = \left[\begin{array}{ccc|c} 5 & 1 & 2 & 34 \\ 0 & 4 & -3 & 12 \\ 10 & -2 & 1 & -4 \end{array}\right]$$

Since in the first column maximum element in absolute value is10 we need to exchange row 1 with row 3.

$$\left[\begin{array}{ccc|c} 5 & 1 & 2 & 34 \\ 0 & 4 & -3 & 12 \\ 10 & -2 & 1 & 4 \end{array}\right] \xleftrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 10 & -2 & 1 & -4 \\ 0 & 4 & -3 & 12 \\ 5 & 1 & 2 & 34 \end{array}\right]$$

So, the pivot for eliminating x is $a_{11} = 10$.

- Now to eliminate y, we need to eliminate in second column below the diagonal.

  Since $a_{22} = 4$ is already larger in absolute value when compared to $a_{32} = 1$

$\therefore$ The pivots element for eliminating y is $a_{22} = 4$ itself

$\therefore$ The pivots for eliminating x and y are 10 and 4 respectively.

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3. **(3.100 to 3.200)**

We know, $x_{n+1} = x_n = x$; when $n \to \infty$

Recursive iteration formula

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{P}{x_n}\right)$$

$$x = \frac{1}{2}\left(x + \frac{P}{x}\right) \quad \Rightarrow \quad \frac{x}{2} = \frac{P}{2x}$$

$$x^2 = P \quad \Rightarrow \quad x = \sqrt{P}$$

$\therefore$ If P = 10, then $x_5 = \sqrt{P} = \sqrt{10} = 3.162$

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4. **(1.66 to 1.68)**

Given

$x^3 - x - 3 = 0$; and $x_0 = 2$

For $f(x) = 0$, Newton-Raphson method is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{(x_k^3 - x_k - 3)}{(3x_k^2 - 1)} = \frac{3x_k^3 - x_k - x_k^3 + x_k + 3}{3x_k^2 - 1}$$

$$\Rightarrow \quad x_{k+1} = \frac{2x_k^3 + 3}{3x_k^2 - 1}$$

For k = 0, $x_1 = \frac{2x_0^3+3}{3x_0^2-1} = \frac{2(2)^3+3}{3(2)^2-1} = \frac{19}{11} = 1.7272$

For k = 1, $x_2 = \frac{2x_1^3+3}{3x_1^2-1} = \frac{2(1.7272)^3+3}{3(1.727)^2-1} = 1.6736$

$\therefore \quad x_2 = 1.6736$

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5. (1 to 1)

Given: $f(x) = 2x^2 - 3x + 3$, $x_0 = 2$

$f'(x) = 4x - 3$

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{(2\times 2^2 - 3\times 2 + 3)}{4\times 2 - 3} = 2 - \frac{5}{5} = 1$$

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6. (10 to 10)

The minimum number of iterations in bisection method for error ε is given by $\frac{|b-a|}{2^n} < \varepsilon$

a : Lower limit of interval

b: Upper limit of interval

n : No of iterations

ε : Error in approximation

$$\frac{|2-1|}{2^n} < 0.001 \quad \Rightarrow \quad 2^n > 1000$$

$\therefore$ n = 10

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7. (5.7 to 5.8)

Given: $y^2 - 2y + 1 = x$ and $\sqrt{x} + y = 5$

$x + \sqrt{y} = ?$

$$\sqrt{x} + y = 5$$

$\Rightarrow \quad \sqrt{x} = 5 - y$

$\Rightarrow \quad x = (5-y)^2 \qquad$ ...(i)

And,

$y^2 - 2y + 1 = x \qquad$ ...(ii)

From equation (i), put expression of x in equation (ii),

$y^2 - 2y + 1 = (5-y)^2$

$\Rightarrow \quad y^2 - 2y + 1 = 25 + y^2 - 10y$

$\Rightarrow \quad 8y = 24$

$\Rightarrow \quad y = 3$

From equation (i)

$x = (5-y)^2$

$\Rightarrow \quad x = (5-3)^2 = 4$

Therefore,

$$x + \sqrt{y} = 4 + \sqrt{3}$$

$$\Rightarrow \quad \boxed{x + \sqrt{y} = 5.732}$$

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8. (0.65 to 0.72)

Given: Equation is,

$$x^3 + x = 1$$

$$\Rightarrow \quad x^3 + x - 1 = 0$$

and initial guess,

$x_0 = 1$

The iterative scheme for f(x) = 0 using Newton-Raphson iterative scheme is,

$$\Rightarrow \quad x_{k+1} = x_k - \frac{(x_k^3 + x_k - 1)}{(3x_k^2 + 1)}$$

$$\Rightarrow \quad x_{k+1} = \frac{2x_k^3 + 1}{3x_k^2 + 1}$$

For k = 1:

$$\Rightarrow \quad x_1 = \frac{2x_0^3 + 1}{3x_0^2 + 1} = \frac{2(1)+1}{3(1)+1} = \frac{3}{4} = 0.75$$

For k = 2:

$$\Rightarrow \quad x_2 = \frac{2x_1^3 + 1}{3x_1^2 + 1} = \frac{2(0.75)^3 + 1}{3(0.75)^2 + 1}$$

$$= \frac{1.84375}{2.6875} = 0.686$$

$\therefore$ The value after 2^{nd} iteration is 0.69.

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9. (0.355 to 0.365)

According to newton Raphson method,

$$x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$$

$f(x) = 3x - e^x + \sin x$

$f'(x) = 3 - e^x + \cos x$

$x_0 = 0.333$

$$\Rightarrow \; x_1 = x_0 - \frac{f(0.333)}{f'(0.333)}$$

$$= 0.333 - \frac{(3 \times 0.33 - e^{0.333} + \sin 0.333)}{(3 - e^{0.333} + \cos 0.333)}$$

$$\Rightarrow \boxed{x_1 = 0.36}$$

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10. (1.53 to 1.59)

Given: $x = 10 \cos x$ and $x_0 = \frac{\pi}{4}$

$\Rightarrow \quad x - 10 \cos x = 0$

For $f(x) = 0$, Newton – Raphson scheme is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{(x_k - 10 \cos x_k)}{(1 + 10 \cdot \sin x_k)}$$

$$\Rightarrow \quad x_{k+1} = \frac{10x_k \cdot \sin x_k + 10 \cos x_k}{(1 + 10 \cdot \sin x_k)}$$

For k = 0,

$$x_1 = \frac{10x_0 \cdot \sin x_0 + 10 \cos x_0}{(1 + 10 \cdot \sin x_0)}$$

$$= \frac{10\left(\frac{\pi}{4} \cdot \frac{1}{\sqrt{2}}\right) + 10\left(\frac{1}{\sqrt{2}}\right)}{\left(1 + 10\left(\frac{1}{\sqrt{2}}\right)\right)} = \frac{10\left(\frac{\pi}{4} + 1\right)}{(10 + \sqrt{2})} = 1.564$$

$\therefore$ First iteration value = x_1 = 1.564

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11. (c)

Given: $f(x) = x^3 + x - 1$ and $x_0 = 1$

By Newton-Raphson scheme,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{(x_k^3 + x_k - 1)}{(3x_k^2 + 1)}$$

$$\Rightarrow \quad x_{k+1} = \frac{2x_k^3 + 1}{3x_k^2 + 1}$$

For k = 0

$$\Rightarrow \quad x_1 = \frac{2x_0^3 + 1}{3x_0^2 + 1} = \frac{2(1)^3 + 1}{3(1)^2 + 1} = \frac{3}{4} = 0.75$$

$\therefore \quad \boxed{\text{First Iteration value} = 0.750}$

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12. (0.29 to 0.31)

Given: $x^3 + 2x^2 + 3x - 1 = 0$ and $x_0 = 1$

For $f(x) = 0$, Newton-Raphson method is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)},$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{\{x_k^3 + 2x_k^2 + 3x_k - 1\}}{3x_k^2 + 4 \cdot x_k + 3}$$

$$\Rightarrow \quad x_{k+1} = \frac{2x_k^3 + 2x_k^2 + 1}{3x_k^2 + 4.x_k + 3}$$

For k = 0, $x_1 = \frac{2x_0^3 + 2x_0^2 + 1}{3x_0^2 + 4.x_0 + 3} = \frac{5}{10} = 0.5$

For k =1;

$$x_2 = \frac{2x_1^3 + 2x_1^2 + 1}{3x_1^2 + 4.x_1 + 3} = \frac{2(0.5)^3 + 2(0.5)^2 + 1}{3(0.5)^2 + 4(0.5) + 3}$$

$$= 0.3043$$

$\therefore$ $\boxed{\text{Value after } 2^{nd} \text{ iteration} = x_2 = 0.304}$

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13. (4.25 to 4.35)

Given equation is $f(x) = x^3 - 5x^2 + 6x - 8 = 0$

Initial guess is $x_0 = 5$

For any function f(x) = 0, newton-Raphson iterative scheme is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{\left(x_k^3 - 5x_k^2 + 6x_k - 8\right)}{\left(3x_k^2 - 10x_k + 6\right)}$$

$$\Rightarrow \quad x_{k+1} = \frac{2x_k^3 - 5x_k^2 + 8}{3x_k^2 - 10x_k + 6}$$

For k = 0; $x_1 = \dfrac{2x_0^3 - 5x_0^2 + 8}{3x_0^2 - 10x_0 + 6}$

$$= \frac{2(125) - 5(25) + 8}{3(25) - 10(5) + 6} = \frac{133}{31} = 4.29$$

$\therefore$ The value at the end of 1st iteration is 4.29.

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14. (2.32 to 2.34)

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4$

$x_0 = 3$

$f(3) = 1,\ f'(3) = 2$

By Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{1}{2} = \frac{5}{2} = 2.5$$

$$(\because \ x_0 = 3,\ f(x_0) = 1,\ f'(x_0) = 2)$$

$$x_1 = \frac{5}{2} \qquad f(x_1) = \left(\frac{5}{2}\right)^2 - 4\left(\frac{5}{2}\right) + 4 = \frac{1}{4}$$

By secant method.

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{3 \times \frac{1}{4} - \frac{5}{2} \times 1}{\frac{1}{4} - 1} = \frac{7}{3} = 2.33$$

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15. (0.3 to 0.4)

Given: $f(x) = -2 + 6x - 4x^2 + 0.5\,x^3$, $x_{ini} = 0$

$f'(x) = 6 - 8x + 1.5x^2$

By Newton Raphson method,

$$x_1 = x_{ini} - \frac{f(x_{ini})}{f'(x_{ini})}$$

$$x_1 = 0 - \frac{(-2)}{6} \qquad \left[\because f(0) = -2,\ f'(0) = 6\right]$$

$$x_1 = \frac{1}{3}$$

$$\Delta x = x_1 - x_{ini} = \frac{1}{3} - 0$$

$$\boxed{\Delta x = \frac{1}{3} = 0.33}$$

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16. (0.53 to 0.56)

Given: $5x - 2\cos x - 1 = 0$

By Newton's Raphson Iterative Scheme

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\Rightarrow \quad x_{k+1} = x_k - \frac{(5x_k - 2\cos x_k - 1)}{(5 + 2\sin x_k)}$$

$$= \frac{2x_k \cdot \sin x_k + 2 \cdot \cos x_k + 1}{5 + 2\sin x_k}$$

Let initial guess is $x_0 = 0$

$$x_1 = \frac{2+1}{5} = 0.6$$

$$x_2 = \frac{2(0.6)\sin(0.6) + 2\cos(0.6) + 1}{5 + 2\sin(0.6)} = 0.543$$

$$x_3 = \frac{2(0.543)\cdot\sin(0.543) + 2\cos(0.543) + 1}{5 + 2\sin(0.543)} = 0.542$$

$\therefore$ The real root is 0.54.

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17. (0.05 to 0.07)

Given: $f(x) = e^x - 1$, $x_0 = 1$

$\Rightarrow f'(x) = e^x$

By newton Raphson method;

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 - \frac{e^1 - 1}{e^1} = 1 - 1 + \frac{1}{e} = \frac{1}{e}$$

now, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$= \frac{1}{e} - \left(\frac{e^{1/e} - 1}{e^{1/e}}\right) = \frac{1}{e} - 1 + \frac{1}{e^{1/e}}$$

$= 0.37 - 1 + 0.69$

$x_2 = 0.06$

We know actual root of $e^x - 1 = 0$ is $x = 0$.

$\therefore$ Absolute error = $|0 - 0.06| = 0.06$

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18. (c)

Given: $f(x) = x^3 + 2x - 1$, $x_0 = 1.2$

$f'(x) = 3x^2 + 2$

By Newton Raphson method;

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.2 - \frac{[1.2^3 + 2\times 1.2 - 1]}{3\times 1.2^2 + 2} = 0.705$$

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19. (b)

Let $f(x_1, x_2) = 10x_2 \sin x_1 - 0.8$

$g(x_1, x_2) = 10x_2^2 - 10x_2 \cos x_1 - 0.6$

$x_1 = 0.0$ and $x_2 = 1.0$

Then Jacobian matrix for $(f, g) \to (x_1, x_2)$

$$J = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \\ \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 10x_2 \cos x_1 & 10\sin x_1 \\ 10x_2 \sin x_1 & 20x_2 - 10\cos x_1 \end{bmatrix}$$

$\therefore$ J at $x_1 = 0.0$ and $x_2 = 1.0$ is,

$$J = \begin{bmatrix} 10\times1\times1 & 10\times 0 \\ 10\times1\times0 & 20\times1 - 10\times1 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

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20. (c)

Given: $f(x) = x + \sqrt{x} - 3 = 0$ and initial guess $x_0 = 2$.

As per Newton Raphson iteration scheme,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{\left(x_k + \sqrt{x_k} - 3\right)}{\left(1 + \frac{1}{2\sqrt{x_k}}\right)}$$

$$\Rightarrow \quad x_{k+1} = \frac{x_k + \frac{\sqrt{x_k}}{2} - x_k - \sqrt{x_k} + 3}{\left(1 + \frac{1}{2\sqrt{x_k}}\right)}$$

$$\Rightarrow \quad x_{k+1} = \frac{3 - \frac{\sqrt{x_k}}{2}}{\left(1 + \frac{1}{2\sqrt{x_k}}\right)}$$

For k = 0

$$x_1 = \frac{3-\frac{\sqrt{x_0}}{2}}{\left(1+\frac{1}{2\sqrt{x_0}}\right)} = \frac{3-\frac{\sqrt{2}}{2}}{\left(1+\frac{1}{2\sqrt{2}}\right)} = 1.6939 \approx 1.694$$

$\therefore$ Value after 1st iteration = 1.694

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21. (a)

Let $f(x) = x^2 - N$

$f'(x) = 2x$

According to Newton – Raphson formula,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = x_i - \frac{x_i^2 - N}{2x_i} = x_i - \frac{x_i}{2} + \frac{N}{2x_i}$$

$$x_{i+1} = \frac{x_i}{2} + \frac{N}{2x_i}$$

$$\boxed{x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)}$$

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22. (a)

Let $f(x) = x^2 - 117$

By Newton Raphson method, $x_{k+1} = x_x - \frac{f(x_k)}{f'(x_k)}$

$$x_{k+1} = x_x - \frac{\left(x_k^2 - 117\right)}{2x_k}$$

$$= x_k - \frac{x_k}{2} + \frac{117}{2x_k} = \frac{x_k}{2} + \frac{117}{2x_k}$$

$$x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$$

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23. (a)

Given: $f(x) = e^x - 1$, $x_0 = -1$

By Newton's method;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{e^{x_0} - 1}{e^{x_0}} \qquad [\because f'(x) = e^x]$$

$$x_1 = -1 - \left(\frac{e^{-1} - 1}{e^{-1}}\right) = \frac{-e^{-1} - e^{-1} + 1}{e^{-1}}$$

$$= \frac{1 - 2e^{-1}}{e^{-1}} = \frac{1 - 2/e}{1/e} = e - 2 = 0.71828$$

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24. (c)

Given: $x = e^{-x}$

$\Rightarrow \quad f(x) = x - e^{-x} = 0$

For $f(x) = 0$, Newton – Raphson scheme is given by

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{\left(x_k - e^{-x_k}\right)}{\left(1 + e^{-x_k}\right)}$$

$$\Rightarrow \quad x_{k+1} = \frac{x_k e^{-x_k} + e^{-x_k}}{(1 + e^{-x_k})} = \frac{e^{-x_k}(1 + x_k)}{(1 + e^{-x_k})}$$

$$\therefore \quad x_{n+1} = (1 + x_n)\frac{e^{-x_n}}{(1 + e^{-x_n})}$$

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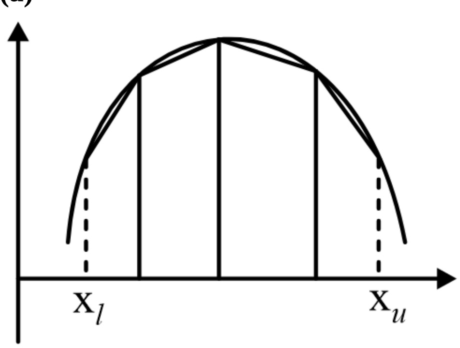
25. (a)



Area under approximated curve will be less as compared to actual smooth function.
Numerically computed area < Actual area
(Trapezoidal Rule)

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26. (d)

Given: $I = \int_1^2 (4x^2 + 2x + 6)dx$

For $\int_a^b f(x)dx$, if f(x) is polynomial of degree ≤ 2, the integration by Simpson's $\frac{1}{3}^{rd}$ rule will be same as the exact value.

∵ $f(x) = 4x^2 + 2x + 6$ is a polynomial of degree '2'

⇒ $I_e = I_s$

∴ $\left|\frac{I_e - I_s}{I_e}\right| \times 100 = 0$

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27. (20.00 to 21.00)

Given: Step size, h = 0.5

By Simpson's 1/3rd Rule, Area (A)

$= \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots)\right]$

$= \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_n\right]$

$= \frac{0.5}{3}\left[5 + 4\times 7.25 + 2\times 10 + 4\times 13.25 + 17.00\right]$

$= \boxed{20.67\text{unit}^2}$

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28. (c)

Given: $f(x) = e^x$, n = 4, a = 0, b = 1

Then $h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

So,

x	0	¼	½	¾	1
f(x)	1	$e^{1/4}$	$e^{1/2}$	$e^{3/4}$	e^1
	y_0	y_1	y_2	y_3	y_n

Using Trapezoidal rule,

$$I = \int_0^1 e^x dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_n\right]$$

$$= \frac{1/4}{2}\left[1 + 2\left(e^{1/4} + e^{1/2} + e^{3/4}\right) + e\right] = \boxed{1.727}$$

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29. (0.38 to 0.42)

Given: $f(x) = 10x - 20x^2$, n = 5, a = 0, b = 0.5

Then $h = \frac{b-a}{n} = \frac{0.5-0}{5} = 0.1$

So,

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	0	0.8	1.2	1.2	0.8	0
	y_0	y_1	y_2	y_3	y_4	y_n

Using Trapezoidal rule, $I = \int_0^{0.5} (10x - 20x^2)dx$

$$= \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + y_3 + y_4)\right]$$

$$= \frac{0.1}{2}\left[0 + 2(0.8 + 1.2 + 1.2 + 0.8) + 0\right]$$

∴ $\boxed{\int_0^{0.5} (10x - 20x^2)dx = 0.40}$

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30. (a)

Given: Integral $\int_4^{5.2} \ln x\,dx$; step-size h = 0.3

x	4	4.3	4.6	4.9	5.2
ln x	ln 4	ln 4.3	ln 4.6	ln 4.9	ln 5.2

By Simpson's $\frac{1}{3}^{rd}$ rule, $\int_4^{5.2} \ln x dx$

$= \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots)\right]$

$= \frac{0.3}{3}\left[(\ln 4 + \ln 5.2) + 4(\ln 4.3 + \ln 4.9) + 2\ln(4.6)\right]$

$= 1.8278$

$\therefore \int_4^{5.2} \ln x dx = 1.83$

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31. (d)

Given: Integral I = $\int_{-1}^{1.4} x|x| dx$ and step size h = 0.6

x	–1	–0.4	0.2	0.8	1.4
x|x|	(y_0) –1	(y_1) –0.16	(y_2) 0.04	(y_3) 0.64	(y_n) 1.96

From Simpson's $\frac{1}{3}^{rd}$ rule, $\int_a^b f(x) dx$ is given by

$= \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdot\cdot)\right]$

$\therefore \quad I = \frac{0.6}{3}\left[(-1 + 1.96) + 4(-0.16 + 0.64) + 2(0.04)\right]$

$= 0.592$

$\therefore \quad \boxed{\int_{-1}^{1.4} x|x| dx = 0.592}$

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32. (b)

The following rules will give zero error for polynomials up to the following degree:

Given, Polynomial is of 3 degree

Rectangular Rule	0 degree (x^0)
Trapezoidal Rule	1 degree (x^1)
Simpson $1/3^{rd}$ Rule	2 degree (x^2)
Simpson $3/8^{th}$ Rule	3 degree (x^3)

Since function is a polynomial of 3 degree hence Only Simpson's rule will give zero error.

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33. (5.1 to 5.5)

Given: Integral $\int_0^{\frac{\pi}{2}} (8 + 4\cos x) dx$ = I (say)

(i) Analytic Value = $\int_0^{\frac{\pi}{2}} (8 + 4\cos x) dx$

$= 8x + 4\sin x\Big|_0^{\frac{\pi}{2}} = 8\left(\frac{\pi}{2}\right) + 4\sin\left(\frac{\pi}{2}\right)$

$= 4(1 + \pi)$ = A (say)

(ii) Using Trapezoidal rule by considering end points,

$\Rightarrow T = \frac{h}{2}\left[(y_0 + y_n)\right]$

$= \frac{\left(\frac{\pi}{2}\right)}{2}\left[(8 + 4\cos x)|_{x=0} + (8 + 4\cos x)|_{x=\frac{\pi}{2}}\right]$

$= \frac{\pi}{4}[16 + 4] = 5\pi$

$\therefore \quad T = 5\pi$

$\% \text{ error} = \left|\frac{A - T}{A}\right| \times 100$

$= \left|\frac{4 + 4\pi - 5\pi}{4 + 4\pi}\right| \times 100 = 5.182\ \%$

$\boxed{\text{Absolute \% error} = 5.2\ \%}$

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34. (63 to 63)

Given: Integral $I = \int_2^4 x^3 dx$

Number of sub-intervals = 2

$\Rightarrow \quad \text{step size} = \frac{4 - 2}{2} = 1 = h$

x	2	3	4
f(x)	$y_0 = 8$	$y_1 = 27$	$y_n = 64$

By Trapezoidal rule,

$$I = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots)\right]$$

$$= \frac{1}{2}[(8+64)+2(27)] = 4 + 32 + 27 = 63$$

$\therefore$ $\boxed{\int_2^4 x^3 dx = 63}$

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35. (a)

Given:

The Points are P (0, 3), Q (0.5, 4), R (1, 5) are collinear.

The difference in the values of $\int_0^1 f(x)dx$ will be zero in between Trapezoidal rule, Simpson's rule and analytical integration.

% Error between Trapezoidal and Simpson's rule = 0

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36. (a)

For a linear function (f(x) = a x + b), the integration of f(x) between any two points using trapezoidal rule gives the same value as the Analytical Integration.

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37. (0.175 to 0.195)

Given

$$\text{Integral} = \int_0^{\pi} (\sin x + \cos x)dx$$

(i) **Analytic Value:** Let analytic value be 'A'

$$A = \int_0^{\pi} (\sin x + \cos x)dx = -\cos x + \sin x |_0^{\pi}$$

$$= +1 + 0 - (-1 + 0) = 2$$

$\Rightarrow$ A = 2

(ii) **Trapezoidal Rule:**

Number of sub-intervals = 3

$\Rightarrow$ step size = $h = \frac{\pi - 0}{3} = \frac{\pi}{3}$

x	0	$\pi/3$	$2\pi/3$	π
f(x)	$y_0 = 1$	$y_1 = \frac{1+\sqrt{3}}{2}$	$y_2 = \frac{-1+\sqrt{3}}{2}$	$y_n = -1$

$$\therefore \int_0^{\pi} (\sin x + \cos x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2)\right]$$

$$= \frac{\pi}{6}\left[1 - 1 + 2\left(\frac{1+\sqrt{3}}{2} + \frac{-1+\sqrt{3}}{2}\right)\right]$$

$$= \frac{\pi}{6}\left(2\sqrt{3}\right) = \frac{\pi}{\sqrt{3}} = 1.8138$$

$\therefore$ $\boxed{\text{Error} = A - 1.8138 = 2 - 1.8138 = 0.186}$

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38. (2 to 2)

Given: $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$

Least number of sub-intervals for Simpson's $\frac{1}{3}^{rd}$ rule is '2'.

$\therefore$ Step size = $\frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
f(x)	$y_0 = \frac{9}{5}$	$y_1 = \frac{3}{20} + \frac{9}{5}$	$y_n = \frac{12}{5}$

$\therefore$ From Simpson's $\frac{1}{3}^{rd}$ rule,

$$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 4y_1\right]$$

$$= \frac{0.5}{3}\left[\frac{9}{5} + \frac{12}{5} + 4\left(\frac{3}{20} + \frac{9}{5}\right)\right]$$

$= 2$ (Value will be as Analytical value)

$$\therefore \boxed{\int_0^1 \left(\frac{3}{5}x^2 + \frac{9}{5}\right)dx = 2}$$

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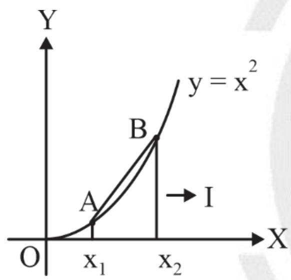
39. (a)

For the function $y = x^2$ between x_1 and x_2



I= exact value of the integral is given by area under the curve $y = x^2$ between x_1 and x_2 bounded by X-axis.

J= Approximate value is obtained by the area under the straight-line AB between x_1 and x_2 bounded by X-axis.

So, $J > I$

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40. (1.36 to 1.37)

Given:

$f(x) = 0.2 + 2.5x – 200x^2 + 675x^3 – 900x^4 + 400x^5$

Step size, h = 0.4

x	0	0.4	0.8
f(x)	0.2	2.456	0.232

By Simpson's 1/3rd Rule,

$$I = \int_0^{0.8} f(x)dx = \frac{h}{3}\left[y_0 + 4y_1 + y_n\right]$$

$$= \frac{0.4}{3}\left[0.2 + 4 \times 2.456 + 0.232\right]$$

$$\boxed{I = 1.367}$$

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41. (21.8 to 22.2)

Given: Step size $h = 0.1$

x	0	0.1	0.2	0.3	0.4
$f(x)$	$y_0 = 0$	$y_1 = 10$	$y_2 = 40$	$y_3 = 90$	$y_n = 160$

From trapezoidal rule,

$$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots)\right]$$

$$\Rightarrow \int_0^{0.4} f(x)dx = \frac{0.1}{2}\left[(0 + 160) + 2(10 + 40 + 90)\right]$$

$$= \frac{0.1}{2}[160 + 280] = 22$$

$$\therefore \boxed{\int_0^{0.4} f(x)dx = 22}$$

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42. (0.68 to 0.70)

Given: $h = 1 \Rightarrow$ Number of sub intervals = 1

$\Rightarrow$ Integration is based on end point

x	1	2
$f(x)$	$y_0 = 1.\ln(0)$	$y_n = 2.\ln 2$

$$\therefore \int_1^2 x \ln x \, dx = \frac{h}{2}\left[(y_0 + y_n)\right]$$

$$= \frac{1}{2}[0 + 2 \cdot \ln 2] = \ln 2 = 0.693$$

$$\boxed{\text{For } h = 1, \int_1^2 x \cdot \ln x \cdot dx = 0.693}$$

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43. (1.1 to 1.2)

Given: Integral $\int_1^3 \frac{1}{x}dx$ and step size h = 1

x	1	2	3
$1/x$	$y_0=1$	$y_1=1/2$	$y_n=1/3$

From Trapezoidal rule

$$\int_1^3 \frac{1}{x}dx = \frac{h}{2}\left[(y_0+y_n)+2y_1\right]$$

$$\Rightarrow \int_1^3 \frac{1}{x}dx = \frac{1}{2}\left[\left(1+\frac{1}{3}\right)+2\left(\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{7}{3}\right] = \frac{7}{6} = 1.167$$

$$\Rightarrow \boxed{\int_1^3 \frac{1}{x}dx = 1.167}$$

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44. (1.74 to 1.76)

Given: Integral I = $\int_{2.5}^{4} \ln x \, dx$

Number of sub intervals = 5

$\Rightarrow$ step size, h = $\frac{4-2.5}{5} = 0.3$

x	2.5	2.8	3.1	3.4	3.7	4.0
ln x	ln 2.5	ln 2.8	ln 3.1	ln 3.4	ln 3.7	ln 4.0

From Trapezoidal rule;

$$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$\int_{2.5}^{4} \ln x \, dx = \frac{0.3}{2}\begin{bmatrix}(\ln 2.5+\ln 4.0)\\ +2(\ln 2.8+\ln 3.1+\ln 3.4+\ln 3.7)\end{bmatrix}$$

= 0.15 (2.3025 + 9.3852) = 1.7533

$$\therefore \boxed{\int_{2.5}^{4} \ln x \, dx = 1.7533}$$

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45. (1.10 to 1.12)

Given: Integral = $\int_{-1}^{1} |x| dx$, no. of sub-intervals = 3

$\therefore$ step size, $h = \frac{1-(-1)}{3} = \frac{2}{3}$

x	–1	–1/3	1/3	1
|x|	1	1/3	1/3	1

By Trapezoidal rule,

$$\int_b^a f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots)\right]$$

$$\Rightarrow \int_{-1}^{1} |x| dx = \frac{\left(\frac{2}{3}\right)}{2}\left[(1+1)+2\left(\frac{1}{3}+\frac{1}{3}\right)\right]$$

$$= \frac{1}{3}\left[2+\frac{4}{3}\right] = \frac{10}{9} = 1.111$$

$$\therefore \boxed{\int_{-1}^{1} |x| dx = 1.111}$$

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46. (0.50 to 0.53)

Given: h = 1

Exact value of integral $\int_0^4 (x^4+10)dx$

$$= \left[\frac{x^5}{5}+10x\right]_0^4 = \frac{4^5}{5}+10\times 4 = 244.8$$

x	0	1	2	3	4
f(x)	10	11	26	91	266

Using Simpson's 1/3rd rule

$$\int_0^4 f(x)dx = \frac{h}{3}\left[y_0+4y_1+2y_2+4y_3+y_n\right]$$

$$= \frac{1}{3}[10+4\times 11+2\times 26+4\times 91+266] = 245.33$$

Approximate value = 245.33

$\therefore$ Magnitude of error = 245.33 – 244.8 = 0.53

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47. (d)

Iteration scheme → Fitting curve → order of fitting polynomial

1. Trapezoidal rule → straight line → first order
2. Simpson's $1/3^{rd}$ rule → quadratic function → second order
3. Simpson's $3/8^{th}$ rule → cubic function → third order.

$$\boxed{\therefore\ P \to 3,\ Q \to 1,\ R \to 2}$$

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48. (d)

Given: $f(x) = \frac{1}{x}$, $h = \frac{1.5 - 0.5}{2} = 0.5$

Exact value of $\int_{0.5}^{1.5} \frac{dx}{x} = [\log x]_{0.5}^{1.5}$ = log 3 = 1.0986

Approximate value by Simpson's rule with 3 point is,

x	0.5	1.0	1.5
f(x)	1/0.5	1/1	1/1.5

$$I = \frac{h}{3}\left[f(0.5) + 4f(1.0) + f(1.5)\right]$$

$$\therefore\ I = \frac{0.5}{3}\left[\frac{1}{0.5} + 4 \times 1 + \frac{1}{1.5}\right] = 1.1111$$

So, approximate value exceeds the exact value by

Approximate value – exact value

= 1.1111 – 1.0986 = 0.012

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49. (c)

Given: integral is $\int_1^3 \frac{1}{x} dx = I(\text{say})$

Number of sub intervals = 2

$\Rightarrow$ step size = $h = \frac{b-a}{n} = \frac{3-1}{2} = 1$

x	1	2	3
$f(x) = 1/x$	$y_0 = 1$	$y_1 = 1/2$	$y_n = 1/3$

Applying Simpson's $\frac{1}{3}^{rd}$ rule

$$\int_1^3 \frac{1}{x} dx = \frac{h}{3}\left[(y_0 + y_n) + 4y_1\right] = \frac{1}{3}\left[\left(1 + \frac{1}{3}\right) + 4\left(\frac{1}{2}\right)\right]$$

$$= \frac{1}{3}\left[3 + \frac{1}{3}\right] = 1 + \frac{1}{9} = 1.111$$

$$\boxed{\therefore \int_1^3 \frac{1}{x} dx = 1.111}$$

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50. (b)

Given: Step size $h = \frac{360 - 0}{6} = 60° = \frac{\pi}{3}$

Angle (degree)	0	60	12	180	240	300	360
Torque (N m)	0	1066	–323	0	323	–355	0

Flywheel energy F.E. = $\int_{\theta=0°}^{\theta=360°} T \cdot d\theta$

Using Simpson's rule, $\int_a^b f(x) dx$

$$= \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots)\right]$$

F.E. =

$$\frac{\pi}{3(3)}\left[(0 + 0) + 4(1066 + 0 - 355)\right] + 2(-323 + 323)$$

$$= 992.74 \text{ J} \quad \Rightarrow \boxed{\int T.d\theta \cong 993 \text{ J}}$$

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51. (a)

Given: h = 0.25

x	0	0.25	0.5	0.75	1.0
f(x)	1	0.9412	0.8	0.64	0.50

According to Simpson's 1/3rd Rule,

$$\int_0^1 f(x)dx = \frac{h}{3}\left[(y_o + y_n) + 2y_2 + 4(y_1 + y_3)\right]$$

$$= \frac{0.25}{3}\left[(1+0.5) + 2\times 0.8 + 4\times(0.9412 + 0.64)\right]$$

$\boxed{I = 0.7854}$

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52. (–0.152 to –0.149)

Given: D.E: $\frac{du}{dt} = 1 - 2tu^2$

$u_{n-1} = 1.75$, h = 0.01, $t_{n-1} = 3.14$

By Euler's Backward method:

$u_n = u_{n-1} + h\, f(u_n, t_n)$ $\quad [\because \ f(u, t) = 1 - 2tu^2]$

$u_n = 1.75 + 0.01\,[1 - 2t_n u_n^2]$

$u_n = 1.75 + 0.01\,[1 - 2 \times 3.15\, u_n^2]$

$u_n = 1.76 - 0.063\, u_n^2$

$0.063\, u_n^2 + u_n - 1.76 = 0$

$u_n = 1.599$

$\therefore$ $u_n - u_{n-1}$ 1.599 –1.75 = – 0.151

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53. (0.999 to 1.001)

Given: $\frac{dy}{dt} + 2y = 0; \quad y(0) = 1$

$\Delta t \rightarrow$ step size for forward Euler method.

For stability of solution of $\frac{dy}{dt} = \lambda y$, solved using forward Euler method,

$|1 + h\lambda| < 1$ where 'h' is step size

$-1 < 1 + h\lambda < 1 \Rightarrow -2 < h\lambda < 0$

$\Rightarrow \quad h \le \frac{2}{|\lambda|}$

$\therefore$ In given problem, $\Delta t \le \frac{2}{|-2|} = 1$

For stability, $\Delta t \le 1$

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54. (6.3 to 6.5)

Given: $\frac{dy}{dx} = 4(x+2) - y$, $f(x, y) = 4(x+2) - y$

h = 0.2

For finding y_2 two iterations has to be followed

	x_0	x_1	x_2
x	1	1.2	1.4
y	3	?	?

By Euler's method

$y_1 = y_0 + hf(x_0, y_0)$

= 3 + 0.2 f(1,3) $\quad \because \ x_0 = 1, y_0 = 3$

= 3 + 0.2 [4 (1+ 2) –3]

$y_1 = 4.8$

$\therefore$ $x_1 = 1.2$, $y_1 = 4.8$

$y_2 = y_1 + h\, f(x_1, y_1)$

= 4.8 + 0.2 f(1.2, 4.8)

= 4.8 + 0.2 [4(1.2 + 2) – 4.8] = 4.8 + 1.6

$y_2 = 6.4$

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55. (b)

If s = α is a root of $\varphi(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0$ for 'k' times,

then $(f(\alpha) = f'(\alpha) = f''(\alpha) = \ldots f^{(k-1)}(\alpha) = 0)$

Given s = –σ is a triple root of ϕ(s) = 0

$\Rightarrow$ $\phi(s) = 0; \phi'(s) = 0; \phi''(s) = 0$ at s = –σ

$\Rightarrow$ $\boxed{\phi(s) = 0; \frac{d\phi}{ds} = 0; \frac{d^2\phi}{ds^2} = 0 \text{ at } s = -\sigma}$

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56. (a)

Given

$$\frac{dy}{dt} = -\pi y$$

For Forward Euler scheme,

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

to have a stable solution for $\frac{dy}{dt} = \lambda y$

Step size *h*, $0 < h < \frac{2}{|\lambda|}$

$\Rightarrow \quad 0 < h < \frac{2}{|-\pi|} \Rightarrow 0 < h < \frac{2}{\pi}$

$\therefore$ For stability, $0 < h < \frac{2}{\pi}$

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57. (a)

Using fourth order central divided difference formula for third derivative

$$f'''(x_i) = \frac{-f(x_{i+3}) + 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) + f(x_{i-3})}{8h^3}$$

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58. (2.55 to 2.65)

Given

$$\frac{dy}{dt} = y \text{ and } y(0) = 1$$

Explicit forward-Euler scheme for a DE $\frac{dy}{dt} = f(t, y)$

where $y(t_0) = y_0$, is given by

$y_{i+1} = y_i + h \cdot f(x_i, y_i)$

$\Rightarrow \quad y_{i+1} = y_i + h \cdot y_i$

$\Rightarrow \quad y_{i+1} = (1 + h).\ y_i$

$\therefore \quad y(0.1) = (1 + h) \cdot y(0)$

$y(0.2) = (1 + h) \cdot y(0.1) = (1 + h)^2 \cdot y(0)$

$y(0.3) = (1 + h) \cdot y(0.2) = (1 + h)^3 \cdot y(0)$

and so on……

So, $y(1) = (1 + h)^{10} \cdot y(0)$

$\Rightarrow \quad y(1) = (1 + 0.1)^{10} \cdot 1 = (1.1)^{10} = 2.59$

$\Rightarrow \quad y(1) = 2.59$

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59. (7.95 to 8.05)

Given: $\frac{du}{dt} = 3t^2 + 1,\ f(u, t) = 3t^2 + 1$

$u_o = 0, t_0 = 0, \Delta t = 2 = h$

By Euler's method

$u_1 = u_0 + h\, f(u_0, t_0)$

$u_1 = u_0 + h\,(3t_0^2 + 1)$

$= 0 + 2(3 \times 0^2 + 1)$

$\boxed{u_1 = 2}$

After first iteration when t = 2 then y = 2.

$$\frac{du}{dt} = 3t^2 + 1$$

Integrating both sides, we get

$$u = 3\frac{t^3}{3} + t + c$$

When t = 0, u = 0

$0 = 0 + 0 + c$

$c = 0$

$u = t^3 + t$

When $t = 2$; $u = 2^3 + 2 = 10$

Absolute error = Exact value – Approximate value

$= 10 - 2 = 8$

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60. (0.060 to 0.063)

Given: $y' = y + 2x - x^2;\ y(0) = 1$

Exact solution is $y = x^2 + e^x$

$\therefore$ At $x = 0.1$, $y(0.1) = (0.1)^2 + e^{0.1} = 1.11517$

Runge kutta 2nd order Method:

Step size h = 0.1

$$y_{i+1}^{(P)} = y_i + h \cdot f(x_i, y_i)$$

$$y_{i+1}^{(C)} = y_i + \frac{h}{2}(f(x_i, y_i) + f(x_i + h, y_{i+1}^{(P)}))$$

Here f(x, y) = y + 2x –x^2.

At x = 0; y = 1

$\Rightarrow y_1^{(P)} = y(0.1)^{(P)} = 1 + 0.1(1 + 2(0) - 0^2)$

= 1 + 0.1(1) = 1.1

$\therefore y_1^{(c)} = 1 + \frac{0.1}{2}[1 + f(0.1, 1.1)]$

$= 1 + \frac{0.1}{2}\left[1 + (1.1 + 2(0.1) - (0.1)^2)\right]$

$= 1 + \frac{0.1}{2}[2.29] = 1.1145$

$\therefore$ y(0.1) = 1.1145

$\therefore \%\ \text{Error} = \left|\frac{y(0.1)_{\text{Exact}} - y(0.1)_{\text{RK}}}{y(0.1)|_{\text{Exact}}}\right| \times 100$

$= \left|\frac{1.11517 - 1.1145}{1.11517}\right| \times 100 \quad = 0.061$

$\boxed{\therefore\ \%\ \text{Error} = 0.061}$

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61. (0.6 to 0.7)

Given equation is $\frac{dx}{dt} = -3x + 2;\ x(0) = 1$

For the DE; $\frac{dx}{dt} = \lambda x + k$ with $x(t_0) = x_0$, to have a stable solution, step size $h \le \frac{2}{|\lambda|}$

$\Rightarrow \quad h \le \frac{2}{|-3|} \Rightarrow h \le \frac{2}{3}$

$\Rightarrow$ h = 0.67

$\therefore$ Largest time step that can be taken without instability is 0.67.

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62. (d)

Given: $\frac{dx}{dt} = 4t + 4 = f(t, x)$

At t = 0, x = x_0; h = Δt = 0.2

From Rk – 4th order method

$$x_{i+1} = x_i + \frac{h}{6}(k_1 + k_2 + k_3 + k_4)$$

where $k_1 = f(t_0, x_0)$

$k_2 = f\left(t_0 + \frac{h}{2}, x_0 + \frac{k_1}{2}\right)$

$k_3 = f\left(t_0 + \frac{h}{2}, x_0 + \frac{k_2}{2}\right)$

$k_4 = f(t_0 + h, x + k)$

$\therefore \quad k_1 = f(0, x_0) = 4(0) + 4 = 4$

$k_2 = f\left(0 + \frac{0.2}{2}, x_0 + \frac{k_1}{2}\right) = 4(0.1) + 4 = 4.4$

$k_3 = f\left(0 + \frac{0.2}{2}, x_0 + \frac{k_2}{2}\right) = 4(0.1) + 4 = 4.4$

$k_4 = f(0 + 0.2, x_0 + k_3) = 4(0.2) + 4 = 4.8$

The increment is given by

$$x_1 - x_0 = \frac{0.2}{6}(4 + 8.8 + 8.8 + 4.8) = 0.88$$

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63. (b)

Numerical Integral

Trapezoidal, Simpson's $\frac{1}{3}^{rd}$, Simpson's $\frac{3}{8}^{th}$ rule,

Solution to transcendental equation

Newton-Raphson Method.

Solution to system of linear equations

Gauss Elimination method

Solution of differential equation

Runge-kutta methods

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64. (d)

Error in central difference is O(h^2).

This mean Error $\propto h^2$

If error for h = 0.03 is 2×10^{-3}

Then, error for h = 0.02 is approximately,

$$= 2\times10^{-3}\times\left(\frac{0.02}{0.03}\right)^2 = \frac{8}{9}\times10^{-3} = \frac{80}{9}\times10^{-4}$$

$$E \approx 9.0\times10^{-4}$$

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65. (b)

Given: $\frac{dy(x)}{dx} - y(x) = x$, Step size h = 0.1

$\Rightarrow \frac{dy}{dx} = x + y = f(x, y)$

Initial condition is y (0) = 0

At $x_0 = 0$, $y_0 = 0$.

Using Euler's first order method,

$y_{i+1} = y_i + h.\ f(x_i, y_i)$

$\therefore$ $y_1 = y_0 + h.\ f(x_0, y_0) = y_0 + h\,(x_0 + y_0)$

$y_1 = 0 + 0.1\,(0 + 0) = 0$

$\Rightarrow$ y (0.1) = 0

y(0.2) = y(0.1) + h.(0.1 + 0)

= 0 + (0.1) (0.1 + 0) = 0.01

y (0.3) = y(0.2) + h(0.2 + 0.01)

= 0.01 + 0.1 (0.2 + 0.01)

= 0.01 + 0.1 (0.21) = 0.01 + 0.021 = 0.031

$\therefore$ At x = 0.3, y(0.3) = 0.031

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66. (c)

Given: Step size $h = 0.01$

At $t = 0$, x = 0, we have to find x at t = 0.01

Using trapezoidal rule, $x(a + nh)$

$$= x(a) + \frac{h}{2}[f(a) + f(a+h) + \ldots.f(a+(n-1)h)]$$

$$\frac{dx}{dt} = e^{-2t}u(t)$$

$$\frac{dx}{dt} = e^{-2t}; \qquad (\because u(t) = 1;\ t > 0)$$

$f(t) = e^{-2t}$

$$x(0.01) = x(0) + \frac{h}{2}[f(0) + f(0.01)]$$

$$= 0 + \frac{0.01}{2}[1 + e^{-0.02}]$$

$x(0.01) = 0.005[1 + e^{-0.02}]$

$x(0.01) = 0.0099$

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67. (c)

When interpolated with all the 'n' points, $\exists$ a unique interpolating polynomial of degree $\le$ n.

$\therefore$ Whether the polynomial is generated using Lagrange's interpolation (or) Newton interpolation method, the polynomial remains same.

$\therefore$ $I_L = I_N$.

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NOTE:

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General Aptitude

General Aptitude

Syllabus

Verbal Aptitude: Basic English grammar: tenses, articles, adjectives, prepositions, conjunctions, verb-noun agreement, and other parts of speech Basic vocabulary: words, idioms, and phrases in context Reading and comprehension Narrative sequencing.

Quantitative Aptitude: Data interpretation: data graphs (bar graphs, pie charts, and other graphs representing data), 2- and 3-dimensional plots, maps, and tables Numerical computation and estimation: ratios, percentages, powers, exponents and logarithms, permutations and combinations, and series Mensuration and geometry Elementary statistics and probability.

Analytical Aptitude: Logic: deduction and induction, Analogy, Numerical relations and reasoning.

Spatial Aptitude: Transformation of shapes: translation, rotation, scaling, mirroring, assembling, and grouping Paper folding, cutting, and patterns in 2 and 3 dimensions.

ME Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4
2010	6	3	0	6
2011	9	0	0	6
2012	7	2	0	6
2013	9	0	0	6
2014 (P1)	7	4	0	4
2014 (P2)	7	5	0	3
2014 (P3)	9	3	0	3
2014 (P4)	8	4	0	3
2015 (P1)	6	2	0	7
2015 (P2)	5	1	0	9
2015 (P3)	7	3	0	5
2016 (P1)	7	3	0	5
2016 (P2)	7	3	0	5
2016 (P3)	4	6	0	5
2017 (P1)	8	1	0	6
2017 (P2)	9	2	0	4
2018 (P1)	11	2	0	2
2018 (P2)	8	5	0	2
2019 (P1)	6	4	0	5
2019 (P2)	7	2	0	6
2020 (P1)	7	2	0	6
2020 (P2)	7	2	0	6
2021 (P1)	7	2	2	4
2021 (P2)	6	6	0	3
2022 (P1)	8	3	1	3
2022 (P2)	9	2	1	3
2023	6	1	2	6

CE Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4
2010	6	3	0	6
2011	9	0	0	6
2012	7	2	0	6
2013	9	2	0	4
2014 (P1)	8	4	0	3
2014 (P2)	4	6	0	5
2015 (P1)	5	3	0	7
2015 (P2)	7	3	0	5
2016 (P1)	6	4	0	5
2016 (P2)	8	2	0	5
2017 (P1)	10	1	0	4
2017 (P2)	6	3	0	6
2018 (P1)	11	2	0	2
2018 (P2)	11	2	0	2
2019 (P1)	3	5	0	7
2019 (P2)	8	0	0	7
2020 (P1)	7	3	0	5
2020 (P2)	7	3	0	5
2021 (P1)	4	3	3	5
2021 (P2)	6	5	1	3
2022 (P1)	7	3	2	3
2022 (P2)	6	5	1	3
2023 (P1)	5	3	1	6
2023 (P2)	7	1	1	6

EE Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4
2010	6	3	0	6
2011	9	0	0	6
2012	7	2	0	6
2013	9	0	0	6
2014 (P1)	6	2	0	7
2014 (P2)	8	0	0	7
2014 (P3)	7	3	0	5
2015 (P1)	7	1	0	7
2015 (P2)	7	3	0	5
2016 (P1)	3	7	0	5
2016 (P2)	7	1	0	7
2017 (P1)	8	3	0	4
2017 (P2)	8	3	0	4
2018	11	2	0	2
2019	7	3	0	5
2020	9	2	0	4
2021	8	2	1	4
2022	7	5	0	3
2023	8	0	0	7

ECE Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4
2010	6	3	0	6
2011	9	0	0	6
2012	7	2	0	6
2013	9	0	0	6
2014 (P1)	7	4	0	4
2014 (P2)	7	5	0	3
2014 (P3)	9	3	0	3
2014 (P4)	8	4	0	3
2015 (P1)	1	5	0	9
2015 (P2)	6	2	0	7
2015 (P3)	5	1	0	9
2016 (P1)	7	3	0	5
2016 (P2)	4	6	0	5
2016 (P3)	7	4	0	4
2017 (P1)	10	4	0	6
2017 (P2)	6	8	0	6
2018	9	0	0	6
2019	6	4	0	5
2020	9	0	0	6
2021	8	2	1	4
2022	6	4	2	3
2023	5	4	0	6

CSE & IT Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4
2010	6	3	0	6
2011	9	0	0	6
2012	7	2	0	6
2013	9	0	0	6
2014 (P1)	6	2	0	7
2014 (P2)	8	0	0	7
2014 (P3)	7	3	0	5
2015 (P1)	7	1	0	7
2015 (P2)	7	3	0	5
2015 (P3)	5	3	0	7
2016 (P1)	6	4	0	5
2016 (P2)	3	7	0	5
2017 (P1)	8	3	0	4
2017 (P2)	8	3	0	4
2018	12	1	0	2
2019	2	8	0	5
2020	7	2	0	6
2021 (P1)	7	2	2	4
2021 (P2)	6	2	3	4
2022	6	6	0	3
2023	6	0	3	6

NOTE:

CHAPTER

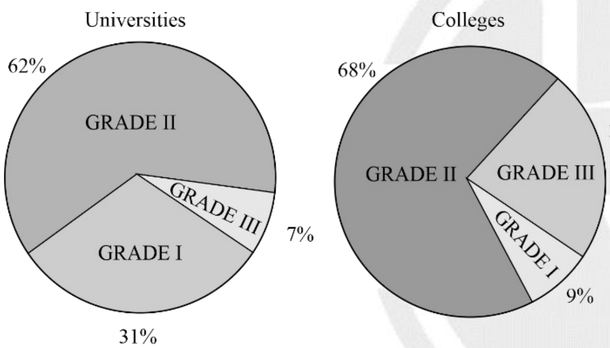
1 QUANTITATIVE APTITUDE

Data Interpretation

1. [MCQ] [GATE-ME-2023: 1M]

A certain country has 504 universities and 25951 colleges. These are categorised into Grades I, II, and III as shown in the given pie charts.

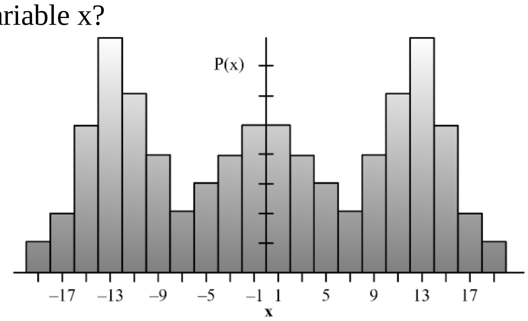
What is the percentage, correct to one decimal place, of higher education institutions (colleges and universities) that fall into Grade III?



(a) 22.7 (b) 23.7
(c) 15.0 (d) 66.8

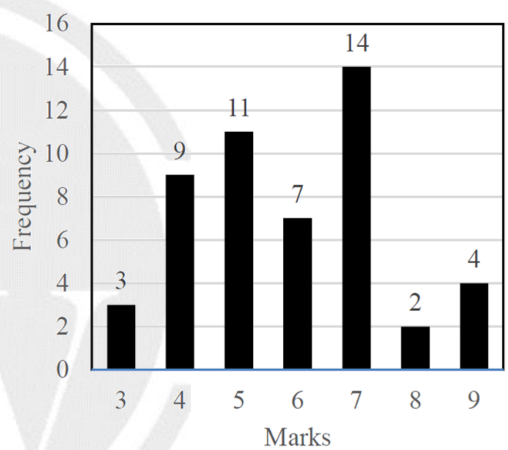
2. [MCQ] [GATE-CE-2023: 2M]

Which one of the options can be inferred about the mean, median and mode for the given probability distribution (i.e., probability mass function), P(x), of a variable x?



(a) mean = median ≠ mode
(b) mean = median = mode
(c) mean ≠ median = mode
(d) mean ≠ mode = median

3. [MCQ] [GATE-CE-2022:2M]



The above frequency chart shows the frequency distribution of marks obtained by a set of students in an exam.

From the data presented above, which one of the following is CORRECT?

(a) mean > mode > median
(b) mode > median > mean
(c) mode > mean > median
(d) median > mode > mean

4. [MCQ] [GATE-CE-2022:1M]

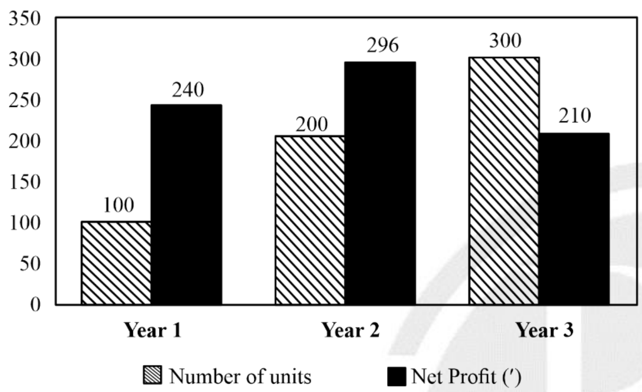
$$x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}.$$

What is the value of $\frac{x + z - y}{y}$?

(a) 0.75 (b) 1.25
(c) 2.25 (d) 3.25

5. [MCQ] [GATE-CS-2021: 2M]

The number of units of a product sold in three different years and the respective net profits are presented in the figure below. The cost/unit in Year 3 was Rs. 1, which was half the cost/unit in Year 2. The cost/ unit in Year 3 was one-third of the cost/unit in Year 1. Taxes were paid on the selling price at 10% ,13% and 15% respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year.

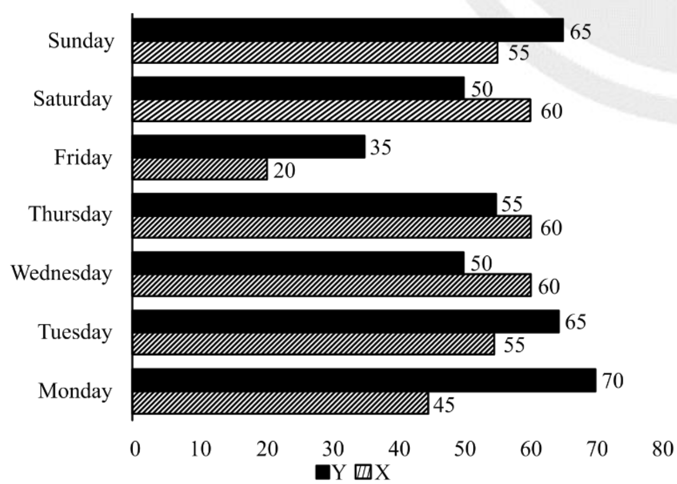


The ratio of the selling price in Year 2 to the selling price in Year 3 is__________.

(a) 4:3 (b) 1:1

(c) 3:4 (d) 1:2

6. [MCQ] [GATE-EC-2021:2M]



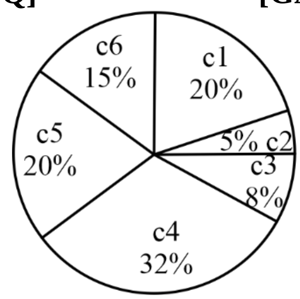
The number of minutes spent by two students, X and Y, exercising every day in a given week are shown in the bar chart above.

The number of days in the given week in which one of the students spent a minimum of 10% more than the other student, on a given day, is

(a) 4 (b) 5

(c) 6 (d) 7

7. [MCQ] [GATE-ME -2021: 2M]



Company	Ratio
C1	3:2
C2	1:4
C3	5:3
C4	2:3
C5	9:1
C6	3:4

The distribution of employees at the rank of executives, across different companies Cl, C2, ... , C6 is presented in the chart given above. The ratio of executives with a management degree to those without a management degree in each of these companies is provided in the table above. The total number of executives across all companies is 10,000.

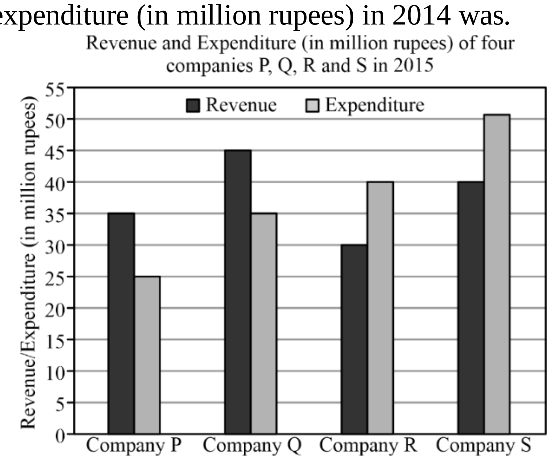
The total number of management degree holders among the executives in companies C2 and CS together is _____ .

(a) 225 (b) 600

(c) 1900 (d) 2500

8. [MCQ] [GATE-EE-2020 : 2M]

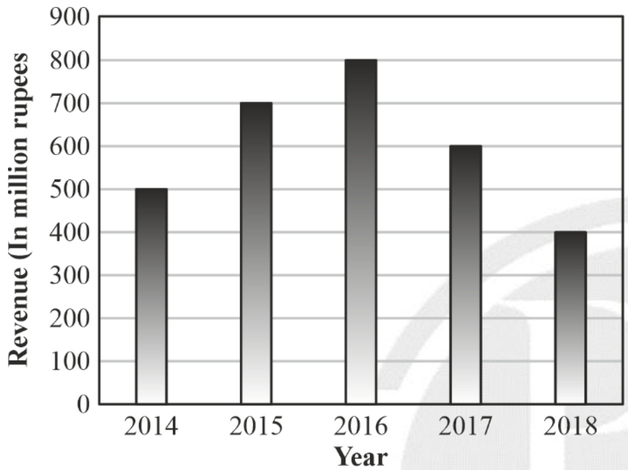
The revenue and expenditure of four different companies P. Q. R and S in 2015 are shown in the figure. If the revenue of company Q in 2015 was 20% more than that in 2014, and company Q had earned a profit of 10% on expenditure in 2014. then its expenditure (in million rupees) in 2014 was.



(a) 32.7 (b) 33.7
(c) 34.1 (d) 35.1

9. [MCQ] [GATE-CS-2020: 2M]

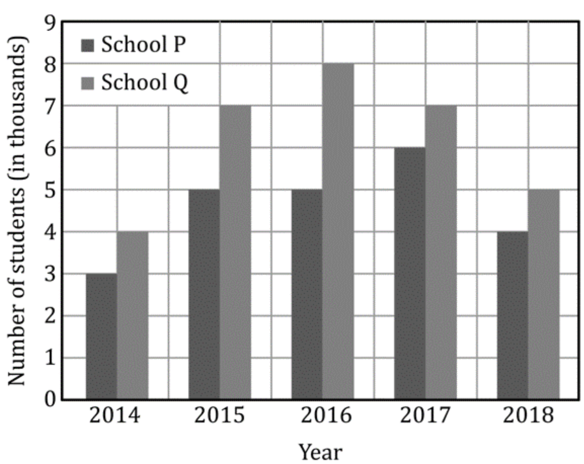
The total revenue of a company during 2014-2018 is shown in the bar graph. If the total expenditure of the company in each year is 500 million rupees, then the aggregate profit or loss (in percentage) on the total expenditure of the company during 2014-2018 is ___.



(a) 16.67% profit (b) 16.67% loss
(c) 20% profit (d) 20% loss

10. [MCQ] [GATE-EC-2020:2M]

The following figure shows the data of students enrolled in 5 years (2014 to 2018) for two schools P and Q. During this period, the ratio of the average number of the students enrolled in school P to the average of the difference of the number of students enrolled in schools P and Q is_____.

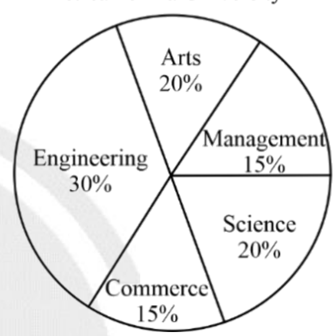


(a) 8 : 23 (b) 23 : 8
(c) 23 : 31 (d) 31 : 23

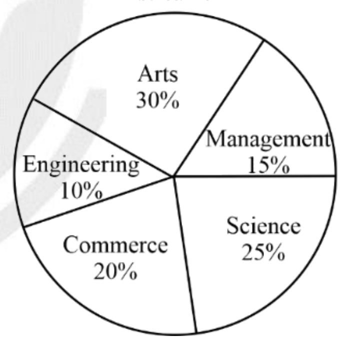
11. [MCQ] [GATE-ME -2020: 2M]

The two pie-charts given below show the data of total students and only girls registered in different streams in a university. If the total number of students registered in the university is 5000, and the total number of the registered girls is 1500; then, the ratio of boys enrolled in Arts to the girls enrolled in Management is

Percentage of students enrolled in different streams in a University



Percentage of girls enrolled in difference streams

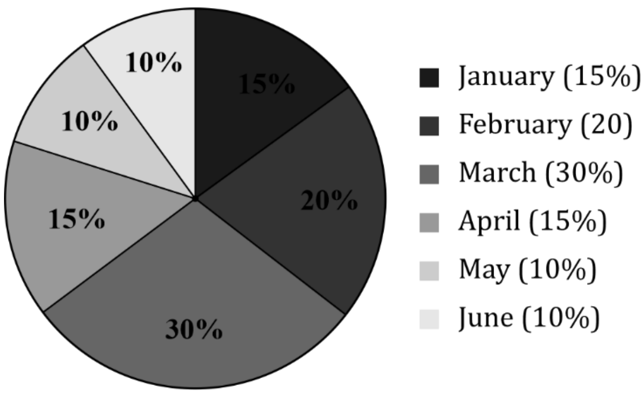


(a) 2 : 1 (b) 9 : 22
(c) 11 : 9 (d) 22 : 9

12. [MCQ] [GATE-CE-2020-2M]

The monthly distribution of 9 Watt LED bulbs sold by two firms X and Y from January to June 2018 is shown in the pie-chart and the corresponding table. If the total number of LED bulbs sold by two firms during April-June 2018 is 50000, then the number of LED bulbs sold by the firm Y during April-June 2018 is _______.

Percentage of 9 Watt LED bulbs sold by the firms X and Y from January 2018 to June, 2018

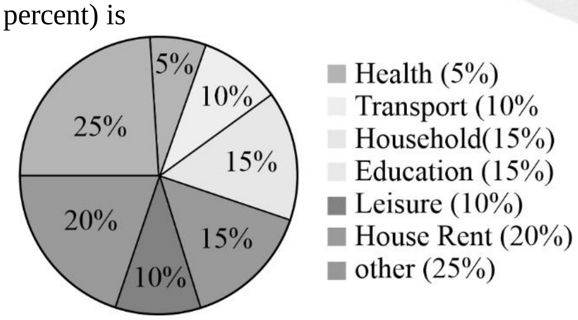


Month	Ratio of LED bulbs sodl by two firms (X : Y)
January	7 : 8
February	2 : 3
March	2 : 1
April	3 : 2
May	1 : 4
June	9 : 11

(a) 11250 (b) 9750
(c) 8750 (d) 8250

13. [MCQ] [GATE-CE-2020: 2M]

The total expenditure of a family, on different activities in a month, is shown in the pie-chart. The extra money spent on education as compared to transport (in percent) is

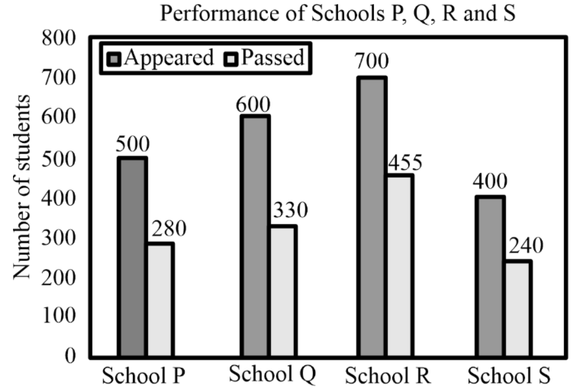


(a) 5 (b) 33.3
(c) 50 (d) 100

14. [MCQ] [GATE-ME -2020: 2M]

The bar graph shows the data of the students who appeared and passed in an examination for four schools P, Q, R and S. The average of success rates (in percentage) of these four schools is _____.

Performance of Schools P, Q, R and S

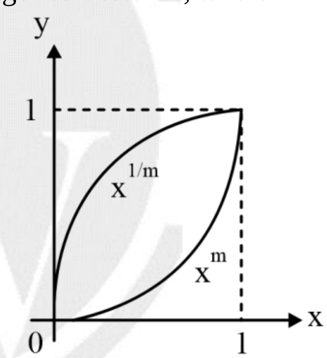


(a) 58.5 % (b) 58.8 %
(c) 59.0 % (d) 59.3 %

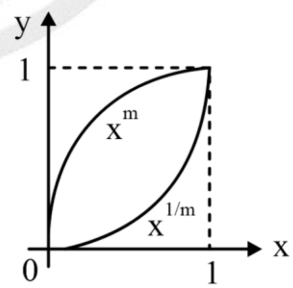
15. [MCQ] [GATE-ME-2020: 2M]

Select the graph that schematically represents BOTH $y = x^m$ and $y = x^{1/m}$ properly in the interval $0 \le x \le 1$, for integer values of m, where $m > 1$.

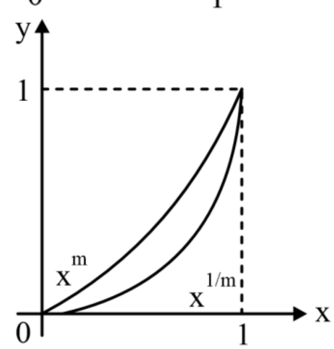
(a)

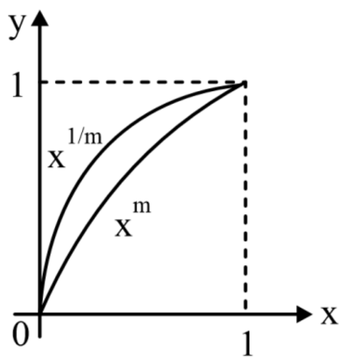


(b)



(c)

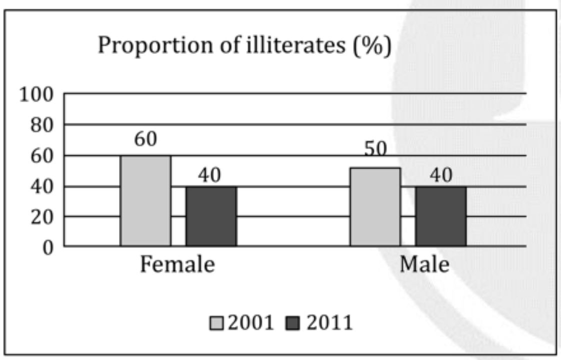


(d) 

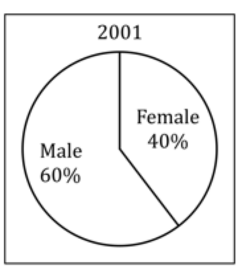
16. [MCQ] [GATE-EC-2019:2M]

The bar graph in Panel (a) shows the proportion of male and female illiterates in 2001 and 2011. The proportions of males and females in 2001 and 2011 are given in Panel (b) and (c), respectively. The total population did not change during this period.

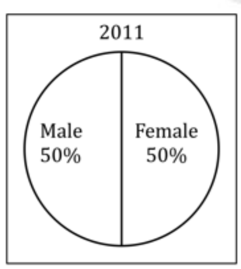
The percentage increase in the total number of literates from 2001 to 2011 is____.



Panel (a)



Panel (b)



Panel (c)

(a) 30.43 (b) 33.43
(c) 34.43 (d) 35.43

17. [MCQ] [GATE-ME-2019: 2M]

Mola is a digital platform for taxis in a city. It offers three types of rides - Pool, Mini and Prime. The Table below presents the number of rides for the past four months. The platform earns one US dollar per ride. What is the percentage share of revenue contributed by Prime to the total revenues of Mola, for the entire duration?

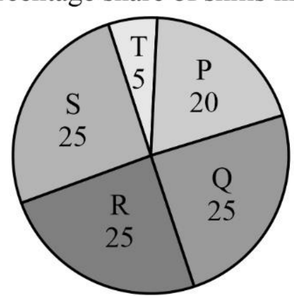
	Month			
Type	January	February	March	April
Pool	170	320	215	190
Mini	110	220	180	70
Prime	75	180	120	90

(a) 16.24 (b) 23.97
(c) 25.86 (d) 38.74

18. [MCQ] [GATE-ME-2019: 2M]

A firm hires employee at five different skill levels P, Q, R, S, T. The shares of employment at these skill levels of total employment in 2010 is given in the pie chart as shown. There were a total of 600 employees in 2010 and the total employment increased by 15% from 2010 to 2016. The total employment at skill levels P, Q and R remained unchanged during this period. If the employment at skill level S increased by 40% from 2010 to 2016, how many employees were there at skill level T in 2016?

Percentage share of skills in 2010



(a) 30 (b) 35
(c) 60 (d) 72

19. [MCQ] [GATE-EE-2018 : 2M]

P, Q, R and S crossed a lake in a boat that can hold a maximum of two persons, with only one set of oars. The following additional facts are available.

(i) The boat held two persons on each of the three forward trips across the lake and one person on each of the two return trips.

(ii) P is unable to row when someone else is in the boat.
(iii) Q is unable to row with anyone else except R.
(iv) Each person rowed for at least one trip.
(v) Only one person can row during a trip.
Who rowed twice?

(a) P (b) Q
(c) R (d) S

20. [MCQ] [GATE-EE-2018 : 2M]

A designer uses marbles of four different colours for his designs. The cost of each marble is the same, irrespective of the colour. The table below shows the percentage of marbles of each colour used in the current design. The cost of each marble increased by 25%. Therefore, the designer decided to reduce equal numbers of marbles of each colour to keep the total cost unchanged. What is the percentage of blue marbles in the new design?

Blue	Black	Red	Yellow
40%	25%	20%	15%

(a) 35.75 (b) 40.25
(c) 43.75 (d) 46.25

21. [MCQ] [GATE-CS-2018: 2M]

In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to gift Rs 750 to each male senior citizen in the town and Rs 1000 to each female senior citizen. Altogether, there were 300 senior citizens eligible for this gift. However, only 8/9th of the eligible men and 2/3rd of the eligible women claimed the gift. How much money (in Rupees) did the philanthropist give away in total?

(a) 1,50,000 (b) 2,00,000
(c) 1,75,000 (d) 1,51,000

22. [MCQ] [GATE-CE-2018:1M]

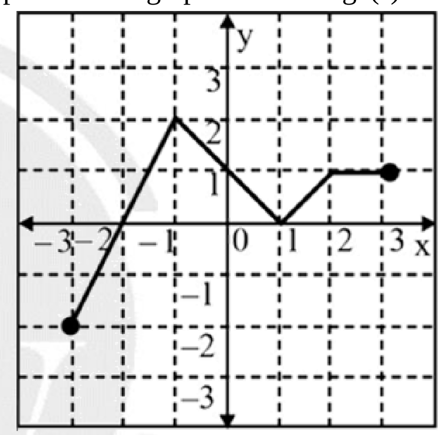
The temperature T in a room varies as a function of the outside temperature T_o and the number of persons in the room p, according to the relation $T = K(\Theta p + T_o)$, where Θ and K are constants. What would be the value of Θ given the following data?

T_0	p	T
25	2	32.4
30	5	42.0

(a) 0.8 (b) 1.0
(c) 2.0 (d) 10.0

23. [MCQ] [GATE-CE-2018: 2M]

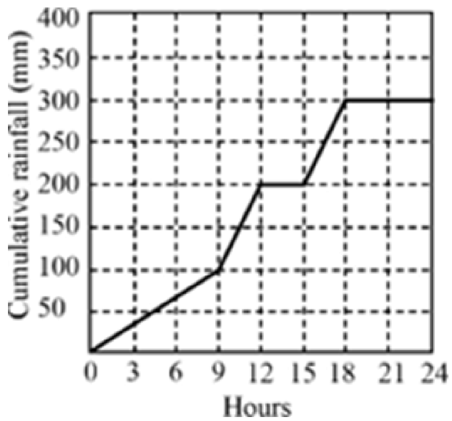
Which of the following function(s) is an accurate description of the graph for the range(s) indicated?



(i) $y = 2x + 4$ for $-3 \le x \le -1$
(ii) $y = |x - 1|$ for $-1 \le x \le 2$
(iii) $y = ||x| - 1|$ for $-1 \le x \le 2$
(iv) $y = 1$ for $2 \le x \le 3$
(a) (i), (ii) and (iii) only.
(b) (i), (ii) and (iv) only.
(c) (i) and (iv) only.
(d) (ii) and (iv) only.

24. [MCQ] [GATE-CE-2018: 2M]

The annual average rainfall in a tropical city is 1000 mm. On a particular rainy day (24-hour period), the cumulative rainfall experienced by the city is shown in the graph. Over the 24-hour period, 50% of the rainfall falling on a rooftop, which had an obstruction free area of 50 m^2, was harvested into a tank. What is the total volume of water collected in the tank in liters?



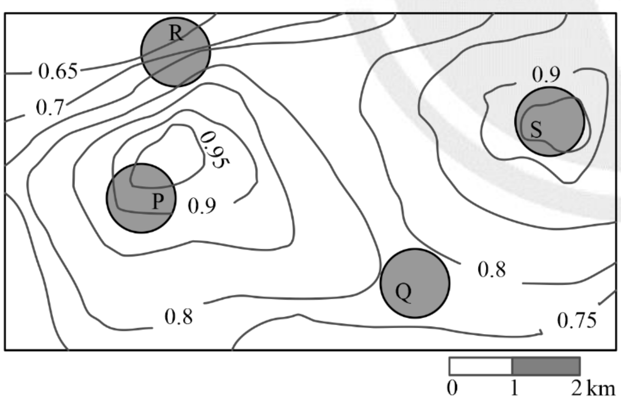
(a) 25,000 (b) 18,750

(c) 7,500 (d) 3,125

25. [MCQ] [GATE-CS,EE-2017 : 2M]

An air pressure contour line joins locations in a region having the same atmospheric pressure. The following is an air pressure contour plot of a geographical region. Contour lines are shown at 0.05 bar intervals in this plot.

If the possibility of a thunderstorm is given by how fast air pressure rises or drops over a region, which of the following regions is most likely to have a thunderstorm?

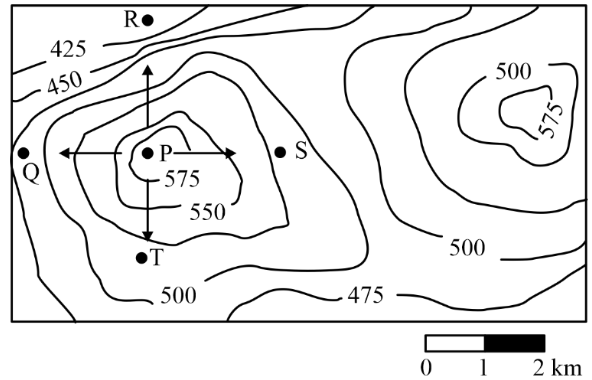


(a) P (b) Q

(c) R (d) S

26. [MCQ] [GATE-EC-2017:2M]

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.



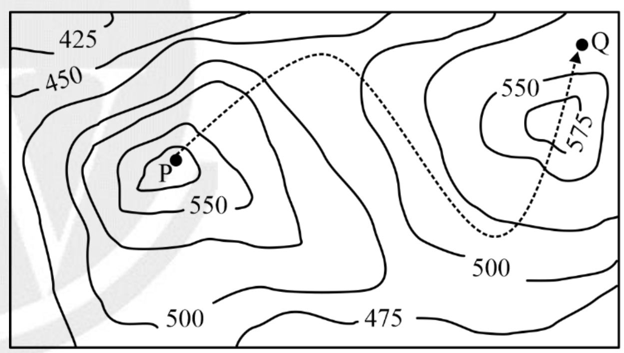
Which of the following is the steepest path leaving from P?

(a) P to Q (b) P to R

(c) P to S (d) P to T

27. [MCQ] [GATE-EC-2017:2M]

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.



The path from P to Q is best described by

(a) Up-Down-Up-Down

(b) Down-Up-Down-Up

(c) Down-Up-Down

(d) Up-Down-Up

28. [MCQ] [GATE-EC-2017:1M]

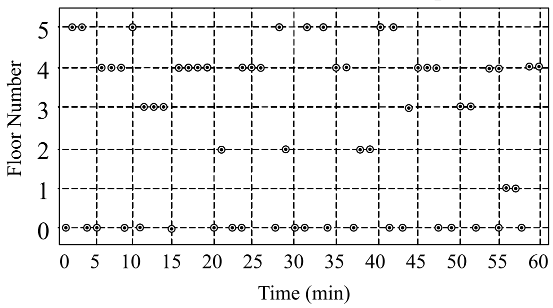
40 % of deaths on city roads may be attributed to drunken driving. The number of degrees needed to represent this as a slice of a pie chart is

(a) 120 (b) 144

(c) 160 (d) 212

29. [MCQ] **[GATE-CE-2017: 2M]**

The points in the graph below represent the halts of a lift for durations of 1 minute, over a period of 1 hour.



Which of the following statements are correct?

i. The elevator never moves directly from any non-ground floor to another non- ground floor over the one-hour period

ii. The elevator stays on the fourth floor for the longest duration over the one hour- period

(a) Only i (b) Only ii
(c) Both i and ii (d) Neither i nor ii

30. [MCQ] **[GATE-CE-2017: 2M]**

Students applying for hostel rooms are allotted rooms in order of seniority. Students already staying in a room will move if they get a room in their preferred list. Preferences of lower ranked applicants are ignored during allocation.

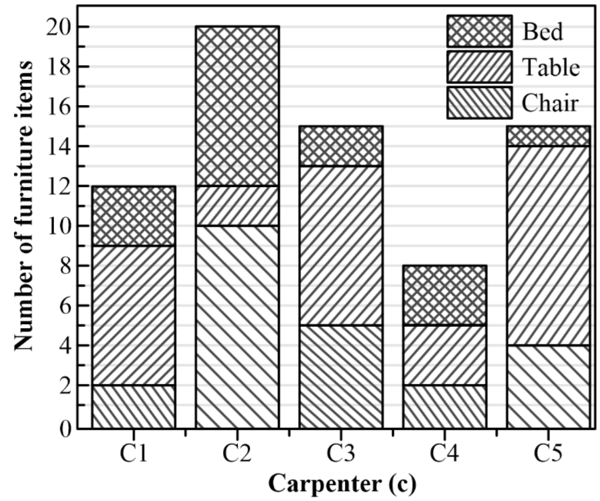
Given the data below, which room will Ajit stay in?

Names	Student seniority	Current room	Room preference list
Amar	1	P	R, S, Q
Akbar	2	None	R, S
Anthony	3	Q	P
Ajit	4	S	Q, P, R

(a) P (b) Q
(c) R (d) S

31. [MCQ] **[GATE-CE-2017: 2M]**

The bar graph below shows the output of five carpenters over one month, each of whom made different items of furniture: chairs, tables, and beds.



Consider the following statements.

i. The number of beds made by carpenter C2 is exactly the same as the number of tables made by carpenter C3.

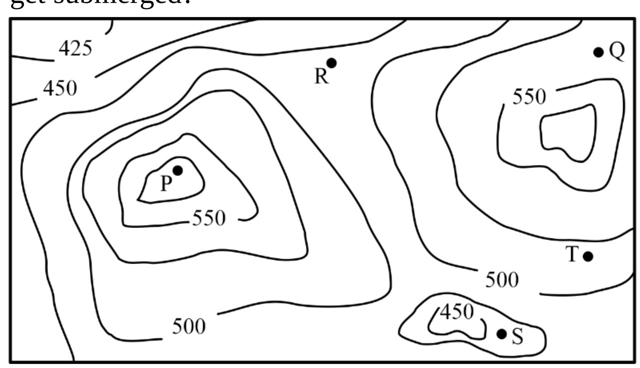
ii. The total number of chairs made by all carpenters is less than the total number of tables.

Which one of the following is true?

(a) Only i (b) Only ii
(c) Both i and ii (d) Neither i nor ii

32. [MCQ] **[GATE-CS,EE-2017 : 2M]**

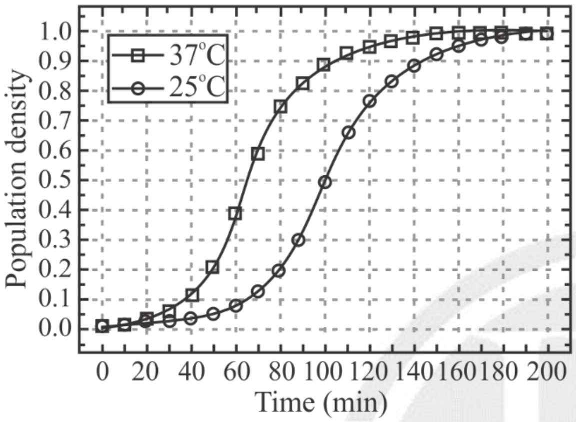
A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. If in a flood, the water level rises to 525 m. which of the villages P, Q, R, S, T get submerged?



(a) P, Q (b) P, Q, T
(c) R, S, T (d) Q, R, S

33. [MCQ] [GATE-ME-2017: 2M]

The growth of bacteria (lactobacillus) in milk leads to curd formation. A minimum bacterial population density of 0.8 (in suitable units) is needed to form curd. In the graph below, the population density of lactobacillus in 1 litre of milk is plotted as a function of time, at two different temperatures, 25°C and 37°C.



Consider the following statements based on the data shown above:

i. The growth in bacterial population stops earlier at 37°C as compared to 25°C

ii. The time taken for curd formation at 25°C is twice the time taken at 37°C

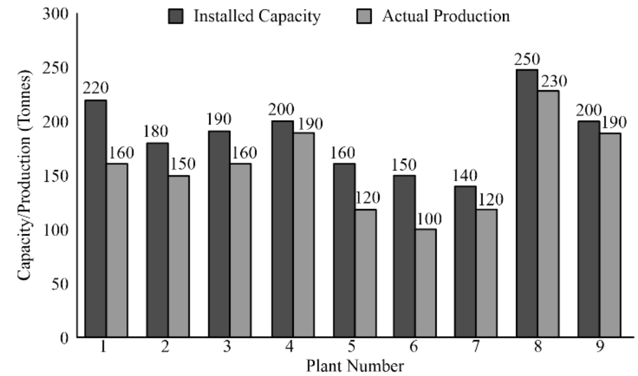
Which one of the following options is correct?

(a) Only i (b) Only ii

(c) Both i and ii (d) Neither i nor ii

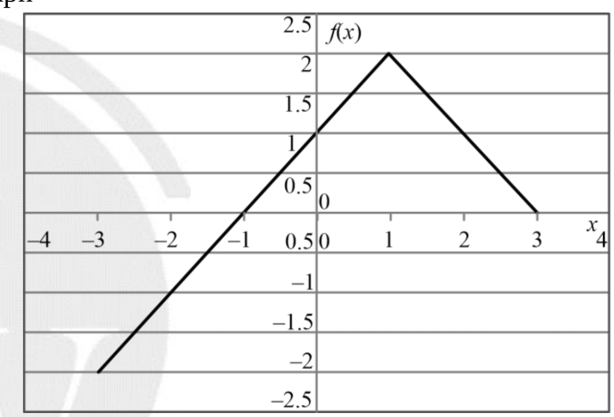
34. [NAT] [GATE-EE-2016 : 2M]

The following graph represents the installed capacity for cement production (in tonnes) and the actual production (in tonnes) of nine cement plants of a cement company. Capacity utilization of a plant is defined as ratio of actual production of cement to installed capacity. A plant with installed capacity of at least 200 tonnes is called a large plant and a plant with lesser capacity is called a small plant. The difference between total production of large plants and small plants, in tonnes is ____.



35. [MCQ] [GATE-CS,EE-2016 : 2M]

Choose the correct expression for f(x) given in the graph



(a) $f(x) = 1 - |x - 1|$ (b) $f(x) = 1 + |x - 1|$

(c) $f(x) = 2 - |x - 1|$ (d) $f(x) = 2 + |x - 1|$

36. [MCQ] [GATE-CS,CE-2016: 2M]

A shaving set company sells 4 different types of razors, Elegance, Smooth, Soft and Executive. Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78 and Executive at Rs. 173 per piece. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter \ Product	Elegance	Smooth	Soft	Executive
Q1	27300	20009	17602	9999
Q2	25222	19392	18445	8942
Q3	28976	22429	19544	10234
Q4	21012	18229	16595	10109

Which product contributes the greatest fraction to the revenue of the company in that year?

(a) Elegance

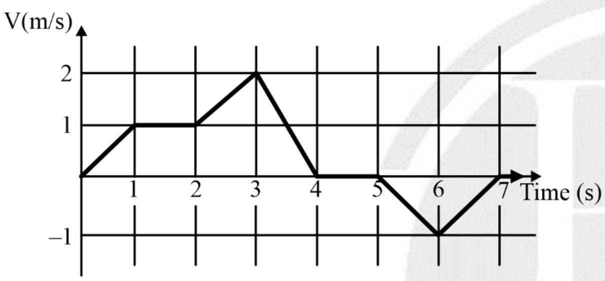
(b) Executive

(c) Smooth

(d) Soft

37. [MCQ] [GATE-EC-2016:2M]

The velocity V of a vehicle along a straight line is measured in m/s and plotted as shown with respect to time in seconds. At the end of the 7 seconds, how much will the odometer reading increase by (in m)?



(a) 0

(b) 3

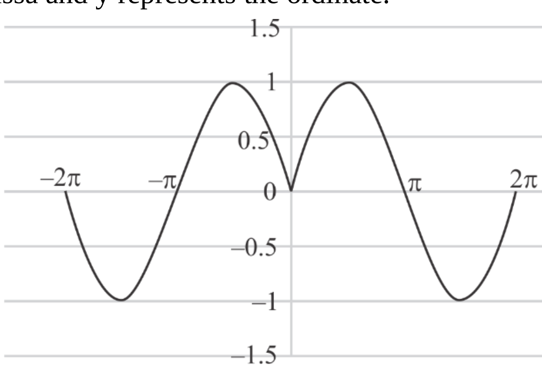
(c) 4

(d) 5

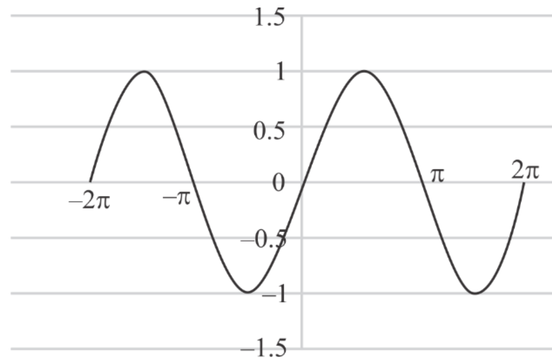
38. [MCQ] [GATE-ME-2016: 2M]

Which of the following curves represents the function $y = \ln\left(\left|e^{[|\sin(|x|)|]}\right|\right)$ for $|x| < 2\pi$? Here, x represents the abscissa and y represents the ordinate.

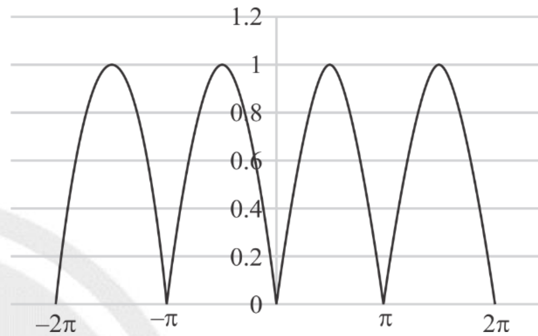
(a)



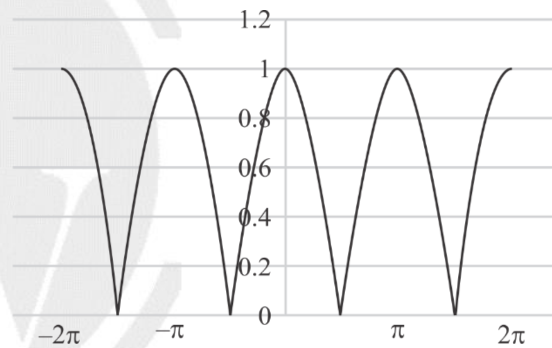
(b)



(c)



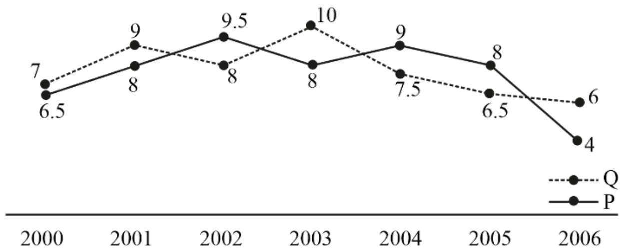
(d)



39. [MCQ] [GATE-CE-2016: 2M]

Two finance companies, P and Q, declared fixed annual rates of interest on the amounts invested with them. The rates of interest offered by these companies may differ from year to year. Year-wise annual rates of interest offered by these companies are shown by the line graph provided below.

If the amounts invested in the companies, P and Q, in 2006 are in the ratio 8:9, then the amounts received after one year as interests from companies P and Q would be in the ratio:



(a) 2 : 3 (b) 3 : 4

(c) 6 : 7 (d) 4 : 3

40. [MCQ] [GATE-EE,CS-2015 : 2M]

The number of students in a class who have answered correctly, wrongly, or not attempted each question in an exam, are listed in the table below. The marks for each question are also listed. There is no negative or partial marking.

What is the average of the marks obtained by the class in the examination?

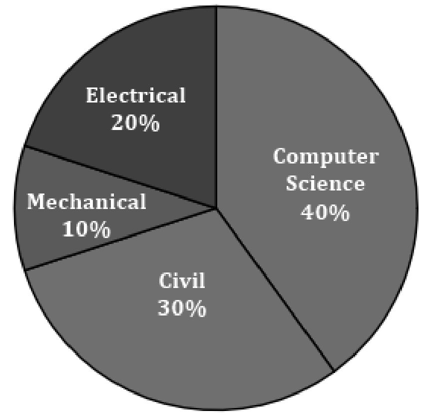
Q.No.	Marks	Answered Correctly	Answered Wrongly	Not Attemped
1	2	21	17	6
2	3	15	27	2
3	1	11	29	4
4	2	23	18	3
5	5	31	12	1

(a) 2.290 (b) 2.970

(c) 6.795 (d) 8.795

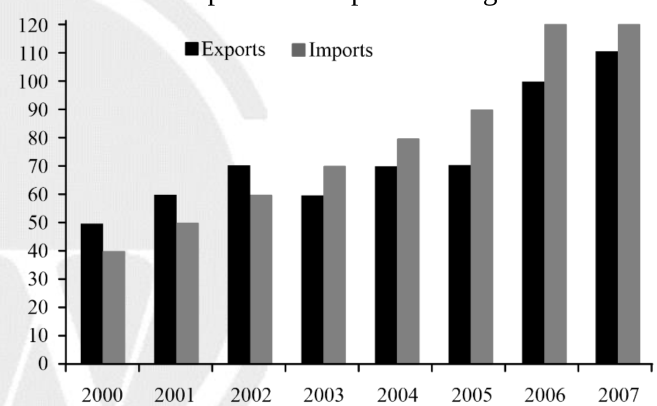
41. [NAT] [GATE-EE,CS-2015 : 2M]

The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is 5:4. There are 40 males in Electrical Engineering. What is the difference between the numbers of female students in the Civil department and the female students in the Mechanical department?



42. [NAT] [GATE-CS,CE-2015: 2M]

The exports and imports (in crores of Rs.) of a country from the year 2000 to 2007 are given in the following bar chart. In which year is the combined percentage increase in imports and exports the highest?



43. [MCQ] [GATE-CE-2015: 2M]

Read the following table giving sales data of five types of batteries for years 2006 to 2012:

Out of the following, which type of battery achieved highest growth between the years 2006 and 2012?

Year	Type I	Type II	Type III	Type IV	Type V
2006	75	144	114	102	108
2007	90	126	102	84	126
2008	96	114	75	105	135
2009	105	90	150	90	75
2010	90	75	135	75	90
2011	105	60	1065	45	120
2012	115	85	160	100	145

(a) Type V
(b) Type III
(c) Type II
(d) Type I

44. [MCQ] [GATE-ME,EC-2015: 1M]

An electric bus has onboard instruments that report the total electricity consumed since the start of the trip as well as the total distance covered. During a single day of operation, the bus travels on stretches M. N, O and P, in that order. The cumulative distances travelled and the corresponding electricity consumption are shown in the Table below:

Stretch	Cumulative distance (km)	Electricity used (kWh)
M	20	12
N	45	25
O	75	45
P	100	57

The stretch where the electricity consumption per km is minimum is

(a) M
(b) N
(c) O
(d) P

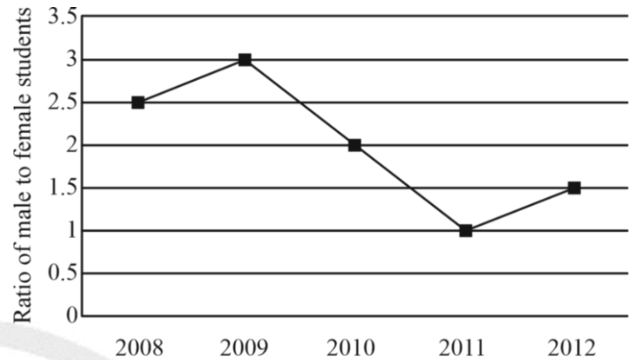
45. [NAT] [GATE-EE,CS-2014 :2 M]

In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own Vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

46. [NAT] [GATE-EE,CS-2014 : 2M]

The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



47. [MCQ] [GATE-EE,CS-2014 : 1M]

The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

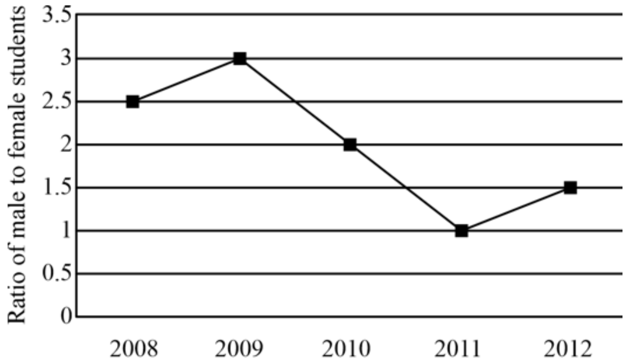
What is the average of the marks obtained by the class in the examination?

Q.No	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

(a) 1.34
(b) 1.74
(c) 3.02
(d) 3.91

48. [MCQ] [GATE-EE,CS-2014 : 2M]

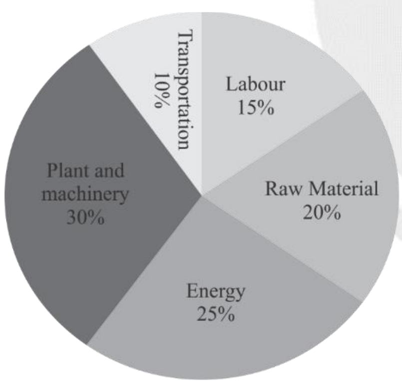
The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?



(a) 1:1 (b) 2:1
(c) 1.5:1 (d) 2.5:1

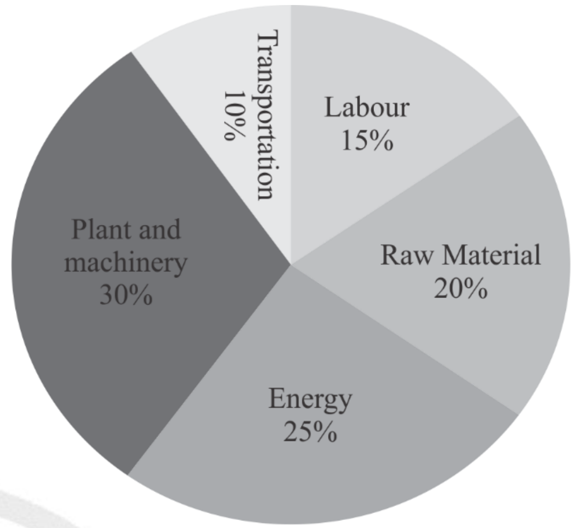
49. [NAT] [GATE-ME,EC-2014: 2M]

A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is Rs. 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. What is the percentage increase in total cost for the company in 2013?



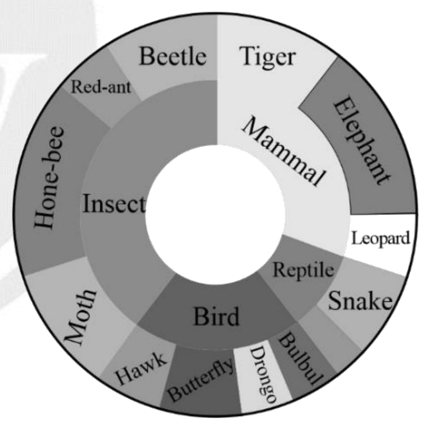
50. [NAT] [GATE-ME,EC-2014: 2M]

A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is Rs. 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. If the company registered a profit of Rs. 10 lakhs in 2012, at what price (in Rs.) was each air purifier sold?



51. [MCQ] [GATE-ME,EC-2014: 2M]

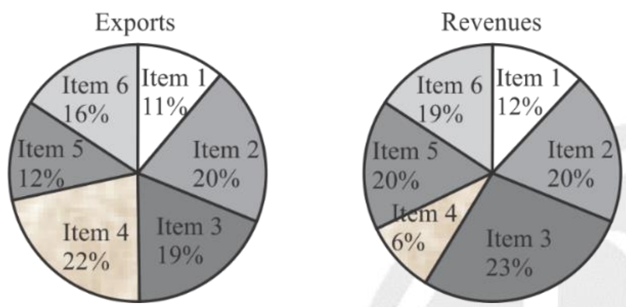
The multi-level hierarchical pie chart shows the population of animals in a reserve forest. The correct conclusions from this information are:



(i) Butterflies are birds
(ii) There are more tigers in this forest than red ants
(iii) All reptiles in this forest are either snakes or crocodiles
(iv) Elephants are the largest mammals in this forest
(a) (i) and (ii) only
(b) (i), (ii), (iii) and (iv)
(c) (i), (iii) and (iv) only
(d) (i), (ii) and (iii) only

52. [MCQ] [GATE-ME,EC-2014: 2M]

The total exports and revenues from the exports of a country are given in the two charts shown below. The pie chart for exports shows the quantity of each item exported as a percentage of the total quantity of exports. The pie chart for the revenues shows the percentage of the total revenue generated through export of each item. The total quantity of exports of all the items is 500 thousand tonnes and the total revenues are 250 crore rupees. Which item among the following has generated the maximum revenue per kg?

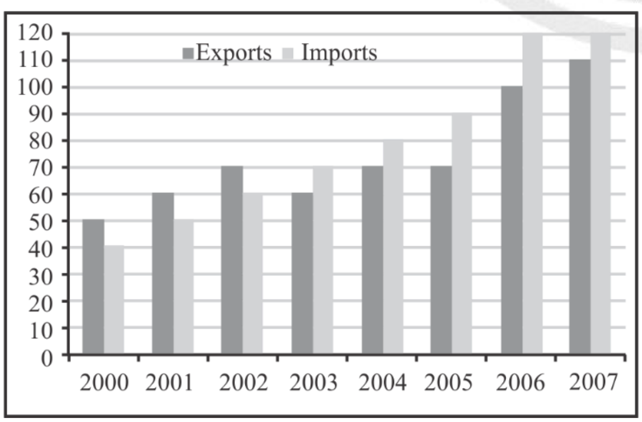


(a) Item 2 (b) Item 3

(c) Item 6 (d) Item 5

53. [MCQ] [GATE-ME,EC-2014: 2M]

The exports and imports (in crores of Rs.) of a country from 2000 to 2007 are given in the following bar chart. If the trade deficit is defined as excess of imports over exports, in which year is the trade deficit 1/5th of the exports?



(a) 2005 (b) 2004

(c) 2007 (d) 2006

54. [MCQ] [GATE-ME,EC-2014: 1M]

The statistics of runs scored in a series by four batsmen are provided in the following table. Who is the most consistent batsman of these four?

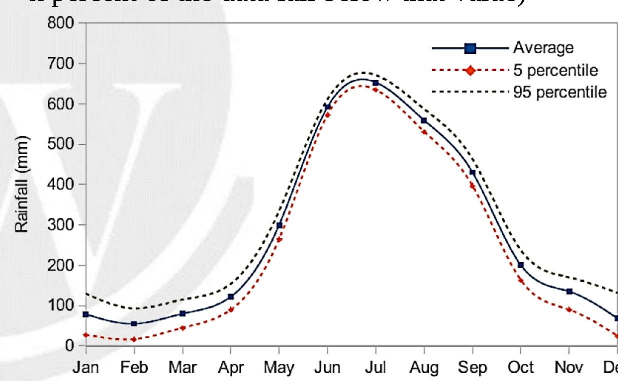
Batsman	Average	Standard deviation
K	31.2	5.21
L	46.0	6.35
M	54.4	6.22
N	17.9	5.90

(a) K (b) L

(c) M (d) N

55. [MCQ] [GATE-CE-2014: 2M]

The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (k percentile is the value such that k percent of the data fall below that value)



(i) On average, it rains more in July than in December

(ii) Every year, the amount of rainfall in August is more than that in January

(iii) July rainfall can be estimated with better confidence than February rainfall

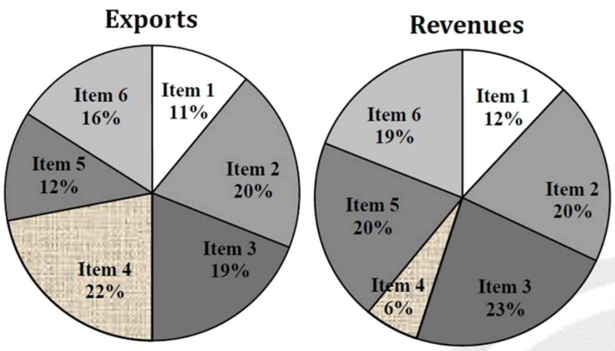
(iv) In August, there is at least 500 mm of rainfall

(a) (i) and (ii) (b) (i) and (iii)

(c) (ii) and (iii) (d) (iii) and (iv)

56. [MCQ] [GATE-CE-2014: 2M]

The total exports and revenues from the exports of a country are given in the two pie charts below. The pie chart for exports shows the quantity of each item as a percentage of the total quantity of exports. The pie chart for the revenues shows the percentage of the total revenue generated through export of each item. The total quantity of exports of all the items is 5 lakh tonnes and the total revenues are 250 crore rupees. What is the ratio of the revenue generated through export of Item 1 per kilogram to the revenue generated through export of Item 4 per kilogram?



(a) 1:2 (b) 2:1
(c) 1:4 (d) 4:1

57. [MCQ] [GATE-CE-2013: 2M]

Following table provides figures (in rupees) on annual expenditure of a firm for two years – 2010 and 2011.
In 2011, which of the following two categories have registered increase by same percentage?

Category	2010	2011
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

(a) Raw material and Salary & wages
(b) Salary & wages and Advertising
(c) Power & fuel and Advertising
(d) Raw material and Research & Development

58. [MCQ] [GATE-EE,EC-2012 : 2M]

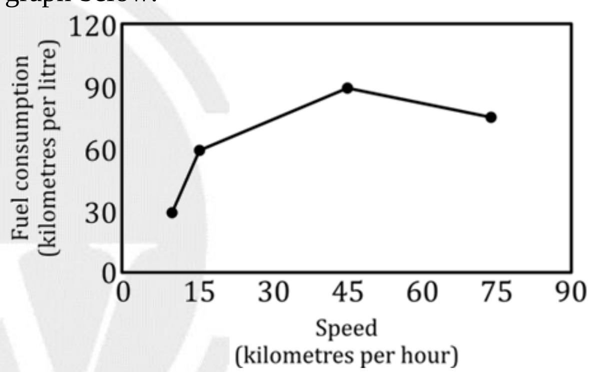
The data given in the following table summarizes the monthly budget of an average household.

Category	Amount (Rs)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is
(a) 10% (b) 14%
(c) 81% (d) 86%

59. [MCQ] [GATE-EC,EE-2011 : 2M]

The fuel consumed by a motorcycle during a journey while traveling at various speeds is indicated in the graph below.



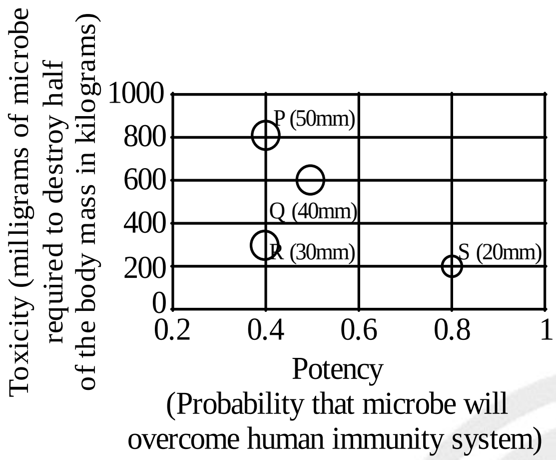
The distances covered during four laps of the journey are listed in the table below

Lap	Distance (Kilometres)	Average Speed (Kilometres per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap
(a) P (b) Q
(c) R (d) S

60. [MCQ] [GATE-ME,CE,CS-2011: 2M]

P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:



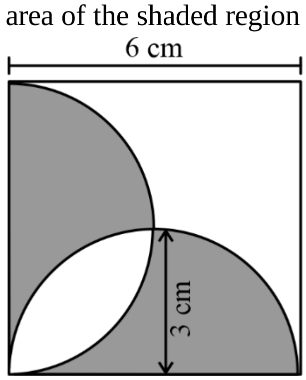
A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

(a) P (b) Q
(c) R (d) S

Numerical Computation

61. [MCQ] [GATE-EE-2023 : 2M]

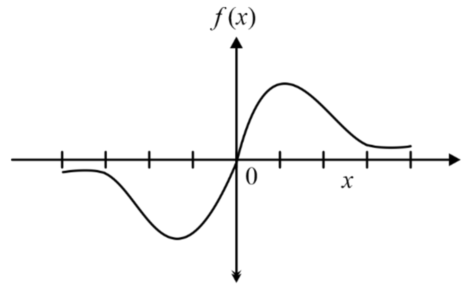
A square with sides of length 6 cm is given. The boundary of the shaded region is defined by two semi-circles whose diameters are the sides of the square, as shown. The area of the shaded region is _______ cm^2.



(a) 6π
(b) 18
(c) 20
(d) 9π

62. [MCQ] [GATE-EE-2023 : 2M]

Which one of the following options represents the given graph?



(a) $f(x) = x^2 2^{-|x|}$ (b) $f(x) = x2^{-|x|}$
(c) $f(x) = |x| 2^{-x}$ (d) $f(x) = x2^{-x}$

63. [MCQ] [GATE-EE-2023 : 1M]

Given a fair six-faced dice where the faces are labelled '1', '2', '3', '4', '5', and '6', what is the probability of getting a '1' on the first roll of the dice and a '4' on the second roll?

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$
(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

64. [MCQ] [GATE-CS-2023:2M]

Consider two function of time (t)

$f(t) = 0.01\, t^2$

$g(t) = 4t$

Where $0 < t < \infty$

Now consider the following two statements:

(i) For some $t > 0$, $g(t) > f(t)$

(ii) There exists a T, such that $f(t) > g(t)$ for all $t > T$

Which one of the following options is TRUE?

(a) Only (i) is correct
(b) Only (ii) is correct
(c) Both (i) and (ii) are correct
(d) Neither (i) nor (ii) is correct

65. [MCQ] [GATE-CS-2023:2M]

f(x) and g(y) are functions of x and y, respectively, and f(x) = g(y) for all real values of x and y. Which one of the following options is necessarily TRUE for all x and y?

(a) $f(x) = 0$ and $g(y) = 0$

(b) $f(x) = g(y) =$ constant

(c) $f(x) \neq$ constant and $g(y) \neq$ constant

(d) $f(x) + g(y) = f(x) - g(y)$

66. [MCQ] [GATE-ME-2023: 2M]

How many pairs of sets (S,T) are possible among the subsets of {1, 2, 3, 4, 5, 6} that satisfy the condition that S is a subset of T?

(a) 729 (b) 728
(c) 665 (d) 664

67. [MCQ] [GATE-ME-2023: 2M]

Consider the following inequalities

$$p^2 - 4q < 4$$
$$3p + 2q < 6$$

Where p and q are positive integers.

The value of (p + q) is ______.

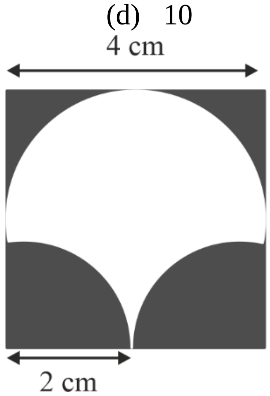
(a) 2 (b) 1
(c) 3 (d) 4

68. [MCQ] [GATE-CE-2023: 2M]

A square of side length 4 cm is given. The boundary of the shaded region is defined by one semi-circle on the top and two circular arcs at the bottom each of radius 2 cm, as shown.

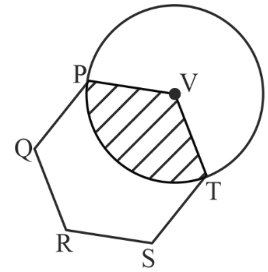
The area of the shaded region is ______ cm^2.

(a) 8 (b) 4
(c) 12 (d) 10



69. [MCQ] [GATE-CE-2023: 1M]

In the given figure, PQRSTV is a regular hexagon with each side of length 5 cm. A circle is drawn with its centre at V such that it passes through P. What is the area (in cm^2) of the shaded region? (The diagram is representative)



(a) $\frac{25\pi}{3}$ (b) $\frac{20\pi}{3}$
(c) 6π (d) 7π

70. [MCQ] [GATE-CE-2023: 2M]

Let a = 30! b = 50! and c = 100! Consider the following numbers: $\log_a c$, $\log_c a$, $\log_b a$, $\log_a b$

Which one of the following inequalities is CORRECT?

(a) $\log_c a < \log_b a < \log_a b < \log_a c$
(b) $\log_c a < \log_a b < \log_b a < \log_b c$
(c) $\log_c a < \log_b a < \log_a c < \log_a b$
(d) $\log_b a < \log_c a < \log_a b < \log_a c$

71. [MCQ] [GATE-EE-2022 : 2M]

There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely.

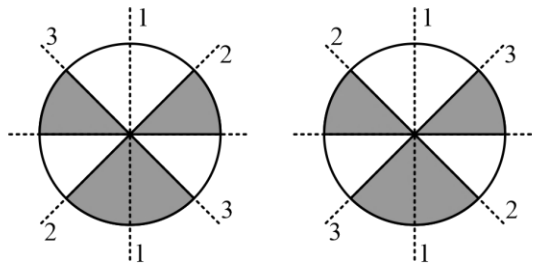
The two dice are thrown once independently at random. What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?

(a) 1/4 (b) 3/4
(c) 1/6 (d) 5/36

72. [MCQ] [GATE-EE-2022 : 1M]

The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random.

What is the probability that the disc DOES NOT retain the same front and rear views after the flipping operation?



Front View Rear View

(a) 0 (b) 1/3
(c) 2/3 (d) 1

73. [MCQ] [GATE-CS-2022:1M]

A function y(x) is defined in the interval [0, 1] on the x-axis as

$$y(x) = \begin{cases} 2 \text{ if } 0 \le x < \frac{1}{3} \\ 3 \text{ if } \frac{1}{3} \le x < \frac{3}{4} \\ 1 \text{ if } \frac{3}{4} \le x \le 1 \end{cases}$$

Which one of the following is the area under the curve for the interval [0, 1] on the x-axis?

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$

(c) $\frac{13}{6}$ (d) $\frac{6}{13}$

74. [MCQ] [GATE-EC-2022:2M]

Four points P (0, 1), Q (0, −3), R (−2, −1), and S (2, −1) represent the vertices of a quadrilateral.

What is the area enclosed by the quadrilateral?

(a) 4 (b) $4\sqrt{2}$

(c) 8 (d) $8\sqrt{2}$

75. [MCQ] [GATE-EC-2022:2M]

Consider the following inequalities.

(i) $2x - 1 > 7$

(ii) $2x - 9 < 1$

Which one of the following expressions below satisfies the above two inequalities?

(a) $x \le -4$ (b) $-4 < x \le 4$

(c) $4 < x < 5$ (d) $x \ge 5$

76. [MCQ] [GATE-EC-2022:1M]

A trapezium has vertices marked as P, Q, R and S (in that order anticlockwise).

The side PQ is parallel to side SR.

Further, it is given that, PQ = 11 cm, QR = 4 cm, RS = 6 cm and SP = 3 cm.

What is the shortest distance between PQ and SR (in cm)?

(a) 1.80 (b) 2.40

(c) 4.20 (d) 5.76

77. [MCQ] [GATE-CE-2022:2M]

Consider the following equations of straight lines:

Line L1: $2x - 3y = 5$

Line L2: $3x + 2y = 8$

Line L3: $4x - 6y = 5$

Line L4: $6x - 9y = 6$

Which one among the following is the correct statement?

(a) L1 is parallel to L2 and L1 is perpendicular to L3

(b) L2 is parallel to L4 and L2 is perpendicular to L1

(c) L3 is perpendicular to L4 and L3 is parallel to L2

(d) L4 is perpendicular to L2 and L4 is parallel to L3

78. [MCQ] [GATE-ME-2022: 2M]

Consider the following functions for non-zero positive integers, p and q.

$$f(p,q) = \underbrace{p \times p \times p \times \ldots\ldots\ldots \times p}_{q \text{ terms}} = p^q; \quad f(p, 1) = p$$

$$g(p,q) = p^{p^{p^{p^{p^{\cdot^{\cdot^{\cdot^{(\text{up to } q \text{ terms})}}}}}}}} \quad ; g(p,1) = p$$

Which one of the following options is correct based on the above?

(a) f (2, 2) = g(2, 2)

(b) f(g(2, 2), 2) < f(2, g(2, 2))

(c) h(2, 1) ≠ f(2, 1)

(d) F(3, 2) > g(3, 2)

79. [MCQ] [GATE-ME-2022: 1M]

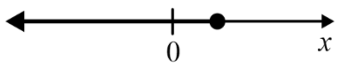
If $f(x) = 2\ln(\sqrt{e^x})$ what is the area bounded by $f(x)$ for the interval [0, 2] on the x-axis?

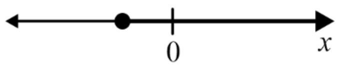
(a) $\frac{1}{2}$ (b) 1

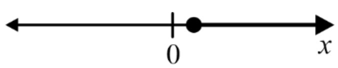
(c) 2 (d) 4

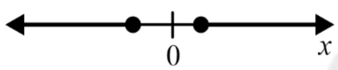
80. [MCQ] [GATE-ME-2022: 1M]

Which one of the following is a representation (not to scale and in bold) of all values of x satisfying the inequality $2 - 5x \le -\frac{6x-5}{3}$ on the real number line?

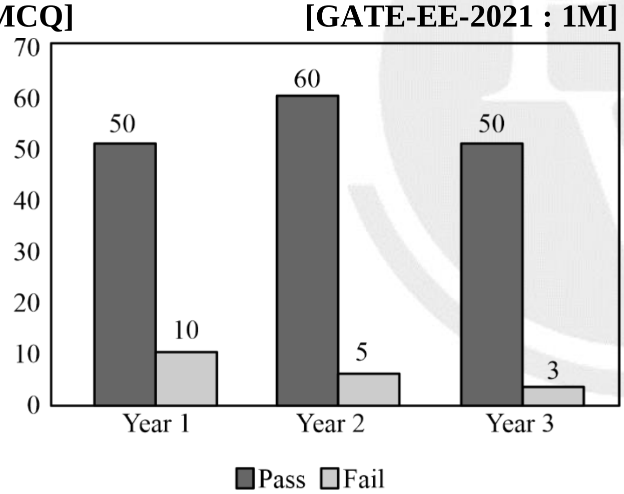
(a) 

(b) 

(c) 

(d) 

81. [MCQ] [GATE-EE-2021 : 1M]



The number of students passing or failing in an exam for a particular subject are presented in the bar chart above. Students who pass the exam cannot appear for the exam again. Students who fail the exam in the first attempt must appear for the exam in the following year. Students always pass the exam in their second attempt. The number of students who took the exam for the first time in the year 2 and the year 3 respectively, are.

(a) 65 and 53 (b) 60 and 50

(c) 55 and 53 (d) 55 and 48

82. [MCQ] [GATE-EE-2021 : 1M]

Let X be a continuous random variable denoting the temperature measured. The range of temperature is [0, 100] degree Celsius and let the probability density function of X be $f(x) = 0.01$ for $0 \le X \le 100$. The mean of X is __________.

(a) 2.5 (b) 5.0

(c) 25.0 (d) 50.0

83. [MCQ] [GATE-CS-2021: 1M]

If θ is the angle, in degrees, between the longest diagonal of the cube and any one of the edges of the cube, then, $\cos\theta =$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

84. [MCQ] [GATE-CS-2021: 1M]

If $\left(x - \frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2 = x + 2$, then the value of x is

(a) 2 (b) 4

(c) 6 (d) 8

85. [MCQ] [GATE-CE-2021: 2M]

A function λ, is defined by

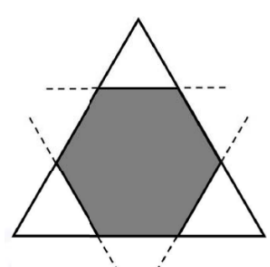
$$\lambda(p, q) = \begin{cases} (p-q)^2, \text{if } p \ge q \\ p + q, \text{ if } p < q \end{cases}$$

The value of the expression λ(–(–3+2),(–2+3))/((–(–2+1))) is:

(a) –1 (b) 0

(c) $\frac{16}{3}$ (d) 16

86. [MCQ] [GATE-EC-2021:2M]



Corners are cut from an equilateral triangle to produce a regular convex hexagon as shown in the figure above.

The ratio of the area of the regular convex hexagon to the area of the original equilateral triangle is

(a) 2 : 3 (b) 3 : 4
(c) 4 : 5 (d) 5 : 6

87. [MCQ] [GATE-EC-2021:2M]

Consider a square sheet of side 1 unit. In the first step, it is cut along the main diagonal to get two triangles. In the next step, one of the cut triangles is revolved about its short edge to form a solid cone. The volume of the resulting cone, in cubic units, is______

(a) $\pi/3$ (b) $2\pi/3$
(c) $3\pi/2$ (d) 3π

88. [MCQ] [GATE-EC-2021:1M]

p and q are positive integers and $\frac{p}{q}+\frac{q}{p}=3$,

then, $\frac{p^2}{q^2}+\frac{q^2}{p^2}=$

(a) 3 (b) 7
(c) 9 (d) 11

89. [MCQ] [GATE-CE-2021: 1M]

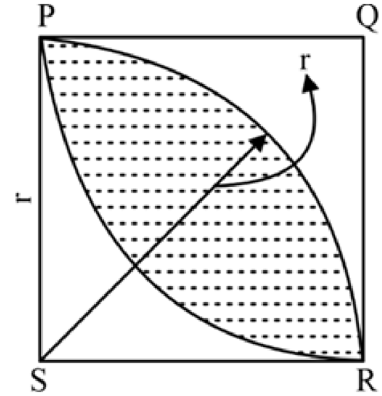
Two identical cube shaped dice each with faces numbered 1 to 6 are rolled simultaneously. The probability that an even number is rolled out on each dice is:

(a) $\frac{1}{36}$ (b) $\frac{1}{8}$
(c) $\frac{1}{12}$ (d) $\frac{1}{4}$

90. [MCQ] [GATE-CE-2021: 2M]

In the figure shown above, PQRS is a square. The shaded portion is formed by the intersection of sectors of circles with radius equal to the side of the square and centers at S and Q.

The probability that any point picked randomly within the square falls in the shaded area is.

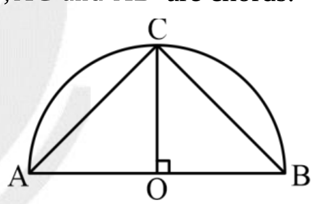


(a) $4-\frac{\pi}{2}$ (b) $\frac{1}{2}$
(c) $\frac{\pi}{2}-1$ (d) $\frac{\pi}{4}$

91. [MCQ] [GATE-EE-2020 : 2M]

Given a semicircle with *O* as the centre, as shown in the figure, the ratio $\frac{\overline{AC}+\overline{CB}}{\overline{AB}}$ is

Where, $\overline{AC}, \overline{AC}$ and $\overline{AB}$ are chords.



(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 2 (d) 3

92. [MCQ] [GATE-EE-2020 : 2M]

In four-digit integer numbers from 1001 to 9999, the digit group "37" (in the same sequence) appears __________ times.

(a) 270 (b) 279
(c) 280 (d) 299

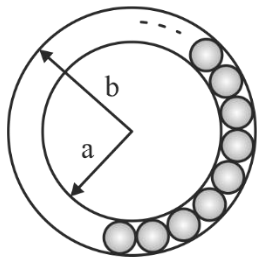
93. [MCQ] [GATE-EE-2020 : 1M]

If P, Q, R, S are four individuals, how many teams of size exceeding one can be formed, with Q as a member?

(a) 5 (b) 6
(c) 7 (d) 8

94. [MCQ] [GATE-CS-2020: 2M]

The figure below shows an annular ring with outer and inner radii as b and a, respectively. The annular space has been painted in the form of blue color circles touching the outer and inner periphery of annular space. If maximum n number of circles can be painted, then the unpainted area available in annular space is _____.



(a) $\pi\left[(b^2-a^2)-\frac{n}{4}(b-a)^2\right]$

(b) $\pi\left[(b^2-a^2)-n(b-a)^2\right]$

(c) $\pi\left[(b^2-a^2)+\frac{n}{4}(b-a)^2\right]$

(d) $\pi\left[(b^2-a^2)+n(b-a)^2\right]$

95. [MCQ] [GATE-CS-2020: 2M]

Two straight lines are drawn perpendicular to each other in X-Yplane. If α and β are the acute angles the straight lines make with the X-axis, then $\alpha + \beta$ is ________.

(a) 60° (b) 90°

(c) 120° (d) 180°

96. [MCQ] [GATE-EC-2020:2M]

a, b, c are real numbers. The quadratic equation $ax^2 - bx + c = 0$ has equal roots, which is β, then

(a) $\beta = b/a$

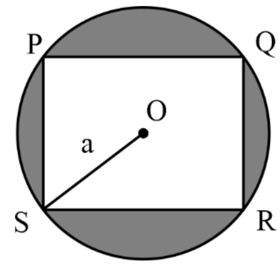
(b) $\beta^2 = ac$

(c) $\beta^3 = bc/(2a^2)$

(d) $b^2 \neq 4ac$

97. [MCQ] [GATE-EC-2020:2M]

A circle with centre O is shown in the figure. A rectangle PQRS of a maximum possible area is inscribed in the circle. If the radius of the circle is a, then the area of the shaded portion is ________.



(a) $\pi a^2 - a^2$ (b) $\pi a^2 - \sqrt{2}a^2$

(c) $\pi a^2 - 2a^2$ (d) $\pi a^2 - 3a^2$

98. [MCQ] [GATE-EC-2020:1M]

A superadditive function f (·) satisfies the following property

$f(x_1 + x_2) \geq f(x_1) + f(x_2)$

Which of the following functions is a superadditive function for x > 1?

(a) e^x (b) $\sqrt{x}$

(c) $1/x$ (d) e^{-x}

99. [MCQ] [GATE-CE-2020: 1M]

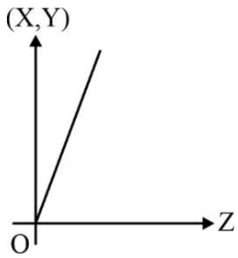
If $f(x) = x^2$ for each $x \in (\infty, \infty)$ then $\frac{f\left(f\left(f(x)\right)\right)}{f(x)}$ is equal to_____.

(a) $f(x)$ (b) $(f(x))^2$

(c) $(f(x))^3$ (d) $(f(x))^4$

100.[MCQ] [GATE-ME-2020: 2M]

An engineer measures THREE quantities X, Y and Z in an experiment. She finds that they follow a relationship that is represented in the figure below: (the product of X and Y linearly varies with Z)



Then, which of the following statements is FALSE?
(a) For fixed Z; X is proportional to Y
(b) For fixed Y; X is proportional to Z
(c) For fixed X; Z is proportional to Y
(d) XY/Z is constant

101.[MCQ] [GATE-ME-2020: 2M]
The sum of the first n terms in the sequence 8, 88, 888, 8888, …… is
(a) $\frac{81}{80}\left(10^n - 1\right) + \frac{9}{8}n$
(b) $\frac{81}{80}\left(10^n - 1\right) - \frac{9}{8}n$
(c) $\frac{80}{81}\left(10^n - 1\right) + \frac{8}{9}n$
(d) $\frac{80}{81}\left(10^n - 1\right) - \frac{8}{9}n$

102.[MCQ] [GATE-ME-2020: 1M]
Define [x] as the greatest integer less than or equal to x, for each x ∈ (–∞, ∞). If y = [x], then area under y for x ∈ [1,4] is _____.
(a) 1 (b) 3
(c) 4 (d) 6

103.[MCQ] [GATE-EE-2019 : 2M]
Given two sets X = {1, 2, 3} and Y = {2, 3. 4}, we construct a set Z of all possible fractions where the numerators belong to set X and the denominators belong to set Y. The product of elements having minimum and maximum values in the set Z is____________.
(a) 1/12
(b) 1/8
(c) 1/6
(d) 3/8

104.[MCQ] [GATE-CE-2019: 1M]
On a horizontal ground, the base of a straight ladder is 6 m away from the base of a vertical pole. The ladder makes an angle of 45° to the horizontal. If the ladder is resting at a point located at one-fifth of the height of the pole from the bottom, the height of the pole is _______ maters.
(a) 15 (b) 25
(c) 30 (d) 35

105.[MCQ] [GATE-EE-2018 : 2M]
A class of twelve children has two more boys than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than boys?
(a) 0 (b) $\frac{325}{864}$
(c) $\frac{525}{864}$ (d) $\frac{5}{12}$

106.[MCQ] [GATE-EE-2018 : 2M]
An e-mail password must contain three characters. The password has to contain one numeral from 0 to 9, one upper case and one lower case character from the English alphabet. How many distinct passwords are possible?
(a) 6,760 (b) 13,520
(c) 40,560 (d) 1,05,456

107.[MCQ] [GATE-EE-2018 : 1M]
Functions F(a, b) and G(a, b) are defined as follows:
$F(a, b) = (a - b)^2$ and $G(a, b) = |a - b|$, where $|x|$ represents the absolute value of x. What would be the value of G(F(1, 3), G(1, 3))?
(a) 2 (b) 4
(c) 6 (d) 36

108.[MCQ] [GATE-EE-2018 : 1M]
The three roots of the equation f(x) = 0 are x = {–2, 0, 3}. What are the three values of x for which f(x – 3) = 0?
(a) –5, –3, 0 (b) –2, 0, 3
(c) 0, 6, 8 (d) 1, 3, 6

109.[MCQ] **[GATE-CS-2018: 1M]**

The area of a square is d. What is the area of the circle which has the diagonal of the square as its diameter?

(a) πd (b) πd^2

(c) $\frac{1}{4}\pi d^2$ (d) $\frac{1}{2}\pi d$

110.[MCQ] **[GATE-CS-2018: 2M]**

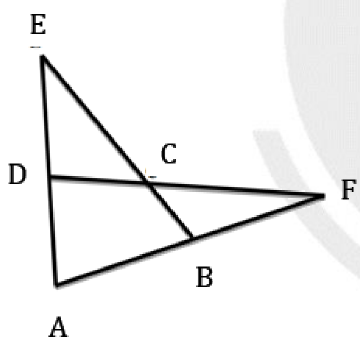
If $pqr \neq 0$ and $p^{-x} = \frac{1}{q}, q^{-y} = \frac{1}{r}, r^{-z} = \frac{1}{p}$, what is the value of the product xyz?

(a) –1 (b) $\frac{1}{pqr}$

(c) 1 (d) pqr

111.[MCQ] **[GATE-CS-2018: 2M]**

In the figure below, ∠DEC + ∠BFC is equal to ____________ .



(a) ∠BCD − ∠BAD (b) ∠BAD + ∠BCF

(c) ∠BAD + ∠BCD (d) ∠CBA+ ∠ADC

112.[MCQ] **[GATE-EC-2018:1M]**

A 1.5 m tall person is standing at a distance of 3 m from a lamp post. The light from the lamp at the top of the post casts her shadow. The length of the shadow is twice her height. What is the height of the lamp post in meters?

(a) 1.5 (b) 3

(c) 4.5 (d) 6

113.[MCQ] **[GATE-CE-2018: 2M]**

$\underbrace{a+a+a+......+a}_{\text{n time}} = a^2b$ and $\underbrace{b+b+b+......+b}_{\text{m time}} = ab^2$,

where a, b, n and m are natural numbers. What is the value of

$$\left(\underbrace{m+m+m+......+m}_{\text{n time}}\right)\left(\underbrace{n+n+n+......+n}_{\text{m time}}\right)$$

(a) $2a^2b^2$ (b) a^4b^4

(c) $ab(a + b)$ (d) $a^2 + b^2$

114.[MCQ] **[GATE-CE-2018: 1M]**

Tower A is 90 m tall and tower B is 140 m tall. They are 100 m apart. A horizontal skywalk connects the floors at 70 m in both the towers. If a taut rope connects the top of tower A to the bottom of tower B, at what distance (in meters) from tower A will the rope intersect the skywalk?

(a) 22.22 (b) 50

(c) 57.87 (d) 77.78

115.[MCQ] **[GATE-CE-2018: 2M]**

For non-negative integers, a, b, c, what would be the value of $a + b + c$ if $\log a + \log b + \log c = 0$?

(a) 3 (b) 1

(c) 0 (d) -1

116.[MCQ] **[GATE-CE-2018: 2M]**

Given that $\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$ for $x \neq y \neq z$

what is the value of the product PQR?

(a) 0 (b) 1

(c) xyz (d) 10^{xyz}

117.[MCQ] **[GATE-ME-2018: 1M]**

The value of the expression

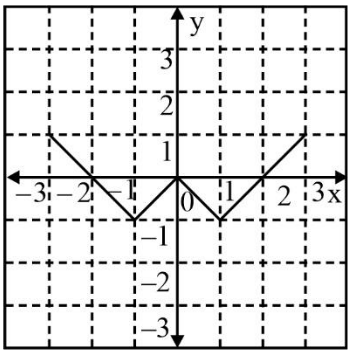
$$\frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uw}$$ is ________.

(a) –1 (b) 0

(c) 1 (d) 3

118.[MCQ] **[GATE-ME-2018: 2M]**

Which of the following functions describe the graph shown in the below figure?



(a) y = ||x| + 1| - 2 (b) y = ||x| - 1| - 1

(c) y = ||x| + 1| - 1 (d) y = ||x - 1| - 1|

119.[MCQ] **[GATE-ME-2018: 2M]**

For integers a, b and c, what would be the minimum and maximum values respectively of
a + b + c if log |a| + log |b| + log |c| = 0?

(a) –3 and 3 (b) –1 and 1

(c) –1 and 3 (d) 1 and 3

120.[MCQ] **[GATE-EE,CE-2017 : 2M]**

The expression $\frac{(x+y)-|x-y|}{2}$ is equal to

(a) The maximum of x and y

(b) The minimum of x and y

(c) 1

(d) None of the above

121.[MCQ] **[GATE-EE,CS-2017 : 2M]**

The number of roots of $e^x + 0.5x^2 - 2 = 0$ in the range [–5, 5] is

(a) 0 (b) 1

(c) 2 (d) 3

122.[MCQ] **[GATE-EC-2017:2M]**

The number of 3-digit numbers such that the digit 1 is never to the immediate right of 2 is

(a) 781 (b) 791

(c) 881 (d) 891

123.[MCQ] **[GATE-EC-2017:2M]**

There are 3 Indians and 3 Chinese in a group of 6 people. How many subgroups of this group can we choose so that every subgroup has at least one Indian?

(a) 56 (b) 52

(c) 48 (d) 44

124.[MCQ] **[GATE-CE-2017: 1M]**

Two dice are thrown simultaneously. The probability that the product of the numbers appearing on the top faces of the dice is a perfect square is

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

125.[MCQ] **[GATE-ME-2017: 2M]**

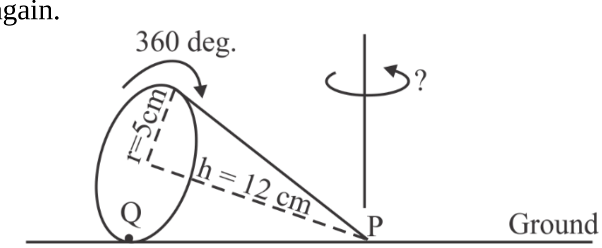
Let S_1 be the plane figure consisting of the points (x, y) given by the inequalities $|x - 1| \le 2$ and $|y + 2| \le 3$. Let S_2 be the plane figure given by the inequalities $x - y \ge -2$, $y \ge 1$, and $x \le 3$. Let S be the union of S_1 and S_2. The area of S is

(a) 26 (b) 28

(c) 32 (d) 34

126.[MCQ] **[GATE-ME-2017: 1M]**

A right-angled cone (with base radius 5 cm and height 12 cm), as shown in the figure below, is rolled on the ground keeping the point P fixed until the point Q (at the base of the cone, as shown) touches the ground again.



By what angle (in radians) about P does the cone travel?

(a) $\frac{5\pi}{12}$ (b) $\frac{5\pi}{24}$

(c) $\frac{24\pi}{5}$ (d) $\frac{10\pi}{13}$

127.[NAT] [GATE-EE-2016 : 2M]

The numeral in the units position of $211^{870} + 146^{127} \times 3^{424}$ is ________.

128.[MCQ] [GATE-EE-2016 : 2M]

Shaquille O' Neal is a 60% career free throw shooter, meaning that he successfully makes 60 free throws out of 100 attempts on average. What is the probability that he will successfully make exactly 6 free throws in 10 attempts?

(a) 0.2508 (b) 0.2816
(c) 0.2934 (d) 0.6000

129.[MCQ] [GATE-EE-2016 : 1M]

If $|9y - 6| = 3$, then $y^2 - 4y/3$ is _____________.

(a) 0 (b) +1/3
(c) –1/3 (d) undefined

130.[MCQ] [GATE-EE,CS-2016 : 1M]

In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$$

(a) n^4 (b) 4^n
(c) 2^{2n-1} (d) 4^{n-1}

131.[MCQ] [GATE-CS,CE-2016: 2M]

If $f(x) = 2x^7 + 3x - 5$, which of the following is a factor of f(x)?

(a) $(x^3 + 8)$ (b) $(x - 1)$
(c) $(2x - 5)$ (d) $(x + 1)$

132.[MCQ] [GATE-CS,CE-2016: 2M]

In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is ________.

(a) 40.00 (b) 46.02
(c) 60.01 (d) 92.02

133.[MCQ] [GATE-EC-2016:2M]

A straight line is fit to a data set (ln x, y). This line intercepts the abscissa at ln x = 0.1 and has a slope of – 0.02. What is the value of y at x = 5 from the fit?

(a) – 0.030 (b) – 0.014
(c) 0.014 (d) 0.030

134.[MCQ] [GATE-EC-2016:2M]

Find the area bounded by the lines $3x + 2y = 14$, $2x - 3y = 5$ in the first quadrant.

(a) 14.95 (b) 15.25
(c) 15.70 (d) 20.23

135. [MCQ] [GATE-ME,EC-2016: 2M]

If $q^{-a} = \frac{1}{r}$ and $r^{-b} = \frac{1}{s}$ and $s^{-c} = \frac{1}{q}$, the value of abc is ______.

(a) $(rqs)^{-1}$ (b) 0
(c) 1 (d) r + q + s

136.[MCQ] [GATE-CE-2016: 2M]

A square pyramid has a base perimeter x, and the slant height is half of the perimeter. What is the lateral surface area of the pyramid?

(a) x^2 (b) $0.75\ x^2$
(c) $0.50\ x^2$ (d) $0.25\ x^2$

137.[MCQ] [GATE-EE,CS-2015 : 1M]

Given Set A = {2, 3, 4, 5} and Set B = {11, 12, 13, 14, 15}, two numbers are randomly selected, one from each set. What is the probability that the sum of the two numbers equals 16?

(a) 0.20 (b) 0.25
(c) 0.30 (d) 0.33

138.[MCQ] [GATE-EE,CS-2015 : 1M]

Consider a function $f(x) = 1 - |x|$ on $-1 \le x \le 1$. The value of x at which the function attains a maximum, and the maximum value of the function are:

(a) 0, –1 (b) –1, 0
(c) 0, 1 (d) –1, 2

139.[MCQ] **[GATE-EE,CS-2015 : 2M]**

If the list of letters. P, R, S, T, U is an arithmetic sequence, which of the following are also in arithmetic sequence?

I. 2P, 2R, 2S, 2T, 2U

II. P – 3. R – 3. S – 3. T – 3, U – 3

III. P^2, R^2, S^2, T^2, U^2

(a) I only (b) I and II

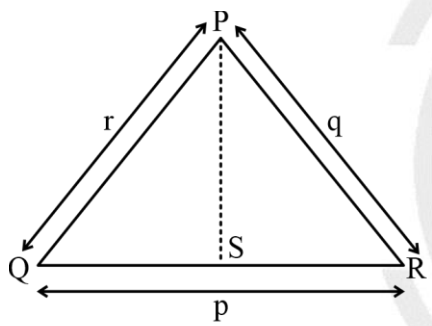
(c) II and III (d) I and III

140.[MCQ] **[GATE-EE,CS-2015 : 2M]**

In a triangle PQR, PS is the angle bisector of ∠QPR and ∠QPS = 60°. What is the length of PS?

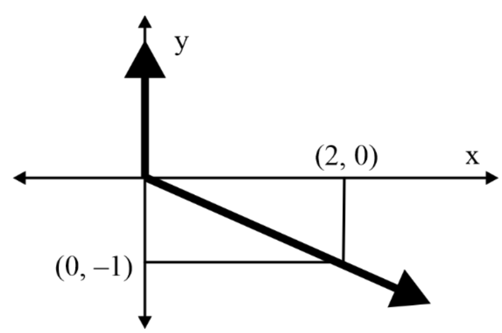
(a) $\frac{(q+r)}{qr}$ (b) $\frac{qr}{(q+r)}$

(c) $\sqrt{(q^2+r^2)}$ (d) $\frac{(q+r)^2}{qr}$



141.[MCQ] **[GATE-CS,CE-2015: 2M]**

Choose the most appropriate equation for the function drawn as thick line, in the plot below.



(a) x = y – |y| (b) x = – (y – |y|)

(c) x = y + |y| (d) x = – (y + |y|)

142.[MCQ] **[GATE-EC-2015:1M]**

If $\log_x (5/7) = -1/3$, then the value of x is

(a) 343/125 (b) 125/343

(c) – 25/49 (d) – 49/25

143.[MCQ] **[GATE-ME-2015: 2M]**

Right triangle PQR is to be constructed in the xy - plane so that the right angle is at P and line PR is parallel to the x-axis. The x and y coordinates of P,Q, and R are to be integers that satisfy the inequalities: $-4 \le x \le 5$ and $6 \le y \le 16$. How many different triangles could be constructed with these properties?

(a) 110 (b) 1,100

(c) 9,900 (d) 10,000

144.[MCQ] **[GATE-ME,EC-2015: 2M]**

log tan 1° + log tan 2° + …………+log tan 89° is ………………

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) –1

145.[MCQ] **[GATE-ME,EC-2015: 1M]**

If x > y > 1, which of the following must be true?

(i) In x > In y (ii) $e^x > e^y$

(iii) $y^x > x^y$ (iv) cos x > cos y

(a) (i) and (ii) (b) (i) and (iii)

(c) (iii) and (iv) (d) (ii) and (iv)

146.[MCQ] **[GATE-ME,EC-2015: 2M]**

If $a^2 + b^2 + c^2 = 1$. then ab + bc + ac lies in the interval

(a) [1, 2/3] (b) [–1/2, 1]

(c) [–1, 1/2] (d) [2, –4]

147.[MCQ] **[GATE-CE-2015: 1M]**

Four cards are randomly selected from a pack of 52 cards. If the first two cards are kings, what is the probability that the third card is a king?

(a) 4/52

(b) 2/50

(c) (1/52) × (1/52)

(d) (1/52) × (1/51) × (1/50)

148.[MCQ] [GATE-EE,CS-2014 : 1M]

The roots of $ax^2 + bx + c = 0$ are real and positive. a, b and c are real, Then $ax^2 + b|x| + c = 0$ has

(a) No roots (b) 2 real roots
(c) 3 real roots (d) 4 real roots

149.[MCQ] [GATE-EE,CS-2014 : 2M]

If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include

(a) 2, 4 (b) 2, 14
(c) 4, 52 (d) 14, 52

150.[MCQ] [GATE-EE,CS-2014 : 2M]

Consider the equation: $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

(a) 1634 (b) 1737
(c) 3142 (d) 3162

151.[MCQ] [GATE-CE-2014: 1M]

If $y = 5x^2 + 3$, then the tangent at $x = 0$, $y = 3$

(a) passes through $x = 0$, $y = 0$
(b) has a slope of +1
(c) is parallel to the x-axis
(d) has a slope of −1

152.[NAT] [GATE-CE-2014: 2M]

The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

153.[NAT] [GATE-CE-2014: 1M]

In any given year, the probability of an earthquake greater than Magnitude 6 occurring in the Garhwal Himalayas is 0.04. The average time between successive occurrences of such earthquakes is ____ years.

154.[MCQ] [GATE-ME,EC-2014: 1M]

Let $f(x, x) = x^n y^m = P$. If x is doubled and y is halved, the new value of f is

(a) $2^{n-m} P$ (b) $2^{m-n} P$
(c) $2(n - m)P$ (d) $2(m - n)P$

155.[MCQ] [GATE-EE,EC-2013 : 2M]

What is the chance that a leap year, selected at random, will contain 53 Saturdays?

(a) 2/7 (b) 3/7
(c) 1/7 (d) 5/7

156.[MCQ] [GATE-EE,EC-2013 : 2M]

The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite sign is

(a) $(-\infty, 0)$ (b) $(0, 1)$
(c) $(1, \infty)$ (d) $(0, \infty)$

157.[MCQ] [GATE-EE,EC-2013 : 2M]

Find the sum to n terms of the series

10 + 84 + 734 + ...

(a) $\frac{9(9^n + 1)}{10} + 1$ (b) $\frac{9(9^n - 1)}{8} + 1$
(c) $\frac{9(9^n - 1)}{8} + n$ (d) $\frac{9(9^n - 1)}{8} + n^2$

158.[MCQ] [GATE-CE-2013: 2M]

X and Y are two positive real numbers such that $2X + Y \le 6$ and $X + 2Y \le 8$. For which of the following values of (X, Y) the function f (X, Y) = 3X + 6Y will give maximum value?

(a) (4/3, 10/3) (b) (8/3, 20/3)
(c) (8/3, 10/3) (d) (4/3, 20/3)

159.[MCQ] [GATE-CE-2013: 2M]

If $|4X-7|=5$ then the values of $2|X|-|-X|$ is:

(a) $2, \frac{1}{3}$ (b) $\frac{1}{2}, 3$
(c) $\frac{3}{2}, 9$ (d) $\frac{2}{3}, 9$

160.[MCQ] [GATE-ME,CS-2013: 2M]

Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+..........\frac{1}{\sqrt{80}+\sqrt{81}}$$

(a) 7 (b) 8
(c) 9 (d) 10

161.[MCQ] [GATE-ME,CE,CS-2012: 2M]

A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation
$Y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(a) 8 metres (b) 10 meters
(c) 12 meters (d) 14 metres

162.[MCQ] [GATE-EE,EC-2012 : 2M]

A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(a) 1/4 (b) 1/16
(c) 7/16 (d) 9/16

163.[MCQ] [GATE-ME,CE,CS-2012: 1M]

The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹50 per unit. The number of units to be produced by the firm such that the profit maximized is

(a) 5 (b) 10
(c) 15 (d) 25

164.[MCQ] [GATE-EE,EC-2011 : 2M]

The sum of *n* terms of the series 4 + 44 + 444 + ... is

(a) $(4/81)\,[10^{n+1} - 9n - 1]$
(b) $(4/81)\,[10^{n-1} - 9n - 1]$
(c) $(4/81)\,[10^{n+1} - 9n - 10]$
(d) $(4/81)\,[10^{n+1} - 9n - 10]$

165.[MCQ] [GATE-EE,EC-2011 : 2M]

Given that f(y) = |y| / y, and q is any non-zero real number, the value of |f(q) – f(–q)| is

(a) 0 (b) –1
(c) 1 (d) 2

166.[MCQ] [GATE-ME,CE,CS-2011: 2M]

The variable cost (V) of manufacturing a product varies according to the euqation V = 4q, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation F = 100/q. How many units should be produced to minimize the total cost (V + F)?

(a) 5 (b) 4
(c) 7 (d) 6

167.[MCQ] [GATE-ME,CS,CE-2011: 1M]

If Log (P) = (1/2) Log (Q) = (1/3) Log (R), then which of the following options is TRUE?

(a) $P^2 = Q^3 R^2$ (b) $Q^2 = PR$
(c) $Q^2 = R^3 P$ (d) $R = P^2 Q^2$

168.[MCQ] [GATE-ME,CE,EE,EC,CS-2010:2M]

If 137 + 276 = 435 how much is 731 + 672?

(a) 534 (b) 1403
(c) 1623 (d) 1513

Numerical Estimation

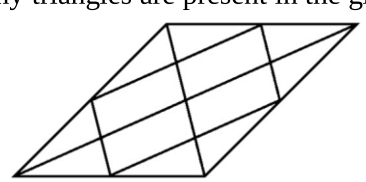
169.[MCQ] [GATE-EE-2023 : 2M]

The digit in the unit's place of the product $3^{999} \times 7^{1000}$ is_______.

(a) 7 (b) 1
(c) 3 (d) 9

170.[MCQ] [GATE-EE-2023 : 1M]

How many triangles are present in the given figure?



(a) 12 (b) 16
(c) 20 (d) 24

171.[MCQ] **[GATE-CS-2023:1M]**

A series of natural numbers $F_1, F_2, F_3, F_4, F_5, F_6, F_7, \ldots$ obeys

$F_{n+1} = F_n + F_{n-1}$ for all integers $n \geq 2$.

If $F_6 = 37$, and $F_7 = 60$, then what is F_1?

(a) 4 (b) 5
(c) 8 (d) 9

172.[MCQ] **[GATE-CS-2023:1M]**

A survey for a certain year found that 90% of pregnant women received medical care at least once before giving birth. Of these women, 60% received medical care from doctors, while 40% received medical care from other healthcare providers. Given this information, which one of the following statements can be inferred with certainty?

(a) More than half of the pregnant women received medical care at least once from a doctor.
(b) Less than half of the pregnant women received medical care at least once from a doctor.
(c) More than half of the pregnant women received medical care at most once from a doctor.
(d) Less than half of the pregnant women received medical care at most once from a doctor.

173.[MCQ] **[GATE-EC-2023:2M]**

Out of 1000 individuals in a town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid-positive individuals. The strategy is to:

(i) Collect saliva samples from all 1000 individuals and randomly group them into sets of 5.
(ii) Mix the samples within each set and test the mixed sample for covid.
(iii) If the test done in (ii) gives a negative result, then declare all the 5 individuals to be covid negative.
(iv) If the test done in (ii) gives a positive result, then all the 5 individuals are separately tested for covid.

Given this strategy, no more than _______ testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

(a) 700 (b) 600
(c) 800 (d) 1000

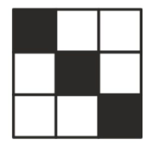
174.[MCQ] **[GATE-EC-2023:1M]**

What is the smallest number with distinct digits whose digits add up to 45?

(a) 123555789 (b) 123457869
(c) 123456789 (d) 99999

175.[MCQ] **[GATE-CE-2023: 1M]**

In how many ways can cells in a 3 × 3 grid be shaded, such that each row and column have exactly one shaded cell? An example of one valid shading is shown.



(a) 2 (b) 9
(c) 3 (d) 6

176.[MCQ] **[GATE-CE-2023: 2M]**

Three husband-wife pairs are to be seated at a circular table that has six identical chairs. Seating arrangements are defined only by the relative position of the people. How many seating arrangements are possible such that every husband sits next to his wife?

(a) 16 (b) 4
(c) 120 (d) 720

177.[MCQ] **[GATE-CE-2023: 2M]**

If x satisfies the equation $4^{8^x} = 256$, then x is equal to ____.

(a) $\frac{1}{2}$ (b) $\log_{16}8$
(c) $\frac{2}{3}$ (d) $\log_4 8$

178.[MCQ] **[GATE-ME-2023: 1M]**

The minute-hand and second-hand of a clock cross each other _______ times between 09:15:00 AM and 09:45:00 AM on a day.

(a) 30 (b) 15
(c) 29 (d) 31

179.[MCQ] [GATE-EE-2022 : 2M]

The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹ 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹ 100.

What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?

(a) 2500 (b) 2250
(c) 1750 (d) 1500

180.[MCQ] [GATE-EE-2022 : 1M]

Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes.

If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?

(a) 5 : 00 PM (b) 5 : 30 PM
(c) 6 : 00 PM (d) 6 : 30 PM

181.[MCQ] [GATE-EE-2022 : 1M]

In a 500 m race, P and Q have speeds in the ratio of 3 : 4. Q starts the race when P has already covered 140 m. What is the distance between P and Q (in m) when P wins the race?

(a) 20 (b) 40
(c) 60 (d) 140

182.[MCQ] [GATE-CS-2022:1M]

Let r be a root of the equation $x^2 + 2x + 6 = 0$.

Then the value of the expression

(r + 2) (r + 3) (r + 4) (r + 5) is

(a) 51 (b) –51
(c) 126 (d) –126

183.[MCQ] [GATE-CS-2022:2M]

In a recently conducted national entrance test, boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates.

Which one of the following is the correct logical inference based on the information provided in the above passage?

(a) Equal number of boys and girls qualified.
(b) Equal number of boys and girls appeared for the test.
(c) The number of boys who appeared for the test is less than the number of girls who appeared.
(d) The number of boys who qualified the test is less than the number of girls who Qualified.

184.[MCQ] [GATE-CS-2022:2M]

A box contains five balls of same size and shape. Three of them are green colored balls and two of them are orange-colored balls. Balls are drawn from the box one at a time. If a green ball is drawn, it is not replaced. If an orange ball is drawn, it is replaced with another orange ball.

First ball is drawn. What is the probability of getting an orange ball in the next draw?

(a) $\frac{1}{2}$ (b) $\frac{8}{25}$
(c) $\frac{19}{50}$ (d) $\frac{23}{50}$

185.[MCQ] [GATE-EC-2022:1M]

A sum of money is to be distributed among P, Q, R, and S in the proportion 5 : 2 : 4 : 3, respectively.

If R gets ₹ 1000 more than S, what is the share of Q (in ₹)?

(a) 500 (b) 1000
(c) 1500 (d) 2000

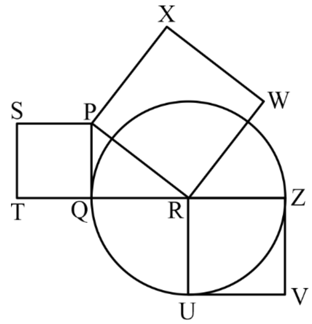
186.[MCQ] [GATE-CE-2022:1M]

If
p : q = 1 : 2
q : r = 4 : 3
r : s = 4 : 5
and u is 50% more than s, what is the ratio p : u?

(a) 2 : 15 (b) 16 : 15
(c) 1 : 5 (d) 16 : 45

187.[MCQ] [GATE-CE-2022:1M]

In the following diagram, the point R is the center of the circle. The lines PQ and ZV are tangential to the circle. The relation among the areas of the squares, PXWR, RUVZ and SPQT is.



(a) Area of SPQT = Area of RUVZ = Area of PXWR
(b) Area of SPQT = Area of PXWR – Area of RUVZ
(c) Area of PXWR = Area of SPQT – Area of RUVZ
(d) Area of PXWR = Area of RUVZ – Area of SPQT

188.[MCQ] [GATE-CE-2022:2M]

P invested ₹ 5000 per month for 6 months of a year and Q invested ₹ x per month for 8 months of the year in a partnership business. The profit is shared in proportion to the total investment made in that year.

If at the end of that investment year, Q receives 4/9 of the total profit, what is the value of x (in ₹)?

(a) 2500 (b) 3000
(c) 4687 (d) 8437

189.[MCQ] [GATE-CE-2022:2M]

In a partnership business the monthly investment by three friends for the first six months is in the ratio 3: 4: 5. After six months, they had to increase their monthly investments by 10%, 15% and 20%, respectively, of their initial monthly investment. The new investment ratio was kept constant for the next six months.

What is the ratio of their shares in the total profit (in the same order) at the end of the year such that the share is proportional to their individual total investment over the year?

(a) 22 : 23 : 24 (b) 22 : 33 : 50
(c) 33 : 46 : 60 (d) 63 : 86 : 110

190.[MCQ] [GATE-CE-2022:1M]

Both the numerator and the denominator of 3/4 are increased by a positive integer, x, and those of 15/17 are decreased by the same integer. This operation results in the same value for both the fractions.

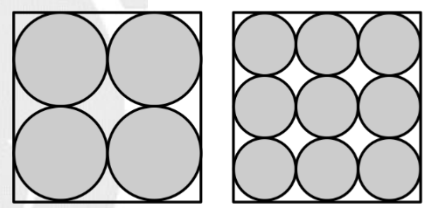
What is the value of x?

(a) 1 (b) 2
(c) 3 (d) 4

191.[MCQ] [GATE-ME-2022: 2M]

Equal sized circular regions are shaded in a square sheet of paper of 1 cm side length. Two cases, case M and case N, are considered as shown in the figures below. In the case M, four circles are shaded in the square sheet and in the case N, nine circles are shaded in the square sheet as shown.

What is the ratio of the areas of unshaded regions of case M to that of case N?



Case M Case N

(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 2 : 1

192.[MCQ] [GATE-ME-2022: 2M]

For the past m days, the average daily production at a company was 100 units per day.

If today's production of 180 units changes the average to 110 units per day, what is the value of m?

(a) 18 (b) 10
(c) 7 (d) 5

193.[MCQ] [GATE-ME-2022: 1M]

A person was born on the fifth Monday of February in a particular year.

Which one of the following statements is correct based on the above information?

(a) The 2nd February of that year is a Tuesday
(b) There will be five Sundays in the month of February in that year
(c) The 1st February of that year is a Sunday
(d) All Mondays of February in that year have even dates

194.[MCQ] [GATE-ME-2022: 2M]

In a 12-hour clock that runs correctly, how many times do the second, minute, and hour hands of the clock coincide, in a 12-hour duration from 3 PM in a day to 3 AM the next day?

(a) 11 (b) 12
(c) 144 (d) 2

195.[MCQ] [GATE-ME-2022: 2M]

An equilateral triangle, a square and a circle have equal areas.

What is the ratio of the perimeters of the equilateral triangle to square to circle?

(a) $3\sqrt{3} : 2 : \sqrt{\pi}$ (b) $\sqrt{(3\sqrt{3})} : 2 : \sqrt{\pi}$
(c) $\sqrt{(3\sqrt{3})} : 4 : 2\sqrt{\pi}$ (d) $\sqrt{(3\sqrt{3})} : 2 : 2\sqrt{\pi}$

196.[MCQ] [GATE-ME-2022: 2M]

A rhombus is formed by joining the midpoints of the sides of a unit square.

What is the diameter of the largest circle that can be inscribed within the rhombus?

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

197.[MCQ] [GATE-ME-2022: 1M]

A person travelled 80 km in 6 hours. If the person travelled the first part with a uniform speed of 10 kmph and the remaining part with a uniform speed of 18 kmph.

What percentage of the total distance is travelled at a uniform speed of 10 kmph?

(a) 28.25 (b) 37.25
(c) 43.75 (d) 50.00

198.[MCQ] [GATE-ME-2022: 1M]

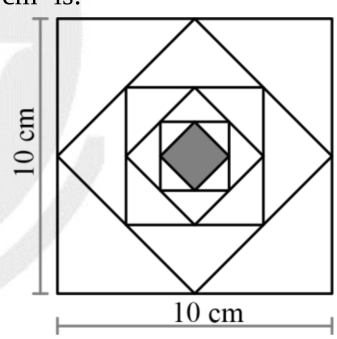
The average of the monthly salaries of M, N and S is ₹ 4000. The average of the monthly salaries of N, S and P is ₹ 5000. The monthly salary of P is ₹ 6000.

What is the monthly salary of M as a percentage of the monthly salary of P?

(a) 50% (b) 75%
(c) 100% (d) 125%

199.[MCQ] [GATE-EE-2021 : 1M]

In the figure shown above, each inside square is formed by joining the midpoints of the sides of the next larger square. The area of the smallest square (shaded) as shown, in cm^2 is:



(a) 12.50 (b) 6.25
(c) 3.125 (d) 1.5625

200.[MCQ] [GATE-EE-2021 : 1M]

Which one of the following numbers is exactly divisible by $(11^{13} + 1)$?

(a) $11^{26} + 1$ (b) $11^{33} + 1$
(c) $11^{39} - 1$ (d) $11^{52} - 1$

201.[MCQ] [GATE-EE-2021 : 1M]

For a regular polygon having 10 sides, the interior angle between the sides of the polygon, in degrees, is:

(a) 396 (b) 324
(c) 216 (d) 144

202.[MCQ] **[GATE-CS-2021: 1M]**

The ratio of boys to girls in a class is 7 to 3.

Among the options below, an acceptable value for the total number of students in the class is:

(a) 21 (b) 37

(c) 50 (d) 73

203.[MCQ] **[GATE-CS-2021: 2M]**

We have 2 rectangular sheets of paper. M and N. of dimensions 6 cm x 1 cm each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is_________.

(a) $\frac{\pi}{2}$ (b) $\frac{3}{\pi}$

(c) $\frac{9}{\pi}$ (d) 3π

204.[MCQ] **[GATE-CS-2021: 2M]**

Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3:4. Discount is calculated as the difference between the marked price and the selling price. The profit percentage is calculated as the ratio of the difference between selling price and cost, to the cost

Items	Cost Rs.	Profit%	Marked Price Rs.
P	5,400	….	5,860
Q	…	25	10,000

(profit % = (Selling price–cost)/cost × 100)

The discount on item Q, as a percentage of its marked price is ________.

(a) 25 (b) 12.5

(c) 10 (d) 5

205.[MCQ] **[GATE-CS-2021: 2M]**

There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag.

The probability that at least two chocolates are identical is _______.

(a) 0.3024 (b) 0.4235

(c) 0.6976 (d) 0.8125

206.[MCQ] **[GATE-CS-2021: 2M]**

The number of students in three classes is in the ratio 3:13:6. If 18 students are added to each class, the ratio changes to 15:35:21.

The total number of students in all the three classes in the beginning was:

(a) 22 (b) 66

(c) 88 (d) 110

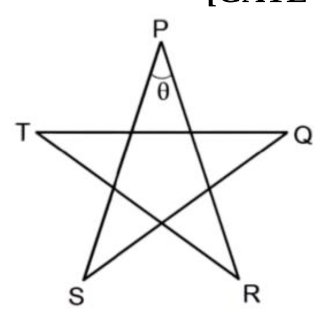
207.[MCQ] **[GATE-EC-2021:1M]**

The current population of a city is 11,02,500. If it has been increasing at the rate of 5% per annum, what was its population 2 years ago?

(a) 9,92,500 (b) 9,95,006

(c) 10,00,000 (d) 12,51,506

208.[MCQ] **[GATE-CE-2021: 2M]**



Five-line segments of equal lengths, PR, PS, QS, QT and RT are used to farm a star as shown in the figure above.

The value of θ, in degrees is____.

(a) 36 (b) 45

(c) 72 (d) 108

209.[MCQ] [GATE-CE-2021: 1M]

Four persons P, Q, R and S are to be seated in a row. R should not be seated at the second position from the left end of the row. The number of distinct seating arrangements possible is:

(a) 6 (b) 9
(c) 18 (d) 24

210.[MCQ] [GATE-CE-2021: 2M]

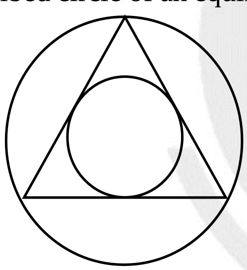
In an equilateral triangle PQR, side PQ is divided into four equal parts, side QR is divided into six equal parts and side PR is divided into eight equal parts. The length of each subdivided part in cm is an integer.

The minimum area of the triangle PQR possible, in cm^2, is

(a) 18 (b) 24
(c) $48\sqrt{3}$ (d) $144\sqrt{3}$

211.[MCQ] [GATE-ME-2021: 2M]

The ratio of the area of the inscribed circle to the area of the circumscribed circle of an equilateral triangle is



(a) $\frac{1}{8}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

212.[MCQ] [GATE-ME-2021: 2M]

A box contains 15 blue balls and 45 black balls. If 2 balls are selected randomly, without replacement, the probability of an outcome in which the first selected is a blue ball and the second selected is a black ball, is ___.

(a) $\frac{3}{16}$ (b) $\frac{45}{236}$
(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

213.[MCQ] [GATE-ME-2021: 1M]

A digital watch X beeps every 30 seconds while watch Y beeps every 32 seconds. They beeped together at 10 AM. The immediate next time that they will beep together is

(a) 10.08 AM (b) 10.42 AM
(c) 11.00 AM (d) 10.00 PM

214.[MCQ] [GATE-ME-2021: 1M]

Five persons P, Q, R, S and T are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and T cannot be seated at either end of the row. P should not be seated adjacent to S. R is to be seated at the second position from the left end of the row. The number of distinct seating arrangements possible is:

(a) 2 (b) 3
(c) 4 (d) 5

215.[MCQ] [GATE-ME-2021: 2M]

Five persons P, Q, R, S and T are sitting in a row not necessarily in the same order. Q and R are separated by one person, and S should not be seated adjacent to Q.

The number of distinct seating arrangements possible is:

(a) 4 (b) 8
(c) 10 (d) 16

216.[MCQ] [GATE-ME-2021: 2M]

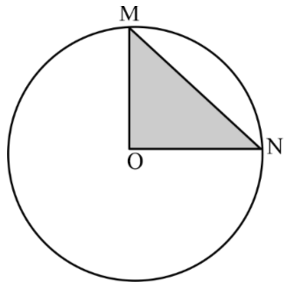
The number of hens, ducks and goats in farm P are 65, 91 and 169, respectively. The total number of hens, ducks and goats in a nearby farm Q is 416. The ratio of hens: ducks: goats in farm Q is 5:14:13. All the hens, ducks and goats are sent from farm Q to farm P.

The new ratio of hens: ducks: goats in farm P is ______.

(a) 5:7:13 (b) 5:14:13
(c) 10:21:26 (d) 21:10:26

217.[MCQ] **[GATE-ME-2021: 1M]**

In the above figure, O is the center of the circle and, M and N lie on the circle.



The area of the right triangle MON is 50 cm^2. What is the area of the circle in cm^2 ?

(a) 2π (b) 50π

(c) 75π (d) 100π

218.[MCQ] **[GATE-EE-2020 : 2M]**

Non-performing Assets (NPAs) of a bank in India is defined as an asset, which remains unpaid by a borrower for a certain period of time in terms of interest, principal, or both. Reserve Bank of India (RBI) has changed the definition of NPA thrice during 1993-2004, in terms of the holding period of loans. The holding period was reduced by one quarter each time. In 1993. the holding period was four quarters (360 days).

Based on the above paragraph, the holding period of loans in 2004 after the third revision was ______ days.

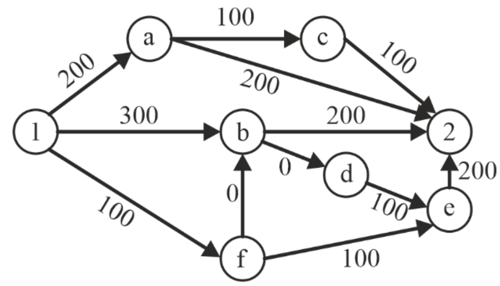
(a) 45 (b) 90

(c) 135 (d) 180

219.[MCQ] **[GATE-CS-2020: 1M]**

There are multiple routes to reach from node 1 to node 2, as shown in the network.

The cost of travel on an edge between two nodes is given in rupees. Nodes ‘a’, ‘b’, ‘c’, ‘d’, ‘e’ and ‘f’ are toll booths. The toll price at toll booths marked 'a' and ‘e' is Rs. 200, and is Rs. 100 for the other toll booths. Which is the cheapest route from node 1 to node 2?



(a) 1-a-c-2 (b) 1-f-b-2

(c) 1-b-2 (d) 1-f-e-2

220.[MCQ] **[GATE-EC-2020:2M]**

It is quarter past three in your watch. The angle between the hour hand and the minute hand is

(a) 0° (b) 7.5°

(c) 15° (d) 22.5°

221.[MCQ] **[GATE-CE-2020: 1M]**

The sum of two positive numbers is 100. After subtracting 5 from each number, the product of the resulting numbers is 0. One of the original numbers is.

(a) 80 (b) 85

(c) 90 (d) 95

222.[MCQ] **[GATE-CE-2020: 1M]**

The ratio of ‘the sum of the odd positive integers from 1 to 100’ to ‘the sum of the even positive integers from 150 to 200’ is_____.

(a) 45 : 95 (b) 1 : 2

(c) 50 : 91 (d) 1 : 1

223.[MCQ] **[GATE-CE-2020: 2M]**

For the year 2019, which of the previous year’s calendar can be used?

(a) 2011 (b) 2012

(c) 2013 (d) 2014

224.[MCQ] **[GATE-CE-2020: 2M]**

The unit’s place in 26591749^{110016} is_____.

(a) 1 (b) 3

(c) 6 (d) 9

225.[MCQ] **[GATE-CE-2020: 2M]**

Insert seven numbers between 2 and 34, such that the resulting sequence including 2 and 34 is an arithmetic progression. The sum of these inserted seven numbers is ____.

(a) 120 (b) 124
(c) 126 (d) 130

226.[MCQ] **[GATE-ME-2020: 2M]**

It was estimated that 52 men can complete a strip in a newly constructed highway connecting cities P and Q in 10 days. Due to an emergency, 12 men were sent to another project. How many numbers of days, more than the original estimate, will be required to complete the strip?

(a) 3 days (b) 5 days
(c) 10 days (d) 13 days

227.[MCQ] **[GATE-ME-2020: 1M]**

There are five levels {P ,Q, R, S,T} in a linear supply chain before a product reaches customers, as shown in the figure

P → Q → R → S → T → Costomers

At each of the five levels, the price of the product is increased by 25%. If the product is produced at level P at the cost of RS. 120 per unit, what is the price paid (in rupees) by the customers?

(a) 187.50 (b) 234.38
(c) 292.96 (d) 366.21

228.[MCQ] **[GATE-EE-2019 : 2M]**

The ratio of the number of boys and girls who participated in an examination is 4:3. The total percentage of candidates who passed the examination is 80 and the percentage of girls who passed is 90. The percentage of boys who passed is.

(a) 55.50 (b) 72.50
(c) 80.50 (d) 90.00

229.[MCQ] **[GATE-EE-2019 : 2M]**

How many integers are there between 100 and 1000 all of whose digits are even?

(a) 60 (b) 80
(c) 100 (d) 90

230.[MCQ] **[GATE-EE-2019 : 1M]**

It takes two hours for a person X to mow the lawn. Y can mow the same lawn in four hours. How long (in minutes) will it take X and Y, if they work together to mow the lawn?

(a) 60 (b) 80
(c) 90 (d) 100

231.[MCQ] **[GATE-CS-2019:1M]**

Two cars start at the same time from the same location and go in the same direction. The speed of the first car is 50 km/h and the speed of the second car is 60 km/h. The number of hours it takes for the distance between the two cars to be 20 km is _____.

(a) 1 (b) 2
(c) 3 (d) 6

232.[MCQ] **[GATE-CS-2019:1M]**

Ten friends planned to share equally the cost of buying a gift for their teacher. When two of them decided not to contribute, each of the other friends had to pay Rs.150 more. The cost of the gift was Rs. _______.

(a) 666 (b) 3000
(c) 6000 (d) 12000

233.[MCQ] **[GATE-EC-2019:2M]**

Two design consultants, P and Q, started working from 8 AM for a client. The client budgeted a total of USD 3000 for the consultants. P stopped working when the hour hand moved by 210 degrees on the clock. Q stopped working when the hour hand moved by 240 degrees. P took two tea breaks of 15 minutes each during her shift, but took no lunch break. Q took only one lunch break for 20 minutes, but no tea breaks. The market rate for consultants is USD 200 per hour and breaks are not paid. After paying the consultants, the client shall have USD _________ remaining in the budget.

(a) 000.00 (b) 166.67
(c) 300.00 (d) 433.33

234.[MCQ] **[GATE-EC-2019:1M]**

It would take one machine 4 hours to complete a production order and another machine 2 hours to complete the same order. If both machines work simultaneously at their respective constant rates, the time taken to complete the same order is ________ hours.

(a) 2/3 (b) 3/4
(c) 4/3 (d) 7/3

235.[MCQ] [GATE-CE-2019: 2M]

An oil tank can be filled by pipe X in 5 hours and pipe Y in 4 hours, each pump working on its own. When the oil tank is full and the drainage hole is open, the oil is drained in 20 hours. If initially the tank was empty and someone started the two pumps together but left the drainage hole open, how many hours will it take for the tank to be filled? (Assume that the rate of drainage is independent of the Head)

(a) 1.50 (b) 2.00
(c) 2.50 (d) 4.00

236.[MCQ] [GATE-CE-2019: 2M]

A square has sides 5 cm smaller than the sides of a second square. The area of the larger square is four times the area of the smaller square. The side of the larger square is ____cm.

(a) 18.50 (b) 15.10
(c) 10.00 (d) 8.50

237.[MCQ] [GATE-CE-2019: 1M]

Suresh wanted to lay a new carpet in his new mansion with an area of 70 × 55 sq. mts. However, an area of 550 sq. mts. had to be left out for flower pots. If the cost of carpet is Rs. 50 per sq. mts. how much money (in Rs.) will be spent by Suresh for the carpet now?

(a) Rs.1,65,000 (b) Rs.1,92,500
(c) Rs.2,75,000 (d) Rs.1,27,500

238.[MCQ] [GATE-CE-2019: 1M]

A retaining wall with measurements 30 m × 12 m × 6 m was constructed with bricks of dimensions 8 cm × 6 cm × 6 cm. If 60% of the wall consists of bricks, the number of bricks used for the construction is ___lakhs.

(a) 30 (b) 40
(c) 45 (d) 75

239.[MCQ] [GATE-CE-2019: 2M]

Mohan, the manager, wants his four workers to work in pairs. No pair should work for more than 5 hours. Ram and John have worked together for 5 hours. Krishna and Amir have worked as a team for 2 hours. Krishna does not want to work with Ram. Whom should Mohan allot to work with John, if he wants all the workers to continue working?

(a) Amir (b) Krishna
(c) Ram (d) None of the three

240.[MCQ] [GATE-CE-2019: 2M]

Population of state X increased by x% and the population of state Y increased by y% from 2001 to 2011. Assume that x is greater than y. Let P be the ratio of the population of state X to state Y in a given year. The percentage increase in P from 2001 to 2011 is.

(a) $\frac{x}{y}$ (b) y - y

(c) $\frac{100(x-y)}{100+x}$ (d) $\frac{100(x-y)}{100+y}$

241.[MCQ] [GATE-ME-2019: 2M]

Two pipes P and Q can fill a tank in 6 hours and 9 hours respectively, while a third pipe R can empty the tank in 12 hours. Initially, P and R are open for 4 hours. Then P is closed and Q is opened. After 6 more hours R is closed. The total time taken to fill the tank (in hours) is ______ .

(a) 13.50 (b) 14.50
(c) 15.50 (d) 16.50

242.[MCQ] [GATE-ME-2019: 2M]

Fiscal deficit was 4% of the GDP in 2015 and that increased to 5% in 2016. If the GDP increased by 10% from 2015 to 2016, the percentage increase in the actual fiscal deficit is _____.

(a) 37.50 (b) 35.70
(c) 25.00 (d) 10.00

243.[MCQ] [GATE-ME-2019: 1M]

The product of three integers X, Y and Z is 192. Z is equal to 4 and P is equal to the average of X and Y. What is the minimum possible value of P?

(a) 6 (b) 7
(c) 8 (d) 9.5

244.[MCQ] [GATE-ME-2019: 2M]

A person divided an amount of Rs. 100,000 into two parts and invested in two different schemes. In one he got 10% profit and in the other he got 12%. If the profit percentages are interchanged with these investments he would have got Rs.120 less. Find the ratio between his investments in the two schemes.

(a) 9 : 16 (b) 11 : 14
(c) 37 : 63 (d) 47 : 53

245.[MCQ] [GATE-ME-2019: 1M]

The sum and product of two integers are 26 and 165 respectively. The difference between these two integers is .

(a) 2 (b) 3
(c) 4 (d) 6

246.[MCQ] [GATE-ME-2019: 1M]

A worker noticed that the hour hand on the factory clock had moved by 225 degrees during her stay at the factory. For how long did she stay in the factory?

(a) 3.75 hours (b) 4 hours and 15 mins
(c) 8.5 hours (d) 7.5 hours

247.[MCQ] [GATE-EE-2018 : 1M]

For what values of k given below is $\frac{(k+2)^2}{k-3}$ an integer?

(a) 4, 8, 18 (b) 4, 10, 16
(c) 4, 8, 28 (d) 8, 26, 28

248.[MCQ] [GATE-CS-2018: 1M]

What would be the smallest natural number which when divided either by 20 or by 42 or by 76 leaves a remainder of 7 in each case?

(a) 3047 (b) 6047
(c) 7987 (d) 63847

249.[MCQ] [GATE-CS-2018: 2M]

In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?

(a) 2:3 (b) 1:1
(c) 3:2 (d) 2:1

250.[MCQ] [GATE-CS-2018: 2M]

A six-sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

(a) Three green faces and four red faces.
(b) Four green faces and three red faces.
(c) Five green faces and two red faces.
(d) Six green faces and one red face.

251.[MCQ] [GATE-EC-2018:2M]

A cab was involved in a hit and run accident at night. You are given the following data about the cabs in the city and the accident.

(i) 85% of cabs in the city are green and the remaining cabs are blue.
(ii) A witness identified the cab involved in the accident as blue.
(iii) It is known that a witness can correctly identify the cab colour only 80% of the time.

Which of the following options is closest to the probability that the accident was caused by a blue cab?

(a) 12% (b) 15%
(c) 41% (d) 80%

252.[MCQ] [GATE-EC-2018:2M]

Two alloys A and B contain gold and copper in the ratios of 2:3 and 3:7 by mass, respectively. Equal masses of alloys A and B are melted to make an alloy C. The ratio of gold to copper in alloy C is ________.

(a) 5:10 (b) 7:13
(c) 6:11 (d) 9:13

253.[MCQ] [GATE-EC-2018:2M]

Leila aspires to buy a car worth Rs. 10,00,000 after 5 years. What is the minimum amount in Rupees that she should deposit now in a bank which offers 10% annual rate of interest, if the interest was compounded annually?

(a) 5,00,000 (b) 6,21,000
(c) 6,66,667 (d) 7,50,000

254.[MCQ] [GATE-EC-2018:1M]

What is the value of $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+....?$

(a) 2 (b) $\frac{7}{4}$
(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

255.[MCQ] [GATE-EC-2018:1M]

If the number 715■423 is divisible by 3 (■ denotes the missing digit in the thousandths place), then the smallest whole number in the place of ■ is ________

(a) 0 (b) 2
(c) 5 (d) 6

256.[MCQ] [GATE-CE-2018: 1M]

Hema's age is 5 years more than twice Hari's age. Suresh's age is 13 years less than 10 times Hari's age. If Suresh is 3 times as old as Hema, how old is Hema?

(a) 14 (b) 17
(c) 18 (d) 19

257.[MCQ] [GATE-CE-2018: 2M]

fruit seller sold a basket of fruits at 12.5% loss. Had he sold it for Rs. 108 more, he would have made a 10% gain. What is the loss in Rupees incurred by the fruit seller?

(a) 48 (b) 52
(c) 60 (d) 108

258.[MCQ] [GATE-CE-2018: 2M]

The price of a wire made of a superalloy material is proportional to the square of its length. The price of 10 m length of the wire is Rs. 1600. What would be the total price (in Rs.) of two wires of lengths 4 m and 6m?

(a) 768 (b) 832
(c) 1440 (d) 1600

259.[MCQ] [GATE-CE-2018: 2M]

Consider a sequence of numbers $a_1, a_2, a_3, \cdots, a_n$ where $a_n=\frac{1}{n}-\frac{1}{n+2}$ for each integer $n > 0$. What is the sum of the first 50 terms?

(a) $\left(1+\frac{1}{2}\right)-\frac{1}{50}$

(b) $\left(1+\frac{1}{2}\right)+\frac{1}{50}$

(c) $\left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)$

(d) $1-\left(\frac{1}{51}+\frac{1}{52}\right)$

260.[MCQ] [GATE-CE-2018: 2M]

A three-member committee has to be formed from a group of 9 people. How many such distinct committees can be formed?

(a) 27 (b) 72
(c) 81 (d) 84

261.[MCQ] [GATE-CE-2018: 2M]

In manufacturing industries, loss is usually taken to be proportional to the square of the deviation from a target. If the loss is Rs. 4900 for a deviation of 7 units, what would be the loss in Rupees for a deviation of 4 units from the target?

(a) 400
(b) 1200
(c) 1600
(d) 2800

262.[MCQ] [GATE-CE-2018: 2M]

A faulty wall clock is known to gain 15 minutes every 24 hours. It is synchronized to the correct time at 9 AM on 11th July. What will be the correct time to the nearest minute when the clock shows 2 PM on 15th July of the same year?

(a) 12:45 PM
(b) 12:58 PM
(c) 1:00 PM
(d) 2:00 PM

263.[MCQ] [GATE-ME-2018: 2M]

An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:

(1) HTHTHT (2) TTHHHT
(3) HTTHHT (4) HHHT__ __.

Which statement describing the last two-coin tosses of the fourth trial has the highest probability of being correct?

(a) Two T will occur.
(b) One H and one T will occur.
(c) Two H will occur.
(d) One H will be followed by one T.

264.[MCQ] [GATE-ME-2018: 2M]

A contract is to be completed in 52 days and 125 identical robots were employed, each operational for 7 hours a day. After 39 days, five-seventh of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

(a) 50 (b) 89
(c) 146 (d) 175

265.[MCQ] [GATE-ME-2018: 2M]

A wire would enclose an area of 1936 m^2, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closest to the sum of the areas enclosed by the two pieces in square meters?

(a) 1096 (b) 1111
(c) 1243 (d) 2486

266.[MCQ] [GATE-ME-2018: 1M]

The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true?

(a) The circle has the largest area.
(b) The square has the largest area.
(c) The equilateral triangle has the largest area.
(d) All the three shapes have the same area.

267.[MCQ] [GATE-ME-2018: 2M]

From the time the front of a train enters a platform, it takes 25 seconds for the back of the train to leave the platform, while travelling at a constant speed of 54 km/h. At the same speed, it takes 14 seconds to pass a man running at 9 km/h in the same direction as the train. What is the length of the train and that of the platform in meters, respectively?

(a) 210 and 140 (b) 162.5 and 187.5
(c) 245 and 130 (d) 175 and 200

268.[MCQ] [GATE-ME-2018: 2M]

Given that a and b are integers and $a + a^2 b^3$ is odd, which one of the following statements is correct?

(a) a and b are both odd
(b) a and b are both even
(c) a is even and b is odd
(d) a is odd and b is even

269.[MCQ] [GATE-ME-2018: 1M]

A number consists of two digits. The sum of the digits is 9. If 45 is subtracted from the number, its digits are interchanged. What is the number?

(a) 63 (b) 72
(c) 81 (d) 90

270.[MCQ] [GATE-ME-2018: 1M]

A rectangle becomes a square when its length and breadth are reduced by 10 m and 5 m, respectively. During this process, the rectangle loses 650 m^2 of area. What is the area of the original rectangle in square meters?

(a) 1125 (b) 2250
(c) 2924 (d) 4500

271.[MCQ] [GATE-EE,CS-2017 : 1M]

Find the smallest number y such that y × 162 is a perfect cube.

(a) 24 (b) 27
(c) 32 (d) 36

272.[MCQ] [GATE-EE,CS-2017 : 1M]

The probability that a k-digit number does NOT contain the digits 0, 5, or 9 is

(a) 0.3^k (b) 0.6^k
(c) 0.7^k (d) 0.9^k

273.[MCQ] [GATE-EE,CS-2017 : 2M]

Arun, Gulab, Neel and Shweta must choose one shirt each from a pile of four shirts coloured red, pink, blue and white respectively. Arun dislikes the colour red and Shweta dislikes the colour white. Gulab and Neel like all the colours. In how many different ways can they choose the shirts so that no one has a shirt with a colour he or she dislikes?

(a) 21 (b) 18
(c) 16 (d) 14

274.[MCQ] [GATE-EE,CS-2017 : 1M]

A test has twenty questions worth 100 marks in total. There are two types of questions. Multiple choice questions are worth 3 marks each and essay questions are worth 11 marks each. How many multiple choice questions does the exam have?

(a) 12 (b) 15
(c) 18 (d) 19

275.[MCQ] [GATE-EE,CS-2017 : 1M]

There are 3 red socks. 4 green socks and 3 blue socks. You choose 2 socks. The probability that they are of the same colour is

(a) 1/5 (b) 7/30
(c) 1/4 (d) 4/15

276.[MCQ] [GATE-EE,CS-2017 : 2M]

X is a 30 digit number starting with the digit 4 followed by the digit 7. Then the number X^3 will have

(a) 90 digits (b) 91 digits
(c) 92 digits (d) 93 digits

277.[MCQ] [GATE-EC-2017:2M]

1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

(a) 3000 (b) 3300
(c) 3600 (d) 3900

278.[MCQ] [GATE-EC-2017:2M]

Trucks (10 m long) and cars (5 m long) go on a single lane bridge. There must be a gap of at least 20 m after each truck and a gap of at least 15 m after each car. Trucks and cars travel at a speed of 36 km/h. If cars and trucks go alternately, what is the maximum number of vehicles that can use the bridge in one hour?

(a) 1440 (b) 1200
(c) 720 (d) 600

279.[MCQ] [GATE-EC-2017:1M]

In the summer, water consumption is known to decrease overall by 25%. A Water Board official states that in the summer household consumption decreases by 20%, while other consumption increases by 70%.

Which of the following statements is correct?

(a) The ratio of household to other consumption is 8/17
(b) The ratio of household to other consumption is 1/17
(c) The ratio of household to other consumption is 17/8
(d) There are errors in the official's statement.

280.[MCQ] [GATE-CE-2017: 1M]

If the radius of a right circular cone is increased by 50%. its volume increases by

(a) 75% (b) 100%

(c) 125% (d) 237.5%

281.[MCQ] [GATE-CE-2017: 1M]

The following sequence of numbers is arranged in increasing order: 1, x, x, x, y, y, 9,16,18. Given that the mean and median are equal, and are also equal to twice the mode, the value of y is

(a) 5 (b) 6

(c) 7 (d) 8

282.[MCQ] [GATE-CE-2017: 2M]

Budhan covers a distance of 19 km in 2 hours by cycling one fourth of the time and walking the rest. The next day he cycles (at the same speed as before) for half the time and walks the rest (at the same speed as before) and covers 26 km in 2 hours. The speed in km/h at which Budhan walks is

(a) 1 (b) 4

(c) 5 (d) 6

283.[MCQ] [GATE-CE-2017: 2M]

The last digit of $(2171)^7 + (2172)^9 + (2173)^{11} + (2174)^{13}$ is

(a) 2 (b) 4

(c) 6 (d) 8

284.[MCQ] [GATE-CE-2017: 2M]

Two machines M1 and M2 are able to execute any of four jobs P, Q, R and S. The machines can perform one job on one object at a time. Jobs P, Q, R and S take 30 minutes, 20 minutes, 60 minutes and 15 minutes each respectively. There are 10 objects each requiring exactly 1 job. Job P is to be performed on 2 objects. Job Q on 3 objects. Job R on 1 object and Job S on 4 objects. What is the minimum time needed to complete all the jobs?

(a) 2 hours (b) 2.5 hours

(c) 3 hours (d) 3.5 hours

285.[MCQ] [GATE-CE-2017: 1M]

What is the value of x when

$$81\times\left(\frac{16}{25}\right)^{x+2}\div\left(\frac{3}{5}\right)^{2x+4}=144$$

(a) 1

(b) –1

(c) –2

(d) Cannot be determined

286.[MCQ] [GATE-ME-2017: 2M]

There are 4 women P, Q, R, S, and 5 men V, W, X, Y, Z in a group. We are required to form pairs each consisting of one woman and one man. P is not to be paired with Z, and Y must necessarily be paired with someone. In how many ways can 4 such pairs be formed?

(a) 74 (b) 76

(c) 78 (d) 80

287.[MCQ] [GATE-ME-2017: 2M]

X bullocks and Y tractors take 8 days to plough a field. If we halve the number of bullocks and double the number of tractors, it takes 5 days to plough the same field. How many days will it take X bullocks alone to plough the field?

(a) 30 (b) 35

(c) 40 (d) 45

288.[MCQ] [GATE-ME-2017: 1M]

P looks at Q while Q looks at R. P is married, R is not. The number of pairs of people in which a married person is looking at an unmarried person is

(a) 0

(b) 1

(c) 2

(d) Cannot be determined

289.[MCQ] [GATE-ME-2017: 1M]

A couple has 2 children. The probability that both children are boys if the older one is a boy is

(a) 1/4 (b) 1/3

(c) 1/2 (d) 1

290.[MCQ] **[GATE-ME-2017: 1M]**

If a and b are integers and a – b is even, which of the following must always be even?

(a) ab (b) $a^2 + b^2 + 1$
(c) $a^2 + b + 1$ (d) ab – b

291.[MCQ] **[GATE-ME-2017: 2M]**

What is the sum of the missing digits in the subtraction problem below?

```
   5 _ _ _ _
 – 4 8 _ 8 9
 -----------
     1 1 1 1
```

(a) 8
(b) 10
(c) 11
(d) Cannot be determined

292.[MCQ] **[GATE-ME-2017: 1M]**

In a company with 100 employees, 45 earn Rs. 20,000 per month, 25 earn Rs. 30,000, 20 earn Rs. 40,000, 8 earn Rs. 60,000, and 2 earn Rs. 150,000. The median of the salaries is

(a) Rs. 20,000 (b) Rs. 30.000
(c) Rs. 32.300 (d) Rs. 40.000

293. [MCQ] **[GATE-EC-2016:1M]**

It takes 10 s and 15 s, respectively, for two trains traveling at different constant speeds to completely pass a telegraph post. The length of the first train is 120 m and that of the second train is 150 m. The magnitude of the difference in the speeds of the two trains (in m/s) is ________.

(a) 2.0 (b) 10.0
(c) 12.0 (d) 22.0

294.[MCQ] **[GATE-CE-2016: 1M]**

(x % of y) + (y % of x) is equivalent to.

(a) 2 % of xy (b) 2 % of (xy/100)
(c) xy % of 100 (d) 100 % of xy

295.[MCQ] **[GATE-CE-2016: 1M]**

The sum of the digits of a two-digit number is 12. If the new number formed by reversing the digits is greater than the original number by 54, find the original number.

(a) 39 (b) 57
(c) 66 (d) 93

296.[MCQ] **[GATE-CE-2016: 2M]**

Ananth takes 6 hours and Bharath takes 4 hours to read a book. Both started reading copies of the book at the same time. After how many hours is the number of pages to be read by Ananth, twice that to be read by Bharath? Assume Ananth and Bharath read all the pages with constant pace.

(a) 1 (b) 2
(c) 3 (d) 4

297.[MCQ] **[GATE-ME,EC-2016: 2M]**

A wire of length 340 mm is to be cut into two parts. One of the parts is to be made into a square and the other into a rectangle where sides are in the ratio of 1:2. What is the length of the side of the square (in mm) such that the combined area of the square and the rectangle is a **MINIMUM?**

(a) 30 (b) 40
(c) 120 (d) 180

298. [MCQ] **[GATE-ME,EC-2016: 1M]**

S, M, E and F are working in shifts in a team to finish a project. M works with twice the efficiency of others but for half as many days as E worked. S and M have 6 hour shifts in a day, whereas E and F have 12 hours shifts. What is the ratio of contribution of M to contribution of E in the project?

(a) 1:1 (b) 1:2
(c) 1:4 (d) 2:1

299. [MCQ] **[GATE-ME,EC-2016: 1M]**

Given $(9 \text{ inches})^{1/2} = (0.25 \text{ yards})^{1/2}$, which one of the following statements is TRUE?

(a) 3 inches = 0.5 yards
(b) 9 inches = 1.5 yards
(c) 9 inches = 0.25 yards
(d) 81 inches = 0.0625 yards

300. [MCQ] [GATE-ME-2016: 2M]

The binary operation □ is defined as a □ b = ab + (a + b), where a and b are any two real numbers. The value of the identity element of this operation, defined as the number x such that a □ x = a, for any a, is ______.

(a) 0 (b) 1
(c) 2 (d) 10

301.[MCQ] [GATE-ME-2016: 2M]

Students taking an exam are divided into two groups, P and Q such that each group has the same number of students. The performance of each of the students in a test was evaluated out of 200 marks. It was observed that the mean of group P was 105, while that of group Q was 85. The standard deviation of group P was 25, while that of group Q was 5. Assuming that the marks were distributed on a normal distribution, which of the following statements will have the highest probability of being TRUE?

(a) No student in group Q scored less marks than any student in group P.
(b) No student in group P scored less marks than any student in group Q.
(c) Most students of group Q scored marks in a narrower range than students in group P.
(d) The median of the marks of group P is 100.

302.[MCQ] [GATE-ME-2016: 1M]

A window is made up of a square portion and an equilateral triangle portion above it. The base of the triangular portion coincides with the upper side of the square. If the perimeter of the window is 6 m, the area of the window in m^2 is ___________.

(a) 1.43 (b) 2.06
(c) 2.68 (d) 2.88

303. [MCQ] [GATE-ME,EC-2016: 2M]

P, Q, R and S are working on a project. Q can finish the task in 25 days, working alone for 12 hours a day. R can finish the task in 50 days, working alone for 12 hours per day. Q worked 12 hours a day but took sick leave in the beginning for two days. R worked 18 hours a day on all days. What is the ratio of work done by Q and R after 7 days from the start of the project?

(a) 10:11 (b) 11:10
(c) 20:21 (d) 21:20

304. [MCQ] [GATE-ME,EC-2016: 2M]

A person moving through a tuberculosis prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis prone zone remains infected but does not show symptoms of disease?

(a) 15 (b) 33
(c) 35 (d) 37

305. [MCQ] [GATE-ME,EC-2016: 1M]

In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits. Of the unripe fruits, 45% are apples. Of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?

(a) 2029198 (b) 2467482
(c) 2789080 (d) 3577422

306.[MCQ] [GATE-EE,CS-2015 : 2M]

The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in m, p, c:

(I) p + m + c = 27/20 (II) p + m + c = 13/20
(III) (p) × (m) × (c) = 1/10

(a) Only relation I is true
(b) Only relation II is true
(c) Relations II and III are true
(d) Relations I and III are true

307.[MCQ] [GATE-EE,CS-2015 : 2M]

If p. q, r, s are distinct integers such that:

f (p, q, r, s) = max (p, q, r, s)

g (p, q, r, s) = min (p, q, r, s)

h (p, q, r, s) = remainder of (p × q) / (r × s) if (p × q) > (r × s) or remainder of (r × s) / (p × q) if (r × s) > (p × q)

Also a function fgh (p. q, r, s) = f (p, q, r, s) × g (p, q, r, s) × h (p, q, r, s)

Also the same operations are valid with two variable functions of the form f (p, q).

What is the value of fg (h (2,5,7,3), 4,6,8)?

(a) 6 (b) 7
(c) 8 (d) 9

308.[MCQ] [GATE-CS,CE-2015: 1M]

A function f(x) is linear and has a value of 29 at x = –2 and 39 at x =3. Find its value at x = 5.

(a) 59 (b) 45
(c) 43 (d) 35

309.[MCQ] [GATE-CE-2015: 2M]

The given question is followed by two statements: select the most appropriate option that solves the question.

Capacity of a solution tank A is 70% of the capacity of tank B. How many gallons of solution are in tank A and tank B?

Statements:

(I) Tank A is 80% full and tank B is 40% full.
(II) Tank A if full contains 14,000 gallons of solution.

(a) Statement I alone is sufficient.
(b) Statement II alone is sufficient.
(c) Either statement I or II alone is sufficient.
(d) Both the statements I and II together are sufficient.

310.[NAT] [GATE-CE-2015: 2M]

How many four digit numbers can be formed with the 10 digits 0, 1,2,..., 9 if no number can start with 0 and if repetitions are not allowed?

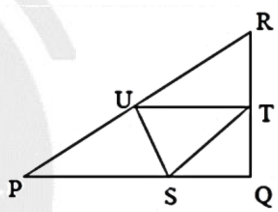
311.[MCQ] [GATE-ME-2015: 2M]

A coin is tossed thrice. Let X be the event that head occurs in each of the first two tosses. Let Y be the event that a tail occurs on the third toss. Let Z be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

(a) X and Y are not independent
(b) Y and Z are dependent
(c) Y and Z are independent
(d) X and Z are independent

312.[NAT] [GATE-ME-2015: 2M]

In the given figure angle Q is a right angle, PS : QS = 3 : 1, RT : QT = 5 : 2 and PU : UR = 1 : 1. If area of triangle QTS is 20 cm², then the area of triangle PQR in cm² is______.



313.[MCQ] [GATE-ME-2015: 1M]

Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?

(a) 20 (b) 10
(c) 8 (d) 5

314.[NAT] [GATE-ME,EC-2015: 2M]

From a circular sheet of paper of radius 30 cm, a sector of 10% area is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is________.

315.[NAT] [GATE-ME,EC-2015: 2M]

A tiger is 50 leaps of its own behind a deer. The tiger takes 5 leaps per minute to the deer's 4. If the tiger and the deer cover 8 meter and 5 metre per leap respectively what distance in meters will the tiger have to run before it catches the deer?

316.[MCQ] [GATE-ME,EC-2015: 1M]

Ram and Ramesh appeared in an interview for two vacancies in the same department The probability of Ram's selection is 1/6 and that of Ramesh is 1/8. What is the probability that only one of them will be selected?

(a) 47/48 (b) 1/4
(c) 13/48 (d) 35/48

317.[NAT] [GATE-EE,CS-2014 : 1M]

If $(z + 1/z)^2 = 98$, compute $(z^2 + 1/z^2)$.

318.[NAT] [GATE-EE,CS-2014 : 2M]

Round-trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is Rs 100, a group of 5 tourists purchasing round-trip tickets will be charged Rs __________.

319.[MCQ] [GATE-EE,CS-2014 : 1M]

What is the average of all multiples of 10 from 2 to 198?

(a) 90 (b) 100
(c) 110 (d) 120

320.[MCQ] [GATE-EE,CS-2014 : 1M]

The value of $\sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$ is

(a) 3.464 (b) 3.932
(c) 4.000 (d) 4.444

321.[MCQ] [GATE-EE,CS-2014 : 2M]

At what time between 6 $a.m.$ and 7 $a.m.$ will the minute hand and hour hand of a clock make an angle closest to 60°?

(a) 6:22 a.m. (b) 6:27 a.m.
(c) 6:38 a.m. (d) 6:45 a.m.

322.[MCQ] [GATE-EE,CS-2014 : 2M]

The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012-2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012-2013 the exchange rate for the USD increased from Rs. 50/USD to Rs. 60/USD. India's GDP in USD during the period 2012-2013

(a) increased by 5 %
(b) decreased by 13%
(c) decreased by 20%
(d) decreased by 11%

323.[NAT] [GATE-CE-2014: 1M]

A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

324.[MCQ] [GATE-CE-2014: 2M]

One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

(a) 3.0 (b) 2.5
(c) 1.5 (d) 1.25

325.[MCQ] [GATE-CE-2014: 1M]

The population of a new city is 5 million and is growing at 20% annually. How many years would it take to double at this growth rate?

(a) 3-4 years (b) 4-5 years
(c) 5-6 years (d) 6-7 years

326. [MCQ] [GATE-ME,EC-2014: 2M]

A five-digit number is formed using the digits 1,3,5,7 and 9 without repeating any of them. What is the sum of all such possible five-digit numbers?

(a) 6666660 (b) 6666600
(c) 6666666 (d) 6666606

327. [MCQ] [GATE-ME,EC-2014: 2M]

Industrial consumption of power doubled from 2000-2001 to 2010-2011. Find the annual rate of increase in percent assuming it to be uniform over the years.

(a) 5.6 (b) 7.2
(c) 10.0 (d) 12.2

328. [NAT] [GATE-ME,EC-2014: 1M]

In a sequence of 12 consecutive odd numbers, the sum of the first 5 numbers is 425. What is the sum of the last 5 numbers in the sequence?

329.[NAT] [GATE-ME-2014: 2M]

A batch of one hundred bulbs is inspected by testing four randomly chosen bulbs. The batch is rejected if even one of the bulbs is defective. A batch typically has five defective bulbs. The probability that the current batch is accepted is _________.

330. [NAT] [GATE-ME,EC-2014: 2M]

A man can row at 8 km per hour in still water. If it takes him thrice as long to row upstream, as to row downstream, then find the stream velocity in km per hour.

331. [MCQ] [GATE-ME,EC-2014: 1M]

In which of the following options will the expression P < M be definitely true?

(a) M < R > P > S (b) M > S < P < F

(c) Q < M < F = P (d) P = A < R < M

332. [MCQ] [GATE-ME,EC-2014: 2M]

It takes 30 minutes to empty a half-full tank by draining it at a constant rate. It is decided to simultaneously pump water into the half-full tank while draining it. What is the rate at which water has to be pumped in so that it gets fully filled in 10 minutes?

(a) 4 times the draining rate
(b) 3 times the draining rate
(c) 2.5 times the draining rate
(d) 2 times the draining rate

333. [NAT] [GATE-ME,EC-2014: 2M]

The sum of eight consecutive odd numbers is 656. The average of four consecutive even numbers is 87. What is the sum of the smallest odd number and second largest even number?

334. [MCQ] [GATE-ME,EC-2014: 1M]

A regular die has six sides with numbers 1 to 6 marked on its sides. If a very large number of throws show the following frequencies of occurrence: 1 $\rightarrow$ 0.167; 2 $\rightarrow$ 0.167; 3 $\rightarrow$ 0.152; 4 $\rightarrow$ 0.166; 5 $\rightarrow$ 0.168; 6 $\rightarrow$ 0.180. We call this die

(a) Irregular (b) Biased

(c) Gaussian (d) Insufficient

335. [MCQ] [GATE-ME,EC-2014: 2M]

You are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head on one face and a tail on the other. You choose a coin at random and toss it, and it comes up heads. The probability that the other face is tails is

(a) 1/4 (b) 1/3

(c) 1/2 (d) 2/3

336. [NAT] [GATE-ME,EC-2014: 2M]

A train that is 280 metres long, travelling at a uniform speed, crosses a platform in 60 seconds and passes a man standing on the platform in 20 seconds. What is the length of the platform in metres?

337.[MCQ] [GATE-EE,EC-2013 : 2M]

A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km per hour over the entire journey is

(a) 30 (b) 36

(c) 40 (d) 24

338.[MCQ] [GATE-EE,EC-2013 : 1M]

In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41°C and of Tuesday to Thursday was 43°C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was

(a) 40 (b) 43

(c) 46 (d) 49

339.[MCQ] [GATE-CE-2013: 1M]

A number is as much greater than 75 as it is smaller than 117. The number is:

(a) 91 (b) 93
(c) 89 (d) 96

340.[MCQ] [GATE-CE-2013: 2M]

A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

(a) 5 (b) 10
(c) 15 (d) 30

341.[MCQ] [GATE-ME,CS-2013: 2M]

The current erection cost of a structure is Rs. 13,200. If the labour wages per day increase by $\frac{1}{5}$ of the current wages and the working hours decrease by $\frac{1}{24}$ of the current period, then the new cost of erection in Rs. is

(a) 16,500 (b) 15,180
(c) 11,000 (d) 10,120

342.[MCQ] [GATE-ME,CS-2013: 2M]

A tourist covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average speed of the tourist in km/h during his entire journey is

(a) 36 (b) 30
(c) 24 (d) 18

343.[MCQ] [GATE-ME,CS-2013: 2M]

Out of all the 2-digit integers betwene 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(a) $\frac{13}{90}$ (b) $\frac{12}{90}$
(c) $\frac{78}{90}$ (d) $\frac{77}{90}$

344.[MCQ] [GATE-ME,CS-2013: 1M]

What will be the maximum sum of 44, 42, 40, ……?

(a) 502 (b) 504
(c) 506 (d) 500

345.[MCQ] [GATE-EE,EC-2012 : 2M]

Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(a) 5 (b) 6
(c) 9 (d) 10

346.[MCQ] [GATE-EE,EC-2012 : 1M]

If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(a) 2.23 (b) 4.33
(c) 11.37 (d) 27.64

347.[MCQ] [GATE-ME,CE,CS-2012: 2M]

Which of the following assertions are CORRECT?

P: Adding 7 to each entry in a list adds 7 to the mean of the list
Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list
R: Doubling each entry in a list doubles the mean of the list
S: Doubling each entry in a list leaves the standard deviation of the list unchanged

(a) P, Q (b) Q, R
(c) P, R (d) R, S

348.[MCQ] [GATE-ME,CE,CS-2012: 2M]

An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable. The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is.

(a) 0.288 (b) 0.334
(c) 0.667 (d) 0.720

349.[MCQ] [GATE-EE,EC-2011 : 2M]

Three friends, R, S and T shared toffee from a bowl. R took $1/3^{rd}$ of the toffees, but returned four to the bowl. S took $1/4^{th}$ of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(a) 38 (b) 31
(c) 48 (d) 41

350.[MCQ] [GATE-EE, ,EC-2011 : 1M]

There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters?

(a) 100 (b) 110
(c) 90 (d) 95

351.[MCQ] [GATE-ME,CE,CS-2011: 2M]

A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4^{th} day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10^{th} day. What is the minimum number of trucks required so that there will be no pending roder at the end of the 5^{th} day?

(a) 4 (b) 5
(c) 6 (d) 7

352.[MCQ] [GATE-ME,CE,CS-2011: 2M]

A container orginally contains 10 litres of pure spirit. From this container 1 litre of spirit is repalced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?

(a) 7.58 litres (b) 7.84 litres
(c) 7 litres (d) 7.29 litres

353.[MCQ] [GATE-ME,CE,EE,EC,CS-2010: 2M]

Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed?

(a) 50 (b) 51
(c) 52 (d) 54

354.[MCQ] [GATE-ME,CE,EE,EC,CS-2010: 2M]

5 skilled workers can build a wall in 20 days, 8 semi-skilled workers can build a wall in 25 days, 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(a) 20 days (b) 18 days
(c) 16 days (d) 15 days

❑❑❑

ANSWER KEY

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(b)
5.	(a)	**6.**	(c)	**7.**	(c)	**8.**	(c)
9.	(c)	**10.**	(b)	**11.**	(d)	**12.**	(Marks to All)
13.	(d)	**14.**	(c)	**15.**	(a)	**16.**	(a)
17.	(b)	**18.**	(c)	**19.**	(c)	**20.**	(c)
21.	(b)	**22.**	(b)	**23.**	(b)	**24.**	(c)
25.	(c)	**26.**	(b)	**27.**	(c)	**28.**	(b)
29.	(d)	**30.**	(b)	**31.**	(c)	**32.**	(c)
33.	(a)	**34.**	(120 to 120)	**35.**	(c)	**36.**	(b)
37.	(d)	**38.**	(c)	**39.**	(d)	**40.**	(c)
41.	(32 to 32)	**42.**	(2006 to 2006)	**43.**	(d)	**44.**	(d)
45.	(48 to 48)	**46.**	(140 to 140)	**47.**	(c)	**48.**	(c)
49.	(22 to 22)	**50.**	(20000 to 20000)	**51.**	(d)	**52.**	(d)
53.	(d)	**54.**	(a)	**55.**	(b)	**56.**	(d)
57.	(d)	**58.**	(d)	**59.**	(b)	**60.**	(d)
61.	(b)	**62.**	(b)	**63.**	(a)	**64.**	(c)
65.	(b)	**66.**	(a)	**67.**	(a)	**68.**	(a)
69.	(a)	**70.**	(a)	**71.**	(a)	**72.**	(c)
73.	(c)	**74.**	(c)	**75.**	(c)	**76.**	(b)
77.	(d)	**78.**	(a)	**79.**	(c)	**80.**	(c)
81.	(d)	**82.**	(d)	**83.**	(b)	**84.**	(b)
85.	(b)	**86.**	(a)	**87.**	(a)	**88.**	(b)
89.	(d)	**90.**	(c)	**91.**	(a)	**92.**	(c)
93.	(c)	**94.**	(a)	**95.**	(b)	**96.**	(c)
97.	(c)	**98.**	(a)	**99.**	(c)	**100.**	(a)
101.	(d)	**102.**	(d)	**103.**	(d)	**104.**	(c)
105.	(Marks to All)	**106.**	(c)	**107.**	(a)	**108.**	(d)
109.	(d)	**110.**	(c)	**111.**	(a)	**112.**	(b)
113.	(b)	**114.**	(a)	**115.**	(a)	**116.**	(b)
117.	(c)	**118.**	(b)	**119.**	(a)	**120.**	(b)
121.	(c)	**122.**	(c)	**123.**	(a)	**124.**	(b)
125.	(c)	**126.**	(d)	**127.**	(7 to 7)	**128.**	(a)
129.	(c)	**130.**	(b)	**131.**	(b)	**132.**	(b)
133.	(a)	**134.**	(b)	**135.**	(c)	**136.**	(d)
137.	(a)	**138.**	(c)	**139.**	(b)	**140.**	(b)
141.	(b)	**142.**	(a)	**143.**	(c)	**144.**	(c)

145.	(a)	**146.**	(b)	**147.**	(b)	**148.**	(d)
149.	(d)	**150.**	(c)	**151.**	(c)	**152.**	(180 to 180)
153.	(25 to 25)	**154.**	(a)	**155.**	(a)	**156.**	(b)
157.	(d)	**158.**	(a)	**159.**	(b)	**160.**	(b)
161.	(b)	**162.**	(c)	**163.**	(a)	**164.**	(c)
165.	(d)	**166.**	(a)	**167.**	(b)	**168.**	(c)
169.	(a)	**170.**	(c)	**171.**	(a)	**172.**	(a)
173.	(a)	**174.**	(c)	**175.**	(d)	**176.**	(a)
177.	(c)	**178.**	(a)	**179.**	(b)	**180.**	(a)
181.	(a)	**182.**	(d)	**183.**	(d)	**184.**	(d)
185.	(d)	**186.**	(d)	**187.**	(b)	**188.**	(b)
189.	(d)	**190.**	(c)	**191.**	(b)	**192.**	(c)
193.	(a)	**194.**	(a)	**195.**	(b)	**196.**	(a)
197.	(c)	**198.**	(a)	**199.**	(c)	**200.**	(d)
201.	(d)	**202.**	(c)	**203.**	(c)	**204.**	(c)
205.	(c)	**206.**	(c)	**207.**	(c)	**208.**	(a)
209.	(c)	**210.**	(d)	**211.**	(c)	**212.**	(b)
213.	(a)	**214.**	(b)	**215.**	(d)	**216.**	(c)
217.	(d)	**218.**	(b)	**219.**	(b)	**220.**	(b)
221.	(d)	**222.**	(c)	**223.**	(c)	**224.**	(a)
225.	(c)	**226.**	(a)	**227.**	(d)	**228.**	(b)
229.	(c)	**230.**	(b)	**231.**	(b)	**232.**	(c)
233.	(b)	**234.**	(c)	**235.**	(c)	**236.**	(c)
237.	(a)	**238.**	(c)	**239.**	(b)	**240.**	(d)
241.	(b)	**242.**	(a)	**243.**	(b)	**244.**	(d)
245.	(c)	**246.**	(d)	**247.**	(c)	**248.**	(c)
249.	(b)	**250.**	(c)	**251.**	(c)	**252.**	(b)
253.	(b)	**254.**	(d)	**255.**	(b)	**256.**	(d)
257.	(c)	**258.**	(b)	**259.**	(c)	**260.**	(d)
261.	(c)	**262.**	(b)	**263.**	(b)	**264.**	(Marks to All)
265.	(c)	**266.**	(a)	**267.**	(d)	**268.**	(d)
269.	(b)	**270.**	(b)	**271.**	(d)	**272.**	(c)
273.	(d)	**274.**	(b)	**275.**	(d)	**276.**	(a)
277.	(c)	**278.**	(a)	**279.**	(d)	**280.**	(c)
281.	(d)	**282.**	(d)	**283.**	(b)	**284.**	(a)
285.	(b)	**286.**	(c)	**287.**	(a)	**288.**	(b)
289.	(c)	**290.**	(d)	**291.**	(d)	**292.**	(b)
293.	(a)	**294.**	(a)	**295.**	(a)	**296.**	(c)

297.	(b)	**298.**	(b)	**299.**	(c)	**300.**	(a)
301.	(c)	**302.**	(b)	**303.**	(c)	**304.**	(c)
305.	(a)	**306.**	(d)	**307.**	(c)	**308.**	(c)
309.	(d)	**310.**	(4536 to 4536)	**311.**	(b)	**312.**	(280 to 280)
313.	(b)	**314.**	(1.9 to 2.2)	**315.**	(800 to 800)	**316.**	(b)
317.	(96 to 96)	**318.**	(850 to 850)	**319.**	(b)	**320.**	(c)
321.	(a)	**322.**	(d)	**323.**	(1300 to 1300)	**324.**	(d)
325.	(a)	**326.**	(b)	**327.**	(b)	**328.**	(495 to 495)
329.	(0.80 to 0.82)	**330.**	(4 to 4)	**331.**	(d)	**332.**	(a)
333.	(163 to 163)	**334.**	(b)	**335.**	(b)	**336.**	(560 to 560)
337.	(c)	**338.**	(c)	**339.**	(d)	**340.**	(a)
341.	(b)	**342.**	(c)	**343.**	(d)	**344.**	(c)
345.	(a)	**346.**	(d)	**347.**	(c)	**348.**	(b)
349.	(c)	**350.**	(a)	**351.**	(c)	**352.**	(d)
353.	(b)	**354.**	(d)				

SOLUTIONS

1. **(a)**

Universities = 504
Colleges = 25951
Percentage of higher education institutions that fall into grade III

$= \dfrac{7 \times 504 + 23 \times 25951}{504 + 25951}$

= 22.7%

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2. **(a)**

Mode = x with highest probability
= –13, 13

Median $= \dfrac{-1+1}{2} = 0$

Mean = 0

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3. **(b)**

Mode = 7 (data with maximum frequency)

Median

$= \left(\dfrac{\frac{n}{2}^{\text{th}} + \left(\frac{n}{2}+1\right)^{\text{th}}}{2} \right) = \dfrac{25^{\text{th}} + 26^{\text{th}}}{2} = \dfrac{6+6}{2} = 6$

Mean

$= \dfrac{3 \times 3 + 9 \times 4 + 11 \times 5 + 7 \times 6 + 14 \times 7 + 2 \times 8 + 4 \times 9}{3 + 9 + 11 + 7 + 14 + 2 + 4}$

$= \dfrac{9 + 36 + 55 + 42 + 98 + 16 + 36}{50}$

$= \dfrac{292}{50} = < 6$

$\therefore$ mode > median > mean

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4. (b)

Given:

$$x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3 = k\,(\text{say})$$

$\Rightarrow$ x = 6k, y = 4k, z = 3k

$$\therefore \quad \frac{x+z-y}{y} = \frac{6k+3k-4k}{4k} = \frac{5}{4} = 1.25$$

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5. (a)

Given cost/unit for year 3 = 1

Cost/unit for year 2 = 2

Cost/unit for year 1 = 3

For year 2

Net profit = selling price – (cost + taxes)

$$296 = SP_2 - \left(2 \times 200 + \frac{13}{100} \times SP_2\right)$$

$SP_2 = 800$

For year 3

$$210 = SP_3 - \left(1 \times 300 + \frac{15}{100} \times SP_3\right)$$

$SP_3 = 600$

Then $\frac{SP_2}{SP_3} = \frac{800}{600} = \frac{4}{3}$

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6. (c)

% of more minutes spent than the other on Sunday

$$= \frac{65-55}{55} \times 100 = 18.18\%$$

% of more minutes spent than the other on Saturday

$$= \frac{60-50}{50} \times 100 = 20\%$$

% of more minutes spent than the other on Friday

$$= \frac{35-20}{20} \times 100 = 75\%$$

% of more minutes spent than the other on Thursday

$$= \frac{60-55}{55} \times 100 = \mathbf{9.09\%}$$

% of more minutes spent than the other on Wednesday

$$= \frac{60-50}{50} \times 100 = 20\%$$

% of more minutes spent than the other on Tuesday

$$= \frac{65-55}{55} \times 100 = 18.18\%$$

% of more minutes spent than the other on Monday

$$= \frac{70-45}{45} \times 100 = 55.56\%$$

So only on Thursday percentage is less than 10 and hence for remaining 6-days, percentage is more than 10.

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7. (c)

Total executives in $C_2 = \frac{5}{100} \times 10000 = 500$

Total executives in $C_5 = \frac{20}{100} \times 10000 = 2000$

Total number of executives with management degree in C_2 & C_5 (from the chart)

$$= \frac{1}{5} \times 500 + \frac{9}{10} \times 2000 = 1900$$

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8. (c)

Revenue of company Q in 2015 = 45 million Rs.
Let the revenue of company Q in 2014 = x million Rs
Then,
The revenue of company Q in 2015 = 1.2x million Rs.
$\therefore$ 1.2x = 45
We get, x = 37.5 million Rs.
As per given in question, company Q had earned a profit of 10%.,

$\because$ Expenditure of Q in 2014 $= \frac{37.5}{1.1} = 34.1$ million Rs.

So, option 'C' is correct.

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9. (c)

Total expenditure during 2014 – 2018

$5 \times 500 = 2500$ million rupees

Total revenue during 2014 – 2018

$= 500 + 700 + 800 + 600 + 400$

$= 3000$ million rupees.

$$\text{Total profit} = \frac{\text{revenue} - \text{expenditure}}{\text{expenditure}}$$

$$= \frac{3000 - 2500}{2500} = 20\%$$

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10. (b)

Average number of students in school P

$$P_{avg} = \frac{3+5+5+6+4}{5} = \frac{23}{5}$$

Average number of students in school Q

$$Q_{avg} = \frac{4+7+8+7+5}{5} = \frac{31}{5}$$

$$\therefore \quad Q_{avg} - P_{avg} = \frac{31}{5} - \frac{23}{5} = \frac{8}{5}$$

$$\therefore \quad P_{avg} : (Q_{avg} - P_{avg}) = \frac{23}{5} : \frac{8}{5} = 23 : 8$$

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11. (d)

Total number of students in art $= \frac{20}{100} \times 5000 = 1000$

Number of girls in art $= \frac{30}{100} \times 1500 = 450$

So, the number of boys in art is = 1000 – 450 = 550

Number of girls in management $= \frac{15}{100} \times 1500 = 225$

So, the required ratio $= \frac{550}{225} = \frac{22}{9}$

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12. (Marks to All)

As per given information
Total bulb sold day two firms X and Y during April-June in year (2018) = 50,000
Assume total numbering of bulb sold by both firm in all months = P

$\therefore \quad 0.15\text{ P} + 0.1\text{ P} + 0.1\text{ P} = 50{,}000$

$0.35\text{ P} = 50{,}000$

$$P = \frac{1000000}{7} = 142857.1429$$

LED bulbs sold by firm Y from April to June 2018

$y_{april} + y_{may} + y_{yane}$

$$= \left(\frac{2}{5} \times 15 + \frac{4}{5} \times 10 + \frac{11}{20} \times 10\right)\% \text{ of P}$$

$= (6 + 8 + 5.5)\%$ of P.

$= 19.5\%$ P

$= \dfrac{19.5 \times 142857.1429}{100} = 27857.42$

Total sold bulb by firm Y from april to yance

$= 27858$

Answer does not match with any of the given options

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13. (d)

Money spent on transport = 0.1 x

Money spent on education = 0.15 x

Extra expenditure $= \dfrac{0.15x - 0.1x}{0.1\ x} \times 100 = 50\%$

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14. (c)

Success rate of each schools:

$P = \dfrac{280}{500} \times 100 = 56\%$

$Q = \dfrac{330}{600} \times 100 = 55\%$

$R = \dfrac{455}{700} \times 100 = 65\%$

$S = \dfrac{240}{400} \times 100 = 60\%$

Average of success rate $= \dfrac{56 + 55 + 65 + 60}{4}$

$= 59\%$

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15. (a)

$\Longrightarrow$ For $y = x^m$, the slope at x = 0 & x = 1

$y' = mx^{m-1}$

$$\boxed{\begin{array}{l} y'(0) = 0 \\ y'(1) = +m \end{array}}$$

$\Longrightarrow$ For $y = x^{1/m}$, the slope at x = 0 & x = 1

$$y' = \frac{1}{m} x^{\frac{1}{m}-1} = \frac{1}{m} \frac{1}{x^{1-\frac{1}{m}}}$$

$$\boxed{\begin{array}{l} y'(0) = \infty \\ y'(1) = \dfrac{1}{m} \end{array}}$$

These conditions are satisfied only by option A

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16. (a)

Let the total population be x.

Number of males in 2001 = 0.60 x

Number of females in 2001 = 0.40 x

Number of illiterates males in 2001

$= 0.50 \times 0.60$ x

= 0.30 x

Number of illiterates females in 2001

$= 0.60 \times 0.40$ x

= 0.24 x

Total number of illiterates in 2001

= 0.30 x + 0.24 x

= 0.54 x

So, total number of literates in 2001 = x – 0.54 x

= 0.46 x

Similarly,

Number of males in 2011 = 0.50 x

Number of females in 2011 = 0.50 x

Number of illiterates males in 2011 $= 0.40 \times 0.50$ x

= 0.20 x

Number of illiterates females in 2011

$= 0.40 \times 0.50$ x

= 0.20 x

Total number of illiterates in 2011

= 0.20 x + 0.20 x

= 0.40 x

So, number of literates in 2011 = x – 0.40 x

$$= 0.60\text{ x}$$

$$\begin{Bmatrix}\text{\% increase}\\ \text{of literates}\end{Bmatrix} = \frac{\begin{Bmatrix}\text{number of}\\ \text{literates in 2011}\end{Bmatrix} - \begin{Bmatrix}\text{number of}\\ \text{literates in 2001}\end{Bmatrix}}{\text{number of literates in 2001}}$$

$$= \frac{0.60x - 0.46x}{0.46x} \times 100 = \frac{14}{46} \times 100 = 30.43\%$$

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17. (b)

The contribution of Pool in total revenues is

170 + 320 + 215 + 190 = 895

The contribution of Mini

110 + 220 + 180 + 70 = 580

The contribution of Prime

75 + 180 + 120 + 90 = 465

The percentage of revenue contributed by Prime

$$= \frac{465}{895 + 580 + 465} \times 100 = 23.97\%$$

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18. (c)

Given, total employee in 2010 was 600, according to the pie chart S had 25% and T had 5% of them.

The increase in total employee is 15% so, the number of employees increased from 2010 to 2016 is:

$$= \frac{15}{100} \times 600 = 90$$

Employee increased in S

$$= \left(\frac{25}{100} \times 600\right) \times \frac{40}{100} = 60$$

Since employment at levels P, Q & R remained unchanged so, employees increased in T is.

90 – 60 = 30

Hence, total number of employees at level T in 2016 is

$$\frac{5}{100} \times 600 + 30 = 60$$

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19. (c)

Q can row only with R, so that let Q and R will go in first forward trip. In this trip Q will row. Now in return trip R will row. Q will not return, it means Q has crossed the lake.

P and R will go in second forward trip. In this trip R will row because P cannot row when anyone else is in the boat. Now in return trip P will row because P is alone in the boat. R will not return, it means R has crossed the lake too.

P and S will go in last forward trip. In this trip S will row, it means P and S have crossed the lake too.

Hence, only R rowed two times.

So, option 'C' is correct.

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20. (c)

Let, the number of marbles = 100

Cost of each marbles = 1 Rs.

As cost increased by 25%,

New cost = 1 + 25% of 1 = 1.25 Rs.

New number of marbles $= \frac{100 \times 1}{1.25} = 80$

Now, consider p number of marbles are reduced of each colour, then

$(40-p)+(25-p)+(20-p)+(15-p)=80$

$100-4p=80 \Rightarrow p=5$

The percentage of blue marbles in new design,

$= \frac{40-5}{80} \times 100 = 43.75\%$

So, option 'C' is correct.

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21. (b)

Given, M + F = 300 →(i)

$= \frac{8}{9} M \times 750 + \frac{2}{3} F \times 1000$

$= \frac{6000}{9} M + \frac{2000}{3} F$

$= \frac{6000}{9} M + \frac{6000}{9} F$

$= \frac{6000}{9}(M+F)$

$TA = \frac{6000}{9} \times 300 = 200000$

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22. (b)

$T = K(\Theta p + T_o)$

For p = 2 & T_o = 25, T = 32.4

$32.4 = K(\Theta \times 2 + 25)$ ______ (1)

For p = 5 & T_o = 30, T = 42.0

$42 = K(\Theta \times 5 + 30)$ ______ (2)

Equation (1) ÷ equation (2)

$\frac{32.4}{42} = \frac{\Theta \times 2 + 25}{\Theta \times 5 + 30}$

$162\,\Theta + 972 = 84\,\Theta + 1050$

$78\,\Theta = 78$

$\Theta = 1$

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23. (b)

(i) $y = 2x + 4$ for $-3 \le x \le -1$

for $\left.\begin{matrix} x=-3, y=-2 \\ x=-1, y=2 \end{matrix}\right\}$ Matches with the graph

(ii) $y = |x-1|$ for $-1 \le x \le 2$

for $\left.\begin{matrix} x=-1, & y=2 \\ x=2, & y=1 \\ x=1, & y=0 \end{matrix}\right\}$ Matches with the graph

(iii) $y = ||x|-1|$ for $-1 \le x \le 2$

for $\left.\begin{matrix} x=-1, & y=0 \\ x=2, & y=1 \end{matrix}\right\}$ Does not match with the graph

(iv) $y = 1$ for $2 \le x \le 3$ } Matches with the graph

So, (i), (ii) & (iv) are accurate descriptions of graph for the range(s) indicated.

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24. (c)

Total rainfall in 24-hour period is 300 mm.

Area of roof top = $50 m^2$

Then volume collected in the tank

= 50% of ($50\ m^2 \times 0.3$ m)

= $7.5\ m^3$ or 7500 liters.

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25. (c)

The air pressure changes by 0.05 in the regions P, Q and S but it changes by 0.20 in the region R. So that region R is most likely to have a thunderstorm.

So, option 'C' is correct.

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26. (b)

Difference in heights in moving P to Q

$= 575 - 475 = 100$ m

Difference in heights in moving P to R

$= 575 - 400 = 175$ m

Difference in heights in moving P to S

$= 575 - 500 = 75$ m

Difference in heights in moving P to T

$= 575 - 500 = 75$ m

$\therefore$ The steepest path is P to R

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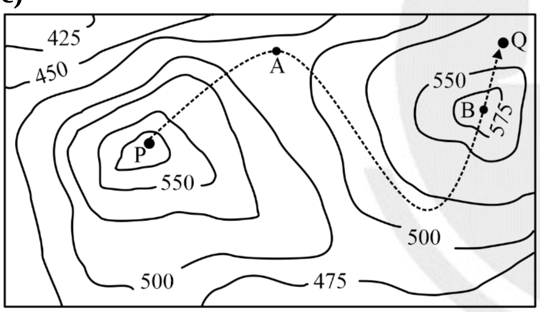
27. (c)



During the path P to A

575m $\rightarrow$ 550m $\rightarrow$ 525m $\rightarrow$ 500m $\rightarrow$ 475m

So, the movement is downwards

During the path A to B

475m $\rightarrow$ 500m $\rightarrow$ 525m $\rightarrow$ 550m $\rightarrow$ 575m

The movement is upwards

During the path B to Q, 575m $\rightarrow$ 550m

The movement is downwards

So, the total movement from P to Q is

Down $\rightarrow$ up $\rightarrow$ Down

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28. (b)

Pie chart is a circle representing total deaths in the city.

$\Rightarrow$ 100% of area $\rightarrow$ 360°

$\therefore$ 40% of area $\rightarrow \dfrac{360° \times 40}{100} = 144°$

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29. (d)

The elevator does move directly from any non-ground floor to another non- ground floor over the one-hour period. Thus (i) is wrong. The elevator stays on the fourth floor for 19 min and stays on the ground floor for 21 min so, statement ii is wrong.

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30. (b)

The allocation of rooms according to their preferences are as below.

Amar – R
Akbar – S
Anthony – P
Ajit – Q

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31. (c)

(i) no. beds by C_2 = no. tables by C_3
8 = 8

(ii) no. of chairs by all < no. of tables by all

$2 + 10 + 5 + 2 + 4 < 7 + 2 + 8 + 3 + 10$

$23 < 30$

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32. (c)

As mentioned in the question that water level rises to 525 m. It clearly indicates that village P and Q will not be submerged because height of these villages is above 525 m. But village R, S, T will get submerged because height of these villages is below 525 m.

So, option 'C' is correct.

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33. (a)

From the graph it is clear that growth of bacteria at 37ºC stops earlier than that at 25ºC. So, statements I is correct.

Curd formation needs minimum density of 0.8 so at 37ºC, Curd form after 90 min and Curd formation at 25º is after 130 min. So statement II is wrong.

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34. (120 to 120)

The plants 1, 4, 8, 9 have installed capacity more than 200 tones, which means these are large plants.

∴ Total production of large plant is

$160 + 190 + 230 + 190 = 770$

The plants 2, 3, 5, 6, 7 have installed capacity less than 200, which means these are small plants.

∴ Total production of small plant is

$= 150 + 160 + 120 + 100 + 120 = 650$

∴ The difference between large and small plants production $= 770 - 650 = 120$

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35. (c)

Function f(x) is zero at $x = -1$ and $x = 3$, also f(x) is 2 at $x = 1$. These three conditions are satisfied by only option (c).

$f(x) = 2 - |x - 1|$

$f(-1) = 2 - |-1 - 1| = 2 - 2 = 0$

$f(3) = 2 - |3 - 1| = 2 - 2 = 0$

$f(1) = 2 - |1 - 1| = 2 - 0 = 2$

So, option 'C' is correct.

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36. (b)

Quarter \ Product	Elegance	Smooth	Soft	Executive
Q1	27300	20009	17602	9999
Q2	25222	19392	18445	8942
Q3	28976	22429	19544	10234
Q4	21012	18229	16595	10109
Total	1,02510	80,059	72186	39,284

Revenue of Elegance → 1,02,510 × 48 = 49,20,480

Revenue of Smooth → 80,059 × 63 = 50,43,717

Revenue of Soft → 72,186 × 78 = 56,30,508

Revenue of Executive → 39,284 × 173 = 67,96,132

Executive contributes the highest to the revenue.

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37. **(d)**

Increase in odometer reading

= Distance travelled by vehicle in 7 seconds

= Area under the V-t graph

$$= \frac{1}{2}\times1\times1+1\times1+1\times1+\frac{1}{2}\times1\times1+\frac{1}{2}\times2\times1 + \frac{1}{2}\times1\times1+\frac{1}{2}\times1\times1$$

$$= \frac{1}{2}+1+1+\frac{1}{2}+1+\frac{1}{2}+\frac{1}{2} = 5\text{ m}$$

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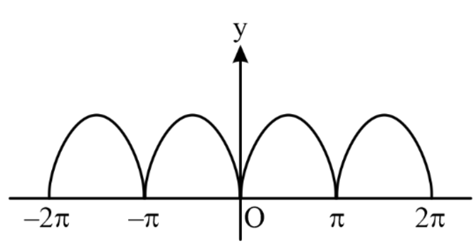
38. **(c)**

e^x = positive for $-\infty < x < +\infty$

$$y = ln\left(\left|e^{\left[\left|\sin(|x|)\right|\right]}\right|\right) = ln\left(e^{\left[\left|\sin(|x|)\right|\right]}\right)$$

$$= \left|\sin|x|\right| ln\,e$$

$$= \left|\sin|x|\right|$$



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39. **(d)**

Given, $\frac{A_P}{A_Q} = \frac{8}{9}$ and rate of interest by P & Q in 2006 are 6% and 4% respectively.

Interest $\propto A \times r$

$$\frac{\text{Interest}_P}{\text{Interest}_Q} = \frac{A_P \times r_P}{A_Q \times r_Q} = \frac{8}{9}\times\frac{6}{4} = \frac{4}{3}$$

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40. **(c)**

Total number of students in the class = 44

Total marks scored by the class,

$$= 21\times2+15\times3+11\times1+23\times2+31\times5 = 299$$

$$\therefore \text{average marks} = \frac{\text{Total marks scored by the class}}{\text{Total number of students in the class}}$$

$$= \frac{299}{44} = 6.795$$

So, option 'C' is correct.

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41. **(32 to 32)**

Number of male students in electrical = 40

Let the number of female students in electrical = F_e

$$\therefore \frac{40}{F_e} = \frac{5}{4} \Rightarrow F_e = 32$$

Therefore, total student in electrical = 40 + 32 = 72

Now, let total student in college = S

Then, $\therefore S\times\frac{20}{100} = 72 \Rightarrow S = 360$

Let the number of female students in civil = F_c

$\therefore F_c = \frac{30}{100} \times 360 \times \frac{4}{9} = 48$

Let the number of female students in mechanical = F_m

$\therefore F_m = \frac{10}{100} \times 360 \times \frac{4}{9} = 16$

Then the difference = 48 – 16 = 32

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42. (2006 to 2006)

⟹ For 2004

$$\left.\begin{array}{l} \text{Export} \rightarrow \frac{10}{60} = \frac{1}{6} \\ \text{Im port} \rightarrow \frac{10}{70} = \frac{1}{7} \end{array}\right] \frac{1}{6} + \frac{1}{7} = \frac{13}{42} \times 100 \Rightarrow 30.95\%$$

⟹ For 2005

Export = 0

$\text{Imp ort} \rightarrow \frac{10}{80} = \frac{1}{8} \times 100 = 12.5\%$

⟹For 2006

$$\left.\begin{array}{l} \text{Export} \rightarrow \frac{30}{70} = \frac{3}{7} \\ \text{Imp ort} \rightarrow \frac{30}{90} = \frac{3}{9} \end{array}\right\} \frac{3}{7} + \frac{3}{9} = \frac{48}{63} = 76.19\%$$

⟹ For 2007

$$\left.\begin{array}{l} \text{Export} = \frac{10}{100} = 10\% \\ \text{Im port} = 0 \end{array}\right\} = 10\%$$

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43. (d)

Growth of type I battery $= \frac{115-75}{75} \times 100 = 53.33\%$

Growth of type II battery = $\frac{85-144}{144} \times 100 = -40.97\%$

Growth of type III battery = $\frac{160-114}{114} \times 100 = 40.35\%$

Growth of type IV battery = $\frac{100-102}{102} \times 100 = -1.96\%$

Growth of type V battery = $\frac{145-108}{108} \times 100 = 34.26\%$

So, type I battery has the highest growth between 2006 to 2012.

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44. (d)

Stretch	Cumulative distance	Electricity used	Electricity/Distance
M	20	12	$= \frac{12}{20} = 0.6$
N	45	25	$= \frac{25-12}{45-20} = 0.52$
O	75	45	$= \frac{45-25}{75-45} = 0.66$
P	100	57	$= \frac{57-45}{100-75} = 0.48$

Electricity consumption per km is minimum in P.

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45. (48 to 48)

Total people have only scooter = 30 + 20 = 50

Total people have both (scooter +car) = 60 + 46 = 106

Total people do not have scooter,

= 40 + 34 + 20 + 50 = 144

Then the percentage of people do not have a scooter,

$$= \frac{144}{300} \times 100 = 48\%$$

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46. (140 to 140)

Let the female students in the 2008 = F

And the male students in the 2008 = M

∴ Ratio of male to female in 2008

$$\Rightarrow \frac{M}{F} = 2.5 \Leftrightarrow M = 2.5F$$

Now the female students in the 2009 = 2F

Now let the male students in the 2009 = M_1

And given that Ratio of male to female in 2009

$$\Rightarrow \frac{M_1}{2F} = 3 \Leftrightarrow M_1 = 6F$$

The increment in percentage of male from 2008 to 2009

$$= \frac{6F - 2.5F}{2.5F} \times 100 = 140\%$$

Scan for Video solution

47. (c)

Total number of students in the class = 44

Total marks scored by the class,

$$= 21 \times 2 + 15 \times 3 + 23 \times 2 = 133$$

$$\therefore \text{average marks} = \frac{\text{Total marks scored by the class}}{\text{Total number of students in the class}}$$

$$= \frac{133}{44} = 3.0227 \simeq 3.02$$

So, option 'C' is correct.

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48. (c)

Let the female students in the 2011 = F

And the male students in the 2011 = M

∴ Ratio of male to female in 2011

$$\Rightarrow \frac{M}{F} = 1 \Leftrightarrow M = F$$

Now the female students in the 2012 = F

Now let the male students in the 2012 = M_1

And given that Ratio of male to female in 2012

$$\Rightarrow \frac{M_1}{F} = 1.5 \Leftrightarrow M_1 = 1.5F$$

Then the ratio of male student from 2012 to 2011

$$= \frac{1.5F}{F} \Rightarrow 1.5:1$$

So, option 'C' is correct.

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49. (22 to 22)

2013 $\rightarrow$ Raw material expenses = 1.3 × 20 = 26%

Other expenses = 1.2 × 80

= 96%

26% + 96% = 122%

122 – 100 = 22%

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50. (20000 to 20000)

0.15x = 4,50,000

x = 4,50,000/0.15

x = 30,00,000 (total manufacturing cost)

Profit = 10,00,000

S.P. = x + Profit = 40,00,000

Each unit S.P. = $\dfrac{40,00,000}{200}$

$\boxed{=20,000}$

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51. **(d)**

The pie chart shows us population of animals not the size of animals so, statement (iv) is false. All other statements are true.

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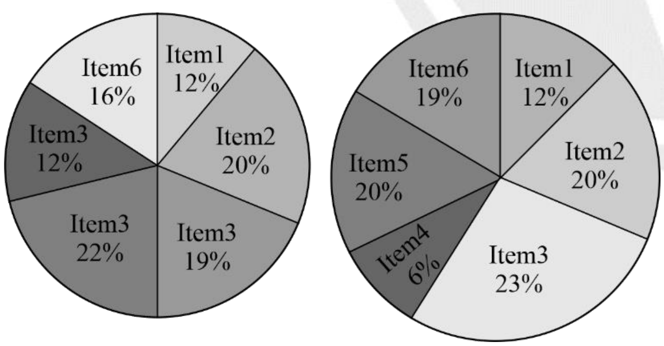
52. **(d)**

$$\text{Revenue generated per kg} \propto \frac{\text{\% of revenue of the item}}{\text{\% of export of the item}}$$

For item $\boxed{2}$, $\dfrac{0.2}{0.2} = 1$

For item $\boxed{3}$, $\dfrac{0.23}{0.19} = 1.210$



For item $\boxed{6}$, $\dfrac{0.19}{0.18} = 1.1875$

For item $\boxed{5}$, $\dfrac{0.2}{0.12} = 1.6$

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53. **(d)**

Given,

$$\text{Trade Deficite (TD)} = \frac{\text{Import (I)} - \text{Export (E)}}{\text{Export (E)}}$$

For 2005, TD = (90-70)/70 = 2/7

Similarly for 2004, TD = 1/7

for 2007, TD = 1/11

for 2006, TD = 1/5

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54. **(a)**

The standard deviation gives an idea about how the data is spread. For a low standard deviation, most of the scores will be near mean value (average).

$$\text{consistency} \propto \frac{1}{\text{standard deviation}}$$

Since standard deviation is lowest for K, hence K is the most consistent batsman.

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55. **(b)**

In July, 5% of rainfall is below 5 percentile line and in January, 5% of rainfall is above 95 percentile line. So, there might be a year for which the rainfall in January is more than that in July. Similarly, there might be a year in which the rainfall in August is below 500 mm. So, statements (ii) and (iv) are not true. The variation in July is very less hence July rainfall can be estimated with more confidence.

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56. **(d)**

Total export = 5×10^8 kgs

Total revenue = 250×10^7 Rs

Item 1 export = $0.11 \times 5 \times 10^8$ kg = 55×10^6 kg

Item 2 export = $0.22 \times 5 \times 10^8$ kg = 1.1×10^8 kg

Item 1 revenue = 0.12 × 250 × 10^7 Rs = 30 × 10^7 Rs
Item 4 revenue = 0.06 × 250 × 10^7 Rs = 15 × 10^7 Rs

Revenue per kg for item 1: - $\dfrac{30\times10^7}{0.55\times10^8}$

Revenue per kg for item 4: - $\dfrac{15\times10^7}{1.1\times10^8}$

$$\Rightarrow \frac{\frac{30\times10^7}{0.55\times10^8}}{\frac{1.5\times10^7}{1.1\times10^8}} = \frac{220}{55} = 4:1$$

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57. (d)

(1) $\dfrac{6240-5200}{5200}\times100 = \uparrow 20\%$

(2) $\dfrac{9450-7000}{7000}\times100 = \uparrow 35\%$

(3) $\dfrac{12600-9000}{9000}\times100 = \uparrow 40\%$

(4) $\dfrac{25000-20000}{20000}\times100 = \uparrow 25\%$

(5) $\dfrac{19500-15000}{15000}\times100 = \uparrow 30\%$

(6) $\dfrac{26400-22000}{22000}\times100 = \uparrow 20\%$

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58. (d)

Category	Amount
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Others	1800
Total	10500

So, total monthly budget of the household = 10500
Total monthly saving = 1500
Then, the percentage of the monthly budget not spent on saving $= \dfrac{9,000}{10,500}\times100 = 85.71\% \approx 86\%$

So, option 'D' is correct.

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59. (b)

For lap 'P'
Average speed = 15 km/hr
Distance travelled = 15 km
Distance travelled per liter at 15 km/hr = 60 km

Fuel consumption per km at 15 km/hr $= \dfrac{1}{60}$ liter

For lap 'Q'
Average speed = 45 km/hr
Distance travelled = 75 km
Distance travelled per liter at 45 km/hr = 90 km

Fuel consumption per km at 45 km/hr $= \dfrac{1}{90}$ liter

For lap 'R'
Average speed = 75 km/hr
Distance travelled = 40 km
Distance travelled per liter at 75 km/hr = 75 km

Fuel consumption per km at 75 km/hr $= \dfrac{1}{75}$ liter

For lap 'S'
Average speed = 10 km/hr
Distance travelled = 10 km
Distance travelled per liter at 10 km/hr = 30 km

Fuel consumption per km at 10 km/hr $= \dfrac{1}{30}$ liter

Hence, from the above four laps, lap 'Q' has least consumption of fuel per km.
So, option 'B' is correct.

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60. (d)

Level of danger $\propto \frac{\text{Growth (G)} \times \text{Potency (P)}}{\text{Toxicity (T)}}$

Where G = area of the circle and P and T can be taken from the graph

From the above formula, level of danger is maximum for S hence will be targeted first.

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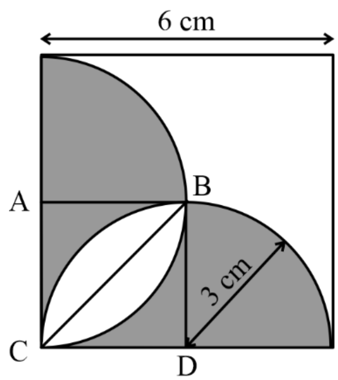
61. (b)



Area of triangle ABC = $\frac{1}{2} \times 3 \times 3 = 4.5 \text{ cm}^2$

Area of quarter circle $= \frac{1}{4}\pi r^2$

$$= \frac{1}{4} \times \pi \times (3)^2 = \frac{9}{4}\pi \text{ cm}^2$$

Now area of arc (blank portion) = $2 \times \left(\frac{9}{4}\pi - 4.5\right)$

$\therefore$ Area of dark portion =

$$2\left(\frac{\pi}{2} \times (3)^2\right) - 2 \times 2\left(\frac{9}{4}\pi - 4.5\right)$$

$$= 9\pi - 9\pi + 18 = 18 \text{ cm}^2$$

$\therefore$ Option 'B' is correct.

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62. (b)

Given graph is an odd function.

From the given all 4 options only option 'B' represents an odd function.

$\therefore$ Option (b) is correct.

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63. (a)

Given Data:

Total number = 6 (i.e. 1, 2, 3, 4, 5, 6)

Probability of getting '1' = P(1) = $\frac{1}{6}$

Probability of getting '4' = P(4) = $\frac{1}{6}$

$\therefore$ As we can see here, both events are independent to each other.

$\therefore$ $P(A \cap B) = P(A) \cdot P(B)$

$P(1 \cap 4) = P(1).\ P(4) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$\therefore$ Option (a) is correct.

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64. (c)

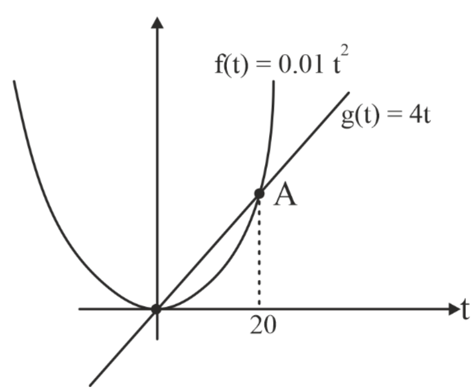
Given $f(t) = 0.01t^2$

$g(t) = 4t$

For point A, g(t) = f(t)

$0.01t^2 = 4t$

$t = 20$



(1) Since for some $t > 0$ $(0 < t < 20)$ $g(t) > f(t)$

(2) After $t = 20$ (say T) $f(t) > g(t)$ for all $t > T$.

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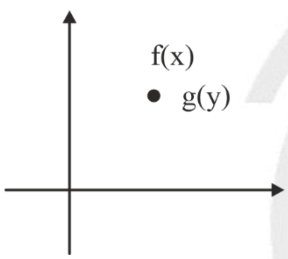
65. (b)

Given $f(x) = g(y)$ for all real values of x & y. So,

$f(4) = g(5)$

or

$f(11) = g(6)$



This is only possible when

$f(4) = f(11) = k$ (constant)

$g(5) = g(6) = k$ (constant)

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66. (a)

No. of pairs of 6-digit subsets & its subsets

= no. of 6-digit subsets × no. of subsets of each 6-digit subset

$= {}^6C_6 \times 2^6 = 64$

No. of pairs of 5-digit subsets & its subsets

= no. of 5-digit subsets × no. of subsets of each 5-digit subset

$= {}^6C_5 \times 2^5 = 6 \times 32 = 192$

No. of pairs of 4-digit subsets & its subset

$= {}^6C_4 \times 2^4 = 15 \times 16 = 240$

No. of pairs of 3-digit subset & its subset

$= {}^6C_3 \times 2^3 = 20 \times 8 = 160$

No. of 2-digit subset & its subset

$= {}^6C_2 \times 2^2 = 15 \times 4 = 60$

No. of 1-digit subset & its subset

$= {}^6C_1 \times 2^1 = 6 \times 2 = 12$

No. of pairs of empty subsets & its subset (ϕ, ϕ)

= 1

Total pairs = 64 + 192 + 240 + 160 + 60 + 12 + 1 = 729

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67. (a)

Positive integers ⇒ 1, 2, 3…………

⇒ For $p + q = 2$ (option A)

The only possibility is $p = q = 1$

This condition satisfies both the inequalities

$p^2 = 4q = -3 < 4$

$3p + 2q = 5 < 6$

⇒ $p + q = 1$ is not possible because zero is not an positive integer.

⇒ $p + q = 3$ can have two possibilities $p = 1, q = 2$ and $p = 2, q = 1$. Both the possibilities do not satisfy both the inequalities.

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68. (a)

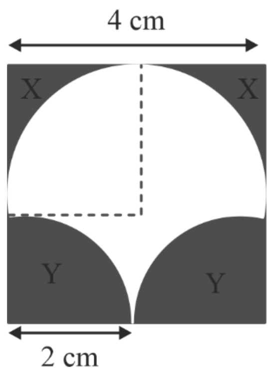
Total shaded area = $2x + 2y$

Area of quadrant (y) $= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2)^2 = \pi \text{cm}^2$

Area of sector (x)

$$= \frac{\text{Area of square}}{4} - \text{Area of quadrant}$$

$$= \frac{4^2}{4} - \pi = \{4 - \pi\}\text{cm}^2$$



Total shaded area = 2x + 2y

$= 2(4 - \pi) + 2\pi = 8\ cm^2$

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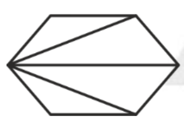
69. (a)



Each angle of a regular hexagon: -

$\Rightarrow \dfrac{(n-2)\times 180}{n} = \dfrac{(6-2)\times 180}{6}$

$\Rightarrow \dfrac{720}{6} = 120°$

Area of sector = $\dfrac{\theta}{360°}\pi r^2$

$= v \propto r = \dfrac{25}{3}\times \pi\ cm^2$

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70. (a)

$\log_n m = \dfrac{\log m}{\log n}$

$\text{Log}_a c = \dfrac{\log 100!}{\log 30!}$, $\log_c a = \dfrac{\log 30!}{\log 100!}$,

$\log_b a = \dfrac{\log 30!}{\log 50!}$, $\log_a b = \dfrac{\log 50!}{\log 30!}$

$\log_c a < \log_b a < \log_a b < \log_a c$

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71. (a)

Q, R, S, T, U, and V are total 6 faces of each dice,
Total possibilities, when two dice are thrown once independently at random = 6 x 6 = 36
Possible outcome combinations due to Q, U, V = 9
i.e. (Q, Q), (Q, U), (U, Q), (Q, V), (V, Q), (U, U), (V, V), (U, V), (V, U).

$\therefore$ Required probability $= \dfrac{9}{36} = \dfrac{1}{4}$

$\therefore$ option (a) is correct.

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72. (c)

As we can observe that in front and rear view of the disc, the view is only symmetrical about 1-1 axis.
Now if we do flip operation, it will retain the same front and rear views in only one case.
It means it will not retain the same front and rear views with respect to 2-2 and 3-3.
Hence out of three cases, two case does not retain the same front and rear views.

$\therefore$ Required probability $= \dfrac{2}{3}$

$\therefore$ option (c) is correct

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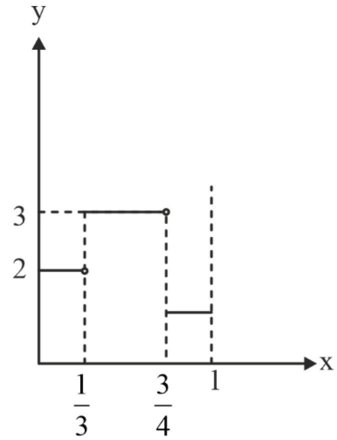
73. (c)

The graph for y(x) is drawn below.

Area under the curve is

$= \frac{1}{3}\times 2+\left(\frac{3}{4}-\frac{1}{3}\right)\times 3+\left(1-\frac{3}{4}\right)\times 1$



$= \frac{2}{3}+\frac{5}{4}+\frac{1}{4} = \frac{8+15+3}{12}=\frac{26}{12}=\frac{13}{6}$

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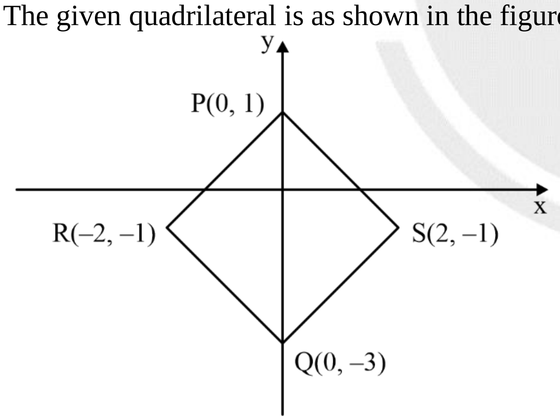
74. (c)

The given quadrilateral is as shown in the figure



Length of side PS = Distance between the points (0,1) and (2, – 1)

$\Rightarrow$ $PS=\sqrt{(0-2)^2+(1+1)^2} = \sqrt{4+4}=\sqrt{8}$

Similarly

$PR=\sqrt{(0+2)^2+(1+1)^2} = \sqrt{8}$

$SQ=\sqrt{(2-0)^2+(-1+3)^2} = \sqrt{8}$

$QR=\sqrt{(0+2)^2+(-3+1)^2} = \sqrt{8}$

That implies all the sides are equal to $\sqrt{8}$ unit

Length of diagonal $PQ=\sqrt{(0-0)^2+(1+3)^2} = 4$

Length of Diagonal $RS=\sqrt{(-2-2)^2+(1+1)^2} = 4$

That implies diagonals are equal to 4 unit.

Hence the given quadrilateral PSQR is a square

$\therefore$ Area = (length of the side)^2

$=(\sqrt{8})^2=8$ square units

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75. (c)

Given: $2x-1>7$

$\Rightarrow$ $2x>8$ $\Rightarrow$ $x>4$

Also, $2x-9<1$ $\Rightarrow$ $2x<10 \Rightarrow x<5$

So, we can write, $4<x<5$

$\Rightarrow$ Hence option (c) is correct

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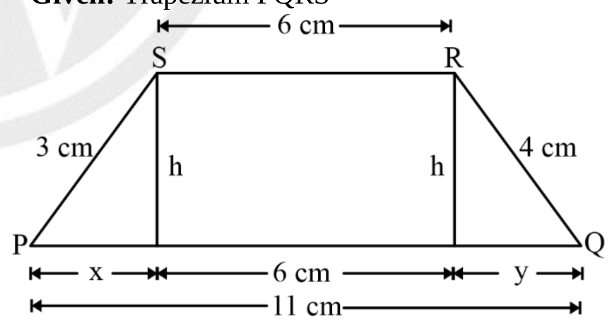
76. (b)

Given: Trapezium PQRS



The shortest distance between PQ and SR will the perpendicular direction h between them.

$\therefore$ PQ = 11 $\Rightarrow$ $x+6+y=11$

$\Rightarrow$ $x+y=5$ …(a)

For the triangles we can write

$h^2+x^2=3^2=9$ …(1)

and $h^2+y^2=4^2=16$ …(2)

Equation (2) – equation (1)

$y^2-x^2=7$ $\Rightarrow$ $(y-x)(y+x)=7$

$$\Rightarrow \; y - x = \frac{7}{5} \qquad \ldots (3)$$

Equation (a) + Equation (3)

$$2y = \frac{7}{5} + 5 = \frac{32}{5} \qquad \Rightarrow y = \frac{16}{5}$$

From equation (2)

$$h^2 = 16 - \left(\frac{16}{5}\right)^2 = 16 - \frac{256}{25} = \frac{144}{25}$$

$$\Rightarrow \quad h = \sqrt{\frac{144}{23}} = \frac{12}{5} = 2.4$$

Scan for Video solution

77. (d)

- Lines are parallel $\Rightarrow$ their slopes are equal.
- Lines are perpendicular $\Rightarrow$ the product of their slopes is –1.

Line L1: $2x - 3y = 5 \Rightarrow y = \frac{2x}{3} - \frac{5}{3}$

slope = $\frac{2}{3}$

Line L2: $3x + 2y = 8 \Rightarrow y = -\frac{3x}{2} + \frac{8}{2}$

slope $= -\frac{3}{2}$

Line L3: $4x - 6y = 5 \Rightarrow y = \frac{4x}{6} - \frac{5}{6}$

slope $= \frac{2}{3}$

Line L4: $6x - 9y = 6 \Rightarrow y = \frac{6x}{9} - \frac{6}{9}$

slope $= \frac{2}{3}$

$\because$ Slope (L4) = Slope (L3) and

Slope (L4) × Slope (L2) = –1, **L4 is perpendicular to L2 and L4 is parallel to L3**

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78. (a)

$$\Rightarrow \left.\begin{array}{l} f(2,2) = 2^2 = 4 \\ g(2,2) = 2^2 = 4 \end{array}\right\} f(2,2) = g(2,2)$$

$$\Rightarrow \; f(g(2,2),2) = f(4,2) = 4^2 = 16$$

$$f(2,g(2,2)) = f(2,4) = 2^4 = 16$$

$$\Rightarrow \left.\begin{array}{l} g(2,1) = 2 \\ f(2,1) = 2 \end{array}\right\} f(2,1) = g(2,1)$$

$$\Rightarrow \left.\begin{array}{l} f(3,2) = 3^2 = 9 \\ g(3,2) = 3^3 = 27 \end{array}\right\} f(3,2) < g(3,2)$$

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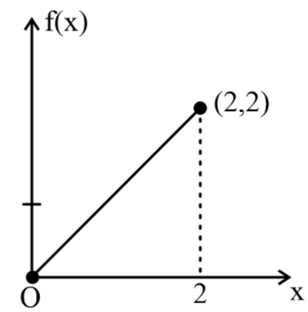
79. (c)

Given,

$$f(x) = 2\ln\sqrt{e^x}$$

$$= 2\ln e^{\frac{x}{2}} = 2\frac{x}{2} = x$$

Area = $\frac{1}{2} 2 \times 2 = 2$



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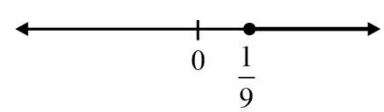
80. (c)

$$2 - 5x \le -\frac{6x - 5}{3}$$

$$6 - 15x \le -6x + 5$$



$$15x - 6x + 5 - 6 \ge 0$$

$$9x - 1 \ge 0$$

$$x \ge \frac{1}{9}$$

This result is similar to option C.

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81. (d)

Number of students appear in year 2 = 60 + 5 = 65
But 10 students who have failed in year 1 are also appear in year 2,
So that, students appear in year 2 for the very first time = 65 – 10 = 55.
Similarly,
Number of students appear in year 3 = 50 + 3 = 53
But 5 students who have failed in year 2 are also appear in year 3,
So that, students appear in year 3 for the very first time = 53 – 5 = 48
So, option 'D' is correct.

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82. (d)

Given f(x) = 0.01 for $0 \le x \le 100$
The mean value of the X will be,

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$

But the function lies in the interval of $0 \le x \le 100$.

Therefore, $\mu = \int_{0}^{100} 0.01x dx = 0.01 \times \left[\frac{x^2}{2}\right]_0^{100} = 50$

So, option 'D' is correct.

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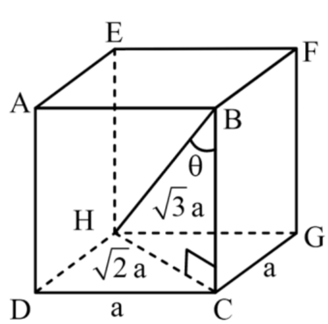
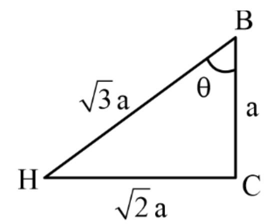
83. (b)



$$\sin\theta = \frac{\sqrt{2}\,a}{\sqrt{3}\,a} = \sqrt{\frac{2}{3}}$$

$$\cos\theta = \frac{a}{\sqrt{3}\,a} = \frac{1}{\sqrt{3}}$$



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84. (b)

$$\left(x - \frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2 = x + 2$$

$$\left(x - \frac{1}{2} + x - \frac{3}{2}\right)\left(x - \frac{1}{2} - x + \frac{3}{2}\right) = x + 2$$

$(2x - 2)(1) = x + 2 \Rightarrow x = 4$

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85. (b)

Given,

$$\lambda(p, q) = \begin{cases} (p-q)^2, \text{ if } p \ge q \\ (p+q), \text{ if } p < q \end{cases}$$

$$f(1) = \lambda\big(-(-3+2), (-2+3)\big) / \Big(\big(-(-2+1)\big)\Big)$$

f(1) = λ(1, 1)
as p = 1 $ q = 1 ; p = q
We can write
$f(1) = \lambda(1, 1) = (1-1)^2 = 0$

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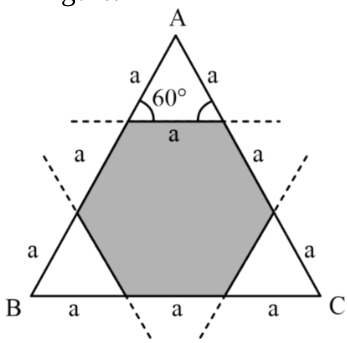
86. (a)

Regular hexagon has equal sides which is equal to the base of corner triangles which are equilateral also as shown in figure.



Area of ΔABC

$$A = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(3a)^2 = \frac{9\sqrt{3}}{4}a^2$$

Area of hexagon

A_H = Area of ΔABC – 3 × Area of corner triangle

$$= \frac{9\sqrt{3}}{4}a^2 - 3\times\frac{\sqrt{3}}{4}.a^2$$

$$= \frac{\sqrt{3}}{4}a^2[9-3] = 6\frac{\sqrt{3}}{4}a^2$$

$$\therefore A_H : A = 6\frac{\sqrt{3}}{4}a^2 \ : \ 9\frac{\sqrt{3}}{4}a^2 \ = 6:9 \ = 2:3$$

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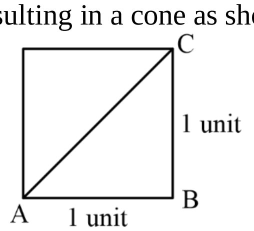
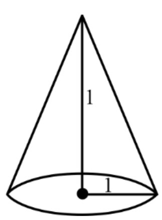
87. (a)

Let the triangle ABC is revolved about side BC, resulting in a cone as shown





So, $r = 1$ unit and $H = 1$ unit

Volume of cone, $V = \frac{1}{3}\pi r^2 H = \frac{\pi}{3}$ cubic unit

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88. (b)

Given: $\frac{p}{q}+\frac{q}{p}=3$ $\Rightarrow \left(\frac{p}{q}+\frac{q}{p}\right)^2 = 3^2$

$$\Rightarrow \frac{p^2}{q^2}+\frac{q^2}{p^2}+2 = 9$$

$$\Rightarrow \frac{p^2}{q^2}+\frac{q^2}{p^2} = 9-2 = 7$$

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89. (d)

P(even on both dice) = P(even on I^{st} & even on II^{nd})

= P(even – I) × P(even – II)

$= \frac{1}{2}\times\frac{1}{2}$

$= \frac{1}{4}$

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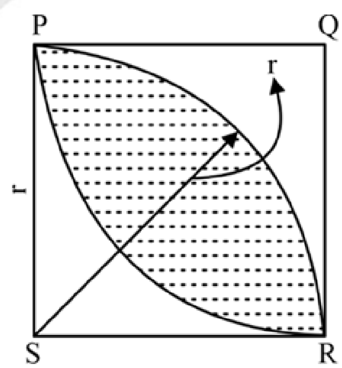
90. (c)



Area of quadrant of a circle $= \frac{\pi r^2}{4}$

Shaded area $= 2\times\frac{\pi r^2}{4} - r^2$

P(Any point in shaded area) $= \frac{\text{Shaded Area}}{\text{Area of square}}$

$$= \frac{\frac{\pi r^2}{4} \times 2 - r^2}{r^2} = \frac{\pi}{2} - 1$$

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91. (a)

In the given figure,

OA = OB = OC = radius of circle

And two right angle triangles are formed with these radii.

$\therefore$ AC = BC = $\sqrt{OA^2 + OC^2} = \sqrt{2}OA = \sqrt{2}OB$

$$\therefore \frac{\overline{AC} + \overline{CB}}{\overline{AB}} = \frac{\sqrt{2}OA + \sqrt{2}OB}{OA + OB} = \sqrt{2}$$

So, option 'A' is correct.

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92. (c)

Four-digit integer numbers from 1001 to 9999, the digit "37" in the same sequence appears in following cases,

Case-1: When 37 is at first two digits,

i.e. 3 7 __ __

In this case, last two digits can be filled by 0 to 9, i.e. total 10 digit. So that total numbers = 10 × 10 = 100.

Case-2: When 37 is at middle two digits,

i.e. __ 3 7 __

In this case, last digit can be filled by 0 to 9, i.e. total 10 digit, but first digit can be filled by 1 to 9, i.e. total 9 digit. So that total numbers = 9 × 10 = 90.

Case-3: When 37 is at last two digits,

i.e. __ __ 3 7

In this case, second digit can be filled by 0 to 9, i.e. total 10 digit, but first digit can be filled by 1 to 9, i.e. total 9 digit. So that total numbers = 9 × 10 = 90.

Total number = 100 + 90 + 90 = 280.

So, option 'C' is correct.

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93. (c)

If Q is compulsory a member of team,

For 2-member team, we need to select 1 member out of 3, therefore number of 2-member team $= {}^3C_1$

For 3-member team, we need to select 2 members out of 3, therefore number of 3-member team $= {}^3C_2$

For 4-member team, we need to select 3 members out of 3, therefore number of 4-member team $= {}^3C_3$

Total number of teams $= {}^3C_1 + {}^3C_2 + {}^3C_3 = 7$

PQ, RQ, SQ, PRQ, PSQ, RSQ, PQRS

So, option 'C' is correct.

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94. (a)

The area annular space = $\pi\,(b^2 - a^2)$

The radius of blue circles $= \frac{(b-a)}{2}$

The area of n blue circles $= \frac{n\pi}{4}(b-a)^2$

The unpainted area in annular space

$$= \pi(b^2 - a^2) - \frac{n\pi}{4}(b-a)^2$$

$$= \pi\left[(b^2 - a^2) - \frac{n}{4}(b-a)^2\right]$$

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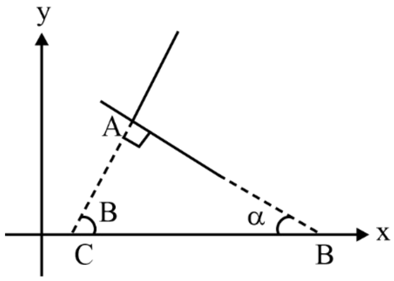
95. (b)



From the graph of ΔABC

$\alpha + \beta + 90° = 180°$

$\alpha+\beta=90°$

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96. (c)

Given: $ax^2 - bx + c = 0$

$$\Rightarrow \quad x = \frac{-(-b) \pm \sqrt{(-b)^2 - 4 \times a \times c}}{2a} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

For equal roots (one repeated root)

$b^2 - 4ac = 0 \qquad \Rightarrow \quad b^2 = 4ac$

And the root is given by

$$\beta = \frac{b}{2a}$$

$$\therefore \ \beta^3 = \frac{b \cdot b^2}{(2a)^3} = \frac{b \cdot 4ac}{8a^3} = \frac{bc}{2a^2}$$

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97. (c)

Given: radius of circle r = a

∵ O is the centre of circle

OP = OQ = OR = OS = radius of circle = a

The length of side 'SR' of square PQRS

$$= \sqrt{a^2 + a^2} = \sqrt{2}\,a$$

Area of shaded region

A = Area of circle – Area of PQRS

$$= \pi a^2 - (\sqrt{2}a)^2 = \pi a^2 - 2a^2$$

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98. (a)

(i) $f(x) = e^x$

$\Rightarrow \quad f(x_1 + x_2) = e^{x_1 + x_2}$

Let $x_1 = 2$, $x_2 = 3$ $\qquad \because \quad x > 1$

For super additive function

$$f(x_1 + x_2) \geq f(x_1) + f(x_2)$$

$\Rightarrow \quad e^{x_1+x_2} \geq e^{x_1} + e^{x_2} \qquad \Rightarrow \quad e^{2+3} \geq e^2 + e^3$

$\Rightarrow \quad e^5 \geq e^2 + e^3 \qquad \Rightarrow \quad 148.4 \geq 27.5$

Hence e^x is a super additive function

Similarly, we can check for $\sqrt{x}$, $\frac{1}{x}$, e^{-x}

$\sqrt{2+3} \geq \sqrt{2} + \sqrt{3} \qquad \Rightarrow \sqrt{5} \geq \sqrt{2} + \sqrt{3}$ which is wrong

$\frac{1}{2+3} \geq \frac{1}{2} + \frac{1}{3} \qquad \Rightarrow \quad \frac{1}{5} \geq \frac{5}{6}$ Which is wrong

And $\frac{1}{e^{2+3}} \geq \frac{1}{e^2} + \frac{1}{e^3}$ which is also wrong

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99. (c)

As per given information

$f(x) = x^2$

$f(x) = x^2$ for × $(-\infty + \infty)$

$f(f(f(x)) = f(f(x^2)) = f(f(x^4))$

$= (x^4)^2 = x^8$

$$\frac{f(f(f(x)))}{f(x)} = \frac{x^8}{x^2} = x^6 = (x^2)^3 = (f(x))^3$$

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100. (a)

Given $X.Y \propto Z$

$X.Y = CZ$

where C is constant.

So, for fixed Z ; $X \propto \frac{1}{Y}$

$\Rightarrow$ For fixed Y ; $X \propto Z$

$\Rightarrow$ For fixed X ; $Z \propto Y$

$\Rightarrow \frac{X \cdot Y}{Z} = C$ (constant)

So, statement in option A is wrong.

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101. (d)

$S = 8 + 88 + 888 + \ldots\ldots.$

$= 8\,(1 + 11 + 111 + \ldots\ldots.)$

$= \frac{8}{9}(9 + 99 + 999 + \ldots\ldots.)$

$= \frac{8}{9}((10-1) + (100-1) + (1000-1) + \ldots\ldots.)$

$= \frac{8}{9}\{(10 + 100 + 1000 + \ldots\ldots.) - n\}$

$= \frac{8}{9}\frac{10(10^n - 1)}{10-1} - \frac{8}{9}n$

$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$

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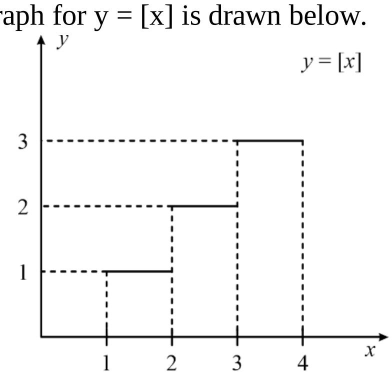
102. (d)

The graph for y = [x] is drawn below.



Area under graph is = 1 × 1 + 2 × 1 + 3 × 1

= 6

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103. (d)

Given two function,

X = {1, 2, 3}

Y = {2, 3, 4}

The combinations are in which numerator belongs to X and denominator belongs to Y are given below,

$$Z = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{3}{2}, \frac{3}{3}, \frac{3}{4}\right\}$$

Minimum value in {Z} = $\frac{1}{4}$

Maximum value in {Z} = $\frac{3}{2}$

$\therefore$ Product $= \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$

So, option 'D' is correct.

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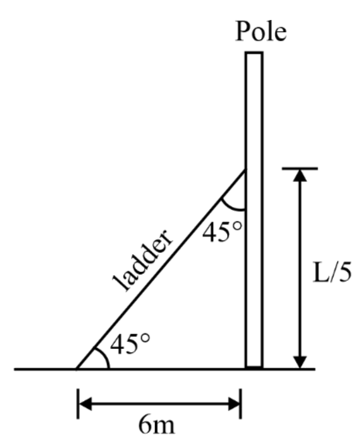
104. (c)

Let the height of pole is L then from the condition given



$$\tan 45° = \frac{L/5}{6} \Rightarrow L = 30 \text{ m}$$

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105. (Marks to All)

Let the number of boys = x
And the number of girls = y
Then, x = y + 2 and x + y = 12
∴ x = 7 and y = 5
Now, in the group of 3 children, in which number of girls will be more than number of boys,
There are only 2 possibilities for this type of group,

(i) 2 girls and 1 boy
(ii) All 3 girls

Now, the number of ways of selection of 2 girls out of 5 girls and 1 boy out of 7 boys

$= {}^5C_2 \times {}^7C_1 = 10 \times 7 = 70$

And, the number of ways of selection of 3 girls out of 5 girls $= {}^5C_3 = 10$

Total ways of selection of 3 students out of 12

$= {}^{12}C_3 = 220$

Required Probability = $= \frac{70+10}{220} = \frac{80}{220} = \frac{4}{11}$

So, no option is correct.
This question was **'Marks to All'** in official IIT answer key.

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106. (c)

Total numeral number = 10

Total lower-case English alphabet = 26

Total upper-case English alphabet = 26

∴ Total password = 10 × 26 × 26 = 6760

Now, these three characters can be interchanging their place in 3! Ways, i.e. 3! = 6 ways.

Hence, Total distinct password = 6760 × 6 = 40560

So, option 'C' is correct.

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107. (a)

Given, $G\left(F(1,3), G(1,3)\right)$

$G\left(F(1,3), G(1,3)\right) = G\left((1-3)^2, |1-3|\right)$

$\Rightarrow G(4,2) = |4-2| = 2$

So, option 'A' is correct.

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108. (d)

Given, $f(x) = 0$ have roots at x = {–2, 0, 3}

Now, $f(x-3) = 0$

New function f(x – 3) = 0 is right shifted by 3, which indicates that value of the function, where its value is zero will be right shifted by three.

$\therefore f(x-3) = 0$ have roots at x = {1, 3, 6}

So, option 'D' is correct.

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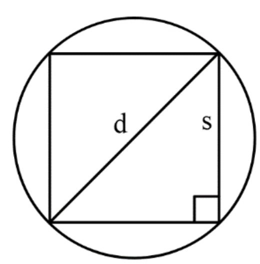
109. (d)



Area of square = d
$(s)^2 = d$

$s = \sqrt{d}$

$(\sqrt{d})^2 + (\sqrt{d})^2 = (\text{diameter})^2$

$d + d = (\text{diameter})^2$

$\text{diameter} = \sqrt{2d}$

$\text{radius} = \dfrac{\sqrt{2d}}{2}$

Area of circle $= \pi r^2$

$= \pi\left(\dfrac{\sqrt{2d}}{2}\right)^2 = \pi\dfrac{2d}{4} = \pi\dfrac{d}{2}$

$= \dfrac{1}{2}\pi d$

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110. (c)

$P^{-x} = \dfrac{1}{q}$

$\Rightarrow \dfrac{1}{P^x} = \dfrac{1}{q}$

$\Rightarrow \boxed{P^x = q}$

$q^{-y} = \dfrac{1}{r}$

$\Rightarrow \dfrac{1}{q^y} = \dfrac{1}{r} \Rightarrow \boxed{q^y = r}$

$r^{-z} = \dfrac{1}{p}$

$\dfrac{1}{r^z} = \dfrac{1}{p}$

$\boxed{r^z = p}$

$P^x = q$

$(P^x)^y = q^y$

$(P^x)^y = r \quad [\because q^y = r]$

$((P^x)^y)^z = r^z$

$P^{xyz} = p \quad [\because r^z = p]$

So, xyz = 1

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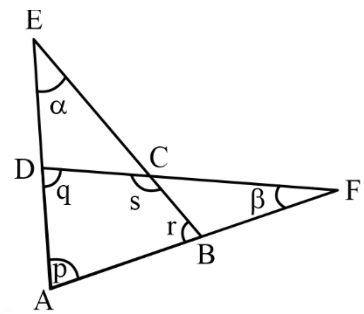
111. (a)



In quadrilateral DCBA

$p + q + r + s = 360°$……..(1)

In ΔAFD

$p + q + \beta = 180°$……..(2)

In ΔAEB

$p + r + \alpha = 180°$…….(3)

From equations 2 & 3

$p + q + \beta + p + r + \alpha = 360°$

From equation 1

$p + q + \beta + p + r + \alpha = p + q + r + s$

$\boxed{\alpha + \beta = s - p}$

Or, $\alpha + \beta = \angle BCD - \angle BAD$, or

$\angle DEC + \angle BFC = \angle BCD - \angle BAD$

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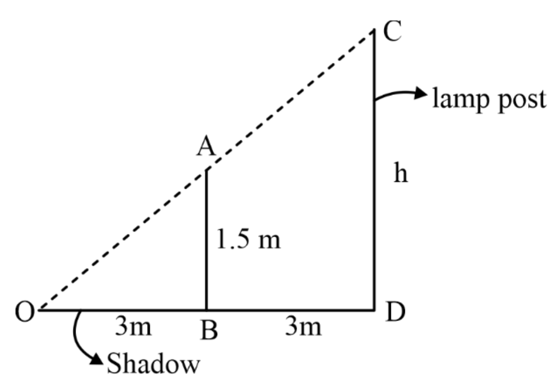
112. (b)



OAB and ODC are similar triangles

$$\frac{AB}{OB} = \frac{CD}{OD} \Rightarrow \frac{1.5}{3} = \frac{h}{6} \qquad \Rightarrow h = 3m$$

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113. (b)

$na = a^2b \Rightarrow n = ab$

$mb = ab^2 \Rightarrow m = ab$

$(mn)\ (nm) \Rightarrow (a^2b^2)\ (a^2b^2) = a^4b^4$

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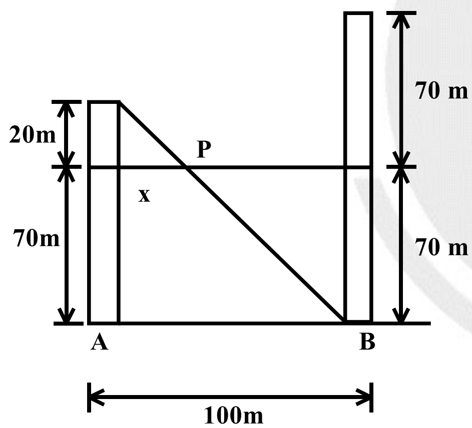
114. (a)

Let the intersection point P is x distance away from tower A. Then from similar triangles.



$$\frac{x}{20} = \frac{(100-x)}{70}$$

$70x = 2000 - 20x$

$90x = 2000$

$x = 22.22$

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115. (a)

$\log a + \log b + \log c = 0$

$\log(a \times b \times c) = 0$

or $\quad a \times b \times c = 1$

$\because$ integers are non-negative

$\therefore$ above condition is only possible if

$a = b = c = 1$

$a + b + c = 3$

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116. (b)

$$\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$$

$\log P = 10(y - z)$

$\log Q = 10(z - x)$

$\log R = 10(x - y)$

$\log P + \log Q + \log R = 10(y - z + z - x + x - y)$

$\log (PQR) = 0 \;$ or $\quad PQR = 1$

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117. (c)

$$\frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uv}$$

$$\frac{1}{\log_u u + \log_u vw} + \frac{1}{\log_v v + \log_v wu} + \frac{1}{\log_w w + \log_w uv}$$

$$\frac{1}{\log_u uvw} + \frac{1}{\log_v uvw} + \frac{1}{\log_w uvw}$$

$$\log_{uvw} u + \log_{uvw} v + \log_{uvw} w$$

$$\log_{uvw} uvw = 1$$

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118. (b)

Let the graph is for $y = f(x)$.

From the graph,

$f(0) = 0,\ f(1) = f(-1) = -1$

This condition is only satisfied by $y = ||x| - 1| - 1$

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119. (a)

$\log |a| + \log |b| + \log |c| = 0$

$\log |a| . |b| . |c| = 0$

$|a| . |b| . |c| = 1$

Since a, b & c are integers hence the values of a, b & c can either be 1 or – 1.

For $(a + b + c)_{min}$ we will have to put a = b = c = –1

$(a + b + c)_{min} = -1 - 1 - 1 = -3$

Maximum value of (a + b + c) can be obtained when a = b = c = 1

$(a + b + c)_{max} = +1 +1 +1 = +3.$

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120. (b)

Case – 1, if x =1, y = 2

$$\frac{(x+y)-|x-y|}{2} = \frac{(1+2)-|1-2|}{2} = 1 = x$$

Case – 2, if x =2, y = 1

$$\frac{(x+y)-|x-y|}{2} = \frac{(2+1)-|2-1|}{2} = 1 = y$$

From the above two cases, it is clear that value of this expression will be minimum value of x and y.

So, option 'B' is correct.

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121. (c)

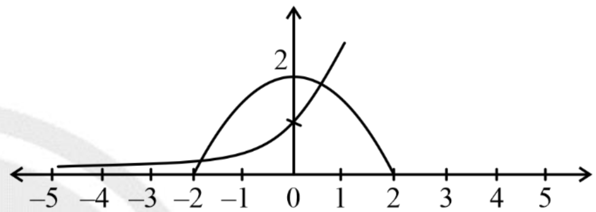
Given,

$\Rightarrow e^x + 0.5x^2 - 2 = 0$

$\Rightarrow e^x = -0.5x^2 + 2$

Above equation can be divide into two parts i.e. L.H.S. is an exponential function and R.H.S. is a parabola.

These two curves can be drawn on same plane. The number of intersections between them will be the roots of the given equation in the range of [–5, 5].



The above graph shows that both curves will intersects two times each other. It means roots of the given equation will be two.

So, option 'C' is correct.

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122. (c)

Total number of 3-digit number = 999 – 99 = 900

Total number of 3-digits numbers with **21** together

21____ : 10 possibility (0 to 9) unit digit

____21 : 9 possibility (1 to 9) hundredth place

= 10 + 9 = 19

So, the required number = 900 – 19 = 881

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123. (a)

Sub groups containing only Indians

$$= 3_{C_1} + 3_{C_2} + 3_{C_3} = 3 + 3 + 1 = 7$$

Subgroups containing one Indian and rest Chinese

$$= 3_{C_1}\left[3_{C_1} + 3_{C_2} + 3_{C_3}\right] = 3[3 + 3 + 1] = 21$$

Sub groups containing two Indian and remaining Chinese

$$= 3_{C_2}\left[3_{C_1} + 3_{C_2} + 3_{C_2}\right] = 21$$

Sub groups containing three Indian and remaining Chinese

$$= 3_{C_3}\left[3_{C_1} + 3_{C_2} + 3_{C_3}\right] = 7$$

Total number of sub groups = 7 + 21 + 21 + 7 = 56

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124. (b)

Perfect square

1, 4, 9, 16, 25, 36

So, the outcomes whose product is perfect square are

(1,1), (2,2), (3,3), (4,4), (5,5), (6,6),

(1,4) & (4,1)

$$P(E) = \frac{8}{36} \Rightarrow \boxed{\frac{2}{9}}$$

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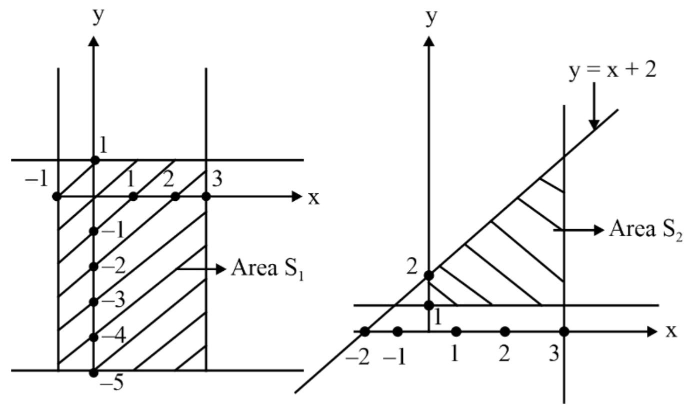
125. (c)

From $|x - 1| \le 2$, x can lie between

$= -1 \le x \le 3$

From $|y + 2| \le 3$, x can lie between

$-5 \le y \le 1$



From $x - y \ge -2$ or $y \le x + 2$, $y \ge 1$ and $x \le 3$, we can find the area S_2.

Area of $S_1 \cup S_2$

$$= 6 \times 4 + \frac{1}{2} 4 \times 4 = 24 + 8 = 32$$

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126. (d)

Given base radius r = 5 cm

Height h = 12 cm

Slanted height $l = \sqrt{5^2 + 12^2}$ = 13 cm

Distance covered by base circle in 2π = Distance covered by slanted height in θ radians.

$2\pi r = \theta \times l$

$$\theta = \frac{2\pi \times 5}{13} = \frac{10\pi}{13}$$

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127. (7 to 7)

Given, $211^{870} + 146^{127} \times 3^{424}$

Unit place of 211^{870} will be 1 always. It's because unit digit of 211 is 1. And we know that if we multiply a number which have unit digit as 1 with itself as many as times then the result will have unit digit as 1.

Unit place of 146^{127} will be 6 always. It's because unit digit of 146 is 6. And we know that if we multiply a number which have unit digit as 6 with same number as many as times then the result will have unit digit as 6.

If unit place of a number is 3 and it multiplied 4 times with itself, it repeats as → 3 → 9 → 7 → 1→ 3 at unit places, which means unit place of 3^{424} is 1.

∴ The unit digit of $211^{870} + 146^{127} \times 3^{424}$ is

$= 1 + (6 \times 1) = 7$

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128. (a)

Probability of success (p) $= \frac{60}{100} = 0.6$

Probability of success (q) $= 1 - 0.6 = 0.4$

∴ Required probability $= {}^{n}C_{r}(p)^{r}(q)^{n-r}$

$= {}^{10}C_{6}(0.6)^{6}(0.4)^{4} = 0.2508$

So, option 'A' is correct.

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129. (c)

Given, $|9y - 6| = 3$

Let, $f(y) = y^2 - \frac{4y}{3}$

Case – 1,

$9y - 6 = 3 \Rightarrow y = 1$

Case – 2,

At, $y = 1 \rightarrow f(y) = (1)^2 - \frac{4\times 1}{3} = -\frac{1}{3}$

At, $y = \frac{1}{3} \rightarrow f(y) = \left(\frac{1}{3}\right)^2 - \frac{4}{3}\times\frac{1}{3} = -\frac{1}{3}$

So, option 'C' is correct.

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130. (b)

Given, $\alpha\beta = 4$

$$\therefore \frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}} = \frac{\alpha^n + \beta^n}{\frac{1}{\alpha^n} + \frac{1}{\beta^n}}$$

$$= \frac{\alpha^n + \beta^n}{\left(\frac{\beta^n + \alpha^n}{\alpha^n\beta^n}\right)} = \frac{\alpha^n + \beta^n}{\left(\frac{\alpha^n + \beta^n}{(\alpha\beta)^n}\right)} = (\alpha\beta)^n$$

$\therefore (\alpha\beta)^n = 4^n$

So, option 'B' is correct.

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131. (b)

$x^3 + 8 = 0 \Rightarrow x^3 = -8 \Rightarrow x = -2$

$f(-2) = 2 \times (-2)^7 + 3(-2) - 5 = 2 \times (-128) - 6 - 5$

$= -256 - 6 - 5$

$= -267$ (not satisfies the equation)

$x - 1 = 0 \Rightarrow x = 1$

$f(1) = 2(1)^7 + 3(1) - 5$

$= 2 + 3 - 5 = 0$ (satisfies the equation)

∴ $x - 1$ is factor of $f(x)$

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132. (b)

Load	Failure
80	10^2
x	5×10^3
40	10^4

5000 is between 100 and 10000 corresponding to the loads of 80 and 40 respectively. Since the variation is exponential hence, the load should be little more than 40.

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133. (a)

Equation of line for abscissa ln x and ordinate y

$y = m (\ln x) + C$

where, m $\rightarrow$ slope, C $\rightarrow$ intercept at y axis

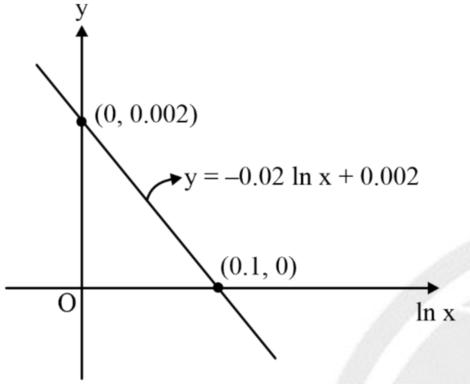
Given m = – 0.02 and for ln x = 0.1, y = 0

$\therefore \quad 0 = -0.02 \times 0.1 + C \quad \Rightarrow \quad C = 0.002$

$\therefore \quad y = -0.02 \ln x + 0.002$

At x = 5, lnx = ln 5

$\therefore \quad y(5) = -0.02 \times \ln 5 + 0.002 = -0.030$



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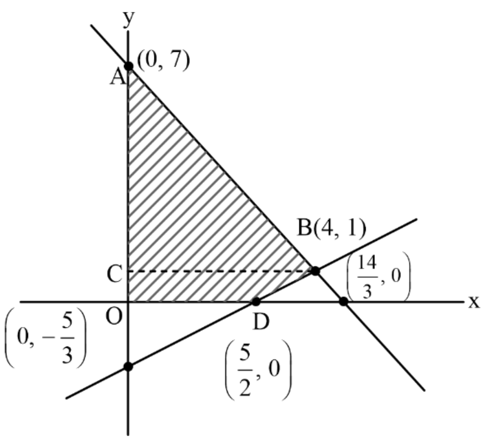
134. (b)

$3x + 2y = 14 \quad \Rightarrow \quad \dfrac{x}{\left(\dfrac{14}{3}\right)} + \dfrac{y}{7} = 1 \quad \ldots (1)$

And $2x - 3y = 5 \quad \Rightarrow \quad \dfrac{x}{\left(\dfrac{5}{2}\right)} + \dfrac{y}{\left(-\dfrac{5}{3}\right)} = 1 \quad \ldots (2)$



Equation (1) × 2 – Equation (2) × 3

$13y = 13 \quad \Rightarrow \quad y = 1$

$\Rightarrow \quad x = \dfrac{5 + 3 \times 1}{2} = 4 \quad \Rightarrow$ Point B (4, 1)

So, the required area is

A = area of ΔABC + Area of trapezium CBDO

$= \dfrac{1}{2} \times 6 \times 4 + \dfrac{5}{2} \times 1 + \dfrac{1}{2} \times 1\left(4 - \dfrac{5}{2}\right)$

$= 12 + 2.5 + \dfrac{3}{4}$ = 15.25 square unit

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135. (c)

Taking log

–a lnq = –1 lnr

$a = \dfrac{\ln r}{\ln q} \quad \ldots (i)$

–b lnr = –1 lns

$a = \dfrac{\ln s}{\ln r} \quad \ldots (ii)$

–c lns = –1lnq

$a = \dfrac{\ln q}{\ln s} \quad \ldots (iii)$

From equations i, ii & iii

a × b × c = 1

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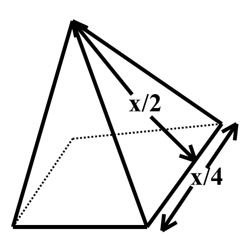
136. (d)



Area of one face

$$= \frac{1}{2}\,\text{base} \times \text{height}$$

$$= \frac{1}{2}\frac{x}{4} \times \frac{x}{2} = \frac{x^2}{16}$$

So, total lateral surface area

$$= 4 \times \left(\frac{x^2}{16}\right) = \frac{x^2}{4} = 0.25\,x^2$$

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137. (a)

Given two sets, A = {2, 3, 4, 5}, B = {11, 12, 13, 14, 15}

Total cases for selection of two numbers, one from each set $= {}^4C_1 \times {}^5C_1 = 20$

Total cases for selection of two numbers, when sum is 16 = 4, i.e. (2, 14), (3, 13), (4, 12), (5, 11)

Required probability $= \frac{\text{favourite}}{\text{Total}} = \frac{4}{20} = 0.20$

So, option 'A' is correct.

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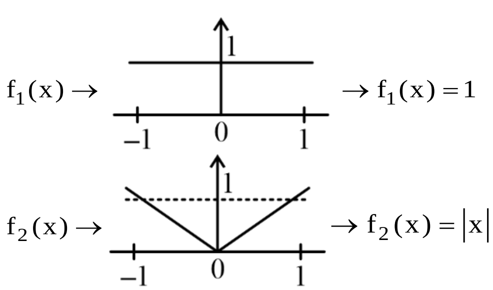
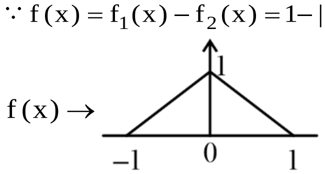
138. (c)



$\because f(x) = f_1(x) - f_2(x) = 1 - |x|$



∴ The above graph represents the function f(x), which attains its maximum value at x = 0 and its maximum value is 1.

So, option '(c)' is correct.

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139. (b)

If we subtract or multiply by a same number in an arithmetic sequence then sequence cannot be change. It will remain an arithmetic sequence, because common difference between the terms will remain same.

Therefore statement (I) and statement (II) are correct.

By squaring each term of an arithmetic sequence, common difference cannot be remained same so that it cannot be an arithmetic sequence.

Therefore statement (III) is incorrect.

So, option 'B' is correct.

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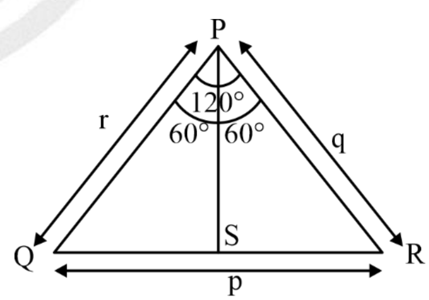
140. (b)



Area of a triangle,

$$= \frac{1}{2} \times (\text{product of any two sides})$$
$$\times \sin(\text{angle formed by same two sides})$$

$$\therefore \text{Area}(\Delta PQR) \Rightarrow \text{Area}(\Delta PQS) + \text{Area}(\Delta PRS)$$

$$\Rightarrow \frac{1}{2} q \times r \times \sin 120° = \frac{1}{2} r \times PS \times \sin 60° + \frac{1}{2} q \times PS \sin 60°$$

$$\Rightarrow q \times r \times \frac{\sqrt{3}}{2} = r \times PS \times \frac{\sqrt{3}}{2} + q \times PS \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow q \times r = r \times PS + q \times PS$$

$$\Rightarrow q \times r = PS(q + r)$$

$$PS = \left[\frac{qr}{r+q}\right]$$

So, option ‘B’ is correct.

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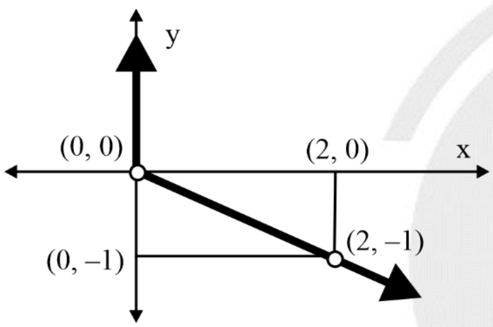
141. (b)



If ‘x = 0, then ‘y’ = 0

If ‘x’ = 2, then ‘y’ = –1

Option A $\Longrightarrow$ x = y – |y|

2 = –1 – |–1| $\Rightarrow$ 2 = –1 – 1

2 = –2 not possible

Option B $\Longrightarrow$ x = – (y – |y|)

2 = – (–1 – |–1|)

2 = 2 correct

Option C $\Longrightarrow$ x = y + |y|

2 = –1 + |–1|

$\boxed{2 = -1 + 1 = 0}$ not possible

Option D $\Longrightarrow$ x = – (y + |y|)

2 = – (–1 + |–1|)

$\boxed{2 = 0}$ not possible.

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142. (a)

We know $\log_b a = x \Rightarrow b^x = a$

$\because \quad \log_x \left(\frac{5}{7}\right) = -\frac{1}{3} \qquad \Rightarrow (x)^{-\frac{1}{3}} = \frac{5}{7}$

$\Rightarrow \quad (x)^{\frac{1}{3}} = \frac{7}{5} \qquad \Rightarrow (x) = \left(\frac{7}{5}\right)^3 = \frac{343}{125}$

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143. (c)

$-4 \le x \le 5$; x can have 10 values

$6 \le y \le 16$; y can have 11 values.

Coordinates of P can be chosen from these values & the number of different coordinates of P will be

$^{10}C_1 \times {}^{11}C_1 = 110$

Since PR line is parallel to x-axis so y-coordinate of R is same as P so the number of different coordinates of R can be

$^{9}C_1 = 9 \left\{\because \begin{array}{l}\text{Coordinate of x can not be}\\ \text{Same as x coordinate of P.}\end{array}\right.$

Similarly, for Q, x-coordinate will be same as x-coordinate of P so number of different coordinates of Q is

$^{10}C_1 = 10 \left\{\because \begin{array}{l}\text{Coordinate of y can not be}\\ \text{Same as y coordinate of P.}\end{array}\right\}$

total number of triangles = 110 × 9 × 10

= 9900

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144. (c)

tan 89° = cot 1°

tan 88° = cot 2°

so

log tan 1° + log tan 2° + ………. log tan 89°

= log (tan 1°, tan 2°. tan 3° ……. tan 45° …… cot 2°. cot 1°)

= log (tan 45°) = log 1 = 0

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145. (a)

For x > y > 1

ln x > ln y and $e^x > e^y$

but $y^x > x^y$ is not always true

If x = 3 & y = 2 ⟹ $2^3 < 3^2$

but when x = 5 & y = 2

$2^5 > 5^2$ and cos x < cos y

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146. (b)

$(a + b + c)^2 \geq 0$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$1 + 2(ab + bc + ca) \geq 0$

$ab + bc + ca \geq -1/2$ ________(1)

$(a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$

$2(a^2 + b^2 + c^2) - 2(ab + bc + ca) \geq 0$

$(a^2 + b^2 + c^2) - (ab + bc + ca) \geq 0$

$1 - (ab + bc + ca) \geq 0$

$(ab + bc + ca) \leq 1$ ________(2)

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147. (b)

Given that first two drawn cards are kings then there will be 2 kings remaining in the pack of 50. So, the probability of 3rd drawn card is king, is

$$P = \frac{2}{50}$$

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148. (d)

Given a quadratic equation, $ax^2 + bx + c = 0$ has real and positive roots,

The roots of a quadratic equation are real and positive if $b^2 - 4ac > 0$, $b^2 - 4ac < -b$, $b < 0$

Now, we know that $|x| = \begin{Bmatrix} x;\ x > 0 \\ -x;\ x < 0 \end{Bmatrix}$

Then, we will get two different equations,

$ax^2 + bx + c = 0$ and $ax^2 - bx + c = 0$

As above we find that, $b^2 - 4ac > 0$, will remain same for both equations, which indicates that both roots of the 2nd equation will be real. So that we will get total 4 real roots.

So, option 'D' is correct.

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149. (d)

Given that x is a real number, then

$$|x^2 - 2x + 3| = 11 \Rightarrow \begin{Bmatrix} x^2 - 2x + 3 = 11 \\ -\left(x^2 - 2x + 3\right) = 11; \end{Bmatrix}$$

Case – 1,

$x^2 - 2x + 3 = 11$

$x^2 - 2x - 8 = 0$

x = 4, – 2

Case – 2,

$-\left(x^2 - 2x + 3\right) = 11$

$x^2 - 2x + 14 = 0$

Above equation will have only complex roots, i.e. no real roots.
Then,
$|-x^3 + x^2 - x|$ at $x = 4$;
$\Rightarrow |-64+16-4| = 52$
$|-x^3 + x^2 - x|$ at $x = -2$;
$\Rightarrow |8+4+2| = 14$
So, option 'D' is correct.

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150. (c)

Given, $(7526)_8 - (Y)_8 = (4364)_8$
$\therefore (Y)_8 = (7526)_8 - (4364)_8$
$= (7526 - 4364)_8 = (3142)_8$
So, option 'C' is correct.

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151. (c)

The derivative of a function gives us the slope of the line tangent to the function at any point on the graph.

$$y = 5x^2 + 3$$
$$\frac{dy}{dx} = 10x$$
$$\frac{dy}{dx} = 0 \left(\text{at } x = 0, y = 3\right)$$

Since, slope is 0 hence, the tangent is parallel to the x-axis.

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152. (180 to 180)

Smallest angle of quadrilateral $\rightarrow$ q
Smallest angle of triangle $\rightarrow$ t

$$t = \frac{2}{3}q$$

Quadrilateral: - $3 : 4 : 5 : 6$
$3x + 4x + 5x + 6x = 360°$
$x = 20°$
$q = 3 \times 20 = 60°$

$$t = \frac{2}{3}(60°) = 40°$$

Largest angle of Δ = 2 × smallest angle of Δ
$= 2 \times t = 2 \times 40° = 80°$
Second largest angle of Δ + largest angle of quadrilateral = $60 + 6x$ ($x = 20°$) = 180

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153. (25 to 25)

P (magnitude > 6) = 0.04
Average time interval: -

$$T = \frac{1}{P} = \frac{1}{0.04} = \frac{100}{4} = 25 \text{ years}$$

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154. (a)

Given: $f(x, y) = x^n y^m = P$
$f(x', y')$

$$= (2x)^n \times \left(\frac{y}{2}\right)^m$$

$= 2^n \times x^n \times y^m \times 2^{-m}$
$= 2^{n-m} \times x^n \times y^m = 2^{n-m} P$

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155. (a)

Number of days in a leap year = 366
Number of weeks in a leap year = 52 weeks + 2 days
It means that there are two odd days in a leap year,
Then there are two possibilities that 53 Saturdays in a leap year.
Case – 1: when a leap year starts with Saturday then 365^{th} day will be 53^{rd} Saturday.

Case – 2: when a leap year starts with Friday then 366th day will be 53rd Saturday.

A leap year can start with any day out of 7, so that total cases of start a leap year is 7. Out of 7, only two cases are those which have 53 Saturday.

$\therefore$ Required Probability = $\frac{2}{7}$

So, option 'A' is correct.

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156. (b)

Given a quadratic equation,

$3x^2 + 2x + p(p-1) = 0$

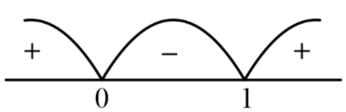
The roots of a quadratic equation are of opposite sign, if product of both roots will be negative or less than zero.

Let roots of a quadratic equation $ax^2 + bx + c = 0$, are α and β, then product of root will be $\alpha\beta = \frac{c}{a}$, then above information can be written as,

$\frac{p(p-1)}{3} < 0$

$p(p-1) < 0$

Using wavy curve,



Then, product of roots will be negative, if value of p will lie in the interval of (0, 1).

So, option 'B' is correct.

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157. (d)

Let the sum equal to 'S'; then

$S = 10 + 84 + 734 + ...$

$S = (9+1) + (9^2+3) + (9^2+5)$ ....

$S = (9 + 9^2 + 9^3 \ldots) + (1 + 3 + 5 \ \ldots)$

$S = \frac{9(9^n - 1)}{(9-1)} + \frac{n}{2}[2 \times 1 + (n-1) \times 2]$

$S = \frac{9(9^n - 1)}{8} + n^2$

So, option 'D' is correct.

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158. (a)

2x + y ≤ 6 ……(1)

x + 2y ≤ 8 ……(2)

Equation (1) - 2×equation 2

$3y \le 10 \Rightarrow y \le \frac{10}{3}$

Putting $y = \frac{10}{3}$ in equation (1)

$2x + \frac{10}{3} \le 6 \Rightarrow \boxed{x \le \frac{4}{3}}$

(4/3, 10/3)

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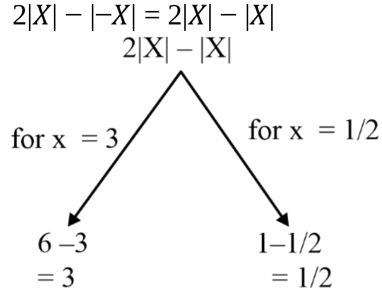
159. (b)

From |4X−7|=5, X = 3 or 1/2

$2|X| - |-X| = 2|X| - |X|$



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160. (b)

$$\text{Sum} = \frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \ldots\ldots \frac{1}{\sqrt{81}+\sqrt{80}}$$

$$= \frac{\sqrt{2}-\sqrt{1}}{\left(\sqrt{2}\right)^2-\left(\sqrt{1}\right)^2} + \frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} \ldots\ldots \frac{\sqrt{81}-\sqrt{80}}{\left(\sqrt{81}\right)^2-\left(\sqrt{80}\right)^2}$$

$$= \cancel{\sqrt{2}} - \sqrt{1} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{4}} - \cancel{\sqrt{3}} + \cancel{\sqrt{5}} - \cancel{\sqrt{4}} \ldots$$

$$\ldots\ldots\ldots + \sqrt{81} - \cancel{\sqrt{80}}$$

$$\text{Sum} = \sqrt{81} - \sqrt{1} = 9 - 1 = 8$$

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161. (b)

$y = 2x - 0.1x^2$

$\rightarrow$ For x = 8 m

y = 16 – 0.1 × 64 = 9.6 m

$\rightarrow$ For x = 10 m

y = 20 – 0.1×100 = 10 m

$\rightarrow$ For x = 12 m

y = 24 – 0.1 × 144 = 9.6 m

$\rightarrow$ For x = 14 m

y = 28 – 0.1 × 196 = 8.4 m

So, the maximum height is 10 m when x = 10

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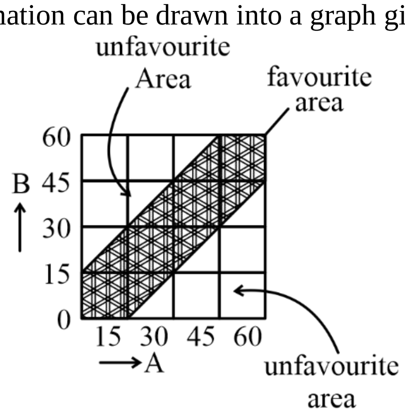
162. (c)

The wait duration cannot be more than 15 minutes, which is applicable for both 'A' and 'B'; the above information can be drawn into a graph given below,



Then the probability that 'A' and 'B' will meet on that day $= \frac{\text{Favourite area}}{\text{Total area}} = \frac{7}{16}$

So, option 'C' is correct.

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163. (a)

CP = $5q^2$, SP = 50q, P = SP – CP

$\boxed{P = 50q - 5q^2}$

$\rightarrow$ For q = 5

P = 50 × 5 – 5 × 25 = 125

$\rightarrow$ For q = 10

P = 500 – 500 = 0

$\rightarrow$ For q = 15

P = 50 × 15 – $5(15)^2$ = -375

$\rightarrow$ For q = 25

P = 50 × 25 – 5 × 625 = -1875

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164. (c)

$$= 4 + 44 + 444 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \frac{4}{9}[9 + 99 + 999 \ldots\ldots\ldots\ldots]$$

$$= \frac{4}{9}\left[(10-1) + (10^2-1) + (10^3-1) \ldots\ldots\right]$$

$$= \frac{4}{9}\left[10 + 10^2 + 10^3 \ldots\ldots -1-1-1 \ldots\ldots\right]$$

$$= \frac{4}{9}\left[\frac{10\left(10^n-1\right)}{(10-1)} - n\right]$$

$$= \frac{4}{81}\left[10^{n+1} - 9n - 10\right]$$

So, option 'C' is correct.

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165. (d)

Given, $f(y) = \frac{|y|}{y}$

Then, $|f(q) - f(-q)| = \left|\frac{|q|}{q} - \frac{|-q|}{-q}\right|$

$= \left|\frac{|q|}{q} + \frac{|-q|}{q}\right| = \left|\frac{|q|}{q} + \frac{|q|}{q}\right|$

$\therefore |-q| = |q| = q$; q is non-zero real number.

$|f(q) - f(-q)| = \left|\frac{q}{q} + \frac{q}{q}\right| = 2$

So, option 'D' is correct.

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166. (a)

Given data: V = 4q; F = 100/q

$\rightarrow V + F = 4q + \frac{100}{q}$

For q = 5 → V + F = 20 + 20 = 40

For q = 4 → V + F = 16 + 25 = 41

For q = 7 → V + F = 28 + 14.28 = 42.28

For q = 6 → V + F = 24 + $16.\overline{6}$ = $40.\overline{6}$

So minimum (V + F) is for q = 5

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167. (b)

$\log P = \frac{1}{2}\log Q = \frac{1}{3}\log R = K$

$\frac{1}{2}\log Q = K \Rightarrow \log Q = 2K \Rightarrow Q = 10^{2K}$

$Q^2 = (10^{2K})^2 = 10^{4K}$

Similarly, $P \times R = 10^K \times 10^{3K} = 10^{4K}$

Hence, $Q^2 = PR$

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168. (c)

$$\begin{array}{r} 137 \\ +276 \\ \hline 435 \end{array}$$

In base 10, 137 + 276 = 413 < 435, so the base is other than 10. To find the base we add unit digits (7 + 6) of the numbers and find out which base can give us 5 in the unit digit of summed value.

7 + 6 = 13 = **8** + 5, so the base is 8, now

$$\begin{array}{r} 731 \\ +672 \\ \hline 1623 \end{array}$$

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169. (a)

Here, if 3 multiplied 4 times with itself, it repeats → 3 → 9 → 7 → 1 at unit places, and again 3 which means unit place of 3^{999} is 7

Now, if 7 multiplied 4 times with itself it repeats → 7 → 9 → 3 → 1 at unit place and again 7 on next move, which means unit place of 7^{1000} is 1.

$\therefore 3^{999} \times 7^{1000}$

$= 7 \times 1 = 7$

$\therefore$ Option (a) is correct.

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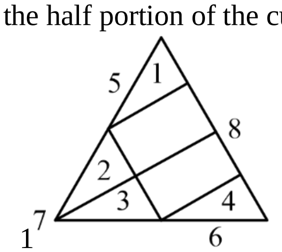
170. (c)

Triangle in the half portion of the curve = 8



Total = 8 × 2 = 16

But when we add both half portions then 4 triangles are also formed,

$\therefore$ Total = 16 + 4 = 20

$\therefore$ Option (c) is correct.

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171. (a)

Given $F_{n+1} = F_n + F_{n-1}$

If we put n = 6

$F_7 = F_6 + F_5$

$F_5 = F_7 - F_6 = 60 - 37 = 23$

$F_4 = F_6 - F_5 = 37 - 23 = 14$

$F_3 = F_5 - F_4 = 23 - 14 = 9$

$F_2 = F_4 - F_3 = 14 - 9 = 5$

$F_1 = F_3 - F_2 = 9 - 5 = 4$

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172. (a)

The percentage of pregnant women received medical care from doctor at least once a year.

$$= \left(\frac{60}{100} \times \frac{90}{100}\right) \times 100$$

$= 54\%$

So, more than half of the pregnant women received medical care at least once from a doctor.

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173. (a)

To find the maximum number of testing kit needed, we have to take worst case scenario i.e, there is only one covid positive among each set of 5 people.

Total number of sets = $\frac{1000}{5} = 200$

So, 200 test kit is needed to first identify the positive 100 sets of 5. Also 5 test kits are needed to identity positive among each set.

So, Total maximum kits required = 200 + 100 × 5 = 700

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174. (c)

Only the numbers 123457869 and 123456789 have distinct digits whose sum add to 45.

Out of these two 123456789 is smallest and hence option (c) is correct.

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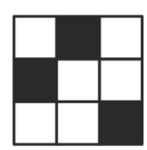
175. (d)

In the top row any one of the three cells can be shaded so, number of ways is 3. Now that a cell in the top row is shaded, we have only 2 number of ways (cells) in which a cell in second row can be shaded. And similarly, there is only one cell which we can shade in the third row. So, total number of ways = 3 × 2 × 1 = 6



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176. (a)

Assuming husband-wife combination as one then total number of ways in which these 3 pairs can be arranged is 2! and the positions of husband wife in each pair can be interchanged in 2 ways and for all three pairs 2^3 so, total number of possible arrangements are

$\Rightarrow 2! \times (2)^3 = 16$

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177. (c)

$$4^{8^x} = 4^4$$

$$8^x = 4$$

$$\left(2^3\right)^x = 2^2$$

$$2^{3x} = 2^2$$

$$\Rightarrow \quad 3x = 2 \qquad \Rightarrow \quad x = \frac{2}{3}$$

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178. (a)

From 9:15 to 9:16 minute hand & second hand will cross each other once, from 9:15 to 9:17 they will cross twice, from 9:15 to 9:18 they will cross thrice, similarly from 9:15 to 9:45 they will cross 30 times.

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179. (b)

Consider the price of an item in store S = x

And the price of same item in store M = y

Therefore, $x = 0.9y$

Store S charges ₹ 150 for delivery,

So, as per given condition,

$y - (x + 150) = 100$

$y - 0.9y - 150 = 100$

$0.1y = 250$

$y = 2500$ ₹

$\therefore$ The price of that item in store S = x

And, we know that $x = 0.9y = 0.9 \times 2500 = 2250$ ₹

$\therefore$ option (b) is correct.

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180. (a)

Given Data:

We have to find the LCM of individual times of every bell to find the same time of the bells to ring together,

$$20 = 2 \times 2 \times 5$$

$$30 = 2 \times 3 \times 5$$

$$50 = 2 \times 5 \times 5$$

Therefore, LCM of 20, 30, 50

$= 2 \times 2 \times 3 \times 5 \times 5 = 300$ minutes = 5 hours

It means they will ring together after 5 hours of 12 pm i.e. at 5 pm.

$\therefore$ option (a) is correct.

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181. (a)

Let the speeds of P and Q are 3x m/sec and 4x m/sec respectively.

Total distance of the race = 500 m

Leading distance of P = 140 m

Distance to be covered by P = 500 – 140 = 360 m

Time taken by P to finish the race after Q starts to run $= \frac{360}{3x} = \frac{120}{x}$ sec

Distance covered by Q in $\frac{120}{x}$ sec

$= \frac{120}{x} \times 4x = 480\text{m}$

$\therefore$ P wins the race by 500 – 480 = 20 m.

$\therefore$ option (a) is correct.

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182. (d)

If r is the root of the equation $x^2 + 2x + 6 = 0$, then it satisfies the equation $r^2 + 2r + 6 = 0$

$$r^2 = -2r - 6$$

$\Longrightarrow$ $(r + 2)(r + 3)(r + 4)(r + 5)$

$=$ $(r^2 + 5r + 6)(r^2 + 9r + 20)$

$=$ $(-2r - 6 + 5r + 6)(-2r - 6 + 9r + 20)$

$=$ $3r(7r + 14)$

$=$ $21(r^2 + 2r) = 21 \times (-6) = -126$

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183. (d)

Given, the boys constituted 65% of those who appeared and girls accounted for 60% of the qualified candidates. Let total candidates are x and qualified candidates are y, then number of girls and number of boys who appeared is given by:

$$\left.\begin{matrix} B_A = 0.65x \\ G_A = 0.35x \end{matrix}\right\} B_A > G_A$$

The number of boys and girls who qualified is given by:

$$\left.\begin{matrix} B_Q = 0.4y \\ G_Q = 0.6y \end{matrix}\right\} B_Q < G_Q$$

From above analysis it is clear that the number of boys who qualified the test is less than the number of girls who Qualified.

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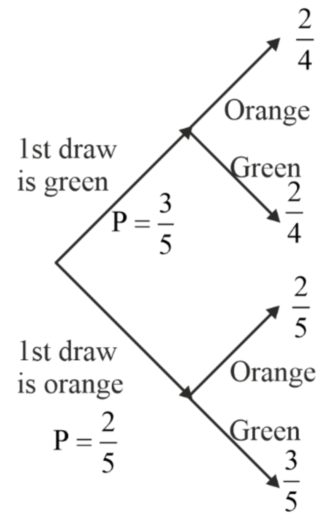
184. (d)



So total probability of 2^{nd} drawn ball is orange.

$$= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{5}$$

$$= \frac{6}{20} + \frac{4}{25} = \frac{30+16}{100} = \frac{46}{100} = \frac{23}{50}$$

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185. (d)

Given: $P : Q : R : S = 5 : 2 : 4 : 3$

Let P' share = 5x, Q' share = 2x, R' share = 4x

S'share = 3x

$\because$ R' share = 1000 + S' Share

$4x = 1000 + 3x \quad \Rightarrow \quad x = 1000$

$\therefore$ Q'share = 2x = 2 × 1000 = ₹ 2000

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186. (d)

Given: u is 50% more than s

$\Rightarrow \quad u = 1.5\,s \quad \Rightarrow \quad \frac{s}{u} = \frac{1}{1.5} \quad \Rightarrow \quad s : u = 1 : 1.5$

$\because \quad p : q = 1 : 2 = 6 : \mathbf{12}$

$q : r = 4 : 3 = \mathbf{12} : 9$

$\Rightarrow \quad p : q : r = 6 : 12 : 9 = 24 : 48 : \mathbf{36}$

$r : s = 4 : 5 \quad = \mathbf{36} : 45$

$\Rightarrow \quad p : q : r : s = 24 : 48 : 36 : \mathbf{45}$

$s : u = \mathbf{45} : 67.5$

$\Rightarrow \quad p : q : r : s : u = 24 : 48 : 36 : 45 : 67.5$

$\therefore \quad p : u = 24 : 67.5 = 240 : 675 = 16 : 45$

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187. (b)

PQ is tangent to circle so, $\angle PQR = 90°$

So, in right angled ΔPQR

$(PR)^2 = (PQ)^2 + (QR)^2$ [Pythagoras theorem]

Area of PXWR = Area of SPQT + Area of RZVU

(Since QR = RZ (radius))

Area of SPQT = Area of PXWR – Area of RZVU

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188. (b)

Let profit be 'T'

Total investment by P = 5000 × 6 = 30000

Total investment by Q = x × 8 = 8x

Q receives profit = $\frac{4}{9}T \Rightarrow$ P receives profit = $\frac{5}{9}T$

$$\frac{\text{Total investment by P}}{\text{Total investment by Q}} = \frac{\text{P 's profit}}{\text{Q 's profit}}$$

$$\frac{30000}{8x} = \frac{5/9T}{4/9T} \quad \Rightarrow x = 3000$$

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189. (d)

	A		B		C
Initial investment ratio	3	:	4	:	5

Investment increased by: -

	10%		15%		20%
New investment ratio	3×1.1	:	4×1.15	:	5×1.2

Total investment ratio: $3 + 3.3 \;:\; 4 + 4.6 \;:\; 5 + 6$

$6.3 : 8.6 : 11 = 63 : 86 : 110$

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190. (c)

Given:

$$\frac{3+x}{4+x} = \frac{15-x}{17-x}$$

$\Rightarrow \quad 51 + 14x - x^2 = 60 + 11x - x^2 \quad \Rightarrow x = 3$

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191. (b)

In case M the radius of each circle is $\frac{1}{4}$cm & in case N, the radius of each circle is $\frac{1}{6}$cm

$$\frac{\text{Unshaded Area in M}}{\text{Unshaded Area in N}}$$

$$= \frac{1^2 - 4\pi \times \left(\frac{1}{4}\right)^2}{1^2 - 9\pi\left(\frac{1}{6}\right)^2} = \left(\frac{4-\pi}{4-\pi}\right)\frac{4}{4} = 1$$

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192. (c)

Given,

For Past m days, average daily production

= 100 units

Today's production = 180

New average daily production

$$110 = \frac{100 \times m + 180}{m + 1}$$

110m + 110 = 100m + 180

10m = 70

$\boxed{m = 7}$

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193. (a)

5 Mondays in a month will require at least 29 days, and for that, the month should start with Monday.

So, 2nd Feb will be Tuesday and 1, 8, 15, 22, 29 will be the dates having Monday.

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194. (a)

(Marks to all in Answer key)

3pm to 11pm, there will be 8 coincidences.

From 11pm to 1am there will be only 1 coincidence.

From 1am to 3am there will 2 coincidences. Hence total of 11 coincidences.

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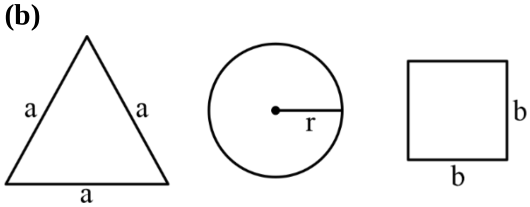
195. (b)



Let

$$A_{\Delta} = A_{O} = A_{\square} = K$$

$$\frac{\sqrt{3}}{4} a^2 = \pi r^2 = b^2 = K$$

$$a = \sqrt{\frac{4K}{\sqrt{3}}}, \; r = \sqrt{\frac{K}{\pi}}, \; b = \sqrt{K}$$

$$P_{\Delta} : P_{\square} : P_{O}$$

$$3a : 4b : 2\pi r$$

$$3\sqrt{\frac{4K}{\sqrt{3}}} : 4\sqrt{K} : 2\pi\sqrt{\frac{K}{\pi}}$$

$$2\sqrt{3\sqrt{3}} : 4 : 2\sqrt{\pi}$$

$$\sqrt{3\sqrt{3}} : 2 : \sqrt{\pi}$$

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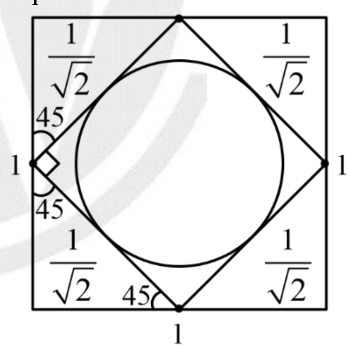
196. (a)

Since the sides of rhombus are of equal length & are perpendicular to each other hence rhombus is a square here.



The side of rhombus is $\frac{1}{\sqrt{2}}$

So, the diameter of largest circle drawn inside rhombus is $\frac{1}{\sqrt{2}}$

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197. (c)

Let distance of first part of the 80 km journey is x km for which the speed is 10 kmph. The speed for second part is 18 kmph.

Total time $= \frac{x}{10} + \frac{(80 - x)}{18}$

$6 = \frac{x}{10} - \frac{x}{18} + \frac{80}{18}$

$x = \left(6 - \frac{80}{18}\right) \times \frac{10 \times 18}{8}$

x = 35

Percentage $= \frac{35}{80} = 43.75\%$

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198. (a)

Given,

$\frac{M + N + S}{3} = 4{,}000$

Or

M + N + S = 12,000 …(a)

$\frac{N + S + P}{3} = 5{,}000$

N + S + P = 15,000 …(b)

From equation (b)

N + S = 15,000 – 6,000 = 9,000

From equation a

M = 12,000 – 9,000 = 3000

$\frac{M}{P} = \frac{3000}{6000} = 50\%$

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199. (c)

As each inner square is formed by joining the mid-point of the side of the outer square then the ratio of the inner square side to outer square side $= \frac{1}{\sqrt{2}}$

This process repeats itself 5 times,

Therefore, the side of smallest square $= \frac{10}{\left(\sqrt{2}\right)^5}$ cm

Then the area of smallest square,

$= \left(\frac{10}{\left(\sqrt{2}\right)^5}\right)^2 = 3.125 \text{ cm}^2$

So, option 'C' is correct.

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200. (d)

Using $a^2 - b^2 = (a + b)(a - b)$,

Only option (d) can be factorize using this property,

$\left(11^{52} - 1\right) = \left(11^{26}\right)^2 - (1)^2 = \left(11^{26} + 1\right)\left(11^{26} - 1\right)$

$= \left(11^{26} + 1\right)\left(11^{13} - 1\right)\left(11^{13} + 1\right)$

As we can see that, $(11^{13} + 1)$ is the factor of $(11^{52} - 1)$,

So that, $(11^{52} - 1)$ is exactly divisible by $(11^{13} + 1)$.

So, option 'D' is correct.

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201. (d)

Sum of all angles of 'n' side polygon $= (n - 2) \times 180°$

Given regular polygon of 10 sides,

∴ Interior angle between two sides,

$$= \left(\frac{n-2}{n}\right) \times 180° = \left(\frac{10-2}{10}\right) \times 180° = 144°$$

So, option 'D' is correct.

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202. (c)

The ratio of boys and girls in the class is 7 : 3. Let us say the number of boys and girls are 7x and 3x respectively. Then total number of students

= 7x + 3x = 10x

So, the number of students should be multiple of 10 hence option (c) 50 is correct

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203. (c)

The radius of cylinder

$$r = \frac{6}{2\pi} \text{cm}$$

Length of the cylinder = l = 1 cm

Volume of cylinder = $\pi r^2 \times l = \pi \left(\frac{36}{4\pi^2}\right) \times 1$

$$V_{cylinder} = \frac{9}{\pi} \text{cm}^3$$

The sheet 6 cm × 1 cm can be cut into 6 small patches of 1 cm × 1 cm each to form a cube. The volume of cube

$V_{cube} = 1 \times 1 \times 1 = 1 \text{ cm}^3$

So, $\frac{V_{cylinder}}{V_{cube}} = \frac{9}{\pi}$

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204. (c)

$$\frac{\text{cost of P}}{\text{cost of Q}} = \frac{3}{4} = \frac{5400}{\text{cost of Q}}$$

Cost of Q = 7200

Since profit on Q is 25% so, selling price of Q (SP_Q)

$SP_Q = 7200 \times 1.25$

$SP_Q = 9000$

Discount on Q

$$= \frac{MP_Q - SP_Q}{MP_Q} = \frac{10000 - 9000}{10000} = 10\%$$

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205. (c)

The probability of no chocolate identical

$$P = \frac{10 \times 9 \times 8 \times 7 \times 6}{10^5} = 0.3024$$

The probability of at least two chocolates identical

Q = 1 – P

= 1 – 0.3024

= 0.6976

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206. (c)

Given the ratio of students in classes A, B and C are 3 : 13 : 6. Let the students in these classes are

A → 3x, B → 13x, c → 6x

(3x + 18) : (13x + 18) : (6x + 18) = 15 : 35 : 21

$$\frac{3x+18}{13x+18} = \frac{15}{35} = \frac{3}{7}$$

21x + 126 = 39x + 54

18x = 72

x = 4

Then total number of students at the beginning

$= 3x + 13x + 6x = 22x$

$= 22 \times 4 = 88$

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207. (c)

Let the population 2 years ago was x.

$\therefore\ x \times 1.05 \times 1.05 = 1102500$

$\Rightarrow \quad x = 10{,}00{,}000$

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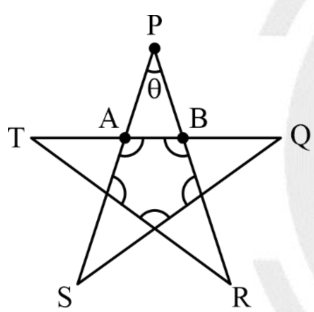
208. (a)



Given,

PR = PS = QS = QT = RT

So inside it will make regular pentagon each angle of

reg pentagon $= \frac{540}{5} = 108^o$

$\angle PAB = 180^0 - 108^0 = 72^0$

$\angle PBA = 180^0 - 108^0 = 72^0$

$\angle APB = \theta = 180^0 - 72^0 - 72^0 = 36$

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209. (c)

1^{st} 2^{nd} 3^{rd} 4^{th}

In 2^{nd} position R cannot sit

So left ways = $3 \times 3! = 18$

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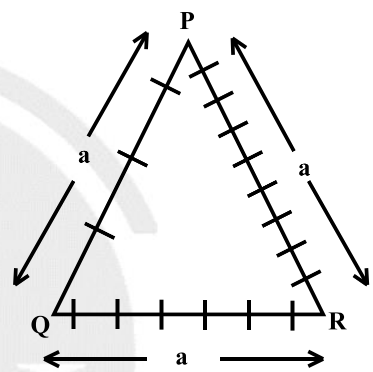
210. (d)

PQ	QR	RP
4	6	8
a	a	a
$\frac{a}{4}$	$\frac{a}{6}$	$\frac{a}{8}$

Let side of equilateral $\Delta = a$



$\frac{a}{4}$ $\quad \frac{q}{6}$ $\quad \frac{a}{8}$

LCM of (4, 6, 8) $\rightarrow$ 24

Area of equilateral $\Delta = \frac{\sqrt{3}}{4}(LCM)^2$

$= \frac{\sqrt{3}}{4}(24)^2$

$= 144\sqrt{3}$

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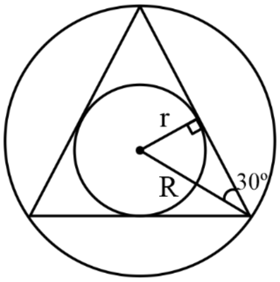
211. (c)

From the diagram below,

$\text{Sin}30^o = \frac{r}{R} = \frac{1}{2}$

$\frac{\pi r^2}{\pi R^2} = \left(\frac{r}{R}\right)^2 = \frac{1}{4}$



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212. (b)

The probability of first selected ball is blue & second selected ball is black, is.

$$= \frac{{}^{15}C_1 \times {}^{45}C_1}{{}^{60}C_1 \times {}^{59}C_1} = \frac{15 \times 45}{60 \times 59} = \frac{45}{236}$$

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213. (a)

$30 = 5 \times 2 \times 3$

$32 = 2^5$

The LCM of 30 & 32 is = $2^5 \times 3 \times 5 = 480$ seconds or 8 minutes

Hence next beep together will be at 10:08 AM

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214. (b)

For the given conditions, the possible arrangements are:

<u>Q</u> <u>R</u> <u>P</u> <u>T</u> <u>S</u>

S R P T Q

S R T P Q

So, total number of possibilities is 3.

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215. (d)

Fixing the positions of Q & R

Q _ R _ _ ⟹ possibilities: $2 \times 2 \times 1 = 4$

∵ S cannot sit adjacent to Q so, at position 2, there can be 2 possibilities $\left(P/T\right)$, in 4 there are 2 possibilities $\left(P \text{ or } T/S\right)$

_Q _ R <u>S</u> ⟹ possibilities: $2 \times 1 \times 1 = 2$

<u>S</u> _ Q _ R ⟹ possibilities: $2 \times 1 \times 1 = 2$

On flipping the positions of Q and R, similarly there will also be 8 possibilities hence total 16 possibilities.

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216. (c)

In farm Q

$5x + 14x + 13x = 416$

$32x = 416$

$x = 13$

After sending to farm P

Hens = $65 + 5 \times 13 = 130$

Ducks = $91 + 14 \times 13 = 273$

Goats = $169 + 13 \times 13 = 338$

H : D : G = 10 : 21 : 26

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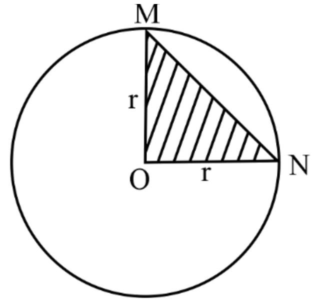
217. (d)

Given area of triangle = 50 cm²



$$\frac{1}{2}r^2 = 50$$

$r = 10$ cm

Area of circle = $\pi r^2 = 100\pi$

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218. (b)

Holding period in 1993 = 360 days

As per the given data, holding period was revised 3 times, each time it was reduced by one quarter. It means it was reduced by 3 quarter in all 3 revision.

Number of days in one quarter = 90

$\therefore$ Days in three quarter = $90 \times 3 = 270$

So that, after third revision, the holding period = 360 – 270 = 90 days

So, option 'B' is correct.

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219. (b)

Cost of route 1 – a – c – 2 = 700

Cost of route 1 – f – b – 2 = 500

Cost of route 1 – b – 2 = 600

Cost of route 1 – f – e – 2 = 700

So, the cheapest route is 1-f-b-2

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220. (b)

Angular speed of hour hand = 0.5 °/minute

Angular speed of minute hand = 6 °/minute

Relative velocity of minute hand w.r.t. hour hand

$= 6 - 0.5 = 5.5$ °/minute

At quarter part three implies minute hand has move 15 min.

So, the angle between the minute hand and hour hand.

= Angle between hands at 3'O clock – Relative angle turned by minute hand in 15 min.

$= 90° - 5.5 \times 15 = 7.5°$

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221. (d)

Let's assume those two positive numbers are A and B

$A + B = 100$

$(A - 5) \times (B - 5) = 0$

$(A - 5) = 0$

$A = 5, B = 95$

Or

$(B - 5) = 0 \Rightarrow B = 5$

$A = 95$

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222. (c)

As per given information

Sum of odd positive between 1,100

$= 1 + 3 + 5 + \ldots.. + 99$

$= \frac{n}{2}[2a + (n-1)d] \quad \because (n = 50)$

$= \frac{50}{2}[2 \times 1 + (49) \times 2]$

$= 25[2 \times 1 \times 88]$

$= 25 \times 100$

Sum of even positive number between 150 to 200 is

$= 150 + 152 + \ldots.+200$

$\frac{n}{2}[2 \times a + (n-1)d] \quad \because (n = 26)$

$\frac{26}{2}[2 \times 150 + (26-1) \times 2]$

$26[150 + 25]$

Ratio =

$$\frac{25\times100}{26\times175}=\frac{100}{26\times7}=\frac{50}{91}$$

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223. (c)

A per given information
In order to use same calender as for the year 2019
First check, is it leap year or not?

$\frac{2019}{400}\neq 0$ therefore it is not leap year

For normal year number of odd days = 1

$\frac{365}{7}$ days = 1 (Remainder)

To find the same calender as for the year 2019, the number of odd days from that year to 2019 must be zero

Total number of odd days
2011 to 2018 =
Odd days (2011) + odd days (2012)+ ……odd days(2018)

$= 1 + 2 + 1 + 1 + 1 + 2 + 1 + 1 = \frac{10}{7} = 3$ days

Hence 2011 not possible
Similary

Odd dyas (2012 – 2018) = $\frac{10-1}{7}=\frac{9}{7}$ = 2 days

Similarly

Odd days (2013-2018) = $\frac{10-1-2}{7}=\frac{7}{7}$ = 0 days

Hence the 2013 year calender is the same as that of the 2019.

Scan for Video solution

224. (a)

Concept of cyclicity
Number = $(26591749)^{110016}$
The cycle of 9^{th} is (9, 18, 729) Hence cycle of 2 term It means 2 term 9 again appear and

Remainder = $\frac{110016}{2}=0$

Therefore, the last term of the cycle is consider as unit digit
Unitl place of 9^{even} = 1

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225. (c)

As per given informatin
1^{st} term → 2, +7 more term+34 → 9^{th} term

$T_n = (a + (n – 1)d$
$34 = 2 + (9 – 1) \times d$
$d = 4$
therefore obtained sereis is:
→ 2, 6, 10, 14, 18, 22, …. 34

$Sn = \frac{n}{2}\ (2a + (n – 1)d)$

$= \frac{9}{2}\ (2 \times 2 + (9 –1) \times 4) = 126$

Scan for Video solution

226. (a)

From Man-Day-Hour-Work formula

$$\frac{m_1d_1h_1}{w_1}=\frac{m_2d_2h_2}{w_2}\Rightarrow\frac{52\times10\times h}{w}=\frac{40\times d_2\times h}{w}$$

Where m = number of men
d = number of days
h = working hours in a day
w = part of work done

d_2 = 13 days

So, 3 more days than the estimated, will be required to complete the strip.

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227. (d)

P	Q	R	S	T	Customers
$120\times1.25 = 150$	$150\times1.25 = 187.5$	$187.5\times1.25 = 234.375$	$234.375\times1.25 = 292.968$	$292.968\times1.25 = 366.21$	

Before entering into next level, the price is increased at that level by 25% hence the T received the product at Rs. 292.968 and after increasing 25% it arrives to customers at Rs. 366.21.

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228. (b)

Let, the number of boys = 4x
And the number of girls = 3x

Then the total passed candidates $= 7x \times \frac{80}{100} = 5.6x$

The number of girls passed $= 3x \times \frac{90}{100} = 2.7x$

$\therefore$ The number of boys passed $= 5.6x - 2.7x = 2.9x$

The percentage of boys passed

$= \frac{2.9x}{4x} \times 100 = 72.5\%$

So, option 'B' is correct.

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229. (c)

The even digits are = 0, 2, 4, 6, 8
The numbers between 100 to 1000 can be a 3-digit number, then we need to fill 3 places (i.e. _ _ _) with 5 digits and repetition of the digits are allowed, but a 3-digit number cannot be starts with 0, so that at hundredth place we have only 4 entries,
$\therefore$ Then the total numbers are = 4 × 5 × 5 = 100
So, option 'C' is correct.

Scan for Video solution

230. (b)

X can mow in 1 hour $= \frac{1}{2}$ portion of lawn

Y can mow in 1 hour $= \frac{1}{4}$ portion of lawn

X and Y both can mow together in 1 hour

$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ portion of lawn

$\therefore$ Time required to X and Y both to mow the complete lawn $= \frac{4}{3}$ hour $= \frac{4}{3} \times 60 = 80$ min

So, option 'B' is correct.

Scan for Video solution

231. (b)

V_A = 50 km/h

V_B = 60 km/h

The relative velocity V= 10 km/h

For a gap of 20km, the time required is

$T = \frac{20}{10} = 2$ hours

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232. (c)

Let the cost of gift is X

For 10 friends, the share on each one will be $\frac{X}{10}$

Now after 2 friends are not contributing then

$\frac{X}{8} - \frac{X}{10} = 150 \implies X = 6000$

Scan for Video solution

233. (b)

Angular velocity of hour hand = 0.5°/minute

So, time taken by hour hand to move 210°

$$= \frac{210}{0.5} = 420 \text{ minutes}$$

Similarly, time taken by hour hand to move 240°

$$= \frac{240}{0.5} = 480 \text{ minutes}$$

So, working time for P = 420 minutes

Working time for Q = 480 minutes

Effective working time for P = 420 – 30 = 390 minutes

Effective working time for Q = 480 – 20 = 460 minutes

Total effective working time for P and Q

$$= \frac{390+460}{60} = \frac{85}{6} \text{ hours}$$

USD paid to consultants = $200 \times \frac{85}{6}$

= 2833.33 USD

So, the remaining USD with the client

= 3000 – 2833.33 = 166.67

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234. (c)

Machine A's 1 hour work = 1/4 part

Another machine B's 1 hour work = 1/2 part

1 hour's work of both the machines working together

$$= \frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4} \text{ part}$$

So, the time taken to complete the task by both machines is 4/3 hours.

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235. (c)

Let 'V' is the volume of tank.

Capacity of pipe X = $+\frac{V}{5}$ litre / hour

Capacity of pipe Y = $+\frac{V}{4}$ litre / hour

Capacity of drainage pipe (Z) = $-\frac{V}{20}$ litre / hour

If both the pumps are started with the drainage hole open then let after T hours, the tank is full, then

$$\left(\frac{V}{5} + \frac{V}{4} - \frac{V}{20}\right)T = V$$

$$\left(\frac{4+5-1}{20}\right)T = 1$$

$$T = \frac{20}{8} = 2.5 \text{ hours}$$

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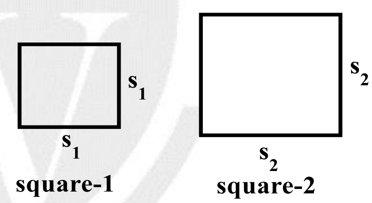
236 (c)



$s_1 = s_2 - 5$ cm ____ (1)

$A_2 = 4A_1$

$s_2^2 = 4\, s_1^2$

$s_2 = 2\, s_1$ _____ (2)

From equation (1) & (2)

$s_1 = 5$ cm

$s_2 = 10$ cm

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237. (a)

Total surface area of mansion = 70 × 55

= 3850 m^2

Area for flower pots = 550 m^2

Carpet area required = 3850 – 550 = 3300 m^2

Cost of carpet = 50 × 3300 = Rs. 1,65,000

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238. (c)

Volume of wall = 30 × 12 × 6 = 2160 m^3

Volume of a brick $= \frac{8}{100} \times \frac{6}{100} \times \frac{6}{100}$

$= 288 \times 10^{-6}\ m^3$

Since, 60% of wall consists of bricks, then number of bricks required

$= \frac{0.6 \times 2160}{288 \times 10^{-6}} = 45 \times 10^5$ or 45 lakhs

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239. (b)

John has already worked with Ram for 5 hours so, Ram can not work with John any more & Ram can not work with Krishna. So, now Ram can only work with Amir. Hence, Krishna should be allotted to work with John.

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240. (d)

⇒ 2001

$$P_1 = \frac{\text{Population of state X in } 2001\,(N_X)}{\text{Population of state Y in } 2001\,(N_Y)} = \frac{N_X}{N_Y}$$

⇒ 2011

$$P_2 = \frac{\left(1 + \frac{x}{100}\right) N_X}{\left(1 + \frac{y}{100}\right) N_Y}$$

Percentage increase in P

$$= \left(\frac{P_2}{P_1} - 1\right) \times 100$$

$$= \left(\frac{1 + \frac{x}{100}}{\left(1 + \frac{y}{100}\right)} - 1\right) 100 \Rightarrow \frac{100\,(x - y)}{100 + y}$$

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241. (b)

Let the total volume of tank is V then capacities of P, Q & R are $\frac{V}{6}, \frac{V}{9}$ and $-\frac{V}{12}$ per hour.

When P & R are opened for 4 hours, then the volume in tank after 4 hour is.

$\left(\frac{V}{6} - \frac{V}{12}\right) \times 4 = \frac{V}{3}$ is filled.

Then P is closed and Q is opened for 6 more hours.

Volume in tank after 6 more hours is.

$\frac{V}{3} + \left(\frac{V}{9} - \frac{V}{12}\right) 6 = \frac{V}{2}$ is filled.

Then R is closed and volume to be filled is $\frac{V}{2}$

$\frac{V}{9} \times x = \frac{V}{2}$

$x = \frac{9}{2}$ or 4.5 hour took by Q to fill $\frac{V}{2}$ volume.

Total time

= 4 + 6 + 4.5 = 14.5 hours

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242. (a)

Let in 2015 then GDP was G_1 & that in 2016 is G_2. Given $G_2 = 1.1\ G_1$, then percentage increase in the actual fiscal deficit is.

$$\frac{5\% \text{ of } G_2 - 4\% G_1}{4\% G_1} = \frac{\frac{5}{100}\times 1.1G_1 - \frac{4}{100}\times G_1}{\frac{4}{100}\times G_1}$$

= 0.375 or 37.5%

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243. (b)

X . Y . Z = 192

$X . Y = \frac{192}{4} = 48$

AM > GM

$\frac{x+y}{2} \geq \sqrt{xy}$

$P \geq \sqrt{48}$

$P \geq 6.928$

P = 7 (Minimum possible value of P for integers X, Y, & Z)

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244. (d)

Let the amounts of two parts are x and (100, 000 – x)

{x × 1.1 + (100,000 – x) × 1.12} – {x × 1.12 + (100,000 – x)1.1} = 120

x(1.1 – 1.12 – 1.12 + 1.1) + 100,000 (1.12 – 1.1) = 120

x = 47,000

So, the required ratio is $\frac{47,000}{100,000 - 47,000} = \frac{47}{53}$

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245. (c)

A + B = 26

A × B = 165

$(A - B)^2 = (A + B)^2 - 4A\times B$

$(A - B)^2 = 26^2 - 4 \times 165$

A – B = 4

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246. (d)

For 1 hour, the hour hand moves by $\frac{360^o}{12} = 30^o$.

So, for 225° $\Rightarrow \frac{225}{30} = 7.5$ hour

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247. (c)

Given, $\frac{(k+2)^2}{k-3}$ will be an integer, if denominator is factor or the numerator,

By option checking, let option (c).

At, $k = 4 \Rightarrow \frac{(4+2)^2}{4-3} = 36$

At, $k = 8 \Rightarrow \frac{(8+2)^2}{8-3} = \frac{100}{5} = 20$

At, $k = 28 \Rightarrow \frac{(28+2)^2}{28-3} = \frac{900}{25} = 36$

So, option ‘C’ is correct.

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248. (c)

$$\begin{array}{c|l} 2 & 20,\ 42,\ 76 \\ \hline 2 & 10,\ 21,\ 38 \\ \hline & \ 5,\ 21,\ 19 \end{array}$$

L.C.M of 20, 42, 76

= 20 × 21 × 19 = 7980

So, the number which leaves a remainder of 7 when divided by 20, 42 and 76 is

= 7980 + 7 = 7987

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249. (b)

Let, the number of invited guests = x

Invited male (I.M) = 0.6x

Invited female (I.F.) = 0.4x

The sum of female attendees (F.A.) and male attendees (M.A.) is 80% of all invited.

Attendees = $\text{F.A.} + \text{M.A.} = 0.8x$

$0.4x + \text{M.A.} = 0.8x$

$\Rightarrow$ M.A. = 0.4x

$$\frac{\text{M.A.}}{\text{F.A.}} = \frac{0.4x}{0.4x} = \frac{1}{1}$$

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250. (c)

Probability of getting green $P(G) = \frac{4}{6} = \frac{2}{3}$
Probability of getting red $P(R) = \frac{2}{6} = \frac{1}{3}$

Option (A)

$$P(G=3) = {}^7C_3\, \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$$

$$= {}^7C_3 \cdot \frac{8}{3^7} = \boxed{\frac{280}{3^7}}$$

Option (B)

$$P(G=4) = {}^7C_4\, \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$$

$${}^7C_4 \cdot \frac{16}{3^7} = \boxed{\frac{560}{3^7}}$$

Option (C)

$$P(G=5) = {}^7C_5 \times \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}$$

$${}^7C_5 \frac{32}{3^7} = \boxed{\frac{672}{3^7}}$$ **(Maximum probability)**

Option (D)

$$P(G=6) = {}^7C_6 \times \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$$

$${}^7C_6 \times \frac{64}{3^7} = \boxed{\frac{448}{3^7}}$$

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251. (c)

The sample space consists of blue cab which he identified as blue and the green cabs which he misidentifies as blue.

$$P\left(\frac{\text{Blue}}{\text{Correct}}\right) = \frac{P\left(\frac{\text{Correct}}{\text{Blue}}\right)}{P\left(\frac{\text{Correct}}{\text{blue}}\right) + P\left(\frac{\text{incorrect}}{\text{green}}\right)}$$

$$= \frac{0.80 \times 0.15}{0.80\times 0.15 + 0.20 \times 0.85}$$

$$= \frac{1200}{2900} \times 100 = 41.37\%$$

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252. (b)

$(\text{Gold})_A : (\text{Copper})_A = 2 : 3$

$(\text{Gold})_B : (\text{Copper})_B = 3 : 7$

Let the alloy C is made by taking x kg of both A and B alloys.

Gold in alloy C, $(\text{Gold})_C$ = Gold in A + Gold in B

$$= \frac{2}{5}x + \frac{3}{10}x = \frac{7x}{10}$$

Copper in alloy C

$(\text{Copper})_C$ = Copper in A + Copper in B

$$= \frac{3}{5}x + \frac{7}{10}x = \frac{13x}{10}$$

$\therefore \quad (\text{Gold})_C : (\text{Copper})_C = \frac{7x}{10} : \frac{13x}{10} = 7 : 13$

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253. (b)

Amount after 5 years A = 10,00,000

Principal amount P =?

We know $A = P\left(1 + \frac{r}{100}\right)^n$

$\Rightarrow 10,00,000 = P\left(1 + \frac{10}{100}\right)^5$

$\Rightarrow \quad P = 620921.3231 \approx 6,21,000$

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254. (d)

Given: Series is $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + ...$

First term a = 1

Common ratio = $\frac{1}{4} \div 1 = \frac{1}{16} \div \frac{1}{4} = ... = \frac{1}{4}$

Sum of infinite geometric progression is given by

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

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255. (b)

A whole number is said to be divisible by 3 if the sum of all its digits is exactly divisible by 3.

Let the number is x.

$\therefore \quad \frac{7+1+5+x+4+2+3}{3} = n \quad$ (n → whole number)

$\Rightarrow \quad x = 3n - 22$

for $x = 0,\ n = \frac{22}{3}$ which is not a whole number

x = 2, n = 8 which is a whole number

x = 5, n = 9 which is a whole number

x = 6, n = $\frac{28}{3}$ which is not whole number

So, the minimum whole number in place of ■ is 2.

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256. (d)

Let the ages of Hema, Hari & Suresh are represented by A, B & C respectively.

$A = 2B + 5$ _____ (1)

$C = 10B - 13$ _____ (2)

$C = 3A$ _____ (3)

From equation 2 & 3

$3A = 10B - 13$ _____ (4)

Equation (4) – equation (1) × (3)

$0 = 4B - 13 - 15$

$B = 7$

From equation (1)

$A = 19$

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257. (c)

$$\frac{\text{cost price} - (\text{Selling price})_1}{\text{cost price}} = 12.5\% = \frac{1}{8}$$

$$\frac{S.P_1}{C.P.} = \frac{7}{8} \quad \text{_____ (1)}$$

For $SP_2 = SP_1 + 108$

$$\frac{SP_2 - CP}{CP} = 10\% = \frac{1}{10}$$

$$\frac{SP_2}{CP} = \frac{11}{10} \quad \text{_____ (2)}$$

Equation (2) – equation (1)

$$\frac{SP_2}{CP} - \frac{SP_1}{CP} = \frac{11}{10} - \frac{7}{8}$$

$$\frac{SP_2 - SP_1}{CP} = \frac{44-35}{40}$$

$$\frac{108}{CP} = \frac{9}{40}$$

$CP = 480$

From equation (1)

$$SP_1 = 480 \times \frac{7}{8} = 420$$

So, the loss in rupees is 480 – 420 = 60

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258. (b)

Given

Price. $P \propto (\text{length, } L)^2$

$P = KL^2$ where K is constant

$1600 = K\ 100 \Rightarrow K = 16$

$P = 16L^2$

Price for two wires of lengths 4 m & 6 m

$P = 16 \times 4^2 + 16 \times 6^2 = 16(4^2 + 6^2) = 832$ Rs.

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259. (c)

$a_1 + a_2 + a_3 + \ldots\ a_{50}$

$$= \left(\frac{1}{1} - \frac{1}{1+2}\right) + \left(\frac{1}{2} - \frac{1}{2+2}\right) + \left(\frac{1}{3} - \frac{1}{3+2}\right) \ldots \left(\frac{1}{50} - \frac{1}{50+2}\right)$$

$$= \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) \ldots\ldots \left(\frac{1}{50} - \frac{1}{52}\right)$$

$$= \frac{1}{1} + \frac{1}{2} - \frac{1}{51} - \frac{1}{52} \quad = 1 + \frac{1}{2} - \left(\frac{1}{51} + \frac{1}{52}\right)$$

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260. (d)

The number of distinct committees of 3-members out of 9 people possible is –

$$^9C_3 = \frac{9 \times 2 \times 7}{3 \times 2 \times 1} = 84$$

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261. (c)

Loss, L ∝ (Deviation, D)^2

L = KD^2 where K is a constant

For D = 7 units, L = Rs. 4900

$4900 = K \times 7^2$

K = 100

L = 100 D^2

for D = 4 units

$L = 100 \times 4^2 =$ Rs. 1600

Scan for Video solution

262. (b)

For every 24 hours, 15 minutes in faulty clock, there is 24 hours in accurate clock

$24.25 \equiv 24$

∵ From 9 AM on 11 July to 2 PM on 15 July, there are (4×24 + 5) hours.

∴ for $(4\times24+5) \equiv \frac{24}{24.25}\times(4\times24+5)$

101 hours in faulty clock ≡ 99.96 hours in accurate clock

∵ The difference in hour in faculty clock & accurate clock is approximately 1.04 hours or 1 hour, 2 minutes.

∴ The correct time when faculty clock shows 2 PM, on 15 July is 12:58 PM in accurate clock on 15 July.

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263. (b)

Probability of two T

$$P(2T) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

Similarly, the probability of two H = $\frac{1}{2}$

Probability of one H & one T

$$= P(TH) + P(HT) = \frac{1}{4}+\frac{1}{4} = \frac{1}{2} \text{ (highest probability)}$$

Probability of one H followed by one T

$$P(HT) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

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264. (Marks to All)

From men-day-hour-work formula

$$\frac{m_1 d_1 h_1}{w_1} = \frac{m_2 d_2 h_2}{w_2}$$

$$\frac{125\times39\times7}{\left(5/7\right)} = \frac{(125+x)\times(52-39)\times8}{\left(2/7\right)}$$

x = 6.25 or 7 more robots.

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265. (c)

Area of square = 1936 m²

$l^2 = 1936$ m²

l = 44 m

Total length of wire = 4l = 176 m.

Let the length of shorter piece after cutting the wire a is

a + 3a = 176 ⟹ a = 44m

So, the lengths of longer & shorter wires are 132m & 44m.

Total area $= \left(\frac{3a}{4}\right)^2 + \pi\left(\frac{a}{2\pi}\right)^2 = 1243.06$ m²

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266. (a)

A circle encloses maximum area for the given parameter among all other shapes.

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267. (d)

Let length of train & platform are a and b in meters respectively.

$\Longrightarrow$ When train passes platform

$$\frac{a+b}{25}=54\times\frac{5}{18}=15\,{}^{m}\!/_{s} \qquad \ldots(1)$$

$\Rightarrow$ When train passes the man

$$\frac{a}{14}=(54-9)\times\frac{5}{18}=12.5\,{}^{m}\!/_{s} \qquad \ldots(2)$$

$\boxed{a = 175 \text{ m}}$

From equation – 1

$a + b = 15 \times 25$

$\boxed{b = 200\text{m}}$

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268. (d)

$a + a^2 b^3$ is odd

$a(1 + ab^3)$ is odd only if a & $(a + ab^3)$ both are odd.

$1 + ab^3$ is odd

or ab^3 is even

a is odd hence b^3 should be even and b^3 is even if b is even. So, a is odd & b is even.

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269. (b)

Let in the two-digit number, y is the units digit and x is the tens digit, then the digit can be written as

$10x + y$

Given, $x + y = 9$

$10x + y – 45 = 10y + x$

$9x = 9y + 45$

$9x = 9 (9 – x) + 45$

$18x = 81 + 45$

$x = 7$

then $y = 2$

$\therefore$ The number is 72

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270. (b)

Let the length & breadth are l & b.

$l – 10 = b – 5$

$b = l – 5$

$lb – (l – 10) (b – 5) = 650$

$l(l – 5) – (l – 10) (l – 10) = 650$

$l^2 – 5l – (l^2 – 20l + 100) = 650$

$15\,l = 750$

$l = 50$

$b = 45$

Area of rectangle = $50 \times 45 = 2250$ m^2

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271. (d)

Given, $y \times 162 = y \times 6 \times 27$

$\Rightarrow y \times 6 \times (3)^3$

Above number will be a perfect cube if y will be 6^2.

$\therefore$ y = 6^2 = 36

So, option 'D' is correct.

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272. (c)

If we consider the one-digit number, then only seven numbers are possible, i.e. 1, 2, 3, 4, 6, 7, 8. Then probability of one-digit number is

$$\left(\frac{7}{10}\right) = 0.7 = (0.7)^1 = (0.7)^k$$

Similarly, if we consider the two-digit number, then only 49 numbers are possible which excludes the digit 0, 5, 9. Then probability of two-digit number is

$$\left(\frac{49}{90}\right) = 0.54 \approx (0.7)^2 = (0.7)^k$$

Similarly, if we consider the three-digit number, then only 343 numbers are possible which excludes the digit 0, 5, 9. Then probability of three-digit number is

$$\left(\frac{343}{900}\right) = 0.38 \approx (0.7)^3 = (0.7)^k$$

If we see this cycle, no option is correct in this question, but if we approximate our answer then only valid option is (c). It may be happened because examiner had taken total two-digit number as 100 but in actual, these are 90. Similarly, total three-digit number as 1000 but in actual, these are 900.

So, option 'C' is correct.

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273. (d)

There are four different people and four different colours, if all four people select any colour out of four, then total 16 cases are possible, but Arun does not like the colour red and Shweta does not like the colour white, so that two cases will reduce. Hence the number of ways they can choose the shirt = 16 – 2 = 14 ways.

So, option 'D' is correct.

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274. (b)

Let,

The number of MCQ questions = p

The number of essay questions = q

$\therefore$ p + q = 20 … (i)

And, 3p + 11q = 100 …(ii)

On solving (i) and (ii), we get

p = 15, q = 5

$\therefore$ The number of MCQ questions = p = 15

So, option 'b' is correct.

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275. (d)

Number of ways, if both socks are Red = ${}^3C_2$

Number of ways, if both socks are Green = ${}^4C_2$

Number of ways, if both socks are Blue = ${}^3C_2$

Number of ways, if both are of any colour = ${}^{10}C_2$

Probability of both socks are of the same colour,

$$= \frac{{}^3C_2 + {}^4C_2 + {}^3C_2}{{}^{10}C_2} = \frac{3+6+3}{45} = \frac{12}{45} = \frac{4}{15}$$

So, option 'D' is correct.

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276. (a)

Smallest 30-digit number, starts with 47 $= 47\times10^{28}$

Greatest 30-digit number, starts with 47 $= 48\times10^{28} - 1$

Now, $\left(47\times10^{28}\right)^3 = 103823\times10^{84}$

And,

$\left(48\times10^{28}-1\right)^3 < \left(110592\times10^{84} = \left(48\times10^{28}\right)^3\right)$

Hence this value will lie in the interval of 103823×10^{84} to 110592×10^{84}. Which clearly indicate that it will be a 90-digit number.

So, option 'A' is correct.

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277. (c)

1200 men and 500 women can build in 2 weeks.

(1200 Men + 500 Women) one-week work $= \frac{1}{2}$ part... (i)

Similarly,

(900 men + 250 women)'s one-week work $= \frac{1}{3}$ part… (ii)

Equation (ii) × 2 – equation (i)

(1800 men – 1200 men)'s one-week work $= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

600 men will complete the work in 6 weeks

Men required to complete the work in one week

$\Rightarrow 600 \times 6 = x \times 1 \quad \Rightarrow \quad x = 3600$

So, 3600 men will complete the work in 1 week.

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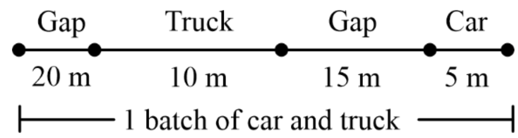
278. (a)



So, distance to be covered by 1 batch of vehicles

$= 20 + 10 + 15 + 5 = 50$ m

$\because$ Speed is 36km/1hr

In 1 hr distance travelled is 36 km = 36000 m

$\because$ 50 m distance is covered by 1 batch

36000 m distance will be covered by $= \frac{1}{50}\times36000$

= 720 batches

So, the maximum number of vehicles

$= 720 \times 2 = 1440$

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279. (d)

water consumption (W) = household consumption (H) + other consumption (O)

In summer, 0.75 W = 0.80 H + 1.70 O

$\Rightarrow$ 0.75 (H + O) = 0.80 H + 1.70 O

$\Rightarrow$ (0.75 – 0.80) H = (1.70 – 0.75) O

$\Rightarrow$ – 0.05 H = 0.95 O $\quad \Rightarrow \frac{H}{O} = -\frac{0.95}{0.05} = -19$

As the ratio of household to other consumption is negative there must be some errors in the official's statement.

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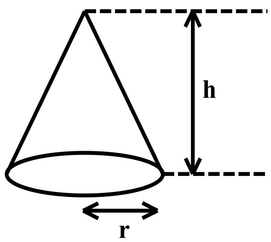
280. (c)

$V = \frac{1}{3}\pi r^2 h$

$= r^2 = 1.5 \times 1.5 = 2.25$

more than 1 by 1.25 thus 125%



Volume of right circular cone

$= \frac{1}{3}\pi r^2 \times h$

$V \propto r^2$

Percentage increase in volume

$= \frac{r_2^2 - r_1^2}{r_1^2} = \frac{(1.5\ r_1)^2 - r_1^2}{r_1^2}$

$= \frac{2.25\ r_1^2 - r_1^2}{r_1^2}$

= 1.25 or 125%

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281. (d)

The sequence of numbers in increasing order

1, x, x, x, y, y, 9, 16, 18

There is total nine numbers so,

Median $\Rightarrow$ 5th term

Median = y

Mode $\Longrightarrow$ highest repeating number = x

Given, median = 2 mode

y = 2x

Mean = $\frac{1 + x + x + x + y + y + 9 + 16 + 18}{9} = y$

$\frac{3x + 2y + 44}{9} = y$

3x + 2y + 44 = 9y

3x + 44 = 7y

3x + 44 = 7(2x) $\{\because y = 2x\}$

11x = 44

x = 4, y = 8

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282. (d)

Let the Budhan speed by cycle is ‘x’ and that by walking is ‘y’ in km/h

$\Rightarrow \quad 19 = x \times \frac{t}{4} + y \times \frac{3t}{4}$

$19 = x\frac{2}{4} + y\frac{3 \times 2}{4}$

38 = x + 3y ______ (1)

$\Rightarrow \quad 26 = x \times \frac{t}{2} + y \times \frac{t}{2}$

$26 = x \times \frac{2}{2} + y \times \frac{2}{2}$

26 = x + y ______ (2)

From equations 1 & 2

2y = 12

y = 6 km/h

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283. (b)

$1 + 2^1 + 3^3 + 4^1$

= 1 + 2+ 7 + 4 = 14

unit digit is 4

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284. (a)

$P \rightarrow 30$ min overall $30 \times 2 = 60$ min

$Q \rightarrow 20$ min overall $20 \times 3 = 60$ min

$R \rightarrow 60$ min overall $60 \times 1 = 60$ min

$S \rightarrow 15$ min overall $15 \times 4 = 60$ min

P	Q	R	S
2	3	1	4

	1st hr	2nd hr
$M_1 \rightarrow$	P P	R
$M_2 \rightarrow$	Q Q Q	S S S S

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285. (b)

$$9^2 \times \frac{\left(\frac{4}{5}\right)^{2x+4}}{\left(\frac{3}{5}\right)^{2x+4}} = 3^2 \times 4^2$$

$$\Rightarrow \left(\frac{4}{3}\right)^{2x+4} = \left(\frac{4}{3}\right)^2 \Rightarrow 2x + 4 = 2 \Rightarrow \boxed{x = -1}$$

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286. (c)

Total number of ways = 5 × 4 × 3 × 2 = 120

Number of ways, in which (P is paired with Z) = 4 × 3 × 2 = 24

Number of ways in which (Y is not paired) = 4 × 3 × 2 × 1 = 24

Number of ways in which (Y is not paired and also P and Z are paired together) = 3 × 2 × 1 = 6

So, the number of ways in which the required pairs can be formed

= 120 – 24 – 24 + 6 = 78

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287. (a)

Let Y tractor is equivalent to AX bullocks. The work done by X bullocks and Y tractors in 8 days will be same as work done by X/2 bullocks and 2Y tractors in 5 days. So,

$$(X + Y)8 = \left(\frac{X}{2} + 2Y\right) \times 5$$

$$(X + AX)8 = \left(\frac{X}{2} + 2AX\right) \times 5$$

$$(1 + A) \times 8 = \left(\frac{1}{2} + 2A\right)5$$

8 + 8A = 2.5 + 10A

A = 2.75

Let 'n' be the number of days taken to plough the field when only X bullocks plough the field. Then similarly,

(X + Y). 8 = X. n

X(1 + A) . 8 =X. n

(1 + 2.75) × 8 = n

$\boxed{n = 30 \text{ days}}$

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288. (b)

Given P looks at Q while Q looks at R. P is married, R is unmarried.

Assuming Q is married.

The pair we get is Q – R, so one pair.

Or Assuming Q is unmarried

Then the pair will be P – Q, here also one pair so in either of the cases, the pair is only 1.

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289. (c)

Let, P(A) : probability of 1[st] child being a boy = $\frac{1}{2}$

P(B) : probability of 2[nd] child being boy = $\frac{1}{2}$

$\because$ Both the events are independent.

$\therefore$ $P(A \cap B) = P(A) \times P(B)$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(B).P(A)}{P(A)} = P(B) = \frac{1}{2}$$

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290. (d)

If a & b are integers and a – b is even then a & b both can either be even or both can be odd integers.

$\Rightarrow$ ab – b

= b(a – 1) will also be even because if a & b are even numbers then a – 1 will be odd & multiplication of odd & even numbers is even.

Similarly if a & b are odd numbers then a – 1 will be even and b(a – 1) will be even.

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291. (d)

$$\begin{array}{r} 5\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{0} \\ -4\ 8\ \underline{9}\ 8\ 9 \\ \hline 1\ 1\ 1\ 1 \end{array} \quad \& \quad \begin{array}{r} 5\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} \\ -4\ 8\ \underline{8}\ 8\ 9 \\ \hline 1\ 1\ 1\ 1 \end{array}$$

There are both possibilities so the sum of missing digits cannot be determined.

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292. (b)

To find median, the 100 employees can be numbered from 1 to 100 in increasing order of their salaries.

1 to 45 earn 20,000, 46 to 70 earn 30,000, 71 to 90 earn 40,000, 91 to 98 earn 60,000 and 99 to 100 earn 150,000. Since 100 is an even number then median

$$= \frac{\left(\frac{100}{2}\right)^{th} \text{employee} + \left(\frac{100}{2}+1\right)^{th} \text{employee}}{2}$$

$$= \frac{50^{th} \text{ employee salary} + 51^{th} \text{ emplyee salary}}{2}$$

$$= \frac{30,000 + 30,000}{2} = 30,000$$

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293. (a)

Time to pass a post = $\frac{\text{Length of train}}{\text{Speed of train}}$

$\Rightarrow \quad 10 = \frac{120}{S_1} \qquad \Rightarrow \quad S_1 = 12$ m/s

Also, $15 = \frac{150}{S_2} \qquad \Rightarrow \quad S_2 = 10$ m/s

So, difference in the speed

$\Delta S = S_1 - S_2 = 12 - 10 = 2$ m/s

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294. (a)

$\Rightarrow$ x% of y + y% of x

$= \frac{x}{100} y + \frac{y}{100} x$

$= \frac{2}{100} xy$ or 2% of xy

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295. (a)

The two-digit number can be represented by "x × 10 + y". where x & y are digits at tens place & units place respectively.

Given, $x + y = 12$ _____ (1)

After reversing, the digits new number is "y × 10 + x".

$(y \times 10 + x) - (x \times 10 + y) = 54$

$9y - 9x = 54$

$y - x = 6$ ______ (2)

From equation (1) & (2)

$x = 3, y = 9$

then the original number is

$= x \times 10 + y$

$= 30 + 9 = 39$

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296. (c)

The reading capacity of Ananth $= \frac{1}{6}$ books/hour

The reading capacity of Bharath $= \frac{1}{4}$ books/hour

Number of pages to be read means number of pages left.

Let after x hours Ananth is left with the number of pages twice that by Bharath

$$\left(1-\frac{x}{6}\right) = 2\left(1-\frac{x}{4}\right)$$

$$\frac{6-x}{6} = \frac{4-x}{2}$$

$6 - x = 12 - 3x$

$2x = 6$

$x = 3$

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297. (b)

$\boxed{\text{Square}}$ $\frac{340-x}{4}$

$\frac{340-x}{4}$

$\boxed{\text{Rectangle}}$ $b = \frac{x}{6}$

$a = \frac{x}{3}$ $a = 2b$

Let the length of two parts are (340 – x) and x.

For rectangle,

$x = 2a + 2b$

Since, b: a = 1:2, hence

$x = 6b$

$b = \frac{x}{6}$ and $a = \frac{x}{3}$

$$\text{Combined Area} = \left(\frac{340-x}{4}\right)^2 + \frac{x}{6} \times \frac{x}{3}$$

$$A = \frac{(340-x)^2}{16} + \frac{x^2}{18}$$

For minimum area, differentiating A with respect to x and equating it to 0, we get the value of x for which area will be minimum.

$$\frac{dA}{dx} = -2\frac{(340-x)}{16} + \frac{2x}{18} = 0$$

$$\frac{340-x}{8} = \frac{x}{9} \Longrightarrow x = 180$$

$\left.\frac{d^2A}{dx^2}\right|_{x=180} = +ve$, hence A is minimum at x = 180.

So, the length of the square corresponding to, minimum combined area is

$$\frac{340-180}{4} = 40 \text{ mm.}$$

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298. (b)

Let x be the number of days E worked. Since the efficiency of M is twice that of others, hence

M's work = $10 \sin 45^{\circ}$

E's work = $\left(12 \frac{\text{Hours}}{\text{Day}} \times x\right) = 12x$

The ratio is 6 : 12 , 1 : 2

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299. (c)

$(9 \text{ inches})^{\frac{1}{2}} = (0.25 \text{ yards})^{\frac{1}{2}}$

Squaring both sides.

9 inches = 0.25 yards

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300. (a)

a □ x = a

ax + (a + x) = a

⇒ For x = 0

a × o + a + o = a

a = a

⇒ For x = 1

a + a + a ≠ a

⇒ For x = 2

2a + a + 2 ≠ a

⇒ For x = 10

10x + a + 10 ≠ a

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301. (c)

For normal distributions

Mean = Median = mode.

Median of P = 105

Median of Q = 85

This shows that most of the students in Q got less marks than P, but not necessarily all.

Standard deviation shows the variation or dispersion of a set of data values. Since standard deviation of P is more than that of Q so marks of students in Q is in a narrower range.

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302. (b)

Perimeter of window = 6m

5a = 6

a = $\frac{6}{5}$

Area of window $= a^2 + \frac{1}{2} a \times \left(\frac{\sqrt{3}}{2} a\right)$

$= a^2\left(1 + \frac{\sqrt{3}}{4}\right) = \left(\frac{6}{5}\right)^2\left(1 + \frac{\sqrt{3}}{4}\right)$

= 2.06 m²

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303. (c)

Capacity of Q to do work per hour per day = $\frac{1}{12 \times 25}$

Capacity of R to do work per hour per day $= \frac{1}{12 \times 50}$

Work done by Q in 5 days working for 12 hours a day $W_Q = \frac{1}{12 \times 25} \times 5 \times 12 = \frac{1}{5}$

Work done by R in 7 days working for 18 hours a day $W_R = \frac{1}{12 \times 50} \times 7 \times 18 = \frac{21}{100}$

So $W_Q : W_R = 20 : 21$

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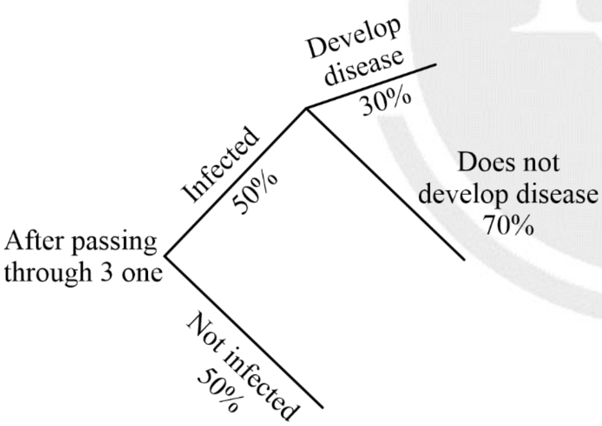
304. (c)



The percentage of people who does not show symptom of disease after being infected.

$= \frac{50}{100} \times \frac{70}{100}$ = 0.35 or 35%

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305. (a)

Out of the total 5692000 fruits, 15% are unripe.

The apples in unripe fruits $= 0.45 \times 0.15 \times 5692000$

= 384210

Out of total fruits, 85% are ripe.

The apple in ripe fruit $= 0.34 \times 0.85 \times 5692000$

= 1644988

Total apples = 384210 + 1644988

= 2029198

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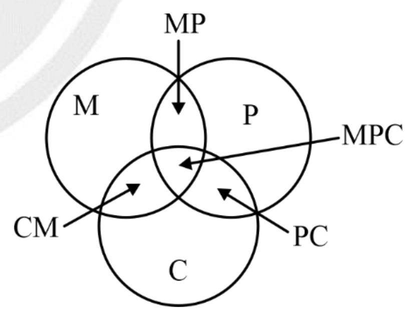
306. (d)

Let;

Probability of passes in Math P(m) = M

Probability of passes in Physics P(p) = P

Probability of passes in Chemistry P(c) = C



Therefore;

The probability of passing in at least one subject,

$$P(m \cup p \cup c) = P(m) + P(p) + P(c) - P(m \cap p) - P(p \cap c) - P(c \cap m) + P(m \cap p \cap c)$$

$$0.75 = M + P + C - MP - PC - CM + MPC \quad ...(i)$$

The probability of passing in at least two subjects,

$$0.5 = P(m \cap p) + P(p \cap c) + P(c \cap m) - 2P(m \cap p \cap c)$$

$$0.5 = MP + PC + CM - 2MPC \quad ...(ii)$$

The probability of passing in exactly two subjects,

$$0.4 = P(m \cap p) + P(p \cap c) + P(c \cap m) - 3P(m \cap p \cap c)$$

$$0.4 = MP + PC + CM - 3MPC \quad ...(iii)$$

From the equation (ii) and (iii), we get

$$MPC \;=\; 0.1 = \frac{1}{10} \quad ...(iv)$$

From the equation (iii) and (iv), we get

$$MP + PC + CM = 0.7 = \frac{14}{20} \quad ...(v)$$

From the equation (i), (iv) and (v), we get

$$M + P + C = 1.3 = \frac{27}{20} \quad ...(vi)$$

So, option 'D' is correct.

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307. (c)

$$h(2,5,7,3) = \frac{7 \times 3}{2 \times 5} \rightarrow \text{Remainder} = 1$$

$$fg\big(h(2,5,7,3),4,6,8\big) = fg(1,4,6,8)$$

$$\Rightarrow f(1,4,6,8) \times g(1,4,6,8) = 8 \times 1 = 8$$

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308. (c)

$f(x) = mx + c$

$f(-2) = -2m + c$

$29 = -2m + c \quad \ldots(i)$

$f(3) = 3m + c$

$39 = 3m + c \quad \ldots(ii)$

(ii) – (i)

$5m = 10$

$$m = \frac{10}{5} = 2$$

From equation (i)

$29 = -4 + c$

$c = 33$

$f(5) = 2 \times 5 + 33$

$= 10 + 33 = 43$

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309. (d)

Volume of A = 70% of volume of B

$V_A = 0.7\, V_B$

Volume of A from statement II

$V_A = 14{,}000$ gallons

Then, $V_B = \frac{14000}{0.7} = 20{,}000$ gallons

Total gallons of solution in A & B from statement I

$= 80\%$ of $V_A + 40\%$ of V_B

$= 0.8 \times 14{,}000 + 0.4 \times 20{,}000$

$= 19{,}200$ gallons

So, to answer the question both the statements together are sufficient.

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310. (4536 to 4536)

Let the four boxes represent four digits. The number of possible digits at each place is shown below the box

$\square \quad \square \quad \square \quad \square$

$9 \times 9 \times 8 \times 7$

So, total number that can be formed, is

$9 \times 9 \times 8 \times 7 = 4536$

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311. (b)

Event Z is that two tails occur at three tosses and event Y is that a tail occurs on the third toss. Event [TTH] is possible in Z but not possible in Y so, Z and Y are dependent.

Events Z and X are opposite to each other so these are dependent.

Events X and Y are not opposing each other hence these events are independent.

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312. (280 to 280)

$\frac{PS}{QS} = \frac{3}{1}$; PS = 3x, QS = 1x

$\frac{RT}{QT} = \frac{5}{2}$; RT = 5y, QT = 2y

Area of ΔQTS

$\frac{1}{2} QS \times QT = 20 cm^2$

$\frac{1}{2}(x \times 2y) = 20 cm^2$

$xy = 20 cm^2$

From ΔPQR

Area = ½ PQ × QR

$= \frac{1}{2}(3x+1x) \times (5y+2y) = \frac{1}{2} 28xy$

$= 14 \times 20 = 280 \ cm^2$

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313. (b)

Number of matches to complete the league round of matches is

$^5C_2 = \frac{5 \times 4}{2} = 10$

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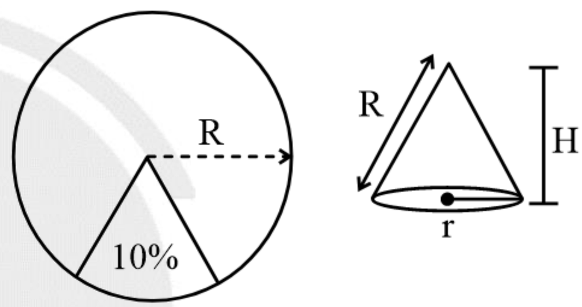
314. (1.9 to 2.2)

Surface area of cone = 0.9 area of a circular sheet



$\pi r R = 0.9 \times \pi R^2$

r = 27 cm

$H = \sqrt{R^2 + r^2}$

$H = \sqrt{30^2 + 27^2} = 13.08 cm$

$\frac{r}{H} = \frac{27}{13.08} = 2.06$

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315. (800 to 800)

Tiger's speed = 5 leaps/min = 5×8 meter/min = 40 meter/min

Dear's speed = 4 leaps/min = 4×5 meter/min = 20 meter/min

Distance between tiger & dear = 50×8 =400 meters.

Time to catch the dear = $\frac{400}{40-20}$ = 20 min

In 20 minutes, tiger will cover 40×20 = 800 meters

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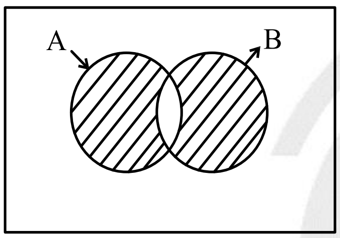
316. (b)

Given $P(A)=\frac{1}{6}$ and $P(B)=\frac{1}{8}$

Probability of only ane of Ram and Ramesh to be selected



= P (A) + P (B) – 2 P (A ∩B)

$=\frac{1}{6}+\frac{1}{8}-2\times\frac{1}{8}\times\frac{1}{6}$

$=\frac{14}{48}-\frac{2}{48}=\frac{12}{48}=\frac{1}{4}$

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317. (96 to 96)

Given $\left(z+\frac{1}{z}\right)^2=98$

Using $(a+b)^2=a^2+b^2+2ab$

$\left(z+\frac{1}{z}\right)^2=z^2+\frac{1}{z^2}+2\times z\times\frac{1}{z}=98$

$z^2+\frac{1}{z^2}+2=98$

$z^2+\frac{1}{z^2}=98-2=96$

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318. (850 to 850)

Charge for single person of two-way fare = 2 × 100

There are total 5 person in the group, then,

Total fare charge without discount = 2 × 100 × 5

Total discount = 10% + 5% = 15 %

∴ Total charge to a group of 5 people

$=1000\times\frac{100-15}{100}=850\,\text{Rs.}$

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319. (b)

From 2 to 198, first multiple of 10 will be 10 and last will be 190.

Let total multiple of 10 in between 2 and 198 is n.

Then, $190=10+(n-1)10 \Rightarrow n=19$

Sum of these 19 numbers = S_n

$S_n=\frac{19}{2}[2\times10+(19-1)\times10]=1900$

Now, the avg of these numbers $=\frac{1900}{19}=100$

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320. (c)

Let, $p=\sqrt{12+\sqrt{12+\sqrt{12+....}}}$

Then, $p=\sqrt{12+p}$

Squaring both sides, we get

$p^2=12+p$

$p^2 - p - 12 = 0$

$\Rightarrow (p-4)(p+3) = 0$

p = 4, –3

Valid value of p will be positive, then p = 4

So, option ‘C’ is correct.

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321. (a)

The required time $= x : (30x \pm \theta) \times \frac{2}{11}$

Where, x = starting time of span

θ = angle between minute and hour hand

Take ‘+’ for 0 to 6 span and ‘–‘for 6 to 12 span

$\therefore \text{Time} = 6 : (30 \times 6 - 60) \times \frac{2}{11} = 6 : 21\frac{9}{11} \approx 6 : 22$

So, option ‘A’ is correct.

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322. (d)

Consider the GDP before year 2012-2013 = x

GDP increased during 2012-2013 by 7%.

∴ GDP during 2012-2013 = 1.07x

Similarly, USD price increased from 50 to 60 during this period, which reduced the value of rupee by a ratio 5/6

$\therefore$ New GDP $= 1.07x \times \frac{5}{6} = 0.8916x$

The percentage change is GDP

$= \frac{0.8916x - x}{x} \times 100$

$= -10.84\% \simeq -11\%$

Negative sign indicates that GDP will decrease by 11% approximately.

So, option ‘D’ is correct.

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323. (1300 to 1300)

Total cost = Fixed cost + Variable cost

= 50000 + 800 Q

= 50000 + 800 × 100

Total cost for = 1,30,000 for 100 tonnes

For 1 tonne = 1,30,000/100 = 1300

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324. (d)

Let the populations in country X and Y are x and y respectively.

x = 3y

$$\begin{array}{cc} x & y \\ \downarrow & \downarrow \\ 3y & y \end{array}$$

The percentage of people taller than 6 ft

$\frac{0.01 \times 3y + 0.02y}{4y} \times 100 = 1.25$

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325. (a)

$5 \to 6 \to 7.2 \to 8.64 \to > 10 \Rightarrow$ 3 to 4 years

$5 \times (1.2)^n = 10$

$(1.2)^n = 2$

$n = \frac{\log 2}{\log 1.2} \Rightarrow n = 3.8$

Scan for Video solution

326. (b)

T.Th	Th	H	T	U
□	□	□	□	□
5 ×	4 ×	3 ×	2 ×	1

= 120

Out of these 120, our 5 possible values will each appear eaual number of times in the unit place, the tens place, the hundreds place etc. So, 120/5 = 24 or there will be 24 numbers starting with 1, 24 starting with 3 and 24 starting with 5 etc.

Sum of the numbers =

24 × [10000 (1 + 3 + 5 + 7 + 9) + 1000 (1 + 3 + 5 + 7 + 9) +100 (1 + 3 + 5 + 7 + 9) + 10 (1 + 3 + 5 + 7 + 9) + (1 + 3 + 5 + 7 + 9)] = 6666600

Scan for Video solution

327. (b)

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$2P = P\left(1+\frac{r}{100}\right)^{10}$$

r = 7.2%

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328. (495 to 495)

1st five numbers = x, x + 2, x + 4, x + 6, x + 8

Sum ⟹ 5x + 20 = 425

$$x = \frac{405}{5} = 81$$

Last five = x + 14, x + 16, x + 18, x + 20, x + 22

Sum ⟹ 81×5 + 90 = $\boxed{495}$

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329. (0.80 to 0.82)

Probability to be defective $\frac{5}{100}$

Probability to be non-defective = $\frac{95}{100} = 0.95$

Probability of batch is accepted = $(0.95)^4 = 0.8145$

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330. (4 to 4)

$$T = \frac{\text{Distance}}{\text{Speed}}$$

Speed of Man = 8 km/h

Speed of Stream = ‘x’ km/h

Since, the time to row upstream is thrice the time to row downstream. Hence,

$$3\left(\frac{\cancel{D}}{8+x}\right) = \frac{\cancel{D}}{8-x}$$

⟹ 24 – 3x = 8 + x

4x = 16

‘x’ = 4 km/h

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331. (d)

In options A and B, the direct relation between P and M is not clear. In option C, P > M. In option D, P < M. So option D is correct.

Scan for Video solution

332. (a)

Let the volume of half tank is 1 litre, then drainage (E) and pumping (F) rates are respectively

$$E = \frac{-1}{30},\ F = \frac{1}{x}$$

Given that the tank should be filled in 10 min, so

$$\frac{1}{x} - \frac{1}{30} = \frac{1}{10}$$

$$\frac{1}{x} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$$

$$\Rightarrow\ x = \frac{30}{4} = 7.5 \text{ min}$$

$$7.5 \times 4 = \boxed{30}$$

Hence water has to be pumped with a rate 4 times that of drainage.

Scan for Video solution

333. (163 to 163)

ODD No.

x, x + 2, x + 4, x + 6, x + 8, x + 10, x + 12, x + 14

sum = 8x + 56 = 656

8x = 600

$x = \boxed{75}$

EVEN No.

y, y + 2, y + 4, y + 6

sum = 4y + 12 = 87 × 4

y = 84

sum of smallest odd number and second largest even number = x + (y + 4) = 75 + (84 + 4) = 163

Scan for Video solution

334. (b)

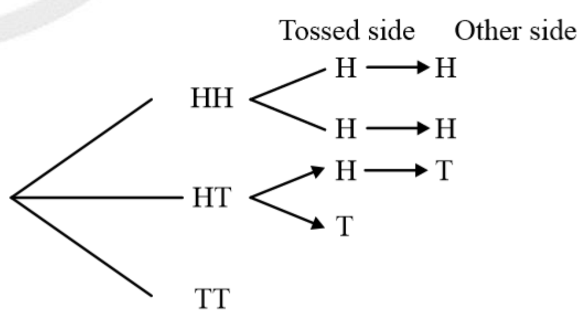
0.167 + 0.167 + 0.152 + 0.166 + 0.168 + 0.180 = 1

Since the probabilities of all the numbers are not same hence the dice is biased.

Scan for Video solution

335. (b)

Given that the tossed side is heads



then the probability of other side being tails is 1/3

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336. (560 to 560)

Train crossing Man

D = 280 m (length of train)

T = 20 sec

Speed (S) = $\frac{280}{20} = 14\text{m / sec}$

Train crossing platform of length P

D = 280 + P

T = 60 sec

S = 14m/sec

D = S × T

280 + P = 14 × 60

$\Rightarrow$ P = 840 – 280 = 560 meter

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337. (c)

$$\text{Average speed} = \frac{\text{Total distance (km)}}{\text{Total time (hr)}}$$

$$\text{Average speed} = \frac{8+6+16}{(15+15+15)\times {}^{1}\!/_{60}}$$

$$\text{Average speed} = \frac{30\times 60}{45} = 40 \text{ km / hr}$$

So, option 'C' is correct.

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338. (c)

Let the temperature of Monday, Tuesday, Wednesday and Thursday are w, x, y, z respectively;

Then, $\frac{w+x+y}{3} = 41$

$\therefore$ w + x + y = 123 .... (i)

And, $\frac{x+y+z}{3} = 43$

$\therefore$ x + y + z = 129 .... (ii)

From equation (i) and (ii),

z – w = 6

Also, given that, z = w + 0.15w = 1.15w

$\therefore$ 1.15w – w = 6

0.15w = 6

w = 40 °C

z = 1.15w = 1.15×40 = 46 °C

So, option 'c' is correct.

Scan for Video solution

339 (d)

$x - 75 = 117 - x$

x = 96

Scan for Video solution

340. (a)

Let n units are sold

$\text{S.P.} - \text{C.P.} = \text{Profit}$

$60 \times n - 100 = 500$

n = 10

$\Rightarrow$ If total cost is increased by 30% then let x% is increased in selling price to maintain the same level of profit.

$$\left(1+\frac{x}{100}\right)60\times 10 - 1.30\times 100 = 500$$

x = 5%

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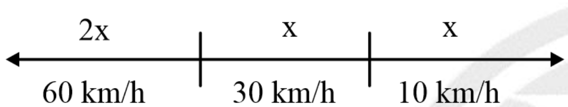
341. (b)

Given that labour wages per day increase by 1/5 and the working hours per day decrease by 1/24, so the new erection cost is

$$= \left(1+\frac{1}{5}\right)\times\left(1-\frac{1}{24}\right)\times 13,200 = 15,180$$

Scan for Video solution

342. (c)



$$\frac{2x}{60}+\frac{x}{30}+\frac{x}{10}=\frac{4x}{V_{avg}}$$

$$\frac{1\cancel{0}\cancel{x}}{6\cancel{0}}=\frac{4\cancel{x}}{V_{avg}}, \Longrightarrow V_{avg} = 6\times 4 = 24 \text{ km/h}$$

Scan for Video solution

343. (d)

Two-digit numbers between 1 to 100 = 90

$$\begin{array}{cc} 14 & 98 \\ 7\times 2 & 7\times 14 \end{array}$$

Numbers divisible by 7 = 7×2, 7×3, 7×4, …..7×14.

So, total numbers divisible by 7 = 13

Not divisible 7 = 90 – 13 = 77

The probability of the selected number is not divisible by 7 = $\frac{77}{90}$

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344. (c)

44 + 42 + 40 + ……….. 2

2 (22 + 21 + 20 + 19 …….. 1)

Sum of n terms of an Arithmetic series

$$= \frac{n(a_1 + a_n)}{2}$$

$$= \cancel{2}\left(\frac{22\times 23}{\cancel{2}}\right) = 506$$

Scan for Video solution

345. (a)

Let,

Total notes of Rs. 20 = x

Total notes of Rs. 10 = y

So, x + y = 14 … (i)

And;

20x + 10y = 230 … (ii)

On solving (i) & (ii);

We get, x = 9, and y = 5

Hence the number of notes of Rs. 10 are 5.

So, option ‘a’ is correct.

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346. (d)

Given;

$(1.001)^{1259} = 3.52$

$(1.001)^{2062} = 7.85$

Then,

$(1.001)^{3321} = (1.001)^{1259} \times (1.001)^{2062}$

$= 3.52 \times 7.85 = 27.632 \approx 27.64$

So, option ‘D’ is correct.

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347. (c)

$$\text{Mean (M)} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$M' = \frac{\sum_{i=1}^{n}(x_i + 7)}{n} = \frac{\sum_{i=1}^{n} x_i}{n} + \frac{7n}{n} = M + 7$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(M - x_i)^2}$$

On doubling each entry, the mean and x_i both will be doubled.

$$\text{Standard deviation} = \sqrt{\frac{1}{N}\sum_{i=1}^{n} 2^2(M - x_i)^2} = 2\sigma$$

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348. (b)

The percentage of total shock absorber which is reliable and made by X = 0.6 × 0.96 = 0.576 or 57.6%

The percentage of total shock absorber which is reliable and made by Y = 0.4 × 0.72 = 0.288 or 28.8%.

The probability that a chosen shock absorber which is reliable, is made by Y

$$= \frac{0.288}{0.576 + 0.288} = \frac{0.288}{0.864}$$

$$\frac{288}{864} = 0.33 = 0.\overline{3}$$

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349. (c)

Only option (c) can be exactly divide by 3 so that it can be the answer. Its because no other option can be divided into three equal parts.

Calculations:

Let the number of toffees = x

R took 1/3rd but returned four, then the remaining toffees in the bowl $= x - \frac{x}{3} + 4 = \frac{2x}{3} + 4$

S took 1/4th but returned three, then the remaining toffees in the bowl

$$= \frac{2x}{3} + 4 - \frac{1}{4}\left(\frac{2x}{3} + 4\right) + 3 = \frac{x}{2} + 6$$

T took ½ of remaining but returned two, then the remaining toffees in the bowl

$$= \frac{1}{2}\left(\frac{x}{2} + 6\right) + 2 = \frac{x}{4} + 5$$

$$\therefore \frac{x}{4} + 5 = 17 \Rightarrow x = 48$$

Scan for Video solution

350. (a)

Let total number of candidates = x

Initial voters of P = 0.4x

Initial voters of Q = 0.6x

As per new change in voters,

New voters of P will be,

= 0.4x – 0.15 × 0.4x + 0.25 × 0.6x = 0.49x

New voters of Q will be,

= 0.6x – 0.25 × 0.6x + 0.15 × 0.4x = 0.51x

∴ 0.51x – 0.49x = 2

x = 100

So, option ‘A’ is correct.

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351. (c)

Let A and B be the number of orders each day and backlogs respectively.

4A + B = 7 × 4 = 28

$10A + B = 3 \times 10 = 30$

Solving above equations we get A=1/3 and B=80/3.

For 5 days to ship the orders

$5A + B = T\times5 \Longrightarrow 5/3 + 80/3 = T\times5$

$T = 5.66$

So, minimum trucks required will be 6.

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352. (d)

Quantity left after n operations = $x\left(1-\frac{y}{x}\right)^n$

Where x and y are initial amount and amount of mixture replaced each time respectively.

$=10 \times 0.9^3$

$= 10 \times 0.729$

$= 7.29$ lit

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353. (b)

Four digits can be named as unit place, ten's place, hundred's place and thousand's place.

Th	H	T	U

If the thousand's place is 3 (fixed) then total number of 4-digit numbers is

$3 \times 3 \times 3 = 27 - 2 = 25$ ($\because$ 3 unique digits are there and two numbers 3222 & 3333 are not possible)

If the thousand's place is 4 (fixed) then total number of 4-digit numbers is

$3 \times 3 \times 3 = 27 - 1 = 26$ ($\because$ 4222 is not possible)

Total numbers possible = 25 + 26 = 51

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354. (d)

Capacities (per day per worker) of skilled (s), semiskilled (ss) and unskilled (us) workers are calculated below:

5 sk = $\frac{1}{20}$ 8 ssk = $\frac{1}{25}$

1sk = $\frac{1}{100}$ 1ssk = $\frac{1}{200}$

10 usk = $\frac{1}{30}$ 1usk = $\frac{1}{300}$

So, 2 skilled, 6 semiskilled and 5 unskilled workers will have combined capacity (per day) of

$$\frac{2}{100}+\frac{6}{200}+\frac{5}{300}=\frac{12+18+10}{600}$$

$$=\frac{40}{600}=\frac{1}{15}$$

So, the time taken to build the wall by this team will be 15 days.

Scan for Video solution

❑❑❑

CHAPTER

2 ANALYTICAL APTITUDE

Logical Reasoning

1. [MCQ] **[GATE-ME-2023: 1M]**

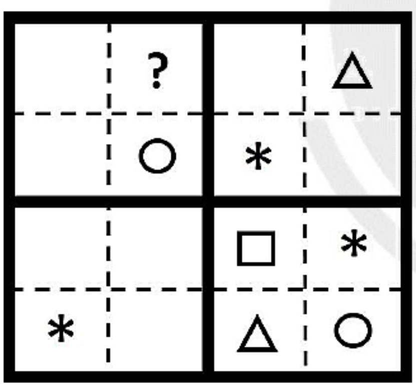
The symbols ○, ∗, Δ and □ are to be filled, one in each box, as shown below.

The rules for filling in the four symbols are as follows.

(1) Every row and every column must contain each of the four symbols.

(2) Every 2×2 square delineated by bold lines must contain each of the four symbols.

Which symbol will occupy the box marked with '?' in the partially filled figure?



(a) ○ (b) ∗

(c) Δ (d) □

2. [**MCQ**] **[GATE-EC-2023:2M]**

A 100 cm × 32 cm rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions 100 cm × 1 cm.

The total number of creases visible when the sheet is unfolded is ______.

(a) 32 (b) 5

(c) 31 (d) 63

3. [**MCQ**] **[GATE-EC-2023:1M]**

In a class of 100 students,

(i) there are 30 students who neither like romantic movies nor comedy movies,

(ii) the number of students who like romantic movies is twice the number of students who like comedy movies, and

(iii) the number of students who like both romantic movies and comedy movies is 20.

How many students in the class like romantic movies?

(a) 40 (b) 20

(c) 60 (d) 30

4. [MCQ] **[GATE-CE-2023: 1M]**

There are 4 red, 5 green, and 6 blue balls inside a box. If N number of balls are picked simultaneously, what is the smallest value of N that guarantees there will be at least two balls of the same colour?

One cannot see the colour of the balls until they are picked.

(a) 4 (b) 15

(c) 5 (d) 2

5. [MCQ] **[GATE-CE-2023: 2M]**

Consider a spherical globe rotating about an axis passing through its poles.

There are three points P, Q and R situated respectively on the equator, north pole and midway between the equator and the north pole in the northern hemisphere. Let P, Q and R move with speeds v_P, v_Q, and v_R, respectively.

Which one of the following options are CORRECT?

(a) $v_P < v_R < v_Q$ (b) $v_P < v_Q < v_R$

(c) $v_P > v_R > v_Q$ (d) $v_P = v_R \neq v_Q$

6. **[MCQ]** **[GATE-CE-2023: 1M]**

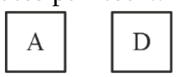
A duck named Donald Duck says "All ducks always lie."

Based only on the information above, which one of the following statements can be logically inferred with certainty?

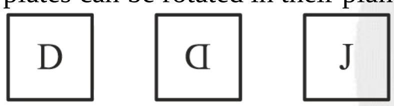
(a) Donald Duck always lies.
(b) Donald Duck always tells the truth.
(c) Donald Duck's statement is true.
(d) Donald Duck's statement is false.

7. **[MCQ]** **[GATE-CS-2023:1M]**

A palindrome is a word that reads the same forwards and backwards. In a game of words, a player has the following two plates painted with letters.



From the additional plates given in the options, which one of the combinations of additional plates would allow the player to construct a five-letter palindrome. The player should use all the five plates exactly once. The plates can be rotated in their plane.

(a) 

(b) 

(c) 

(d) 

8. **[MCQ]** **[GATE-ME-2022: 1M]**

Four girls P, Q, R and S are studying languages in a University. P is learning French and Dutch. Q is learning Chinese and Japanese. R is learning Spanish and French. S is learning Dutch and Japanese.

Given that: French is easier than Dutch; Chinese is harder than Japanese; Dutch is easier than Japanese, and Spanish is easier than French.

Based on the above information, which girl is learning the most difficult pair of languages?

(a) P (b) Q
(c) R (d) S

9. **[MCQ]** **[GATE-EC-2022:2M]**

In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P: R has copied in the exam.
Statement of Q: S has copied in the exam.
Statement of R: P did not copy in the exam.
Statement of S: Only one of us is telling the truth.
Statement of T: R is telling the truth.

The investigating team had authentic information that S never lies.

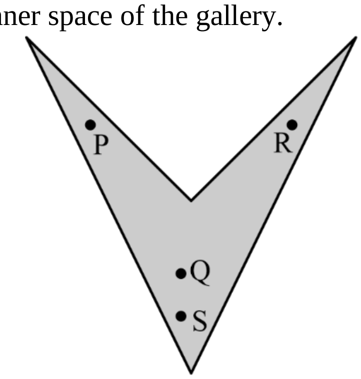
Based on the information given above, the person who has copied in the exam is

(a) R (b) P
(c) Q (d) T

10. **[MCQ]** **[GATE-EC-2022:1M]**

An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard.

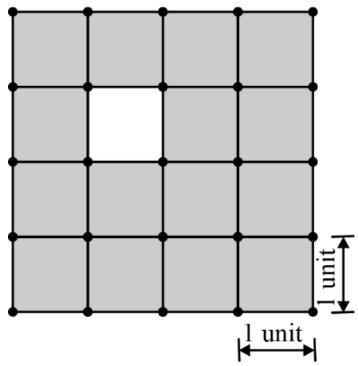
If the security guard does not move around the posted location and has a 360° view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery.



(a) P and Q
(b) Q
(c) Q and S
(d) R and S

11. [**MCQ**] **[GATE-EC-2022:1M]**

The figure shows a grid formed by a collection of unit squares. The unshaded unit square in the grid represents a hole.



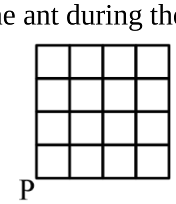
What is the maximum number of squares without a "hole in the interior" that can be formed within the 4 × 4 grid using the unit squares as building blocks

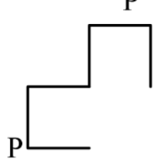
(a) 15
(b) 20
(c) 21
(d) 26

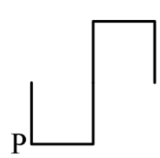
12. [MCQ] [GATE-EE-2022: 2M]

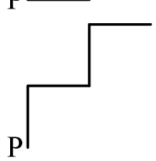
An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases.

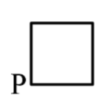
Which one of the following is a part of a possible trajectory of the ant during the movement?



(a) 

(b) 

(c) 

(d) 

13. [MCQ] [GATE-EE-2022: 2M]

The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order.

Consider the following statements:

★ The line segment joining R and S is longer than the line segment joining P and Q.
★ The line segment joining R and S is perpendicular to the line segment joining P and Q.
★ The line segment joining R and U is parallel to the line segment joining T and Q.

Based on the above statements, which one of the following options is CORRECT?

(a) The line segment joining R and T is parallel to the line segment joining Q and S
(b) The line segment joining T and Q is parallel to the line joining P and U
(c) The line segment joining R and P is perpendicular to the line segment joining U and Q
(d) The line segment joining Q and S is perpendicular to the line segment joining R and P

14. [MCQ] [GATE-EE-2022 : 1M]

Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some bottles are cups.
Statement 2: All cups are knives.
Conclusion I: Some bottles are knives.
Conclusion II: Some knives are cups.
Conclusion III: All cups are bottles.
Conclusion IV: All knives are cups.

Which one of the following options can be logically inferred?

(a) Only conclusion I and conclusion II are correct
(b) Only conclusion II and conclusion III are correct
(c) Only conclusion II and conclusion IV are correct
(d) Only conclusion III and conclusion IV are correct

15. [MCQ] [GATE-CE-2022:1M]

Given the statements:

★ P is the sister of Q.

★ Q is the husband of R

★ R is the mother of S.

★ T is the husband of P.

Based on the above information, T is ______ of S.

(a) the grandfather (b) the father

(c) an uncle (d) a brother

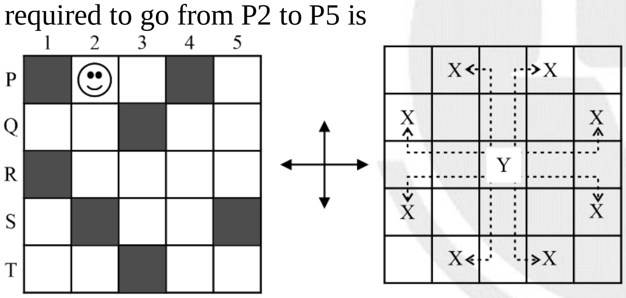
16. [MCQ] [GATE-CE-2022:2M]

In the square grid shown on the left, a person standing at P2 position is required to move to P5 position.

The only movement allowed for a step involves, "two moves along one direction followed by one move in a perpendicular direction". The permissible directions for movement are shown as dotted arrows in the right.

For example, a person at a given position Y can move only to the positions marked X on the right.

Without occupying any of the shaded squares at the end of each step, the minimum number of steps required to go from P2 to P5 is



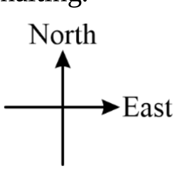
(a) 4 (b) 6

(c) 5 (d) 7

17. [MCQ] [GATE-CE-2022:2M]

An ant walks in a straight line on a plane leaving behind a trace of its movement. The initial position of the ant is at point P facing east.

The ant first turns 72° anticlockwise at P, and then does the following two steps in sequence exactly FIVE times before halting.

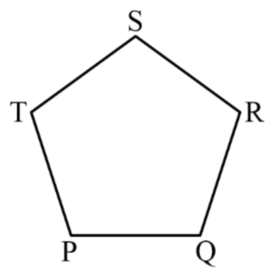


1. moves forward for 10 cm.

2. turns 144° clockwise.

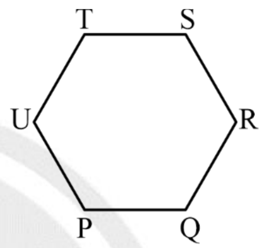
The pattern made by the trace left behind by the ant is

(a)



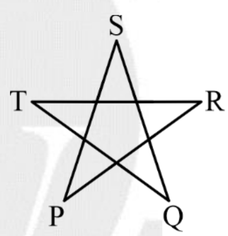
PQ = QR = RS = ST = TP = 10 cm

(b)



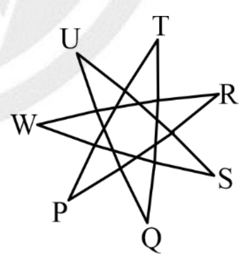
PQ = QR = RS = ST = TU = UP = 10 cm

(c)



SQ = QT = TR = RP = PS = 10 cm

(d)



SW = WR = RP = PT = TQ = QU = US = 10 cm

18. [MCQ] [GATE-CS-2022:2M]

The corners and mid-points of the sides of a triangle are named using the distinct letters P, Q, R, S, T and U, but not necessarily in the same order. Consider the following statements:

- The line joining P and R is parallel to the line joining Q and S.

- P is placed on the side opposite to the corner T.
- S and U cannot be placed on the same side.

Which one of the following statements is correct based on the above information?

(a) P cannot be placed at a corner
(b) S cannot be placed at a corner
(c) U cannot be placed at a mid-point
(d) R cannot be placed at a corner

19. [MCQ] [GATE-ME-2021: 1M]

Ms. X came out of a building through its front door to find her shadow due to the morning sun falling to her right side with the building to her back. From this, it can be inferred that building is facing

(a) North (b) East
(c) West (d) South

20. [MCQ] [GATE-EE-2021 : 1M]

Seven cars P, Q, R, S, T, U and V are parked in a row not necessarily in that order. The cars T and U should be parked next to each other. The cars S and V also should be parked next to each other, whereas P and Q cannot be parked next to each other. Q and S must be parked next to each other. R is parked to the immediate right of V. T is parked to the left of U.

Based on the above statements, the only INCORRECT option given below is:

(a) There are two cars parked in between Q and V.
(b) Q and R are not parked together.
(c) V is the only car parked in between S and R.
(d) Car P is parked at the extreme and.

21. [MCQ] [GATE-CE-2021: 1M]

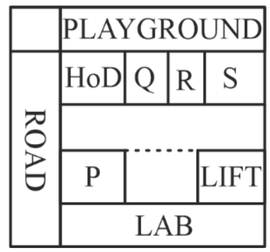
Four persons P, Q, R and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and R cannot sit adjacent to each other. S should be seated to the right of Q. The number of distinct seating arrangements possible is:

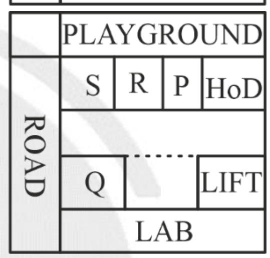
(a) 2 (b) 4
(c) 6 (d) 8

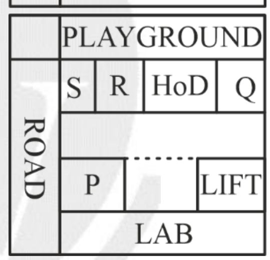
22. [MCQ] [GATE-CE-2020: 1M]

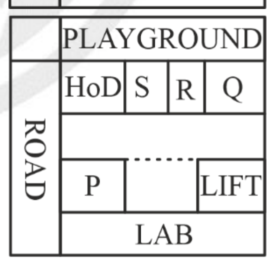
After the inauguration of the new building, the Head of the Department (HoD) collated faculty preferences for office space. P wanted a room adjacent to the lab. Q wanted to be close to the lift. R wanted a view of the playground and S wanted a corner office.

Assuming that everyone was satisfied, which among the following shows a possible allocation?

(a)


(b)


(c)


(d)


23. [MCQ] [GATE-CE-2020: 2M]

Five friends P, Q, R, S and T went camping. At night, they had to sleep in a row inside the tent. P, Q and T refused to sleep next to R since he snored loudly. P and S wanted to avoid Q as he usually hugged people in sleep.

Assuming everyone was satisfied with the sleeping arrangements, what is the order in which they slept?

(a) RSPTQ (b) SPRTQ
(c) QRSPT (d) QTSPR

24. [MCQ] **[GATE-ME-2019: 2M]**

M and N had four children P, Q, R and S. Of them, only P and R were married. They had children X and Y respectively. If Y is a legitimate child of W, which one of the following statements is necessarily FALSE?

(a) M is the grandmother of Y
(b) R is the father of Y
(c) W is the wife of R
(d) W is the wife of P

25. [MCQ] **[GATE-ME-2019: 2M]**

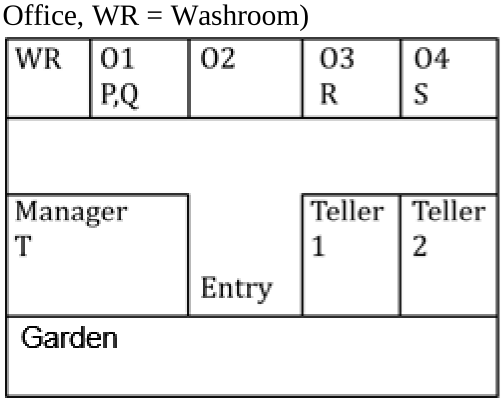
Under a certain legal system, prisoners are allowed to make one statement. If their statement turns out to be true then they are hanged. If the statement turns out to be false then they are shot. One prisoner made a statement and the judge had no option but to set him free. Which one of the following could be that statement?

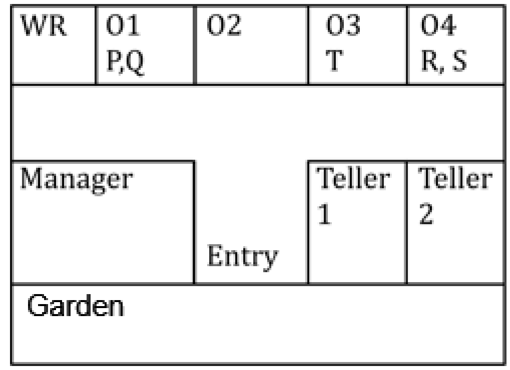
(a) I did not commit the crime
(b) I committed the crime
(c) I will be shot
(d) You committed the crime

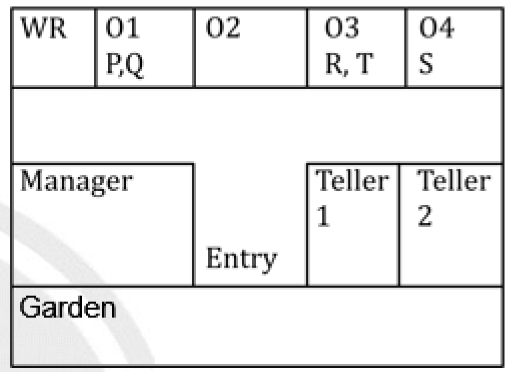
26. [MCQ] **[GATE-EC-2019:2M]**

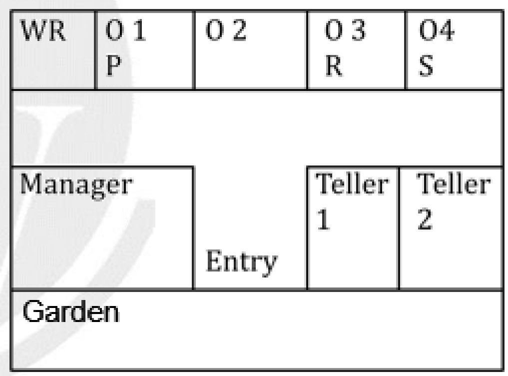
Five people P, Q, R, S and T work in a bank. P and Q don't like each other but have to share an office till T gets a promotion and moves to the big office next to the garden. R, who is currently sharing an office with T wants to move to the adjacent office with S, the handsome new intern. Given the floor plan, what is the current location of Q, R and T?

(O = Office, WR = Washroom)

(a)


(b)


(c)


(d)


27. [MCQ] **[GATE-EC-2019:1M]**

Five different books (P, Q, R, S, T) are to be arranged on a shelf. The books R and S are to be arranged first and second, respectively from the right side of the shelf. The number of different orders in which P, Q and T may be arranged is______.

(a) 2 (b) 6
(c) 12 (d) 120

28. [MCQ] **[GATE-EC-2019:1M]**

When he did not come home, she _________ him lying dead on the roadside somewhere.

(a) concluded (b) looked

(c) notice (d) pictured

29. [**MCQ**] **[GATE-EC-2019:2M]**

Four people are standing in a line facing you. They are Rahul, Mathew, Seema and Lohit. One is an engineer, one is a doctor, one a teacher and another a dancer. You are told that:

1. Mathew is not standing next to Seema
2. There are two people standing between Lohit and the engineer
3. Rahul is not a doctor
4. The teacher and the dancer are standing next to each other
5. Seema is turning to her right to speak to the doctor standing next to her.

Who among them is an engineer?

(a) Seema (b) Lohit

(c) Rahul (d) Mathew

30. **[MCQ]** **[GATE-CE-2019: 2M]**

P. Q. R. S and T are related and belong to the same family. P is the brother of S. Q is the wife of P. R and T are the children of the siblings P and S respectively. Which one of the following statements is necessarily FALSE

(a) S is the aunt of R

(b) S is the aunt of T

(c) S is the sister-in-law of Q

(d) S is the brother of P

31. **[MCQ]** **[GATE-CS-2019:2M]**

The police arrested four criminals – P, Q, R and S. The criminals knew each other. They made the following statements:

P says "Q committed the crime."

Q says "S committed the crime."

R says "I did not do it."

S says "What Q said about me is false."

Assume only one of the arrested four committed the crime and only one of the statements made above is true. Who committed the crime?

(a) P (b) Q

(c) R (d) S

32. **[MCQ]** **[GATE-CS-2019:2M]**

Three of the five students are allocated to a hostel put in special requests to the warden. Given the floor plan of the vacant rooms, select the allocation plan that will accommodate all their requests.

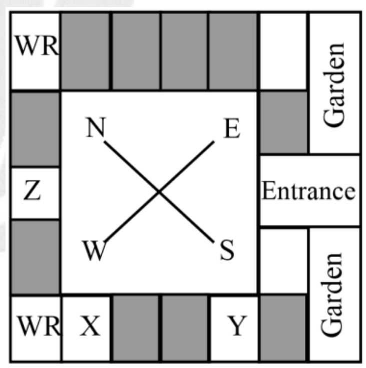
Request by X: Due to pollen allergy, I want to avoid a wing next to the garden.

Request by Y: I want to live as far from the washrooms as possible, since I am very sensitive to smell.

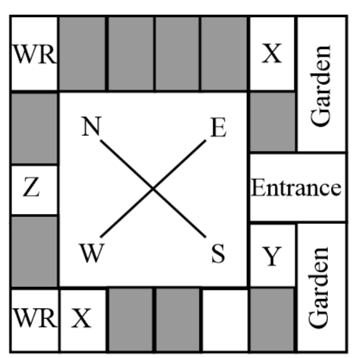
Request by Z: I believe in Vaastu and so I want to stay in the South-west wing.

The shaded rooms are already occupied. WR is washroom.

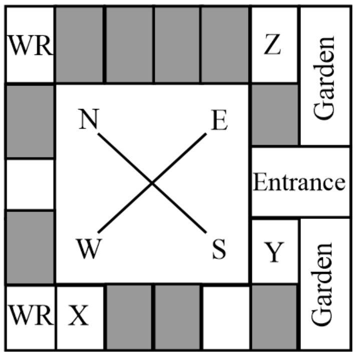
(a)



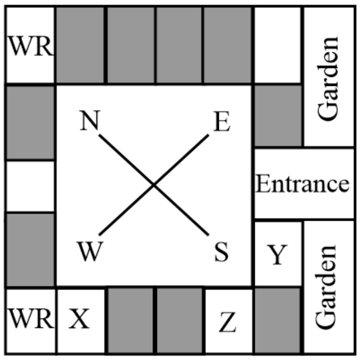
(b)



(c)



(d)



33. [MCQ] [GATE-ME-2017: 2M]

All people in a certain island are either 'Knights' or 'Knaves' and each person knows every other person's identity. Knights NEVER lie, and knaves ALWAYS lie.

P says "Both of us are knights". Q says "None of us are knaves".

Which one of the following can be logically inferred from the above?

(a) Both P and Q are knights
(b) P is a knight; Q is a knave
(c) Both P and Q are knaves
(d) The identities of P, Q cannot be determined

34. [MCQ] [GATE-ME-2017: 1M]

P, Q, and R talk about S's car collection. P states that S has at least 3 cars. Q believes that S has less than 3 cars. R indicates that to his knowledge, S has at least one car. Only one of P, Q and R is right. The number of cars owned by S is

(a) 0 (b) 1
(c) 3 (d) Cannot be determined

35. [MCQ] [GATE-EC-2017:2M]

Each of P, Q, R, S, W, X, Y and Z has been married at most once. X and Y are married and have two children P and Q. Z is the grandfather of the daughter S of P. Further, Z and W are married and are parents of R. Which one of the following must necessarily be FALSE?

(a) X is the mother-in-law of R
(b) P and R are not married to each other
(c) P is a son of X and Y
(d) Q cannot be married to R

36. [MCQ] [GATE-EC-2017:1M]

A rule states that in order to drink beer, one must be over 18 years old. In a bar. there are 4 people. P is 16 years old. Q is 25 years old. R is drinking milkshake and S is drinking a beer. What must be checked to ensure that the rule is being followed?

(a) Only P's drink
(b) Only P's drink and S's age
(c) Only S's age
(d) Only P's drink, Q's drink and S's age

37. [MCQ] [GATE-EC-2017:2M]

S, T, U, V, W, X, Y and Z are seated around a circular table. T's neighbours are Y and V. Z is seated third to the left of T and second to the right of S. U's neighbours are S and Y: and T and W are not seated opposite each other. Who is third to the left of V?

(a) X (b) W
(c) U (d) T

38. [MCQ] [GATE-CS, EE-2017: 1M]

Rahul, Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali. Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other?

(a) Rahul and Murali
(b) Srinivas and Arul
(c) Srinivas and Murali
(d) Srinivas and Rahul

39. [MCQ] **[GATE-CS, EE-2017 : 2M]**

Six people are seated around a circular table. There are at least two men and two women. There are at least three right-handed persons. Every woman has a left-handed person to her immediate right. None of the women are right-handed. The number of women at the table is

(a) 2 (b) 3

(c) 4 (d) None of the above

40. [MCQ] **[GATE-CS, EE-2017 : 1M]**

There are five buildings called V, W, X, Y and Z in a row (not necessarily in that order). V is to the West of W. Z is to the East of X and the West of V. W is to the West of Y. Which is the building in the middle?

(a) V (b) W

(c) X (d) Y

41. [MCQ] **[GATE-CS, EE-2017 : 2M]**

There are three boxes. One contains apples, another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labelled. If you are permitted to open just one box and then pull out and inspect only one fruit. which box would you open to determine the contents of all three boxes?

(a) The box labelled 'Apples'

(b) The box labelled 'Apples and Oranges'

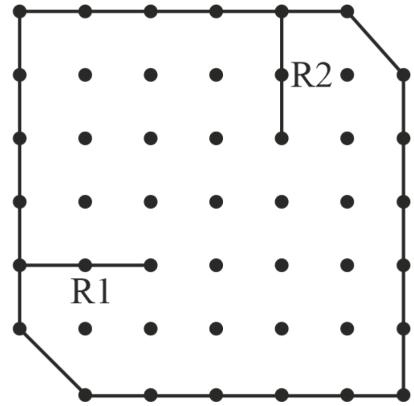
(c) Thee box labelled 'Oranges'

(d) Cannot be determined

42. [MCQ] **[GATE-CS-2017:1M]**

A plot of land must be divided between four families. They want their individual plots to be similar in shape, not necessarily equal in area. The land has equally spaced poles, marked as dots in the below figure. Two ropes, R1 and R2, are already present and cannot be moved.

What is the least number of additional straight ropes needed to create the desired plots? A single rope can pass through three poles that are aligned in a straight line.



(a) 2 (b) 4

(c) 5 (d) 3

43. [MCQ] **[GATE-CE-2017: 1M]**

Four cards lie on a table. Each card has a number printed on one side and a colour on the other. The faces visible on the cards are 2, 3, red and blue.

Proposition: If a card has an even value on one side, then its opposite face is red.

The cards which MUST be turned over to verify the above proposition are:

(a) 2, red (b) 2, 3, red

(c) 2, blue (d) 2, red, blue

44. [MCQ] **[GATE-ME, EC-2016: 2M]**

M and N start from the same location. M travels 10 km East and then 10 km North-East. N travels 5 km South and then 4 km South-East. What is the shortest distance (in km) between M and N at the end of their travel?

(a) 18.60 (b) 22.50

(c) 20.61 (d) 25.00

45. [MCQ] **[GATE-ME, EC-2016: 2M]**

Leela is older than her cousin Pavithra. Pavithra's brother Shiva is older than Leela. When Pavithra and Shiva are visiting Leela, all three likes to play chess. Pavithra wins more often than Leela does.

Which one of the following statements must be TRUE based on the above?

(a) When Shiva plays chess with Leela and Pavithra, he often loses.
(b) Leela is the oldest of the three.
(c) Shiva is a better chess player than Pavithra.
(d) Pavithra is the youngest of the three.

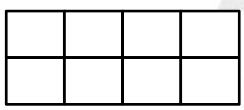
46. [**MCQ**] **[GATE-EC-2016:1M]**

M has a son Q and a daughter R. He has no other children. E is the mother of P and daughter-in law of M. How is P related to M?

(a) P is the son-in-law of M
(b) P is the grandchild of M
(c) P is the daughter in law of M
(d) P is the grandfather of M

47. [MCQ] [GATE-CS, EE-2016 : 2M]

In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?



(a) 21 (b) 27
(c) 30 (d) 36

48. [MCQ] [GATE-CS, EE-2016 : 2M]

All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

(i) Ooty is not a hill-station.
(ii) No hill-station can have more than one lake.

(a) (i) only (b) (ii) only
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

49. [MCQ] [GATE-CS, CE-2016: 1M]

If 'relftaga' means carefree, 'otaga' means careful and 'fertaga' means careless, which of the following could mean 'aftercare'?

(a) zentaga (b) tagafer
(c) tagazen (d) relffer

50. [MCQ] [GATE-ME-2015: 1M]

Tanya is older than Eric.
Cliff is older than Tanya
Eric is older than cliff.
If the first two statements are true, than the third statement is:

(a) True (b) False
(c) Uncertain (d) Data insufficient

51. [**MCQ**] **[GATE-EC-2015:2M]**

A cube of side 3 units is formed using a set of smaller cubes of side 1 unit. Find the proportion of the number of faces of the smaller cubes visible to those which are NOT visible.

(a) 1 : 4 (b) 1 : 3
(c) 1 : 2 (d) 2 : 3

52. [MCQ] [GATE-EE, CS-2015: 2M]

Four branches of a company are located at M, N, O, and P. M is north of N at a distance of 4 km. P is south of O at a distance of 2 km N is southeast of O by 1 km What is the distance between M and P in km?

(a) 5.34 (b) 6.74
(c) 28.5 (d) 45.49

53. [MCQ] [GATE-CS, CE-2015: 2M]

The head of a newly formed government desires to appoint five of the six selected members P, Q, R, S, T and U to portfolios of Home, Power, Defense, Telecom, and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets either Power or Telecom, then she must get the other one T insists on a portfolio if P gets one

Which is the valid distribution of portfolios?

(a) P-Home Q-Power. R-Defense. S-Telecom. T-Finance
(b) R-Home, S-Power. P-Defense, Q-Telecom, T-Finance
(c) P-Home. Q-Power. T-Defense. S-Telecom. U-Finance
(d) Q Home. U Power. T-Defense. R-Telecom P-Finance

54. [MCQ] [GATE-ME, EC-2014: 2M]

If 'KCLFTSB' stands for 'best of luck' and 'SHSWDG' stands for 'good wishes', which of the following indicates 'ace the exam'?

(a) MCHTX (b) MXHTC

(c) XMHCT (d) XMHTC

55. [MCQ] [GATE-ME, EC-2014: 2M]

Lights of four colors (red, blue, green, yellow) are hung on a ladder. On every step of the ladder there are two lights. If one of the lights is red, the other light on that step will always be blue. If one of the lights on a step is green, the other light on that step will always be yellow. Which of the following statements is not necessarily correct?

(a) The number of red lights is equal to the number of blue lights

(b) The number of green lights is equal to the number of yellow lights

(c) The sum of the red and green lights is equal to the sum of the yellow and blue lights

(d) The sum of the red and blue lights is equal to the sum of the green and yellow lights

56. [NAT] [GATE-EE, CS-2014 : 2M]

When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? ___________.

57. [MCQ] [GATE-EE, CS-2014 : 2M]

A dance programme is scheduled for 10.00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is

(a) Students should come at 9.00 a.m. and parents should come at 10.00 a.m

(b) Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.

(c) Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 a.m.

(d) Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

58. [MCQ] [GATE-CE-2014: 1M]

Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

59. [MCQ] [GATE-CE-2014: 2M]

In a group of four children, Som is younger to Riaz. Shiv is elder to Ansu. Ansu is youngest in the group. Which of the following statements is/are required to find the eldest child in the group?

Statements

1. Shiv is younger to Riaz.
2. Shiv is elder to Som.

(a) Statement 1 by itself determines the eldest child.

(b) Statement 2 by itself determines the eldest child.

(c) Statements 1 and 2 are both required to determine the eldest child.

(d) Statements 1 and 2 are not sufficient to determine the eldest child.

60. [MCQ] [GATE-CE-2013: 2M]

Abhishek is elder to Savar.

Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

(a) Abhishek is elder to Anshul
(b) Anshul is elder to Abhishek
(c) Abhishek and Anshul are of the same age
(d) No conclusion follows

61. [MCQ] [GATE-EC, EE-2012 : 2M]

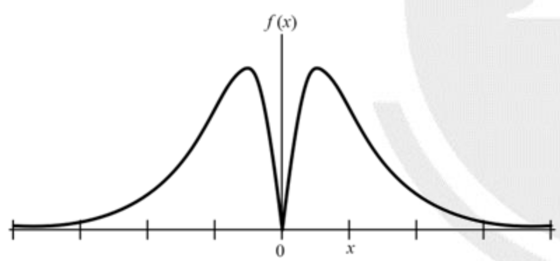
There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

(a) 2 (b) 3
(c) 4 (d) 8

Logical Venn Diagrams

62. [MCQ] [GATE-EC-2023:2M]

Which one of the following options represents the given graph?



(a) $f(x) = x^2\, 2^{-|x|}$ (b) $f(x) = x\, 2^{-|x|}$
(c) $f(x) = |x|\, 2^{-x}$ (d) $f(x) = x\, 2^{-x}$

63. [MCQ] [GATE-ME-2022: 2M]

Given below are three conclusions drawn based on the following three statements.

Statement 1: All teachers are professors.

Statement 2: No professor is a male.

Statement 3: Some males are engineers.

Conclusion I: No engineer is a professor.

Conclusion II: Some engineers are professors.

Conclusion III: No male is a teacher.

Which one of the following options can be logically inferred?

(a) Only conclusion III is correct
(b) Only conclusion I and conclusion II are correct
(c) Only conclusion II and conclusion III are correct
(d) Only conclusion I and conclusion III are correct

64. [MCQ] [GATE-CE-2022:1M]

A survey of 450 students about their subjects of interest resulted in the following outcome.

- 150 students are interested in Mathematics.
- 200 students are interested in Physics.
- 175 students are interest in Chemistry.
- 50 students are interest in Mathematics and Physics.
- 60 students are interested in Physics and Chemistry.
- 40 students are interest in Mathematics and Chemistry.
- 30 students are interested in Mathematics, Physics and Chemistry.
- Remaining students are interested in Humanities.

Based on the above information, the number of student's interest in Humanities is.

(a) 10 (b) 30
(c) 40 (d) 45

65. [MCQ] [GATE-CE-2022:2M]

Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some soaps are clean.

Statement 2: All clean objects are wet.

Conclusion I: Some clean objects are soaps.

Conclusion II: No clean object is a soap.

Conclusion III: Some wet objects are soaps.

Conclusion IV: All wet objects are soaps.

Which one of the following options can be logically inferred?

(a) Only conclusion I is correct
(b) Either conclusion I or conclusion II is correct
(c) Either conclusion III or conclusion IV is correct
(d) Only conclusion I and conclusion III are correct

66. **[MCQ]** **[GATE-CS-2022:1M]**
Given below are four statements.
Statement 1: All students are inquisitive.
Statement 2: Some students are inquisitive.
Statement 3: No student is inquisitive.
Statement 4: Some students are not inquisitive.
From the given four statements, find the two statements that CANNOT BE TRUE simultaneously, assuming that there is at least one student in the class.
(a) Statement 1 and Statement 3
(b) Statement 1 and Statement 2
(c) Statement 2 and Statement 4
(d) Statement 3 and Statement 4

67. **[MCQ]** **[GATE-CE-2021: 1M]**
$\oplus$ and $\odot$ are two operators on numbers p and q such that $p \oplus q = \frac{p^2+q^2}{pq}$ and $p \odot q = \frac{p^2}{q}$,. If $x \oplus y = 2 \odot 2$, then x =
(a) $\frac{y}{2}$ (b) y
(c) $\frac{3y}{2}$ (d) 2y

68. **[MCQ]** **[GATE-CE-2021: 1M]**
$\oplus$ and $\odot$ are two operators on numbers p and q such that $p \odot q = p - q$, and $p \oplus q = p \times q$.
Then, $(9 \odot (6 \oplus 7)) \odot (7 \oplus (6 \odot 5)) =$
(a) 40 (b) –26
(c) –33 (d) –40

69. **[MCQ]** **[GATE-CE-2021: 2M]**
On a planar field, you travelled 3 units East from a point O. Next you travelled 4 units South to arrive at point P. Then you travelled from P in the North-East direction such that you arrive at a point that is 6 units East of point O. Next, you travelled in the North-West direction, so that you arrive at point Q that is 8 units North of point P.
The distance of point Q to point O, in the same units, should be_____
(a) 3 (b) 5
(c) 4 (d) 6

70. **[MCQ]** **[GATE-ME-2021: 2M]**
Given below are two statements 1 and 2, and two conclusions I and II.
Statement 1: All entrepreneurs are wealthy.
Statement 2: All wealthy are risk seekers.
Conclusion I: All risk seekers are wealthy.
Conclusion II: Only some entrepreneurs are risk seekers.
Based on the above statements and conclusions, which one of the following options is CORRECT?
(a) Only conclusion I is correct
(b) Only conclusion II is correct
(c) Neither conclusion I nor II is correct
(d) Both conclusions I and II are correct

71. [**MCQ]** **[GATE-EC-2021:2M]**
Given below are two statements and two conclusions.
Statement 1: All purple are green.
Statement 2: All black are green.
Conclusion I: Some black are purple.
Conclusion II: No black is purple.
Based on the above statements and conclusions, which one of the following options is logically CORRECT?
(a) Only conclusion I is correct.
(b) Only conclusion II is correct
(c) Either conclusion I or II is correct.
(d) Both conclusion I and II are correct

72. **[MCQ]** **[GATE-CE-2021: 1M]**
In a company, 35% of the employees drink coffee, 40% of the employees drink tea and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?
(a) 15 (b) 25
(c) 35 (d) 40

73. **[MCQ]** **[GATE-CE-2021: 2M]**
Among the option given below, the statements that logically follows from the two statements 1 and 2 above, is:
1. Some football players play cricket.
2. All cricket players play hockey.
(a) No football player plays hockey.
(b) Some football players play hockey.
(c) All football players play hockey.
(d) All hockey players play football.

74. **[MCQ]** **[GATE-CS-2021: 2M]**
Given below are two statements 1 and 2, and two conclusions I and II.

Statement 1: All bacteria are microorganisms.

Statement 2: All pathogens are microorganisms.

Conclusion I: Some pathogens are bacteria.

Conclusion II: All pathogens are not bacteria.

Based on the above statements and conclusions, which one of the following options is logically CORRECT?

(a) Only conclusion I is correct

(b) Only conclusion II is correct

(c) Either conclusion I or II is correct.

(d) Neither conclusion I nor II is collect.

75. [MCQ] [GATE-CE-2020: 1M]

If 0, 1, 2, …..,7, 8, 9 are coded as O, P, Q,….,V, W, X, then 45 will be coded as____.

(a) TS
(b) ST
(c) SS
(d) SU

76. [MCQ] [GATE-CE-2020: 2M]

In a school of 1000 students, 300 students play chess and 600 students play football. If 50 students play both chess and football, the number of students who play neither is____.

(a) 200 (b) 150

(c) 100 (d) 5

77. [MCQ] [GATE-CE-2019: 1M]

If E = 10; J = 20; O = 30; and T = 40, what will be P + E + S + T?

(a) 51 (b) 82

(c) 120 (d) 164

78. [MCQ] [GATE-CE-2019: 2M]

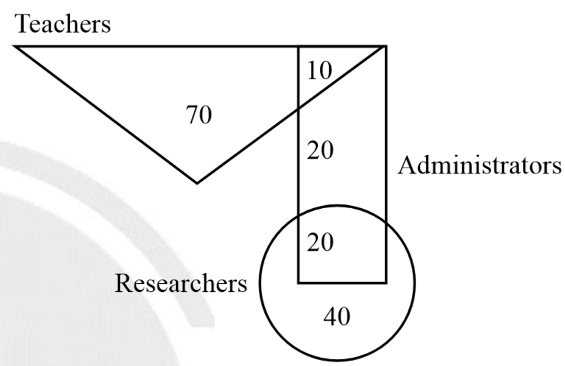
In a sports academy of 300 people, 105 play only cricket, 70 play only hockey, 50 play only football, 25 play both cricket and hockey, 15 play both hockey and football and 30 play both cricket and football. The rest of them play all three sports. What is the percentage of people who play at least two sports?

(a) 23.30 (b) 25.00

(c) 28.00 (d) 50.00

79. [MCQ] [GATE-CS-2019:2M]

In the given diagram, teachers are represented in the triangle, researchers in the circle and administrators in the rectangle. Out of the total number of the people, the percentage of administrators shall be in the range of _______.



(a) 0 to 15 (b) 16 to 30

(c) 31 to 45 (d) 46 to 60

80. [MCQ] [GATE-CS-2019:2M]

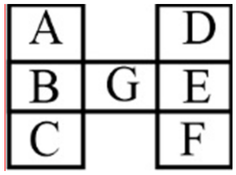
In a college. there are three student clubs. Sixty students are only in the Drama club. 80 students are only in the Dance club. 30 students are only in the Maths club. 40 students are in both Drama and Dance clubs. 12 students are in both Dance and Maths clubs. 7 students are in both Drama and Maths clubs, and 2 students are in all the clubs. If 75% of the students in the college are not in any of these clubs, then the total number of students in the college is _____.

(a) 1000 (b) 975

(c) 900 (d) 225

81. [MCQ] [GATE-CE-2018: 2M]

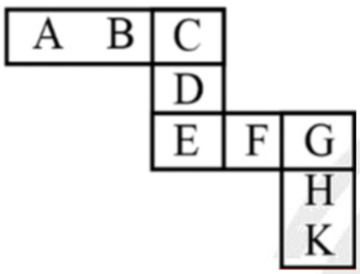
Each of the letters arranged as below represents a unique integer from 1 to 9. The letters are positioned in the figure such that (A × B × C), (B × G × E) and (D × E × F) are equal. Which integer among the following choices cannot be represented by the letters A, B, C, D, E, F or G?



(a) 4 (b) 5
(c) 6 (d) 9

82. [MCQ] [GATE-CE-2018: 2M]

Each of the letters in the figure below represents a unique integer from 1 to 9. The letters are positioned in the figure such that each of (A + B + C), (C + D + E), (E + F + G) and (G + H + K) is equal to 13. Which integer does E represent?



(a) 1
(b) 4
(c) 6
(d) 7

83. [MCQ] [GATE-ME-2018: 2M]

Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

(a) 0 (b) 2
(c) 4 (d) 8

84. [MCQ] [GATE-ME-2018: 2M]

Consider the following three statements:
(i) Some roses are red.
(ii) All red flowers fade quickly.
(iii) Some roses fade quickly.
Which of the following statements can be logically inferred from the above statements?

(a) If (i) is true and (ii) is false, then (iii) is false.
(b) If (i) is true and (ii) is false, then (iii) is true.
(c) If (i) and (ii) are true, then (iii) is true.
(d) If (i) and (ii) are false, then (iii) is false.

85. [MCQ] [GATE-EC-2017:1M]

500 students are taking one or more courses out of Chemistry, Physics, and Mathematics. Registration records indicate course enrolment as follows: Chemistry (329), Physics (186), Mathematics (295), Chemistry and Physics (83), Chemistry and Mathematics (217), and Physics and Mathematics (63). How many students are taking all 3 subjects?

(a) 37 (b) 43
(c) 47 (d) 53

86. [MCQ] [GATE-EC-2017:1M]

Some tables are shelves. Some shelves are chairs. All chairs are benches. Which of the following conclusions can be deduced from the preceding sentences?

i. At least one bench is a table
ii. At least one shelf is a bench
iii. At least one chair is a table
iv. All benches are chairs

(a) Only i
(b) Only ii
(c) Only ii and iii
(d) Only iv

87. [MCQ] [GATE-CE-2017: 1M]

Consider the following sentences:
All benches are beds. No bed is a bulb. Some bulbs are lamps.
Which of the following can be inferred?
i. Some beds are lamps.
ii. Some lamps are beds.

(a) Only i (b) Both i and ii
(c) Only ii (d) Neither i nor ii

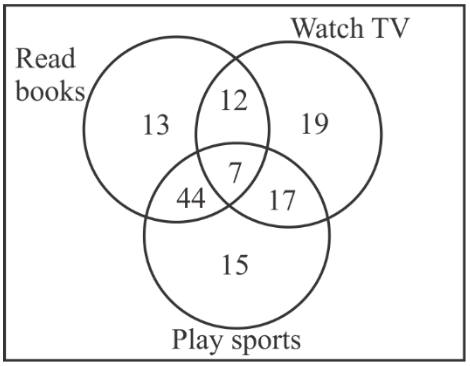
88. [MCQ] [GATE-CE-2017: 2M]

P. Q. R. S. T and U are seated around a circular- table. R is seated two places to the right of Q. P is seated three places to the left of R. S is seated opposite U. If P and U now switch seats, which of the following must necessarily be true?

(a) P is immediately to the right of R
(b) T is immediately to the left of P
(c) T is immediately to the left of P or P is immediately to the right of Q
(d) U is immediately to the right of R or P is immediately to the left of T

89. [MCQ] [GATE-ME, EC-2016: 2M]

The Venn diagram shows the preference of the student population for leisure activities.



From the data given, the number of students who like to read books or play sports is

(a) 44 (b) 51

(c) 79 (d) 108

90. [MCQ] [GATE-ME-2016: 1M]

Fact: If it rains, then the field is wet.

Read the following statements:

(i) It rains

(ii) The field is not wet

(iii) The field is wet

(iv) It did not rain

Which one of the options given below is NOT logically possible, based on the given fact?

(a) If (iii), then (iv) (b) If (i), then (iii).

(c) If (i), then (ii). (d) If (ii), then (iv).

91. [MCQ] [GATE-CS, EE-2016: 2M]

Among 150 faculty members in an institute, 55 are connected with each other through Facebook and 85 are connected through WhatsApp. 30 faculty members do not have Facebook or WhatsApp accounts. The number of faculty members connected only through Facebook accounts is.

(a) 35 (b) 45

(c) 65 (d) 90

92. [MCQ] [GATE-CE-2016: 2M]

Fact 1: Humans are mammals.

Fact 2: Some humans are engineers.

Fact 3: Engineers build houses.

If the above statements are facts, which of the following can be logically inferred?

I. All mammals build houses.

II. Engineers are mammals.

III. Some humans are not engineers.

(a) II only (b) III only

(c) I, II and III (d) I only

93. [MCQ] [GATE-CE-2015: 1M]

Mr. Vivek walks 6 meters North-east, then turns and walks 6 meters South-east, both at 60 degrees to east. He further moves 2 meters South and 4 meters West. What is the straight distance in metres between the point he started from and the point he finally reached?

(a) $2\sqrt{2}$ (b) 2

(c) $\sqrt{2}$ (d) $1\sqrt{2}$

94. [MCQ] [GATE-ME-2015: 2M]

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

I. No manager is a leader.

II. All leaders are executives.

Conclusions:

I. No manager is an executive.

II. No executive is a manager

(a) Only conclusion I follows.

(b) Only conclusion II follows.

(c) Neither conclusion I nor II follows.

(d) Both conclusions I and II follow.

95. [MCQ] [GATE-ME, EC-2015: 2M]

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

I. All film stars are playback singers

II. All film directors are film stars.

Conclusions:

I. All film directors arc playback singers.

II. Some film stars are film directors.

(a) Only conclusion I follows.

(b) Only conclusion II follows

(c) Neither conclusion I nor II follows.

(d) Both conclusions I and II follow.

96. [MCQ] [GATE-CE-2015: 2M]

There are 16 teachers who can teach Thermodynamics (TD), 11 who can teach Electrical Sciences (ES), and 5 who can teach both TD and Engineering Mechanics (EM). There are a total of 40 teachers. 6 cannot teach any of the three subjects, i.e. EM, ES or TD. 6 can teach only ES. 4 can teach all three subjects, i.e. EM, ES and TD. 4 can teach ES and TD. How many can teach both ES and EM but not TD?

(a) 1 (b) 2
(c) 3 (d) 4

97. [MCQ] [GATE-CE-2014: 2M]

Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

(a) ALRVX (b) EPVZB
(c) ITZDF (d) OYEIK

98. [MCQ] [GATE-CE-2014: 2M]

X is 1 km northeast of Y. Y is 1 km southeast of Z. W is 1 km west of Z. P is 1 km south of W. Q is 1 km east of P. What is the distance between X and Q in km?

(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) 2

99. [MCQ] [GATE-ME, EC-2014: 1M]

Read the statements:

All women are entrepreneurs.

Some women are doctors.

Which of the following conclusions can be logically inferred from the above statements?

(a) All women are doctors
(b) All doctors are entrepreneurs
(c) All entrepreneurs are women
(d) Some entrepreneurs are doctors

100. [MCQ] [GATE-ME,CE,EE,EC,CS-2010: 1M]

25 persons are in a room, 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is

(a) 2 (b) 17
(c) 13 (d) 3

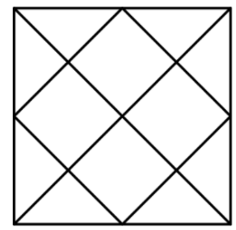
101. [NAT] [GATE-CE-2014: 2M]

10% of the population in a town is HIV^+. A new diagnostic kit for HIV detection is available; this kit correctly identifies HIV^+ individuals 95% of the time, and HIV^- individuals 89% of the time. A particular patient is tested using this kit and is found to be positive. The probability that the individual is actually positive is _______.

Numerical Relations

102. [MCQ] [GATE-EC-2023:1M]

How many rectangles are present in the given figure?

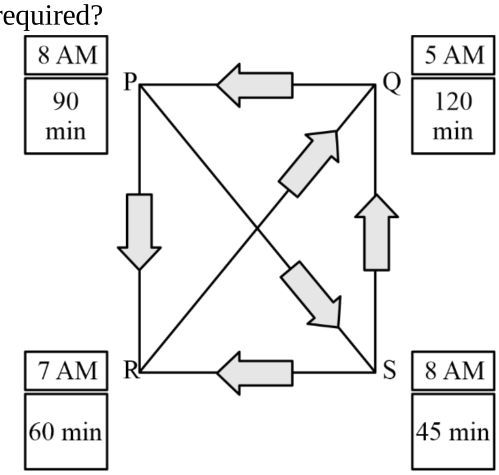


(a) 8 (b) 9
(c) 10 (d) 12

103. [MCQ] [GATE-ME-2022: 2M]

Four cities P, Q, R and S are connected through one-way routes as shown in the figure. The travel time between any two connected cities is one hour. The boxes beside each city name describe the starting time of first train of the day and their frequency of operation. For example, from city P, the first trains of the day start at 8 AM with a frequency of 90 minutes to each of R and S. A person does not spend additional time at any city other than the waiting time for the next connecting train.

If the person starts from R at 7 AM and is required to visit S and return to R, what is the minimum time required?



(a) 6 hours, 30 minutes
(b) 3 hours, 45 minutes
(c) 4 hours, 30 minutes
(d) 5 hours, 15 minutes

104. [MCQ] [GATE-ME-2021: 2M]

Consider a square sheet of side 1 unit. The sheet is first folded along the main diagonal. This is followed by a fold along its line of symmetry. The resulting folded shape is again folded along its line of symmetry. The area of each face of the final folded shape, in square units, equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(c) $\frac{1}{16}$ (d) $\frac{1}{32}$

105. [MCQ] [GATE-ME-2021: 1M]

The front door of Mr. X's house faces East. Mr. X leaves the house, walking 50 m straight from the back door that is situated directly opposite to the front door. He then turns to his right, walks for another 50 m and stops. The direction of the point Mr. X is now located at with respect to the starting point is

(a) South-East (b) North-East
(c) West (d) North-West

106. [MCQ] [GATE-ME-2021: 1M]

If $\oplus \div \odot = 2$; $\oplus \div \Delta = 3$; $\odot + \Delta = 5$; $\Delta \times \otimes = 10$, then the value of $(\otimes - \oplus)^2$, is :

(a) 0 (b) 1
(c) 4 (d) 16

107. [MCQ] [GATE-ME-2021: 1M]

If $\begin{cases} "\oplus" \mathit{means} "-", \\ "\otimes" \mathit{means} "\div", \\ "\Delta" \mathit{means} "+", \\ "\nabla" \mathit{means} "\times", \end{cases}$

then, the value of the expression $\Delta 2 \oplus 3 \Delta ((4 \otimes 2) \nabla 4) =$

(a) –1 (b) –0.5
(c) 6 (d) 7

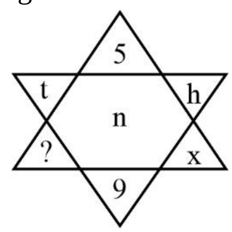
108. [MCQ] [GATE-CS-2020: 2M]

If values of P = 3, R = 27, T = 243, then Q + S = ___________.

(a) 40 (b) 80
(c) 90 (d) 110

109. [MCQ] [GATE-ME-2020: 2M]

Find the missing element in the following figure



(a) d (b) e
(c) w (d) y

110. [MCQ] [GATE-ME-2020: 2M]

P, Q, R and S are to be uniquely coded using α and β. If P is coded as αα and Q as αβ, then R ans S, respectively, can be coded as

(a) βα and αβ (b) ββ and αα
(c) αβ and ββ (d) βα and ββ

111. [MCQ] [GATE-EE-2020 : 2M]

Select the next element of the series: Z, WV, RQP, __________.

(a) LKJI (b) JIHG
(c) KJIH (d) NMLK

112. [MCQ] [GATE-ME-2019: 1M]

If IMHO = JNIP; IDK = JEL; and SO = TP, then IDC = ____ .

(a) JDE (b) JED
(c) JDC (d) JCD

113. [MCQ] [GATE-EE-2019 : 2M]

Consider five people – Mita, Ganga, Rekha, Lakshmi and Sana. Ganga is taller than both Rekha and Lakshmi. Lakshmi is taller than Sana. Mita is taller than Ganga.

Which of the following conclusions are true?

1. Lakshmi is taller than Rekha
2. Rekha is shorter than Mita
3. Rekha is taller than Sana
4. Sana is shorter than Ganga

(a) 1 and 3 (b) 3 only
(c) 2 and 4 (d) 1 only

114. [MCQ] [GATE-EE-2019 : 1M]

The missing number in the given sequence 343. 1331,_______, 4913 is

(a) 3375 (b) 2744
(c) 2197 (d) 4096

115. [MCQ] [GATE-EE-2018 : 2M]

In a certain code, AMCF is written as EQGJ and NKUF is written as ROYJ. How will DHLP be written in that code?

(a) RSTN (b) TLPH
(c) HLPT (d) XSVR

116. [MCQ] [GATE-ME-2018: 2M]

A house has a number which needs to be identified. The following three statements are given that can help in identifying the house number.

I. If the house number is a multiple of 3, then it is a number from 50 to 59.

II. If the house number is NOT a multiple of 4, then it is a number from 60 to 69.

III. If the house number is NOT a multiple of 6, then it is a number from 70 to 79.

What is the house number?

(a) 54 (b) 65
(c) 66 (d) 76

117. [MCQ] [GATE-ME-2018: 1M]

Find the missing group of letters in the following series:

BC, FGH, LMNO, _____

(a) UVWXY (b) TUVWX
(c) STUVW (d) RSTUV

118. [MCQ] [GATE-CS-2018: 1M]

What is the missing number in the following sequence?

2, 12, 60, 240, 720, 1440, _____, 0

(a) 2880

(b) 1440

(c) 720

(d) 0

119. [MCQ] [GATE-CS, CE-2016: 1M]

A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is ___________.

(a) 56 (b) 64
(c) 72 (d) 96

120. [MCQ] [GATE-CS, CE-2016: 2M]

Consider the following statements relating to the level of poker play of four players P, Q, R and S.

I. P always beats Q
II. R always beats S
III. S loses to P only sometimes
IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

(i) P is likely to beat all the three other players
(ii) S is the absolute worst player in the set

(a) (i) only (b) (ii) only
(c) (i) and (ii) (d) neither (i) nor (ii)

121. [MCQ] [GATE-EE-2016 : 1M]

Pick the odd one out in the following:

13, 23, 33, 43, 53

(a) 23 (b) 33
(c) 43 (d) 53

122. [MCQ] [GATE-CS, EE-2016 : 1M]

Pick the odd one from the following options.

(a) CADBE (b) JHKIL
(c) XVYWZ (d) ONPMQ

123. [MCQ] [GATE-EC-2016:2M]

A flat is shared by four first year undergraduate students. They agreed to allow the oldest of them to enjoy some extra space in the flat. Manu is two months older than Sravan, who is three months younger than Trideep. Pavan is one month older than Sravan. Who should occupy the extra space in the flat?

(a) Manu (b) Sravan
(c) Trideep (d) Pavan

124. [**MCQ**] **[GATE-EC-2016:1M]**

The number that least fits this set: (324, 441, 97 and 64) is __________.

(a) 324 (b) 441

(c) 97 (d) 64

125. [MCQ] **[GATE-ME, EC-2016: 2M]**

Two and a quarter hour back, when seen in a mirror, the reflection of a wall clock without number markings seemed to show 1:30. What is the actual current time shown by the clock?

(a) 8:15 (b) 11:15

(c) 12:15 (d) 12:45

126. [MCQ] **[GATE-ME-2016: 2M]**

Find the missing sequence in the letter series.

B, FH, LNP, _ _ _ _.

(a) SUWY (b) TUVW

(c) TVXZ (d) TWXZ

127. [MCQ] **[GATE-ME, EC-2016: 1M]**

Michael lives 10 km away from where I live. Ahmed lives 5 km away and Susan lives 7 km away from where I live. Arun is farther away than Ahmed but closer than Susan from where I live. From the information provided here, what is one possible distance (in km) at which I live from Arun's place?

(a) 3.00 (b) 4.99

(c) 6.02 (d) 7.01

128. [MCQ] **[GATE-CS, CE-2015: 1M]**

If ROAD is written as URDG. then SWAN should be written as:

(a) VXDQ

(b) VZDQ

(c) VZDP

(d) UXDQ

129. [MCQ] **[GATE-EE, CS-2015 : 1M]**

Based on the given statements, select the most appropriate option to solve the given question.

If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements I : Each step is 3/4 foot high

Statements II : Each step is 1 foot wide

(a) Statement I alone is sufficient, but statement II alone is not sufficient

(b) Statement II alone is sufficient, but statement I alone is not sufficient.

(c) Both statements together are sufficient, but neither statement alone is sufficient

(d) Statement I and II together are not sufficient

130. [MCQ] **[GATE-EE, CS-2015 : 1M]**

Based on the given statements, select the most appropriate option to solve the given question.

What will be the total weight of 10 poles each of same weight?

Statements:

(I) One fourth of the weight of a pole is 5 Kg.

(II) The total weight of these poles is 160 kg more than the total weight of two poles.

(a) Statement I alone is not sufficient

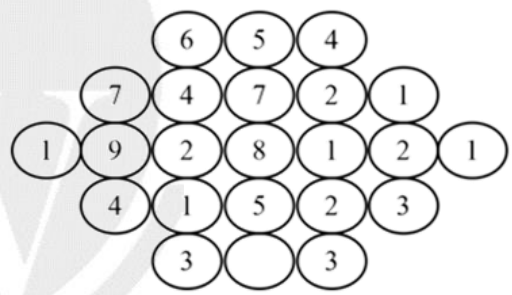
(b) Statement II alone is not sufficient

(c) Either I or II alone is sufficient

(d) Both statements I and II together are not sufficient.

131. [**NAT**] **[GATE-EC-2015:2M]**

Fill in the missing value



132. [**MCQ**] **[GATE-EC-2015:1M]**

Operators □, ♦ and → are defined by: a □ b = $\frac{a-b}{a+b}$; a ♦ b = $\frac{a+b}{a-b}$; a → b = ab.

Find the value of (66 □ 6) → (66 ♦ 6).

Find the value of (66 □ 6) → (66 ♦ 6).

(a) –2 (b) –1

(c) 1 (d) 2

133. [MCQ] **[GATE-ME, EC-2015: 1M]**

Find the missing sequence in the letter series below:

A, CD, GHI,? UVWXY

(a) LMN (b) MNO

(c) MNOP (d) NOPQ

134. [MCQ] [GATE-EE, CS-2014 : 2M]

Which number does not belong in the series below?
2, 5, 10, 17, 26, 37, 50, 64

(a) 17 (b) 37
(c) 64 (d) 26

135. [MCQ] [GATE-ME, EC-2014: 2M]

Find the next term in the sequence: 13M, 17Q, 19S, ___

(a) 21W (b) 21V
(c) 23W (d) 23V

136. [MCQ] [GATE-ME, EC-2014: 2M]

Find the next term in the sequence: 7G, 11K, 13M, ________

(a) 15Q (b) 17Q
(c) 15P (d) 17P

137. [NAT] [GATE-ME, EC-2014: 1M]

The next term in the series 81, 54, 36, 24, … is ________

138. [MCQ] [GATE-ME, EC-2014: 2M]

Find the odd one from the following group:

W,E,K,O I,Q,W,A F,N,T,X N,V,B,D

(a) W,E,K,O (b) I,Q,W,A
(c) F,N,T,X (d) N,V,B,D

139. [NAT] [GATE-ME, EC-2014: 1M]

Fill in the missing number in the series.

2 3 6 15 _____ 157.5 630

140. [NAT] [GATE-ME, EC-2014: 1M]

What is the next number in the series?

12 35 81 173 357 _____

141. [MCQ] [GATE-ME, CE, CS-2012: 2M]

Given the sequence of terms, AD CG FK JP, the next term is

(a) OV (b) OW
(c) PV (d) PW

142. [MCQ] [GATE-ME,CE,EE,EC,CS-2010: 2M]

Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e., brothers and sisters). All were born on 1st January. The age difference between any two successsive siblings (that is born one after another) is less than 3 years.

Given the following facts:

(i) Hari's age + Gita's age > Irfan's age + Saira's age

(ii) The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.

(iii) There are no twins.

In what order were they born (oldest first)?

(a) HSIG (b) SGHI
(c) IGSH (d) IHSG

❑❑❑

ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(a)
5.	(c)	**6.**	(d)	**7.**	(b)	**8.**	(b)
9.	(b)	**10.**	(c)	**11.**	(b)	**12.**	(c)
13.	(a)	**14.**	(a)	**15.**	(c)	**16.**	(c)
17.	(c)	**18.**	(b)	**19.**	(d)	**20.**	(a)
21.	(c)	**22.**	(c)	**23.**	(a)	**24.**	(d)
25.	(c)	**26.**	(c)	**27.**	(b)	**28.**	(d)
29.	(d)	**30.**	(b)	**31.**	(c)	**32.**	(d)
33.	(d)	**34.**	(a)	**35.**	(d)	**36.**	(b)
37.	(a)	**38.**	(c)	**39.**	(a)	**40.**	(a)
41.	(b)	**42**	(d)	**43.**	(c)	**44.**	(c)
45.	(d)	**46.**	(b)	**47.**	(c)	**48.**	(d)
49.	(c)	**50**	(b)	**51.**	(c)	**52.**	(a)
53.	(b)	**54.**	(b)	**55.**	(d)	**56.**	(6 to 6)
57.	(b)	**58.**	(b)	**59.**	(a)	**60.**	(d)
61.	(a)	**62.**	(a)	**63.**	(a)	**64.**	(d)
65.	(d)	**66.**	(a)	**67.**	(b)	**68.**	(d)
69.	(b)	**70.**	(c)	**71.**	(c)	**72.**	(c)
73.	(b)	**74.**	(d)	**75.**	(b)	**76.**	(b)
77.	(c)	**78.**	(b)	**79.**	(c)	**80.**	(c)
81.	(b)	**82.**	(b)	**83.**	(c)	**84.**	(c)
85.	(d)	**86.**	(b)	**87**	(d)	**88.**	(c)
89.	(d)	**90.**	(c)	**91.**	(a)	**92.**	(b)
93.	(a)	**94.**	(c)	**95.**	(d)	**96.**	(a)
97.	(d)	**98.**	(c)	**99.**	(d)	**100.**	(d)
101.	(0.4896 to 0.04896)	**102.**	(c)	**103.**	(a)	**104.**	(b)
105.	(d)	**106.**	(b)	**107.**	(d)	**108.**	(c)
109.	(a)	**110.**	(d)	**111.**	(c)	**112.**	(b)
113.	(c)	**114.**	(c)	**115.**	(c)	**116.**	(d)
117.	(b)	**118.**	(b)	**119.**	(d)	**120.**	(d)
121.	(b)	**122.**	(d)	**123.**	(c)	**124.**	(c, d)
125.	(d)	**126.**	(c)	**127.**	(c)	**128.**	(b)
129.	(a)	**130.**	(c)	**131.**	(3 to 3)	**132.**	(c)
133.	(c)	**134.**	(c)	**135.**	(c)	**136.**	(b)
137.	(16 to 16)	**138.**	(d)	**139.**	(45 to 45)	**140.**	(725 to 725)
141.	(a)	**142.**	(b)				

SOLUTIONS

1. (b)

The box obtained after filling the symbols is.

□	*	○	△
△	○	*	□
○	△	□	*
*	□	△	○

Hence the symbol in place of ? is *. Option (B) is correct.

Scan for Video solution

2. (c)

Number of creases after 1st folding = 1

Number of creases after 2nd folding = 2^1

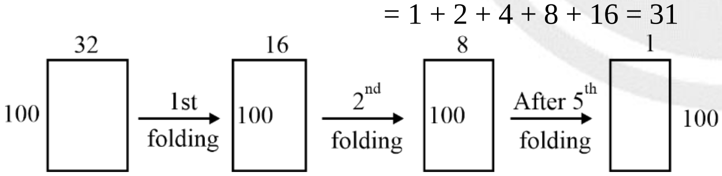
Number of creases after 3rd folding = 2^2

Number of creases after 4th folding = 2^3

Number of creases after 5th folding = 2^4

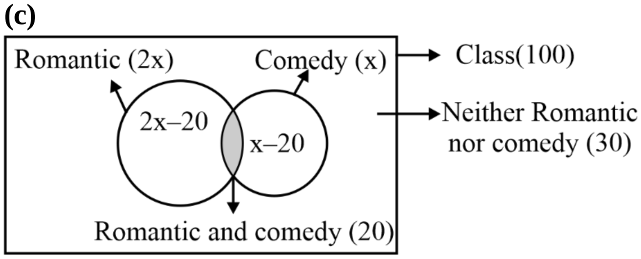
Total number of creases after unfolding

= 1 + 2 + 4 + 8 + 16 = 31



Scan for Video solution

3. (c)



n (neither romantic nor comedy)

= total students – n (Romantic ∪ comedy)

$30 = 100 - (2x + x - 20) \Rightarrow 3x - 20 = 70$

$x = \frac{90}{3} = 30$

n(romantic) = 2x = 2 × 30 = 60

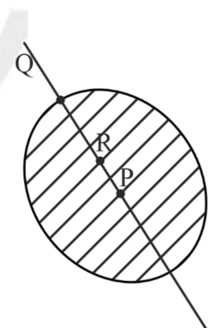
Scan for Video solution

4. (a)

There are balls of three different colours. In worst case the three picked ball are of different colours. But as the fourth ball is picked that has to be one of these three colours so, out of four balls there are definitely two balls of same colour. Hence, the minimum number of N is 4.

Scan for Video solution

5. (c)



For rotational motion, the speed is given by $v = r\omega$

Where, r is the distance of the point from the axis of rotation and ω is the angular speed which is same for all the points here hence,

$v \propto r$

$\because r_P > r_R > r_Q$

$\therefore v_P > v_R > v_Q$

Scan for Video solution

6. **(d)**

If Donald Duck statements is true then he is also lying which contradicts the statement. So, Donald duck's statement is wrong.

Scan for Video solution

7. **(b)**

To construct 5 letter pelindrom out of 5 alphabets, first alphabet should be same as 5^{th} one and 2^{nd} alphabet should be same as 4^{th} one while 3^{rd} alphabet should be unique.

In options (A), (C) & (D) this condition can not be fulfilled. For option (B), the possible pelindromes are:

R A D A R

A R D R A

Scan for Video solution

8. **(b)**

From the given data, the languages in descending difficulty order is given below.

From the given data, the language in descending order of difficulty is given below.

Chinese > Japanese > Dutch > French > Spanish

Since girl Q is studying Chinese and Japanese hence Q is learning the most difficulty pair of languages.

Scan for Video solution

9. **(b)**

It is given that S never lier and S is saying **"only one of us is telling the truth"** implies all the other's statements of P, Q, R & T are not true. So, we can say

(i) R has not copied in the exam

(ii) S has not copied in the exam

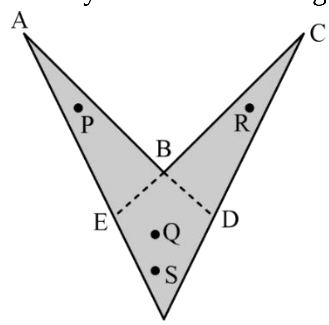
(iii) P did copy in the exam

(iv) R is not telling the truth

Scan for Video solution

10. **(c)**

As the walls are opaque, the security guard from position P and R, cannot see the region CDB and ABE respectively as shown in the figure



From position Q and S, the whole gallery can be watched.

Scan for Video solution

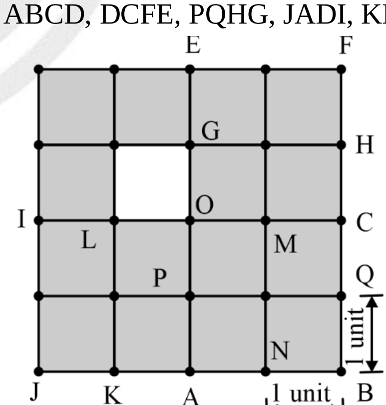
11. **(b)**

Number of squares without hole of size (1 × 1)

= 4 × 4 – 1 = 15

Number of squares without hole of size (2 × 2)

= ABCD, DCFE, PQHG, JADI, KNML= 5



∴ Total number of squares without hole

= 15 + 5 = 20

Scan for Video solution

12. (c)

The ant starts at the position P, and move up to top-right corner of the grid, the only possible way in the option is given in option (c).

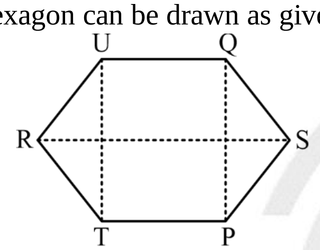
It is because in option (a) and (b), starting position of the ant is not P so that they both eliminated and in option (d) final destination is not given,

$\therefore$ option (c) is correct.

Scan for Video solution

13. (a)

As per the given statement in the question, a regular convex hexagon can be drawn as given below,



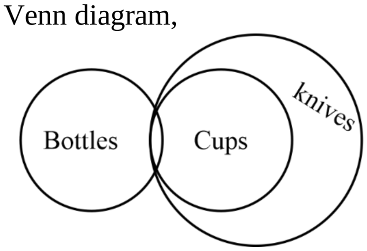
From the above figure, we can see that the line segment joining R and T is parallel to the line segment joining Q and S. And other given options is not satisfied.

$\therefore$ option (a) is correct.

Scan for Video solution

14. (a)

Given two statements can be represented in the given Venn diagram,



From the Venn diagram, only conclusion (I) and conclusion (II) follows.

$\therefore$ option (a) is correct

Scan for Video solution

15. (c)

$$T^{+} \sim P^{-} \cdot Q^{+} \sim \underset{\substack{\downarrow \\ s}}{R^{-}}$$

. $\rightarrow$ Sibling   + $\rightarrow$ male

– $\rightarrow$ female   ~ $\rightarrow$ couple

$\therefore$ T is an uncle of s.

Scan for Video solution

16. (c)

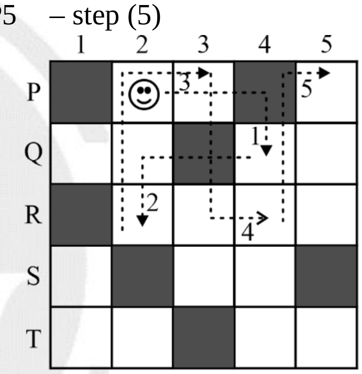
P2 to Q4 – step (1)

Q4 to R2 – step (2)

R2 to P3 – step (3)

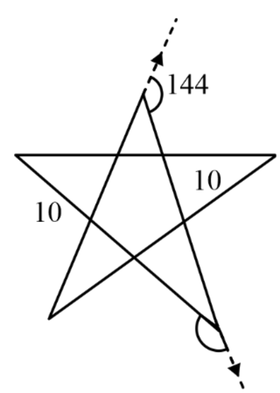
P3 to R4 – step (4)

R4 to P5 – step (5)



Scan for Video solution

17. (c)

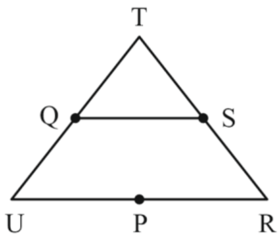


After turning 72° anticlockwise at P the ant will walk 10 cm along PS. At S it takes a 144° clockwise turn and then move along SQ and it repeat these two operations 5 times before halting.

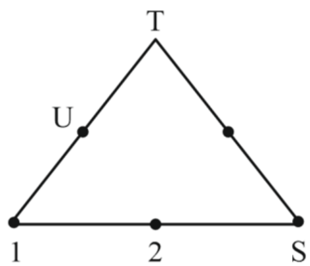
Scan for Video solution

18. (b)

From the given three conditions, the possible triangle is drawn below.



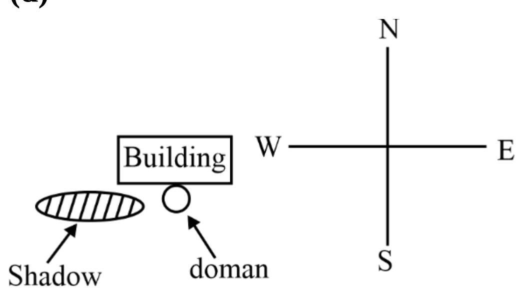
If we have to put S at a corner then according to the condition III, the possible triangle will be.



Since P is at the side opposite to corner T so, P can possibly at 1 or 2 but this will not satisfy condition (I), hence S can not be at the corner.

Scan for Video solution

19. (d)



So, the building is facing south.

Scan for Video solution

20. (a)

From the given information, the following pattern can be written,

(i) PTUQSVR

(ii) TUQSVRP

(iii) QSVRTUP

(iv) QSVRPTU

From the above pattern, it is clear that, only one car is parked between Q and V.

So, option 'a' is correct.

Scan for Video solution

21. (c)

Given, P & R cannot sit adjacent to each other and S should be seated to the right side of Q

Hence, PQRS, PQSR, RQPS, RQSP, QRSP, QPRS

To separate R & S,

Q first and S at third place = 2 ways

Q second and S at third place = 2 ways

Q second and S at fourth place = 2 ways.

Total = 6 ways.

Scan for Video solution

22. (c)

From given information the best allocation of faculties is in option c.

P $\rightarrow$ near lab

Q $\rightarrow$ near lift

R $\rightarrow$ near play ground

S $\rightarrow$ at the corner

Scan for Video solution

23. (a)

As per given information

Condition 1: P, W, & T refused to sleep next to R.

By applying condition 1 only option (a) is satisfying

Condition 2: P and S avoid Q.

Again option (a) satisfied condition 2 as well.

The arrangement of sleeping is RSPTQ.

Option (a) RSPTQ

Scan for Video solution

24. (d)

∵ Y is the child of R & W

∴ W cannot be the wife of P. So, option (D) is false other three options are possibilities which can be true.

Scan for Video solution

25. (c)

If the prisoner gave the statement "I will be shot" then considering it to be true, the person should be hanged which is contradictory and considering it to be false the person should be shot which is again contradictory. So, the judge set him free.

Scan for Video solution

26. (c)

It is given that currently P and Q are having same office and R and T also having same office.

So, option (c) is correct.

Scan for Video solution

27. (b)

Out of (P, Q, R, S, T), S and R are fixed at first & second places from right, so the remaining P, Q, T can be arranged as ${}^3P_3 = 3! = 6$ ways

Scan for Video solution

28. (d)

Concluded: Arrive at a judgement or opinion by reasoning

Looked: Think of or regard in a specified way

Notice: The fact of observing or paying attention to something.

Pictured: Form a mental image of someone or something

Scan for Video solution

29. (d)

There are two people between Lohit and Engineer

↓ Facing towards

↑ ↑ ↑ ↑

Engineer Lohit

Seema turns to her right to speak to doctor

↑ ↑ ↑ ↑

Engineer Seema Lohit / Doctor

Mathew is not standing next to seema

↑ ↑ ↑ ↑

Mathew / Engineer Seema Lohit / Doctor

So, the Rahul and seema both can be either teacher or dancer as teacher and dancer are next to each other.

Scan for Video solution

30. (b)

Since, R and T are the children of P and S respectively. So, S can be either father or mother of T hence, statement in option B is False.

Scan for Video solution

31. (c)

If P's statement is correct then all other statements are wrong and so R's statements is also wrong which contradicts the P's statement.

Similaly, if we consider Q's and R's statements correct once at a time then there will be contradictions, but if we consider S's statements correct then there will not be any contradiction and R will the criminal.

Scan for Video solution

32. (d)

Options B is wrong as X's condition is not satisfied

Options A is wrong as Z's conditions is not satisfied.

Options C is wrong as Z's condition is not satisfied.

Options D satisfies all the conditions of X, Y, Z.

Scan for Video solution

33. (d)

P says "Both of us are knights"

Q says "None of us are knave"

If P and Q are knights then this condition supports both the statements.

If P & Q are knave then this condition also supports both the statements.

So, no clear indication for P & Q.

Scan for Video solution

34. (a)

If S has 0 cars, then only Q is right.

If S has 1 car, then both Q & R are right.

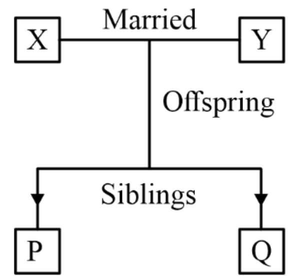
If S has 3 cars, then both P & R are right.

Since only one of P, Q and R can be right so S has 0 cars.

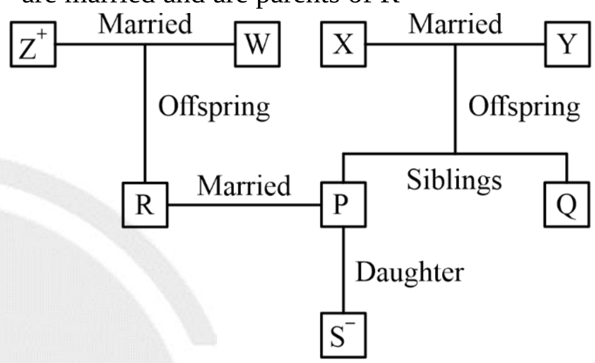
Scan for Video solution

35. (d)

X and Y are married and have two children P and Q



Z is grandfather of the daughters S of P and Z and W are married and are parents of R



So, P and R are a couple and since everyone is married at most once, Q cannot be married to R.

Scan for Video solution

36. (b)

Given: Age/drink for all the persons are

$P \rightarrow 16$ /?, $Q \rightarrow 25$ /?

$R \rightarrow$? / milkshake $S \rightarrow$?/ beer

Rule for beer is age should be over 18 years.

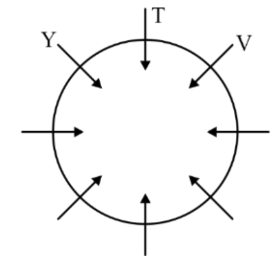
So, we need to check P's drink and S's age to follow the rule.

Scan for Video solution

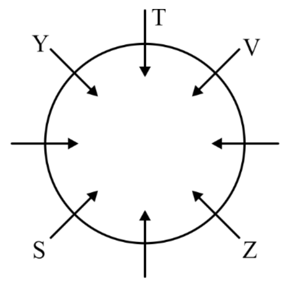
37. (a)

Given:

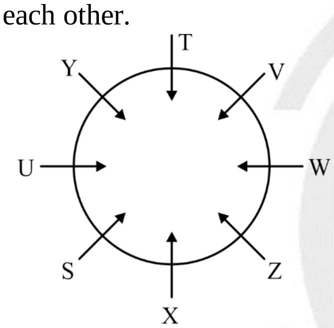
T's neighbours are Y and V



Z is seated third to the left of T and second to the right of S



U's neighbours are S and Y: T and W are not seated opposite each other.

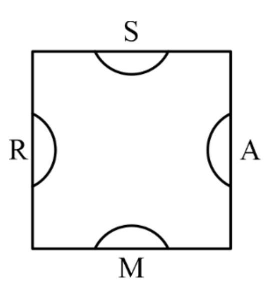


∴ The person third to the left of V is X.

Scan for Video solution

38. (c)

The given information can be drawn in a figure given below,



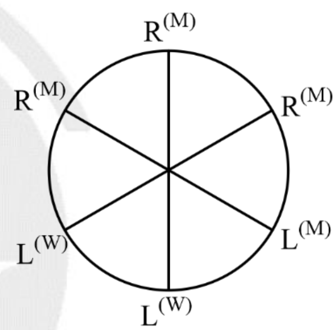
So, option (c) is correct.

Scan for Video solution

39. (a)

As clearly mentioned that there are at least two men and two women, and given that none of the woman is right handed. It means two women is left handed. Also given that there are 3 right handed person. It means, there are at least 3 men because no woman can be right handed.

Now every woman has a left-handed person to her immediate right. It means both women are sitting next to each other and third person sitting right to 2nd woman is also a left-handed person, which cannot be a woman because if it is a woman then next person will be a left-handed person. But there are at least 3 right handed person. So that there are 4 men and 2 women.

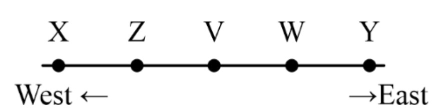


So, option 'A' is correct.

Scan for Video solution

40. (a)

The given information can be drawn in a figure given below,



We can see in the above figure that the building 'V' is in the middle

So, option 'A' is correct.

Scan for Video solution

41. (b)

Given Data:

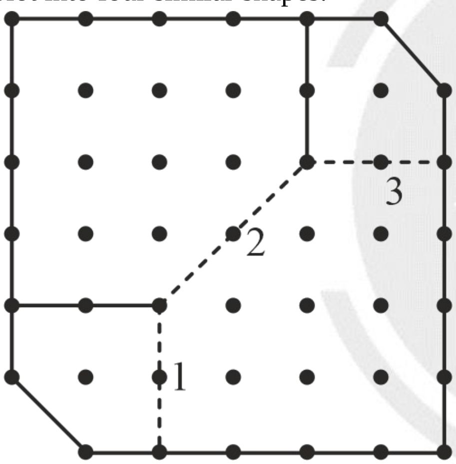
In this case, you will choose the box which was labeled as apples and oranges. It is because, all are labeled as wrong so that this box must be a single fruit box, i.e. apples box or oranges box. By this way one box can be identified, now other two can be identified easily because other two also wrongly labeled in which one has single fruit and other has two fruits.

So, option 'b' is correct.

Scan for Video solution

42. (d)

We can use minimum of 3 additional ropes to divide the plot into four similar shapes.



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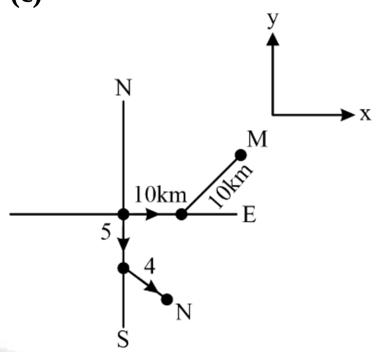
43. (c)

Card with number 2 should be turned to check if the colour is red or blue. If the colour is red then proposition is true. Now a blue-coloured card should be turned to check if the number is even of odd. If the number is odd then the proposition is true.

$p \Rightarrow q$	Direct	for '$\Rightarrow$'
$p \Rightarrow q$	Converse	Direct & contrapositive
$rp \Rightarrow rq$	Inverse	are true
$rq \Rightarrow rp$	Contrapositive	Boy $\Rightarrow$Human being

Scan for Video solution

44. (c)



The position of M is $10+10\cos 45^{\circ}$ in x-direction and $10\sin 45^{\circ}$ in y-direction i.e. (17.07, 7.07).

Similarly, the position of N is (2.83, –7.83)

The shortest distance

$$= \sqrt{(17.07-2.83)^2 + (7.07-(-7.83))^2}$$

$$= \sqrt{202.78+222.01}$$

= 20.61 km

Scan for Video solution

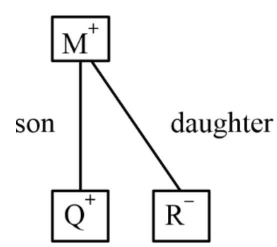
45. (d)

From the passage it is clear that Shiva is oldest and Pavithra is youngest. Hence option D is correct.

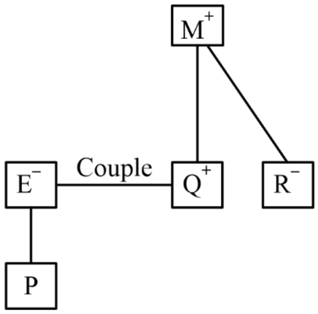
Scan for Video solution

46. (b)

M has a son Q and a daughter R and he has no other children



E is the mother of P and daughter-in law of M



As gender of P is not known, we can say P is the grandchild of M.

Scan for Video solution

47. (c)

To form a rectangle, we require 2 horizontal and two vertical lines. Here we need to select, 2 out of 5 vertical and 2 out of 3 horizontal lines.

Therefore, Number of rectangles = ${}^5C_2 \times {}^3C_2 = 30$

So, option 'C' is correct.

Scan for Video solution

48. (d)

Ooty is a hill-station or not which is not mentioned here so that statement (i) is wrong.

Also, it is not mentioned that no hill-station can have more than one lake, so that statement (ii) is wrong.

So, option 'd' is correct.

Scan for Video solution

49. (c)

From the observation of the given words, it is clear that 'taga' refers to 'care'.

Fertaga $\rightarrow$ careless

Tagafer $\rightarrow$ lesscare

So, the correct word for 'aftercare' is 'tagazen'.

Scan for Video solution

50. (b)

From the two statements it is clear that cliff is oldest and Eric is youngest. So, statement three is wrong as Eric is younger than cliff.

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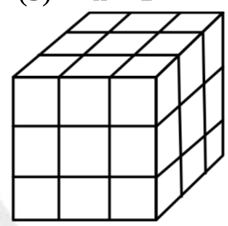
51. (c)

$$\begin{Bmatrix}\text{Volume of}\\ \text{unit 3 cube}\end{Bmatrix} = \begin{Bmatrix}\text{number of}\\ \text{unit 1 cube}\end{Bmatrix} \times \begin{Bmatrix}\text{volume of}\\ \text{unit 1 cube}\end{Bmatrix}$$

$(3)^3 = n \times 1^3 \qquad \Rightarrow n = 27$



Total number of faces of unit 1 cube = 6 × 27 = 162

Number of faces of larger cube which are visible = 6

Each face of unit 3 cube = 9 faces of unit 1 cube

So, number of faces of unit 1 which are visible

= 9 × 6 = 54

Hence number of faces of unit 1 which are not visible

= 162 – 54 = 108

Required ratio = 54 : 108 = 1 : 2

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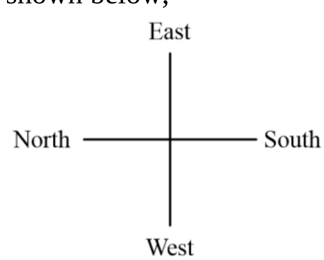
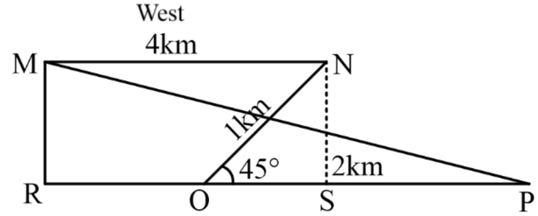
52. (a)

The given information can be drawn into a figure as shown below,





In the ΔNOS,

$$\sin 45° = \frac{OS}{1} \Rightarrow OS = \frac{1}{\sqrt{2}} km$$

$$PS = OP - OS = \left(2 - \frac{1}{\sqrt{2}}\right) km$$

$$PR = MN + PS = 4 + \left(2 - \frac{1}{\sqrt{2}}\right) = \left(6 - \frac{1}{\sqrt{2}}\right) km$$

And, $MR = NS = OS = \left(\frac{1}{\sqrt{2}}\right) km$

$$MP = \sqrt{PR^2 + MR^2}$$

$$MP = \sqrt{\left(6 - \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 5.34 km$$

So, option 'A' is correct.

Scan for Video solution

53. (b)

According to the given conditions, if S gets any portfolio, then U does not want any portfolio so, option C is wrong. R wants either Home or Finance or no portfolio so, options A and D is also wrong. If S gets either Power or Telecom then Q wants other one. All these conditions are satisfied by option B.

Scan for Video solution

54. (b)

BEST OF LUCK

$\rightarrow$ BSTFLCK

$\rightarrow$ KCLFTSB

GOOD WISHES $\rightarrow$ GDWSHS

SHSWDG

ACE THE EXAM $\rightarrow$ CTHXM

MXHTC

Scan for Video solution

55. (d)

Since red light is hung with blue and green light is hung with yellow hence, the number of red lights = number of blue lights the number of green lights = number of yellow lights and the sum of red and green lights = sum of blue and yellow lights.

Scan for Video solution

56. (6 to 6)

As given 4 corner points of a tetrahedron forms four planes, Now, if we add a point which increase the count to 5. We know that more than 2 points are required to form a plane. To make maximum number of planes, we require minimum 3 points to form a plane. So that total number of planes will be ${}^5C_3 = 10$ planes.

∴ New internal planes = 10 – 4 = 6

Scan for Video solution

57. (b)

All the students, who are participating have to come at 9:00 am with their parents, but the other students and their parents should come at 10:00 am, therefore option (b) is correct.

Scan for Video solution

58. (B)

Since, Eswar does not live on a floor immediately above or immediately below Bhola so, options A, C & D are eliminated. All other conditions are fulfilled in option B is correct.

Scan for Video solution

59. (a)

If only statement I is provided then we can confidently say that Riaz is eldest but we cannot determine between Shiv & Som that who is elder.

So, statement I is sufficient to determine the eldest of four.

Riaz > Shiv > Som > Ansu.

Scan for Video solution

60. (d)

Abhishek > Savar

Anshul > Savar

From these two information, we can decide between Abhishek and Anshul that who is elder.

Scan for Video solution

61. (a)

Case – 1

Take any 6 bags and put 3, 3 bags on each side of weighting balance. If both are equal then put rest two bags each on both side, this will indicate heavier bag. So, only two weighing's are required.

Case – 2

Take any 6 bags and put 3, 3 bags on each side of weighting balance. If both are not equal then rest two are equal of weight then no need to compare them, now out of three bags which was heavier in 1^{st} weighting, put 2 bags on each side, if they are equal then remaining 3^{rd} will be heavier or if they are not equal then one will be heavier. So, only two weighing's are required.

So, option 'A' is correct.

Scan for Video solution

62. (a)

(a) $\rightarrow f(x) = x^2 2^{-|x|} \Rightarrow f(-x) = (-x)^2\, 2^{-|-x|} = x^2\, 2^{-|x|} = f(x)$

$\rightarrow$ **even**

(b) $\rightarrow f(x) = x\, 2^{-|x} \Rightarrow f(-x) = -x\, 2^{-|-x|} = -x\, 2^{-|x|} = -f(x)$

$\rightarrow$ **Not even**

(c) $\rightarrow f(x) = |x|\, 2^{-x} \Rightarrow f(-x) = |-x| 2^{-(-x)} = |x| 2^{x} \neq f(x)$

$\rightarrow$ **Not even**

(d) $\rightarrow f(x) = x\, 2^{-x} \Rightarrow f(-x) = (-x)\, 2^{-(-x)} = -x\, 2^{x} \neq f(x)$

$\rightarrow$ **Not even**

so, as the given graph is an even function, hence option (a) is correct.

Scan for Video solution

63. (a)

Conclusion I is wrong as some engineers can be professors.

Conclusion II is also wrong because it is not certain that can some engineers be professors are not.

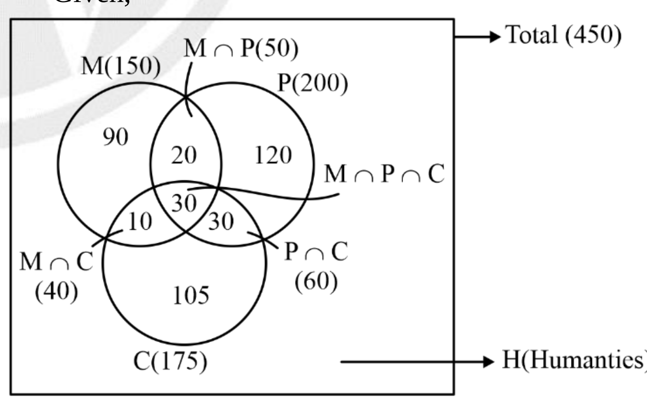
Conclusion III is correct because No male is professors so also no male can be teacher.

So, option A is correct.

Scan for Video solution

64. (d)

Given,



$n(H) = \text{Total} - n(M \cup P \cup C)$

$= 450 - (150+120+30+105) = 450 - 405$

$n(H) = 45$

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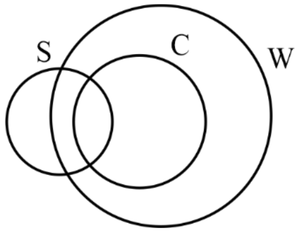
65. (d)

Some
- → At least one (≥ 1)
- → Not none
- → Some may be all also.



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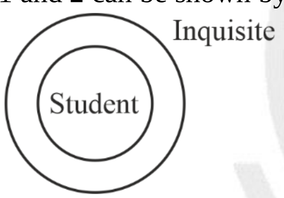
66. (a)

Statements 'All students are inquisitee' and 'No student is inquisitee' cannot be true simultaneously. Statements 1 and 2 can be shown by single diagram.



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67. (b)

Given data:

$$p \text{ r } q = \frac{p^2+q^2}{pq},\ p \text{ u } q = \frac{p^2}{q}$$

$\$\ x \oplus y = 2 \odot 2$

By observing given data we can write

$$x \text{ r } y = \frac{x^2+y^2}{xy}\ \$\ 2 \odot 2 = \frac{2^2}{2} = 2$$

Now putting these values in given x, y equation:

$$\frac{x^2+y^2}{xy} = 2 \Rightarrow x^2 + y^2 - 2xy$$

$$(x-y)^2 = 0 \Rightarrow x = y$$

Scan for Video solution

68. (d)

Given,

$\Rightarrow\ p \odot q = p - q$

$\Rightarrow\ p \oplus q = p * q$

From these equations we can observer

$\odot = -\ \$\ \oplus = \times$

Put these values in given equation

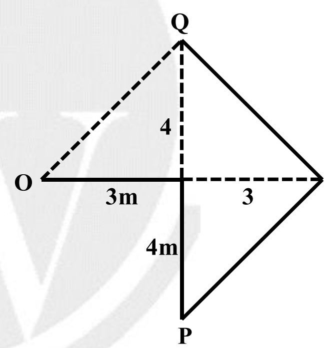
$(9 \odot (6 \oplus 7)) \odot (7 \oplus (6 \odot 5))$

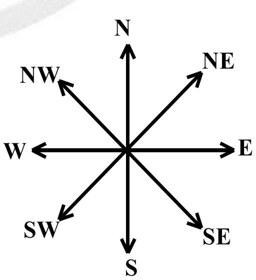
$(9 - (6 * 7)) - (7 * (6 - 5))$

$9 - 42 - 7 = -40$

Scan for Video solution

69. (b)





Using Pythagoras theorem:

$(OQ)^2 = 3^2 + 4^2$

$OQ = \sqrt{9 + 16} = 5$

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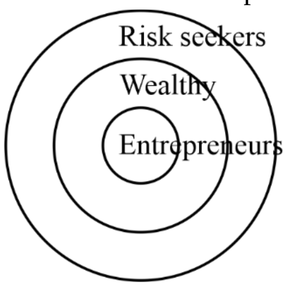
70. (c)

Conclusion I is wrong because only some risk seekers are wealthy conclusion II is also wrong because all the entrepreneurs are risk seekers.



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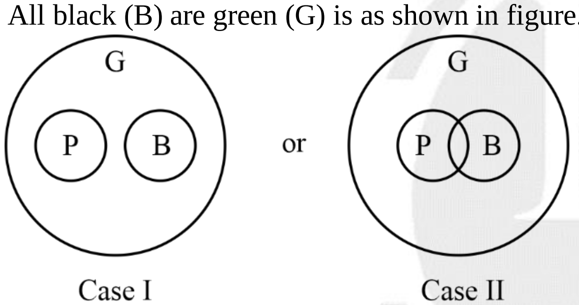
71. (c)

Possible cases for all purple (P) are green (G) and All black (B) are green (G) is as shown in figure.



So, if we take case I, conclusion (ii) is true but conclusion (i) is false.

If we take case II, conclusion (i) is true but conclusion (ii) is false.

So, either conclusion I or II is correct not both.

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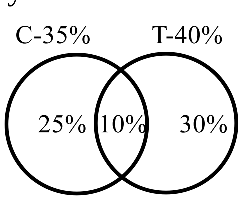
72. (c)

Given data:

% of employees drink coffee = 35%

% of employees drink coffee = 40%

% of employees drink both = 10%



100% - (30% + 10% + 25%)

= 35%

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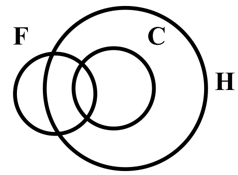
73. (b)

Some football player plays hockey.



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74. (d)

Statement I is not correct as there is also a possibility of no pathogens are bacteria.

Statement II is not correct as this indicates at least few pathogens are not bacterial

So, option (d) is correct.

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75. (b)

$\underset{0}{O}, \underset{1}{P}, \underset{2}{Q}, \underset{3}{R}, \underset{4}{S}, \underset{5}{T}, \underset{6}{U}, \underset{7}{V}, \underset{8}{W}, \underset{9}{X}$

$TS \rightarrow 54$

$ST \rightarrow 45$

$SS \rightarrow 44$

$SU \rightarrow 46$

So, correct option is (b).

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76. (b)

As per given information

Sample size = n(s) = 100 students

Students play chess n(A) = 300

Students play foot ball n(B) = 600

Students play both game $n(A \cap B) = 50$
Students who atleast play one of the game:
$n(A \cup B) = n(A + n(B) - n(A \cap B)$
$= 300 + 600 - 50$
$= 900 - 50 = 850$
Number of students who play neither of the game
$n(s) - n(A \cup B) = 1000 - 850 = 150$

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77. **(c)**
$E = 10 \Rightarrow 5 \times 2$
$J = 20 \Rightarrow 10 \times 2$
$O = 30 \Rightarrow 15 \times 2$
$T = 40 \Rightarrow 20 \times 2$
Then,
$P = 16 \times 2 = 32$
$S = 19 \times 2 = 38$
$P + E + S + T = 32 + 10 + 38 + 40 = 120$

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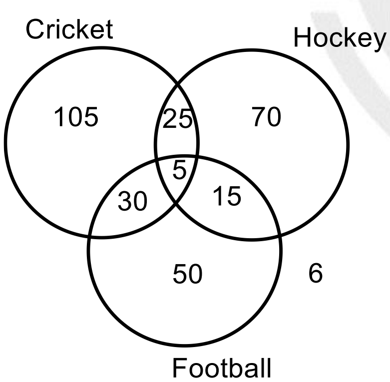
78. **(b)**



From given scenario, the Venn diagram is drawn below. Percentage of people who play at least two games

$$= \frac{25+15+30+5}{300} \times 100 = 25\%$$

Scan for Video solution

79. **(c)**
Total number of people = 70 + 10 + 20 + 20 + 40 = 160
Total administrator = 50

$$\% \text{ of administrators} = \frac{50}{160} \times 100 = 31.25\%$$

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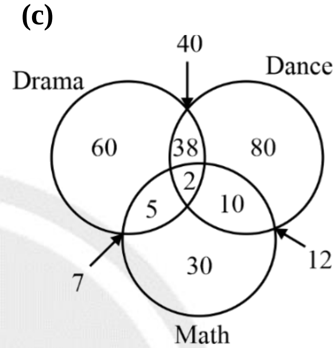
80. **(c)**



From the venn diagram, total students in the clubs = 60 + 80 + 30 + 38 + 10 + 5 + 2 = 225
Since 25% of all the students are in clubs, then

$$225 = \frac{25}{100} \times x \Rightarrow x = 900$$

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81. **(b)**
Let us assume A = 5, then in $A \times B \times C$, the possible unit digits are 0 or 5. But according to the condition $(A \times B \times C) = (B \times G \times E)$ which mean $B \times G \times E$ should also have 0 or 5 at its unit digit which is only possible when one of the digits (B or G or E) is 5. Which is not possible as each letter should represent unique integer 1 to 9.
$\Rightarrow$ If we consider B = 5 then same case will happen with $D \times E \times F$ i.e., one of these 3 letters should represent 5 to get unit digit 0 or 5 which is not possible.

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82. (b)

$(A + B + C) + (C + D + E) + (E + F + G) + (G + H + K) = 4 \times 13$

$\because A + B + C + D + E + F + G + H + K = 1 + 2 + \ldots 9 = 45$

$\therefore \quad 45 + C + E + G = 4 \times 13$

$C + E + G = 7$

Only possible digits we can assign to C, E & G are {1, 2, 4}.

If E = 1, then either C = 2 or G = 2 which means D should be 10 or F should be 10 respectively to make the summation 13 which is not possible as number should be among 1 to 9. So, out of (1, 2, 4) only 4 can be the correct option.

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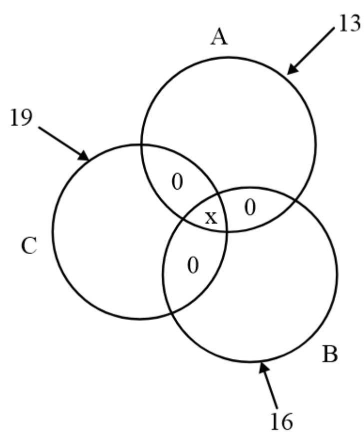
83. (c)

Let x students watches all three movies, then from Venn diagram-



Total number of students = 40

$13 + 16 + 19 - 2x = 40$

{$\because$ x is counted three times so 2x is subtracted}

$2x = 48 - 40$

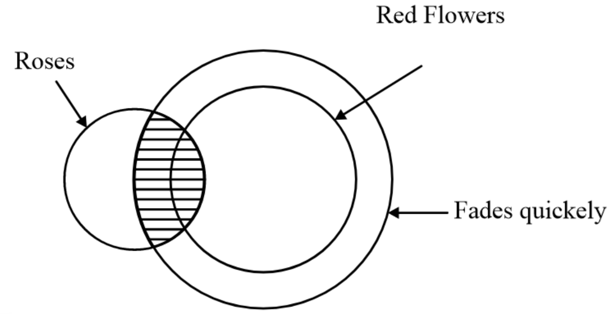
$\boxed{x = 4}$

Scan for Video solution

84. (c)

Using statements (i) and (ii) we can draw following Venn diagram and shaded portion representing that same roses fade quickly (statement III).

So, if statement (i) & (ii) are true, then statement (iii) is also true.



Scan for Video solution

85. (d)

Given: Chemistry(C), Physics(P), and Mathematics(M)

$n(C) = 329$, $n(P) = 186$, $n(M) = 295$

$n(C \cap P) = 83$, $n(C \cap M) = 217$

$n(P \cap M) = 63$, $n(P \cup C \cup M) = 500$

$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap C) + n(P \cap C \cap M)$

$500 = 186 + 329 + 295 - 83 - 217 - 63 + n(P \cap C \cap M) \Rightarrow n(P \cap C \cap M) = 53$

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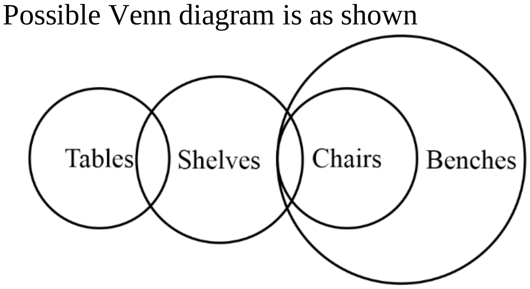
86. (b)

Possible Venn diagram is as shown



Conclusion (i) is false as "No bench is a table" is a possibility.

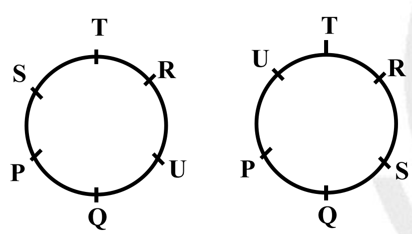
Conclusion (ii) is true as "Some chair will always be a table" in all possible cases.
Conclusion (iii) is false as "No chair is a table" is a possibility.
Conclusion (iv) is false as "Some benches are not chair" is a possibility.

Scan for Video solution

87. (d)
From both the statements it is not certain that some beds are lamps or some lamps are beds or no bed is a lamp.
So, neither (i) nor (ii) is true.

Scan for Video solution

88. (c)

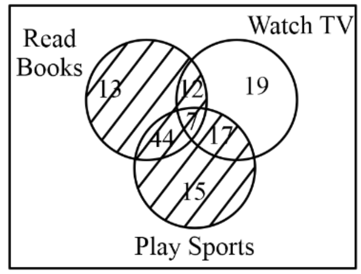


Case-1 **Case-2**

As per the given conditions, two cases as shown above, are possible. If P & U now switch seats, then it is clear that T is immediate left of P or P is immediately to the right of Q.

Scan for Video solution

89. (d)
The shaded area represents the students who like to read books or play sports. Hence the number of students



= 13 + 12 + 44 + 7 + 15 + 17 = 108

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90. (c)
If it rains then the field is wet & if the field is not wet then it did not rain, so option B & D are possible. If the field is wet it does not necessarily due to rain so option A is possible but option C is not possible.

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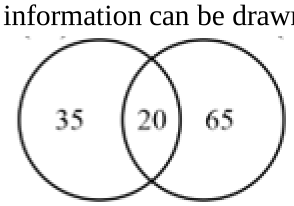
91. (a)
Total faculty members = 150
Faculty members do not have any Facebook or WhatsApp = 30
Faculty members have Facebook or WhatsApp = 150 – 30 = 120

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$120 = 55 + 85 - P(A \cap B)$

$P(A \cap B) = 20$

The above information can be drawn as,



Which means that only Facebook users is 35.
So, option 'A' is correct.

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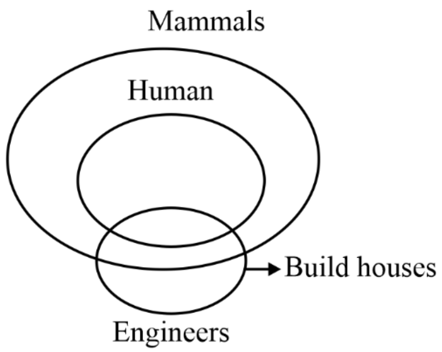
92. (b)

From the given facts, we can draw Venn diagram as shown below.



So, we can certainly say that some humans are not engineers.

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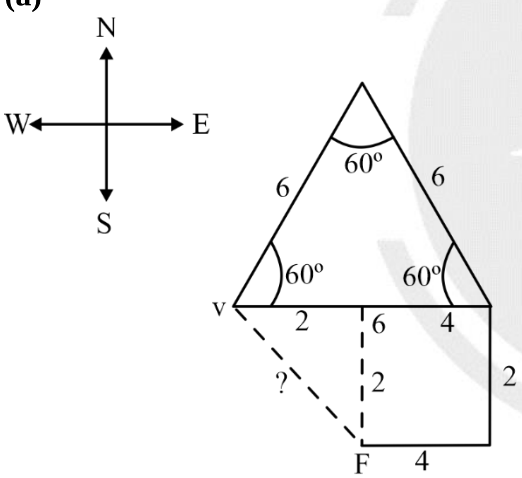
93. (a)



$VF = \sqrt{2^2 + 2^2}$

$VF = 2\sqrt{2}$

Scan for Video solution

94. (c)

statements:

No manager is leader

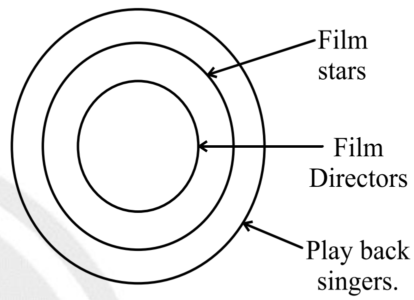
All leaders are executives

but manager can be executive and executive can be manager. So, both the conclusions do not follow.

Scan for Video solution

95. (d)

From the Venn diagram it can be analyzed that all film directors are playback singers and some film stars are film directors.



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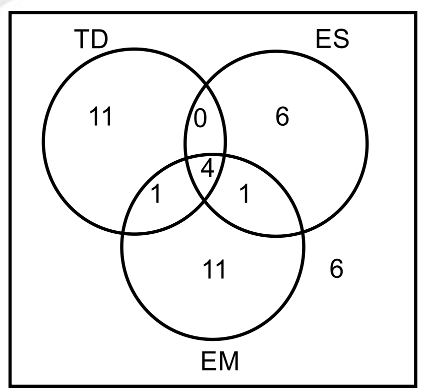
96. (a)

From the given conditions we can draw the Venn diagram as shown below. From Venn diagram, we can say that the number of teachers who can teach ES & EM but not TD is 1.



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97. (d)

A	L	R	V	X
3	3	3	3	3
E	P	V	Z	B
3	3	3	3	3
I	T	Z	D	F
5	4	4	4	4
O	Y	E	I	K

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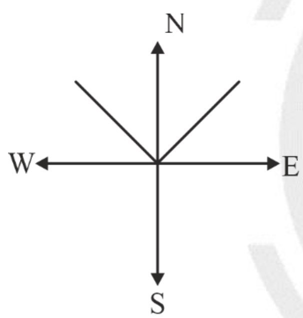
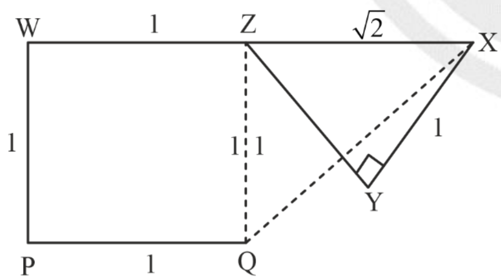
98. (c)

The positions of X, Y, Z, W, P and Q are shown below.





$$XZ = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$XQ = \sqrt{1^2 + \left(\sqrt{2}\right)^2} = \sqrt{3}$$

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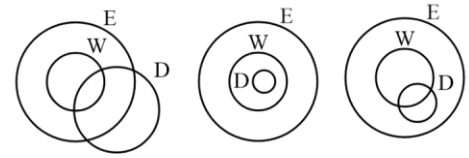
99. (d)

The possible venn diagrams from the statements



It is clear that if all women are entrepreneurs and some women are doctors then some entrepreneurs will definetely be doctors

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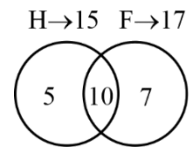
100. (d)



Total number of persons = 25

The number of persons playing either Hockey or Football or both = 5 + 10 + 7 = 22

The number of persons playing neither hockey nor football is:

25 – 22 = 3

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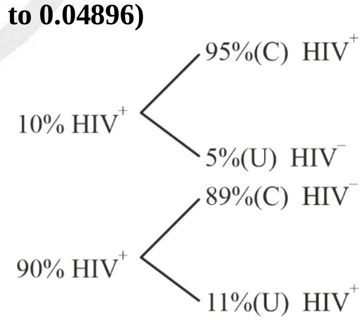
101. (0.4896 to 0.04896)



P (A/B)

$$P\left(\text{actually HIV}^+ / \text{HIV}^+\right)$$

$$= \frac{0.1 \times 0.95}{0.1 \times 0.95 + 0.9 \times 0.11} = \frac{0.0950}{0.194} = 0.4896$$

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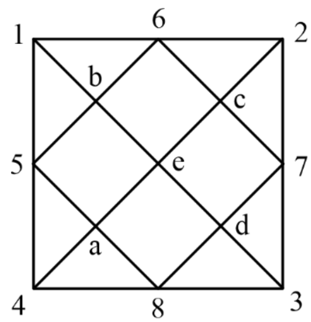
102. (c)



Note: A square is a special type of rectangle because it has all the characteristic of a rectangles.

Number of rectangles of size unit 1×2

= '56ca', '8ac7', '8db5, '7db6' = 4

Number of rectangles (square) of size 1×1

= '5aeb', '6ceb', '7dec', '8acd' = 4

Number of rectangles (square) of size 2×2

= '1 2 3 4', '5 6 7 8' = 2

$\therefore$ Total number of rectangles = 4 + 4 + 2 = 10

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103.(a)

The only way for the person to move from R to S & returning to R is.

$$\underbrace{R \rightarrow Q}_{1\text{ hour}} \underbrace{\rightarrow P}_{1\text{ hour}} \underbrace{\rightarrow S}_{1\text{ hour}} \underbrace{\rightarrow R}_{1\text{ hour}}$$

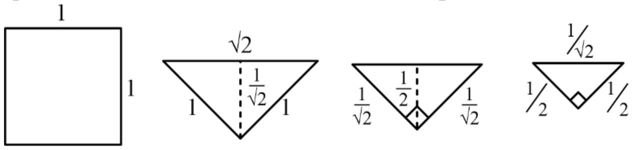
1 hour 1 hour $\frac{1}{2}$ hour

Person will board the train at times 7am (R), 9am (Q), 11am (P), 12:30pm (S) and reaches the station R after 6:30 hours.

Scan for Video solution

104.(b)

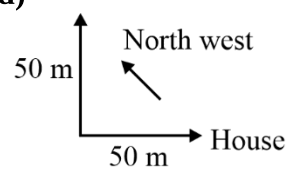
When the sheet is folded as mentioned in the question, we act the final shape as below.

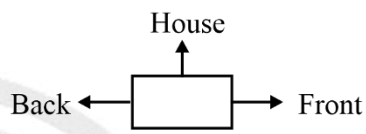


The area of final shape = $\frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

Scan for Video solution

105.(d)





So, the end location of Mr. X from is House is North-West

Scan for Video solution

106.(b)

$\frac{\oplus}{\odot} = 2, \frac{\oplus}{\Delta} = 3$

$\frac{\odot}{\Delta} = \frac{3}{2}$ & $\odot + \Delta = 5$

$\therefore \odot = 3, \Delta = 2$

$\Delta \times \otimes = 10$

$\otimes = \frac{10}{\Delta} = \frac{10}{2} = 5$

$\therefore (\otimes - \oplus)^2 = (5 - 2 \times 3)^2 = 1$

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107.(d)

$\Delta 2 \oplus 3\Delta\left((4 \otimes 2)\nabla 4\right)$

$+ 2 - 3 + ((4 \div 2) \times 4)$

$-1 + (2 \times 4) = 7$

Scan for Video solution

108. (c)

Q = 3P = 3 × 3 = 9

R = 3Q = 3 × 9 = 27

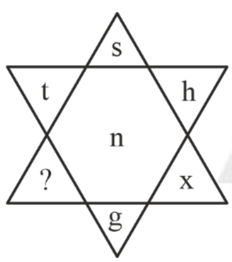
S = 3R = 3 × 27 = 81

T = 3S = 3 × 81 = 243

So, Q + S = 9 + 81 = 90.

Scan for Video solution

109. (a)



The position of h, n & t alphabets are 8, 14 & 20. So, the difference of the numbers of n-h & t-n are 6 & 6. Similarly, the difference x-n is 10 so, the missing element should be 'd' so that n-d = 10.

Scan for Video solution

110. (d)

There are 4 codes that can be made by α and β.

α α, α β, βα and β, β.

P is coded as αα and Q as αβ so, R and S can be coded as βα and ββ.

So, option d is correct.

Scan for Video solution

111. (c)

Given sequence following a special pattern.

Z – 3 = WV, V – 4 = RQP, P – 5 = KJIH

Or, (26), (23, 22), (18, 17, 16), (11,10,9,8)

So, option 'C' is correct.

Scan for Video solution

112. (b)

$$\begin{array}{cccc} I & M & H & O \\ \downarrow{+1} & \downarrow{+1} & \downarrow{+1} & \downarrow{+1} \\ J & N & I & P \end{array} \qquad \begin{array}{ccc} I & D & K \\ \downarrow{+1} & \downarrow{+1} & \downarrow{+1} \\ J & E & L \end{array}$$

Similarly

$$\begin{array}{ccc} I & D & C \\ \downarrow{+1} & \downarrow{+1} & \downarrow{+1} \\ J & E & D \end{array}$$

(b) (Makes to all in answer key)

Scan for Video solution

113. (c)

From the given information, it is concluded that Mita is tallest, as Ganga is taller than Laxmi and Rekha then Rekha must be shorter than Mita. As Laxmi is taller than Sana, then Sana must be shorter than Ganga. Statement (2) and (4) are true. Statement (1) and (3) are invalid so these are not true.

So, option 'C' is correct.

Scan for Video solution

114. (c)

Given series is the cube of consecutive prime numbers,

$7^3, 11^3, 13^3, 17^3$ = 343, 1331, 2197, 4913

Missing term = 2197

So, option 'C' is correct.

Scan for Video solution

115. (c)

The given two sequence AMCF = EQGJ and NKUF = ROYJ, following the same pattern. In this pattern, all letters have difference of 3 letters at their respective places. i.e. A = E, M = Q, C = G and F = J

Therefore, DHLP = HLPT

So, option 'C' is correct.

Scan for Video solution

116. (d)

Multiple of 3 between 50 to 59 are {51, 54, 57}, all three are non-multiple of 4 & 6.

Non-multiple of 4 between 60 to 69 are {61, 62, 63, 65, 66, 67, 69}, no number is a multiple of 6 and non-multiple of 3.

Non-multiple of 6 between 70 to 79 are {70, 71, 73, 74, 75, 76, 77, 79}, only 76 is the multiple of 6 and non-multiple of 3.

Scan for Video solution

117. (b)

In the series, before BC, one alphabet (A) is missing, before FGH, two alphabets (D & E) are missing, before LMNO three alphabets are missing so after LMNO, four alphabets will be missing so next term of the series starts with T and will have five alphabets in that term, so it will be TUVWX.

Scan for Video solution

118. (b)

$$2 \xrightarrow{\times 6} 12 \xrightarrow{\times 5} 60 \xrightarrow{\times 4} 240 \xrightarrow{\times 3} 720 \xrightarrow{\times 2} 1440 \xrightarrow{\times 1} \underline{1440} \xrightarrow{\times 0} 0$$

So, the missing number is 1440.

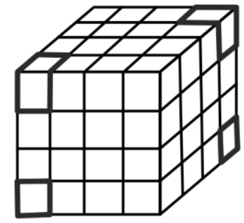
Scan for Video solution

119. (d)

Surface area of 1 face

4 × 4 = 16

Surface area of 6 faces = 16 × 6 = 96



Removing one cubic block from each corner will not change the surface area of the cube. So, the surface area is 96 unit^2.

Scan for Video solution

120. (d)

P always beats Q, Q always beats R, R always beats S but S loses to P only sometimes. From above observation, we can say that P is not the best player among all the players and S loses to P only sometimes so, S is not the worst player.

Scan for Video solution

121. (b)

Given series contains prime numbers 13, 23, 43, 53, but 33 is not a prime number. So that odd one is 33. So, option 'B' is correct.

Scan for Video solution

122. (d)

Option (a), (b), (c) following the same pattern, but only option (d) following a different pattern.

Hence, odd one is option (d).

Pattern: –2, +3, –2, +3 …

(a) C – 2 = A + 3 = D – 2 = B + 3 = E

(b) J – 2 = H + 3 = K – 2 = I + 3 = L

(c) X – 2 = V + 3 = Y – 2 = W + 3 = Z

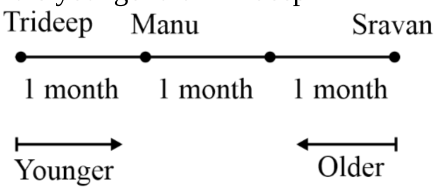
(d) O – 2 = M + 3 = P – 2 = N + 3 = Q

So, option 'd' is correct.

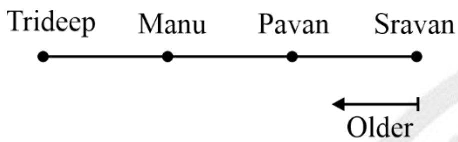
Scan for Video solution

123. (c)

Manu is 2 months older than Sravan, who (Sravan) is 3 months younger than Trideep



Pavan is one-month older than Sravan



So, we can see that Trideep is the Oldest

Scan for Video solution

124. (c, d)

$324 = 18^2$, $441 = 21^2$, $64 = 8^2$

So, 97 is not perfect square and only prime number hence least fits the given set.

Also,

$324 \rightarrow 3+2+4 = 9 = 3^2$ = perfect square

$441 \rightarrow 4+4+1 = 9 = 3^2$ = perfect square

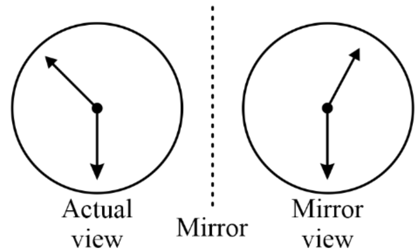
$97 \rightarrow 9 + 7 = 16 = 4^2$ = perfect square

$64 \rightarrow 6 + 4 = 10$ = not perfect square

So, option (c) and (d) both are the least fitting in the set.

Scan for Video solution

125. (d)



Two and quarter hours back, if in mirror view, the time is 1:30 then in actual clock it is 10:30. So after 2 hour, 15 minutes, the current time will be 12:45.

Scan for Video solution

126. (c)

$$\underbrace{B,}\ \xrightarrow{+4}\ \underbrace{F\ H,}_{+2}\ \xrightarrow{+4}\ \underbrace{L\ N\ P,}_{+2\ +2}\ \xrightarrow{+4}\ \underbrace{T\ V\ X\ Z}_{+2\ +2\ +2}$$

Scan for Video solution

127. (c)

Arun lives farther away than Ahmed (5 km) but closer than Suson (7 km). So, the possible distance of Susan is 6.02 km.

Scan for Video solution

128. (b)

$\overset{+3}{R}\ \overset{+3}{O}\ \overset{+3}{A}\ \overset{+3}{D} \rightarrow U\ R\ D\ G$

$\overset{+3}{S}\ \overset{+3}{W}\ \overset{+3}{A}\ \overset{+3}{N} \rightarrow V\ Z\ D\ Q$

Scan for Video solution

129. (a)

As per given data, we have to cover only height to go from first floor to second floor. So that we have taken care only height of the step. As there is nothing mentioned about the width to be taken, then we can ignore the width of the step.

Hence only statement (I) is alone sufficient and statement (II) in not sufficient in any condition.

Total steps $= \frac{9}{3/4} = \frac{9\times 4}{3} = 12$

So, option ‘A’ is correct.

Scan for Video solution

130. (c)

Let the weight of a pole = x kg

As per data given in statement (I):

$\frac{x}{4} = 5 \Rightarrow x = 20\text{kg}$

As per data given in statement (II):

$10x = 2x + 160 \Rightarrow x = 20\text{kg}$

From either statement (I) or statement (II), we can calculate the weight of single pole. It means we can identify the weight of 10 poles.

Hence, either statement (I) or statement (II) is sufficient.

So, option ‘C’ is correct.

Scan for Video solution

131. (3 to 3)

The pattern followed

$$5 = \frac{6+4}{2}$$

$$7 = \frac{7+4+2+1}{2}$$

$$8 = \frac{1+9+2+1+2+1}{2}$$

$$\vdots$$

So, $x = \frac{3+3}{2} = 3$

Scan for Video solution

132. (c)

$$(66 \square 6) \rightarrow (66 \blacklozenge 6) = \left(\frac{66-6}{66+6}\right) \rightarrow \left(\frac{66+6}{66-6}\right)$$

$$= \frac{60}{72} \times \frac{72}{60} = 1$$

Scan for Video solution

133.(c)

+2 +3 +4 +5

A , C D , G H I , M N O P , UV WX Y

+1 +1 +1 +1 +1 +1

Scan for Video solution

134. (c)

2, 5, 10, 17, 26, 37, 50, 64

2, 2 + 3, 5 + 5, 10 + 7, 17 + 9, 26 + 11, 37 + 13, 50 + 15,

2, 5, 10, 17, 26, 37, 50, 65

Here a next odd integer is added to previous term.

Hence, 64 is wrong term.

So, option ‘C’ is correct.

Scan for Video solution

135. (c)

13, 17, 19 (23) (Prime No.)

M + 6 $\rightarrow$ S similarly Q + 6 $\rightarrow$ W

Hence thw next term in the series is 23W

Scan for Video solution

136. (b)

Next prime no. is 17 and G + 6 $\rightarrow$ 13 so K + 6 $\rightarrow$ Q.

So, the term in the sequence is 17Q

Scan for Video solution

137. (16 to 16)

$\frac{81}{1.5} = 54, \frac{54}{1.5} = 36, \frac{36}{1.5} = 24, \frac{24}{1.5} = 16$

Scan for Video solution

138. (d)

W , E , K , O
23, 5, 11, 15
+8, +6 +4

I , Q , W , A
9, 17, 23, 1
+8, +6 +4

F , N , T , X
6, 14, 20, 24
+8, +6 +4

N , V , B , D
14, 22, 2, 4
+8, +6 +2

Scan for Video solution

139. (45 to 45)

2 3 6 15 $\boxed{45}$ 157.5 630
×1.5 ×2 ×2.5 ×3 ×3.5 ×4

Scan for Video solution

140. (725 to 725)

×2 +11 ×2+11 ×2+11 ×2+11 ×2+11
12, 35, 81, 173, 357,____

$357 \times 2 + 11 = 725$

Scan for Video solution

141. (a)

+2 +3 +4
AD CG FK JP
+3 +4 +5

J + 5 = O

P + 6 = V

So, the correct term is OV

Scan for Video solution

142. (b)

Given:

H + G > I + S

G – S = 1 OR S – G = 1

(b) SGHI	(c) IGSH	(d) IHSG
S = G + 1	G – S = 1	S = 1 + G
H + G > I + S	G = 1 + S	H + G > I + S
H + G > I + G + 1	H + G > I + S	H + G > I + 1 + G
H > I + 1 (satisfies)	H + 1 + S > I + S	H > I + 1 (does not satisfy)
	H + 1 > I (does not satisfy)	

Scan for Video solution

❑❑❑

CHAPTER

3 SPATIAL APTITUDE

Transformations of Shapes

1. [MCQ] [GATE-CS-2023:1M]

Looking at the surface of a smooth 3-dimensional object from the outside, which one of the following options is TRUE?

(a) The surface of the object must be concave everywhere.

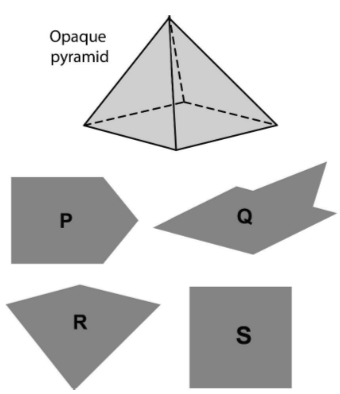
(b) The surface of the object must be convex everywhere.

(c) The surface of the object may be concave in some places and convex in other places.

(d) The object can have edges, but no corners.

2. [MCQ] [GATE-ME-2023: 2M]

An opaque pyramid (shown below), with a square base and isosceles faces, is suspended in the path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. The pyramid can be reoriented in any direction within the light beam. Under these conditions, which one of the shadows P, Q, R, and S is NOT possible?



(a) P (b) Q

(c) R (d) S

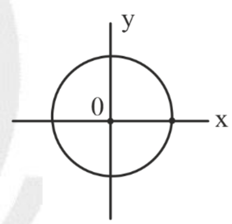
3. [MCQ] [GATE-CE-2023:1M]

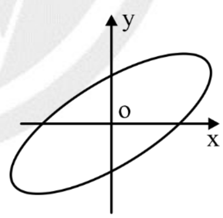
Consider a circle with its centre at the origin (O), as shown. Two operations are allowed on the circle.

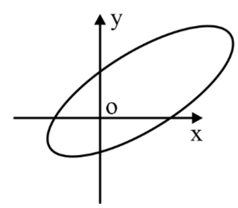
Operation 1: Scale independently along the x and y axes.

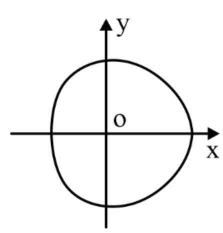
Operation 2: Rotation in any direction about the origin.

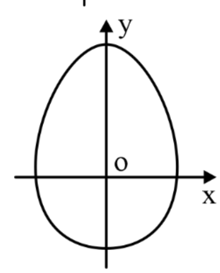
Which figure among the options can be achieved through a combination of these two operations on the given circle?



(a) 

(b) 

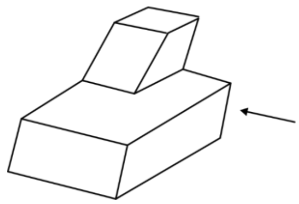
(c) 

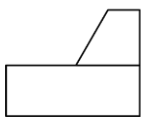
(d) 

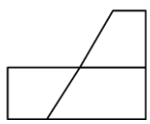
4. [MCQ] [GATE-ME-2022: 1M]

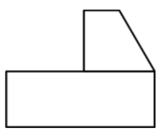
A block with a trapezoidal cross-section is placed over a block with rectangular cross section as shown above.

Which one of the following is the correct drawing of the view of the 3D object as viewed in the direction indicated by an arrow in the below figure?



(a) 

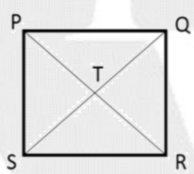
(b) 

(c) 

(d) 

5. [MCQ] [GATE-EC-2022:2M]

Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.



Let X, Y and Z represent the following operations:

X: rotation of the square by 180 degree with respect to the S-Q axis.

Y: rotation of the square by 180 degree with respect to the P-R axis.

Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T.

Consider the following three distinct sequences of operation (which are applied in the left to right order).

(1) XYZZ

(2) XY

(3) ZZZZ

Which one of the following statements is correct as per the information provided above?

(a) The sequence of operations (1) and (2) are equivalent

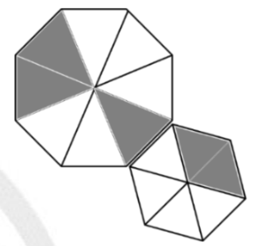
(b) The sequence of operations (1) and (3) are equivalent

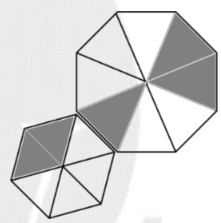
(c) The sequence of operations (2) and (3) are equivalent

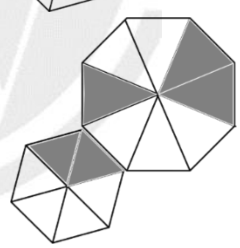
(d) The sequence of operations (1), (2) and (3) are equivalent

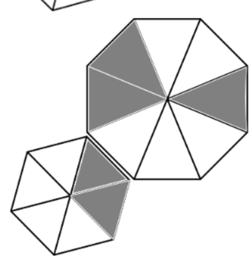
6. [MCQ] [GATE-CE-2022:1M]

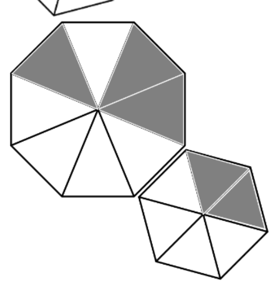
For the picture shown below, which one of the following is the correct picture representing reflection with respect to the mirror shown as the dotted line?



(a) 

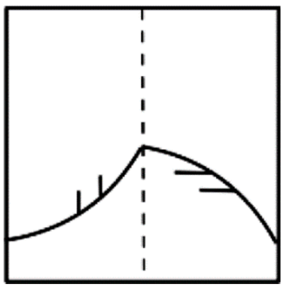
(b) 

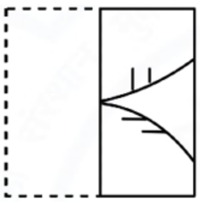
(c) 

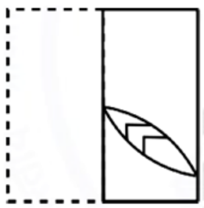
(d) 

7. [MCQ] [GATE-CS-2021: 1M]

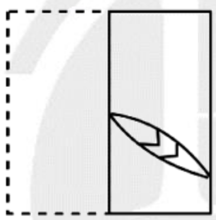
A transparent square sheet shown below is folded along the dotted line. The folded sheet will look like________.



(a) 

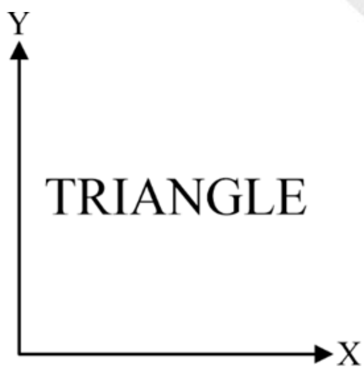
(b) 

(c) 

(d) 

8. [MCQ] [GATE-CE-2021: 1M]

The mirror image of the above text about the X-axis is.



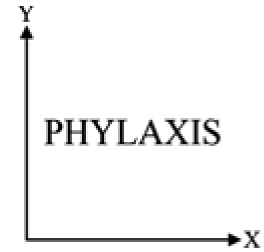
(a) 

(b) 

(c) 

(d) 

9. [MCQ] [GATE-CE-2021: 1M]



The mirror image of the below text about the X-axis is.

(a) 

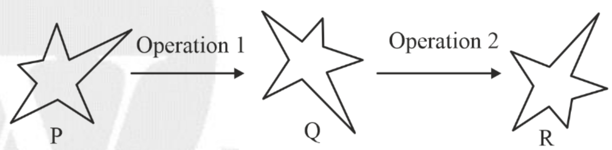
(b) 

(c) 

(d) 

10. [MCQ] [GATE-CS-2017:1M]

Which one of the options best describes the transformation of the 2-dimensional figure P to Q, and then to R, as shown?



(a) Operation 1: A clockwise rotation by 90° about an axis perpendicular to the plane of the figure
Operation 2: A reflection along a horizontal line.

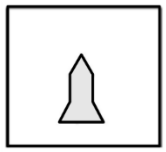
(b) Operation 1: A counterclockwise rotation by 90° about an axis perpendicular to the plane of the figure
Operation 2: A reflection along a horizontal line.

(c) Operation 1: A clockwise rotation by 90° about an axis perpendicular to the plane of the figure
Operation 2: A reflection along a vertical line.

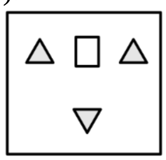
(d) Operation 1: A counterclockwise rotation by 180° about an axis perpendicular to the plane of the figure
Operation 2: A reflection along a vertical line.

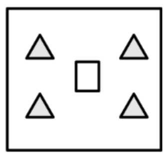
Classification and Grouping

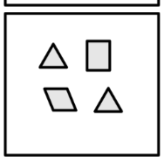
11. [MCQ] [GATE-ME-2022: 1M]



Which one of the groups given below can be assembled to get the shape that is shown above using each piece only once without overlapping with each other? (rotation and translation operations may be used).

(a) 

(b) 

(c) 

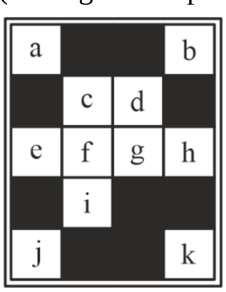
(d) 

Patterns in 2D & 3D

12. [MCQ] [GATE-CE-2023: 1M]

A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line.

The figure below consists of 20-unit squares arranged as shown. In addition to the given black squares, upto 5 more may be coloured black. Which one among the following options depicts the minimum number of boxes that must be coloured black to achieve two lines of symmetry? (The figure is representative)



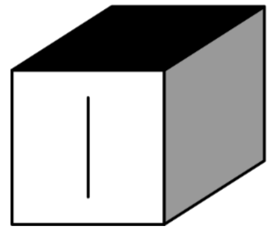
(a) d (b) c, d, i

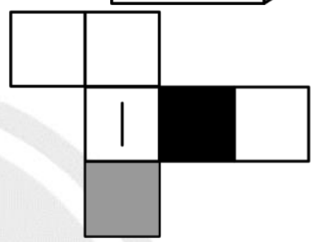
(c) c, i (d) c, d, i, f, g

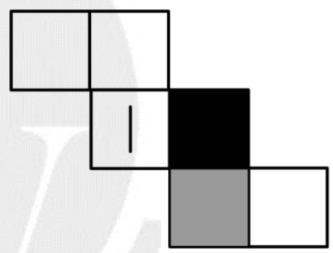
13. [MCQ] [GATE-CE-2022:2M]

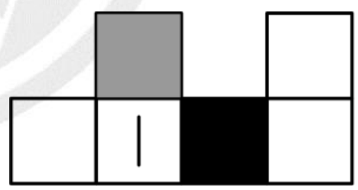
Consider a cube made by folding a single sheet of paper of appropriate shape. The interior faces of the cube are all blank. However, the exterior faces that are not visible in the above view may not be blank.

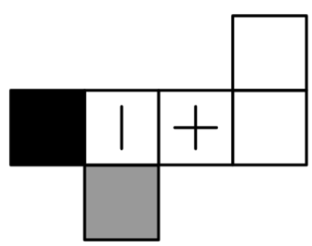
Which one of the following represents a possible unfolding of the cube?



(a) 

(b) 

(c) 

(d) 

14. [MCQ] [GATE-CE-2021: 2M]

Consider two rectangular sheets, Sheet M and Sheet N of dimensions 6 cm x 4 cm each.

Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.

Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.

Folding operation 1 is carried out on Sheet M three times.

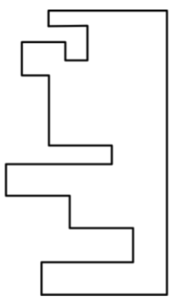
Folding operation 2 is carried out on Sheet N three times.

The ratio of perimeters of the final folded shape of Sheet N to the final folded shape of Sheet M is____.

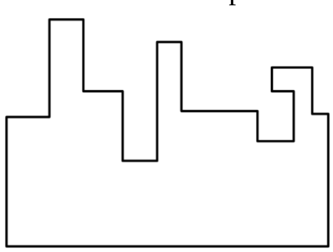
(a) 13 : 7 (b) 3 : 2

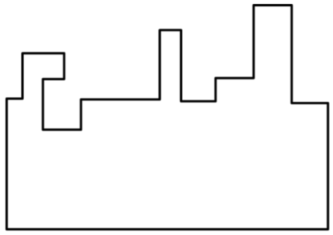
(c) 7 : 5 (d) 5 : 13

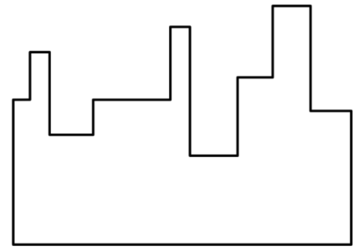
15. [MCQ] [GATE-ME-2021: 2M]

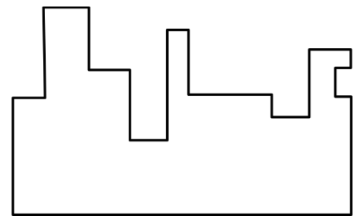


A jigsaw puzzle has 2 pieces. One of the pieces is shown above. Which one of the given options for the missing piece when assembled will form a rectangle? The piece can be moved, rotated or flipped to assemble with the above piece.

(a) 

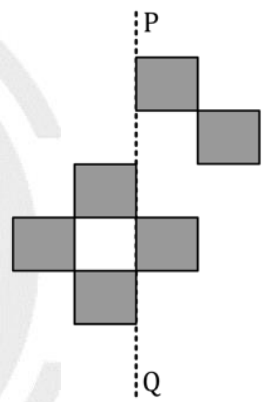
(b) 

(c) 

(d) 

16. [MCQ] [GATE-EC-2021:1M]

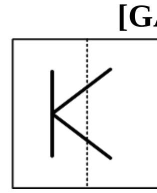
The least number of squares that must be added so that the line P-Q becomes the line of symmetry is



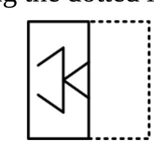
(a) 4 (b) 3

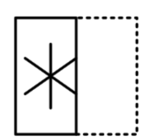
(c) 6 (d) 7

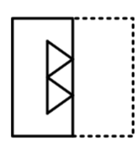
17. [MCQ] [GATE-EE-2021: 1M]

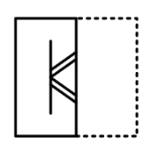


A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like

(a) 

(b) 

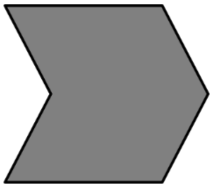
(c) 

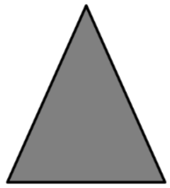
(d) 

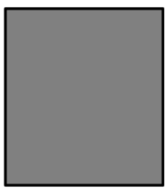
18. [MCQ] [GATE-CS-2021: 1M]

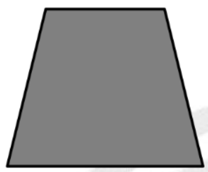
A polygon is convex if, for every pair of points, P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon.

Which one of the following is <u>NOT</u> a convex polygon?

(a) 

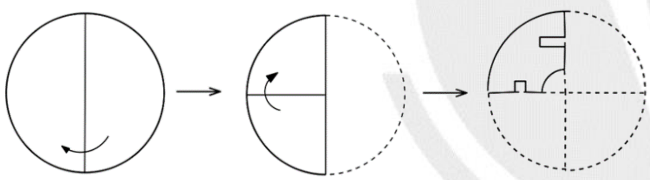
(b) 

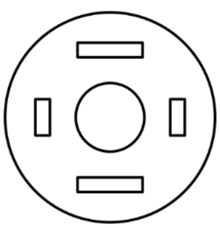
(c) 

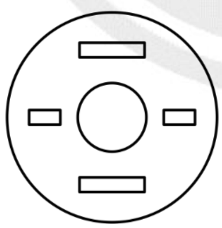
(d) 

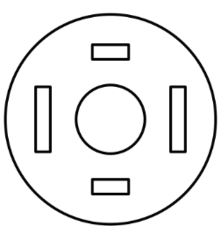
19. [MCQ] [GATE-CS-2021: 1M]

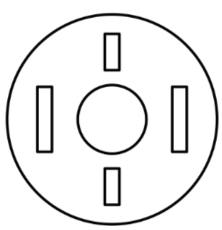
A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like _______.



(a) 

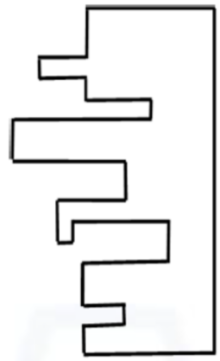
(b) 

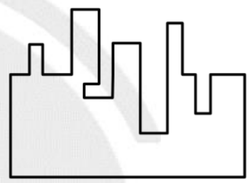
(b) 

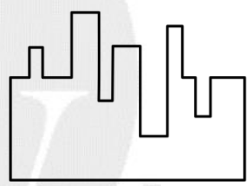
(d) 

20. [MCQ] [GATE-CS-2021: 2M]

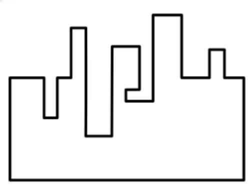
A jigsaw puzzle has 2 pieces. One of the pieces is shown below. Which one of the given options for the missing piece when assembled will form a rectangle? The piece can be moved, rotated or flipped to assemble with the below piece.



(a) 

(b) 

(c) 

(d) 

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ANSWER KEY

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(a)
5.	(b)	**6.**	(a)	**7.**	(b)	**8.**	(b)
9.	(b)	**10.**	(a)	**11.**	(b, c)	**12.**	(b)
13.	(MTA)	**14.**	(a)	**15.**	(a)	**16.**	(c)
17.	(c)	**18.**	(a)	**19.**	(a)	**20.**	(a)

SOLUTIONS

1. (c)

If the object has edges, then it surely shall have corners: option D wrong

Just seeing the surface of SMOOTH 3D object from outside, we cannot say it has concave or convex faces so, options A & B are also wrong and option C is correct

Scan for Video solution

2. (b)

Shadow Q is not possible

Depending upon the angle of incidence of the light, the shadows P, R and S can be obtained.

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3. (a)

Even after both the operations, the centre of the new shape will also lie on the origin. When operation 1 is performed then we can get an ellipse and when operation 2 is performed then a tilted ellipse with centre at origin can be achieved, hence option A is correct.

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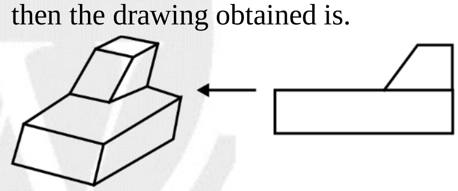
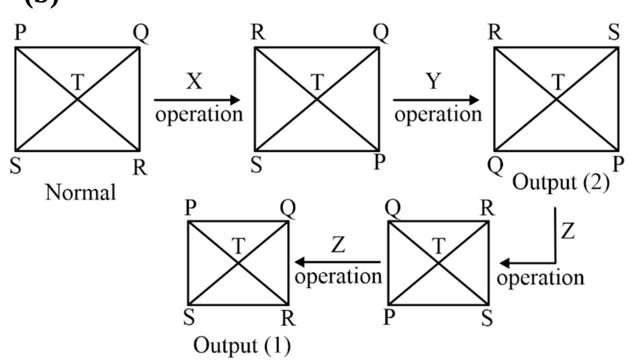
4. (a)

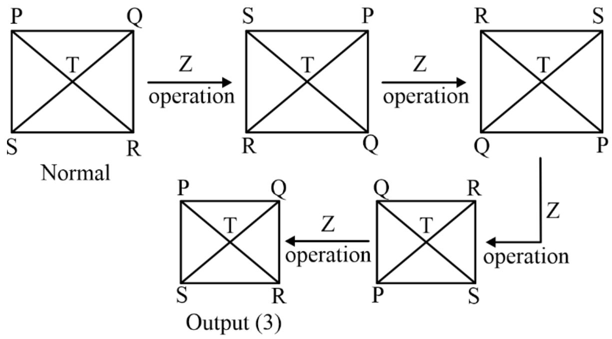
When this 3D object is viewed in the direction of arrow, then the drawing obtained is.



Scan for Video solution

5. (b)





So, output (1) and (3) are equivalent.

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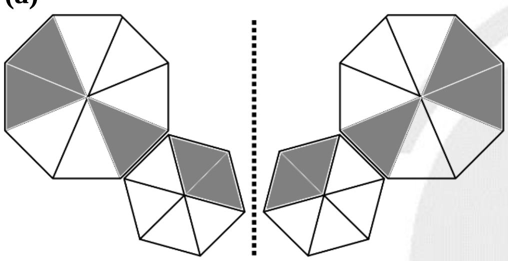
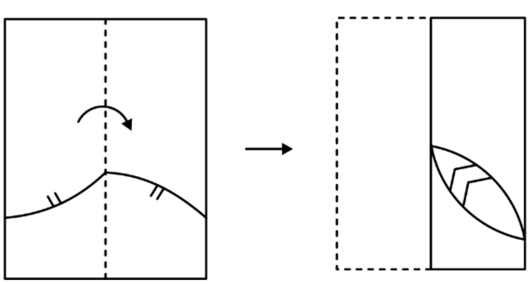
6. **(a)**



Scan for Video solution

7. **(b)**



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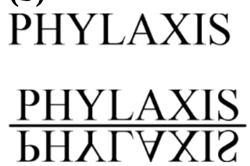
8. **(b)**

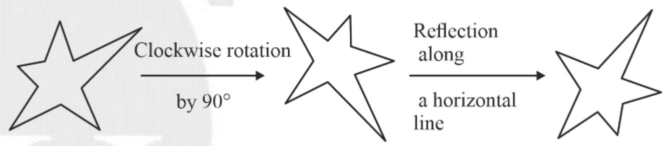
Scan for Video solution

9. **(b)**



Scan for Video solution

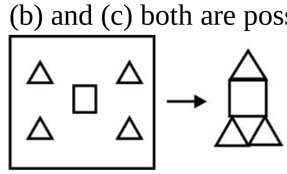
10. **(a)**

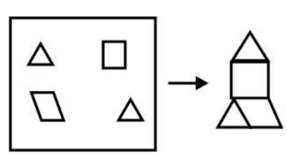


Scan for Video solution

11. **(b, c)**

(b) and (c) both are possible





Scan for Video solution

12. (b)

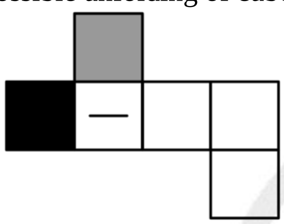
The boxes to be coloured black to achieve two lines of symmetry are c, d and i.

Scan for Video solution

13. (MTA)

The black shade should be in the direction of line and the grey surface should be in the direction perpendicular to the line.

One of the possible unfolding of cube is shown below.



No option matches with the unfolding.

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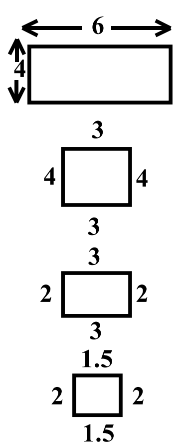
14. (a)



<u>**M**</u>

6, 4

3, 4

3, 2

1.5, 2

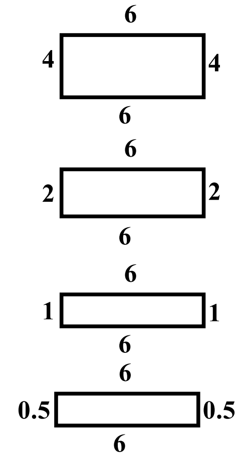
<u>**N**</u>

6, 4

6, 2

6, 1

6, 0.5



Perimeter = 2(a + b)

Sheet M's perimeter = 2(1.5 + 2) = 7

Sheet N's perimeter = 2(6 + 0.5) = 13

$$\frac{\text{ratio of sheet N perimeter}}{\text{ratio of sheet M perimeter}} = \frac{13}{7}$$

Scan for Video solution

15. (a)

The piece in option A when flipped and rotated, will form a rectangle on assembling with first part.

Scan for Video solution

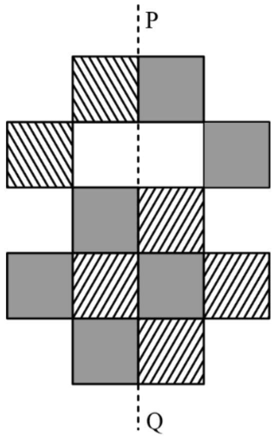
16. (c)

For lower part of the figure to be symmetric about PQ, we have to add 1 square on the left side and 3 squares on the right side as shown in figure.

For upper part, we have to add 2 squares on the left side only as shown in figure.

So, total number of minimum squares needed is

= 1 + 3 + 2 = 6



Scan for Video solution

17. (c)

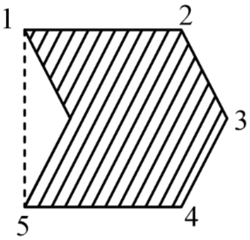
As the paper is folded with respect to dotted line, it will form two triangles vertically one above other. Which is available in option (c).

So, option ‘C’ is correct.

Scan for Video solution

18. (a)

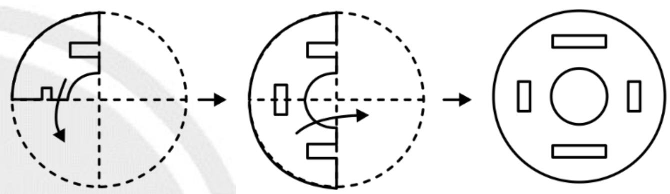
In polygon as shown in figure below, if line from 1 to 5 is drawn then it lies outside the polygon hence it is not a convex polygon



Scan for Video solution

19. (a)

The unfolding of the punched paper is shown below.



From the above diagram it is clear that the unfolded paper will be like in option A.

Scan for Video solution

20. (a)

The piece in option A if flipped and rotated by 90° can fit in the piece in question.

Scan for Video solution

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CHAPTER

4 VERBAL APTITUDE

Basic English Grammar

1. **[MCQ]** **[GATE-ME-2023:1M]**

He did not manage to fix the car himself, so he _______ in the garage.

(a) got it fixed (b) getting it fixed
(c) gets fixed (d) got fixed

2. **[MCQ]** **[GATE-EE-2023:1M]**

Rafi told Mary, "I am thinking of watching a film this weekend."

The following reports the above statement in indirect speech:

Rafi told Mary that he _______ of watching a film that weekend.

(a) thought (b) is thinking
(c) am thinking (d) was thinking

3. **[MCQ]** **[GATE-CE-2023:1M]**

The line ran______ the page right through the center and divide the page into two

(a) across (b) of
(c) between (d) about

4. **[MCQ]** **[GATE-CE-2023:1M]**

"I have not decided what I will do this evening I ________ visit friend."

(a) mite (b) would
(c) might (d) didn't

5. **[MCQ]** **[GATE-CE-2022:1M]**

The movie was funny and I______.

(a) Could help laughing
(b) Couldn't help laughed
(c) Couldn't help laughing
(d) Could helped laughed

6. **[MCQ]** **[GATE-ME-2021:1M]**

Consider the following sentences:

(i) The number of candidates who appear for the GATE examination is staggering.
(ii) A number of candidates from my class are appearing for the GATE examination.
(iii) The number of candidates who appear for the GATE examination are staggering.
(iv) A number of candidates from my class is appearing for the GATE examination.

Which of the above sentences are grammatically CORRECT?

(a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iii) (d) (ii) and (iv)

7. **[MCQ]** **[GATE-ME-2021:1M]**

Consider the following sentences:

(i) After his surgery, Raja hardly could walk.
(ii) After his surgery, Raja could barely walk.
(iii) After his surgery, Raja barely could walk.
(iv) After his surgery, Raja could hardly walk.

Which of the above sentences are grammatically CORRECT?

(a) (i) and (ii) (b) (i) and (iii)
(c) (iii) and (iv) (d) (ii) and (iv)

8. **[MCQ]** **[GATE-EC-2021:1M]**

Consider the following sentences:

(i) I woke up from sleep.
(ii) I woked up from sleep.
(iii) I was woken up from sleep
(iv) I was wakened up from sleep.

Which of the above sentences are grammatically CORRECT?

(a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iii) (d) (i) and (iv)

9. [MCQ] [GATE-EE-2021:1M]

The people ___________ were at the demonstration were from all sections of society.

(a) whose (b) which
(c) who (d) whom

10. [MCQ] [GATE-CE-2021:1M]

Getting to the top is ______ than staying on top

(a) more easy (b) much say
(c) easiest (d) easier

11. [MCQ] [GATE-CE-2021:2M]

Statement: Either P marries Q or X marries Y

Among the options below, the logical NEGATION of the above statements.

(a) P does not marry Q and X marries Y.
(b) Neither P marries Q nor X marries Y.
(c) X does not marry Y and P marries Q.
(d) P marries Q and X marries Y.

12. [MCQ] [GATE-CE-2021:1M]

(i) Arun and Aparna are here.
(ii) Arun and Aparna is here.
(iii) Arun's families is here.
(iv) Arun's family is here.

Which of the above sentence are grammatically CORRECT?

(a) (i) And (ii) (b) (ii) And (iv)
(c) (i) And (iv) (d) (iii) And (iv)

13. [MCQ] [GATE-CS-2021:1M]

Consider the following sentences:

(i) Everybody in the class is prepared for the exam.
(ii) Babu invited Danish to his home because he enjoys playing chess.

Which of the following is the CORRECT observation about the above two sentences?

(a) (i) is grammatically correct and (ii) is unambiguous
(b) (i) is grammatically incorrect and (ii) is unambiguous"
(c) (i) is grammatically correct and (ii) is ambiguous"
(d) (i) is grammatically incorrect and (ii) is ambiguous

14. [MCQ] [GATE-CS-2021:1M]

Gauri said that she can play the keyboard_________ her sister.

(a) as well as (b) as better as
(c) as nicest as (d) as worse as

15. [MCQ] [GATE-ME-2020:1M]

While I agree ____his proposal this time, I do not often agree ____him

(a) to, with (b) with, to
(c) with, with (d) to, to

16. [MCQ] [GATE-ME-2020:1M]

Jofra Archer, the England fast bowler, is _____ than accurate.

(a) more fast (b) faster
(c) less fast (d) more faster

17. [MCQ] [GATE-EC-2020:1M]

He was not only accused of theft _______of conspiracy.

(a) rather (b) but also
(c) but even (d) rather than

18. [MCQ] [GATE-EC-2020:1M]

The untimely loss of life is a cause of serious global concern as thousands of people get killed _____ accidents every year while many other die _______ diseases like cardio vascular disease, cancer, etc.

(a) in, of (b) from, of
(c) during, from (d) from, from

19. [MCQ] [GATE-EE-2020:1M]

People were prohibited _______ their vehicles near the entrance of the main administrative building.

(a) to park
(b) from parking
(c) parking
(d) to have parked

20. [MCQ] [GATE-EE-2020:1M]

This book, including all its chapters, _________ interesting. The students as well as the instructor _________ in agreement about it.

(a) is, was
(b) are, are
(c) is, are
(d) were, was

21. [MCQ] [GATE-CE-2020:1M]

Rescue teams deployed _____ disaster hit areas combat ____ a lot of difficulties to save the people.

(a) with, at
(b) in, with
(c) with, with
(d) to, to

22. [MCQ] [GATE-CS-2020:1M]

Raman is confident of speaking English _______ six months as he has been practicing regularly ______ the last three weeks.

(a) during, for
(b) for, since
(c) for, in
(d) within, for

23. [MCQ] [GATE-ME-2019:1M]

Once the team of analysts identify the problem, we ______ in a better position to comment on the issue.

Which one of the following choices CANNOT fill the given blank?

(a) will be
(b) were to be
(c) are going to be
(d) might be

24. [MCQ] [GATE-ME-2019:1M]

_____I permitted him to leave, I wouldn't have had any problem with him being absent, ______I?

(a) had, wouldn't
(b) have, would
(c) had, would
(d) have, wouldn't

25. [MCQ] [GATE-EC-2019:1M]

The boat arrived ___________ dawn.

(a) in
(b) at
(c) on
(d) under

26. [MCQ] [GATE-EC-2019:1M]

The strategies that the company __________ to sell its products _________ house-to-house marketing.

(a) use, includes
(b) uses, include
(c) used, includes
(d) uses, including

27. [MCQ] [GATE-EE-2019:1M]

The passengers were angry ______ the airline staff about the delay.

(a) on
(b) about
(c) with
(d) towards

28. [MCQ] [GATE-CE-2019:1M]

They have come a long way in _______ trust among the users.

(a) creating
(b) created
(c) creation
(d) create

29. [MCQ] [GATE-CE-2019:1M]

The CEO's decision to quit was as shocking to the Board as it was to ________.

(a) i
(b) me
(c) my
(d) myself

30. [MCQ] [GATE-CE-2019:1M]

Hima Das was _____ only Indian athlete to win _____ gold for India.

(a) the, many
(b) the, a
(c) an, a
(d) an, the

31. [MCQ] [GATE-CS-2019:1M]

The expenditure on the project ______ as follows: equipment Rs.20 lakhs, salaries Rs. 12 lakhs, and contingency Rs.3 lakhs.

(a) break down
(b) break
(c) breaks down
(d) breaks

32. [MCQ] [GATE-CS-2018:1M]

"From where are they bringing their books? ________ bringing _______ books from _____."

The words that best fill the blanks in the above sentence are

(a) their, they're, there
(b) they're, their, there
(c) there, their, they're
(d) they're, there, there

33. [MCQ] [GATE-ME-2017:1M]

The ways in which this game can be played _______ potentially infinite.

(a) is
(b) is being
(c) are
(d) are being

34. [MCQ] [GATE-ME-2017:1M]

He was one of my best __________ and I felt his loss _________.

(a) friend, keenly (b) friends, keen
(c) friend, keener (d) friends, keenly

35. [MCQ] [GATE-CS, EE-2017:1M]

After Rajendra Chola returned from his voyage to Indonesia, he ___________ to visit the temple in Thanjavur.

(a) was wishing (b) is wishing
(c) wished (d) had wished

36. [MCQ] [GATE-CS, EE-2017:1M]

Saturn is ____________to be seen on a clear night with the naked eye.

(a) enough bright (b) bright enough
(c) as enough bright (d) bright as enough

37. [MCQ] [GATE-EC-2017:1M]

It is _____ to read this year's textbook _______ the last year's.

(a) easier, than (b) most easy, than
(c) easier, from (d) easiest, from

38. [MCQ] [GATE-EC-2017:1M]

I_____made arrangements had I _____ informed earlier.

(a) could have, been (b) would have, being
(c) had, have (d) had been, been

39. [MCQ] [GATE-CE-2017:1M]

The bacteria in milk are destroyed when it _______ heated to 80 degree Celsius.

(a) would be (b) will be
(c) is (d) was

40. [MCQ] [GATE-CE-2017:1M]

The event would have been successful if you ______ able to come.

(a) are (b) had been
(c) have been (d) would have been

41. [MCQ] [GATE-ME, EC-2016:1M]

Based on the given statements, select the appropriate option with respect to grammar and usage.

Statements

(i) The height of Mr. X is 6 feet.
(ii) The height of Mr. Y is 5 feet.
(a) Mr. X is longer than Mr. Y.
(b) Mr. X is more elongated than Mr. Y.
(c) Mr. X is taller than Mr. Y.
(d) Mr. X is lengthier than Mr. Y.

42. [MCQ] [GATE-ME-2016:1M]

The unruly crowd demanded that the accused be _____________ without trial.

(a) hanged (b) hanging
(c) hankering (d) hung

43. [MCQ] [GATE-ME-2016:1M]

The volume of a sphere of diameter 1 unit is ________ than the volume of a cube of side 1 unit.

(a) least (b) less
(c) lesser (d) low

44. [MCQ] [GATE-ME, EC-2016:1M]

Which of the following is CORRECT with respect to grammar and usage?

Mount Everest is ____________.

(a) the highest peak in the world
(b) highest peak in the world
(c) one of highest peak in the world
(d) one of the highest peaks in the world

45. [MCQ] [GATE-CS, EE-2016:1M]

The man who is now Municipal Commissioner worked as ____________.

(a) the security guard at a university
(b) a security guard at the university
(c) a security guard at university
(d) the security guard at the university

46. [MCQ] [GATE-EC-2016:1M]

An apple costs Rs. 10. An onion costs Rs. 8.

Select the most suitable sentence with respect to grammar and usage.

(a) The price of an apple is greater than an onion.
(b) The price of an apple is more than onion.
(c) The price of an apple is greater than that of an onion.
(d) Apples are more costlier than onions.

47. [MCQ] [GATE-CE-2016:1M]

If I were you, I _______ that laptop. It's much too expensive.

(a) won't buy (b) shan't buy
(c) wouldn't buy (d) would buy

48. [MCQ] [GATE-CE-2016:1M]

Choose the most appropriate set of words from the options given below to complete the following sentence _________ is a will ______ is a way.

(a) wear, there, their (b) were, their, there
(c) where, there, there (d) where, their, their

49. [MCQ] [GATE-CS, CE-2016:1M]

Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

(a) I will not leave the place until the minister does not meet me.
(b) I will not leave the place until the minister doesn't meet me.
(c) I will not leave the place until the minister meet me.
(d) I will not leave the place until the minister meets me.

50. [MCQ] [GATE-ME-2015:2M]

Select the appropriate option in place of underlined part of the sentence.

<u>Increased productivity necessary</u> reflects greater efforts made by the employees.

(a) Increase in productivity necessary
(b) Increase productivity is necessary
(c) Increase in productivity necessarily
(d) No improvement required

51. [MCQ] [GATE-ME, EC-2015:2M]

Ram and Shyam shared a secret and promised to each other that it would remain between them. Ram expressed himself in one of the following ways as given in the choices below. Identify the correct way as per standard English.

(a) It would remain between you and me.
(b) It would remain between I and you.
(c) It would remain between you and I
(d) It would remain with me.

52. [MCQ] [GATE-ME, EC-2015:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

If the athlete had wanted to come first in the race, he_________________ several hours every day.

(a) should practice
(b) should have practiced
(c) practiced
(d) should be practicing

53. [MCQ] [GATE-ME, EC-2015:2M]

In the following sentence certain parts are underlined and marked P, Q and R. One of the parts may contain certain error or may not be acceptable in standard written communication. Select the part containing an error. Choose D as your answer if there is no error.

The student corrected <u>all the errors</u> (P) that <u>the instructor marked</u> (Q) on the <u>answer book</u> (R).

(a) P (b) Q
(c) R (d) No Error

54. [MCQ] [GATE-ME, EC-2015:1M]

What is the adverb for the given word below?

Misogynous

(a) Misogynousness (b) Misogynity
(c) Misogynously (d) Misogynous

55. [MCQ] [GATE-ME, EC-2015:1M]

Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence.

Dhoni, as well as the other team members of Indian team, _____ present on the occasion

(a) were (b) was
(c) has (d) have

56. [MCQ] [GATE-EE, CS-2015:1M]

We __________ our friend's birthday and we ______________ how to make it up to him

(a) completely forgot – don't just know
(b) forgot completely - don't just know
(c) completely forgot - just don't know
(d) forgot completely - just don't know

57. [MCQ] [GATE-EE, CS-2015:1M]

Didn't you buy ______________ when you went shopping?

(a) any paper
(b) much paper
(c) no paper
(d) a few paper

58. [MCQ] [GATE-EE, CS-2015:2M]

Out of the following four sentences, select the most suitable sentence with respect to grammar and usage

(a) Since the report lacked needed information, it was of no use to them
(b) The report was useless to them because there were no needed information in it.
(c) Since the report did not contain the needed information, it was not real useful to them
(d) Since the report lacked needed information, it would not had been useful to them

59. [MCQ] [GATE-EC-2015:2M]

The following question presents a sentence, part of which is underlined. Beneath the sentence you find four ways of phrasing the underlined part. Following the requirements of the standard written English, select the answer that produces the most effective sentence.

Tuberculosis, together with its effects, <u>ranks one of the leading causes of death</u> in India

(a) ranks as one of the leading causes of death
(b) rank as one of the leading causes of death
(c) has the rank of one of the leading causes of death
(d) are one of the leading causes of death

60. [MCQ] [GATE-CS, CE-2015:1M]

The Tamil version of _________________ John Abraham-starrer *Madras Cafe* _______ cleared by the Censor Board with no cuts last week but the film's distributors __________ no takers among the exhibitors for a release in Tamil Nadu ______this Friday.

(a) Mr., was, found, on
(b) a, was, found, at
(c) the, was, found, on
(d) a, being, find, at

61. [MCQ] [GATE-ME, EC-2014:1M]

'Advice' is ________________.

(a) a verb
(b) a noun
(c) an adjective
(d) both a verb and a noun

62. [MCQ] [GATE-ME, EC-2014:1M]

Which of the options given below best completes the following sentence?

She will feel much better if she ________________.

(a) will get some rest
(b) gets some rest
(c) will be getting some rest
(d) is getting some rest

63. [MCQ] [GATE-ME, EC-2014:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

Communication and interpersonal skills are ________ important in their own ways.

(a) each
(b) both
(c) all
(d) either

64. [MCQ] [GATE-ME, EC-2014:1M]

Choose the most appropriate phrase from the options given below to complete the following sentence.

The aircraft __________ take off as soon as its flight plan was filed.

(a) is allowed to
(b) will be allowed to
(c) was allowed to
(d) has been allowed to

65. [MCQ] [GATE-EE, CS-2014:1M]

Who ___________ was coming to see us this evening?

(a) you said
(b) did you say
(c) did you say that
(d) had you said

66. [MCQ] [GATE-EE, CS-2014:1M]

While trying to collect an envelope from under the table,
I II
Mr. X fell down and was losing consciousness
III IV

Which one of the above underlined parts of the sentence is NOT appropriate?

(a) I (b) II
(c) III (d) IV

67. [MCQ] [GATE-EE, CS-2014:1M]

If she ______________ how to calibrate the instrument, she ______________ done the experiment.

(a) knows, will have
(b) knew, had
(c) had known, could have
(d) should have known, would have

68. [MCQ] [GATE-CE-2014:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

A person suffering from Alzheimer's disease ______ short-term memory loss.

(a) experienced (b) has experienced
(c) is experiencing (d) experiences

69. [MCQ] [GATE-ME, CS-2013:1M]

Choose the grammatically INCORRECT sentence:

(a) He is of Asian origin
(b) They belonged to Africa
(c) She is an European.
(d) They migrated from India to Australia

70. [MCQ] [GATE-ME, CS-2013:1M]

Were you a bird, you _______ in the sky.

(a) would fly (b) shall fly
(c) should fly (d) shall have flown

71. [MCQ] [GATE-EC, EE-2013:1M]

Choose the grammatically CORRECT sentence:

(a) Two and two add four
(b) Two and two become four
(c) Two and two are four
(d) Two and two make four.

72. [MCQ] [GATE-CE-2013:1M]

The professor ordered to the student to go out of the class
I II III IV

Which of the above underlined parts of the sentence is grammatically incorrect?

(a) I (b) II
(c) III (d) IV

73. [MCQ] [GATE-ME, CE, CS-2012:1M]

Choose the grammatically INCORRECT sentence:

(a) They gave us the money back less the service charges of Three Hundred rupees.
(b) This country's expenditure is not less than that of Bangladesh.
(c) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.
(d) This country's expenditure on educational reforms is very less

74. [MCQ] [GATE-ME, CE, CS-2012:1M]

Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one ________ was hurt in the stampede

(a) that (b) which
(c) who (d) whom

75. [MCQ] [GATE-EC, EE-2012:1M]

One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?

I requested that he should be given the driving test today instead of tomorrow.

(a) requested that (b) should be given
(c) the driving test (d) instead of tomorrow

76. [MCQ] [GATE-EC, EE-2012:1M]

Choose the most appropriate alternative from the options given below to complete the following sentence:

If the tired soldier wanted to lie down, he _______ the mattress out on the balcony

(a) should take (b) shall take
(c) should have taken (d) will have taken

77. [MCQ] [GATE-ME, CE, CS-2011:1M]

Choose the most appropriate word(s) from the options given below to complete the following sentence.

I contemplated ______ Singapore for my vacation but decided against it.

(a) to visit (b) having to visit
(c) visiting (d) for a visit

Basic English Vocabulary

78. [MCQ] [GATE-ME-2023:1M]

Planting : Seed : : Raising : _____

(By word meaning)

(a) Child (b) Temperature
(c) Height (d) Lift

79. [MCQ] [GATE-EE-2023:1M]

Permit : _______ : : Enforce : Relax

(By word meaning)

(a) Allow (b) Forbid
(c) License (d) Reinforce

80. [MCQ] [GATE-EC-2023:1M]

Courts : _______: : Parliament : Legislature

(By word meaning)

(a) Judiciary (b) Executive
(c) Governmental (d) Legal

81. [MCQ] [GATE-EC-2023:1M]

"I cannot support this proposal. My ________ will not permit it."

(a) conscious (b) consensus
(c) conscience (d) consent

82. [MCQ] [GATE-CE-2023:1M]

Kind : ____ : : Often : Seldom

(By word meaning)

(a) Cruel (b) Variety
(c) Type (d) Kindred

83. [MCQ] [GATE-CE-2023:1M]

Eject : Insert : : Advance : ____

(By word meaning)

(a) Advent (b) Progress
(c) Retreat (d) Loan

84. [MCQ] [GATE-CE-2023:2M]

Based only on the truth of the statement 'some humans are intelligent', which one of the following options can be logically inferred with certainty?

(a) No human is intelligent.
(b) All humans are intelligent.
(c) Some non-humans are intelligent.
(d) Some intelligent beings are humans.

85. [MCQ] [GATE-CS-2023:1M]

We reached the station late, and _______ missed the train.

(a) near (b) nearly
(c) utterly (d) mostly

86. [MCQ] [GATE-CS-2023:1M]

Kind : _______ : : Often : Frequently

(By word meaning)

(a) Mean (b) Type
(c) Cruel (d) Kindly

87. [MCQ] [GATE-CS-2023:2M]

Which one of the following sentence sequences creates a coherent narrative?

(i) Once on the terrace, on her way to her small room in the corner, she notices the man right away.
(ii) She begins to pant by the time she has climbed all the stairs.
(iii) Mina has bought vegetables and rice at the market, so her bags are heavy.
(iv) He was leaning against the parapet, watching the traffic below.

(a) (i), (ii), (iv), (iii)
(b) (ii), (iii), (i), (iv)
(c) (iv), (ii), (i), (iii)
(d) (iii), (ii), (i), (iv)

88. [MCQ] [GATE-ME-2022:1M]

Writing too many things on the ________ while teaching could make the students get_______.

(a) bored / board (b) board / bored
(c) board / board (d) bored / bored

89. [MCQ] [GATE-ME-2022:1M]

After playing _________ hours of tennis, I am feeling _________ tired to walk back.

(a) too / too (b) too / two
(c) two / two (d) two / too

90. [MCQ] [GATE-EE-2022:1M]

As you grow older, an injury to your _________ may take longer to _________.

(a) heel/heel (b) heal/heel
(c) heal/heal (d) heel/heal

91. [MCQ] [GATE-EC-2022:1M]

Mr. X speaks _________ Japanese _________ Chinese.

(a) neither / or (b) either / nor
(c) neither / nor (d) also / but

92. [MCQ] [GATE-CE-2022:1M]

You should ______ when to say _______

(a) no/no (b) no/know
(c) know/know (d) know/no

93. [MCQ] [GATE-CS-2022:1M]

The _________ is too high for it to be considered _________.

(a) fair/ fare (b) fair/ fair
(c) fare/ fare (d) fare/ fair

94. [MCQ] [GATE-EC-2021:1M]

Nostalgia is to *anticipation* as _______ is to _______.

Which one of the following options maintains a similar logical relation in the above sentence?

(a) present, past (b) future, past
(c) past, future (d) future, present

95. [MCQ] [GATE-EE-2021:1M]

Oasis is to sand as island is to ______________.

Which one of the following options maintains a similar logical relation in the above sentence?

(a) stone (b) land
(c) water (d) mountain

96. [MCQ] [GATE-CS-2021:1M]

______ is to surgery as writer is to ________

Which one of the following options maintains a similar logical relation in the above sentence?

(a) Plan, outline (b) Hospital, library
(c) Doctor, book (d) Medicine, grammar

97. [MCQ] [GATE-CS-2021:1M]

Pen : Write : : Knife : ___________

Which one of the following options maintains a similar logical relation in the above?

(a) Vegetables (b) Sharp
(c) Cut (d) Blunt

98. [MCQ] [GATE-ME-2020:1M]

In one of the greatest innings ever seen in 142 years of Test history, Ben Stokes upped the tempo in a five-and-a-half hour long stay of 219 balls including 11fours and 8 sixes that saw him finish on a 135 not out as England squared the five-match series.

Based on their connotations in the given passage, which one of the following meanings DOES NOT match?

(a) upped = increased (b) squared = lost
(c) tempo = enthusiasm (d) saw = resulted in

99. [MCQ] [GATE-ME-2020:1M]

Select the word that fits the analogy:

White : Whitening : : Light

(a) Lightning (b) Lightening
(c) Lighting (d) Enlightening

100. [MCQ] [GATE-ME-2020:1M]

The recent measure to improve the output would ____the level of production to our satisfaction.

(a) increase (b) decrease
(c) speed (d) equalize

101.[MCQ] [GATE-ME-2020:1M]

I do not think you know the case well enough to have opinions. Having said that, I agree with your other point. What does the phrase "having said that" mean in the given text?

(a) as opposed to what I have said
(b) despite what I have said
(c) in addition to what I have said
(d) contrary to what I have said

102.[MCQ] [GATE-ME-2020:1M]

Select the word that fits the analogy:

Build : Building : : Grow : ______

(a) Grown (b) Grew
(c) Growth (d) Growed

103.[MCQ] [GATE-ME-2020:1M]

He is known for his unscrupulous ways. He always sheds ______ tears to deceive people.

(a) fox's (b) crocodile's
(c) crocodile (d) fox

104.[MCQ] [GATE-EE-2020:1M]

Stock markets _________ at the news of the coup.

(a) poised (b) plunged
(c) plugged (d) probed

105.[MCQ] [GATE-EE-2020:1M]

Select the word that fits the analogy:

Do : Undo : : Trust : _________

(a) Entrust (b) Intrust
(c) Distrust (d) Untrust

106.[MCQ] [GATE-EC-2020:1M]

Select the word that fits the analogy:

Explicit : Implicit : : Express: ___________.

(a) Impress (b) Repress
(c) Compress (d) Suppress

107.[MCQ] [GATE-CE-2020:1M]

Select the word that fits the analogy:

Fuse : Fusion :: Use : _____.

(a) Usage (b) User
(c) Uses (d) Usion

108.[MCQ] [GATE-CE-2020:2M]

It is a common criticism that most of the academicians live in their _____ so, they are not aware of the real-life challenges.

(a) homes (b) ivory towers
(c) glass palaces (d) big flats

109.[MCQ] [GATE-CE-2020:1M]

His hunger for reading is insatiable. He reads indiscriminately. He is most certainly a/an ____ reader.

(a) all-round (b) precocious
(c) voracious (d) wise

110.[MCQ] [GATE-CE-2020:1M]

Select the most appropriate word that can replace the underlined word without changing the meaning of the sentence:

Now a days, most children have a tendency to <u>belittle</u> the legitimate concerns of their parents.

(a) disparage (b) applaud
(c) reduce (d) begrudge

111.[MCQ] [GATE-CS-2020:1M]

His knowledge of the subject was excellent, but his classroom performance was _______.

(a) extremely poor (b) good
(c) desirable (d) praiseworthy

112.[MCQ] [GATE-CS-2020:1M]

Select the word that fits the analogy:

Cook : Cook :: Fly :

(a) Flyer (b) Flying
(c) Flew (d) Flighter

113.[MCQ] [GATE-ME-2019:1M]

Are there enough seats here? There are ______ People here than I expected.

(a) many (b) most
(c) least (d) more

114.[MCQ] [GATE-ME-2019:1M]

A final examination is the _______ of a series of evaluations that a student has to go through.

(a) culmination (b) consultation
(c) desperation (d) insinuation

115.[MCQ] [GATE-ME-2019:1M]

The minister avoided any mention of the issue of women's reservation in the private sector. He was accused of ______the issue

(a) collaring (b) skirting
(c) tying (d) belting

116.[MCQ] [GATE-ME-2019:1M]

John Thomas, an ________writer, passed away in 2018.

(a) Imminent (b) Prominent
(c) Eminent (d) Dominant

117.[MCQ] [GATE-EE-2019:1M]

Newspapers are a constant source of delight and recreation for me. The _________ trouble is that I read __________ many of them.

(a) even, quite (b) even, too
(c) only, quite (d) only, too

118.[MCQ] [GATE-EE-2019:1M]

I am not sure if the bus that has been booked will be able to _______ all the students.

(a) sit (b) deteriorate
(c) fill (d) accommodate

119.[MCQ] [GATE-CE-2019:1M]

The lecture was attended by quite _______ students, so the hall was not very ________.

(a) a few, quite (b) few, quiet
(c) a few, quiet (d) few, quite

120.[MCQ] [GATE-CE-2019:1M]

Daytime temperature in Delhi can __________ 40°C.

(a) get (b) stand
(c) reach (d) peak

121.[MCQ] [GATE-CE-2019:1M]

The growth rate of ABC motors in 2017 was the same _________ XYZ motors in 2016

(a) as off (b) as those of
(c) as that off (d) as that of

122.[MCQ] [GATE-CE-2019:2M]

"The increasing interest in tribal characters might be a mere coincidence, but the timing is of interest. None of this, though, is to say that the tribal hero has arrived in Hindi cinema, or that the new crop of characters represents the acceptance of the tribal character in the industry. The films and characters are too few to be described as a pattern."

What does the word 'arrived' mean in the paragraph above?

(a) reached a terminus (b) came to a conclusion
(c) attained a status (d) went to a place

123.[MCQ] [GATE-CS-2019:1M]

The search engine's business model ______ around the fulcrum of trust.

(a) revolves (b) plays
(c) sinks (d) bursts

124.[MCQ] [GATE-CS-2019:1M]

A court is to a judge as ________ is to a teacher.

(a) a student (b) a punishment
(c) a syllabus (d) a school

125.[MCQ] [GATE-ME-2018:1M]

"The judge's standing in the legal community, though shaken by false allegations of wrongdoing, remained _________."

The word that best fills the blank in the above sentence is

(a) undiminished (b) damaged
(c) illegal (d) uncertain

126.[MCQ] [GATE-ME-2018:1M]

"The dress _________ her so well that they all immediately _________ her on her appearance."

The words that best fill the blanks in the above sentence are

(a) complemented, complemented
(b) complimented, complemented
(c) complimented, complimented
(d) complemented, complimented

127.[MCQ] [GATE-ME-2018:1M]

"Her _______ should not be confused with miserliness; she is ever willing to assist those in need."

The word that best fills the blank in the above sentence is

(a) cleanliness (b) punctuality
(c) frugality (d) greatness

128.[MCQ] [GATE-ME-2018:1M]

"Going by the _________ that many hands make light work, the school _______ involved all the students in the task."

The words that best fill the blanks in the above sentence are

(a) principle, principal (b) principal, principle
(c) principle, principle (d) principal, principal

129.[MCQ] [GATE-EE-2018:1M]

"A common misconception among writers is that sentence structure mirrors thought; the more _________ the structure, the more complicated the ideas".

The word that best fills the blank in the above sentence is

(a) detailed (b) simple
(c) clear (d) convoluted

130.[MCQ] [GATE-EE-2018:1M]

"Since you have gone off the ________, the _________ sand is likely to damage the car."

The words that best fill the blanks in the above sentence are

(a) course, coarse (b) course, course
(c) coarse, course (d) coarse, coarse

131.[MCQ] [GATE-EC-2018:1M]

"Even though there is a vast scope for its _____tourism has remained a/an ________ area."

The words that best fill the blanks in the above sentence are

(a) improvement, neglected
(b) rejection, approved
(c) fame, glum
(d) interest, disinterested

132.[MCQ] [GATE-EC-2018:1M]

"By giving him the last ______ of the cake, you will ensure lasting _______ in our house today."

The words that best fill the blanks in the above sentence are

(a) peas, piece (b) piece, peace
(c) peace, piece (d) peace, peas

133.[MCQ] [GATE-CE-2018:1M]

"The driver applied the _____ as soon as she approached the hotel where she wanted to take a _______.

(a) brake, break (b) break, break
(c) brake, brake (d) break, brake

134.[MCQ] [GATE-CE-2018:1M]

"It is no surprise that every society has had codes of behavior. However the nature of these codes is often_______"

The word that best fills the blank in the above sentence is

(a) unpredictable (b) simple
(c) expected (d) strict

135.[MCQ] [GATE-CE-2018:1M]

"His face ______ with joy when the solution of the puzzle was _____ to him."

The words that best fill the blanks in the above sentence are

(a) shone, shown (b) shone, shone
(c) shown, shone (d) shown, shown

136.[MCQ] [GATE-CE-2018:1M]

"Although it does contain some pioneering ideas one would hardly characterize the work as______.

The word that best fills the blank in the above sentence is

(a) innovative (b) single
(c) dull (d) boring

137.[MCQ] [GATE-CS-2018:1M]

"A _________ investigation can sometimes yield new facts, but typically organized ones are more successful."

The word that best fills the blank in the above sentence is

(a) meandering (b) timely
(c) consistent (d) systematic

138.[MCQ] [GATE-ME-2017:2M]

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters."

Which of the following is closest in meaning to 'cleaving'?

(a) Deteriorating (b) Arguing
(c) Departing (d) Splitting

139.[MCQ] [GATE-ME-2017:1M]

If you choose plan P, you will have to ______ plan Q, as these two are mutually _______.

(a) forgo, exclusive (b) forget, inclusive
(c) accept, exhaustive (d) adopt, intrusive

140.[MCQ] [GATE-ME-2017:1M]

As the two speakers became increasingly agitated, the debate became _________.

(a) lukewarm (b) poetic
(c) forgiving (d) heated

141.[MCQ] [GATE-CS, EE-2017:1M]

Research in the workplace reveals that people work for many reasons_____________.

(a) money beside (b) beside money
(c) money besides (d) besides money

142.[MCQ] [GATE-CS, EE-2017:1M]

Choose the option with words that are not synonyms.

(a) aversion, dislike (b) luminous, radiant
(c) plunder, loot (d) yielding, resistant

143.[MCQ] [GATE-EC-2017:1M]

The ninth and the tenth of this month are Monday and Tuesday ____________.

(a) figuratively (b) retrospectively
(c) respectively (d) rightfully

144.[MCQ] [GATE-EC-2017:2M]

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages: for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".

Here, the word 'antagonistic' is closest in meaning to

(a) Impartial (b) argumentative
(c) separated (d) hostile

145.[MCQ] [GATE-EC-2017:1M]

She has a sharp tongue and it can occasionally turn _____.

(a) hurtful (b) left
(c) methodical (d) vital

146.[MCQ] [GATE-CE-2017:1M]

______ with someone else's email account is now a very serious offence.

(a) involving (b) Assisting
(c) Tampering (d) Incubating

147.[MCQ] [GATE-CE-2017:1M]

There was no doubt that their work was <u>thorough</u>

Which of the words below is closest in meaning to the underline word above?

(a) Pretty (b) Complete
(c) Sloppy (d) Haphazard

148.[MCQ] [GATE-ME, EC-2016:1M]

After India's cricket world cup victory in 1985, Shrotria who was playing both tennis and cricket till then, decided to concentrate only on cricket. And the <u>rest is history</u>.

What does the underlined phrase mean in this context?

(a) History will rest in peace
(b) Rest is recorded in history books
(c) Rest is well known
(d) Rest is archaic

149.[MCQ] [GATE-ME, EC-2016:1M]

The students ___________ the teacher on teachers' day for twenty years of dedicated teaching.

(a) facilitated (b) felicitated
(c) fantasized (d) facillitated

150.[MCQ] [GATE-ME-2016:1M]

Choose the statement(s) where the underlined word is used correctly:

(i) A prone is a dried plum.
(ii) He was lying prone on the floor.
(iii) People who eat a lot of fat are prone to heart disease.

(a) (i) and (iii) only (b) (iii) only
(c) (i) and (ii) only (d) (ii) and (iii) only

151.[MCQ] [GATE-ME, EC-2016:1M]

The policeman asked the victim of a theft, "What did you _______ ?"

(a) loose (b) lose
(c) loss (d) louse

152.[MCQ] [GATE-ME, EC-2016:1M]

Despite the new medicine's ______________ in treating diabetes, it is not ______________widely.

(a) effectiveness --- prescribed
(b) availability --- used
(c) prescription --- available
(d) acceptance --- proscribed

153.[MCQ] [GATE-EE-2016:1M]

Identify the correct spelling out of the given options:

(a) Managable (b) Manageable
(c) Mangaeble (d) Managible

154.[MCQ] [GATE-EE-2016:1M]

The chairman requested the aggrieved shareholders to ______________ him.

(a) bare with (b) bore with
(c) bear with (d) bare

155.[MCQ] [GATE-CS, EE-2016:1M]

Nobody knows how the Indian cricket team is going to <u>cope with</u> the difficult and seamer-friendly wickets in Australia.

Choose the option which is closest in meaning to the underlined phrase in the above sentence.

(a) put up with (b) put in with
(c) put down to (d) put up against

156.[MCQ] [GATE-CS, EE-2016:1M]

Find the odd one in the following group of words. mock, deride, praise, jeer

(a) mock (b) deride
(c) praise (d) jeer

157.[MCQ] [GATE-EC-2016:1M]

The Buddha said, "Holding on to anger is like <u>grasping</u> a hot coal with the intent of throwing it at someone else; you are the one who gets burnt."

Select the word below which is closest in meaning to the word underlined above.

(a) Burning (b) Igniting
(c) Clutching (d) Flinging

158.[MCQ] [GATE-CE-2016:1M]

He turned a deaf ear to my request

What does the underlined phrasal verb mean?

(a) ignored (b) appreciated
(c) twisted (d) returned

159.[MCQ] [GATE-CS, CE-2016:1M]

A rewording of something written or spoken is a ______________.

(a) paraphrase (b) paradox
(c) paradigm (d) paraffin

160.[MCQ] [GATE-CS, CE-2016:1M]

Archimedes said, "Give me a lever long enough and a fulcrum on which to place it, and I will move the world."

The sentence above is an example of a ___________ statement.

(a) figurative (b) collateral
(c) literal (d) figurine

161.[MCQ] [GATE-ME-2015:1M]

Fill in the blank with the correct idiom/phrase.

That boy from the town was a _____ in the sleepy village.

(a) Dog out of herd (b) Sheep form the heap
(c) fish out of water (d) bird from the flock

162.[MCQ] [GATE-ME-2015:1M]

Choose the statement where underlined word is used correctly.

(a) When the teacher eludes to different authors, he is being elusive.
(b) When the thief keeps eluding the police, he is being elusive.
(c) Matters that are difficult to understand, identify or remember are allusive.
(d) Mirages can be allusive, but a better way to express them is illusory

163.[MCQ] [GATE-ME-2015:1M]

Choose the appropriate word/phrase, out of the four option given below, to complete the following sentence:

Apparent lifelessness______ dormant life.

(a) harbours (b) leads to
(c) supports (d) affects

164.[MCQ] [GATE-ME, EC-2015:1M]

Choose the correct verb to fill in the blank below:

Let us _______________.

(a) introvert (b) alternate
(c) atheist (d) altruist

165.[MCQ] [GATE-ME, EC-2015:1M]

Choose the most suitable one word substitute for the following expression:

Connotation of a road or way

(a) Pertinacious (b) Viaticum
(c) Clandestine (d) Ravenous

166.[MCQ] [GATE-ME, EC-2015:1M]

Choose the word most similar in meaning to the given word:

Awkward

(a) Inept (b) Graceful
(c) Suitable (d) Dreadful

167.[MCQ] [GATE-EE, CS-2015:1M]

Which of the following options is the closest in meaning to the sentence below?

She enjoyed herself immensely at the party.

(a) She had a terrible time at the party
(b) Site had a horrible time at the party
(c) She had a terrific time at the party
(d) She had a terrifying time at the party

168.[MCQ] [GATE-EE, CS-2015:1M]

Which one of the following combinations is incorrect?

(a) Acquiescence – Submission
(b) Wheedle - Roundabout
(c) Flippancy – Lightness
(d) Profligate – Extravagant

169.[MCQ] [GATE-EE, CS-2015:2M]

Select the alternative meaning of the underlined part of the sentence.

The chain snatchers took to their heels when the police party arrived.

(a) took shelter in a thick jungle
(b) open indiscriminate fire
(c) took to flight
(d) unconditionally surrendered

170.[MCQ] [GATE-EE, CS-2015:1M]

Choose the statement where underlined word is used correctly.

(a) The industrialist had a personnel jet
(b) I write my experience in my personnel diary
(c) All personnel are being given the day off
(d) Being religious is a personnel aspect

171.[MCQ] [GATE-EE, CS-2015:1M]

A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is

(a) Fabric (b) Textile
(c) Fibre (d) Apparel

172.[MCQ] [GATE-EC-2015:1M]

Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:

Frogs _______________.

(a) croak (b) roar
(c) hiss (d) patter

173.[MCQ] [GATE-EC-2015:1M]

Choose the word most similar in meaning to the given word:

Educe

(a) Exert (b) Educate
(c) Extract (d) Extend

174.[MCQ] [GATE-EC-2015:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

The principal presented the chief guest with a __________ as token of appreciation.

(a) momento (b) memento
(c) momentum (d) moment

175.[MCQ] [GATE-CE-2015:1M]

Choose the most appropriate word from the option given below to complete the following sentence.

The official answered _________ that the complaints of the citizen would be looked into.

(a) respectably (b) respectfully
(c) reputably (d) respectively

176.[MCQ] [GATE-CE-2015:1M]

Choose the statement where underlined word is used correctly.

(a) The minister <u>insured</u> the victims that everything would be all right.
(b) He <u>ensured</u> that the company will not have to bear any loss
(c) The actor got himself <u>ensured</u> against any accident.
(d) The teacher <u>insured</u> students of good results

177.[MCQ] [GATE-CE-2015:1M]

Which word is not a synonym for the word vernacular?

(a) Regional (b) Indigenous
(c) Indigent (d) colloquial

178.[MCQ] [GATE-CE-2015:2M]

The word similar in meaning to 'dreary' is

(a) cheerful (b) dreamy
(c) hard (d) dismal

179.[MCQ] [GATE-CS, CE-2015:1M]

Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything _____________ to the requirements of the exam.

(a) related (b) extraneous
(c) outside (d) useful

180.[MCQ] [GATE-CS, CE-2015:1M]

Select the pair that best expresses a relationship similar to that expressed in the pair:

Children : Pediatrician

(a) Adult : Orthopaedist
(b) Females : Gynaecologist
(c) Kidney : Nephrologist
(d) Skin : Dermatologist

181.[MCQ] [GATE-ME, EC-2014:1M]

While receiving the award, the scientist said, "I feel vindicated". Which of the following is closest in meaning to the word 'vindicated'?

(a) Punished (b) Substantiated
(c) Appreciated (d) Chastened

182.[MCQ] [GATE-ME, EC-2014:1M]

After the discussion, Tom said to me, 'Please revert!'. He expects me to __________.

(a) retract (b) get back to him
(c) move in reverse (d) retreat

183.[MCQ] [GATE-ME, EC-2014:1M]

Which of the following options is the closest in meaning to the word underlined in the sentence below?

In a democracy, everybody has the freedom to <u>disagree</u> with the government.

(a) Dissent (b) Descent
(c) Decent (d) Decadent

184.[MCQ] [GATE-ME, EC-2014:1M]

The value of one U.S. dollar is 65 Indian Rupees today, compared to 60 last year. The Indian Rupee has ____________.

(a) depressed (b) depreciated
(c) appreciated (d) stabilized

185.[MCQ] [GATE-ME, EC-2014:1M]

Choose the most appropriate pair of words from the options given below to complete the following sentence.

She could not _____ the thought of _________ the election to her bitter rival.

(a) bear, loosing (b) bare, loosing
(c) bear, losing (d) bare, losing

186.[MCQ] [GATE-ME, EC-2014:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

Many ancient cultures attributed disease to supernatural causes. However, modern science has largely helped _________ such notions.

(a) impel (b) dispel
(c) propel (d) repel

187.[MCQ] [GATE-EE, CS-2014:1M]

Which of the following options is the closest in meaning to the phrase underlined in the sentence below?

It is fascinating to see life forms <u>cope with</u> varied environmental conditions.

(a) Adopt to (b) Adapt to
(c) Adept in (d) Accept with

188.[MCQ] [GATE-EE, CS-2014:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

He could not understand the judges awarding her the first prize, because he thought that her performance was quite __________.

(a) superb (b) medium
(c) mediocre (d) exhilarating

189.[MCQ] [GATE-EE, CS-2014:1M]

In a press meet on the recent scam, the minister said, "The buck stops here". What did the minister convey by the statement?

(a) He wants all the money
(b) He will return the money
(c) He will assume final responsibility
(d) He will resist all enquiries

190.[MCQ] [GATE-EE, CS-2014:1M]

Choose the most appropriate phrase from the options given below to complete the following sentence.

India is a post-colonial country because

(a) It was a former British colony
(b) Indian Information Technology professionals have colonized the world
(c) India does not follow any colonial practices
(d) India has helped other countries gain freedom

191.[MCQ] [GATE-EE, CS-2014:1M]

Match the columns

	Column -I		Column-II
1.	Eradicate	P.	Misrepresent
2.	Distort	Q.	Soak completely
3.	Saturate	R.	Use
4.	Utilize	S.	Destroy utterly

(a) $1\rightarrow S, 2\rightarrow P, 3\rightarrow Q, 4\rightarrow R$
(b) $1\rightarrow P, 2\rightarrow Q, 3\rightarrow R, 4\rightarrow S$
(c) $1\rightarrow Q, 2\rightarrow R, 3\rightarrow S, 4\rightarrow P$
(d) $1\rightarrow S, 2\rightarrow P, 3\rightarrow R, 4\rightarrow Q$

192.[MCQ] [GATE-EE, CS-2014:1M]

Choose the word that is opposite in meaning to the word “coherent”.

(a) Sticky
(b) Well-connected
(c) Rambling
(d) Friendly

193.[MCQ] [GATE-CE-2014:1M]

A student is required to demonstrate a high level of comprehension of the subject especially in the social sciences.

The word closest in meaning to comprehension is

(a) Understanding
(b) Meaning
(c) Concentration
(d) Stability

194.[MCQ] [GATE-CE-2014:1M]

Choose the most appropriate word from the option given below to complete the following sentence.

One of his biggest ______ was his ability to forgive

(a) Vice (b) Virtues
(c) Choices (d) Strength

195.[MCQ] [GATE-CE-2014:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

________ is the key to their happiness; they are satisfied with what they have.

(a) Contentment (b) Ambition
(c) Perseverance (d) Hunger

196.[MCQ] [GATE-CE-2014:1M]

Which of the following option is the closest in meaning to the sentence below?

As a women, I have no country

(a) Women have no country.
(b) Women are not citizens of any country
(c) Women's solidarity knows no national boundaries
(d) Women of all countries have equal legal rights

197.[MCQ] [GATE-ME, CS-2013:1M]

Which one of the following options is the closest in meaning to the word given below?

Nadir

(a) Highest (b) Lowest
(c) Medium (d) Integration

198.[MCQ] [GATE-ME, CS-2013:1M]

Complete the sentence:

Universalism isto particularism as diffuseness is to ______.

(a) specificity (b) neutrality
(c) generality (d) adaptation

199.[MCQ] [GATE-EC, EE-2013:1M]

They were requested not to **quarrel** with others.

Which one of the following options is the closest in meaning to the word quarrel?

(a) Make out (b) Call out
(c) Dig out (d) Fall out

200.[MCQ] [GATE-EC, EE-2013:1M]

Complete the sentence:

Dare ________________ mistakes.

(a) commit (b) to commit
(c) committed (d) committing

201.[MCQ] [GATE-EC, EE-2013:1M]

Statement: You can always give me a ring whenever you need.

Which one of the following is the best inference from the above statement?

(a) Because I have a nice caller tune.
(b) Because I have a better telephone facility.
(c) Because a friend in need is a friend indeed.
(d) Because you need not pay towards the telephone bills when you give me a ring.

202.[MSQ] [GATE-CE-2013:1M]

Which of the following option is the closest in meaning to the word given below:

Primeval

(a) Modern (b) Historic
(c) Primitive (d) Antique

203.[MCQ] [GATE-CE-2013:1M]

Friendship no matter how _______ it is, has its limitations

(a) cordial (b) intimate
(c) secret (d) pleasant

204.[MCQ] [GATE-CE-2013:1M]

Select the pair that best expresses a relationship similar to that expressed in the pair:

Medicine: Health

(a) Science: Experiment
(b) Wealth: Peace
(c) Education: Knowledge
(d) Money: Happiness

205.[MCQ] [GATE-ME, CE, CS-2012:1M]

Which one of the following options is the closest in meaning to the word given below?

Mitigate

(a) Diminish (b) Divulge
(c) Dedicate (d) Denote

206.[MCQ] [GATE-ME, CE, CS-2012:1M]

Choose the most appropriate alternative from the options given below to complete the following sentence:

Despite several ________ the mission succeeded in its attempt to resolve the conflict

(a) attempts (b) setbacks
(c) meetings (d) delegations

207.[MCQ] [GATE-EC, EE-2012:1M]

Which one of the following options is the closest in meaning to the word given below?

Latitude

(a) Eligibility (b) Freedom
(c) Coercion (d) Meticulousness

208.[MCQ] [GATE-EC, EE-2012:1M]

Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his________ was impressive.

(a) beggary (b) nomenclature
(c) jealousy (d) nonchalance

209.[MCQ] [GATE-ME, CE, CS-2011:2M]

Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer form losses through death and parting.

Based on the above passage which topic would not be included in a unit on bereavement?

(a) How to write a letter of condolence
(b) What emotional stages are passed through in the healing process
(c) What the leading causes of death are
(d) How to give support to a grieving friend

210.[MCQ] [GATE-ME, CE, CS-2011:1M]

Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or _____.

(a) hyperbolic (b) restrained
(c) argumentative (d) indifferent

211.[MCQ] [GATE-ME, CE, CS-2011:1M]

Which of the following options is the closest in the meaning to the word below:

Inexplicable

(a) Incomprehensible (b) Indelible
(c) Inextricable (d) Infallible

212.[MCQ] [GATE-ME, CE, CS-2011:1M]

Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Amalgamate

(a) merge (b) split
(c) collect (d) separate

213.[MCQ] [GATE-EC, EE-2011:1M]

Choose the most appropriate word from the options given below to complete the following sentence:

Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which ___________ treatments are unsatisfactory.

(a) similar (b) most
(c) uncommon (d) available

214.[MCQ] [GATE-EC, EE-2011:1M]

Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Frequency

(a) periodicity (b) rarity
(c) gradualness (d) persistency

215.[MCQ] [GATE-EC, EE-2011:1M]

Choose the most appropriate word from the options given below to complete the following sentence:

It was her view that the country's problems had been ____________ by foreign technocrats, so that to invite them to come back would be counter-productive.

(a) identified (b) ascertained
(c) exacerbated (d) analysed

216.[MCQ] [GATE-EC, EE-2011:1M]

The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair:

Gladiator : Arena

(a) Dancer : stage (b) Commuter : train
(c) Teacher : classroom (d) Lawyer : courtroom

217.[MCQ] [GATE-ME,CE,EE,EC,CS-2010:1M]

Choose the most approximate word from the options given below to complete the following sentence:

His rather casual remarks on politics ______ his lack of seriousness about the subject.

(a) masked (b) belied
(c) betrayed (d) suppressed

218.[MCQ] [GATE-ME,CE,EE,EC,CS-2010:1M]

Which of the following options is the closest in meaning to the word below:

Circuitous

(a) Cyclic (b) Indirect
(c) Confusing (d) Crooked

219.[MCQ] [GATE-ME,CE,EE,EC,CS-2010:1M]

The question below consists of a pair related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed : Worker

(a) Fallow : Land (b) Unaware : Sleeper
(c) Wit : Jester (d) Renovated : House

220.[MCQ] [GATE-ME,CE,EE,EC,CS-2010:1M]

Choose the most approximate word from the options given below to complete the following sentence:

If we manage to ______ our natural resources, we would leave a better planet for our children.

(a) uphold (b) restrain
(c) cherish (d) conserve

Case Studies

221.[MCQ] [GATE-ME-2023:2M]

Which one of the sentence sequences in the given options creates a coherent narrative?

(i) I could not bring myself to knock.
(ii) There was a murmur of unfamiliar voices coming from the big drawing room and the door was firmly shut.
(iii) The passage was dark for a bit, but then it suddenly opened into a bright kitchen.
(iv) I decided I would rather wander down the passage.

(a) (iv), (i), (iii), (ii) (b) (iii), (i), (ii), (iv)
(c) (ii), (i), (iv), (iii) (d) (i), (iii), (ii), (iv)

222.[MCQ] [GATE-ME-2023:2M]

In a recently held parent-teacher meeting, the teachers had very few complaints about Ravi. After all, Ravi was a hardworking and kind student. Incidentally, almost all of Ravi's friends at school were hardworking and kind too. But the teachers drew attention to Ravi's complete lack of interest in sports. The teachers believed that, along with some of his friends who showed similar disinterest in sports, Ravi needed to engage in some sports for his overall development.

Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

(a) All of Ravi's friends are hardworking and kind.
(b) No one who is not a friend of Ravi is hardworking and kind.
(c) None of Ravi's friends are interested in sports.
(d) Some of Ravi's friends are hardworking and kind.

223.[MCQ] [GATE-EE-2023:2M]

Which one of the options does NOT describe the passage below or follow from it?

We tend to think of cancer as a 'modern' illness because its metaphors are so modern. It is a disease of overproduction, of sudden growth, a growth that is unstoppable, tipped into the abyss of no control. Modern cell biology encourages us to imagine the cell as a molecular machine. Cancer is that machine unable to quench its initial command (to grow) and thus transform into an indestructible, self-propelled automaton. [Adapted from The Emperor of All Maladies by Siddhartha Mukherjee]

(a) It is a reflection of why cancer seems so modern to most of us.
(b) It tells us that modern cell biology uses and promotes metaphors of machinery.
(c) Modern cell biology encourages metaphors of machinery, and cancer is often imagined as a machine.
(d) Modern cell biology never uses figurative language, such as metaphors, to describe or explain anything.

224.[MCQ] [GATE-EE-2023:2M]

Students of all the departments of a college who have successfully completed the registration process are eligible to vote in the upcoming college elections. However, by the time the due date for registration was over, it was found that suprisingly none of the students from the Department of Human Sciences had completed the registration process. Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with certainty?

(i) All those students who would not be eligible to vote in the college elections would certainly belong to the Department of Human Sciences.
(ii) None of the students from departments other than Human Sciences failed to complete the registration process within the due time.
(iii) All the eligible voters would certainly be students who are not from the Department of Human Sciences.

(a) (i) and (ii) (b) (i) and (iii)
(c) only (i) (d) only (iii)

225.[MCQ] [GATE-EE-2023:1M]

A recent survey shows that 65% of tobacco users were advised to stop consuming tobacco. The survey also shows that 3 out of 10 tobacco users attempted to stop using tobacco. Based only on the information in the above passage, which one of the following options can be logically inferred with certainty?

(a) A majority of tobacco users who were advised to stop consuming tobacco made an attempt to do so.
(b) A majority of tobacco users who were advised to stop consuming tobacco did not attempt to do so.
(c) Approximately 30% of tobacco users successfully stopped consuming tobacco.
(d) Approximately 65% of tobacco users successfully stopped consuming tobacco.

226.[MCQ] [GATE-EC-2023:2M]

Which one of the following options can be inferred from the given passage alone?

When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

[Excerpt from *The Truth about Stories* by T. King]

(a) It is a child's description of what he or she likes.
(b) It is an adult's memory of what he or she liked as a child.
(c) The child in the passage read stories about interplanetary travel only in parts.
(d) It teaches us that stories are good for children.

227.[MCQ] [GATE-EC-2023:2M]

Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.

Based only on the information provided above, which one of the following options can be logically inferred with *certainty*?

(a) All creatures with claws are animals.
(b) Some creatures with claws are non-ferocious.
(c) Some non-ferocious creatures have claws.
(d) Some ferocious creatures are creatures with claws.

228.[MCQ] [GATE-CS-2023:2M]

The country of Zombieland is in distress since more than 75% of its working population is suffering from serious health issues. Studies conducted by competent health experts concluded that a complete lack of physical exercise among its working population was one of the leading causes of their health issues. As one of the measures to address the problem, the Government of Zombieland has decided to provide monetary incentives to those who ride bicycles to work.

Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

(a) All the working population of Zombieland will henceforth ride bicycles to work.
(b) Riding bicycles will ensure that all of the working population of Zombieland is free of health issues.
(c) The health experts suggested to the Government of Zombieland to declare riding bicycles as mandatory.
(d) The Government of Zombieland believes that riding bicycles is a form of physical exercise.

229.[MCQ] [GATE-CE-2023:2M]

Elvesland is a country that has peculiar beliefs and practices. They express almost all their emotions by gifting flowers. For instance, if anyone gifts a white flower to someone, then it is always taken to be declaration of one's love for that person. In a similar manner, the gifting of a yellow flower to someone often means that one is angry with that person.

Based only on the information provided above, which one of the following sets of statements(s) can be logically inferred with certainly?

(i) In Elevesland, one always declares one's love by gifting a white flower.
(ii) In Elvesland, all emotions are declared by gifting flowers.
(iii) In Elvesland, sometimes one expresses one's anger by gifting a flower that is not yellow.
(iv) In Elvesland, sometimes one expresses one's love by gifting a white flower.

(a) Only (ii) (b) (i), (ii) and (iii)
(c) (i), (iii) and (iv) (d) Only (iv)

230.[MCQ] [GATE-CE-2023:2M]

Based only on the following passage, which one of the options can be inferred with certainty?

When the congregation sang together, Apenyo would also join, though her little screams were not quite audible because of the group singing. But whenever there was a special number, trouble would begin. Apenyo would try singing along, much to the embarrassment of her mother. After two or three such mortifying Sunday evenings, the mother stopped going to church altogether until Apenyo became older and learnt to behave.

At home too, Apenyo never kept quiet; she hummed or made up silly songs to sing by herself, which annoyed her mother at times but most often made her become pensive. She was by now convinced that her daughter had inherited her love of singing from her father who had died unexpectedly away from home.

[Excerpt from These Hills Called Home by Temsula Ao]

(a) The mother was embarrassed about her daughter's singing at home.
(b) The mother's feelings about her daughter's singing at home were only of annoyance.
(c) The mother was not sure if Apenyo had inherited her love of singing from her father.
(d) When Apenyo hummed at home, her mother tended to become thoughtful.

231.[MCQ] [GATE-CE-2023:2M]

The James web telescope, recently launched in space, is giving humankind unprecedented access to the depths of time by imaging very old stars formed almost 13 billion years ago. Astrophysicists and cosmologists believe that this odyssey in space may even shed light on the existence of dark matter. Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet "dark" in dark matter.

Based on the above paragraph, which one of the following statements is FASLE?

(a) No other telescope has captured images of stars older than those captured by the James web telescope
(b) People other than astrophysical and cosmologists may also believe in the existence of dark matter.
(c) The James Web telescope could be of use in the research on dark matter.
(d) If dark matter was known to interact via the strong interaction, then the epithet "dark" would be justified.

232.[MCQ] [GATE-CE-2022:2M]

Healthy eating is a critical component of healthy aging. When should one start eating healthy? It turns out that it is never too early. For example, babies who start eating healthy in the first year are more likely to have better overall health as they get older.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Healthy eating is important for those with good health condition, but not for others
(b) Eating healthy can be started at any age, earlier the better
(c) Eating healthy and better overall health are more correlated at a young age, but not elder age
(d) Healthy eating is more important for adults than kids

233.[MCQ] [GATE-CE-2022:2M]

In the last few years, several new shopping malls were opened in the city. The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Fewer people are visiting the malls but spending more
(b) More people are visiting the malls but not spending enough
(c) More people are visiting the malls and spending more
(d) Fewer people are visiting the malls and not spending enough

234.[MCQ] [GATE-ME-2022:2M]

Fish belonging to species S in the deep sea have skins that are extremely black (ultra-black skin). This helps them not only to avoid predators but also sneakily attack their prey. However, having this extra layer of black pigment results in lower collagen on their skin, making their skin more fragile.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Having ultra-black skin is only advantageous to species S
(b) Species S with lower collagen in their skin are at an advantage because it helps them avoid predators
(c) Having ultra-black skin has both advantages and disadvantages to species S
(d) Having ultra-black skin is only disadvantageous to species S but advantageous only to their predators

235.[MCQ] [GATE-ME-2022:2M]

Humans are naturally compassionate and honest. In a study using strategically placed wallets that appear "lost", it was found that wallets with money are more likely to be returned than wallets without money. Similarly, wallets that had a key and money are more likely to be returned than wallets with the same amount of money alone. This suggests that the primary reason for this behavior is compassion and empathy.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Wallets with a key are more likely to be returned because people do not care about money
(b) Wallets with a key are more likely to be returned because people relate to suffering of others
(c) Wallets used in experiments are more likely to be returned than wallets that are really lost
(d) Money is always more important than keys

236.[MCQ] [GATE-EE-2022:2M]

Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Humans engage in altruism due to guilt but not empathy
(b) Humans engage in altruism due to empathy but not guilt
(c) Humans engage in altruism due to group identification but not empathy
(d) Humans engage in altruism due to empathy but not familiarity

237.[MCQ] [GATE-EC-2022:2M]

Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

(a) Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous
(b) Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects
(c) Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous
(d) Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence

238.[MCQ] [GATE-CS-2022:2M]

Some people believe that “what gets measured, improves”. Some others believe that “what gets measured, gets gamed”. One possible reason for the difference in the beliefs is the work culture in organizations. In organizations with good work culture, metrics help improve outcomes. However, the same metrics are counterproductive in organizations with poor work culture.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) Metrics are useful in organizations with poor work culture
(b) Metrics are useful in organizations with good work culture
(c) Metrics are always counterproductive in organizations with good work culture
(d) Metrics are never useful in organizations with good work culture

239.[MCQ] [GATE-ME-2021:2M]

The world is going through the worst pandemic in the past hundred years. The air travel industry is facing a crisis, as the resulting quarantine requirement for travelers led to weak demand.

In relation to the first sentence above, what does the second sentence do?

(a) Restates an idea from the first sentence.
(b) Second sentence entirely contradicts the first sentence.
(c) The two statements are unrelated.
(d) States an effect of the first sentence.

240.[MCQ] [GATE-ME-2021:2M]

Oxpeckers and rhinos manifest a symbiotic relationship in the wild. The oxpeckers warn the rhinos about approaching poachers, thus possibly saving the lives of the rhinos. Oxpeckers also feed on the parasitic ticks found on rhinos.

In the symbiotic relationship described above, the primary benefits for oxpeckers and rhinos respectively are,

(a) Oxpeckers get a food source, rhinos have no benefit.
(b) Oxpeckers save their habitat from poachers while the rhinos have no benefit.
(c) Oxpeckers get a food source, rhinos may be saved from the poachers.
(d) Oxpeckers save the lives of poachers, rhinos save their own lives.

241.[MCQ] [GATE-ME-2021:1M]

"The increased consumption of leafy vegetables in the recent months is a clear indication that the people in the state have begun to lead a healthy lifestyle"

Which of the following can be logically inferred from the information presented in the above statement?

(a) The people in the state did not consume leafy vegetables earlier.
(b) Consumption of leafy vegetables may not be the only indicator of healthy lifestyle.
(c) Leading a healthy lifestyle is related to a diet with leafy vegetables.
(d) The people in the state have increased awareness of health hazards causing by consumption of junk foods.

242.[MCQ] [GATE-EE-2021:1M]

The importance of sleep is often overlooked by students when they are preparing for exams. Research has consistently shown that sleep deprivation greatly reduces the ability to recall the material learnt. Hence, cutting down on sleep to study longer hours can be counterproductive.

Which one of the following statements is the CORRECT inference from the above passage?

(a) Sleeping well alone is enough to prepare for an exam. Studying has lesser benefit.
(b) Students are efficient and are not wrong in thinking that sleep is a waste of time.
(c) If a student is extremely well prepared for an exam, he needs little or no sleep.
(d) To do well in an exam, adequate sleep must be part of the preparation.

243.[MCQ] [GATE-EC-2021:2M]

Computers are ubiquitous. They are used to improve efficiency in almost all fields from agriculture to space exploration. Artificial intelligence (AI) is currently a hot topic. AI enables computers to learn, given enough training data. For humans, sitting in front of a computer for long hours can lead to health issues.

Which of the following can be deduced from the above passage?

(i) Now a days, computers are present in almost all places.
(ii) Computers cannot be used for solving problems in engineering.
(iii) For humans, there are both positive and negative effects of using computers.
(iv) Artificial intelligence can be done without data.

(a) (ii) and (iii)
(b) (ii) and (iv)
(c) (i), (iii) and (iv)
(d) (i) and (iii)

244.[MCQ] [GATE-CE-2021:2M]

Humans have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So far as we know, no other species possesses this ability. This skill is so important that we have different words to refer to its different flavors, such as imagination, invention and innovation.

Based on the above passage, which one of the following is TRUE?

(a) No species possess the ability to construct worlds in their minds.
(b) The terms imagination, invention and innovation refer to unrelated skills.
(c) We do not know of any species other than humans who possess the ability to construct mental worlds.
(d) Imagination, invention and innovation are unrelated to the ability to construct mental worlds.

245.[MCQ] [GATE-CE-2021:2M]

The author said, "Musicians rehearse before their concerts. Actors rehearse their roles before the opening of a new play. On the other hand, I find it strange that many public speakers think they can just walk on to the stage and start speaking. In my opinion, it is no less important for public speakers to rehearse their talks."

Based on the above passage, which one of the following is TRUE?

(a) The author is of the opinion that rehearsing is important for musicians, actors and public speakers.
(b) The author is of the opinion that rehearsing is less important for public speakers than for musicians and actors.
(c) The author is of the opinion that rehearsing is more important only for musicians than public speakers.
(d) The author is of the opinion that rehearsal is more important for actors than musicians.

246.[MQ] **[GATE-CS-2021:2M]**

Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality.

Which one of the following statements summarizes the passage?

(a) The proposed AOM addresses the core problems that cause obesity.
(b) If obesity reduces, poverty will naturally reduce, since obesity causes poverty.
(c) AOM are addressing the core problems and are likely to succeed.
(d) AOM are addressing the problem superficially.

247.[MCQ] **[GATE-CS-2021:2M]**

Listening to music during exercise improves exercise performance and reduces discomfort. Scientists researched whether listening to music while studying can help students learn better and the results were inconclusive. Students who needed external stimulation for studying fared worse while students who did not need any external stimulation benefited from music.

Which one of the following statements is the CORRECT inference of the above passage?

(a) Listening to music has no effect on learning and a positive effect on physical exercise.
(b) Listening to music has a clear positive effect both on physical exercise and on learning.
(c) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.
(d) Listening to music has a clear positive effect on learning in all students. Music has a positive effect only in some students who exercise.

248.[MCQ] **[GATE-ME-2020:2M]**

Climate change and resilience deal with two aspects - reduction of sources of non-renewable energy resources and reducing vulnerability of climate change aspects. The terms 'mitigation' and 'adaptation' are used to refer to these aspects, respectively.

Which of the following assertions is best supported by the above information?

(a) Mitigation deals with consequences of climate change.
(b) Adaptation deals with causes of climate change.
(c) Mitigation deals with actions taken to reduce the use of fossil fuels.
(d) Adaptation deals with actions taken to combat green-house gas emissions.

249.[MCQ] **[GATE-ME-2020:2M]**

Crowd funding deals with mobilisation of funds for a project from a large number of people, who would be willing to invest smaller amounts through web-based platforms in the project.

Based on the above paragraph, which of the following is correct about crowd funding?

(a) Funds raised through unwilling contributions on web-based platforms.
(b) Funds raised through large contributions on web-based platforms.
(c) Fund raised through coerced contributions on web-based platform.
(d) Fund raised through voluntary contribution on web-based platform.

250.[MCQ] [GATE-EC-2020:2M]

The global financial crisis in 2008 is considered to be the most serious world-wide financial crisis, which started with the sub-prime lending crisis in USA in 2007. The subprime lending crisis led to the banking crisis in 2008 with the collapse of Lehman Brothers in 2008. The sub-prime lending refers to the provision of loans to those borrowers who may have difficulties in repaying loans, and it arises because of excess liquidity following the East Asian crisis.

Which one of the following sequences shows the correct precedence as per the given passage?

(a) East Asian crisis → subprime lending crisis → banking crisis → global financial crisis.
(b) Subprime lending crisis → global financial crisis → banking Crisis → East Asian crisis.
(c) Banking crisis → subprime lending crisis → global financial Crisis → East Asian crisis.
(d) Global financial crisis → East Asian crisis → Banking crisis → subprime lending crisis.

251.[MCQ] [GATE-EC-2020:1M]

The Canadian constitution requires that equal importance be given to English and French. Last year, Air Canada lost a lawsuit, and had to pay a six-figure fine to a French-speaking couple after they filed complaints about formal in-flight announcements in English lasting 15 seconds, as opposed to informal 5 second message in French.

The French-speaking couple were upset at _________.

(a) The in-flight announcements being made in English.
(b) The English announcements being clearer than the French ones.
(c) The English announcements being longer than the French ones.
(d) Equal importance being given to English and French.

252.[MCQ] [GATE-CE-2020:2M]

The American psychologist Howard Gardner expounds that human intelligence can be subcategorised into multiple kinds, in such a way that individuals differ with respect to their relative competence in each kind. Based on this theory, modem educationists insist on prescribing multi-dimensional curriculum and evaluation parameters that enable development and assessment of multiple intelligences.

Which of the following statements can be inferred from the given text?

(a) Howard Gardner insists that the teaching curriculum and evaluation needs to be multidimensional.
(b) Howard Gardner wants to develop and assess the theory of multiple intelligences.
(c) Modern educationists want to develop and assess the theory of multiple intelligences.
(d) Modern educationists insist that the teaching curriculum and evaluation needs to be multi-dimensional.

253.[MCQ] [GATE-CE-2020:2M]

Nominal interest rate is defined as the amount paid by the borrower to the lender for using the borrowed amount for a specific period of time. Real interest rate calculated on the basis of actual value (inflation-adjusted), is approximately equal to the difference between nominal rate and expected rate of inflation in the economy.

Which of the following assertions is best supported by the above information?

(a) Under high inflation, real interest rate is low and borrowers get benefited.
(b) Under low inflation, real interest rate is high and borrowers get benefited.
(c) Under high inflation, real interest rate is low and lenders get benefited.
(d) Under low inflation, real interest rate is low and borrowers get benefited.

254.[MCQ] [GATE-CE-2020:1M]

Select the word that fits the analogy:

Partial : Impartial :: Popular

(a) Impopular (b) Dispopular
(c) Mispopular (d) Unpopular

255.[MCQ] [GATE-CS-2020:1M]

The dawn of the 21st century witnessed the melting glaciers oscillating between giving too much and too little to billions of people who depend on them for fresh water. The UN climate report estimates that without deep cuts to man-made emissions, at least 30% of the northern hemisphere's surface permafrost could melt by the end of the century. Given this situation of imminent global exodus of billions of people displaced by rising seas, nation-states need to rethink their carbon footprint for political concerns, if not for environmental ones.

Which one of the following statements can be inferred from the given passage?

(a) Nation-states do not have environmental concerns.
(b) Nation-states are responsible for providing fresh water to billions of people.
(c) Billions of people are responsible for man-made emissions.
(d) Billions of people are affected by melting glaciers.

256.[MCQ] [GATE-CS-2020:2M]

Goods and Services Tax (GST) is an indirect tax introduced in India in 2017 that is imposed on the supply of goods and services, and it subsumes all indirect taxes except few. It is a destination –based tax imposed on goods and services used, and it is not imposed at the point of origin from where goods come. GST also has a few components specific to state governments, central government and Union Territories (UTs).

Which one of the following statements can be inferred from the given passage?

(a) GST is imposed on the production of goods and services.
(b) GST includes all indirect taxes.
(c) GST does not have a components specific to UT.
(d) GST is imposed at the point of usage of goods and services.

257.[MCQ] [GATE-ME-2019:2M]

X is an online media provider. By offering unlimited and exclusive online content at attractive prices for a loyalty membership, X is almost forcing its customers towards its loyalty membership. If its loyalty membership continues to grow at its current rate, within the next eight years more households will be watching X than cable television.

Which one of the following statements can be inferred from the above paragraph?

(a) Most households that subscribe to X' loyalty membership discontinues watching cable television
(b) Non-members prefer to watch cable television
(c) Cable television operators don't subscribe to X's loyalty membership
(d) The X is canceling accounts of non-members

258.[MCQ] [GATE-ME-2019:2M]

While teaching a creative writing class in India, I was surprised at receiving stories from the students that were all set in distant places: in the American West with cowboys and in Manhattan penthouses with clinking ice cubes. This was, till an eminent Caribbean writer gave the writers in the once-colonised countries the confidence to see the shabby lives around them as worthy of being "told".

The writer of the passage is surprised by the creative writing assignments of his students. Because __________ .

(a) Some of the students had written stories set in foreign places
(b) None of the students had written stories set in India
(c) None of the students had written about ice cubes and cowboys
(d) Some of the students had written about ice cubes and cowboys

259.[MCQ] [GATE-ME-2019:2M]

Congo was named by Europeans. Congo's dictator Mobuto later changed the name of the country and the river to Zaire with the objective of Africanising names of persons and spaces. However, the name Zaire was a Portuguese alteration of *Nzadi o Nzere,* a local

African term meaning ‘River that swallows Rivers’. Zaire was the Portuguese name for the Congo river in the 16th and 17th centuries.

Which one of the following statements can be inferred from the paragraph above?

(a) Mobuto was not entirely successful in Africanising the name of his country
(b) The term *Nzadi o Nzere* was of Portuguese origin
(c) Mobuto’s desire to Africanise names was prevented by the Portuguese
(d) As a dictator Mobuto ordered the Portuguese to alter the name of the river to Zaire

260.[MCQ] [GATE-EE-2019:2M]

An award-winning study by a group of researchers suggests that men are as prone to buying on impulse as women but women feel more guilty about shopping.

Which one of the following statements can be inferred from the given text?

(a) Some men and women indulge in buying on impulse
(b) All men and women indulge in buying on impulse
(c) Few men and women indulge in buying on impulse
(d) Many men and women indulge in buying on impulse

261.[MCQ] [GATE-EC-2019:2M]

“Indian history was written by British historians - extremely well documented and researched, but not always impartial. History had to serve its purpose: Everything was made subservient to the glory of the Union Jack. Latter-day Indian scholars presented a contrary picture.”

From the text above, we can infer that:

Indian history written by British historians_________

(a) was well documented and not researched but was always biased
(b) was not well documented and researched and was always biased
(c) was well documented and researched but was sometimes biased
(d) was not well documented and researched and was sometimes biased

262.[MCQ] [GATE-CE-2019:2M]

The Newspaper reports that over 500 hectares of tribal land spread across 28 tribal settlements in Mohinitampuram forest division have already been “alienated”. A top forest official said. “First the tribals are duped out of their land holdings. Second, the families thus rendered landless are often forced to encroach further into the forests”.

On the basis of the information available in the paragraph, _____ is/are responsible for duping the tribals.

(a) Forest officials
(b) Landless families
(c) The Newspaper
(d) It cannot be inferred who

263.[MCQ] [GATE-CE-2019:2M]

“Popular Hindi fiction, despite or perhaps because of-its wide reach, often does not appear in our cinema. As ideals that viewers are meant to look up to rather than identify with. Hindi film protagonists usually read books of aspirational value. Textbooks. English books or high value literature.”

Which one of the following CANNOT be inferred from the paragraph above?

(a) Though popular Hindi fiction has wide reach it often does not appear in movies.
(b) Protagonists in Hindi movies, being ideals for viewers read only books of aspirational value
(c) Textbooks, English books or high literate have aspirational value but not popular Hindi fiction
(d) People do not look up to writers of textbooks, English books or high value literature

264.[MCQ] [GATE-CE-2019:2M]

The new cotton technology', Bollgard-II. with herbicide tolerant traits has developed into a thriving business in India. However, the commercial use of this technology is not legal in India. Notwithstanding that, reports indicate that the herbicide tolerant cotton had been purchased by farmers at an average of Rs 200 more than the control price of ordinary cotton, and planted in 15% of the cotton growing area in the 2017 Kharif season.

Which one of the following statements can be inferred from the given passage?

(a) Farmers want to access the new technology if India benefits from it

(b) Farmers want to access the new technology even if it is not legal

(c) Farmers want to access the new technology for experimental purposes

(d) Farmers want to access the new technology by paying high price

265.[MCQ] [GATE-CS-2019:2M]

"A recent High Court judgement has sought to dispel the idea of begging as a disease — which leads to its stigmatization and criminalization — and to regard it as a symptom. The underlying disease is the failure of the state to protect citizens who fall through the social security net."

Which one of the following statements can be inferred from the given passage?

(a) Beggars are lazy people who beg because they are unwilling to work

(b) Beggars are created because of the lack of social welfare schemes

(c) Begging is an offence that has to be dealt with firmly

(d) Begging has to be banned because it adversely affects the welfare of the state

266.[MCQ] [GATE-EC-2018:2M]

A coastal region with unparalleled beauty is home to many species of animals. It is dotted with coral reefs and unspoilt white sandy beaches. It has remained inaccessible to tourists due to poor connectivity and lack of accommodation. A company has spotted the opportunity and is planning to develop a luxury resort with helicopter service to the nearest major city airport. Environmentalists are upset that this would lead to the region becoming crowded and polluted like any other major beach resorts.

Which one of the following statements can be logically inferred from the information given in the above paragraph?

(a) The culture and tradition of the local people will be influenced by the tourists.

(b) The region will become crowded and polluted due to tourism.

(c) The coral reefs are on the decline and could soon vanish.

(d) Helicopter connectivity would lead to an increase in tourists coming to the region.

267.[MCQ] [GATE-EC-2018:2M]

The Cricket Board has long recognized John's potential as a leader of the team. However, his on-field temper has always been a matter of concern for them since his junior days. While this aggression has filled stadium with die-hard fans, it has taken a toll on his own batting. Until recently, it appeared that he found it difficult to convert his aggression into big scores. Over the past three seasons though, that picture of John has been replaced by a cerebral, calculative and successful batsman-captain. After many years, it appears that the team has finally found a complete captain.

Which of the following statements can be logically inferred from the above paragraph?

(i) Even as a junior cricketer, John was considered a good captain.

(ii) Finding a complete captain is a challenge.

(iii) Fans and the Cricket Board have differing views on what they want in a captain.

(iv) Over the past three seasons John has accumulated big scores.

(a) (i), (ii) and (iii) only (b) (iii) and (iv) only

(c) (ii) and (iv) only (d) (i), (ii), (iii) and (iv)

268.[MCQ] [GATE-ME-2017:2M]

Two very famous sportsmen Mark and Steve happened to be brothers, and played for country K. Mark teased James, an opponent from country E. "There is no way you are good enough to play for your country." James replied, "Maybe not, but at least I am the best player in my own family."

Which one of the following can be inferred from this conversation?

(a) Mark was known to play better than James
(b) Steve was known to play better than Mark
(c) James and Steve were good friends
(d) James played better than Steve

269. [MCQ] [GATE-ME-2017:2M]

"Here, throughout the early 1820s, Stuart continued to fight his losing battle to allow his sepoys to wear their caste-marks and their own choice of facial hair on parade, being again reprimanded by the commander-in-chief. His retort that 'A stronger instance than this of European prejudice with relation to this country has never come under my observations' had no effect on his superiors."

According to this paragraph, which of the statements below is most accurate?

(a) Stuart's commander-in-chief was moved by this demonstration of his prejudice.
(b) The Europeans were accommodating of the sepoys' desire to wear their caste-marks.
(c) Stuart's 'losing battle' refers to his inability to succeed in enabling sepoys to wear caste-marks.
(d) The commander-in-chief was exempt from the European prejudice that dictated how the sepoys were to dress.

270. [MCQ] [GATE-CS, EE-2017:2M]

"The hold of the nationalist imagination on our colonial past is such that anything inadequately or improperly nationalist is just not history."

Which of the following statements best reflects the author's opinion?

(a) Nationalists are highly imaginative.
(b) History is viewed through the filter of nationalism.
(c) Our colonial past never happened.
(d) Nationalism has to be both adequately and properly imagined.

271. [MCQ] [GATE-CS, EE-2017:2M]

"We lived in a culture that denied any merit to literary works, considering them important only when they were handmaidens to something seemingly more urgent - namely ideology. This was a country where all gestures, even the most private, were interpreted in political terms."

The author's belief that ideology is not as important as literature is revealed by the word:

(a) 'culture' (b) 'seemingly'
(c) 'urgent' (d) 'political'

272. [MCQ] [GATE-EC-2017:2M]

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages: for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters."

Which of the following statements best reflects the author's opinion?

(a) An intimate association does not allow for the necessary perspective.
(b) Matters are recorded with an impartial perspective.
(c) An intimate association offers an impartial perspective.
(d) Actors are typically associated with the impartial recording of matters.

273. [MCQ] [GATE-CE-2017:2M]

The old concert hall was demolished because of fears that the foundation would be affected by the construction of the new metro line in the area. Modern technology for underground metro construction tried to mitigate the impact of pressurized air pockets created by the excavation of large amounts of soil. But even with these safeguards, it was feared that the soil below the concert hall would not be stable.

From this, one can infer that

(a) The foundations of old buildings create pressurized air pockets underground, which are difficult to handle during metro construction.
(b) Metro construction has to be done carefully considering its impact on the foundations of existing buildings.

(c) Old buildings in an area form an impossible hurdle to metro construction in that area.
(d) Pressurized air can be used to excavate large amounts of soil from underground areas.

274.[MCQ] [GATE-CE-2017:1M]

Bhaichung was observing the pattern of people entering and leaving a car service centre. There was a single window where customers were being served. He saw that people inevitably came out of the centre in the order that they went in. However, the time they spent inside seemed to vary a lot: some people came out in a matter of minutes while for others it took much longer.

From this, what can one conclude?

(a) The centre operates on a first-come-first-served basis, but with variable service times, depending on specific customer needs.
(b) Customers were served in an arbitrary order, since they took varying amounts of time for service completion in the centre.
(c) Since some people came out within a few minutes of entering the centre, the system is likely to operate on a last-come-first-served basis.
(d) Entering the centre early ensured that one would have shorter service times and most people attempted to do this.

275.[MCQ] [GATE-CE-2017:2M]

A map shows the elevations of Darjeeling, Gangtok, Kalimpong, Pelling and Siliguri. Kalimpong is at a lower elevation than Gangtok. Pelling is at a lower elevation than Gangtok. Pelling is at a higher elevation than Siliguri. Darjeeling is at a higher elevation than Gangtok.

Which of the following statements can be inferred from the paragraph above?

(i) Pelling is at a higher elevation than Kalimpong
(ii) Kalimpong is at a lower elevation than Darjeeling
(iii) Kalimpong is at a higher elevation than Siliguri
(iv) Siliguri is at a lower elevation than Gangtok

(a) Only (ii) (b) Only (ii) and (iii)
(c) Only (ii) and (iv) (d) Only (iii) and (iv)

276.[MCQ] [GATE-ME, EC-2016:2M]

Social science disciplines were in existence in an amorphous form until the colonial period when they were institutionalized. In varying degrees, they were intended to further the colonial interest. In the time of globalization and the economic rise of postcolonial countries like India, conventional ways of knowledge production have become obsolete.

Which of the following can be logically inferred from the above statements?

(i) Social science disciplines have become obsolete.
(ii) Social science disciplines had a pre-colonial origin.
(iii) Social science disciplines always promotes colonialism.
(iv) Social science must maintain disciplinary boundaries.

(a) (ii) only (b) (i) and (iii) only
(c) (ii) and (iv) only (d) (iii) and (iv) only

277.[MCQ] [GATE-ME-2016:2M]

A smart city integrates all modes of transport, uses clean energy and promotes sustainable use of resources. It also uses technology to ensure safety and security of the city, something which critics argue, will lead to a surveillance state.

Which of the following can be logically inferred from the above paragraph?

(i) All smart cities encourage the formation of surveillance states.
(ii) Surveillance is an integral part of a smart city.
(iii) Sustainability and surveillance go hand in hand in a smart city.
(iv) There is a perception that smart cities promote surveillance.

(a) (i) and (iv) only (b) (ii) and (iii) only
(c) (iv) only (d) (i) only

278.[MCQ] [GATE-ME, EC-2016:2M]

In a world filled with uncertainty, he was glad to have many good friends. He had always assisted them in times of need and was confident that they would reciprocate. However, the events of the last week proved him wrong.

Which of the following inference(s) is/are logically valid and can be inferred from the above passage?

(i) His friends were always asking him to help them.
(ii) He felt that when in need of help, his friends would let him down.
(iii) He was sure that his friends would help him when in need.
(iv) His friends did not help him last week.

(a) (i) and (ii) (b) (iii) and (iv)
(c) (iii) only (d) (iv) only

279.[MCQ] [GATE-EE-2016:2M]

Sourya committee had proposed the establishment of Sourya Institutes of Technology (SITs) in line with Indian Institutes of Technology (IITs) to cater to the technological and industrial needs of a developing country.

Which of the following can be logically inferred from the above sentence?

Based on the proposal,

(i) In the initial years, SIT students will get degrees from IIT.
(ii) SITs will have a distinct national objective.
(iii) SIT like institutions can only be established in consultation with IIT.
(iv) SITs will serve technological needs of a developing country.

(a) (iii) and (iv) only
(b) (i) and (iv) only
(c) (ii) and (iv) only
(d) (ii) and (iii) only

280.[MCQ] [GATE-EE-2016:2M]

A poll of students appearing for masters in engineering indicated that 60 % of the students believed that mechanical engineering is a profession unsuitable for women. A research study on women with masters or higher degrees in mechanical engineering found that 99 % of such women were successful in their professions.

Which of the following can be logically inferred from the above paragraph?

(a) Many students have misconceptions regarding various engineering disciplines.
(b) Men with advanced degrees in mechanical engineering believe women are well suited to be mechanical engineers.
(c) Mechanical engineering is a profession well suited for women with masters or higher degrees in mechanical engineering.
(d) The number of women pursuing higher degrees in mechanical engineering is small.

281.[MCQ] [GATE-EE-2016:1M]

R2D2 is a robot. R2D2 can repair aeroplanes. No other robot can repair aeroplanes.

Which of the following can be logically inferred from the above statements?

(a) R2D2 is a robot which can only repair aeroplanes.
(b) R2D2 is the only robot which can repair aeroplanes.
(c) R2D2 is a robot which can repair only aeroplanes.
(d) Only R2D2 is a robot.

282.[MCQ] [GATE-CS, EE-2016:2M]

Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous.

Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) The author believes that computers are not good for us.
(ii) Mobile computers and the internet are both intended inventions

(a) (i) only (b) (ii) only
(c) both (i) and (ii) (d) neither (i) nor (ii)

283.[MCQ] [GATE-EC-2016:2M]

The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern. It is estimated that inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction in the number of people infected with rabies.

Which of the following can be logically inferred from the above sentences?

(a) The number of people in India infected with rabies is high.
(b) The number of people in other parts of the world who are infected with rabies is low.
(c) Rabies can be eradicated in India by vaccinating 70 % of stray dogs.
(d) Stray dogs are the main source of rabies worldwide

284.[MCQ] [GATE-CS, CE-2016:2M]

Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

(a) India is a country of exactly seventeen languages.
(b) Linguistic pluralism is the only indicator of a nation's diversity.
(c) Indian currency notes have sufficient space for all the Indian languages.
(d) Linguistic pluralism is strong evidence of India's diversity.

285.[MCQ] [GATE-CE-2016:2M]

Today, we consider Ashoka as a great ruler because of the copious evidence he left behind in the form of stone carved edicts. Historians tend to correlate greatness of a king at his time with the availability of evidence today.

Which of the following can be logically inferred from the above sentences?

(a) Emperors who do not leave significant sculpted evidence are completely forgotten.
(b) Ashoka produced stone carved edicts to ensure that later historians will respect him.
(c) Statues of kings are a reminder of their greatness.
(d) A king's greatness, as we know him today, is interpreted by historians..

286.[MCQ] [GATE-ME, EC-2015:2M]

Ms. X will be in Bagdogra from 01/05/2014 to 20/05/2014 and from 22/05/2014 to 31/05/2014. On the morning of 21/05/2014, she will reach Kochi via Mumbai.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(a) Ms. X will be in Kochi for one day, only in May.
(b) Ms. X will be in Kochi for only one day in May
(c) Ms. X will be only in Kochi for one day in May
(d) Only Ms. X will be in Kochi for one day in May

287.[MCQ] [GATE-ME, EC-2015:2M]

In the following question, the first and the last sentence of the passage are in order and numbered 1 and 6. The rest of the passage is split into 4 parts and numbered as 2, 3, 4 and 5. These 4 parts are not arranged in proper order. Read the sentences and arrange them in a logical sequence to make a passage and choose the correct sequence from the given options.

1. On Diwali, the family rises early in the morning
2. The whole family, including the young and the old enjoy doing this.
3. Children let off fireworks later in the night with their friends.
4. At sunset, the lamps are lit and the family performs various rituals.
5. Father, mother, and children visit relatives and exchange gifts and sweets.
6. Houses look so pretty with lighted lamps all around.

(a) 2,5,3,4 (b) 5,2,4,3
(c) 3,5,4,2 (d) 4,5,2,3

288.[MCQ] [GATE-ME, EC-2015:2M]

Lamenting the gradual sidelining of the arts in school curricula, a group of prominent artists wrote to the Chief Minister last year, asking him to allocate more funds to support arts education in schools. However, no such increase has been announced in this year's Budget. The artists expressed their deep anguish at their request not being approved, but many of them remain optimistic about funding in the future.

Which of the statement(s) below is/are logically valid and can be inferred from the above statements?

(i) The artists expected funding for the arts to increase this year.
(ii) The Chief Minister was receptive to the idea of increasing funding for the arts.
(iii) The Chief Minister is a prominent artist.
(iv) Schools are giving less importance to arts education nowadays
(a) (iii) and (iv)
(b) (i) and (iv)
(c) (i), (ii) and (iv)
(d) (i) and (iii)

289.[MCQ] [GATE-EE, CS-2015:2M]

The given statement is followed by some courses of action. Assuming the statement to be true, decide the correct option.

Statement:

There has been a significant drop in the water level in the lakes supplying water to the city.

Course of action:

(I) The water supply authority should impose a partial cut in supply to tackle the situation.
(II) The government should appeal to all the residents through mass media for minimal use of water.
(III) The government should ban the water supply in lower areas.
(a) Statements I and II follow
(b) Statements I and III follow
(c) Statements II and III follow
(d) All statements follow

290.[MCQ] [GATE-EC-2015:2M]

Humpty Dumpty sits on a wall every day while having lunch. The wall sometimes breaks. A person sitting on the wall falls if the wall breaks.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(a) Humpty Dumpty always falls while having lunch
(b) Humpty Dumpty does not fall sometimes while having lunch
(c) Humpty Dumpty never falls during dinner
(d) When Humpty Dumpty does not sit on the wall, the wall does not break

291.[MCQ] [GATE-EC-2015:2M]

Read the following paragraph and choose the correct statement.

Climate change has reduced human security and threatened human well-being. An ignored reality of human progress is that human security largely depends upon environmental security. But on the contrary, human progress seems contradictory to environmental security. To keep up both at the required level is a challenge to be addressed by one and all. One of the ways to curb the climate change may be suitable scientific innovations, while the other may be the Gandhian perspective on small scale progress with focus on sustainability.

(a) Human progress and security are positively associated with environmental security.
(b) Human progress is contradictory to environmental security.
(c) Human security is contradictory to environmental security.
(d) Human progress depends upon environmental security.

292.[MCQ] [GATE-CS, CE-2015:2M]

Alexander turned his attention towards India, since he had conquered Persia

Which one of the statements below is logically valid and can be inferred from the above sentence?

(a) Alexander would not have turned his attention towards India had he not conquered Persia.
(b) Alexander was not ready to rest on his laurels and wanted to march to India.
(c) Alexander was completely in control of his army and could command it to move towards India
(d) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.

293.[MCQ] [GATE-CS, CE-2015:2M]

Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament. He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addressed this problem, success at the highest level will continue to elude him.

Which of the statement(s) below is/are logically valid and can be inferred from the above passage?

(i) He was already a successful batsman at the highest level.
(ii) He has to improve his temperament in order to become a great batsman.
(iii) He failed to make many of his good starts count.
(iv) Improving his technical skills will guarantee success.

(a) (iii) and (iv)
(b) (ii) and (iii)
(c) (i), (ii) and (iii)
(d) (ii) only

294. [MCQ] [GATE-ME, EC-2014:1M]

"India is a country of rich heritage and cultural diversity."

Which one of the following facts best supports the claim made in the above sentence?

(a) India is a union of 28 states and 7 union territories.
(b) India has a population of over 1.1 billion.
(c) India is home to 22 official languages and thousands of dialects.
(d) The Indian cricket team draws players from over ten states.

295. [MCQ] [GATE-ME, EC-2014:2M]

For submitting tax returns, all resident males with annual income below Rs 10 lakh should fill up Form P and all resident females with income below Rs 8 lakh should fill up Form Q. All people with incomes above Rs 10 lakh should fill up Form R, except nonresidents with income above Rs 15 lakhs, who should fill up Form S. All others should fill Form T. An example of a person who should fill Form T is

(a) A resident male with annual income Rs 9 lakh
(b) A resident female with annual income Rs 9 lakh
(c) A non-resident male with annual income Rs 16 lakh
(d) A non-resident female with annual income Rs 16 lakh

296. [MCQ] [GATE-EE, CS-2014:2M]

The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures.

What can be inferred from this passage?

(a) The Palghat gap is caused by high rainfall and high temperatures in southern Tamil Nadu and Kerala
(b) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low-lying
(c) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala
(d) Higher summer temperatures result in higher rainfall near the Palghat Gap area

297. [MCQ] [GATE-EE, CS-2014:2M]

Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and schizophrenia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.

On which of the following assumptions does the statement above rely?

(a) Strategies are now available for eliminating psychiatric illnesses
(b) Certain psychiatric illnesses have a genetic basis
(c) All human diseases can be traced back to genes and how they are expressed
(d) In the future, genetics will become the only relevant field for identifying psychiatric illnesses

298. [MCQ] [GATE-EE, CS-2014:2M]

The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to the south and west and Lithuania to the east respectively. Which of the statements below can be inferred from this passage?

(a) Kaliningrad was historically Russian in its ethnic make up
(b) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia
(c) Koenigsberg was renamed Kaliningrad, as that was its original Russian name
(d) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

299.[MCQ] [GATE-EE, CS-2014:2M]

The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region. Which one of the following statements, if true, does not contradict this conclusion?

(a) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent
(b) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency
(c) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
(d) The number of people with malarial fever (also contracted from mosquito bites) has increased this year

300.[MCQ] [GATE-EE, CS-2014:2M]

By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.

Which of the following inference(s) may be drawn from the above passage?

(i) Our understanding of the universe changes based on the positions of stars
(ii) Paradigm shifts usually occur at the beginning of centuries
(iii) Stars are important objects in the universe
(iv) Experimental evidence was important in confirming this paradigm shift

(a) (i), (ii) and (iv) (b) (iii) only
(c) (i) and (iv) (d) (iv) only

301.[MCQ] [GATE-CE-2014:2M]

Moving into a world of big data will require us to change our thinking about the merits of exactitude. To apply the conventional mindset of measurement to the digital, connected world of the twenty-first century is to miss a crucial point. As mentioned earlier, the obsession with exactness is an artefact of the information-deprived analog era. When data was sparse, every data point was critical, and thus great care was taken to avoid letting any point bias the analysis. *From "BIG DATA" Viktor Mayer-Schonberger and Kenneth Cukier*

The main point of the paragraph is:

(a) The twenty-first century is a digital world
(b) Big data is obsessed with exactness
(c) Exactitude is not critical in dealing with big data
(d) Sparse data leads to a bias in the analysis

302.[MCQ] [GATE-CE-2014:1M]

Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(a) Rajan has decided to work only in a group.
(b) Rajan and Sajan were formed into a group against their wishes.
(c) Sajan had decided to give in to Rajan's request to work with him.
(d) Rajan had believed that Sajan and he would be working together.

303.[MCQ] [GATE-ME, CS-2013:2M]

After several defeats in wars, Robert Bruce went in exile and wated to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit sucide. And then, Bruce went back again and won many a battle, and the rest is history. Which of the following assertions is best supported by the abvoe information?

(a) Failure is the pillar of success
(b) Honesty is the best policy
(c) Life begins and ends with adventures
(d) No adversity justifies giving up hope

304.[MCQ] [GATE-EC, EE-2013:2M]

Statement: There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals, socialists, and so on.

Which one of the following is the best inference from the above statement?

(a) The emergence of nationalism in colonial India led to our Independence.
(b) Nationalism in India emerged in the context of colonialism.
(c) Nationalism in India is homogeneous.
(d) Nationalism in India is heterogeneous.

305.[MCQ] [GATE-EC, EE-2012:2M]

One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.

Which one of the following statements best sums up the meaning of the above passage?

(a) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances
(b) The legions were treated inhumanly as if the men were animals
(c) Discipline was the armies' inheritance from their seniors
(d) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

306.[MCQ] [GATE-EC, EE-2011:2M]

The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.

It can be inferred from the passage, that horses were

(a) given immunity to diseases
(b) generally quite immune to diseases
(c) given medicines to fight toxins
(d) given diphtheria and tetanus serums

307.[MCQ] [GATE-ME, CE, CS-2011:2M]

Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be availabe for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement?

(a) Gender-discriminatory
(b) Xenophobic
(c) Not designed to make the post attractive
(d) Not gender-discriminatory

308.[MCQ] [GATE-ME,CE,EE,EC,CS-2010:2M]

Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their causes. Which of the following statements best sums up the meaning of the above passage:

(a) Modern warfare has resulted civil strife
(b) Chemical agents are useful in modern warfare
(c) Use of chemical agents in warfare would be undesirable
(d) People in military establishments like to use chemical agents in war

❑❑❑

ANSWER KEY

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(c)
5.	(c)	**6.**	(a)	**7.**	(d)	**8.**	(b)
9.	(c)	**10.**	(d)	**11.**	(b)	**12.**	(c)
13.	(c)	**14.**	(a)	**15.**	(a)	**16.**	(a)
17.	(b)	**18.**	(a)	**19.**	(b)	**20.**	(c)
21.	(b)	**22.**	(d)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(b)	**27.**	(c)	**28.**	(a)
29.	(b)	**30.**	(b)	**31.**	(c)	**32.**	(b)
33.	(c)	**34.**	(d)	**35.**	(c)	**36.**	(b)
37.	(a)	**38.**	(a)	**39.**	(c)	**40.**	(b)
41.	(c)	**42.**	(a)	**43.**	(b)	**44.**	(a)
45.	(b)	**46.**	(c)	**47.**	(c)	**48.**	(c)
49.	(d)	**50.**	(c)	**51.**	(a)	**52.**	(b)
53.	(b)	**54.**	(c)	**55.**	(b)	**56.**	(c)
57.	(a)	**58.**	(a)	**59.**	(a)	**60.**	(c)
61.	(b)	**62.**	(b)	**63.**	(b)	**64.**	(c)
65.	(b)	**66.**	(d)	**67.**	(c)	**68.**	(d)
69.	(c)	**70.**	(a)	**71.**	(d)	**72.**	(b)
73.	(d)	**74.**	(a)	**75.**	(b)	**76.**	(a)
77.	(c)	**78.**	(a)	**79.**	(b)	**80.**	(a)
81.	(c)	**82.**	(a)	**83.**	(c)	**84.**	(d)
85.	(b)	**86.**	(b)	**87.**	(d)	**88.**	(b)
89.	(d)	**90.**	(d)	**91.**	(c)	**92.**	(d)
93.	(d)	**94.**	(c)	**95.**	(c)	**96.**	(c)
97.	(c)	**98.**	(b)	**99.**	(b)	**100.**	(a)
101.	(b)	**102.**	(c)	**103.**	(c)	**104.**	(b)
105.	(c)	**106.**	(b)	**107.**	(a)	**108.**	(b)
109.	(c)	**110.**	(a)	**111.**	(a)	**112.**	(a)
113.	(d)	**114.**	(a)	**115.**	(b)	**116.**	(c)
117.	(d)	**118.**	(d)	**119.**	(c)	**120.**	(c)
121.	(d)	**122.**	(c)	**123.**	(a)	**124.**	(d)
125.	(a)	**126.**	(d)	**127.**	(c)	**128.**	(a)

129.	(d)	**130.**	(a)	**131.**	(a)	**132.**	(b)
133.	(a)	**134.**	(a)	**135.**	(a)	**136.**	(a)
137.	(a)	**138.**	(d)	**139.**	(a)	**140.**	(d)
141.	(d)	**142.**	(d)	**143.**	(c)	**144.**	(d)
145.	(a)	**146.**	(c)	**147.**	(b)	**148.**	(c)
149.	(b)	**150.**	(d)	**151.**	(b)	**152.**	(a)
153.	(b)	**154.**	(c)	**155.**	(a)	**156.**	(c)
157.	(c)	**158.**	(a)	**159.**	(a)	**160.**	(a)
161.	(c)	**162.**	(b)	**163.**	(a)	**164.**	(b)
165.	(b)	**166.**	(a)	**167.**	(c)	**168.**	(b)
169.	(c)	**170.**	(c)	**171.**	(d)	**172.**	(a)
173.	(c)	**174.**	(b)	**175.**	(b)	**176.**	(b)
177.	(c)	**178.**	(d)	**179.**	(b)	**180.**	(b)
181.	(b)	**182.**	(b)	**183.**	(a)	**184.**	(b)
185.	(c)	**186.**	(b)	**187.**	(b)	**188.**	(c)
189.	(c)	**190.**	(a)	**191.**	(a)	**192.**	(c)
193.	(a)	**194.**	(b)	**195.**	(a)	**196.**	(c)
197.	(b)	**198.**	(a)	**199.**	(d)	**200.**	(b)
201.	(c)	**202.**	(c)	**203.**	(b)	**204.**	(c)
205.	(a)	**206.**	(b)	**207.**	(b)	**208.**	(d)
209.	(c)	**210.**	(b)	**211.**	(a)	**212.**	(b)
213.	(d)	**214.**	(b)	**215.**	(c)	**216.**	(a)
217.	(b)	**218.**	(b)	**219.**	(a)	**220.**	(d)
221.	(c)	**222.**	(d)	**223.**	(d)	**224.**	(d)
225.	(b)	**226.**	(b)	**227.**	(d)	**228.**	(d)
229.	(d)	**230.**	(d)	**231.**	(d)	**232.**	(b)
233.	(b)	**234.**	(c)	**235.**	(b)	**236.**	(c)
237.	(d)	**238.**	(b)	**239.**	(d)	**240.**	(c)
241.	(c)	**242.**	(d)	**243.**	(d)	**244.**	(c)
245.	(a)	**246.**	(d)	**247.**	(c)	**248.**	(c)
249.	(d)	**250.**	(a)	**251.**	(c)	**252.**	(d)
253.	(a)	**254.**	(d)	**255.**	(d)	**256.**	(d)
257.	(a)	**258.**	(b)	**259.**	(a)	**260.**	(a)
261.	(c)	**262.**	(d)	**263.**	(b)	**264.**	(b)
265.	(b)	**266.**	(d)	**267.**	(c)	**268.**	(b)
269.	(c)	**270.**	(b)	**271.**	(b)	**272.**	(a)
273.	(b)	**274.**	(a)	**275.**	(c)	**276.**	(a)
277.	(c)	**278.**	(b)	**279.**	(c)	**280.**	(c)

281.	(b)	**282.**	(d)	**283.**	(a)	**284.**	(d)
285.	(d)	**286.**	(b)	**287.**	(b)	**288.**	(b)
289.	(a)	**290.**	(b)	**291.**	(b)	**292.**	(a)
293.	(b)	**294.**	(c)	**295.**	(b)	**296.**	(c)
297.	(b)	**298.**	(b)	**299.**	(d)	**300.**	(d)
301.	(c)	**302.**	(d)	**303.**	(d)	**304.**	(d)
305.	(a)	**306.**	(b)	**307.**	(d)	**308.**	(c)

SOLUTIONS

1. (a)

He did not manage to fix the car himself, so he got it fixed in the garage.

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2. (d)

Direct Speech - Indirect Speech

Present Continuous Tense - Past Continuous Tense

∴ Indirect Speech of the given sentence will be,

Rafi told Mary that he **was thinking** of watching a film that weekend.

∴ Option (d) is correct.

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3. (a)

When it goes from one side to other, we use the word 'across'.

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4. (c)

As it is tentatively, thus may past 'might' will be used.

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5. (c)

'couldn't help laughing' is used when a person finds situation very funny.

Scan for Video solution

6. (a)

"The number" is followed by a singular verb while "A number" is followed by plural verb hence sentences I & II are correct.

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7. (d)

Hardly or barely comes before the main verb of a sentence. But when there is a model or auxiliary verb, hardly/barely usually comes after it.

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8. (b)

Wake: emerge or cause to emerge from sleep. Its second and third form is woke and woken respectively.

So, statements (i) and (iii) are correct.

9. (c)

Whose is used for belongs to someone,
Which is a relative cause
Who is use for a person/object
Whom is a preposition
$\therefore$ Option (c) is correct.

10. (d)

The use of 'than' suggest that it should be followed by comparative adjective degree. The comparative adjective degree of easy is 'easier'

11. (b)

Negation means opposite,
Opposite of either is neither.
Opposite of or is nor.
Hence, "Neither P marries Q nor X marries Y."

12. (c)

In statement (ii) "is" is incorrect.
In statement (iii) "families" is incorrect.
So (i) and (iv) are correct.

13. (c)

1st sentence is grammatically correct as everybody or everyone here is considered a single group so singular form of verb 'is' is used.

Second sentence is ambiguous because it is not clear that who among the two, enjoy to play chess.

14. (a)

The correct option is 'as well as' because it is in first degree. Option B and D are is comparative degree so should be followed by 'than'.

15. (a)

While I agree <u>to</u> his proposal this time, I do not often agree <u>with</u> him.

'Agree with' is used for a person, and the 'agree to' is used for a non-person.

16. (a)

Jofra Archer, the England fast bowler, is <u>more fast</u> than accurate.

In the sentence, we are comparing the two different qualities of the bowler: speed & accuracy. For such comparison, we use 'more + positive degree adjective'.

17. (b)

The construction **not only … but (also)** is called a co relative conjunction. It is used to present two related pieces of information. Both pieces of information are being presented as surprising or unexpected, with the second one being even more surprising than the first.

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18. (a)

'In' is used to indicate opinion, belief, feeling, event etc and **'of'** is used if someone dies of a specific disease or some reasons which is well-known. **'From'** is used if something leads to death.

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19. (b)

If something is prohibited then it will be from someone,

So that from parking is the suitable word in this sentence.

∴ Option (b) is correct

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20. (c)

This book is a singular noun and the students as well as the instructor, it means two person so that it is a plural noun, then in 1st blank space 'is' and in 2nd 'are' will be used.

∴ Option (c) is correct.

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21. (b)

'in' is used to indicate location or position. 'with' is used in opposition to or against. Thus option (b) is the correct answer.

Scan for Video solution

22. (d)

'Within' is used where something will occur in a specified period of time. 'For' is used for duration of time (Hour, Days, Week, Month) whereas 'Since' is used for specific point in time.

Scan for Video solution

23. (b)

The first part of the sentence is in future tense

So, "were to be" cannot be used because it is in past tense. All other options can be used.

Scan for Video solution

24. (c)

Had I permitted him to leave, I wouldn't have had any problem with him being absent, would I?

The sentence is in past tense and having third form of verb (V_3) hence 'had' will be there. Since the second part of the sentence contains 'wouldn't' so, the question tab 'would I' will be used.

Scan for Video solution

25. (b)

At is used to refer to time or place.

Scan for Video solution

26. (b)

The subject **company** is singular so the verb will be singular also i.e. **uses**. And as the subject **products** is plural the associated verb will be plural i.e., **include.**

Scan for Video solution

27. (c)

In this sentence 'with' is the suitable word for anger/fight with someone.

So, option (c) is correct.

Scan for Video solution

28. (a)

The sentence should be in present perfect continuous tense, therefore verb+ing is to be used, i.e. 'Creating'.

Scan for Video solution

29. (b)

The given sentence has a blank in object. The objective pronoun is 'me'.

Scan for Video solution

30. (b)

When a specific or definite noun is mentioned, 'the' article is used. For indefinite 'a' and 'an' is used based on consonant and vowel respectively. Here the first blank should consist definite article 'the' and second blank indefinite article 'a'.

Scan for Video solution

31. (c)

For singular noun, singular verb is used while for plural noun, plural verb is used.

'The expenditure' is singular noun so singular verb 'breaks down' should be used.

Scan for Video solution

32. (b)

They're $\rightarrow$ They are

Their $\rightarrow$ Pronounce for they

There $\rightarrow$ Denotes the place

Scan for Video solution

33. (c)

The ways in which this game can be played <u>are</u> infinite.

Since the subject of the sentence is plural hence 'are' will be used.

Scan for Video solution

34. (d)

He was one of my best <u>friends</u> and I felt his loss <u>keenly</u>.

'One of the' is used to describe something/someone from the same group.

Keenly means intensely, in a lager manner.

Scan for Video solution

35. (c)

Here first sentence in past perfect the second must be in past indefinite, so wished is the second form of wish which is used in past indefinite tense.

So, option 'c' is correct.

Scan for Video solution

36. (b)

Enough is used after the adjective or adverb and before noun, so that bright enough is suitable for this sentence.

So, option 'b' is correct.

Scan for Video solution

37. (a)

The comparative forms of the adjective "easy" are " easier" and "easiest" not "Most easier".

Than is used in comparisons as a conjunction.

Scan for Video solution

38. (a)

We use 'could have' to talk about something of a past possibility that didn't happen. And we use third form of verb with had.

Scan for Video solution

39. (c)

For facts or universal truth, simple present tense is used. So 'is'.

Scan for Video solution

40. (b)

Here 'If clause' should have past perfect tense, therefore 'Had been' is suitable in the given blank.

Scan for Video solution

41. (c)

'Tall' is used to measure height of a person or object.

'Long' is used to measure length of a thing.

Mr. X is taller than Mr. Y.

Scan for Video solution

42. (a)

The unruly crowd demanded that the correct be <u>hanged</u> without trial.

Scan for Video solution

43. (b)

The volume of a sphere of diameter 1 unit is less than, the volume of a cube of side 1 unit.

Lesser than would be incorrect because lesser & than both imply a comparison making them redundant. Less is comparative form of little.

Scan for Video solution

44. (a)

Mountain Everest is the highest peak in the world.

'Highest peak' is specific so we need to use the definite article 'The' before it.

Scan for Video solution

45. (b)

Security guard is a common man so that 'a' will be used as an article, and 'the' will be use with university so that option (b) is correct.

Scan for Video solution

46. (c)

We use **"than"** when we compare two objects or things directly with each other, as in "Your laptop is more powerful than my laptop" or, more succinctly, "Your laptop is more powerful than mine."

On the other hand, we use **"than that of"** when we compare not the two objects or things directly but an attribute, possession, or part of theirs, as in "The processor of your laptop is more powerful than that of mine."

More costlier is wrong because we cannot use "two comparative degrees together in a single sentence".

Scan for Video solution

47. (c)

When 'If clause' is in past tense 'main clause' uses the word like 'could / would / should'. Here the blank should have negative, thus suitable word in blank is 'wouldn't buy'.

Scan for Video solution

48. (c)

This is famous quote 'Where there is a will, there is a way.' It means if someone has strong wish to do something, they would definitely find a way to do it.

Scan for Video solution

49. (d)

It is useless to use negative sentence after until. Until is not followed by a negative sentence.

Also, ministor is singular thus singular verb 'meets' is to be used.

Scan for Video solution

50. (c)

The underline part can be replaced with "Increase in productivity necessarily" so the complete sentence is 'Increase in productivity necessarily reflects greater efforts made by the employees.'

Scan for Video solution

51. (a)

"It would remain between you & me" is the correct statement.

Scan for Video solution

52. (b)

The most appropriate word from the options given below is "should have practiced"

If the athlete wanted to come first in the race, he should have practiced several hours every day.

Scan for Video solution

53. (b)

Part Q is wrong. It should be "the instructor had marked"

Scan for Video solution

54. (c)

Misogynous: Showing feelings of hating women. The adverb of misogynous is "Misogynously". Adverb is the word or phrase that modifies an adjective, verb or a word group.

Scan for Video solution

55. (b)

Dhoni as well as other members of Indian team, was present on the occasion. The noun preceding "as well as" carries stronger emphasis than the noun following it.

Scan for Video solution

56. (c)

Completely must be used before verb so 'completely forgot' must be used in the 1st blank space. Just cannot be in between the don't and know, therefore, 'just don't know will be used in 2nd blank space.
So, option (c) is correct.

Scan for Video solution

57. (a)

Much is used for uncountable things, no/few are not inferred with the statement, so that only option (a) is left, which is correct.

Scan for Video solution

58. (a)

(a) Since the report lacked needed information, it was of no use to them

(b) The report was useless to them because there **was** no needed information in it.

(c) Since the report did not contain the needed information, it was not **really** useful to them

(d) Since the report lacked needed information, it would not **have** been useful to them

So, option (a) is correct.

Scan for Video solution

59. (a)

If connective/ appositives like **along with**, **together with**, **as well as, accompanied by** etc are used to combine two subjects, the verb agrees with the subject mentioned first.
So, verb must agree with Tuberculosis which is a singular and hence **ranks** should be used.

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60. (c)

The suitable words to complete the sentence are:

The Tamil version of the John Abraham-starrer *Madras Cafe* was cleared by the Censor Board with no cuts last week but the film's distributors found no takers among the exhibitors for a release in Tamil Nadu on this Friday.

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61. (b)

Advice is a noun that means guidance whereas advise is a verb which means to guide someone.

Scan for Video solution

62. (b)

In the given sentence "If clause" is given. In this, the principal clause is in future tense (will/shall) so the conditional clause (if) should be in present indefinite tense.
Hence "gets some rest" is the correct option.

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63. (b)

Since comunication and interpersonal skills are considered together here hence 'both' is the correct word.
'Each' is used for separate things, 'all' is not used for 2 things and 'either' is used to present alternative hence these three words are not appropriate here.

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64. (c)

The second part of the sentence is in past tense so first sentence should also be in past tense.

Scan for Video solution

65. (b)

This is an interrogative sentence, so that who is a question word, therefore 'did' is helping verb. Now that cannot be used here, therefore, option (b) is correct.

Scan for Video solution

66. (d)

Part (iv) is not appropriate, because the sentence is in the past that it must be 'lost consciousness'

Scan for Video solution

67. (c)

If first clause in the past perfect then main clause must be in the past indefinite.

Therefore, had known and could have best suited words in this sentence.

Scan for Video solution

68. (d)

For facts or universal truth, simple present tense is used. Thus 'experiences' is the suitable word.

Scan for Video solution

69. (c)

'An European' is wrong. It should be 'A European'.

Scan for Video solution

70. (a)

In a conditional sentence, if the part of the sentence containing "If I" is in past then resultant part of the sentence will contain 'would'.

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71. (d)

The sentence 'Two and two make four' is correct as the plural verb make agree with the plural compound subject 'two and two'.

So, option (d) is correct.

Scan for Video solution

72. (b)

'to' is used before the verb, here 'ordered to' is incorrect. It should be 'ordered'.

Scan for Video solution

73. (d)

In statement (D), 'very less' should be 'much less' or very little. Very less implies that it is less than something else or less than it was before.

Scan for Video solution

74. (a)

That $\rightarrow$ Essential clause

Which $\rightarrow$ Non – essential clause

Who and whom are for people not dogs.

Scan for Video solution

75. (b)

The sentence is in active voice as the speaker requested it, "should be given" is in passive voice and is an incorrect usage. It should be 'should give' for should be given.

Hence option (b) correct.

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76. (a)

For excreting outcome that are completely unrealistic or will not likely happen is future, are the simple past tense in the if-clause and an auxiliary model verb (e.g, could, should, would, might) in the main clause (the one that express the unrealistic or unlikely outcome). Hence option (a) correct.

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77. (c)

Contemplate is a transitive verb so it is followed by gerund (a verb form which functions as noun) hence the correct sentence is "I contemplated **visiting** Singapore for my vacation but decided against it."

Scan for Video solution

78. (a)

Planting : Seed :: Raising : Child

Planting a Seed is similar to raising a child

Scan for Video solution

79. (b)

'Enforce' and 'Relax' are opposite words to each other

∴ 'Permit' and 'Forbid' also have the same relation (opposite) to each other.

∴ Option (b) is correct.

Scan for Video solution

80. (a)

The legislature works in the parliament, similarly the judiciary performs their duty or work in the courts.

Scan for Video solution

81. (c)

Conscious: aware of and responding to one's surroundings

Consensus: a general agreement

Conscience: a person's moral sense of right and wrong, viewed as acting as a guide to one's behaviour.

Consent: permission for something to happen or agreement to do something.

Scan for Video solution

82. (a)

Seldom is antonym of often similarly cruel is antonym of kind.

Scan for Video solution

83. (c)

Insert is antonym of eject similarly retreat is antonym of advance.

Scan for Video solution

84. (d)

Since, some humans are intelligent means at least one human is intelligent so, option A is wrong. Option B is wrong because may be some humans are not intelligent. Option C is wrong because may be all the intelligent are humans.

Scan for Video solution

85. (b)

We reached the station late, and <u>nearly</u> missed the train.

Utterly → Completely

Mostly → Almost all the time.

Scan for Video solution

86. (b)

Frequently is the synonym of often similarly type is the synonym of kind refering to classification.

Scan for Video solution

87. (d)

The correct sequence that creates a coherent narrative is – (iii), (ii), (i), (iv).

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88. (b)

Writing too many things on the board while teaching could make the students get bored.

Scan for Video solution

89. (d)

After playing two hours of tennis, I am feeling too tired to walk back.

Scan for Video solution

90. (d)

Heel = back part of the foot

Heal = to become healthy again

∴ Option (d) is correct.

Scan for Video solution

91. (c)

Neither/nor is used together to state "two or more things that are untrue or won't happen".

Scan for Video solution

92. (d)

No → rejection

Know → to be aware

Scan for Video solution

93. (d)

Fare → The amount paid for a journey on a public transport.

Fair → Impartial and just.

So, correct sentence is

The fare is too high for it to be considered fair.

Scan for Video solution

94. (c)

Nostalgia: Excessively sentimental yearning for return to or of some past period or irrecoverable condition.

Anticipation: A feeling of excitement about something that is going to happen in the near future.

Scan for Video solution

95. (c)

Oasis is the place in the sand which is wet.

Same island is the place in the water which is dry.

So, option (c) is correct.

Scan for Video solution

96. (c)

Doctor performs the surgery similarly writer writes the book.

Scan for Video solution

97. (c)

Pen is used to write similarly knife is used to cut.

Scan for Video solution

98. (b)

Squared the series means to have same number of wins/ points as the opponent in the series.

Scan for Video solution

99. (b)

Whitening makes the objects white similarly lightening makes the objects light (in weight).

Scan for Video solution

100. (a)

The recent measures to improve the output would <u>increase</u> the level of production to our satisfaction

Scan for Video solution

101. (b)

In the sentence "Having said that" means despite what I have said.

Scan for Video solution

102. (c)

Build is a verb and building is a noun similarly grow is a verb and growth is a noun

Scan for Video solution

103. (c)

Shed crocodile tear means to pretend to be sad. Hence the correct sentence is "He is known for his unscrupulous ways. He always sheds <u>crocodile</u> tears to deceive people."

Scan for Video solution

104. (b)

Coup = seizure of power,

Poised = poison

Plunged = rise/fall

Plugged is use to switch on some electrical equipment.

Rise or fall is the part of stock market,

∴ Option (b) is correct.

Scan for Video solution

105. (c)

Do and undo are opposite to each other,

So, opposite word of trust is distrust.

∴ Option (c) is correct

Scan for Video solution

106. (b)

Explicit describes something that is very clear and without vagueness or ambiguity. **Implicit** often functions as the opposite, referring to something that is understood, but not described clearly or directly.

Express: To make known (as an idea, emotion or opinion)

Repress: To repress is to hold something back or to prevent an act of volition, especially by force.

Suppress: To suppress something is to put an end to it forcibly, among other things.

Scan for Video solution

107. (a)

'Fuse' is verb and 'Fusion' is its noun form. Similarly, 'Use' is verb and 'Usage' is its noun form.

Scan for Video solution

108. (b)

Ivory towers → unaware of reality. Thus option (b) is the answer.

Scan for Video solution

109. (c)

Voracious → doing something with enthusiasm.

Precocious → Advance, Ahead.

Thus suitable word for the given blank is voracious.

Scan for Video solution

110. (a)

Disparage → To lower reputation

Applaud → praise

Reduce → decrease

Begrudge → to be envy or jealous

Scan for Video solution

111. (a)

'But' in the sentence shows that last part of sentence contradicts to the first part of sentence. So correct option is 'extremely poor'.

Scan for Video solution

112. (a)

Here 'cook' is verb then 'cook' is noun.

Cook (subject) cooks (verb) the food similarly, the flyer flies the aircraft.

Scan for Video solution

113. (d)

In the sentence, 'more' is correct because 'than' comes for comparative degree.

"Are there enough seats here? There are more people than I expected."

Scan for Video solution

114. (a)

Culmination means highest or end point of something especially attained after a long time.

So, correct sentence is "A final examination is the culmination" of a series of evaluations that a student has to go through.

Scan for Video solution

115. (b)

The verb ‘skirt’ means avoiding the discussion of a subject hence the correct sentence is:

The minister avoided any mention of the issue of women’s reservation in the private sector. He was accused of skirting the issue.

Scan for Video solution

116. (c)

John Thomas an eminent writer, passes away in 2018.

Eminent means famous and respected.

After ‘an’ the word should start with vowel sound.

Scan for Video solution

117. (d)

In first sentence, discussion about delight or joy but in second sentence, it is for trouble, so even cannot be used here. Because both sentences have some different meaning, trouble is due to news paper so it should be too.

So, option (d) is correct.

Scan for Video solution

118. (d)

Deteriorate means decline or worse. So, it cannot be the proper word in this sentence.

Accommodate means occupied something, or providing sufficient space. Therefore, accommodate is the suitable word to this sentence.

Scan for Video solution

119. (c)

‘quite a few’ means a fairly large number. ‘Quiet’ means silence. Thus option (c) is appropriate in the given blanks.

Scan for Video solution

120. (c)

‘Reach’ is appropriate word for the given blank.

It means extend or stretch out. Peak is highest point or top.

Scan for Video solution

121. (d)

In comparison ‘off’ word is inappropriate. Also in a singular noun ‘those’ can’t be used. Thus, the blank should consist ‘as that of’.

Scan for Video solution

122. (c)

From the passage, the meaning of word “arrived” can be taken as ‘attained a status’.

Scan for Video solution

123. (a)

The word ‘revolve’ in the sentence infer to ‘an element which is treated as the most important’ so correct sentence is.

The search engine’s business model revolves around the fulcrum of trust.

Scan for Video solution

124. (d)

A court is to a judge as school is to a teacher.

The workplace for a judge is a court

The workplace for a teacher is a school.

Scan for Video solution

125. (a)

Undiminished: Not reduced or lessened.

"The judge's standing in the legal community, through shaken by false allegation of wrongdoing, remained undiminished.

Scan for Video solution

126. (d)

Complement: Refers to something that completes something else.

Compliment: A polite expression of praise or admire.

So, complete sentence is "The dress Complemented her so well that they all immediately Complimented her on her appearance.

Scan for Video solution

127. (c)

Miserliness: Extreme meanness or excessive desire to save money (Negative meaning)

Frugality: Quality of being economic with money (Positive meaning)

Her Frugality should not be confused with miserliness, she is ever willing to assist those in need.

Scan for Video solution

128. (a)

Going by the Principle that many hands make light work, then school Principal involved all the students in the task.

Principle $\rightarrow$ A fundamental truth.

Principal $\rightarrow$ The most important person in an organization.

Scan for Video solution

129. (d)

As per given in the question, that ideas must be complicated means confused, so that detailed/simple/clear words are not suitable. Convoluted means difficult which is inferred to complicated.

So, option (d) is correct.

Scan for Video solution

130. (a)

Course = way or direction or path

Coarse = rough area or harsh land

"Since you have gone off the way, the rough area sand is likely to damage the car."

So, option (a) is correct.

Scan for Video solution

131. (a)

Improvement: A thing that makes something better.

Neglected: Not receiving proper attention.

Glum: Looking or feeling dejected

Disinterested: Not influenced by considerations of personal advantage.

"Even though there is a vast scope for its **Improvement,** tourism has remained a **Neglected** area."

Scan for Video solution

132. (b)

Piece: a portion of an object produced by cutting or breaking the whole.

Peace: A lack of conflict and freedom from fear of violence between individuals or groups.

Peas: A type of vegetable.

Scan for Video solution

133. (a)

Brake → Attempt to stop

Break → relax / rest.

Thus the sentence should be 'The driver applied the <u>brake</u> as soon as she approached the hotel where she wanted to take a <u>break</u>.'

Scan for Video solution

134. (a)

As we cannot be aware of the codes or rules of different society, thus the proper word here in blank should be 'Unpredictable'.

Scan for Video solution

135. (a)

Shone → shine → glitter

Shown → show → exhibit.

Hence the correct option is (a).

Scan for Video solution

136. (a)

'Pioneering ideas' means new ideas. Thus, the suitable word is 'Innovative'.

Scan for Video solution

137. (a)

Meandering → Indirect, not organized

Scan for Video solution

138. (d)

Cleaving mean splitting.

Scan for Video solution

139. (a)

Forgo: go without (something desirable)

Exclusive: can not occur at the same time (events).

So, the correct sentence is.

If you choose plan P, you will have to <u>forgo</u> plan Q, as these two are <u>mutually</u> exclusive.

Scan for Video solution

140. (d)

As the two speakers become increasingly agitated, the debate become <u>heated</u>.

Here heated means angry, furious, fierce, intense.

Scan for Video solution

141. (d)

As per research, people doing work for many reasons besides money.

So, option (d) is correct.

Scan for Video solution

142. (d)

Aversion means strong dislike,
Luminous or radiant means lighting
Plunder or loot means robbing
Yielding means giving and resistance means stopping or to oppose.
So, option (d) is correct.

Scan for Video solution

143. (c)

Figuratively: used to indicate a departure from a literal use of words; metaphorically
Retrospectively: with consideration of past events or situations
Respectively: Separately or individually and in the order already mentioned
Rightfully: In accordance with a legitimate right.

Scan for Video solution

144. (d)

Antagonistic: Showing or feeling active opposition or hostility towards someone or something.
Impartial: Treating all rivals or disputants equally.
Argumentative: using or characterized by systematic reasoning
Hostile: Showing or feeling opposition or dislike; unfriendly

Scan for Video solution

145. (a)

Sharp tongued means to be someone who often criticizes and speaks in a severe way.

Scan for Video solution

146. (c)

Involving → participation

Assisting → Guiding

Tampering → Modification/change

Incubating → keeping warm the egg to make it hatch.

Thus 'tampering' is suitable word in the given blank.

Scan for Video solution

147. (b)

Pretty → Lovely

Sloppy → careless

Haphazard → Not systematic

Thorough means systematic and complete. Thus option (b) is correct.

Scan for Video solution

148. (c)

The phrase 'rest is history' means rest is well known.

Scan for Video solution

149. (b)

Felicitated: Congratulated.

The students felicitated the teacher on teacher's day for twenty years of dedicated teaching.

Scan for Video solution

150. (d)

Prone: Likely to suffer from, lying flat especially face downwards.

Prune is a fruit which is a dried plum.

So, statement (ii) & (iii) are correct.

Scan for Video solution

151. (b)

The verb that follows 'did' should be used in its 1^{st} form. So, lose is the correct word.

Scan for Video solution

152. (a)

The correct option to create a meaningful sentence is A

Despite the new medicine's <u>effectiveness</u> in treating diabetes, it is not <u>prescribed</u> widely.

Scan for Video solution

153. (b)

Manage + able = manageable, is the correct word here, so that So, option (b) is correct.

Scan for Video solution

154. (c)

Bare means without cover
Bear means tolerate with
So that bear with is best suited in this statement.
So, option (c) is correct.

Scan for Video solution

155. (a)

Cope with means deal with or put up with
So, option (a) is correct.

Scan for Video solution

156. (c)

Mock means to copy someone or deride or fun as someone or jeer.
Praise means to compliment someone.
Praise is the odd one here
So, option (c) is correct.

Scan for Video solution

157. (c)

Grasp: To take and hold (something) with your fingers, hand etc.
Clutching: Clutch suggests anxiety in seizing or grasping and may imply less success in holding.

Scan for Video solution

158. (a)

Turning deaf ear is not listening or ignoring. Thus option (a) is correct.

Scan for Video solution

159. (a)

Paraphrase $\rightarrow$ Restatement of something already written or spoken

Paradox $\rightarrow$ A Self-contradictory statement

Paradigm $\rightarrow$ A specific pattern or model

Paraffin $\rightarrow$ A wax like solid obtained from petroleum.

So, the correct word to complete the sentence is 'paraphrase'.

Scan for Video solution

160. (a)

Figurative → Representing examples from real life.

Collateral → A security for loan repayment.

Literal → Simple meaning of given word or statement.

Figurine → Human stature.

So, the correct word to complete the sentence is 'figurative'.

Scan for Video solution

161. (c)

The meaning of idiom "fish out of water" is 'a person who feels awkward or unhappy because they are in a situation that is not familiar or because they are different from the people around them. Other options have no meaning.

Scan for Video solution

162. (b)

Elusive: Difficult to find, achieve.

Allusive: To describe something that makes a reference, hint.

Elude: Scape from or avoid.

So, the correct statement is 'When the thief keeps eluding the police, he is being elusive.'

Scan for Video solution

163. (a)

Harbour: Keep a thought or felling (typically a negative one) in one's mind, so, the sentence will be "apparent lifelessness harbours dormant life.

Scan for Video solution

164. (b)

Introvert, atheist and altruist all are nouns hence 'alternate' is the correct answer as it is a verb.

Scan for Video solution

165. (b)

Connotation: connotations of a particular word are the ideas or qualities which the word make you think of.

Pertinacious: holding firmly to an opinion

Viaticum: provision for a journey

Clandestine: kept secret or done secretively

Ravenous: Extremely hungry

Scan for Video solution

166. (a)

Awkward: Causing difficulty, hard to deal with

Inept: Generally incompetent (showing no skills)

Graceful: having elegance

Suitable: Right for the purpose or situation.

Dreadful: Causing great suffering

So, most similar word to awkward is 'inept'.

Scan for Video solution

167. (c)

Immensely means great/terrific/good time.
Terrible means bad/horrible/terrifying
So, option (c) is correct.

Scan for Video solution

168. (b)

(a) Acquiescence/acceptancel/submission - same words. .

(b) Wheedle/flat/direct and Roundabout/not direct – opposite words

(c) Flippancy/Lightness/lack of respect – same words

(d) Profligate/Extravagant/reckless – same words

So, option (b) is correct.

Scan for Video solution

169. (c)

'took to their heels' means run away or took to flight
So, option (c) is correct.

Scan for Video solution

170. (c)

Personnel means employees or staff
Personal means private or individual

(a) The industrialist had a **personel** jet

(b) I write my experience in my **personel** diary

(c) All **personnel** are being given the day off

(d) Being religious is a **personel** aspect

So, option (c) is correct.

Scan for Video solution

171. (d)

Fabric/textile = cloth made by fibre
Fibre = cotton or thread by which fabric is made
Apparel = attire/garments/shirt/trousers/shirt
So, option (d) is correct.

Scan for Video solution

172. (a)

Croak: A characteristic deep hoarse sound made by a frog or a crow.

Scan for Video solution

173. (c)

Educe: Bring out / extract or develop something latent or potential.

Scan for Video solution

174. (b)

Memento: An object kept as a reminder of a person or event.

Scan for Video solution

175. (b)

Respectably→ Acceptable standard.
Respectfully→ with respect
Respectively → As mentioned in order or sequence
Reputably → Image or standard
Hence the suitable word in the blank is 'Respectfully'.

Scan for Video solution

176. (b)

Insured → to give protection

Ensured → to approve/make sure.

Thus in statement (b) the word is used correctly

Scan for Video solution

177. (c)

Vernacular → Native Speech, Common language

Regional → particular area

Indigenous → originally natural, native

Indigent → poor

Colloquial → used is ordinary language.

Except 'Indigent' all other words have same meaning.

Scan for Video solution

178. (d)

Dreary → Dull or boring

Dismal → Less attractive or without hope

Thus option (d) is correct.

Scan for Video solution

179. (b)

Extraneous → Irreverent/ which is not related

So, the correct word that fits in the sentence is 'extraneous'.

Scan for Video solution

180. (b)

Pediatrician → Doctor for kids

Orthopaedist → Bone doctor

Gynaecologist → Doctor for women

Nephrologist → Kidney doctor

Dermatologist → Skin doctor

Scan for Video solution

181. (b)

Vindicated → justified

Substantiated → justified

Chastened → to bring down

So, the correct word will be 'substantiated'

Scan for Video solution

182. (b)

Revert → reply

Retract → to take the statement back

Get back to him → reply

Move in reverse → to move backward

Retreat → moving away

So, the correct word will be 'Get back to him'.

Scan for Video solution

183. (a)

Dissent → having opinions that are different from majority

Descent → to go downward

Decent → proper/civilized

Decadent → to decline

So, the correct word will be 'dissent'.

Scan for Video solution

184. (b)

Depressed $\rightarrow$ Sad

Depreciated $\rightarrow$ Loosing value

Appreciated $\rightarrow$ Praise

Stabilized $\rightarrow$ To be stable

Hence most appropriate word is 'depreciated'.

Scan for Video solution

185. (c)

Bear $\rightarrow$ to carry/stand a thought

Bare $\rightarrow$ uncovered part of the body

Losing $\rightarrow$ to set free

Losing $\rightarrow$ suffering a loss.

So, the appropriate pair of word is 'Bear, Losing'

Scan for Video solution

186. (b)

Impel $\rightarrow$ To force someone

Dispel $\rightarrow$ To dissappear

Propel $\rightarrow$ To push

Repel $\rightarrow$ To get back

Hence most appropriate word is 'dispel'.

Scan for Video solution

187. (b)

'cope with' means to face someone or to deal with or adjust with

Adopt to means accepting legally

Adapt to means to adjust with

Adept in means skillful

Accept with means agree with

$\therefore$ Option (b) is correct

Scan for Video solution

188. (c)

The person was not sure about her performance that she should be awarded by first prize because her performance was mediocre.

$\therefore$ Option (c) is correct.

Scan for Video solution

189. (c)

'The buck stops here' means that I will take the responsibility.

$\therefore$ Option (c) is correct.

Scan for Video solution

190. (a)

Post means after any duration,

In early days, due to the exisance of the British colony, now India is a post-colonial country.

$\therefore$ Option (a) is correct.

Scan for Video solution

191. (a)

Eradicate means to remove completely which is equivalent to destroy utterly. 1 $\rightarrow$ S

Distort means to mislead or Misrepresent. 2 $\rightarrow$ P

Saturate means Thorley soaked. 3 $\rightarrow$ Q

Utilize means thoroughly use. 4 $\rightarrow$ R

$\therefore$ Option (a) is correct.

Scan for Video solution

192. (c)

Coherent means reasonable or logical or clear meaning.

Sticky means attachable

Rambling means confusing

Therefore, option (c) is correct.

Scan for Video solution

193. (a)

Comprehension →Ability to understand or grasp. The correct answer is 'Understanding'.

Scan for Video solution

194. (b)

Vice → bad behavior

Virtues → good behavior. Thus the correct option is 'Virtues'

Scan for Video solution

195. (a)

Contentment → satisfaction

Ambition → goal, aim

Perseverance → to be firm

Hunger → desire to eat. Thus the appropriate word is 'Contentment.

Scan for Video solution

196. (c)

'As a women, I have no country' means there cannot be boundaries or barriers to divide women. All women are united. Thus option (c) is the closest in the meaning to the given sentence.

Scan for Video solution

197. (b)

Nadir → The lowest or the most unsuccessful point in a situation.

Scan for Video solution

198. (a)

Universalism → All

Particularism → Specific

Diffuseness → Spread in direction

Specificity → The quality of belonging uniquely to a particular subject

Neutrality → Neutral in quarrel

Generality → General

Adaptation → Oneself Suitable to environment

Scan for Video solution

199. (d)

Quarrel means to have an angry argument or disagreement.

Make out means manage with some difficulty/to see or hear someone or something.

Call out means for gathering or to deal with an emergency or provide a service.

Dig out means to remove earth or debris from by digging, to hollow out by digging, to find by searching.

Fall out means quarrel or to cut off relations over a quarrel. Former friends who have fallen out.

∴ Option (d) is correct.

Scan for Video solution

200. (b)

A preposition must be connected between these two words so that sentence will be dare to commit mistakes.

∴ Option (b) is correct.

Scan for Video solution

201. (c)

A friend in need is a friend indeed” is a proverb which means a person who helps at a difficult time is a person who you can rely on.

So, option (c) is correct.

Scan for Video solution

202. (c)

Primeval → Ancient / past

Modern → Recent/ present

Historic → famous/significant

Primitive → Earliest/ Ancient

Antique → old valuable thing/ Treasure.

Thus Primeval means ‘Primitive’.

Scan for Video solution

203. (b)

Cordial → friendly

Intimate → familiar/close. Thus, the option (b).

Scan for Video solution

204. (c)

Medicine is taken to improve health similarly education is taken to improve knowledge.

Scan for Video solution

205. (a)

Mitigate → to reduce

Diminish → reduce the size

Divulge → make something

Dedicate → to offer

Denote → to represent

Scan for Video solution

206. (b)

Setbacks → an event that make someone to lose progress.

The correct sentence is “Despite several **setbacks** the mission succeeded in its attempt to resolve the conflict.”

Scan for Video solution

207. (b)

Latitude: scope for freedom of action or thought

Eligibility: the state of having the right to do or obtain something through satisfaction of the appropriate conditions.

Freedom: the power or right to act, speak, or think as one wants.

Coercion: the practice of persuading someone to do something by using force or threats.

Meticulousness: marked by extreme or excessive care in the consideration or treatment of details

Hence option (b) is correct.

Scan for Video solution

208. (d)

Beggary: a state of extreme poverty

Nomenclature: the devising or choosing of names for things, especially in a science or other discipline.

Jealousy: the state or feeling of being jealous.

Nonchalance: (of a person or manner) feeling or appearing casually calm and relaxed; not displaying anxiety, interest, or enthusiasm

Hence option (d) is correct.

Scan for Video solution

209. (c)

The topics in options a, b and d should be included to deal with bereavement and grief to help students in times of losses though death and parting but topic in option c cannot help.

Scan for Video solution

210. (b)

Understated → Not trying to attract people

Tentative → Uncertain or with hesitation

Restrained → Doing something in limit

Scan for Video solution

211. (a)

Inexplicable → Puzzled, Unclear, Unsolved

Incomprehensible → Not able to understood

Indelible → Cannot be erased

Inextricable → Which cannot be separated

Infallible → In capable of doing something

Scan for Video solution

212. (b)

Amalgamate → To unite, to merge

So opposite word will be split.

Scan for Video solution

213. (d)

The sentence implies human genes are to be manipulated only to correct diseases for which the satisfactory treatments are not available.

Hence option (d) is correct.

Scan for Video solution

214. (b)

Frequency: the number of times something is repeated, or the fact of something happening often.

Periodicity: the quality of character of being periodic; the tendency to recur at intervals.

Rarity: (of an event, situation or condition) not occurring very often.

Gradualness: occurring or developing slowly or by small increments.

Persistency: the fact of continuing in an opinion or course of action in spite of difficulty or opposition.

So, 'Rarity' is opposite to 'Frequency' and hence option (b) is correct.

Scan for Video solution

215. (c)

The sentence implies that foreign technocrats were harmful to the country. So, only suitable opinion is **Exacerbate** – meaning aggravate or make things worse or increase the severity

$\therefore$ Option (c) is correct.

Scan for Video solution

216. (a)

Arena is the combat place of the gladiator or some other animals. It means gladiator performs in the arena. The same way a dancer performs on the stage.

So, option (a) is correct.

Scan for Video solution

217. (b)

Belied $\rightarrow$ Being in contradiction or unable to give a true impression.

Scan for Video solution

218. (b)

Circuitous $\rightarrow$ Route longer than direct way.

Scan for Video solution

219. (a)

Fallow $\rightarrow$ refers to the land which is ploughed but left for a period without being sown in order to restore fertility.

Worker who is not working is unemployed similarly land which is not producing anything is fallow.

Scan for Video solution

220. (d)

(a) Uphold $\rightarrow$ Cause to remain

(b) Restrain $\rightarrow$ Keep under control

(c) Cherish $\rightarrow$ Be fond of

(d) Conserve $\rightarrow$ Keep in safety

Scan for Video solution

221. (c)

The correct sequence of sentence that creates a coherent narrative is: II $\rightarrow$ I $\rightarrow$ IV $\rightarrow$ III.

Scan for Video solution

222. (d)

In the paragraph it is given that almost all of Ravi's Friends at school were hand working and kind too. So, it is not necessary that Ravi's Friends outside the school are hardworking and king too, and also almost 'all' infer 'some' so, from the passage it is certain that "Some of Ravi's Friends are hardworking and kind."

Scan for Video solution

223. (d)

From the conclusion of the given passage, we find that option a, b, c, describe the passage so option a, b, c is incorrect.

Option (d) is correct because modern cell biology uses figurative language, such as metaphors, to describe or explain anything.

$\therefore$ Option (d) is correct.

Scan for Video solution

224. (d)

It may be possible that student, who would not eligible to vote will belong to any other department not only to the Human Sciences, hence statement (i) is not correct.

It may be possible that student, who failed to complete the registration process within time will belong to any other department not only to the Human Sciences, hence statement (ii) is not correct.

Statement (iii) is correct because no student from the department of Human Sciences was eligible to vote as they failed to complete the registration within time.

∴ Hence only statement (iii) is correct.

∴ Option (d) is correct.

Scan for Video solution

225. (b)

As given in the statement that 65% of the tobacco users were advised to stop consuming tobacco but only 3 out 10 users attempted to stop using tobacco, it means only 30% of that 65% tobacco users attempted to stop using tobacco. Which indicates that majority of the tobacco users who were advised to stop consuming tobacco did not attempted to do so. Only some of the majority attempted to stop using tobacco.

∴ Option (b) is correct.

Scan for Video solution

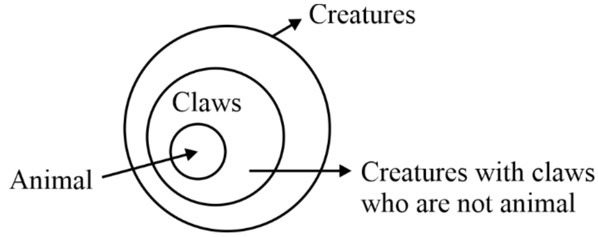
226. (b)

The sentence "when I was a kid" implies it is an adult's memory of what he or she liked as a child.

Scan for Video solution

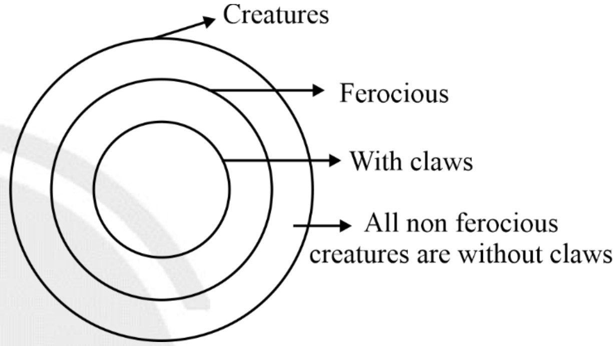
227. (d)

Option (a) is wrong because

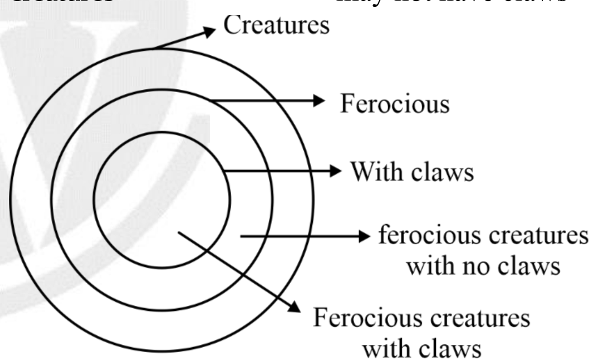


Option (b) is wrong because "All creatures that have clause are ferocious".

Option (c) is wrong because



Option (d) is right because "same ferocious creatures may not have claws"



Scan for Video solution

228. (d)

From reading the paragraph we certainly can infer that the government of zombieland believes that riding bicycles is a form of physical exercise.

Scan for Video solution

229. (d)

In Elvesland, gifting a white flower to someone is taken as declaration of love but one not always declare one's love by gifting white flower. Yellow flower is gifted to show anger. So, statement iv is correct.

Scan for Video solution

230. (d)

The mother's feelings about her daughter's singing at home were not only of annoyance but most often this made the mother pensive (thoughtful). So, option (d) is correct.

Scan for Video solution

231. (d)

"Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet "dark" in dark matter." So, option (d) is false and all other are true.

Scan for Video solution

232. (b)

"For example, babies who start eating healthy in the first year are more likely to have better overall health as they get older.**" implies Eating healthy can be started at any age, earlier the better.**

Scan for Video solution

233. (b)

"The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low.**"** Implies **More people are visiting the malls but not spending enough.**

Scan for Video solution

234. (c)

Having ultra black skin is advantageous because it helps them to avoid predators & also attack their prey. It also has disadvantage because it makes their skin fragile.

Scan for Video solution

235. (b)

In the passage it is clear that the primary reason for people to return the wallet is empathy which means the ability to understand the feeling of another. So, wallets with a key are more likely to be returned because people relate suffering of others.

Scan for Video solution

236. (c)

As given in the passage that altruism may be attributed to empathy or guilt has now been rejected. So that option (a), (b) and (d) is not correct.

Humans engage in altruism due to group identification and familiarity

$\therefore$ Option (c) is correct.

Scan for Video solution

237. (d)

Controlling mosquitoes using chemicals may have undesired consequences implies **"Using chemicals to kill mosquitoes may have undesired consequences"**

And it remains to be seen if this novel approach has unforeseen consequences implies **"it is not clear if using genetically modified mosquitoes has any negative consequence".**

Scan for Video solution

238. (b)

The passage clearly indicates that the matrics are useful in the orgazinations with good work culture. So, option (c) is wrong and options (a) and (d) also wrong as discussed above.

Scan for Video solution

239. (d)

In the two given sentences, it is observed that facing a crisis is the effect of pandemic, hence option (d) is correct.

Scan for Video solution

240. (c)

In the symbiotic relationship described in passage, the primary benefit for Oxpeckers is that they get food source & that for rhinos is that they may by saved from the poachers.

Scan for Video solution

241. (c)

From the passage it is clear that the indication of healthy lifestyle is related to consumption of leafy vegetable.

Scan for Video solution

242. (d)

The conclusion of the passage is that to do well in an exam, adequate sleep must be part of the preparation.

$\therefore$ Options (d) is correct.

Scan for Video solution

243. (d)

"Computers are ubiquitous" implies **Nowadays, computers are present in almost all places**. "They are used to improve efficiency in almost all and fields from agriculture to space exploration. For humans, sitting in front of a computer for long hours can lead to health issues" implies **for humans, there are both positive and negative effects of using computers**. "AI enables computers to learn, given enough training data" implies

Artificial intelligence cannot be done without data.

So, option (d): (i) & (iii) is correct.

Scan for Video solution

244. (c)

We do not know of any species other than humans who posses the ability to construct mental worlds.

Scan for Video solution

245. (a)

The author is of the opinion that rehearsing is important for musicians, actors and public speakers.

Scan for Video solution

246. (d)

The passage indicates that the core problems that causes obesity is poverty and income inequality. Since AOM is not addressing the core problems, so AOM is superficially addressing the problem.

Scan for Video solution

247. (c)

Since it is clear from the passage that only some students who did not need any external stimulation, benefited from music so option c is correct.

Scan for Video solution

248. (c)

Mitigation deals with actions taken to reduce the use of fossils fuels.

Adaptation deals with the process of adjustment to actual climate, and its effects in order to moderate harm due to climate change.

Scan for Video solution

249. (d)

In a crowd funding, fund is raised from a large number of people willing to (voluntarily) contribute on web-based platforms.

Scan for Video solution

250. (a)

It is clear from the passage that **world-wide financial crisis** started with the sub-prime lending crisis in USA in 2007. And the subprime lending crisis led to the **banking crisis** in 2008 with the collapse of Lehman Brothers in 2008. The **sub-prime lending crisis** arised because of excess liquidity following the **East Asian crisis.**

Scan for Video solution

251. (c)

The French-speaking couple were upset at the English announcements being longer than the French ones.

Scan for Video solution

252. (d)

As per given information

Theory given by the American psychologist Howard Gardner. Based on this theory, Modern educationist insist that the teaching curriculum and evaluation needs to be multidimension. Therefore option D is correct.

Scan for Video solution

253. (a)

As the inflation is high real interest rate will be low borrow get benefited.

As the under low inflation, the interset rate will be high lenders get benefited. Therefore option A is correct.

Scan for Video solution

254. (d)

Partial : Impartial : : Popular : unpopular

Scan for Video solution

255. (d)

From the passage, melting glaciers are affecting billions of people and nation-states of UN need to rethink their carbon footprint, infers that some nation-states are not very serious for environment change.

Scan for Video solution

256. (d)

It is clearly mentioned in passage that GST is not imposed on the point of origin rather it is imposed on point of destination/usage.

Scan for Video solution

257. (a)

From the paragraph it can be inferred that most households that subscribe X's loyalty membership discontinue watching cable television.

Scan for Video solution

258. (b)

The teacher was surprised because he/she received the stories from the students that were **all** set in distant places and none of the students had written stories set in India.

Scan for Video solution

259. (a)

Since the name Zaira was a Portuguese alteration of *Nzadi o Nzere* which is an African term. So, Mobuto was not entirely successful in Africanising the name of his country.

Scan for Video solution

260. (a)

In the question statement about men are prone to buying on impulse as women but not about many/few/all men/women but it can be equivalent to some men and women, therefore option (a) is correct.

Scan for Video solution

261. (c)

"Indian history was written by British historians- extremely well documented and researched, but not always impartial" implies **Indian history written by British historians was well documented and researched but was sometimes biased.**

Scan for Video solution

262. (d)

In the passage it is not mentioned who duped the tribals out of their land holdings. Hence, option D is correct.

Scan for Video solution

263. (b)

In the given paragraph it is said 'Protagonists usually read books of aspirational values'. Here the word usually does not mean only. They may read other books too. Therefore option (b) cannot be inferred.

Scan for Video solution

264. (b)

From the passage it is clear that the new cotton technology, Bollgard-II is not legal in India, but farmers are using this technology, this indicates that farmers want to access the new technology even if it is not legal.

Scan for Video solution

265. (b)

The passage infers that beggers are created because of the lack of social welfare schems.

Scan for Video solution

266. (d)

"It has remained inaccessible to tourists due to poor connectivity and lack of accommodation. A company has spotted the opportunity and is planning to develop a luxury resort with helicopter service to the nearest major city airport" implies **Helicopter connectivity would lead to an increase in tourists coming to the region.**

Scan for Video solution

267. (c)

"However, his on-field temper has always been a matter of concern for them since his junior days" **implies john's temper as a player not captain.** "After many years, it appears that the team has finally found a complete captain" implies **finding a complete captain is a challenge.** "Over the past three seasons though, that picture of John has been replaced by a cerebral, calculative and successful batsman-captain" implies **Over the past three seasons John has accumulated big scores.**

Scan for Video solution

268. (b)

James was the best player in his own family this means Mark was not best in his family so, Steve was known to play better than Mark.

Scan for Video solution

269. (c)

Stuart's demonstration had no effect on his superior so option A is not correct. The Europeans were prejudiced so option B is not correct Stuart was reprimanded by commander-in-chief, so option D is also not correct. So, option (c) is correct.

Scan for Video solution

270. (b)

As mentioned in the statement inadequately or improperly nationalist is just not history, but it is viewed through the filter of nationalism. Which is not based on the imagination or past.
So, option (b) is correct.

Scan for Video solution

271. (b)

The author's belief that ideology is not as important as literature is revealed by the word seemingly.
So, option (b) is correct.

Scan for Video solution

272. (a)

"for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters." Implies **An intimate association offers partial perspective and hence matters are not recorded with an impartial perspective.**

Actors are typically associated with the partial recording of matters.

So, we can say "An intimate association does not allow for the necessary perspective."

Scan for Video solution

273. (b)

From the passage, air pockets are created by excavation of large amount of soil. So, option A & D is wrong. Old buildings do not form impossible hurdle to metro construction in that area.

Scan for Video solution

274. (a)

Since, the customers came out of the centre in the order that they went in infers that the service pattern in first-come-first-served basis.

Scan for Video solution

275. (c)

Kalimpong < Gangtok

Pelling < Gangtok

Pelling > Siliguri

Darjeeling > Gangtok

From the above conditions, statements (ii) & (iv) are correct.

Scan for Video solution

276. (a)

From the paragraph it can be inferred that social science disciplines had a pre-colonial origin because the disciplines were in existence in an amorphous form until the colonial period.

Scan for Video solution

277. (c)

From the paragraph it can be inferred that surveillance state is an perception of use of technology to ensure safety & security of smart city.

Scan for Video solution

278. (b)

From the passage it can be inferred that he was sure that his friends would help him when in need but his friends did not help him last week.

Scan for Video solution

279. (c)

It is not mentioned that SIT student will get an IIT degree so that statement (i) is incorrect.

Any institute but not only SIT can only be established in consultation with IIT so that statement (iii) is also incorrect.

SIT is doing this as a national objective so that they can provide good student to the country by which any country can develop.

So, option (c) is correct.

Scan for Video solution

280. (c)

As per the passage the woman who had done their masters in mechanical engineering are successful.

Which means that only option (c) is best suited this statement. As most of the people have believed that masters in mechanical engineering are not suitable for woman so that option (b) is wrong,

Option (a) and (d) do not have any sense with the passage.

So, option (c) is correct.

Scan for Video solution

281. (b)

R2D2 is a robot. R2D2 can repair aeroplanes. No other robot can repair aeroplanes.

The conclusion of the above statement is R2D2 is the only robot which can repair aeroplanes.

So, option (b) is correct.

Scan for Video solution

282. (d)

Author believes that computers have advantages as well as disadvantages, so that statement (i) is not correct.

Mobile computers and the internet are both unintended inventions, so that statement (ii) is not correct.

So, option (d) is correct.

Scan for Video solution

283. (a)

"The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern" implies **the number of people in India infected with rabies is high**.

Also inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction not eradication.

Scan for Video solution

284. (d)

From the passage it can be inferred that the linguistic pluralism is one of the indicators of nation's diversity. In India there are 22 officially registered spoken languages. Linguistic pluralism is strong evidence of India's diversity.

Scan for Video solution

285. (d)

From the passage "Historians tend to correlate greatness of a king at his time with the availability of evidence today" infers that a king's greatness as we know him today, is interpreted by historians.

Scan for Video solution

286. (b)

From the paragraph it can be inferred that Ms. x will reach Kochi on 21 may & will be Bagdogra on 22 may but she may visit Kochi again in June or any other month.

Scan for Video solution

287. (b)

The correct order of the sentences for a meaningful paragraph is

1, 5, 2, 4, 3, 6.

Scan for Video solution

288. (b)

From the paragraph its can be inferred that the artists expected funding for the arts to increase this year & schools are giving less importance to arts education nowadays.

Scan for Video solution

289. (a)

The government shouldnot ban the water supply in lower areas. Therefore statement (iii) is not correct.

So, option 'a' is correct.

Scan for Video solution

290. (b)

Because the wall sometimes breaks, we can say **Humpty Dumpty does not fall sometimes while having lunch.**

Scan for Video solution

291. (b)

"But on the contrary, human progress seems contradictory to environmental security" implies **Human progress is contradictory to environmental security**.

Scan for Video solution

292. (a)

From the statement, it can be inferred that since Alexander conquered Persia, he headed for India. Now if he had not conquered Persia then he would not move towards India.

Scan for Video solution

293. (b)

From the passage it is clear that he already had all the technical skills and to become a great batsman he has to work on his temperament. He was guilty of throwing away his wickets several times after working hard to lay a strong foundation.

Scan for Video solution

294. (c)

In option C, 22 official languages and thousands of dialects represents the rich heritage and cultural diversity of India.

Scan for Video solution

295. (b)

Male above 10 lakhs $\rightarrow$ FORM-R, Male below 10 lakhs $\rightarrow$ FORM-P

Female above 10 lakhs $\rightarrow$ FORM-R, Female below 8 lakhs $\rightarrow$ FORM-Q

Non-residence with income above 15 lakhs $\rightarrow$ FORM-S

All others should fill the FORM-T

From the options, a residence female with annual income Rs. 9 lakhs will be in others and so will the FORM-T.

Scan for Video solution

296. (c)

Palghat is surrounded by the hills which is lower than the hilly terrain to its north and south. Which impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.

$\therefore$ Option (c) is correct.

Scan for Video solution

297. (b)

Certain psychiatric illnesses have a genetic basis i.e. depression and schizophrenia, and consequently.

∴ Option (b) is correct.

Scan for Video solution

298. (b)

Before world war 2, Kaliningrad was a German majority population area, but currently it is a Russian territory even it is not close to the Russia. Therefore,

∴ Option (b) is correct.

Scan for Video solution

299. (d)

The dengue cases were increases because municipal authorities have concluded that measures to control the mosquito population have failed in this region. Therefore, the number of people with malarial fever (also contracted from mosquito bites) has increased this year.

∴ Option (d) is correct.

Scan for Video solution

300. (d)

In the above statement only, statement (iv) inferred with the passage, therefore, option (d) is correct.

Scan for Video solution

301. (c)

From the passage, the main point we can take is that exactitude is not critical in dealing with big data.

Scan for Video solution

302. (d)

From the sentences it can be inferred that Rajan had believed that Sajan and he would be working together but Sajan decided to do the project on his own.

Scan for Video solution

303. (d)

From the paragraph it can be inferred that 'No adversity justifies giving up hope'. The spider failed many times but that did not deter it to refrain from making attempts. This event encouraged Bruce not to lose hope and he won many battles thereafter.

Scan for Video solution

304. (d)

There were different streams of freedom movement in colonial India, so that we can say the nationalism in India is heterogeneous or they are of different thinking.

∴ Option (d) is correct.

Scan for Video solution

305. (a)

"In the legions, military' law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them" implies "Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances".

∴ Option (a) is correct.

Scan for Video solution

306. (b)

In the passage doctors are injecting toxin to horses because they know that horses will develop immunity automatically. They are not giving any immunity to horses.

$\therefore$ Option (b) is correct.

Scan for Video solution

307. (d)

In the advertisement there is nothing mention about gender so the advertisement is not gender-discriminatory.

Scan for Video solution

308. (c)

From the paragraph it can be inferred that use of chemical agents would be undesirable in warfare.

Scan for Video solution

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NOTE:

NOTE: