ascending order – возрастающий порядок
mathematical expectation, mean value – математическое ожидание
rod – стержень; **on the average** – в среднем; **deviation** – отклонение
concentration – кучность; **to strike** – поражать
dispersion, variance – дисперсия, рассеяние
mean square deviation – среднее квадратическое отклонение
numerical characteristic – числовая характеристика

Exercises for Seminar 7

7.1. Two balls are chosen randomly from an urn containing 8 white, 4 black and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value?

7.2. The probability of working each of four combines without breakages during a certain time is equal to 0,9. Compose the law of distribution of a random variable X – the number of combines working trouble-free. Find the mathematical expectation, the dispersion and the mean square deviation of the random variable X.

The answer: $M(X) = 3,6;\ D(X) = 0,36;\ \sigma(X) = 0,6.$

7.3. The probability of birth of a boy in a family is equal to 0,515. Compose the law of distribution of a random variable X – the number of boys in families having four children. Find the mathematical expectation, the dispersion and the mean square deviation.

The answer: $M(X) = 2,06;\ D(X) = 0,999;\ \sigma(X) = 1,0.$

7.4. There are 6 masters of sports in a group of 10 sportsmen. One selects (under the circuit without replacement) 3 sportsmen. Compose the law of distribution of a random variable X – the number of masters of sports of the selected sportsmen. Find the mathematical expectation of the random variable X.

The answer: $M(X) = 1,8$.

7.5. A shooter makes shots in a target before the first hit. The probability of hit in the target at each shot is equal to 0,7. Compose the law of distribution of a random variable X – the number of shots made by the shooter. Find the most probable number of cartridges (patrons) given to the shooter.

The answer: $k_0 = 1$.

7.6. The mathematical expectation of a random variable X is equal to 8. Find the mathematical expectation of the following random variables: a) $X - 4$; b) $3X + 4$.

7.7. The dispersion of a random variable X is equal to 8. Find the dispersion of the following random variables: a) $X - 2$; b) $3X + 2$.

7.8. Independent random variables X and Y have the following distributions:

X	2	4	6
p	0,3	0,5	0,2

Y	3	4
p	0,4	0,6

Compose the law of distribution of the random variable $Z = X + Y$. Find the mathematical expectation, the dispersion and the mean square deviation of the random variable Z.

The answer: $M(Z) = 7,4;\ D(Z) = 2,2$.

7.9. Find the mathematical expectation and the dispersion of random variable $Z = 4X - 2Y$ if $M(X) = 5$, $M(Y) = 3$, $D(X) = 4$, $D(Y) = 6$. The random variables X and Y are independent.

The answer: $M(Z) = 14$; $D(Z) = 88$.

7.10. A total of 4 buses carrying 148 students from the same school arrives at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus. Which of $M(X)$ or $M(Y)$ do you think is larger? Why? Compute $M(X)$ and $M(Y)$.

Exercises for Homework 7

7.11. Two dice are rolled. Let X equal the sum of the 2 dice. What are the possible values of X, and what are the probabilities associated with each value?

7.12. The probability that a buyer will make a purchase in a shop is equal to 0,4. Compose the law of distribution of a random variable X – the number of buyers who have made a purchase if the shop was visited by 3 buyers. Find the mathematical expectation, the dispersion and the mean square deviation of the random variable X.

The answer: $M(X) = 1,2$; $D(X) = 0,72$; $\sigma(X) = 0,85$.

7.13. A buyer attends shops for purchasing the necessary goods. The probability that the goods are in a certain shop is equal to 0,4. Compose the law of distribution of a random variable X – the number of shops which will be attended by the buyer from four possible. Find the most probable number of shops which will be visited by the buyer.

The answer: $1 \le k_0 \le 2$.

7.14. A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number (mathematical expectation) of defective items in the sample.

The answer: 0,6.

7.15. A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you win – \$1.00 (that is, you lose \$1.00). Calculate the mathematical expectation and the dispersion of the amount you win (marble – мрамор; to withdraw – извлекать).

The answer: $M(X) = -1/15$; $D(X) = 49/45$.

7.16. The mathematical expectation of a random variable X is equal to 7. Find the mathematical expectation of the following random variables:

a) $X + 6$; b) $4X - 3$.

7.17. The dispersion of a random variable X is equal to 9. Find the dispersion of the following random variables: a) $X + 6$; b) $2X - 7$.

7.18. Independent random variables X and Y have the following distributions:

X	2	4	6
p	0,3	0,5	0,2

Y	3	4
p	0,4	0,6

Compose the law of distribution of the random variable $V = XY$. Find the mathematical expectation, the dispersion and the mean square deviation of the random variable V.

The answer: $M(V) = 13,68$; $D(V) = 29,3376$.

7.19. Find the mathematical expectation and the dispersion of random variables:

a) $Z = 2X - 4Y$; b) $Z = 3X + 5Y$

if $M(X) = 5$, $M(Y) = 3$, $D(X) = 4$, $D(Y) = 6$. The random variables X and Y are independent.

The answer: a) $M(Z) = -2$; $D(Z) = 112$; b) $M(Z) = 30$; $D(Z) = 186$.