1)

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} \qquad \hat{\beta} = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}$$

$$y_i = Y_i - \bar{Y}$$
$$x_i = X_i - \bar{X}$$
$$\hat{y}_i = \hat{Y}_i - \bar{Y}$$

$$\sum_{i=1}^{n} Y_i = n\hat{\alpha} + \hat{\beta}\sum_{i=1}^{n} X_i$$
$$\sum_{i=1}^{n} X_i Y_i = \hat{\alpha}\sum_{i=1}^{n} X_i + \hat{\beta}\sum_{i=1}^{n} X_i^2$$

$$\boxed{\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_i - \bar{X})^2} = \frac{\sum_{i=1}^{n} (X_i Y_i - X_i \bar{Y} - \bar{X} Y_i + \bar{X}\bar{Y})}{\sum_{i=1}^{n} (X_i^2 - 2X_i\bar{X} + \bar{X}^2)}$$

$$= \frac{\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \bar{Y} - \sum_{i=1}^{n} \bar{X} Y_i + \sum_{i=1}^{n} \bar{X}\bar{Y}}{\sum_{i=1}^{n} X_i^2 - 2\sum_{i=1}^{n} X_i \bar{X} + \sum_{i=1}^{n} \bar{X}^2} = \frac{\sum_{i=1}^{n} X_i Y_i - \bar{Y}\sum_{i=1}^{n} X_i - \bar{X}\sum_{i=1}^{n} Y_i + \bar{X}\bar{Y}\sum_{i=1}^{n} 1}{\sum_{i=1}^{n} X_i^2 - 2\bar{X}\sum_{i=1}^{n} X_i + \bar{X}^2\sum_{i=1}^{n} 1}$$

$$= \frac{\sum_{i=1}^{n} X_i Y_i - \bar{Y}\sum_{i=1}^{n} X_i - \bar{X}\sum_{i=1}^{n} Y_i + \bar{X}\bar{Y}n}{\sum_{i=1}^{n} X_i^2 - 2\bar{X}\sum_{i=1}^{n} X_i + \bar{X}^2 n} \quad \frac{1/n}{1/n} \quad \frac{\frac{1}{n}\sum_{i=1}^{n} X_i Y_i - \bar{Y}\bar{X} - \bar{X}\bar{Y} + \bar{X}\bar{Y}}{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - 2\bar{X}^2 + \bar{X}^2}$$

$$= \frac{\frac{1}{n}\sum_{i=1}^{n} X_i Y_i - \bar{Y}\bar{X}}{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2} = \frac{\frac{1}{n}\sum_{i=1}^{n} X_i Y_i - \frac{1}{n^2}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_i\right)^2} \quad \frac{n^2}{n^2} \quad = \boxed{\frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}}$$

$\therefore$ Son equivalentes.

2 Verifique analíticamente que la media de Y es igual a la media de sus estimaciones.

$$\overline{Y} = \overline{\hat{Y}}$$

$$\overline{Y} = \hat{\alpha} + \hat{\beta}\,\overline{X} \qquad \hat{Y}_i = \hat{\alpha} + \hat{\beta}\,X_i$$

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i \Rightarrow \frac{\sum_{i=1}^{n} \hat{Y}_i}{n} = \frac{\sum_{i=1}^{n} \hat{\alpha} + \sum_{i=1}^{n} \hat{\beta} X_i}{n} \Rightarrow \overline{\hat{Y}} = \frac{\hat{\alpha} n}{n} + \hat{\beta}\,\overline{X}$$

$$\Rightarrow \overline{\hat{Y}} = \hat{\alpha} + \hat{\beta}\,\overline{X} = \overline{Y}$$

$$\therefore \quad \overline{Y} = \overline{\hat{Y}}$$

3 Verifique que la suma simple de los residuales es nula.

$$\sum_{i=1}^{n} e_i = 0$$

$$y_i = Y_i - \overline{Y}$$
$$x_i = X_i - \overline{X}$$
$$\hat{y}_i = \hat{Y}_i - \overline{Y}$$

$$e_i = y_i - \hat{y}_i$$
$$= y_i - \hat{\beta}\ x_i$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$

e $\overline{Y} = \hat{\alpha} + \hat{\beta}\overline{X}$ se estima
las variables involucradas e

$$\sum_{i=1}^{n} Y_i = n\hat{\alpha} + \hat{\beta}\sum_{i=1}^{n} X_i$$
$$\sum_{i=1}^{n} X_i Y_i = \hat{\alpha}\sum_{i=1}^{n} X_i + \hat{\beta}\sum_{i=1}^{n} X_i^2$$

$$\boxed{\hat{\alpha} = \overline{Y} + \hat{\beta}\overline{X}}$$

$$\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} (y_i - \hat{y}_i) = \sum_{i=1}^{n} (y_i - \hat{\beta} x_i) = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta} x_i = \sum_{i=1}^{n} y_i - \hat{\beta} \sum_{i=1}^{n} x_i$$

$$= \sum_{i=1}^{n} (Y_i - \overline{Y}) - \hat{\beta} \sum_{i=1}^{n} (X_i - \overline{X}) = \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} \overline{Y} - \hat{\beta}\left(\sum_{i=1}^{n} X_i - \sum_{i=1}^{n} \overline{X}\right)$$

$$= \sum_{i=1}^{n} Y_i - n\overline{Y} - \hat{\beta}\left(\sum_{i=1}^{n} X_i - \overline{X} n\right) = \sum_{i=1}^{n} Y_i - n\left(\frac{\sum_{i=1}^{n} Y_i}{n}\right) - \hat{\beta}\left(\sum_{i=1}^{n} X_i - n\left(\frac{\sum_{i=1}^{n} X_i}{n}\right)\right)$$

$$= \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} Y_i - \hat{\beta}\left(\sum_{i=1}^{n} X_i - \sum_{i=1}^{n} X_i\right) = 0 - \hat{\beta}\, 0 = 0$$