Age-Structured Population Model of Female Bobcats

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Background

- Endangered bobcat in Florida
  - 16 age classes
  - Average lifespan of wild bobcat is 7 years
- How will this bobcat population will change over the next 10 years?
  - Best and worst case scenarios
  - Catastrophes

Assumptions

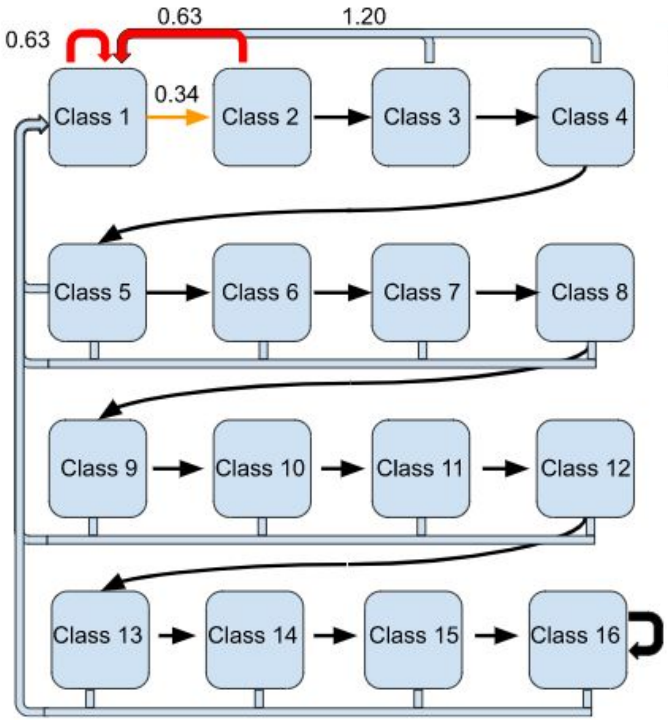
- Only female bobcats
- Only 16 age classes
- Given survival and reproductive rates accurate
- A time step is 1 year
  - 10 time steps to get to 10 years
- Catastrophe is a stochastic process
  - Every year there is an equal probability of a catastrophe
  - Random behavior is simulated with a uniform distribution from 0 to 1, where anything less than or equal to 1/25 is treated as a catastrophe year.

Model Structure

- Three models: best case, worst case, stochastic case

- For each:
  - Population distribution vs. time as time $\to \infty$?
  - Population distribution vs. time after 10 years?

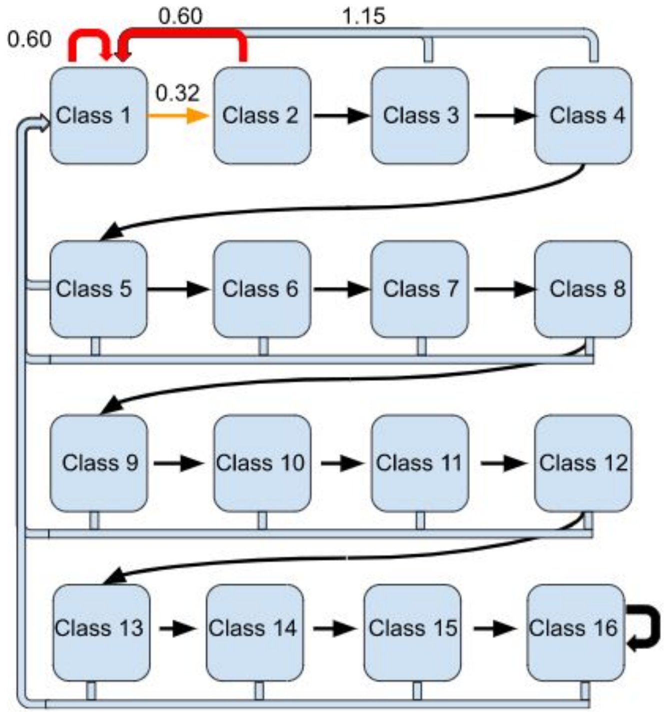
Model Structure: Model 1 (Best Case)



Blue arrows/lines: 1.20
Black arrows: 0.71

	Class 1		…		Class 16
Class 1	0.63	0.63	1.20	…	1.20
Class 2	0.34	0	0	…	0
⋮	0	0.71	0	…	…
	…	0	…	0	0
Class 16	0	…	0	0.71	0.71

Model Structure: Model 2 (Worst Case)



Blue arrows/lines: 1.15
Black arrows: 0.68

	Class 1		…		Class 16
Class 1	0.60	0.60	1.15	…	1.15
Class 2	0.32	0	0	…	0
	0	0.68	0	…	…
⋮	…	0	…	0	0
Class 16	0	…	0	0.68	0.68

Modeling Population at Time = 10 Years

- Since *L* is constant with time:

$$\begin{pmatrix} \text{Class 1 counts} \\ \text{Class 2 counts} \\ \cdot \\ \cdot \\ \cdot \\ \text{Class 16 counts} \end{pmatrix}_{n=10} = \begin{bmatrix} 0.63 & 0.63 & 1.20 & \dots & 1.20 \\ 0.34 & 0 & 0 & \dots & 0 \\ 0 & 0.71 & 0 & \dots & \dots \\ \dots & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0.71 & 0.71 \end{bmatrix}^{10} \begin{pmatrix} \text{Class 1 counts} \\ \text{Class 2 counts} \\ \cdot \\ \cdot \\ \cdot \\ \text{Class 16 counts} \end{pmatrix}_{n=0}$$

Modeling Population as Time $\to \infty$

- Long-Term Growth Rate:

  $\lambda = eig(L) \to \lambda$ is population growth from year to year:

  $P(n+1) = \lambda * P(n)$ for large n

- Long-Term Population Distribution:

  $\vec{\lambda} = eig(L) \to \vec{\lambda}$ is normalized so entries sum to 1

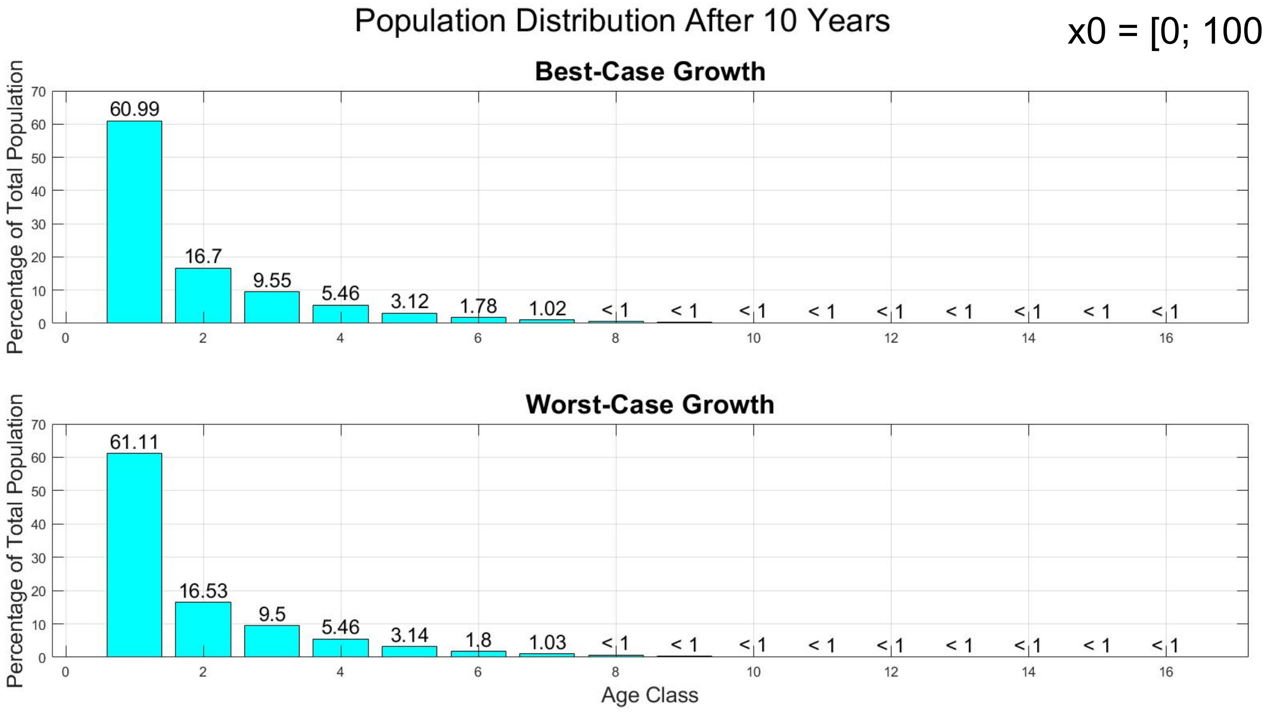
Results and Analysis

Population at *n* = *0*: x0 = [0; 100; 50; 50; 25; 10; 0; …; 0]

	Best Case Growth Model			Worst Case Growth Model		
	Dominant Eigenvalue:	1.24 (24% growth rate)		**Dominant Eigenvalue:**	1.18 (18% growth rate)	
	Dominant Eigenvector (Long-Term)	**Population Distribution (After 10 Years)**		**Dominant Eigenvector (Long-Term)**	**Population Distribution (After 10 Years)**	
		(Percentage)	(Count)		(Percentage)	(Count)
Class 1	61 %	61 %	1818	61 %	61 %	1130
Class 2	17 %	17 %	498	17 %	17 %	305
Class 3	9.6 %	9.6 %	284	9.5 %	9.5 %	175
Class 4	5.5 %	5.5 %	162	5.5 %	5.5 %	101
Class 5	3.1 %	3.1 %	93	3.1 %	3.1 %	58
Class 6	1.8 %	1.8 %	53	1.8 %	1.8 %	33
Class 7	1.0 %	1.0 %	30	1.0 %	1.0 %	19
Class 8	0.58 %	0.59 %	17	0.60 %	0.61 %	11

	Best Case Growth Model			Worst Case Growth Model		
	Dominant Eigenvalue:	1.24 (24% growth rate)		**Dominant Eigenvalue:**	1.18 (18% growth rate)	
	Dominant Eigenvector (Long-Term)	**Population Distribution (After 10 Years)**		**Dominant Eigenvector (Long-Term)**	**Population Distribution (After 10 Years)**	
		(Percentage)	(Count)		(Percentage)	(Count)
Class 9	0.33 %	0.35 %	10	0.34 %	0.36 %	6
Class 10	0.19 %	0.17 %	4	0.20 %	0.17 %	3
Class 11	0.11 %	0.00 %	0	0.11 %	0.00 %	0
Class 12	0.063 %	0.11 %	3	0.065 %	0.11 %	2
Class 13	0.036 %	0.055 %	1	0.037 %	0.057 %	1
Class 14	0.020 %	0.055 %	1	0.022 %	0.057 %	1
Class 15	0.012 %	0.027 %	0	0.012 %	0.029 %	0
Class 16	0.016 %	0.011 %	0	0.017 %	0.011 %	0
		Total:	2,974		Total:	1,845

Results and Analysis



x0 = [0; 100; 50; 50; 25; 10; 0; …; 0]

Total:	2,974

Total:	1,845

Model Structure: Model 3 (Stochastic Case)

- Simulating a 10-year pattern of growth; repeat 100 times. Any of these 1,000 years could be a catastrophe year.
- RNG (uniform distribution) to determine catastrophe/no catastrophe
- Given 4% chance of catastrophe, catastrophe occurs if RNG ≤ 0.04
- Best case growth unless catastrophe year
- Catastrophe year: Reproduction → 70% of best case
  Survival → at best case rate

Model Structure: Model 3 (Stochastic Case)

- Best case growth unless catastrophe year

No catastrophe: same as best case

0.63	0.63	1.20	…	1.20
0.34	0	0	…	0
0	0.71	0	…	…
…	0	…	0	0
0	…	0	0.71	0.71

Catastrophe: Reproduction → 70% best case
Survival: same as best case

0.441	0.441	0.84	…	0.84
0.34	0	0	…	0
0	0.71	0	…	…
…	0	…	0	0
0	…	0	0.71	0.71

Modeling Population at Time = 10 Years

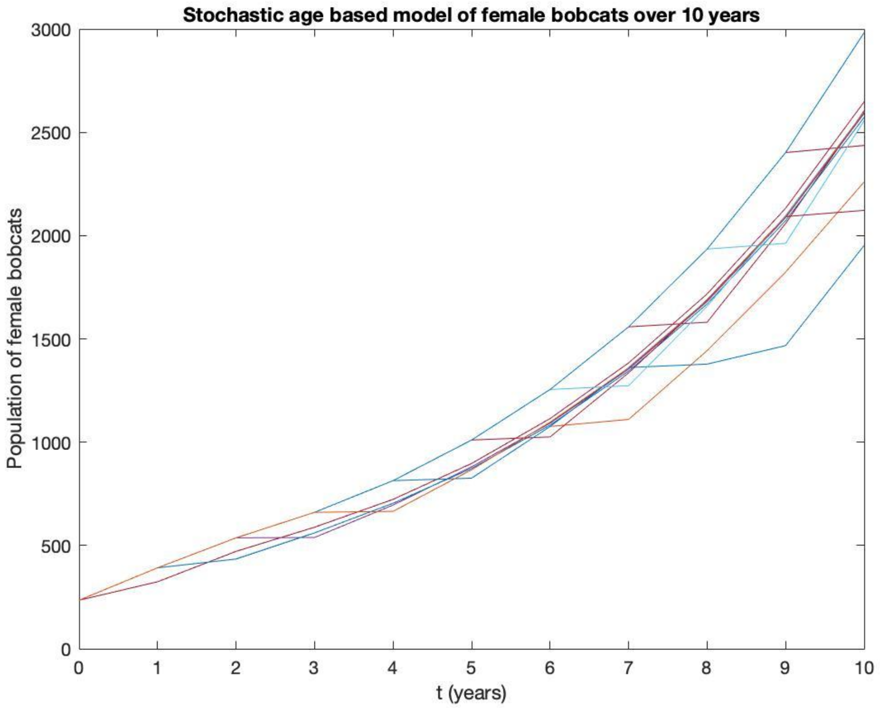
- Population transition, year *n* to year *n+1*:

$$\begin{pmatrix} \text{Class 1 counts} \\ \text{Class 2 counts} \\ . \\ . \\ . \\ \text{Class 16 counts} \end{pmatrix}_{n+1} = \begin{array}{|c|c|c|c|c|} \hline 0.63 & 0.63 & 1.20 & \ldots & 1.20 \\ \hline 0.34 & 0 & 0 & \ldots & 0 \\ \hline 0 & 0.71 & 0 & \ldots & \ldots \\ \hline \ldots & 0 & \ldots & 0 & 0 \\ \hline 0 & \ldots & 0 & 0.71 & 0.71 \\ \hline \end{array} \begin{pmatrix} \text{Class 1 counts} \\ \text{Class 2 counts} \\ . \\ . \\ . \\ \text{Class 16 counts} \end{pmatrix}_{n}$$

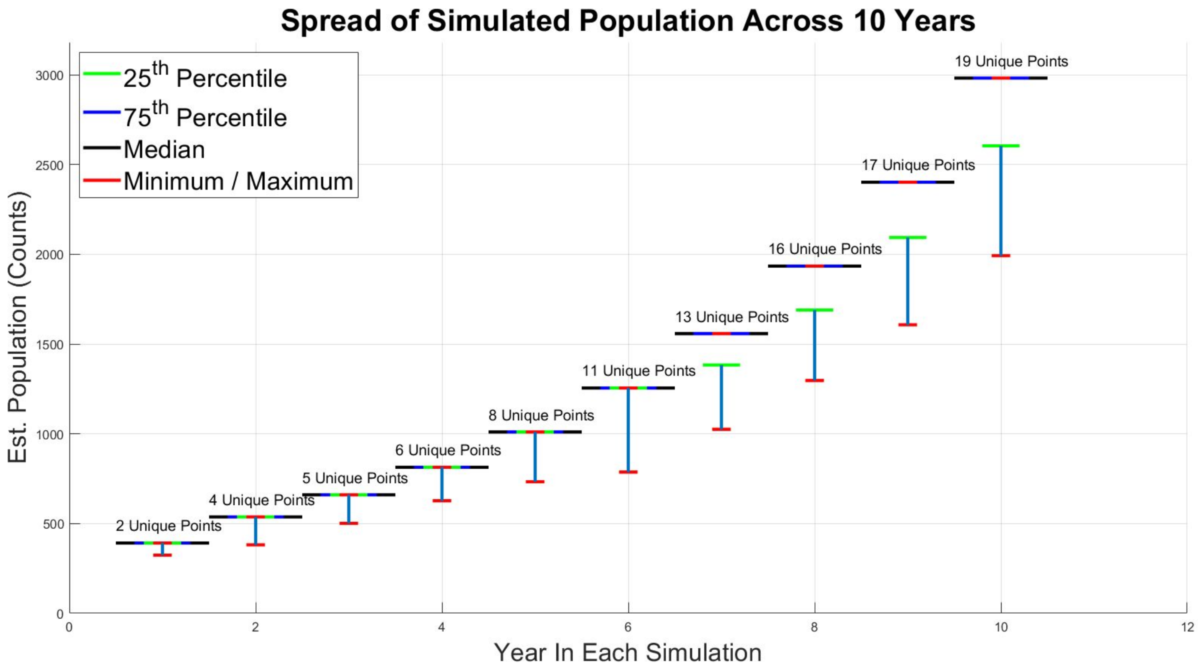
Modeling Population as Time → ∞

- Stochastic nature → time-varying Leslie matrices
- Find dominant eigenvalue/eigenvectors for both matrices
- Model catastrophe 100% of time and 0% of time - bounds actual outcome

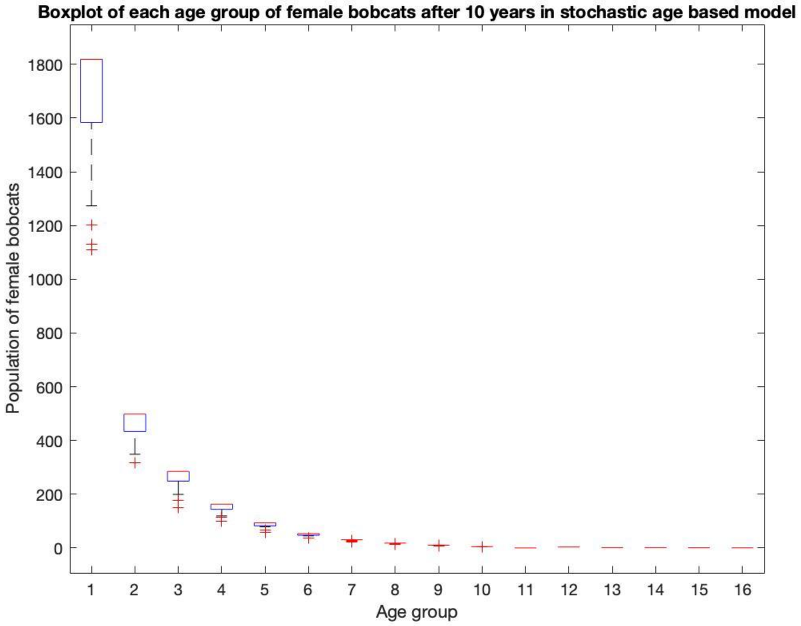
Results and Analysis



Results and Analysis



Results and Analysis



Model Strengths and Limitations

- Only female can’t generalize to full population
  - Need male rates
- Catastrophe 4% every year
  - Low probability and assumed uniform distribution
  - Different distributions and percentage
- 100 trials may not be sufficient
  - Increase # trials
  - Would help with lack of variation
- Fundamental simple model
  - Fixed transition rates may not be reflective of population

Summary

- Bobcat population is expected to grow, despite being endangered
  - Stochastic model/ simulation
  - Best and worst-case scenarios
  - Bobcats are currently classified as “Least Concern”
    - Not endangered today
- Little variation
  - 100 runs
  - Best and worst case scenarios

Citations

[1] James Cox, Randy Kautz, Maureen MacLaughlin, and Terry Gilbert. Closing the Gaps in Florida's Wildlife Conservation System. Office of Environmental Services, Florida Game, and Fresh Water Fish Commission, Tallahassee, Florida, 1994.

[2] D. Mooney and R. Swift, A Course in Mathematical Modeling, American Mathematical Society, Classroom Resource Materials 13 (1999) p. 40–42.

[3] Frunzete, Codrin. Bobcat Lifespan: How Long Do Bobcats Live? Misfit Animals, 2022 https://misfitanimals.com/bobcats/bobcats-lifespan/.

Questions?