Assignment

FFT / DFT

(b) [1, 1, 2, 2]

for 4-point DFT

$$F(k) = \sum_{t=0}^{n-1} f(t) \cdot e^{\left(\frac{-2\pi k t}{n}\right) i} \quad k \in \{0, 1, 2, \ldots\}$$

for $n = 4$ $k \in \{0, 1, 2, 3\}$

$f(0) + f(2) + f(1) + f(3)$

$F(0) = f(0) + f(1) + f(2) + f(3)$

$F(1) = \sum_{0}^{3} f(t) \cdot e^{\left(\frac{-\pi t}{2}\right) i} = \underline{f(0) - f(2)} - i\,\underline{(f(1) - f(3))}$

$f(2) = \sum_{0}^{3} f(t) \cdot e^{\left(\frac{-2\pi t}{2}\right) i} = f(0) - f(1) + f(2) - f(3)$

$\Rightarrow \underline{f(0) + f(2)} - \underline{f(1) - f(3)}$

$f(3) = \sum_{0}^{3} f(t) \cdot e^{\left(\frac{-3\pi t}{2}\right) i} \; f(0) - f(2) + i\,(f(1) \ldots$

$\Rightarrow f(0) - f(2) + i\,(f(1) - f(3))$

(a) $f(0) = \sum_{0}^{3} \quad \underline{f(0) + f(2)} + f(1) + f(3)$

$f(1)$ $= f(0) - f(2) - i\,(f(1) - f(3))$

$f(2)$ $\underline{f(0) + f(2)} - f(1) - f(3)$

$f(3)$ $f(0) - f(2) + i\,(f(1) - f(3))$

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$f(0) = 1 \quad f(2) = 3$
$f(1) = 2 \quad f(4) = 2$

(6) (a)

$[1, 2, 3, 2]$.

$\hookrightarrow$ 4 point DFT

$n = 4 \quad k \in \{0, 1, 2, 3\}$

$F(k) = \sum_{t=0}^{n-1} f(t) \cdot e^{\left(-\frac{2\pi k t}{n}\right) i}$ ← element

$F(0) = f(0) + f(2) + f(1) + f(3)$

$1 + 3 + 2 + 2 = 8.$

$F(1) = \sum_{t=0}^{3} f(t)\, e^{-\left(\frac{2\pi k t}{2}\right) i} = f(0) - f(2) - i(f(1) - f(3))$

$= 1 - 3 - i(2 - 2)$

$= 1 - 3 - 0 = -2.$

$F(2) = \sum_{t=0}^{3} f(t)\, e^{-\left(\frac{2\pi k t}{2}\right) i} = f(0) + f(2) - f(1) - f(3)$

$= 1 + 3 - 2 - 2$

$4 - 4 = 0$

$F(3) = \sum_{t=0}^{3} f(t)\, e^{-\left(\frac{2\pi k t}{2}\right) i} = f(0) - f(2) + i(f(1) - f(3))$

$= 1 - 3 + i(2 - 2)$

$= -2$

DFT $\rightarrow \langle 8 \; -2 \; 0 \; -2 \rangle$

(b) $[1, 1, 2, 2]$

$f(0) = f(0) + f(2) + f(1) + f(3)$

~~1 + 3 +~~

$1 + 2 + 1 + 2 = 6.$

$F(1) = f(0) - f(2) - i(f(1) - f(3))$

$1 - 2 - i(1 - 2) = -1 + i$

$f(2) \quad f(0) + f(2) - f(1) - f(3)$

$1 - 2 - 1 - 2 = -4$

$f(3) \quad f(0) - f(2) + i(f(1) - f(3))$

$= 1 - 2 + i(1 - 2)$

$\Rightarrow -1 + i(-1) = -1 - i$

DFT $\langle 6, -1+i, -4, -1-i \rangle$

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6 (c)

The Fourier Transform of a periodic sinusoidal wave is a pair of impulses (Direct delta function) located at the positive and negative freq. corresponding to the freq of the sinusoidal wave. The magnitude of each impulse represents the amplitude of the sinusoidal wave, and phase angle of the sinosoid determines the phase shift of these impulses.

mathematic if we have periodic sinosoidal wave $x(t)$ defined as. phase angle.

$$\boxed{x(t) = \underbrace{A}_{\text{amplitude}} \cdot \cos(2\pi \underbrace{f_0}_{\text{freq}} \underbrace{t}_{\text{time}} + \phi)}$$

Then the Fourier Transform $X(f)$ of $x(t)$ is

$$X(f) = A \cdot \left(\tfrac{1}{2}\delta(f-f_0) + \tfrac{1}{2}\delta(f+f_0)\right)$$

$\delta(f) \rightarrow$ Direc delta

$f_0 \rightarrow$ freq.

Now how the Fourier Transform changes with the phase angle

1) Phase angle $\phi = 0$

$\hookrightarrow$ sinosoidal wave start

(2) $\phi > 0$

$\hookrightarrow$ sinosoidal wave starts at a value greater than zero

(3) $\phi < 0$

$\hookrightarrow$ sinosoidal wave start at a value less than zero overflow & underflow

int = 5
float = 3

7

1) 101.11 × 011.101

First we convert Binary into decimal:

$2^{-1} \cdot 2^{-2}$ 101.11 = 5.75

$\frac{1}{2}+\frac{1}{4}$ 011.101 = 3.625

$\frac{3}{4}$ 5.75 × 3.625 = 20.84375

$\frac{5}{2^3}$ in binary 10100.11001

.625 ↳ result exceed the capacity

$\frac{5}{8}$ 2^3 5-bit integer but not 3-bit fraction part (fraction = 5-bit)

∴ overflow condition will occur.

2) 111.01 × 100.10

111.01 = 7.25

$\frac{1}{2^2}$ 100.10 = 4.5

$\frac{1}{4}$ .25 7.25 × 4.5 = 32.625

$\frac{1}{2}$ 100000.101

↳ It is follow the overflow condition.

3) ~~75~~ 011.110 × 101.111 .75

⇒ 011.110 = 3 . $\frac{6}{8}$ = 3.75

101.111 = 5 . $\frac{7}{8}$ (.875) = 5.875

3.25 × 5.875 = 22.03125

10110.00001

↳ Exceed the capacity

so overflow