$^nc_r \, p^r q^{n-r}$

Ex-1

An infinite seq of independent trials is to be performed. each trial results in a success with probability $p$ and a failure with probability $1-p$ what is the prob that

a). at least 1 success occures in the first $n$ trials

$^nc_0 p^0 q^{n-0}$

For no trial $\quad ^nc_0 \, p^0 (1-p)^{n-0}$

$(1-p)^n$

atleast 1 success $= 1-(1-p)^n$

b)) Exactly $k$ success occur in the first $n$ trials.

$^nc_k p^k q^{n-k}$

Soln $\quad ^nc_k . p^k (1-p)^{n-k}$

c). All trials result in successes

Soln. $\quad p^n$

Ex-2:

A system composed of $n$ seperate components is said to be a parallel system if it functions when at least one of the components functions.
for such a system if component $i$, which is independent of the other. components. functions with probability $P_i$ $i=1,2,3 \cdots n$. what is the probability that the system functions

system not working $= (1-P_1)(1-P_2)(1-P_3) \cdots (1-P_n)$

$\therefore$ Correctly work $\quad 1-\prod_{i=1}^{n}(1-P_i)$.

Ex-3: Independent trials consisting of rolling a pair of fair dice are performed. what is the probability that an outcome of 5 appears an outcome of 7 when the outcome of a roll is the sum of the dice.

$n(S) = 36$

(2,3) (3,2) (4,1) (1,4)

4,3 3,4 5,2 2,5 6,1 1,6

$P(5 \text{ on any trial}) = \frac{4}{36}$ & $P(7 \text{ on any trial}) = \frac{6}{36}$. ✓

Ex-4

Independent trials resulting in a success with probability $P$ and a failure with prob $1-P$ are performed. what is the prob that $n$ successes occure before $m$ failure.

Soln.

$P_{n,m}$ the prob that $n$ success occur before $m$ failures.

$$P_{n,m} = P\, P_{n-1,m} + (1-P)\, P_{n,m} \quad n \geq 1 \quad m \geq 1$$

$$P_{n,0} = 0$$
$$P_{0,m} = 1$$

Exactly $k$ successes in first $(n+m-1)$ trials.

$$\binom{m+n-1}{k} p^k (1-p)^{m+n-1-k}.$$

$$P_{n,m} = \sum_{k=n}^{m+n-1} \binom{m+n-1}{k} p^k (1-P)^{m+n-1-k}.$$

## Random var :-

Discrete - set of outcomes is a sequence :-

Probability mass function: $p(a) = P(X=a)$

Cumulative distribution function: $F(a) = P(X<=a)$

Ex: Flipping two heads.

outcome : {HH TT. HT. TH}.

$P(1) = 1/2$

$P(2) = 1/3$

$P(3) = 1/6$

$F(<1) = 0$

$F(1 \le x \le 2) = 1/2$

$F(2 \le x < 3) = 5/6$

$F(3 \le x) = 1$

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$ [For pdf -]

$\Rightarrow$ Area under curve is 1

$\longrightarrow$ Continuous Random variable :-

$P(a \le x \le b) = \int_a^b f(x)$.

$P(x \ge a) = \int_a^{\infty} f(x)\,dx$.

$P(a \le x \le a) = \int_a^a f(x) = 0$

$P(x \le a) \quad \int_{-\infty}^{a} f(x)$.

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

Ex-1

$$f(x) = \begin{cases} c(4x-2x^2) & 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

1) value of c

2) $P(1 \le x \le 2)$ & $P(0 \le x \le 0.5)$

For pdf

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_0^2 c(4x-2x^2)\,dx = 1$$

$$\Rightarrow c\left[\frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} - \frac{2x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow c\left[\left(2\cdot 2^2 - \frac{2}{3}\cdot 2^3\right) - 0\right] = 1$$

$$\Rightarrow c\left[8 - \frac{16}{3}\right] = 1$$

$$\Rightarrow 8c\left[\frac{3-2}{3}\right] = 1$$

$$\Rightarrow 8c \times \frac{1}{3} = 1$$

$$\Rightarrow c = 3/8$$

(2) $P(1 \le x \le 2)$

$$\int_1^2 c(4x-2x^2)\,dx$$

$$\Rightarrow \frac{3}{8}\left[2x^2 - \frac{2x^3}{3}\right]_1^2$$

$$\frac{3}{8}\left[\left(8-\frac{16}{3}\right) - \left(2-\frac{2}{3}\right)\right]$$

$$\Rightarrow \frac{3}{8}\left[6 - \frac{16}{3} + \frac{2}{3}\right]$$

$$\frac{3}{8}\left[6 - \frac{14}{3}\right]$$

$$\frac{\cancel{3}}{\cancel{8}_2} \times \frac{\cancel{4}}{\cancel{3}} = \frac{1}{2}$$

2 II

$$\int_0^{1/2} c(4x-2x^2)\,dx$$

$$\Rightarrow \frac{3}{8}\left[2x^2 - \frac{2x^3}{3}\right]_0^{1/2}$$

$$\Rightarrow \frac{3}{8}\left[2\cdot\frac{1}{4} - \frac{2\cdot 1}{3\times 8_4}\right] = \frac{3}{8}\left(\frac{1}{2} - \frac{1}{12}\right) = \frac{\cancel{3}}{8} \times \frac{5}{\cancel{12}_4} = 5/32$$

Ex-2

Determine unknown Constant for the following density functions:

$\sqrt{2} : 1.414$

$\sqrt{3} : 1.73$

$$f(x) = \begin{cases} ① \; c(1-e^{-x}) & x \text{ is +ve real no } x \geq 0 \\ ② \; (c-x) & 0 < x < c \\ ③ \; c \cdot x & 0 < x < 1 \\ ④ \; c(1-x^2) & 0 < x < 1 \end{cases}$$

① $\Rightarrow \int_0^\infty c(1-e^{-x})$

$\Rightarrow$

if $x$ is a probability density function then

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty}$$

① $\Rightarrow \int_0^\infty c(1-e^{-x}) + \int_0^1 \cancel{c\,x}\, dx = 1$

$\Rightarrow c\left[x + \frac{e^{-x}}{-1}\right]_0^\infty = 1$

$\infty + \frac{1}{\infty}$

$\Rightarrow c\left[(\infty + e^{-\infty}) - (0 + e^{-0})\right] = 1$

$\infty + 0$

$\Rightarrow c\left[(\infty + 0) - (1)\right] = 1$

$\Rightarrow c \cdot \infty - c = 1$

$c = -1$ [ $c \cdot \infty \to 0$ $\to$ undefined assume $f^n \to 0$ ]

② $\int_0^c (c-x)\, dx = 1$

$\Rightarrow \left[cx - \frac{x^2}{2}\right]_0^c = 1$

$\Rightarrow c^2 - \frac{c^2}{2} = 1$

$\Rightarrow c^2(1-\frac{1}{2}) = 1$

$\Rightarrow c^2(\frac{1}{2}) = 1$

1.414

$\Rightarrow c = \sqrt{2}$

$E(x) = \Sigma x\, p(x)$

3) $\int_0^1 c\,x\,dx = 1$

$\Rightarrow \left[\frac{c\,x^2}{2}\right]_0^1 = 1$

$\Rightarrow c\left[\frac{1}{2} - 0\right] = \frac{c}{2} = \frac{r_2}{2} = \frac{1}{r_2} = 1$

$c = 2$

(4) $\int_0^1 c\,(1-x^2)\,dx = 1$

$\Rightarrow c\left[x - \frac{x^3}{3}\right]_0^1 = 1$

$\Rightarrow c\left[\left(1-\frac{1}{3}\right) - (0-0)\right] = 1$

$\Rightarrow c\left(\frac{2}{3}\right) = 1$

$\Rightarrow c = 3/2$

$P(x_1, y_1)$ $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$(x_2, y_2)$

R.M
V.L mam

(1) A distance metric [$d(x,y)$ : distance b/w points x and y] should satisfy the following conditions:

(i) $d(x,x) = 0$

(ii) $d(x,y) = d(y,x)$

(iii) $d(x,z) <= d(x,y) + d(y,z)$

verify if Eu. Euclidean distance in 2D and n-dimensional space meet above condition.

$$d_{Ec}(P,Q) = \sqrt{\sum_{i=1}^{n} (q_i - p_i)^2}$$

1) $d(x,x) = 0$

$$\sqrt{(x-x)^2} = 0$$

The distance b/w a point & itself should be zero in both 2D & n-dimensional space. the distance b/w a point & itself is indeed 0, so this condition holds.

(ii) $d(x,y) = d(y,x)$

→ This condition states that the distance b/w two points should be the same regardless of the order in which they are considered. E.D. satisfies this property in both 2D & n dimensional.

iii) $d(x,z) <= d(x,y) + d(y,z)$ also known as the triangle inequality ~~it satisfi~~ it states that the sum of the distance b/w two points via third point should be greater than or equal to the direct distance b/w two points.

The E.D satisfies the △ inequality in both 2D & n-dimensional. all given condition hold.

euclidian distance

$P(Y=0) = P(0,0) + P(1,0)$ = .3 + .1 = (.40)

(2) marginal pmf Y = 1

$P(Y=1) = P(0,1) + P(1,1)$

$\Rightarrow .25 + .35 = .60$

X 3 new

Y 2 used but still working

2 defective

Sol^n $Y = 0$

$$P(X=0/Y=0) = \frac{P(0,0)}{P(0,0)+P(1,0)}$$

$$\Rightarrow \frac{.30}{.3+.10} = \frac{.30}{.40} = \frac{3}{4} = .75$$

$$P(X=1/Y=0) = \frac{P(1,0)}{P(0,0)+P(1,0)}$$

$$\Rightarrow \frac{.10}{.30+.10} = \frac{1}{40} = .25$$

b) $Y = 1$

$$P(X=1/Y=1) = \frac{P(1,1)}{P(0,1)+P(1,1)}$$

$$\Rightarrow \frac{.35}{.25+.35} = \frac{.35}{.60} = 7/12$$

.5833

pmf of X given (a) Y = 0
(b) Y = 1

a) .25, .75

b) .5833, .4167

$$P(X=0/Y=1) = \frac{P(0,1)}{P(0,1)+P(1,1)}$$

$$\Rightarrow \frac{.25}{.60} = .4167$$

Q-4 What are the various sampling methods to select a representative sample from a given population? List the shortcomings of each method. Describe in your own words why Random Sampling is a better choice.

Sol^n (1) Random sampling.

(2) Systematic sampling

(3) Stratified sampling

(4) Cluster sampling

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(5) Convenience Sampling

(1) R.S :- non if properly implemented.

(2) S.S :- may introduce bias if there's an underlying pattern in the population.

(3) S.S :- Requires prior knowledge of population strata may be ~~implemented~~ impractical. if strata are not clearly defined.

(4) clusture sampling :- (C.S)
may lead to less precision compared to other methods if clustures are not representative.

(5) C.S :- 1
Highly prone to selection bias, may not represent the population accurately.

Random Sampling is a better choice bcoz it ensure that every member of the population has an equal chance of being selected thus reducing the potential for bias. It also allows for statistical inference and generalization to the population.

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Q-5

$\Rightarrow f(x,y) = 2e^{-2x}e^{-y}$ for $x \geq 0$ & $y \geq 0$

a) $P(X>1, Y>1) : \int_{x=1}^{\infty}\int_{y=1}^{\infty} 2e^{-2x}e^{-y}\,dy\,dx$

$$\Rightarrow 2\int_{x=1}^{\infty} e^{-2x}\left[\frac{e^{-y}}{-1}\right]_1^{\infty} dx$$

$\frac{1}{e^{\infty}}$ 0

$$-2e^{-2x}\left[e^{-\infty} - e^{-1}\right]$$

$$0-1$$

$$\int_{x=1}^{\infty} 2e^{-2x}\,dx$$

$$\Rightarrow 2\left[\frac{e^{-2x}}{-2}\right]_1^{\infty}$$

$$\Rightarrow -2\left[e^{-2\times\infty} - e^{-2}\right]$$

$$-2\left[0 - e^{-2}\right]$$

$$\Rightarrow 2e^{-2}$$

b) $P(X \geq y) = \int_{x=0}^{\infty}\int_{y=0}^{x} 2e^{-2x}e^{-y}\,dy\,dx$

$$\left[\frac{e^{-y}}{-1}\right]_0^{x}$$

$$\left[\frac{e^{-x} - e^{-0}}{-1}\right] = (1-e^{-x})$$

Probability 3

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$$\int_{x=0}^{\infty} 2e^{-2x}(1-e^{-x})\,dx$$

$$\Rightarrow 2\int_0^{\infty}\left(e^{-2x}-e^{-3x}\right)dx$$

$$\Rightarrow 2\left[\frac{e^{-2x}}{-2}-\frac{e^{-3x}}{-3}\right]_0^{\infty}$$

$$\Rightarrow 2\left[\left(\frac{e^{-2\times\infty}}{-2}-\frac{e^{-3\times\infty}}{-3}\right)-\left(\frac{1}{-2}-\frac{1}{-3}\right)\right]$$

$$\Rightarrow 2\left[(0-0)-\left(-\frac{1}{2}+\frac{1}{3}\right)\right]$$

$$2\left[\frac{1}{2}-\frac{1}{3}\right]$$

$$2\left[\frac{3-2}{6}\right]=2\cdot\frac{1}{6}=1/3$$

(c) $P(X<a)$ a is non negative:

$$P(X<a)=\int_{x=0}^{a}\int_{y=0}^{\infty}2e^{-2x}\cdot e^{-y}\,dy\,dx.$$

$$\left[\frac{e^{-y}}{-1}\right]_0^{\infty}$$

$$\Rightarrow\left[\frac{e^{-\infty}-e^{-0}}{-1}\right]$$

$$\frac{0-1}{-1}=1\int_0^{a}2e^{-2x}dx.$$

sum. $B=x$

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·3875

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1

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$$= 2\left[\frac{e^{-2x}}{-2}\right]_0^a$$

$$\Rightarrow \frac{2}{-2}\left[e^{-2a} - e^0\right]$$

$$-\left[e^{-2a} - 1\right]$$

$$\left(1 - e^{-2a}\right)$$

d) $\underset{a \to \infty}{P(Y > a)}$ a is non negative.

$$P(Y > a) = \int_{x=0}^{\infty}\int_{y=a}^{\infty} 2e^{-2x} e^{-y} \, dy \, dx$$

$$\left[\frac{e^{-y}}{-1}\right]_a^{\infty}$$

$$\frac{e^{-\infty} - e^{-a}}{-1}$$

$$e^{-a}.$$

$$\int_0^{\infty} 2e^{-2x} \cdot e^{-a} \, dx.$$

$$\cancel{2}e^{-a}\left[\frac{e^{-2x}}{-\cancel{2}}\right]_0^{\infty}$$

$$-e^{-a}\left[e^{-\infty} - e^{-0}\right]$$

$$-e^{-a}(0 - 1)$$

$$\Rightarrow e^{-a}$$