Exercises for Seminar 8

8.1. Let the law of distribution of a discrete random variable be given:

X	1	4	6	8
P	0,1	0,3	0,4	0,2

Find the integral function of the random variable X and construct its graph.

8.2. Find the integral function of distribution of the random variable X – the number of hits in a target if three shots were made with the probability of hit in the target equal 0,8 for each shot.

8.3. A continuous random variable X is given by the integral function:

$$F(x) = \begin{cases} 0 & if \quad x \le 0, \\ \dfrac{x^3}{125} & if \quad 0 < x \le 5, \\ 1 & if \quad x > 5. \end{cases}$$

Determine:

a) the probability of hit of the random variable into the interval (2; 3);

b) the mathematical expectation, the dispersion and the mean square deviation of the random variable X.

8.4. A random variable X is given by the integral function:

$$F(x) = \begin{cases} 0 & if \quad x \le -2, \\ \dfrac{x}{4} + \dfrac{1}{2} & if \quad -2 < x \le 2, \\ 1 & if \quad x > 2. \end{cases}$$

Find the probability that in result of a trial the random variable X will take on the value:

(a) less than 0;

(b) less than 1;

(c) no less than 1;

(d) being in the interval (0; 2).

8.5. The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda e^{-x/100} & if \quad x \ge 0 \\ 0 & if \quad x < 0 \end{cases}$$

What is the probability that

(a) a computer will function between 50 and 150 hours before breaking down;

(b) it will function less than 100 hours?

Direction: Take e equal 2,718281.

The answer: a) 0,3834; b) 0,632.

8.6. The lifetime in hours of a certain kind of radio tube is a random variable having a probability density function given by

$$f(x) = \begin{cases} 0 & if \quad x \le 100 \\ \dfrac{100}{x^2} & if \quad x > 100 \end{cases}$$

What is the probability that exactly 2 of 5 such tubes in a radio set will have to be replaced within the first 150 hours of operation? Assume that the events E_i, i = 1, 2, 3, 4, 5, that the i^{th} such tube will have to be replaced within this time, are independent.

The answer: 80/243.

8.7. Let X be a random variable with probability density function

$$f(x) = \begin{cases} C(1-x^2) & if \quad -1 < x < 1 \\ 0 & otherwise \end{cases}$$

(a) What is the value of C?

(b) What is the cumulative distribution function of X?

The answer: a) 3/4.

8.8. Compute $M(X)$ if X has a density function given by

$$f(x) = \begin{cases} \frac{1}{4} x e^{-x/2} & if \quad x > 0 \\ 0 & otherwise \end{cases}$$

The answer: 4.

8.9. A random variable X is given by the differential function:

$$f(x) = \begin{cases} 0 & if \quad x \le 0, \\ \frac{4a - 2x}{3a^2} & if \quad 0 < x \le a, \\ 0 & if \quad x > a. \end{cases}$$

Find:

(a) the integral function;

(b) the probability of hit of the random variable into the interval (a/6; a/3).

The answer: b) 7/36.

8.10. A random variable X is given by the integral function:

$$F(x) = \begin{cases} 0 & if \quad x \le 2, \\ \frac{x^3 - 8}{19} & if \quad 2 < x \le 3, \\ 1 & if \quad x > 3. \end{cases}$$

Find:

(a) the differential function;

(b) the probability of hit of the random variable X into the interval (2,5; 3);

(c) the mathematical expectation, the dispersion and the mean square deviation of the random variable X.

The answer: b) 0,599; c) $M(X)$ = 2,566; $D(X)$ = 0,079.

Exercises for Homework 8

8.11. Let the law of distribution of a discrete random variable be given:

X	– 2	5	7	9
p	0,4	0,3	0,2	0,1

Find the integral function of the random variable X and construct its graph.

8.12. The probability of passing the first exam by a student is 0,7, the second exam – 0,6 and the third exam – 0,8. Find the integral function of the random variable *X* – the number of exams passed by the student. Determine *M(X).*

The answer: M(X) = 2,1.

8.13. The density function of *X* is given by

$$f(x)=\begin{cases} a+bx^2 & if \quad 0\le x\le 1 \\ 0 & otherwise \end{cases}$$

If *M(X)* = 3/5, find *a* and *b*.

The answer: a = 3/5; *b* = 6/5.

8.14. A system consisting of one original unit and a spare can function for a random amount of time *X*. If the density of *X* is given (in units of months) by

$$f(x)=\begin{cases} Cxe^{-x/2} & if \quad x>0 \\ 0 & if \quad x\le 0 \end{cases}$$

what is the probability that the system functions for at least 5 months (a spare – запасной элемент)?

The answer: 0,616.

8.15. Suppose that *X* is a continuous random variable whose probability density function is given by

$$f(x)=\begin{cases} C(4x-2x^2) & if \quad 0<x<2 \\ 0 & otherwise \end{cases}$$

(a) What is the value of *C*?
(b) Find *P*(*X* > 1).
The answer: a) 3/8; b) 1/2.

8.16. Find *M(X)* and *D(X)* when the density function of *X* is

$$f(x)=\begin{cases} 2x & if \quad 0\le x\le 1 \\ 0 & otherwise \end{cases}$$

The answer: M(X) = 2/3; *D(X)* = 1/18.

8.17. Let *X* be a random variable with probability density function

$$f(x)=\begin{cases} C(2x-x^3) & if \quad 0<x<\frac{5}{2} \\ 0 & otherwise \end{cases}$$

(a) What is the value of *C*?
(b) What is the cumulative distribution function of *X*?
The answer: a) – 64/225.

8.18. Compute *M(X)* if *X* has a density function given by

$$f(x)=\begin{cases} \frac{50}{x^3} & if \quad x>5 \\ 0 & otherwise \end{cases}$$

The answer: 10.

8.19. A random variable X is given by the differential function:

$$f(x)=\begin{cases} 0 & if \quad x \le 0, \\ x^3+x & if \quad 0<x\le\sqrt{\sqrt{5}-1}, \\ 0 & if \quad x>\sqrt{\sqrt{5}-1}. \end{cases}$$

Find:

(a) the integral function;

(b) the probability of hit of the random variable into the interval (1; 1,1).

The answer: b) 0,221.

8.20. A random variable X is given by the integral function:

$$F(x)=\begin{cases} 0 & if \quad x \le 1, \\ \dfrac{x^2}{2}-\dfrac{x}{2} & if \quad 1<x\le 2, \\ 1 & if \quad x>2. \end{cases}$$

Find:

(a) the differential function;

(b) the probability of hit of the random variable X into the interval (1; 1,5);

(c) the mathematical expectation, the dispersion and the mean square deviation of the random variable X.

The answer: b) 0,375; c) $M(X)$ = 19/12; $D(X)$ = 11/144.