Research methodology

LU-Decomposition

A system of linear eqn with n variables. $AX = B$.

Consistency

- Unique soln → Trivial soln, $|A| \neq 0$

  $r(A) = r(B) = r = n$

- Infinitely many soln. → Non-Trivial soln, $|A| = 0$

  $r(A) = r(B) = r < n$

Inconsistency.

$r(A) \neq r(B)$

$2x + 3y = 7$. slop $= -2/3$

$3x - 2y = 4$ slop $= 3/2$

$x = 2, y = 1$

* $5x + 3y = 2$ $\times 2$

$2x + 7y = -5$ $\times 5$

$10x + 6y = 4$

$10x + 35y = -25$

$-29y = 29$

$y = -1$

$x = 1$

* $2x + 3y = 7$

$3x - 2y = 4$

$$\rightarrow \begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$x - 2y + z = 5$

$5x + 3y + z = 4$

$2x + 7y + 2z = -1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 5 & 3 & 1 \\ 2 & 7 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ -1 \end{bmatrix}$$

→ if the no of equation $<$ no of unknowns the system is underdetermined.
An underdetermined system] has infinitely many sol^n including none if the equations are inconsistent.

→ if the no of equations exceeds the no of n of unknowns. the system is overdetermined.

→ For lower triangular matrix $a_{ij} = 0$ if $i \le j$. ($i < j$)
" upper " " " if $i > j$

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Forward substitution:-

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & -5 & 0 & 0 \\ -3 & 2 & 7 & 0 \\ 2 & 1 & -2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ -9 \\ 7 \\ 8 \end{pmatrix}$$

$2x_1 = 4$ —— (1)

$3x_1 - 5x_2 = -9$ —— (2)

$-3x_1 + 2x_2 + 7x_3 = 7$ —— (3)

$2x_1 + x_2 - 2x_3 - 3x_4 = 8$ —— (4)

$x_1 = 2$

$3x_1 - 5x_2 = -9$
$3 \times 2 - 5x_2 = -9$
$x_2 = 3$

$-6 + 6 + 7x_3 = 7$
$x_3 = 1$

$4 + 3 - 2 - 3x_4 = 8$
$-3x_4 = 3$
$x_4 = -1$

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Forward →

$$\begin{pmatrix} 3 & 0 & 0 & 0 \\ 3 & -5 & 0 & 0 \\ 2 & -3 & 5 & 0 \\ -2 & 1 & 4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 12 \\ 2 \\ 7 \\ -5 \end{pmatrix}$$

$3x_1 = 12$

$3x_1 - 5x_2 = 2$

$2x_1 - 3x_2 + 5x_3 = 7$

$-2x_1 + x_2 + 4x_3 + 3x_4 = -5$

$\Rightarrow x_1 = 4$

$\Rightarrow 12 - 5x_2 = 2$

$\Rightarrow -5x_2 = -10$

$\Rightarrow x_2 = 2$

$\Rightarrow$

$\Rightarrow 8 - 6 + 5x_3 = 7$

$\Rightarrow x_3 = 1$

$-8 + 2 + 4 + 3x_4 = -5$

$\Rightarrow x_4 = -1$

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Forward →

$$\begin{pmatrix} 5 & 0 & 0 & 0 \\ 2 & -5 & 0 & 0 \\ 2 & 2 & -3 & 0 \\ -3 & 6 & 4 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 20 \\ 3 \\ 4 \\ 8 \end{pmatrix}$$

Soln. $5x_1 = 20$

$2x_1 - 5x_2 = 3$

$2x_1 + 2x_2 - 3x_3 = 4$

$-3x_1 + 6x_2 + 4x_3 + 3x_4 = 8$

$x_1 = 4$

~~2x~~

$8 - 5x_2 = 3$

$x_2 = 1$

$8 + 2 - 3x_3 = 4$

$\Rightarrow x_3 = 2$

$-12 + 6 + \cancel{8} + 3x_4 = \cancel{8}$

$x_4 = 2$.

$x_1 = 0A + 1m.$
$x_2 = 1A + 2m.$
$x_3 = 2A + 3m.$

$x_k = (k-1)A + km.$

Complexity $= O(n^2) = \Theta(n^2)$

Backword Substitution $\rightarrow$

$$\begin{pmatrix} 2 & 1 & -2 & 2 \\ 0 & 7 & 1 & 5 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -11 \\ 14 \\ 5 \\ 6 \end{pmatrix}$$

$-3x_4 = 6$
$3x_3 + 2x_4 = 5$
$7x_2 + x_3 + 5x_4 = 14.$
$2x_1 + x_2 - 2x_3 + 2x_4 = -11$

$x_4 = -2$
$3x_3 - 4 = 5$
$x_3 = 3$

$7x_2 + 3 - 10 = 14$
$7x_2 = 21$
$x_2 = 3$
$2x_1 + 3 - 6 - 4 = -11$
$2x_1 - 11 = -11$
$2x_1 = 0$
$x_1 = 0$

$2x_1 = -4$
$x_1 = -2$

$x_1 = 0A + 1m$
$x_2 = 1A + 2m.$
$x_3 = 2A + 3m.$
$\vdots$
$x_{n-k} = kA + (k+1)m.$
T.C $= O(n^2) = \Theta(n^2).$

word $\rightarrow$

$$\begin{pmatrix} 3 & 1 & 2 & 4 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \\ 9 \\ -3 \end{pmatrix}$$

$3x_1 + x_2 + 2x_3 + 4x_4 = 6$
$2x_2 + 4x_3 + 3x_4 = -1$
$5x_3 - 4x_4 = 9$
$3x_4 = -3$

$3x_4 = -3$
$x_4 = -1$
$\cancel{5x_3 + 12 = 9}$
$\cancel{5x_3 = -3}$
$\cancel{x_3 = -3/5}$
$5x_3 + 4 = 9$
$x_3 = 1$

$2x_2 + 4 - 3 = -1$
$x_2 = -1$
$3x_1 - 1 + 2 - 4 = 6$
$\Rightarrow 3x_1 = 9$
$x_1 = 3$

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$$\begin{pmatrix} 5 & 1 & 6 & 4 \\ 0 & 2 & 3 & -4 \\ 0 & 0 & -2 & 5 \\ 0 & 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 11 \\ 9 \end{pmatrix}$$

$\rightarrow$

$$5x_1 + x_2 + 6x_3 + 4x_4 = 6$$
$$2x_2 + 3x_3 - 4x_4 = -2$$
$$-2x_3 + 5x_4 = 11$$
$$3x_4 = 9$$

$x_4 = 3$

$-2x_3 + 15 = 11$

$-2x_3 = -4$

$x_3 = 2$

$2x_2 + 6 - 12 = -2$

$x_2 = 2$

$5x_1 + 2 + 12 + 12 = 6$

$5x_1 = -20$

$x_1 = -4$

$m = LU$

$$Q = m^{-1} \begin{bmatrix} m_{11} & m_{12} & m_{13} & \cdots & m_{n1} \\ m_{21} & m_{22} & & & m_{n2} \\ \vdots & & & & \\ m_{n1} & m_{n2} & & \cdots & m_{nn} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & \vdots \\ L_{31} & L_{32} & \cdots & L_{3n} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & \cdot & U_{1n} \\ 0 & U_{22} & - & U_{2n} \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & U_{nn} \end{bmatrix}$$

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

$m_{11} = L_{11}U_{11}$

$m_{21} = L_{21}U_{11}$

$m_{31} = L_{31}U_{11}$

$m_{12} = L_{11}U_{12}$

$m_{13} = L_{11}U_{13}$

$m_{22} = L_{21}U_{12} + L_{22}U_{22}$ $\quad m_{33} =$

$m_{32} = L_{31}U_{12} + L_{32}U_{22}$

$m_{23} = L_{21}U_{13} + L_{22}U_{23}$

$m_{33} = L_{31}U_{13} + L_{32}U_{23} + L_{33}U_{33}$

assumption :-

$L_{11} = 1 \quad U_{11} = m_{11}$ ✓

$U_{11} = 1 \quad L_{11} = m_{11}$ ✓

$m = LU$

$$m = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 5 & 10 \\ 6 & 12 & 14 \end{pmatrix} \qquad m = LU = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & 5 & 10 \\ 6 & 12 & 14 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 0 & U_{22} & U_{23} \\ 0 & U_{32} & U_{33} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 10 \\ 3 & 12 & 14 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 0 & U_{22} & U_{23} \\ 0 & U_{32} & U_{33} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 10 \\ 3 & 12 & 14 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & L_{22} & L_{23} \\ 3 & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ 0 & U_{22} & U_{23} \\ 0 & U_{32} & U_{33} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 3 \\ 2 & 5-2 & 10 \\ 3 & 12 & 14 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 3 \\ 2 & 3 & 10-6 \\ 3 & 12 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 3 \\ 2 & 3 & 4 \\ 3 & 12-3 & 14 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 3 \\ 2 & 3 & 4 \\ 3 & 9 & 14 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 3 & 4 \\ 3 & 9 & 14-9 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 3 & 4 \\ 3 & 9 & 5 \end{pmatrix}$$

Ans:- $x = 1$, $y = 2$, $z = 3$

Q: solve the following system of eqn by L.U Decomposition:

$$x + 5y + z = 14$$
$$2x + y + 3z = 13$$
$$3x + y + 4z = 17$$

$M = LU$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{31} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix} \quad \begin{matrix} Ax = B \\ Mx = B \end{matrix}$$

$M = LU$

$Mx = B$

$LUx = B \quad [M = LU]$

$LY = B \quad [Ux = Y]$

$M = L \cdot U$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}.$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{11}L_{21} & U_{12}L_{21}+U_{22} & U_{13}U_{21}+U_{23} \\ U_{11}L_{31} & U_{12}L_{31}+U_{22}L_{32} & U_{13}L_{31}+U_{23}L_{32}+U_{33} \end{bmatrix}$$

$U_{11} = 1 \quad U_{12} = 5 \quad U_{13} = 1$

$U_{11} L_{21} = 2$
$L_{21} = 2$

$U_{12} L_{21} + U_{22} = 1$
$5 \times 2 + U_{22} = 1$
$U_{22} = -9$

$U_{13} U_{21} + U_{23} = 3$
$U_{13} \times 2 + U_{23} = 3$
$2 + U_{23} = 3$
$U_{23} = 1$

$U_{11} L_{31} = 3$
$L_{31} = 3$

$U_{12} L_{31} + U_{22} L_{32} = 1$
$5 \times 3 + 0 - 9 \times L_{32} = 1$
$15 - 9L_{32} = 1$
$L_{32} = 14/9.$

$U_{13} L_{31} + U_{23} L_{32} + U_{33} = 4$
$1 \times 3 + 1 \times \frac{14}{9} + U_{33} = 4$
$3 + \frac{14}{9} + U_{33} = 4$
$\frac{27+14}{9} + U_{33} = 4$
$U_{33} = 4 - \frac{41}{9}$
$U_{33} = \frac{36-41}{9} \quad -5/9.$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

$LY = B.$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$\Rightarrow \quad y_1 = 14$

$2y_1 + y_2 = 13$
$28 + y_2 = 13$
$y_2 = -15$

$3y_1 + \frac{14}{9} y_2 + y_3 = 17$
$42 + \frac{14}{9} \times (-15) + y_3 = 17$
$14(3 + \frac{-15}{9}) + y_3 = 17$
$14(\frac{27-15}{9}) + y_3 = 17$
$\Rightarrow \frac{14 \times 12^4}{9_3} + y_3 = 17$
$\frac{56}{3} + y_3 = 17$
$y_3 = 17 - \frac{56}{3} = \frac{51-56}{3} = \frac{-5}{3}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$UX = Y.$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

$\Rightarrow$ (1) $x + 5y + z = 14$

(2) $-9y + z = -15$

(3) $-5/9\, z = -\frac{5}{3}$

from eqn (3)

$$-\frac{5}{9} z = -\frac{5}{3}$$

$$z = \frac{9}{3} = 3$$

$-9y + 3 = -15$

$-9y = -18$

$y = 2$

$x + 5y + z = 14$

$x + 10 + 3 = 14$

$x = 1$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 2 & -10 & 2 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 0 & -4 & 2 \end{bmatrix} \xrightarrow{R_3 \to R_3 + 4R_2} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 14 \end{bmatrix}$$

L

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -4 & 1 \end{bmatrix}$$

$$LU: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 14 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 2 & -10 & 2 \end{bmatrix}$$

Q.2 PPT. (mam → wrong answer)

$$M = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & -7 \end{pmatrix}. \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ +2 & 1 & 0 \\ +3 & 0 & 1 \end{pmatrix}.$$

↓ Convert U

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & -7 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 3 \\ 0 & -1 & -2 \\ 3 & 3 & -7 \end{pmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \overset{U}{\begin{pmatrix} 1 & 1 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -16 \end{pmatrix}}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 1 & 1 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -16 \end{pmatrix}}_{U}$$

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & 9-16 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & -7 \end{pmatrix}}_{M} \checkmark$$

$$\begin{pmatrix} \\ \\ \end{pmatrix}$$

performed. what is the probability that an outcome of ... is ... of 1 when the outcome of a roll is

$A = LU \quad L: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 10 \\ 6 & 12 & 14 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 4 \\ 0 & 9 & 5 \end{bmatrix}$$

$$\downarrow R_3 \to R_3 - 3R_2$$

L:

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & -7 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 10 \\ 6 & 12 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & -7 \end{bmatrix}$$

Q 2

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & -7 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\downarrow R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - 3R_1$$

$-7 - (-9)$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

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$LU = \begin{cases} A & \text{unit matrix} \\ & \text{permutation matrix} \end{cases}$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 2 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 2 & 7 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 7 \\ 2 & 4 & 5 \end{bmatrix} = L \cdot U$$

Convert this matrix to UT matrix by using echlon form

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 7 \\ 2 & 4 & 5 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -2 \\ 0 & 0 & -1 \end{bmatrix} \longrightarrow U$$

Select the multiplication factor and inverse sign. For LT matrix

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 2 & 7 \end{pmatrix} < LU$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 2 & 7 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 0 & -4 & -2 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

U

$$\boxed{\underline{LU} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -2 \\ 0 & 0 & -1 \end{bmatrix}}$$

$AA^{-1} = I$

AI

$\frac{1}{A}$ $AI = 1$

For calculating inverse matrix

$$LU = \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} = A^{-1}$$

$$\begin{bmatrix} B_{12} \\ B_{22} \\ B_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$LUX = b$

Forward Calculation

$$\begin{bmatrix} B_{13} \\ B_{23} \\ B_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Forward Calculation: $LUX = b$

$LY = b$

(1) backword Calculation: $UX = Y$.

$$\begin{bmatrix}1&0&0\\3&1&0\\2&0&1\end{bmatrix}\begin{bmatrix}1&2&3\\0&-4&-2\\0&0&-1\end{bmatrix} = L.U$$

Forward Subsitution:

$$\begin{bmatrix}1&0&0\\3&1&0\\2&0&1\end{bmatrix}\begin{bmatrix}Y_1\\Y_2\\Y_3\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$Y_1 = 1$

$3Y_1 + Y_2 = 0 \quad Y_2 = -3$

$2Y_1 + Y_3 = 0 \quad Y_3 = -2$

$$\begin{bmatrix}1\\-3\\-2\end{bmatrix}$$

Again backword subsitution

$$\begin{bmatrix}1&2&3\\0&-4&-2\\0&0&-1\end{bmatrix}\begin{bmatrix}Q_{11}\\Q_{21}\\Q_{31}\end{bmatrix} = \begin{bmatrix}1\\-3\\-2\end{bmatrix}$$

$Q_{11} + 2Q_{21} + 3Q_{31} = 1$

$-4Q_{21} - 2Q_{31} = -3$

$-Q_{31} = -2$

$Q_{31} = 2$

$Q_{21} = -1/4$

$Q_{11} = -4.5 = -\frac{9}{2}$

$$\begin{bmatrix}Q_{11}\\Q_{21}\\Q_{31}\end{bmatrix} = \begin{bmatrix}-4.5\\-1/4\\2\end{bmatrix}$$

For 2nd forward:

$$\begin{bmatrix}1&0&0\\3&1&0\\2&0&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$Y_1 = 0$, $Y_2 = 1$, $Y_3 = 0$

backword

$$\begin{bmatrix}1&2&3\\0&-4&-2\\0&0&-1\end{bmatrix}\begin{bmatrix}Q_{12}\\Q_{22}\\Q_{32}\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$Q_{12} = .5$

$Q_{22} = -.25$

$Q_{32} = 0$

$$\begin{bmatrix}Q_{12}\\Q_{22}\\Q_{32}\end{bmatrix} = \begin{bmatrix}.5\\-.25\\0\end{bmatrix}$$

For 3rd!

Forward

$$\begin{bmatrix}1&0&0\\3&1&0\\2&0&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$y_1 = 1$, $y_2 = 0$, $Y_3 = 1$

backword

$$\begin{bmatrix}1&2&3\\0&-4&-2\\0&0&-1\end{bmatrix}\begin{bmatrix}Q_{13}\\Q_{23}\\Q_{33}\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$Q_{13} + 2Q_{23} + 3Q_{33} = 0$

$-4Q_{23} - 2Q_{33} = 0$

$-Q_{33} = 1$

$Q_{33} = -1$, $Q_{23} = .5$

$Q_{13} = 2$

$$\begin{bmatrix}Q_{13}\\Q_{23}\\Q_{33}\end{bmatrix} = \begin{bmatrix}2\\0.5\\-1\end{bmatrix}$$

inverse of matrix:

$$\begin{bmatrix}Q_{11}&Q_{12}&Q_{13}\\Q_{21}&Q_{22}&Q_{23}\\Q_{31}&Q_{32}&Q_{33}\end{bmatrix} = \begin{bmatrix}-4.5&.5&2\\-.25&-.25&.5\\2&0&-1\end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$\begin{bmatrix}-4.5&.5&2\\2&0&-1\end{bmatrix}$$

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$$\begin{bmatrix}-4.5&.5&2\\2&0&-1\\-.25&-.25&.5\end{bmatrix}$$

$$\begin{bmatrix}-4.5&2&.5\\-.25&.5&-.25\\2&-1&0\end{bmatrix}$$

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