PHYS 20323/60323: Fall 2021

Assignment #7

Due: Friday Oct. 22, 2021

NOTE: Please show all work so that I have the opportunity to properly evaluate your work, or to award partial credit.

1. *The quadratic equation*

   (a) Write a program that takes as input three numbers, a, b, and c, and prints out the two solutions to the quadratic equation $ax^2 + bx + c = 0$ using the standard formula:
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1}$$
   Use your program to compute the solutions of $0.001x^2 + 1000x + 0.001 = 0$.

   (b) There is another way to write the solutions to a quadratic equation. Multiplying top and bottom of the solution above by $-b \mp \sqrt{b^2 - 4ac}$, show that the solutions can also be written as:
   $$x = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} \tag{2}$$
   Add further lines to your program to print these values in addition to the earlier ones and again use the program to solve $0.001x^2 + 1000x + 0.001 = 0$. What do you see? How do you explain it?

   (c) Using what you have learned, write a new program that calculates both roots of a quadratic equation accurately in all cases.

For full credit: upload your program to Github and turn in your answers to part (b) and a printout of it in action, showing the solution of the equation $0.001x^2 + 1000x + 0.001 = 0$.

This is a good example of how computers dont always work the way you expect them to. If you simply apply the standard formula for the quadratic equation, the computer will sometimes get the wrong answer. In practice the method you have worked out here is the correct way to solve a quadratic equation on a computer, even though its more complicated than the standard formula. If you were writing a program that involved solving many quadratic equations this method might be a good candidate for a user-defined function: you could put the details of the solution method inside a function to save yourself the trouble of going through it step by step every time you have a new equation to solve.

2. *Simpson's Rule* Write a function that implements intergration using Simpson's Rule. Use it to evaluate the intergral $\int_0^2 (x^4 - 2x + 1)\, dx$, using 10, 100, and 1000 slices

3. *Heat capacity of a solid*

   Debyes theory of solids gives the heat capacity of a solid at temperature T to be

$$C_V = 9V\rho\kappa_B \left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx \tag{3}$$

   where V is the volume of the solid, ρ is the number density of atoms, κ_B is Boltzmann's constant, and Θ_D is the so-called Debye temperature, a property of solids that depends on their density and speed of sound.

   (a) Write a Python function `cv(T)` that calculates C_V for a given value of the temperature, for a sample consisting of 1000 cubic centimeters of solid aluminum, which has a number density of $\rho = 6.022 \times 10^{28}$ m^{-3} and a Debye temperature of $\Theta_D = 428$ K. Use the trapezoidal rule to evaluate the integral with N = 1000 sample points. em Hint: The value of the integrand at $x = 0$ is zero.

   (b) Use your function to make a graph of the heat capacity as a function of temperature from $T = 5$ K to $T = 500$ K.

   For full credit: Upload your program plus the plotted output that shows heat capacity as a function of temperature.