Prob 1

Introduction to probability.

T = 10

Ex-2-a.

Soln

m = 4
c = 3
H = 2
E = 1

i) All the books dealing with the same subject are together on the shelf.

Soln:

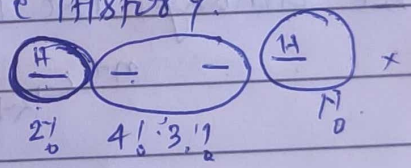
Arrange All mathematics books = 4!
" " chemistry " = 3!
" " History " = 2!
" " English " = 1!

Total arrangement = 4! × 3! × 2! × 1! = 288

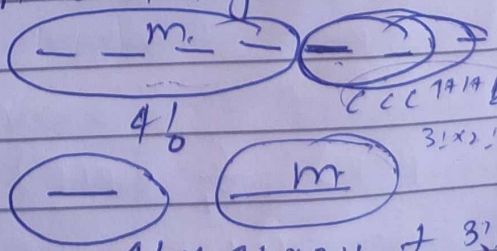
H1 H2 Same subjects are together = 288.

ii) First & last books are History.

2 History at the end. Ans. 2! × 7!



iii) All mathematics books placed together:



4! × 3! × 2! + 3! × 2! × 1! × 4!

⇒ 24 × 12 + 12 × 24

⇒ 24 (12 + 12)

⇒ 576 ways

2-(a) $B = 6$.

$G = 4$

i) Different ranking : $10!$

$= 10!$

ii) What is the prob that all girls outrank boys.

$$\frac{4! \times 6!}{10!} = \frac{\cancel{4!} \times 6!}{10 \times 9 \times 8 \times 7 \times \cancel{6!}} = \frac{1}{210}$$

iii) 2 girls are placed in top 3 ranks

$$\frac{{}^4C_2 \times {}^6C_1 \times 3! \times 7!}{10!}$$

$$\Rightarrow \frac{6 \times 6 \times 6 \times 7!}{10 \times 9 \times 8 \times 7!} = 3/10$$

$\Rightarrow$

2-(c) n item a random sample of size K is to be selected. What is the prob a given item will be among the k selected.

$${}^{n-1}C_{k-1} : \frac{k}{n}$$

2(d) 6 B players.

6x-1 same question 6 W players.

~~Ex 4~~ Exercise

a) (i) 7 horses $\{h_1, h_2 \dots h_7\}$.

h_2 being winner & h_5 being the last

h_2 - - - - - h_5

$|S| = 7!$

$$\frac{5!}{7!} = \frac{1}{42}$$

(ii) h_3 finishing before h_6.

$$\Rightarrow \frac{2! \times 5!}{7!} = 1/2$$

(iii) h_2 being neither the first nor the last.

$$\Rightarrow \text{(not } h_2\text{)} - - - - - \text{(not } h_2\text{)}$$

$$\frac{5 \times 6!}{7!} = 5/7.$$

b) *

3-Red. 12

4-Black. Chose 3 balls.

5-White.

Selecting all balls of diff colors

$$\frac{3}{12} \times \frac{4}{11} \times \frac{5}{10}$$

(ii) Two Balls of same color

~~BBR~~ BRB RBB	RRB	RRW	~~RRB~~	WWR	WWB
3	3	3			

6 RRW

BBW 3! 6

(1) RRB

$$\frac{3}{12} \times \frac{2}{11} \times \frac{4}{10} \times 3$$

(2 red 1 black)

RWB

WWB

(ii) (2 W 1 balls)

$\left(\frac{5}{12} \times \frac{4}{11} \times \frac{4}{10}\right) \times 3$

(iii) RRW

$\left(\frac{3}{12} \times \frac{2}{11} \times \frac{5}{10}\right) \times 3$

(iv) WWR

$\left(\frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}\right) \times 3$

(v) BBR

$\left(\frac{4}{12} \times \frac{3}{11} \times \frac{3}{10}\right) \times 3$

(vi) BBW.

BWB
WBB

$\left(\frac{4}{12} \times \frac{3}{11} \times \frac{5}{10}\right) \times 3$

$\Rightarrow$

(3) All balls of same colour :-

doubt $\frac{3}{12} + \frac{4}{12} + \frac{5}{12}$

$\Rightarrow$

odd of an Event :-

6 R.
3 B.

$P(R) = \frac{6}{9}$, $P(F) = 1 - \frac{6}{9} = 3/9$.

Ans: $(6/9) \div 3/9 = 2$

↳ It is 2 times likely that red ball would be picked.

Ex-1

6 student xi

6 " xii

First pair $^{12}C_2$

2nd pair $^{10}C_2$

No of ways of selecting all pairs

$^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times {}^{6}C_2 \times {}^{4}C_2 \times {}^{2}C_2$

$\frac{12\times11}{2} \times \frac{10\times9}{2} \times \frac{8\times7}{2} \times \frac{6\times5}{2} \times \frac{4\times3}{2} \times \frac{2\times1}{2}$

$\Rightarrow \frac{12!}{2^6}$

$$Cov(X,Y) = \sum_{k=1}^{N} \frac{(x_k - \mu_x)(y_k - \mu_y)}{N-1}$$

(2) Covariance b/w two var X & Y is defined as

$$Cov(X,Y) = \sum_{k=1}^{N} \frac{(x_k - \mu_k)(y_k - \mu_k)}{N-1}$$

using above definition prove that

i) $Cov(aX, Y) = a\, Cov(Y, X)$

ii) $Cov(X+a, Y) = Cov(Y, X)$

iii) $Cov(X+Y, Z) = Cov(X, Z) + Cov(Y, Z)$

soln:- i) $Cov(aX, Y) = a\, Cov(Y, X)$

L.H.S

$$Cov(aX, Y) = \frac{1}{N-1} \sum_{k=1}^{N} (aX_k - a\mu_x)(Y_k - \mu_y)$$

$$\Rightarrow \frac{a}{N-1} \sum_{k=1}^{N} (x_k - \mu_x)(y_k - \mu_y)$$

$$\Rightarrow a \cdot Cov(Y, X)$$

ii) $Cov(X+a, Y) = Cov(Y, X)$

$$\because Cov(X, Y) = \sum_{k=1}^{N} \frac{(x_k - \mu_x)(y_k - \mu_y)}{N-1}$$

Let's $Cov(X+a, Y) \Rightarrow \frac{1}{N-1} \sum_{k=1}^{N} (x_k + a - \mu_x)(y_k - \mu_y)$

$\Rightarrow$ Expanding this expression we get

$Cov(X+a, Y) = Cov(Y, X)$

iii) $Cov(X+Y, Z) = Cov(X, Z) + Cov(Y, Z)$

$\Rightarrow$ ~~LHS~~ $Cov(X, Y) = \frac{1}{M-1} \sum_{K=1}^{M} (x_K - \mu_X)(y_K - \mu_K)$

L.H.S $Cov(X+Y, Z) = \frac{1}{M-1} \sum_{K=1}^{M} (x_K + y_K - \mu_X - \mu_Y)(z_K - \mu_Z)$

$\Rightarrow \frac{1}{M-1} \sum_{K=1}^{M} ((x_K - \mu_X) + (y_K - \mu_Y))(z_K - \mu_Z)$

$\Rightarrow \frac{1}{M-1} \sum_{K=1}^{M} (x_K - \mu_X)(z_K - \mu_Z) + (y_K - \mu_Y)(z_K - \mu_Z)$

$\Rightarrow Cov(X, Z) + Cov(Y, Z)$

proved.

(3)

(12) joint probability mass fn of X & Y. $P(i, j) = P\{X = i, Y = j\}$

$P(0,0) = .3$

$P(0,1) = .25$

$P(1,0) = .10$

$P(1,1) = .35$

(11) Calculate the conditional probability mass function of X given that $Y = 0$ (1)

$Y = 1$ (2)

① marginal PMF of X: P(Y=0) = P(0,0) + P(1,0) = .3 + .1 = .4

② marginal PMF of Y: P(Y=1) = P(0,1) + P(1,1) ⇒ .25 + .35 = .60

3 new
2 used but still working
2 defective

Soln $y = 0$

$$P(X=0/Y=0) = \frac{P(0,0)}{P(0,0)+P(1,0)}$$

$$\Rightarrow \frac{.30}{.3+.10} = \frac{.30}{.40} = \frac{3}{4} = .75$$

$$P(X=1/Y=0) = \frac{P(1,0)}{P(0,0)+P(1,0)}$$

$$\Rightarrow \frac{.10}{.30+.10} = \frac{1}{40} = .25$$

b) $y = 1$

$$P(X=1/Y=1) = \frac{P(1,1)}{P(0,1)+P(1,1)}$$

$$\Rightarrow \frac{.35}{.25+.35} = \frac{.35}{.60} = 7/12 = .5833$$

pmf of X given (a) Y=0 (b) Y=1

a) .25, .75

b) .5833, .4167

$$P(X=0/Y=1) = \frac{P(0,1)}{P(0,1)+P(1,1)}$$

$$\Rightarrow \frac{.25}{.60} = .4167$$

Q-4 What are the various sampling methods to select a representative sample from a given population? List the shortcomings of each method. Describe in your own words why Random Sampling is a better choice.

Soln ① Random sampling.
② systematic sampling
③ stratified sampling
④ cluster sampling